Turbulence

Introduction to Plasma and Fusion Course

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Reach out if you have questions!

My Worldline

$$
\begin{aligned}
[P_\mu, P_\nu] &= 0,\\
[D, K_\mu] &= -K_\mu,\\
[D, P_\mu] &= P_\mu,\\
[K_\mu, K_\nu] &= 0,\\
[K_\mu, P_\nu] &= \eta_{\mu\nu} D - i M_{\mu\nu},
\end{aligned}
$$

where tokamak?

what tokamak?

covid graduation

Ancient History

Undergrad

Grad School

Princeton Days

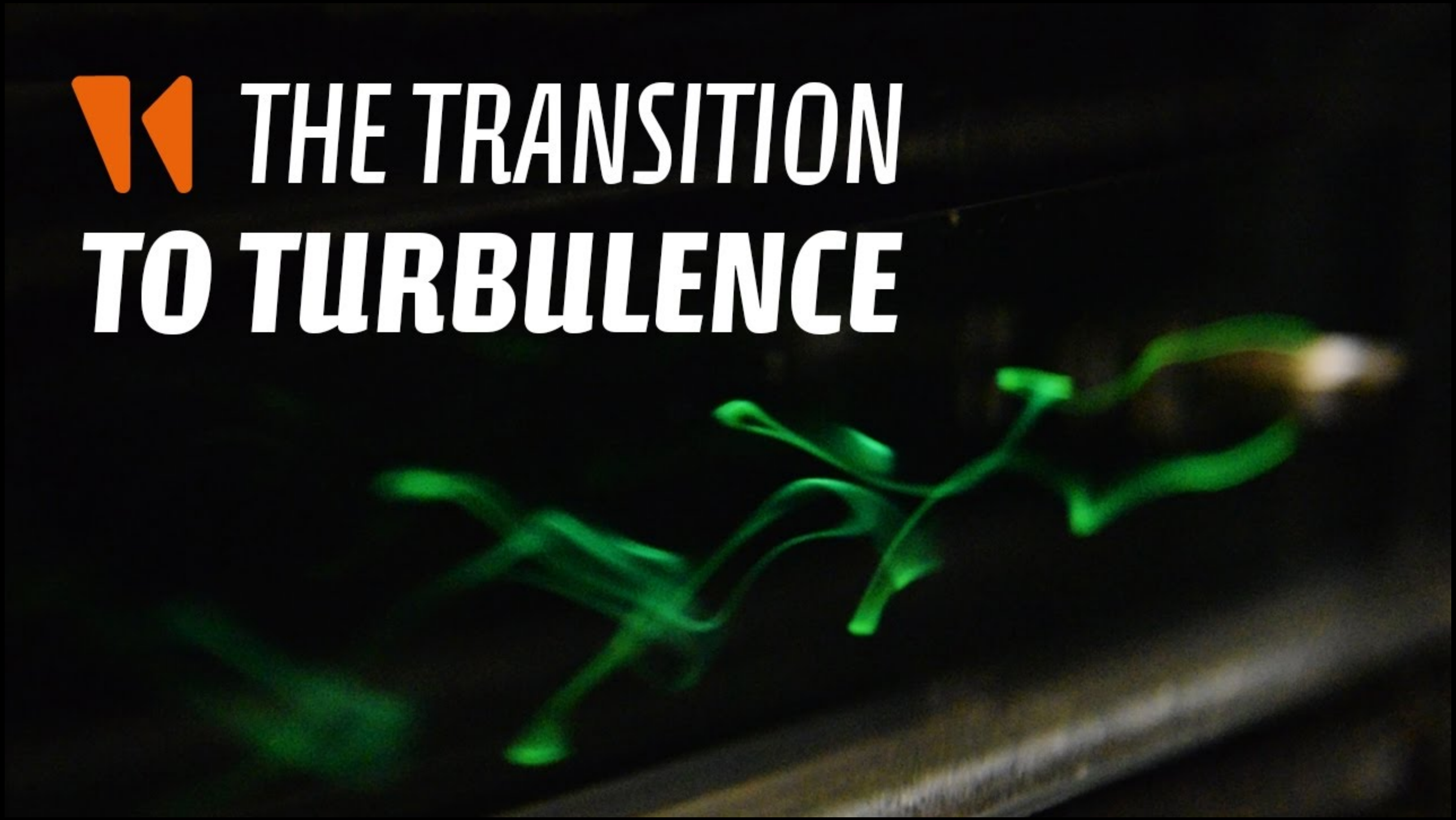
Present

Lecture Topics

- Quick overview of fluid turbulence
- Turbulence and transport in magnetic confinement fusion

Fluid Turbulence Overview

Transition to Turbulence



laminar flow —> .... —> turbulent flow

Transition to Turbulence

laminar flow —> .... —> turbulent flow

- Turbulent flow transition at a critical flow velocity, u_{crit}, or length scale L_{crit}
- Described by conservation of momentum:

viscosity

$$\rho \underbrace{\left(\frac{\partial \mathbf{u}}{\partial t} + \mathbf{u} \cdot \nabla \mathbf{u} \right)}_{\text{derivative moving with fluid particle}} + \nabla p = \rho \nu \nabla^2 \mathbf{u}$$

density

pressure

flow velocity

Transition to Turbulence

laminar flow —> .... —> turbulent flow

- Turbulent flow transition at a critical flow velocity, $u_{\rm crit}$, or length scale $L_{\rm crit}$
- Described by conservation of momentum:

$$\rho\left(\frac{\partial \mathbf{u}}{\partial t} + \mathbf{u}\cdot\nabla\mathbf{u}\right) + \nabla p = \rho\nu\,\nabla^2\mathbf{u}$$

- Balancing convective derivative with viscosity gives Reynolds number:

$$\frac{\rho\mathbf{u}\cdot\nabla\mathbf{u}}{\rho\nu\,\nabla^2\mathbf{u}} \sim \frac{u^2/L}{\nu u/L^2} = \frac{uL}{\nu} \qquad \boxed{\mathrm{Re} \equiv \frac{uL}{\nu}}$$

Transition to Turbulence

laminar flow —> .... —> turbulent flow

- Empirically, turbulence onset at $\mathrm{Re} \sim 10^4$
- Critical length scale $L_{\mathrm{crit}} \sim 10^4 \frac{\nu}{u}$: make system sufficiently large, turbulence arises
- L_{crit} effect also occurs in fusion plasmas!

Turbulent cascade

Turbulence cascades energy from injection scale to dissipative scale

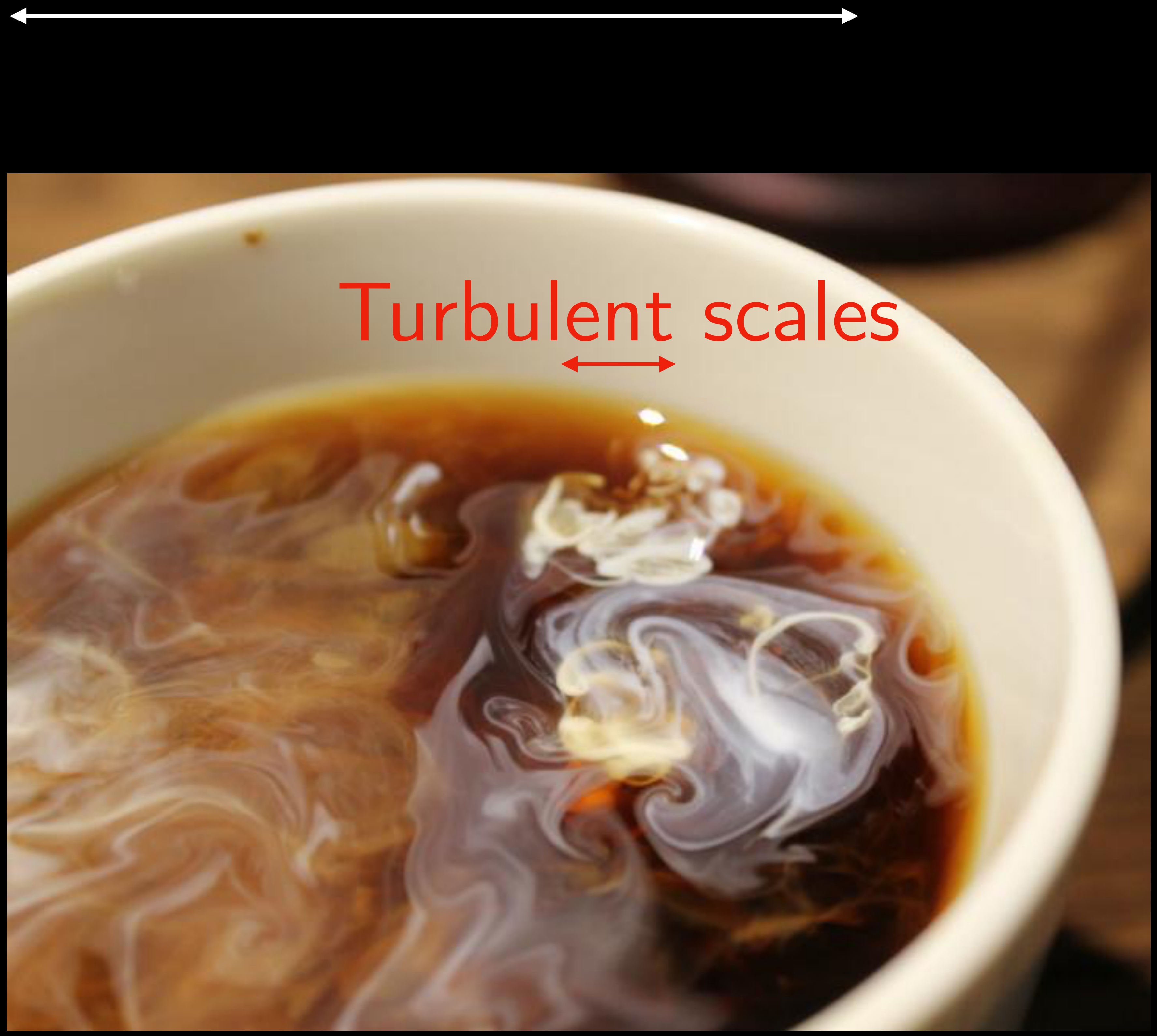
Injection scale L

$\longleftrightarrow$

Turbulent cascade

Turbulence cascades energy from injection scale to dissipative scale

Injection scale L



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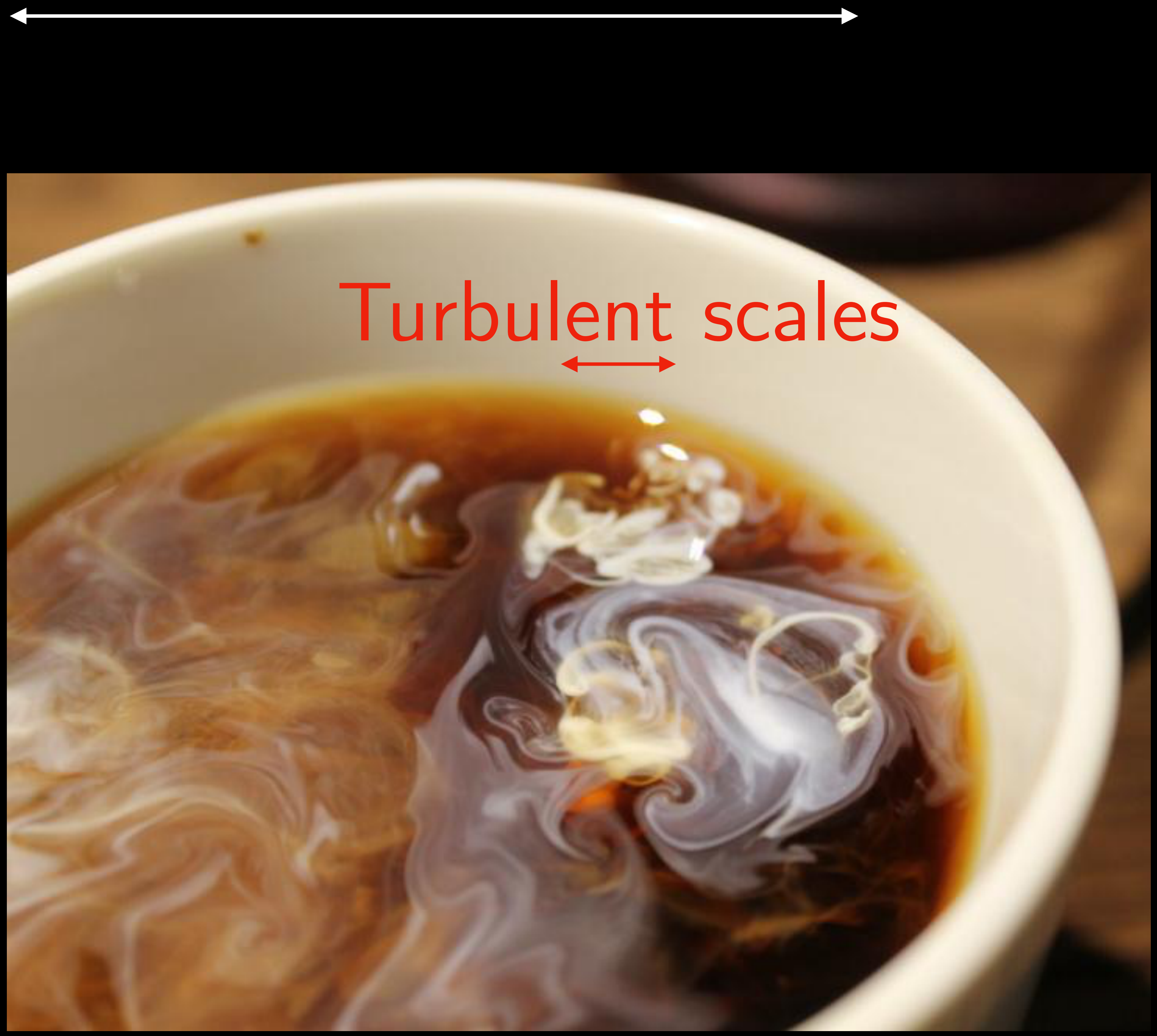


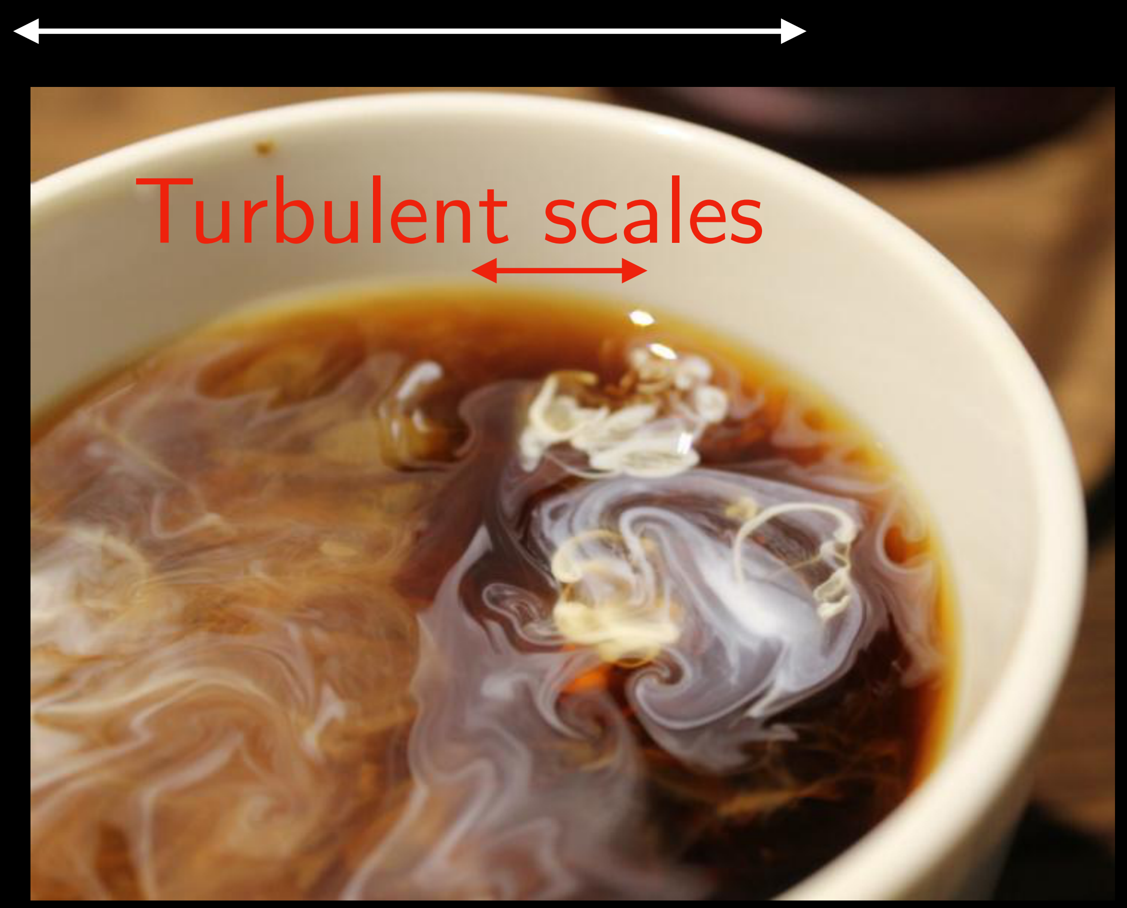
Dissipative scale too small to see

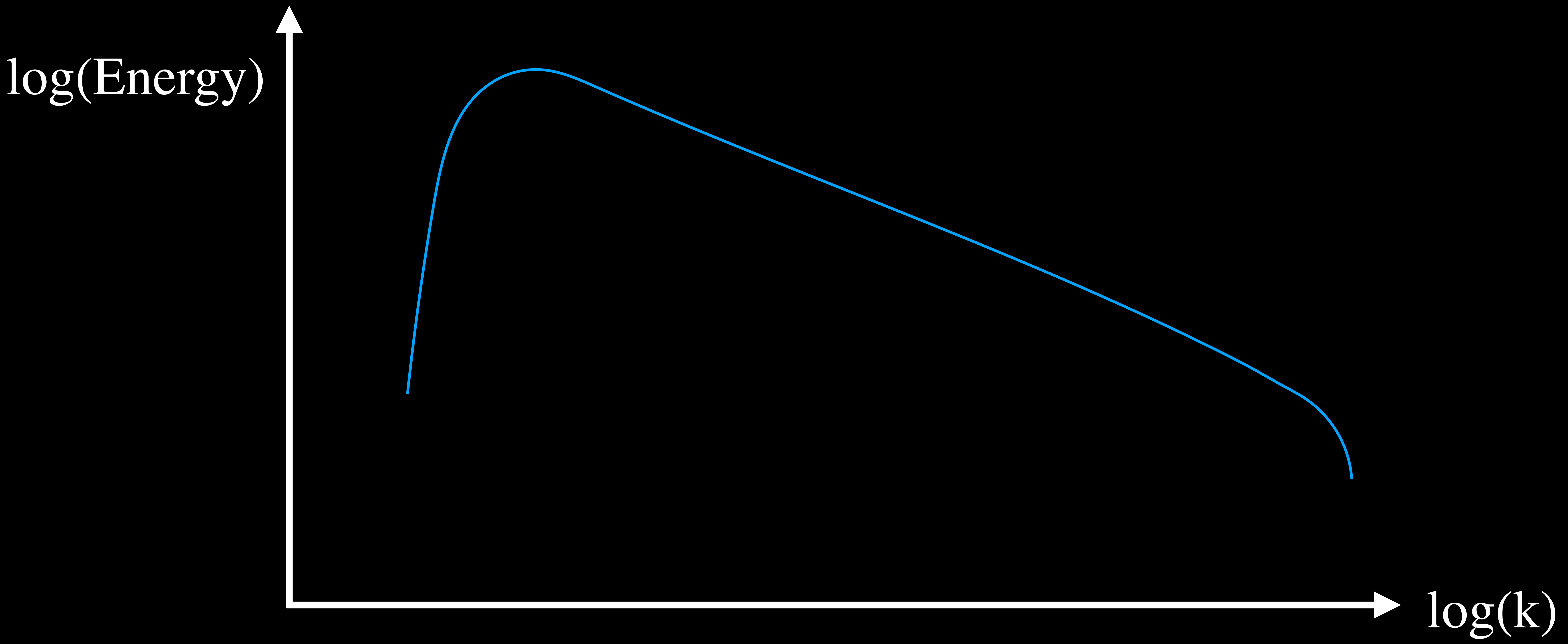
Turbulent cascade

Turbulence cascades energy from injection scale to dissipative scale

Plot energy versus wavenumber k

Injection scale L





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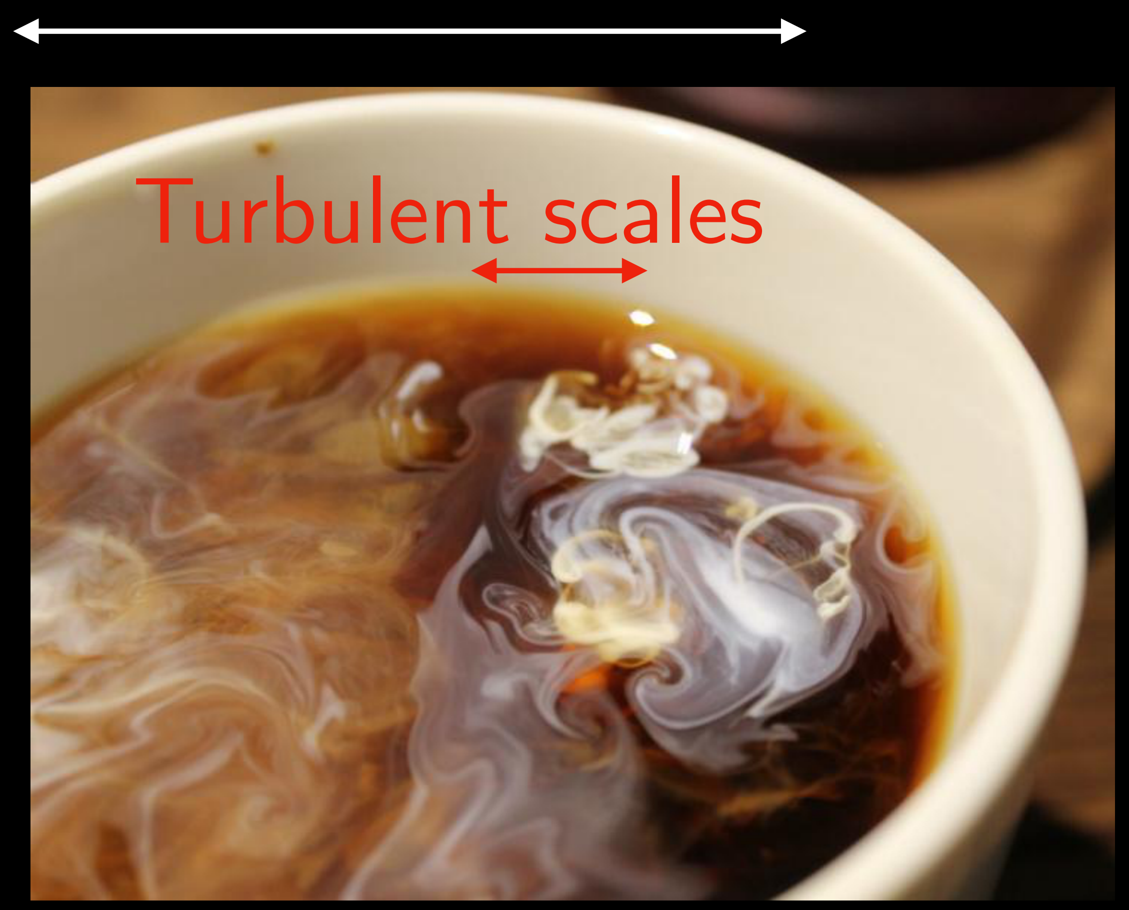

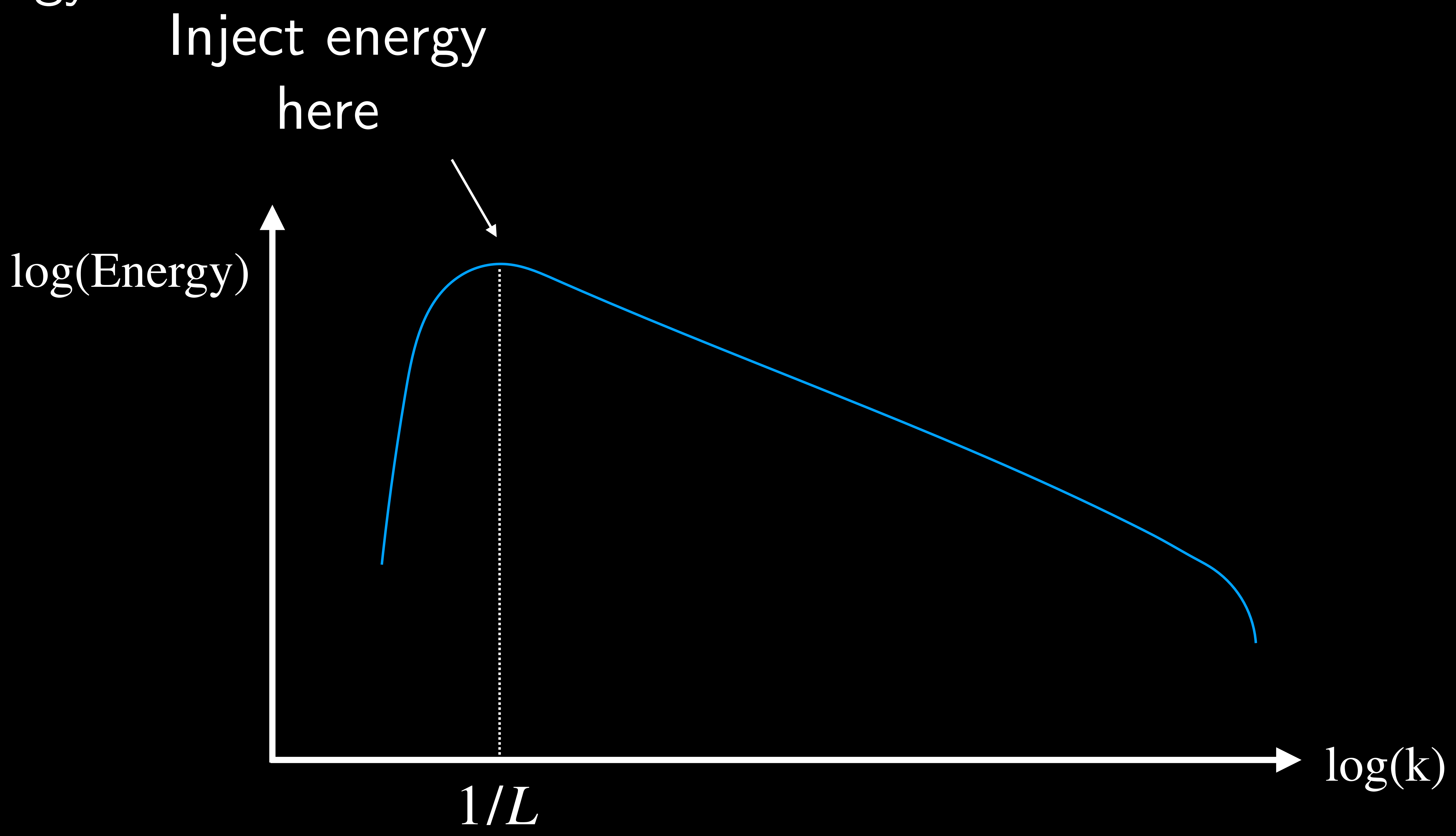

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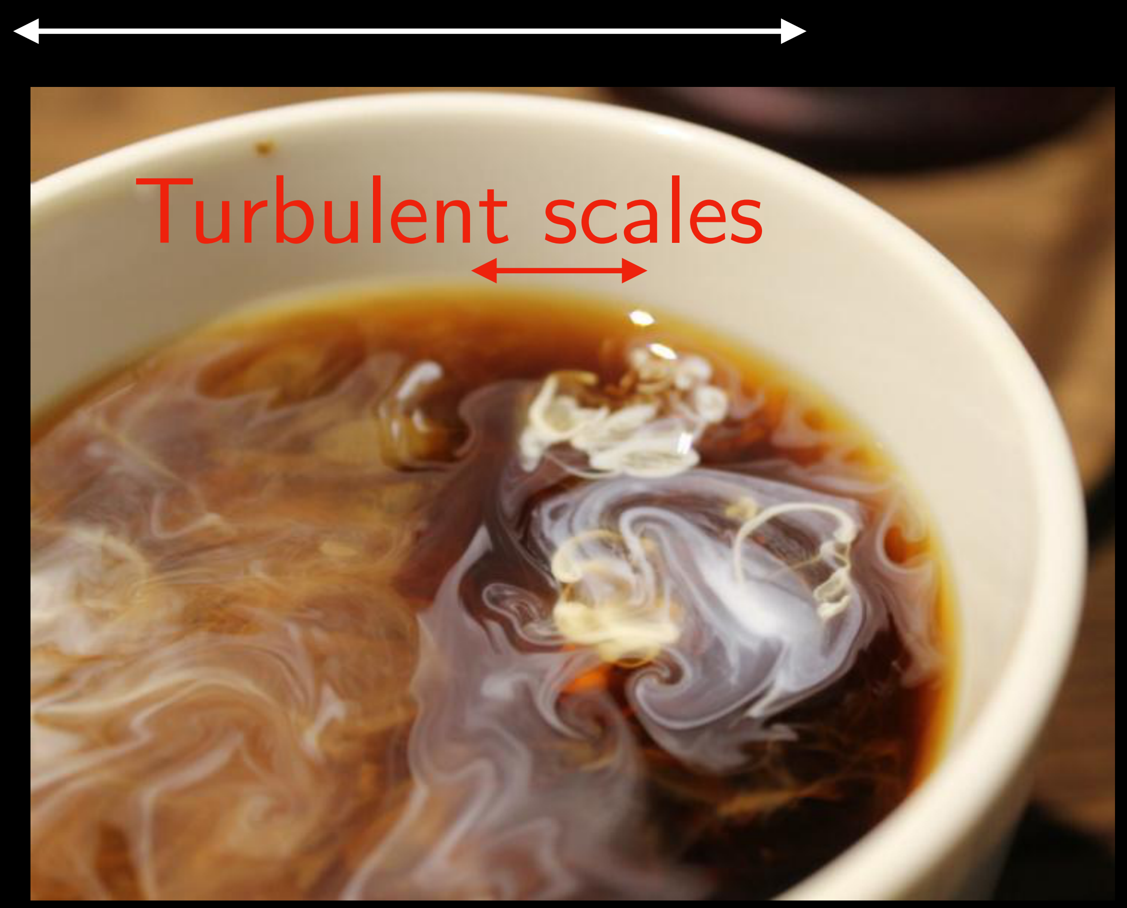
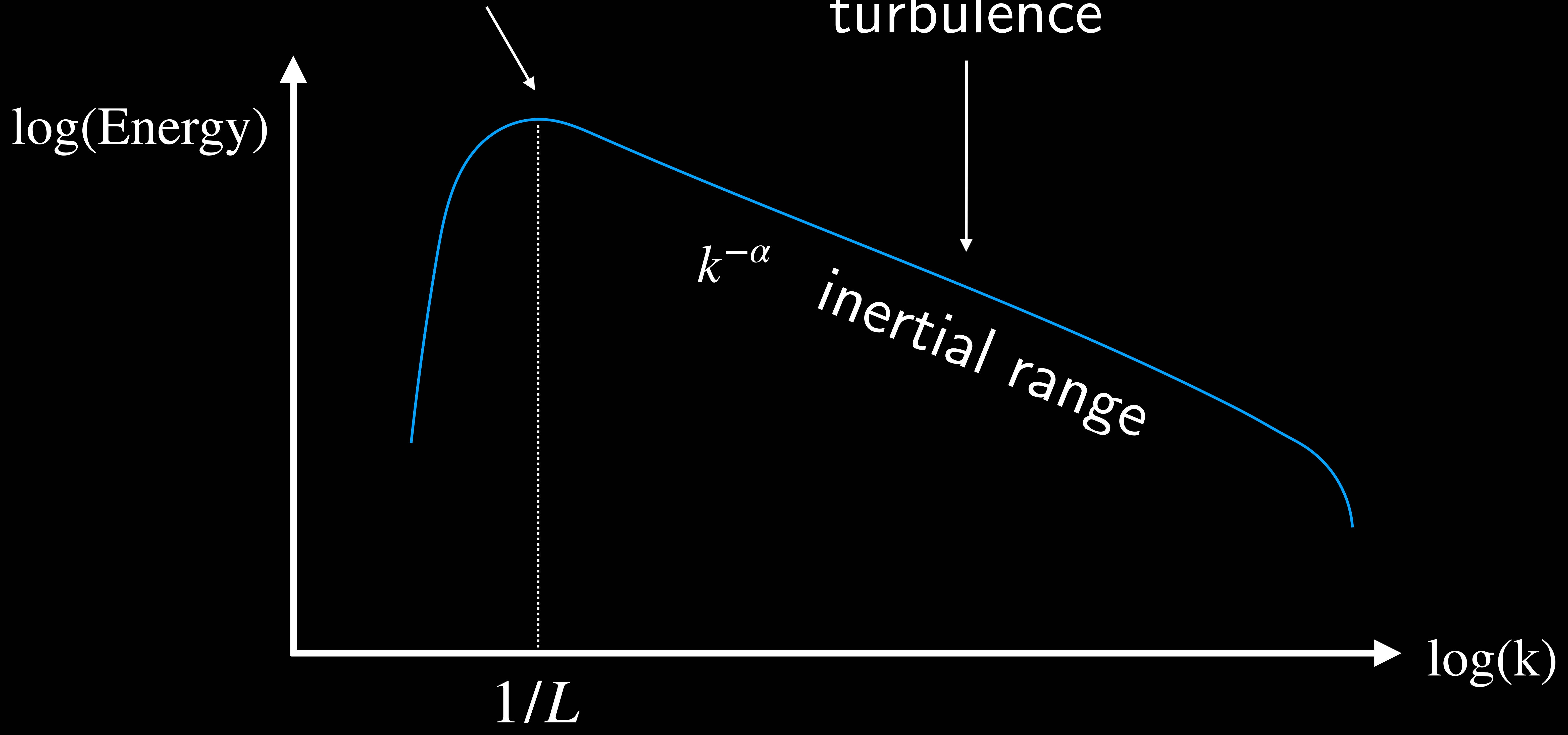


Inject energy
here

Here be
turbulence



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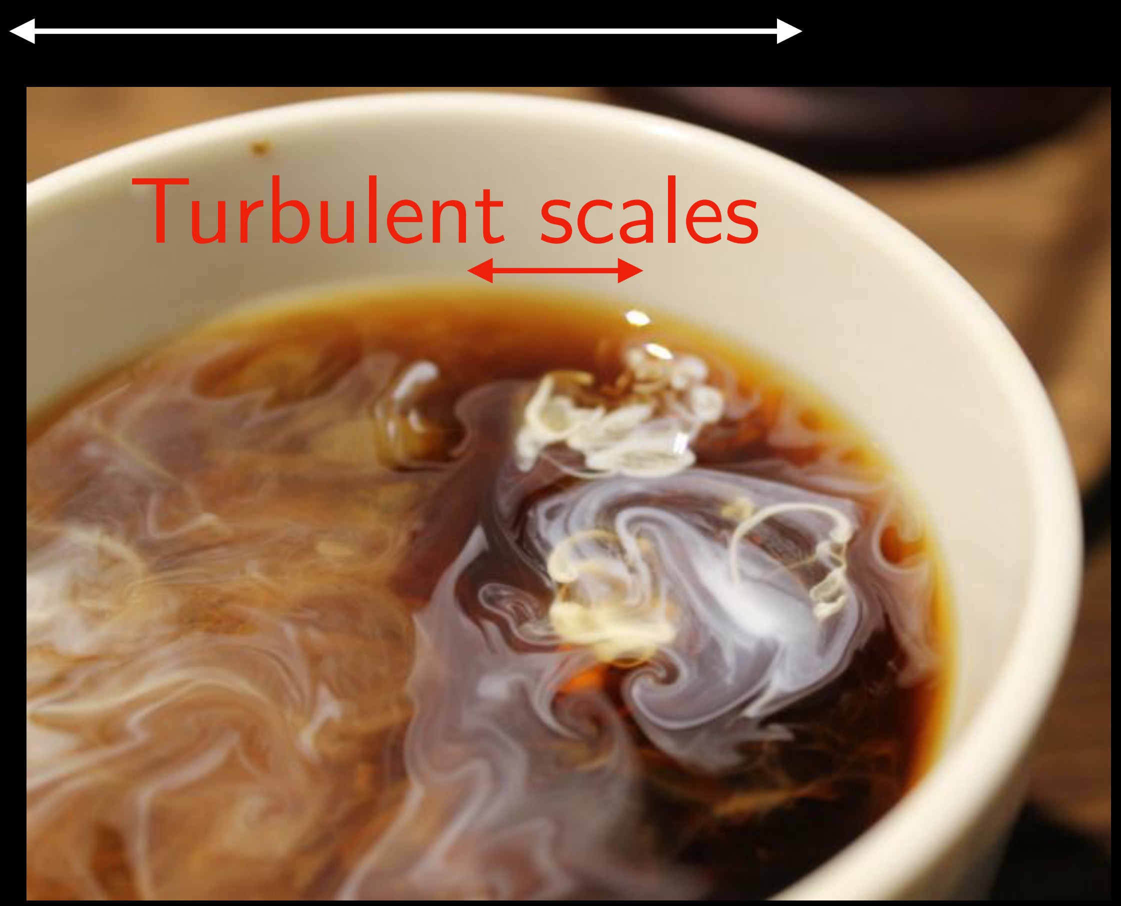
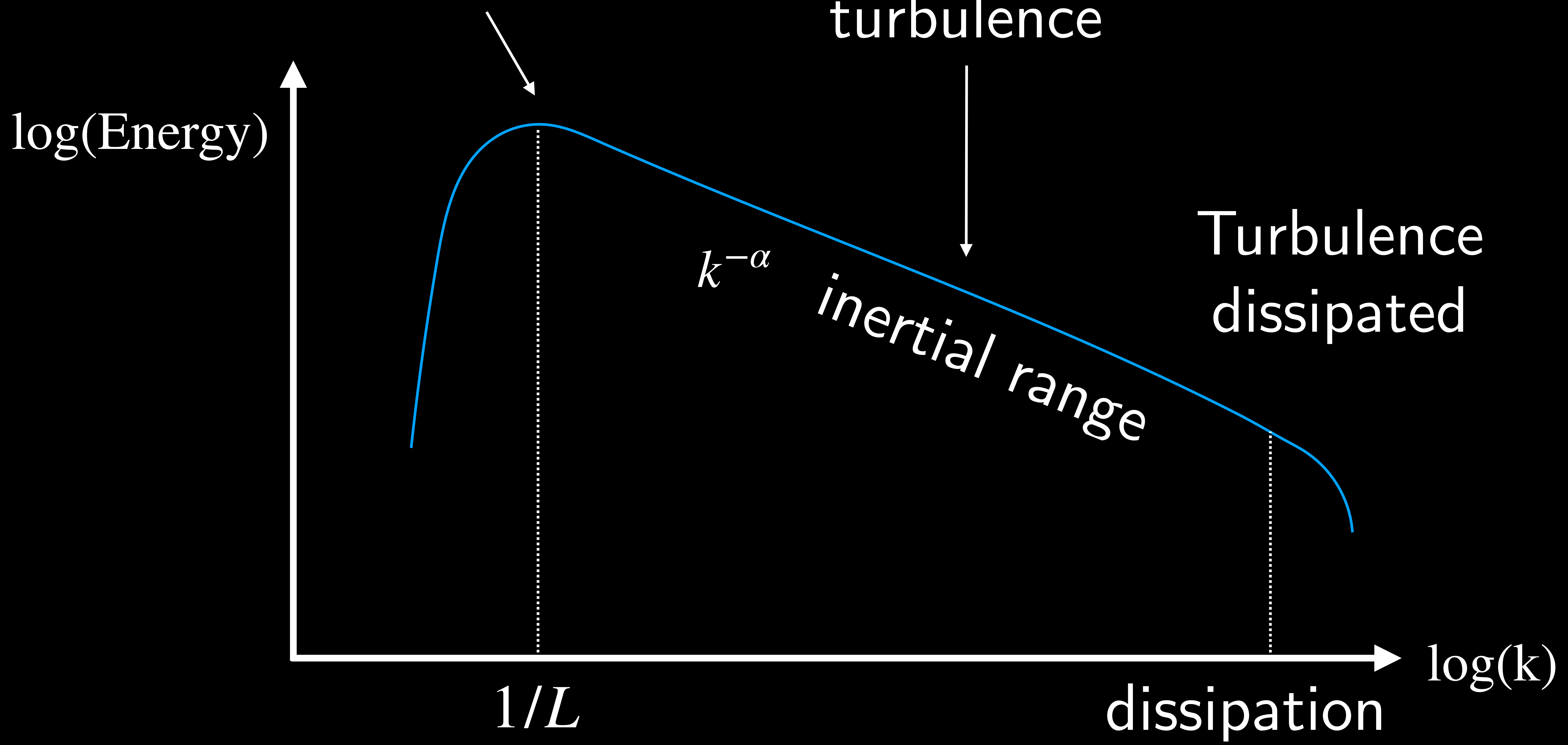



Inject energy here

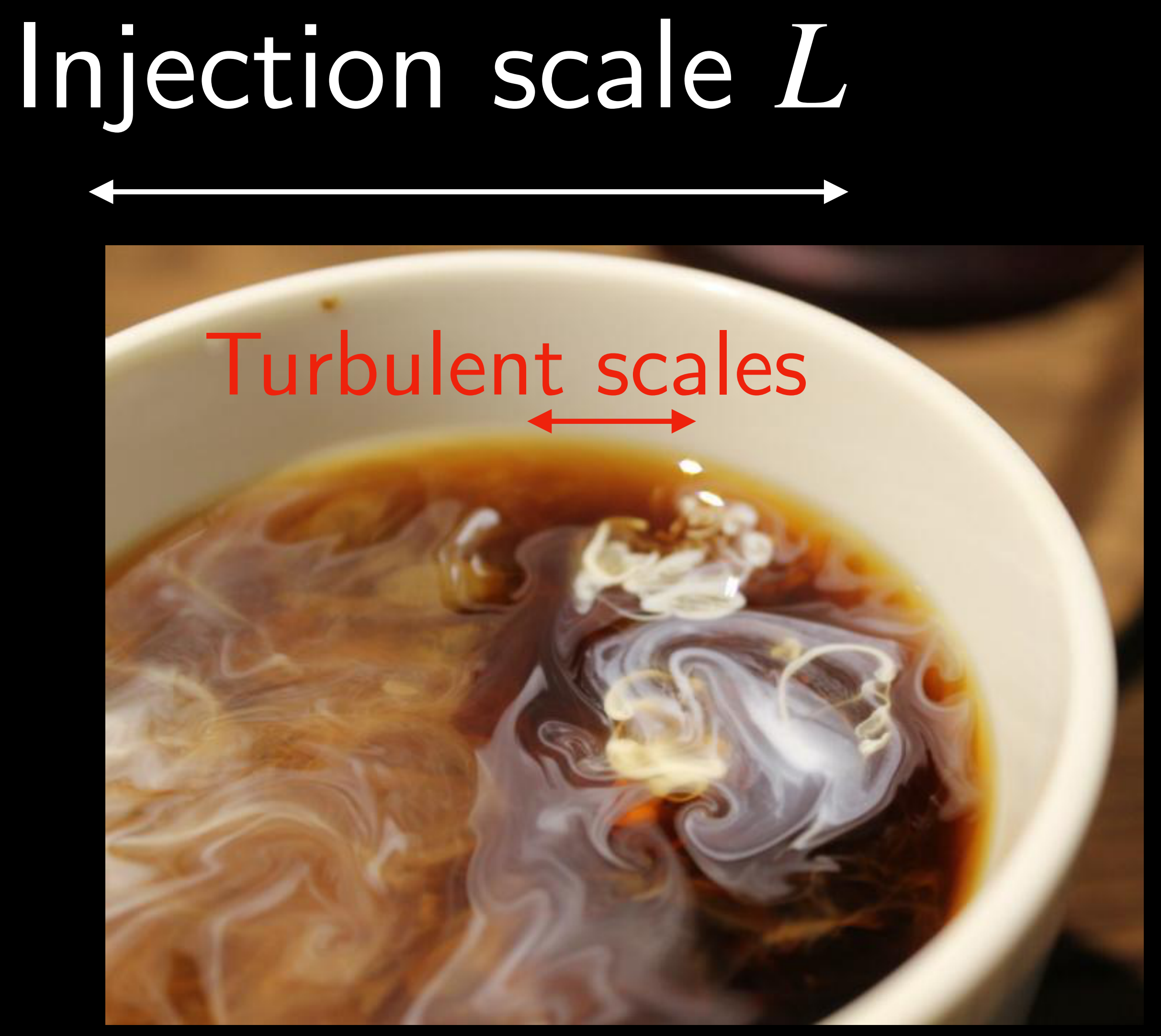
Here be turbulence

Turbulence dissipated

Turbulent cascade

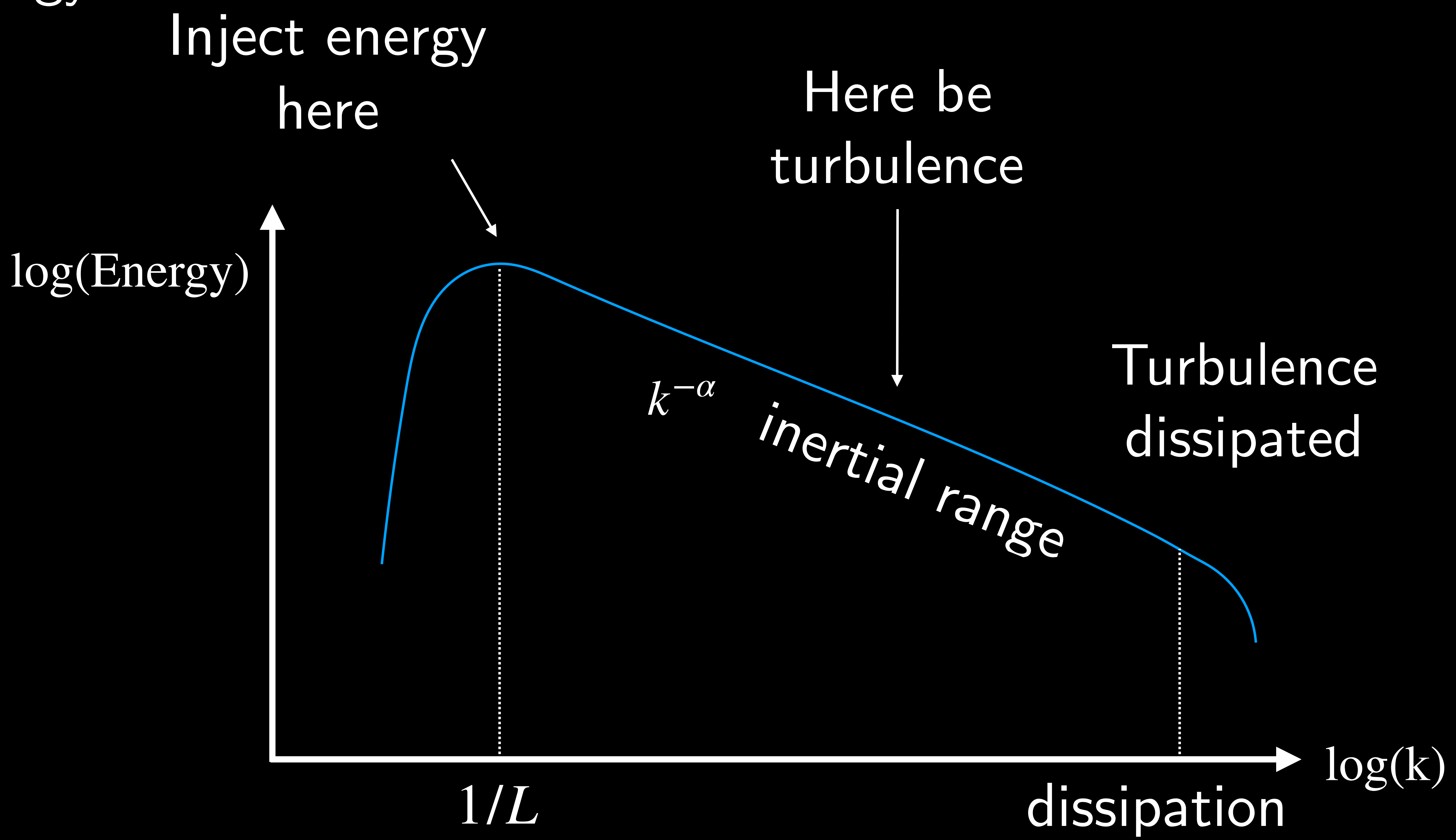
Turbulence cascades energy from injection scale to dissipative scale

Plot energy versus wavenumber k



Injection scale L

Turbulent scales



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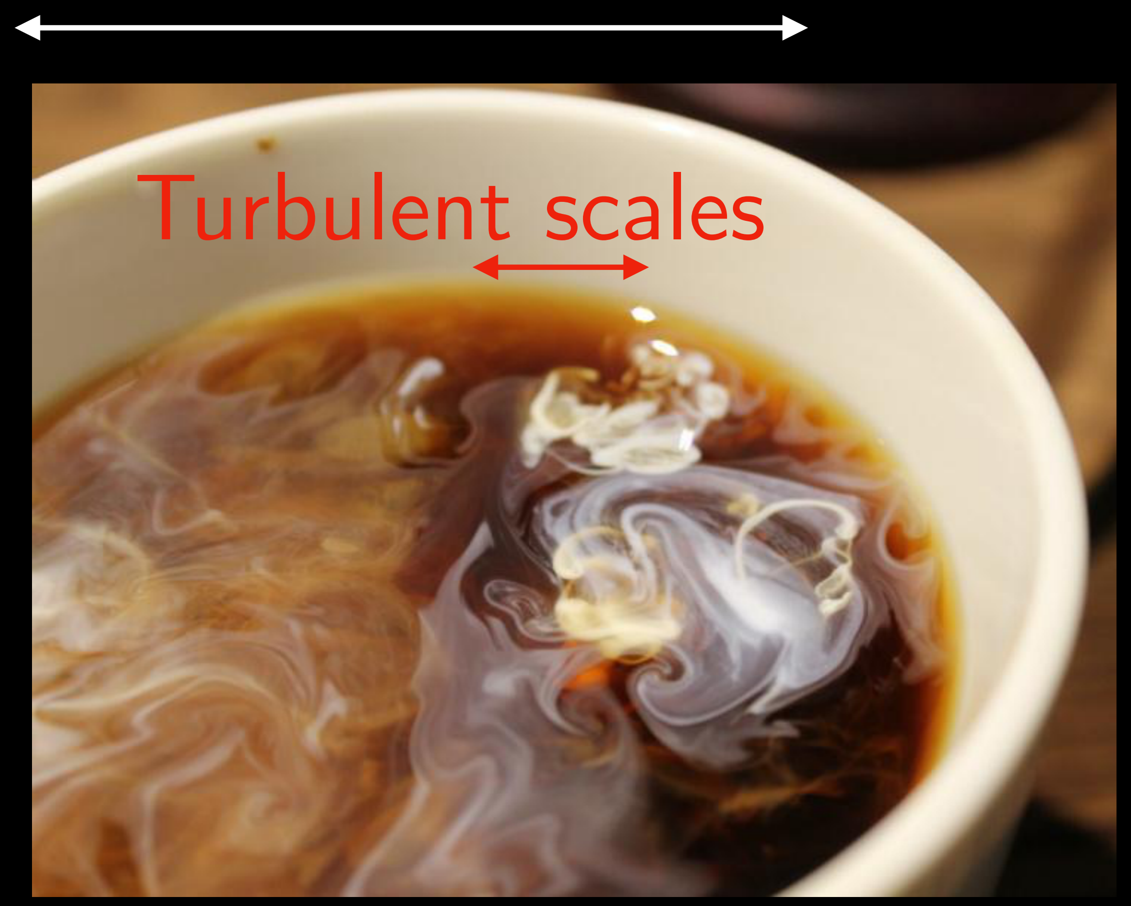

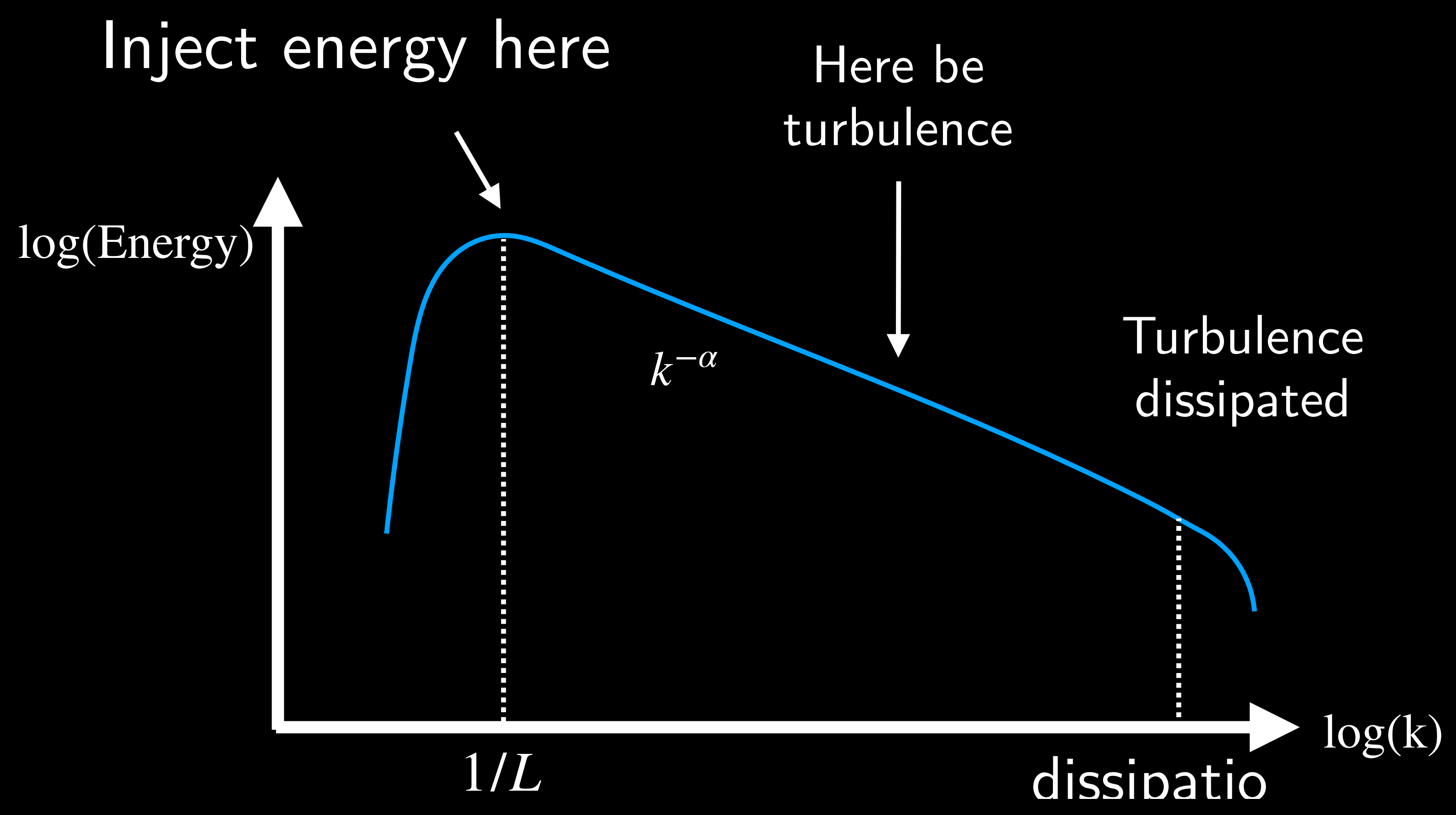

- Turbulence cascades energy from injection scale to a dissipative scale
- A lot more to say about fluid turbulence...
- Some techniques and observations can be carried over to plasmas, but with significant differences in assumptions.

Turbulence and Transport in Magnetic Confinement Fusion

Turbulence and Transport in Magnetic Confinement Fusion

Background

- 1968: T-3 Russian tokamak, record temperature > 1 keV (11.6 million Kelvin).
- 1970-80s: plasma temperatures lower than expected

Turbulence and Transport in Magnetic Confinement Fusion

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Turbulence and Transport in Magnetic Confinement Fusion

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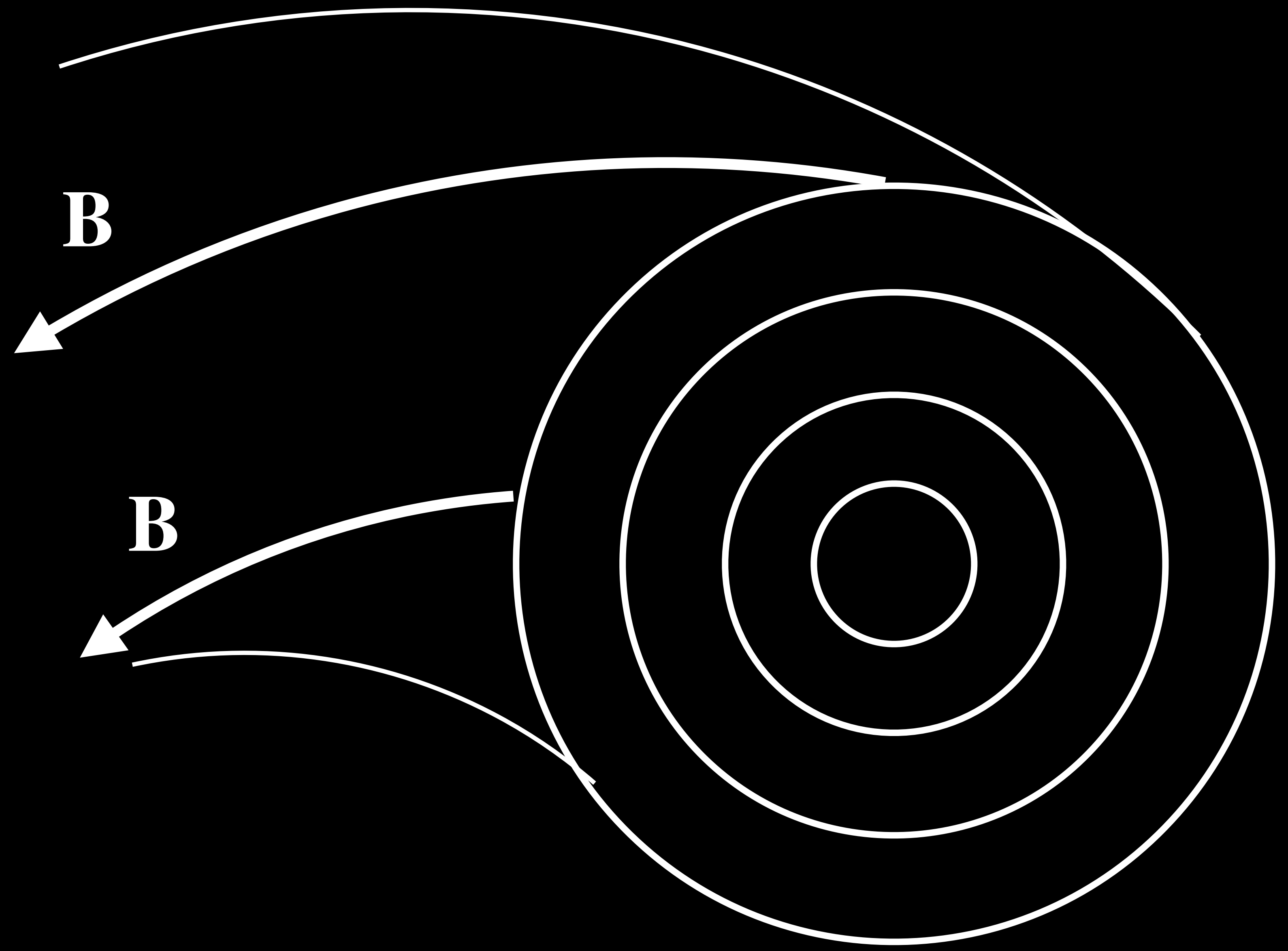
- 1970-80s: plasma temperatures lower than expected, why?

Due to rapid heat diffusion across field lines!



Cutaway of
tokamak
plasma

Turbulence and Transport in Magnetic Confinement Fusion

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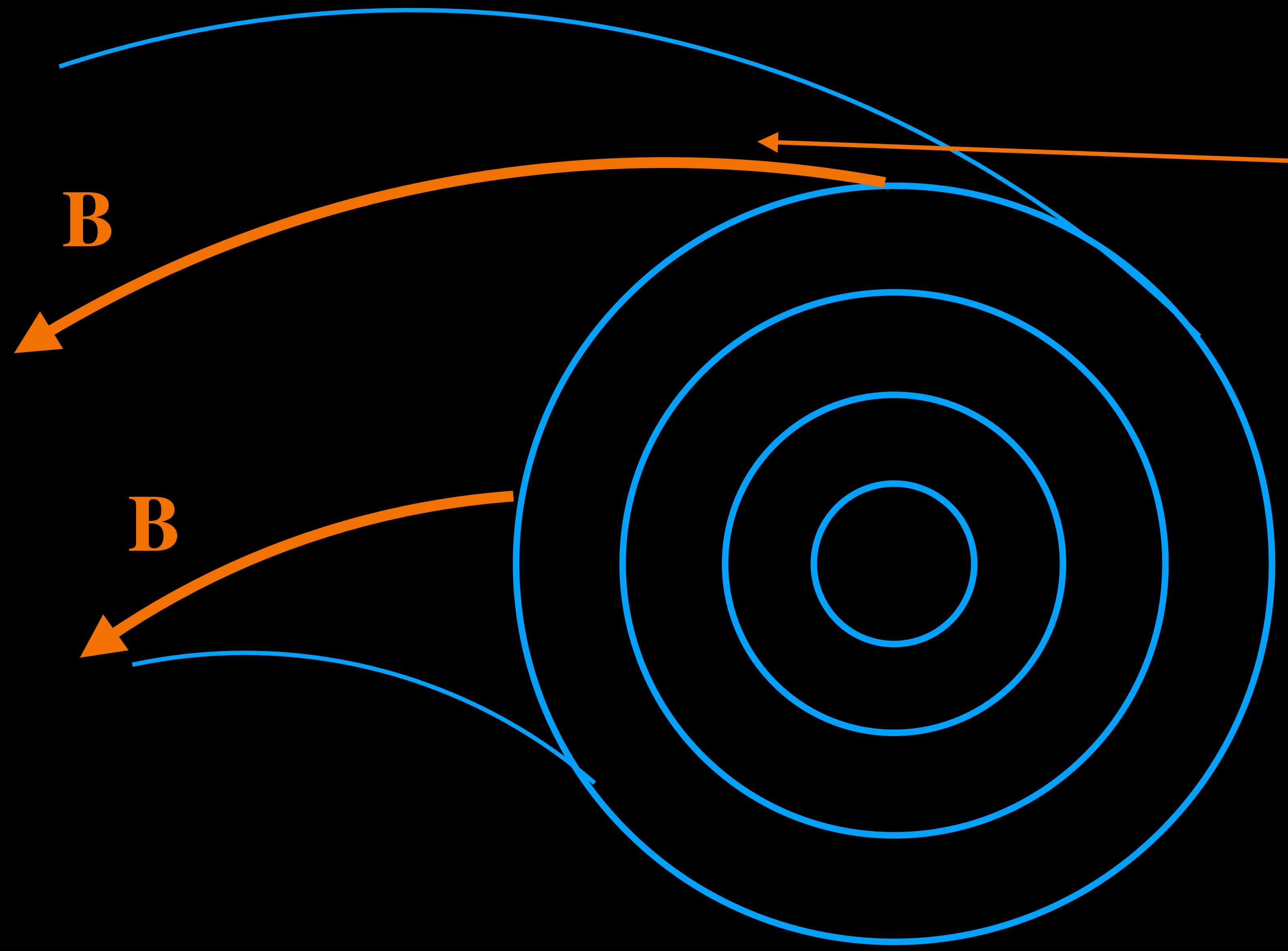

Magnetic field lines trace out ‘flux surfaces’

Cutaway of tokamak plasma

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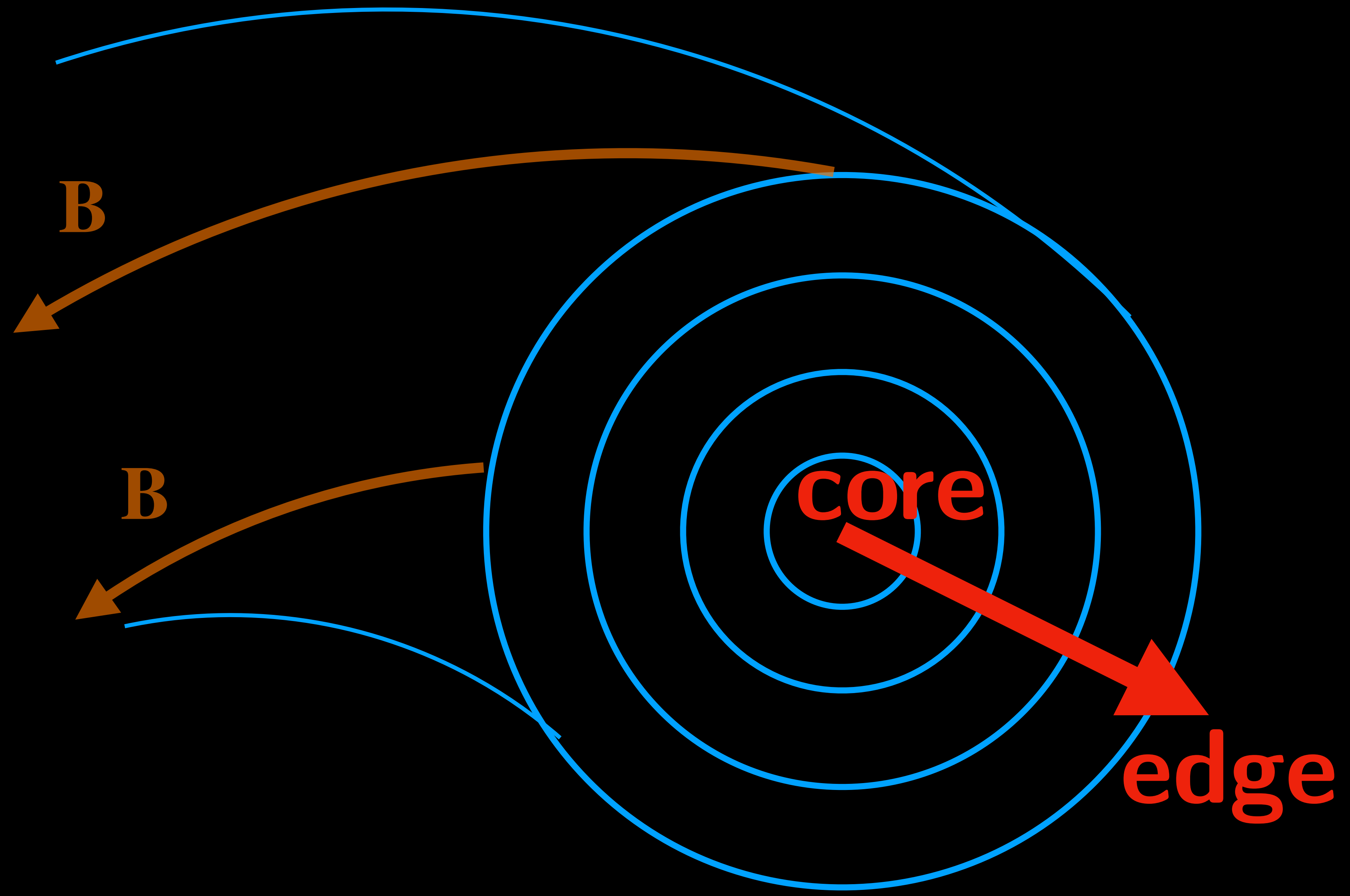

Cutaway of tokamak plasma

Due to <u>diffusion</u>, heat '<u>leaks</u>' from plasma core to edge

Turbulence and Transport in Magnetic Confinement Fusion

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Due to rapid heat diffusion across field lines!

Let's consider three mechanisms for heat transport across field lines

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Classical

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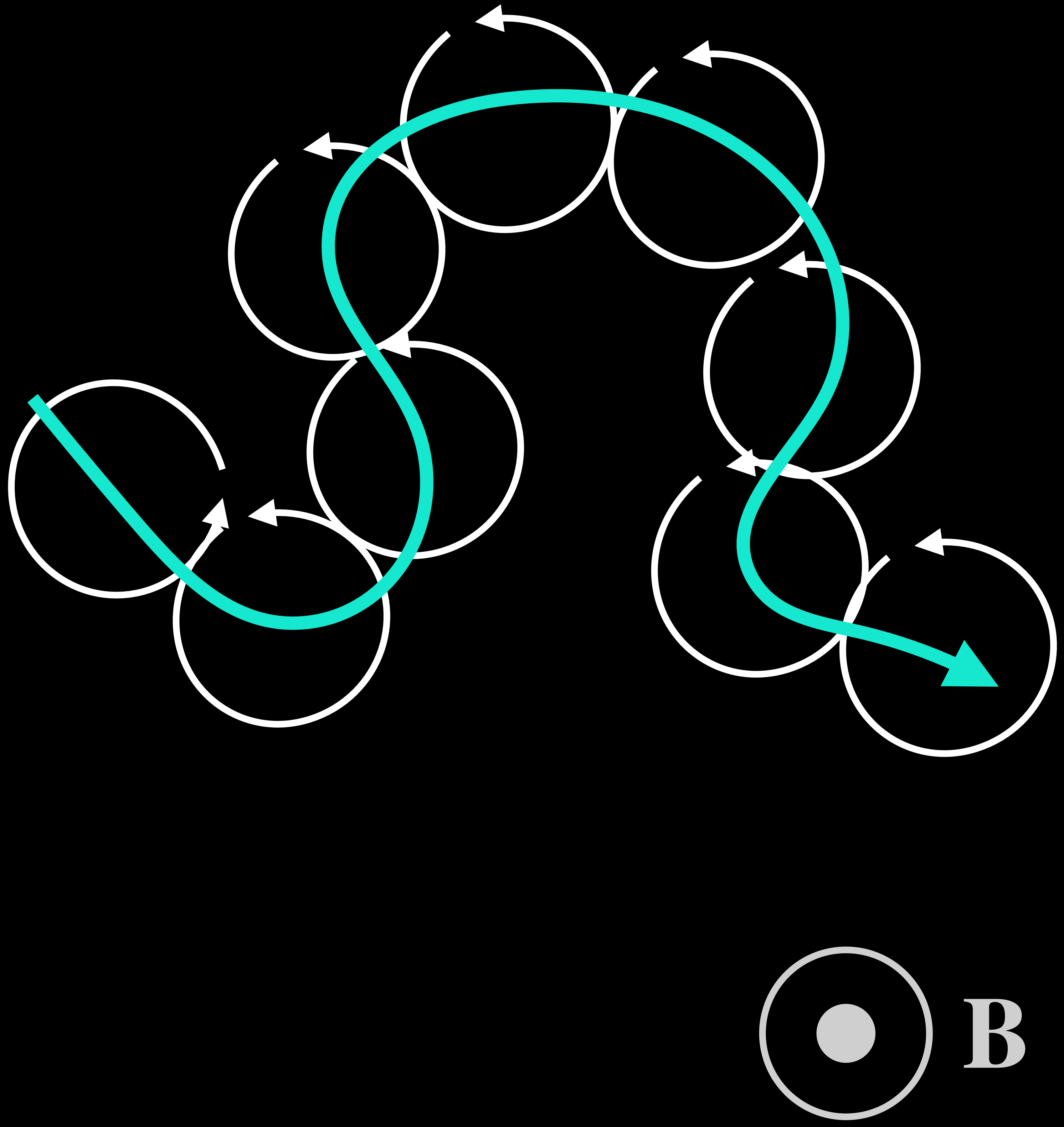
Classical

Idea:

in magnetic field $\mathbf{B}$, ions have gyroradius ρ_i, collide with frequency ν_i that knocks them from orbit

→random walk diffusion



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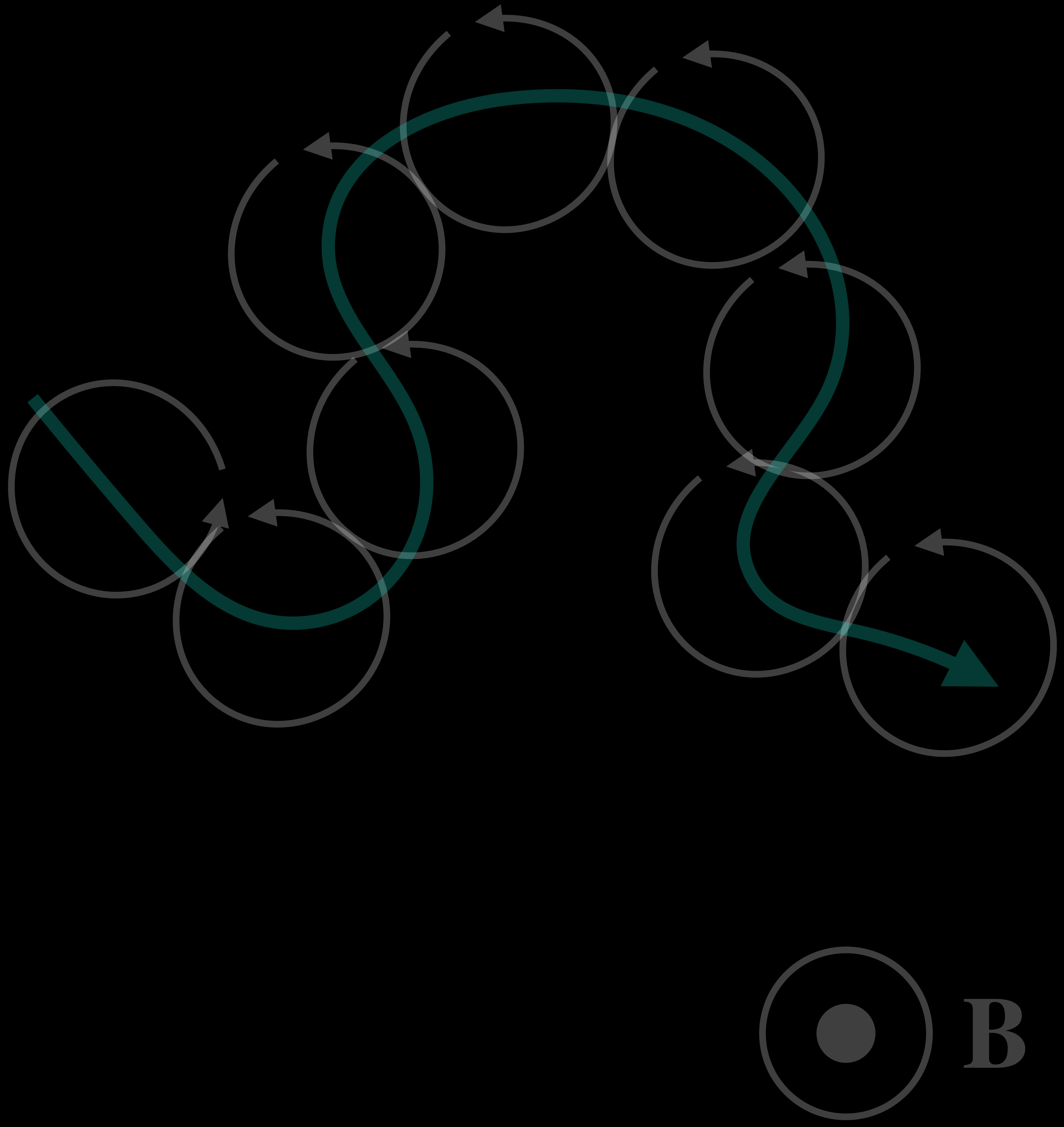
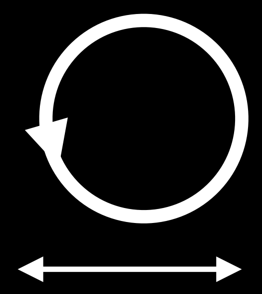


Classical



ρ_i = *ion gyroradius*

$$\boxed{D_{\mathrm{c}} \sim \rho_i^2 \nu_i}$$

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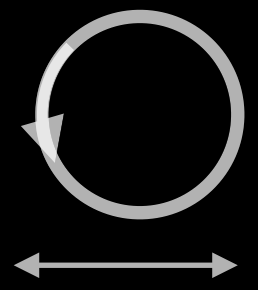

$$\boxed{D_c \sim \rho_i^2 \nu_i}$$

Neoclassical

Idea:

Classical diffusion +
toroidal geometry!

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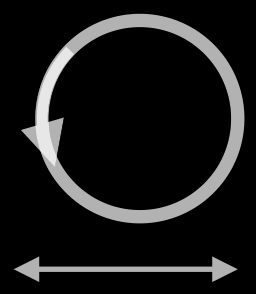

$$\boxed{D_c \sim \rho_i^2 \nu_i}$$

Neoclassical

$$\boxed{D_{nc} \sim q^2 D_c}$$

Toroidal geometry
factor $q \gg 1$

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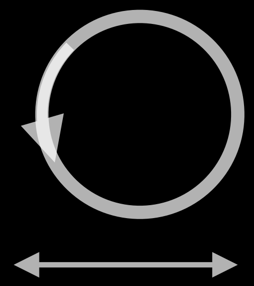
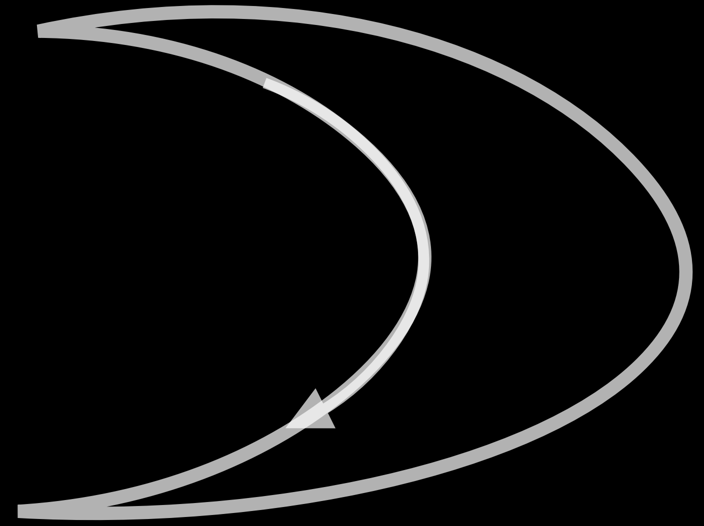


$$D_{\mathrm{c}} \sim \rho_i^2 \nu_i$$

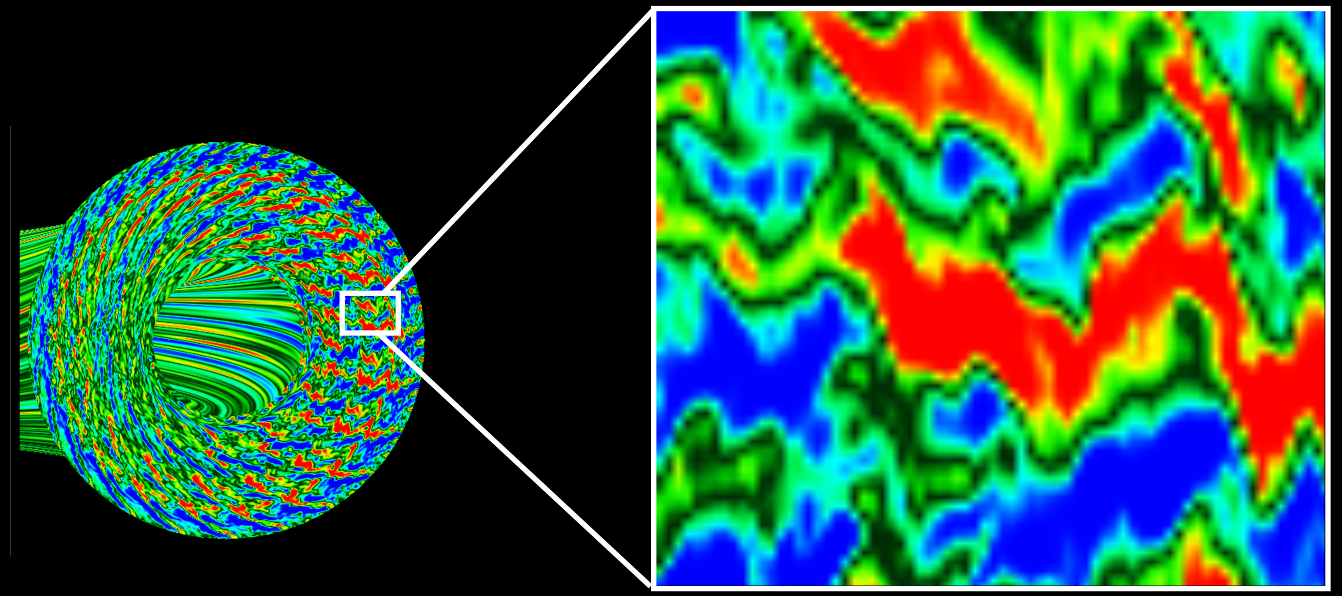
ρ_i



$$D_{\mathrm{nc}} \sim q^2 D_c$$

Anomalous/Turbulent

Credit:
https://w3.pppl.gov/~hammett/collaborators/mbeer/



Tokamaks: $D_{\mathrm{turb}} \gg D_{\mathrm{nc}}$

Stellarators: $D_{\mathrm{turb}} \gtrsim D_{\mathrm{nc}}$

Turbulence and Transport in Magnetic Confinement Fusion

Transport equations

- Density, momentum, and energy transport described by transport equations

temperature

Energy equation:

$$\frac{3}{2}n\frac{dT}{dt} + \nabla \cdot \mathbf{q} = P$$

heat flux $[\mathrm{W/m^2}]$

energy sources and sinks $[\mathrm{W/m^3}]$

particle density $[\mathrm{\#/m^3}]$

Turbulence and Transport in Magnetic Confinement Fusion

Transport equations

- Density, momentum, and energy transport described by transport equations

$$\frac{3}{2}n\frac{dT}{dt} + \nabla \cdot \mathbf{q} = P$$

Note: in this lecture, we consider heat transport perpendicular to magnetic field

Parallel heat transport typically ‘classical,’ important in tokamak edge, not covered here

Turbulence and Transport in Magnetic Confinement Fusion

Transport equations

- Density, momentum, and energy transport described by transport equations
- Steady state

$$\cancel{\frac{3}{2}n\frac{dT}{dt}} \quad \nabla \cdot \mathbf{q} = P$$

Heat flux due to plasma turbulence $\sim$ Plasma heating, radiative losses

Turbulence and Transport in Magnetic Confinement Fusion

Transport equations

- Density, momentum, and energy transport described by transport equations
- Steady state

$$\nabla \cdot \mathbf{q} = P$$

Higher diffusion coefficient —> higher heat flux

$$\mathbf{q} = -nD\nabla T$$

—> larger heating sources to balance power loss at fixed gradient ∇T

Turbulence and Transport in Magnetic Confinement Fusion

Transport equations

$$\mathbf{q} = -nD\nabla T$$

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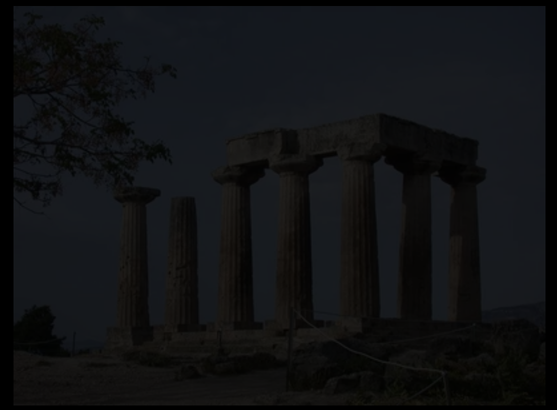
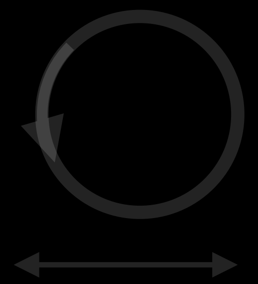
—> larger sources to balance power loss at fixed gradient ∇T





$D_\mathrm{c} \sim \rho_i^2 \nu_i$

ρ_i

$$\mathbf{q} \simeq -nD_\mathrm{turb}\nabla T$$

$D_\mathrm{nc} \sim q^2 D_c$

Anomalous/Turbulent

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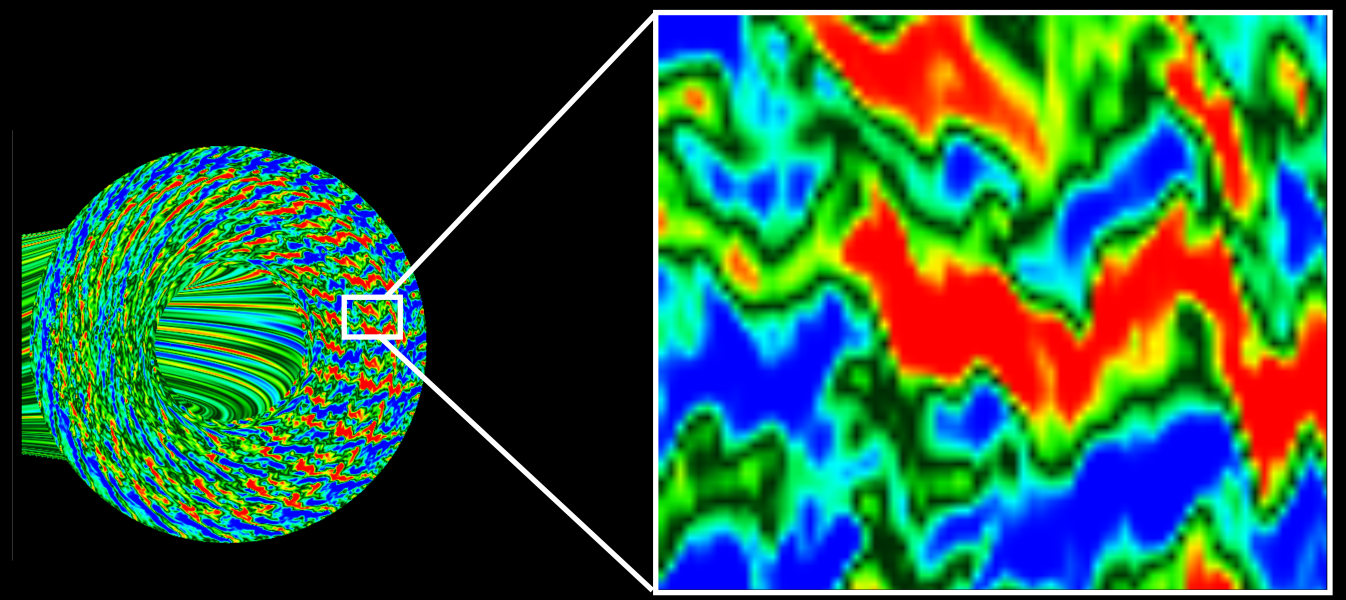

Tokamaks: $D_\mathrm{turb} \gg D_\mathrm{nc}$

Stellarators: $D_\mathrm{turb} \gtrsim D_\mathrm{nc}$

Turbulence and Transport in Magnetic Confinement Fusion

Plasma Turbulence

- Turbulence is primary physics reason we do not yet have fusion power plant
- Plasma instabilities create turbulent eddies that rapidly move energy from the core to the edge

Turbulence and Transport in Magnetic Confinement Fusion

Plasma Turbulence

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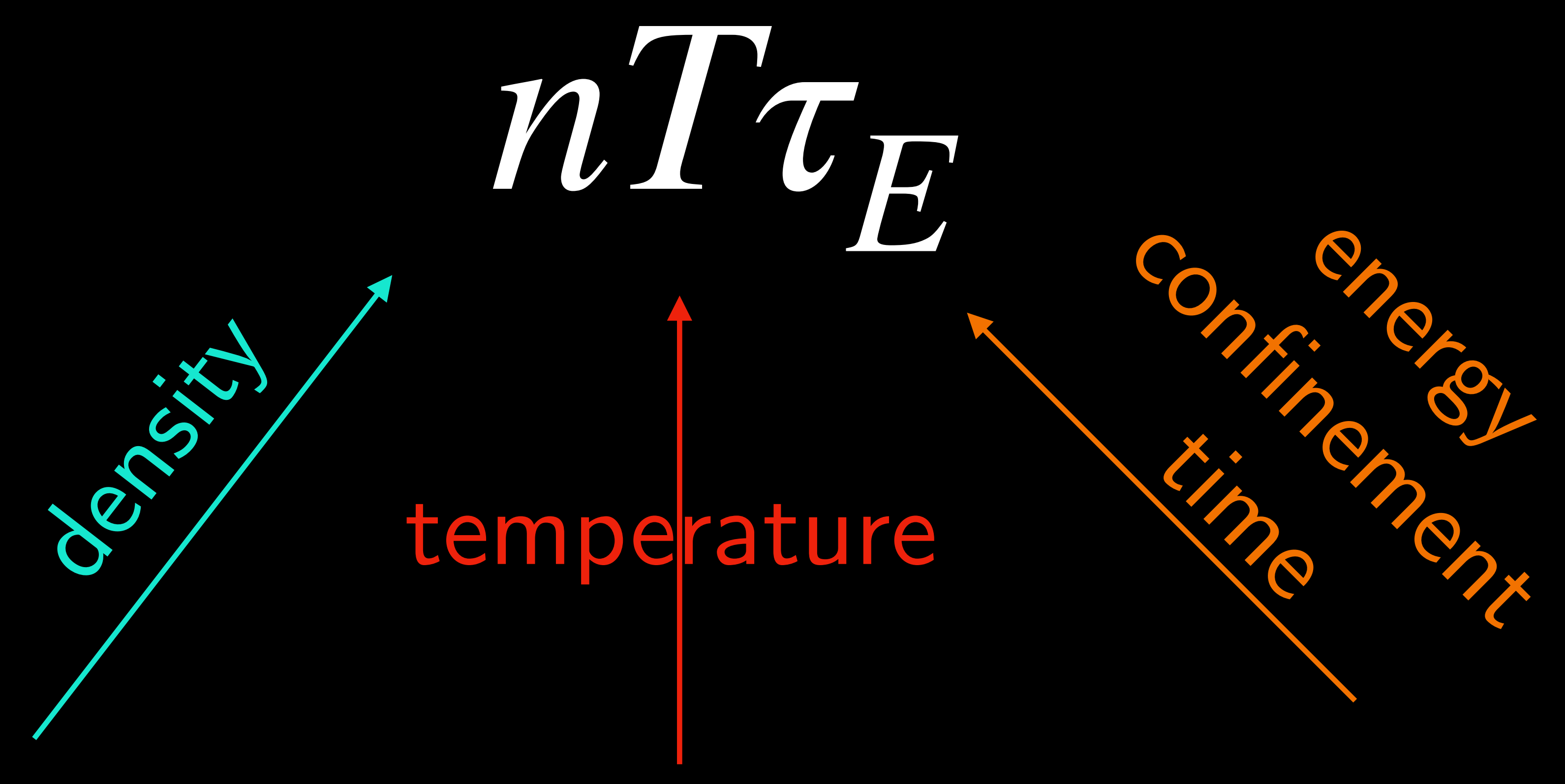
- In later lectures, you will learn that for self-sustaining plasma burn: triple product $nT\tau_E$ must exceed minimum value



$$nT\tau_E$$

plasma must be: dense hot well-confined

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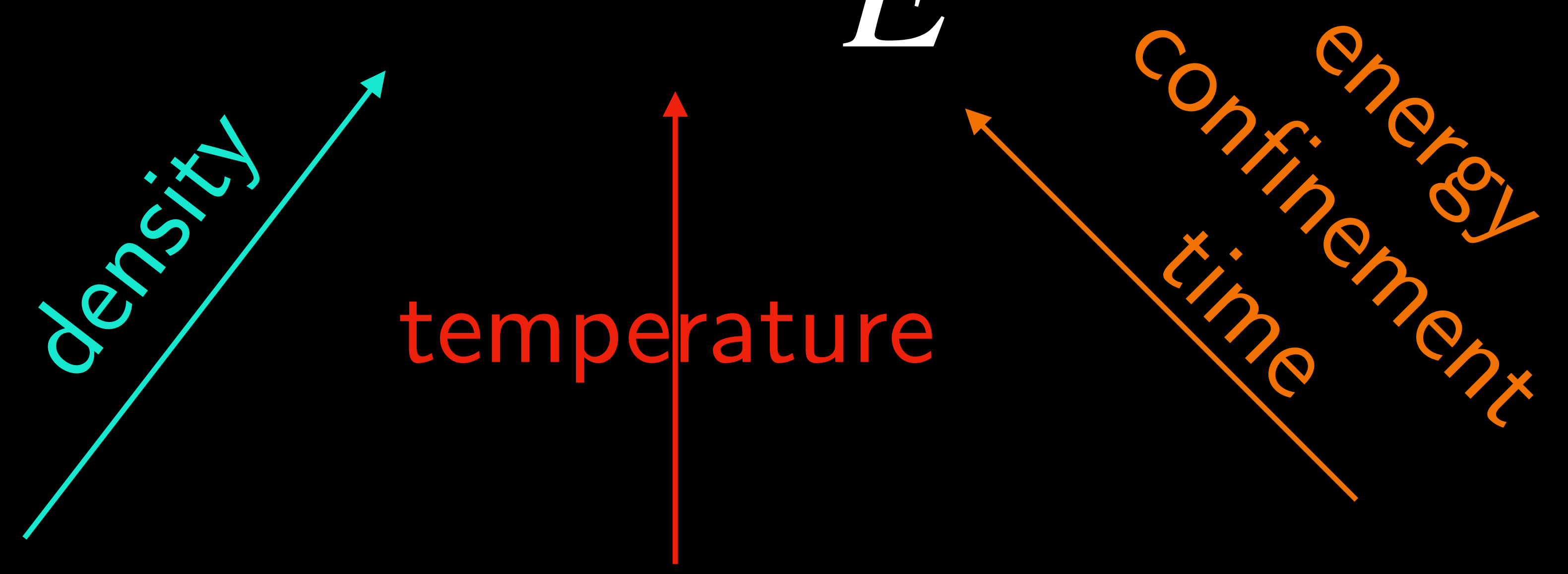
$$nT\tau_E$$



plasma must be: dense hot well-confined

<u>Plasma instability and turbulence mostly degrades T and τ_E</u>

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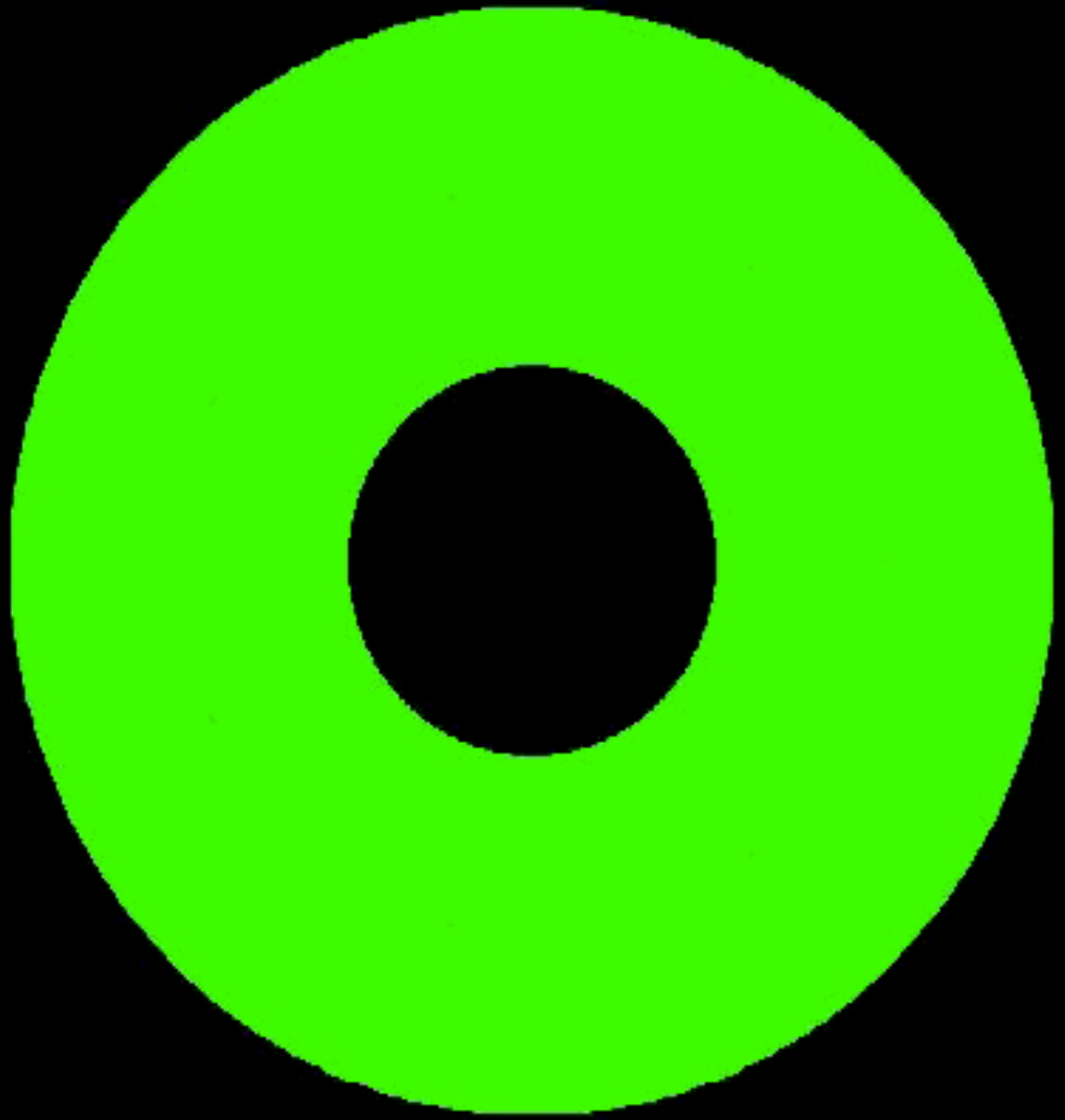

Gyrokinetic simulation of tokamak turbulence

credit: https://w3.pppl.gov/~hammett/viz/viz.html

Turbulence and Transport in Magnetic Confinement Fusion

Plasma Turbulence

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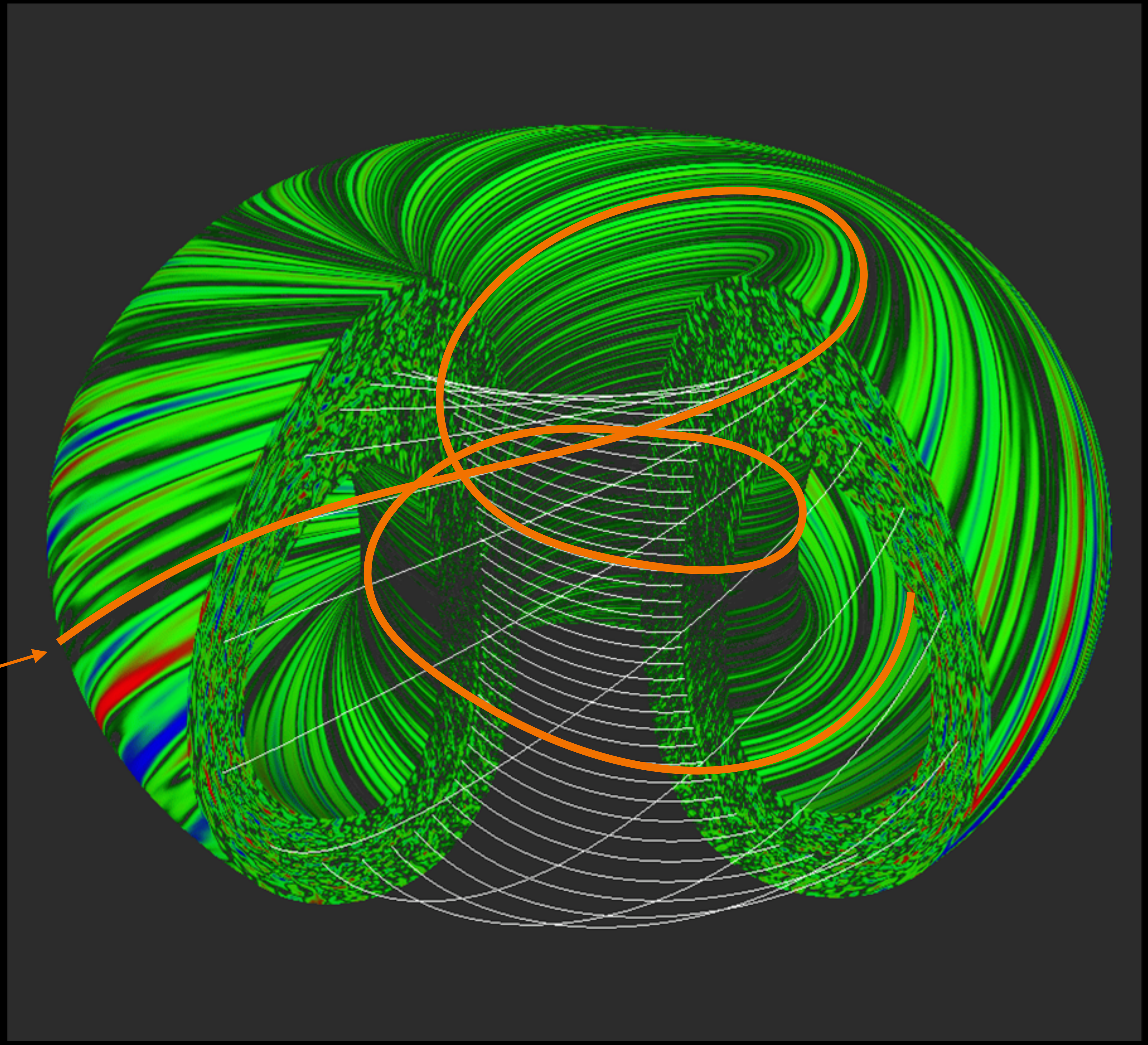
- Plasma instabilities create turbulent eddies that rapidly move energy from the core to the edge
- Turbulence highly anisotropic: structures elongated along field lines, short across field lines



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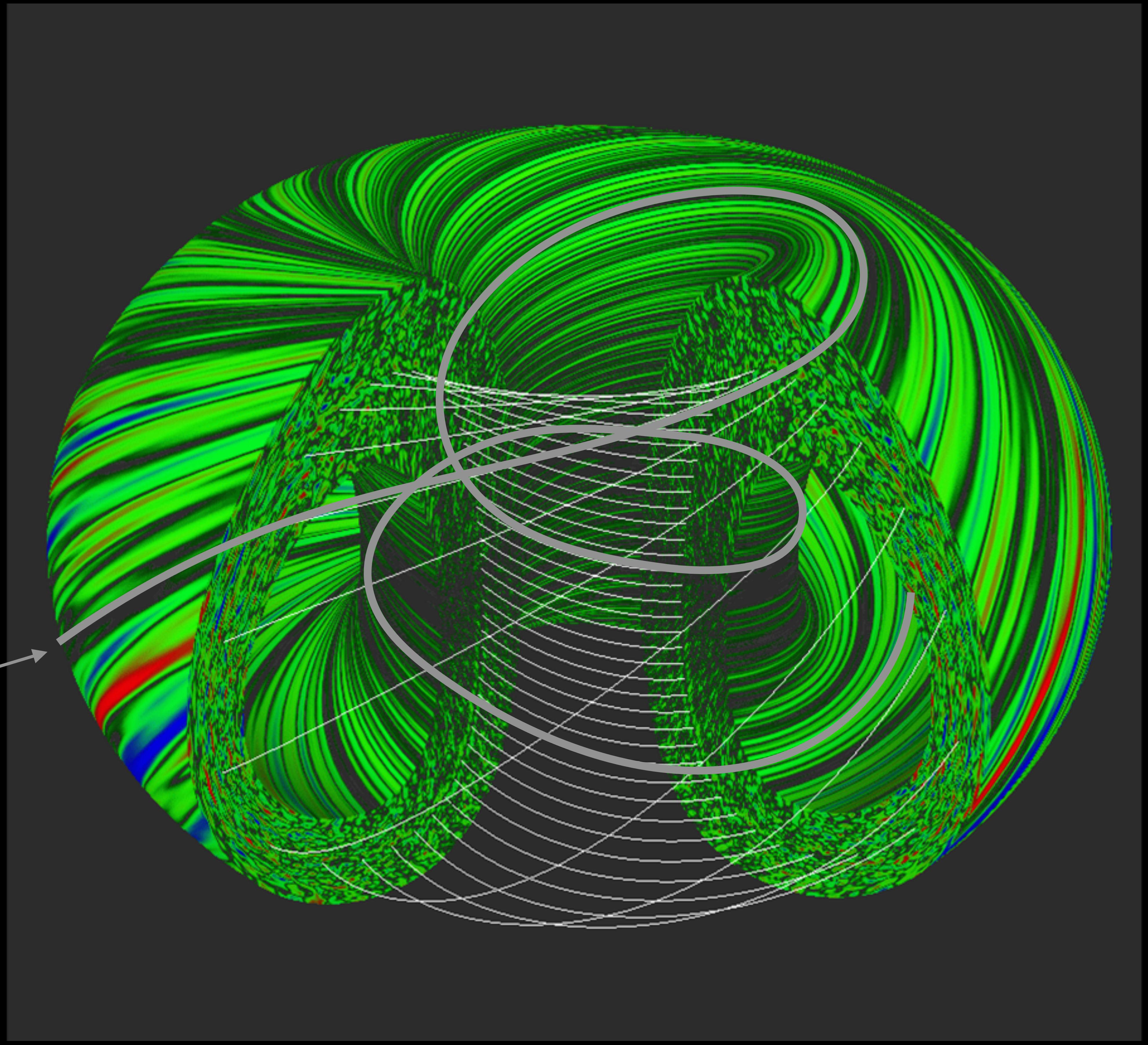
- Turbulence highly anisotropic: structures elongated along field lines, short across field lines
- Let's estimate turbulent transport arising from these structures



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Turbulence and Transport in Magnetic Confinement Fusion

GyroBohm Transport

Turbulence and Transport in Magnetic Confinement Fusion

GyroBohm Transport

- Initial turbulent electrostatic potential fluctuation ϕ^t in magnetic field $\mathbf{B}$ with pressure gradient ∇p.

Turbulence and Transport in Magnetic Confinement Fusion

GyroBohm Transport

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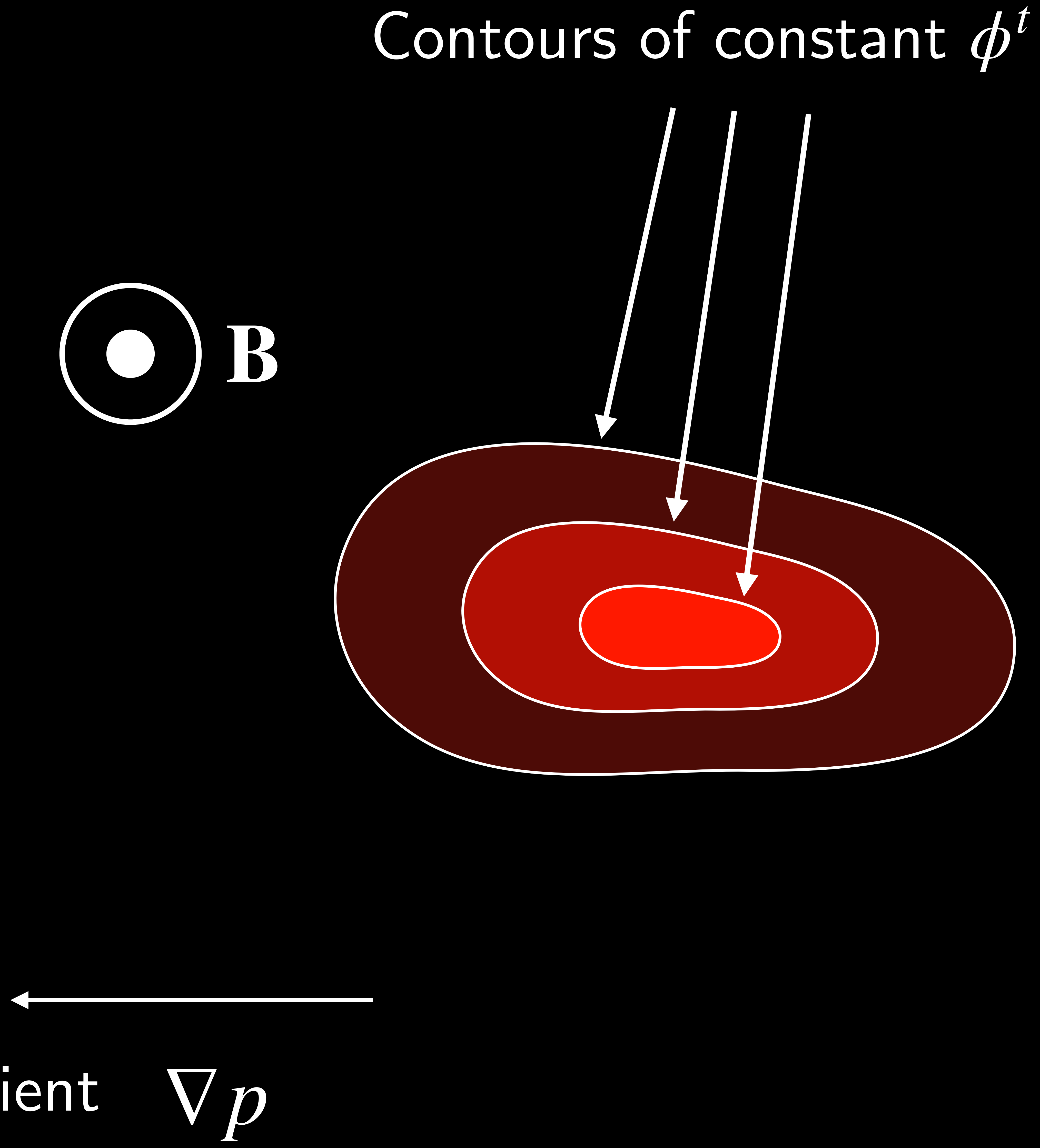
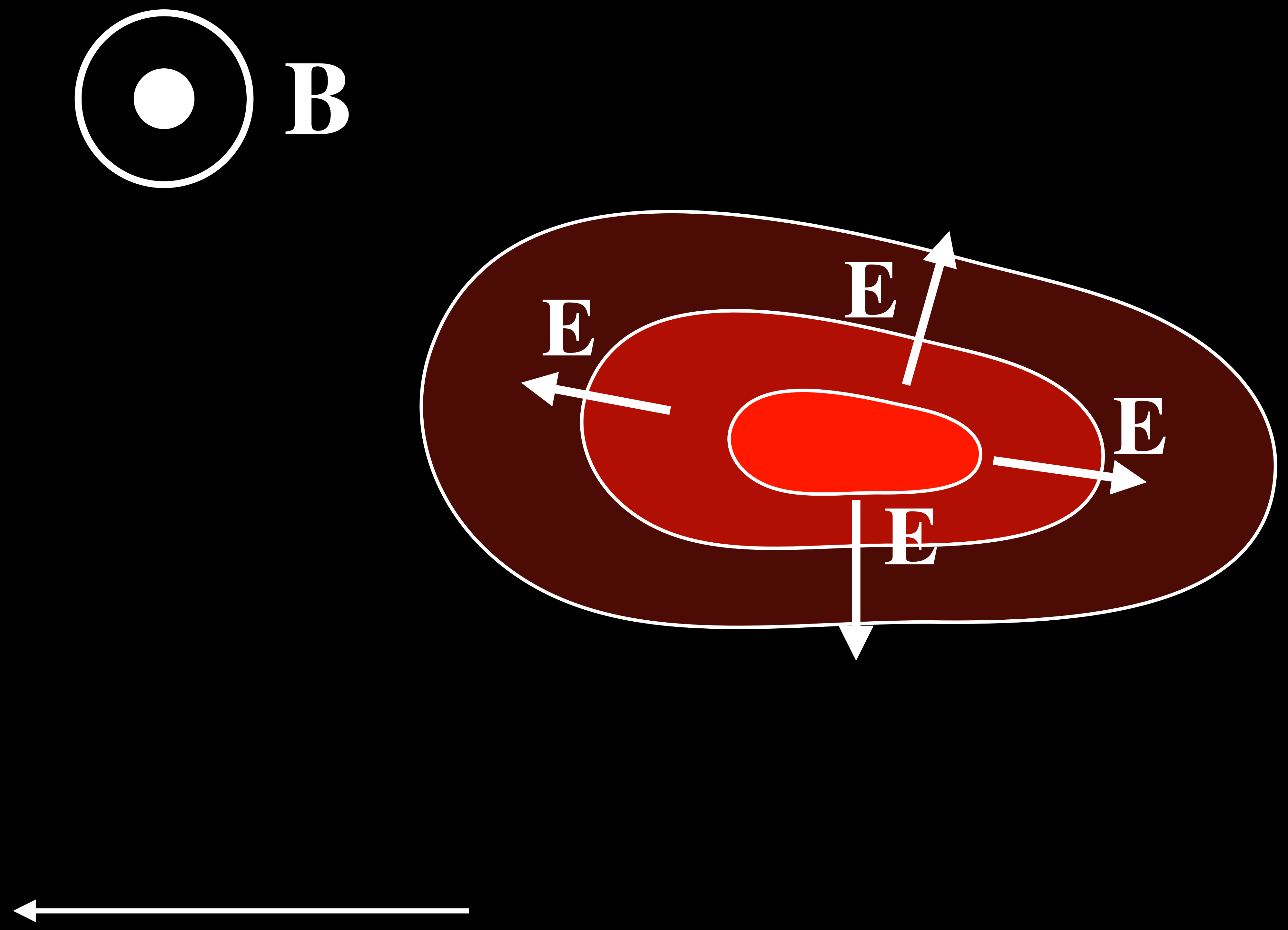


Turbulence and Transport in Magnetic Confinement Fusion

GyroBohm Transport

- → electric field $\mathbf{E} = -\nabla \phi^t$

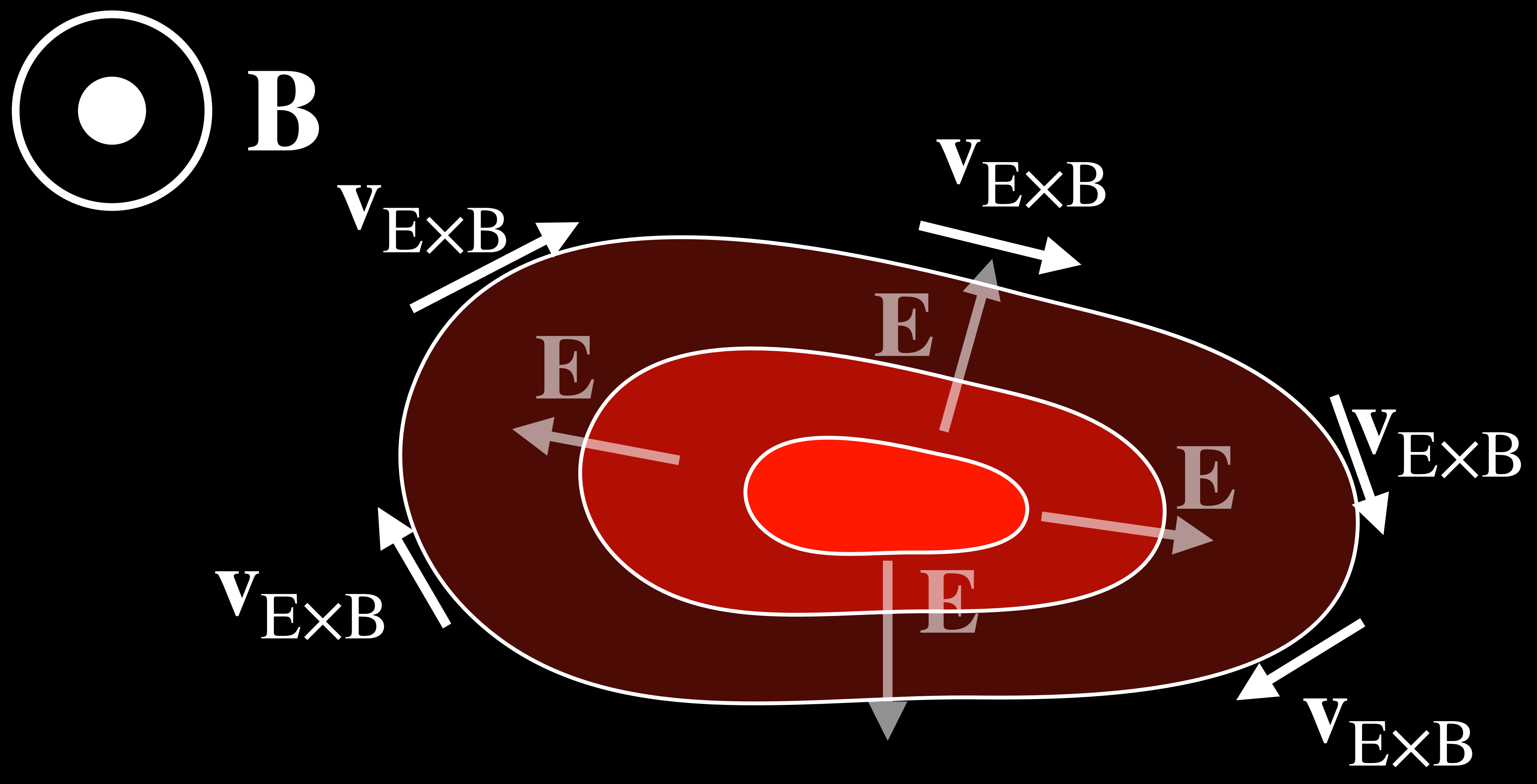


pressure gradient ∇p

Turbulence and Transport in Magnetic Confinement Fusion

GyroBohm Transport

- → $\mathbf{v}_{\mathrm{E\times B}}$ drift perpendicular to both E and B

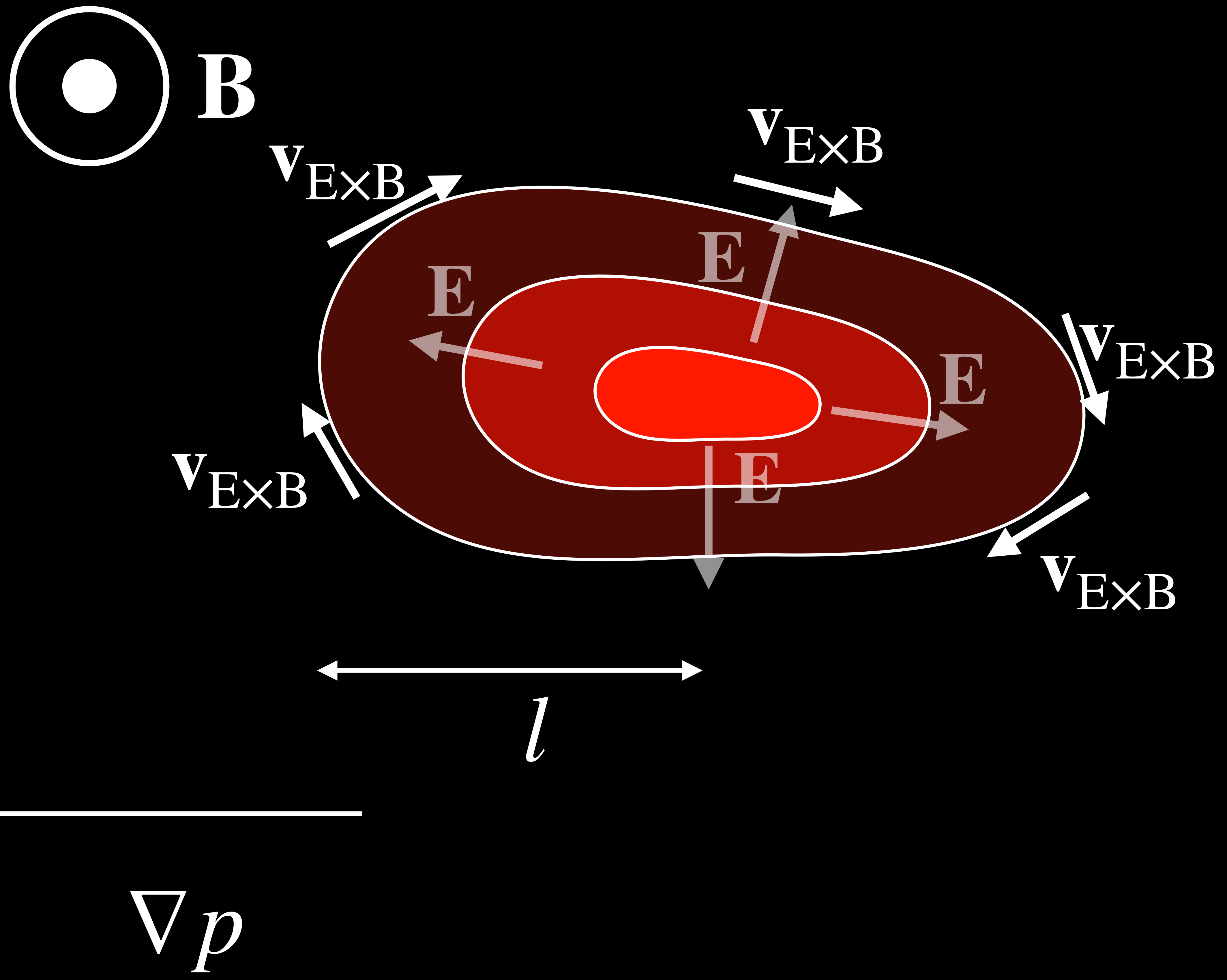


pressure gradient ∇p

Turbulence and Transport in Magnetic Confinement Fusion

GyroBohm Transport

- → $\mathbf{v}_{\mathrm{E\times B}}$ drift perpendicular to both E and B
- $\mathbf{v}_{\mathrm{E\times B}} = \dfrac{\mathbf{B}\times\nabla\phi^t}{B^2} \sim \dfrac{\phi^t}{Bl}$

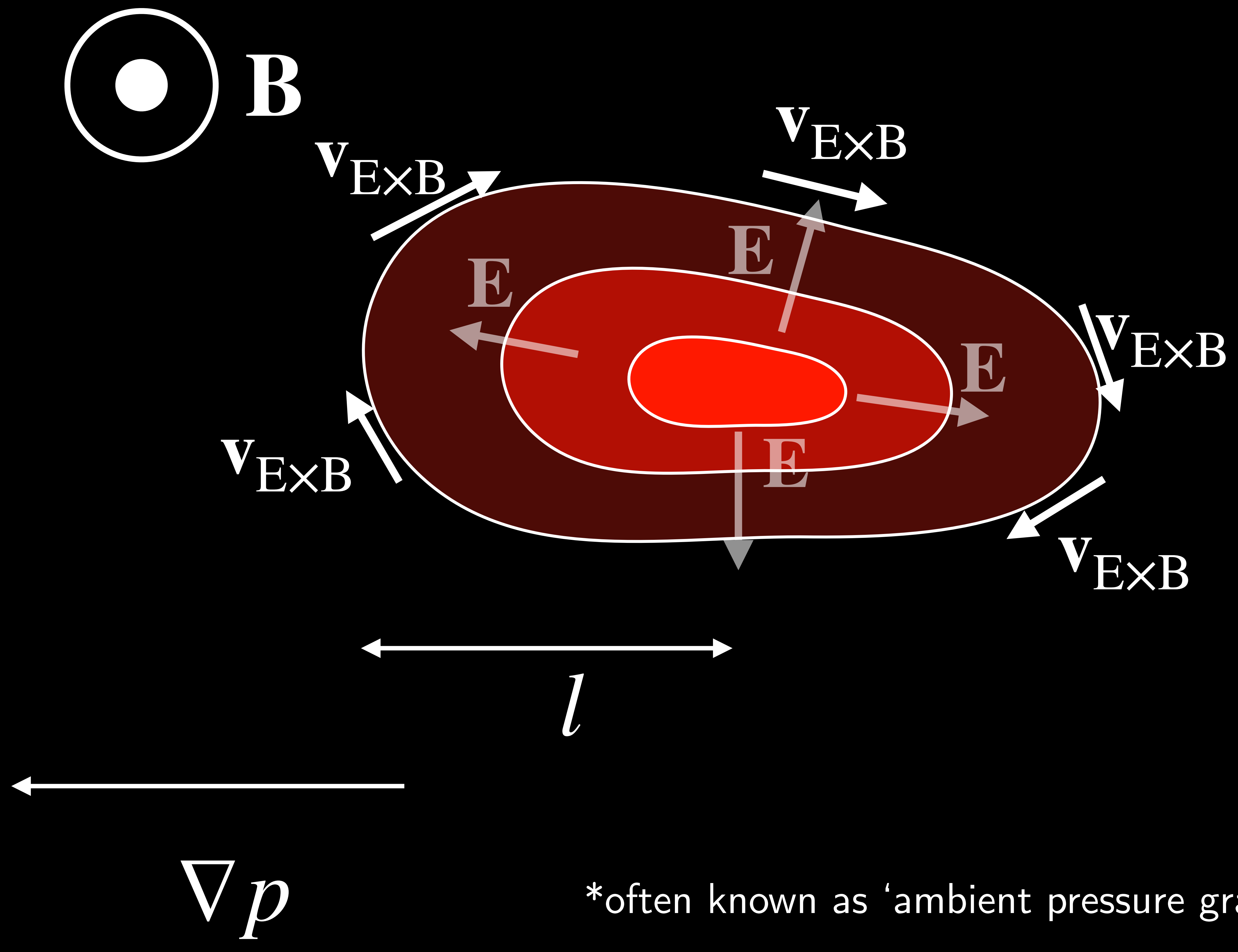


Turbulence and Transport in Magnetic Confinement Fusion

GyroBohm Transport

- → $\mathbf{v}_{\mathrm{E\times B}}$ drift perpendicular to both E and B
- $\mathbf{v}_{\mathrm{E\times B}} = \dfrac{\mathbf{B}\times\nabla\phi^t}{B^2} \sim \dfrac{\phi^t}{Bl}$
- Perturbed potential energy comparable to plasma thermal energy*:

  $$(L/l)e\phi^t \sim T = \frac{1}{2}mv_t^2$$



*often known as 'ambient pressure gradient argument'

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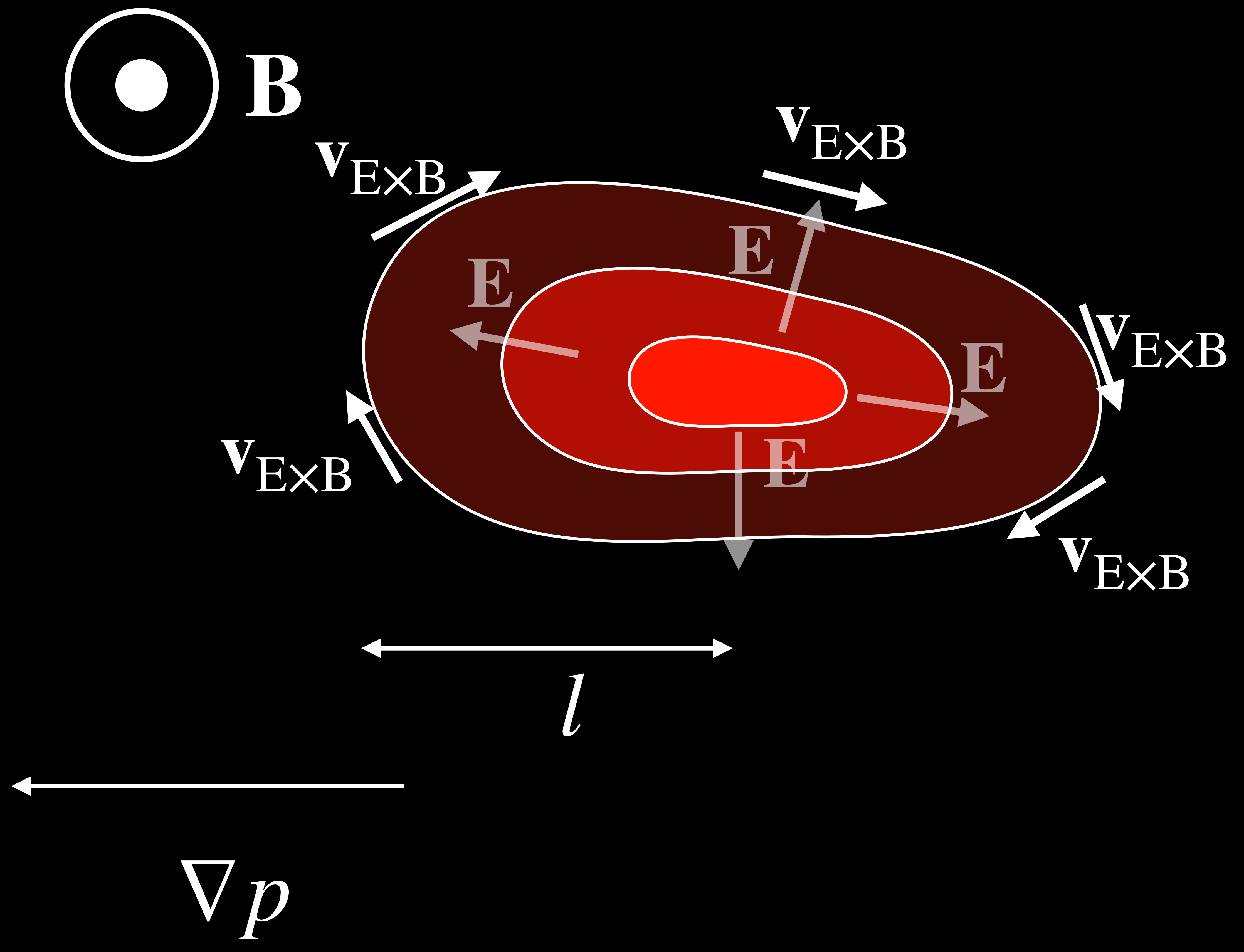
GyroBohm Transport

- → $\mathbf{v}_{\mathrm{E\times B}}$ drift perpendicular to both E and B
- $\mathbf{v}_{\mathrm{E\times B}} = \dfrac{\mathbf{B}\times\nabla\phi^t}{B^2} \sim \dfrac{\phi^t}{Bl} \sim \dfrac{T}{eB}\dfrac{1}{L}$
- Perturbed potential energy comparable to plasma thermal energy:

$$(L/l)e\phi^t \sim T = \frac{1}{2}mv_t^2$$



Turbulence and Transport in Magnetic Confinement Fusion

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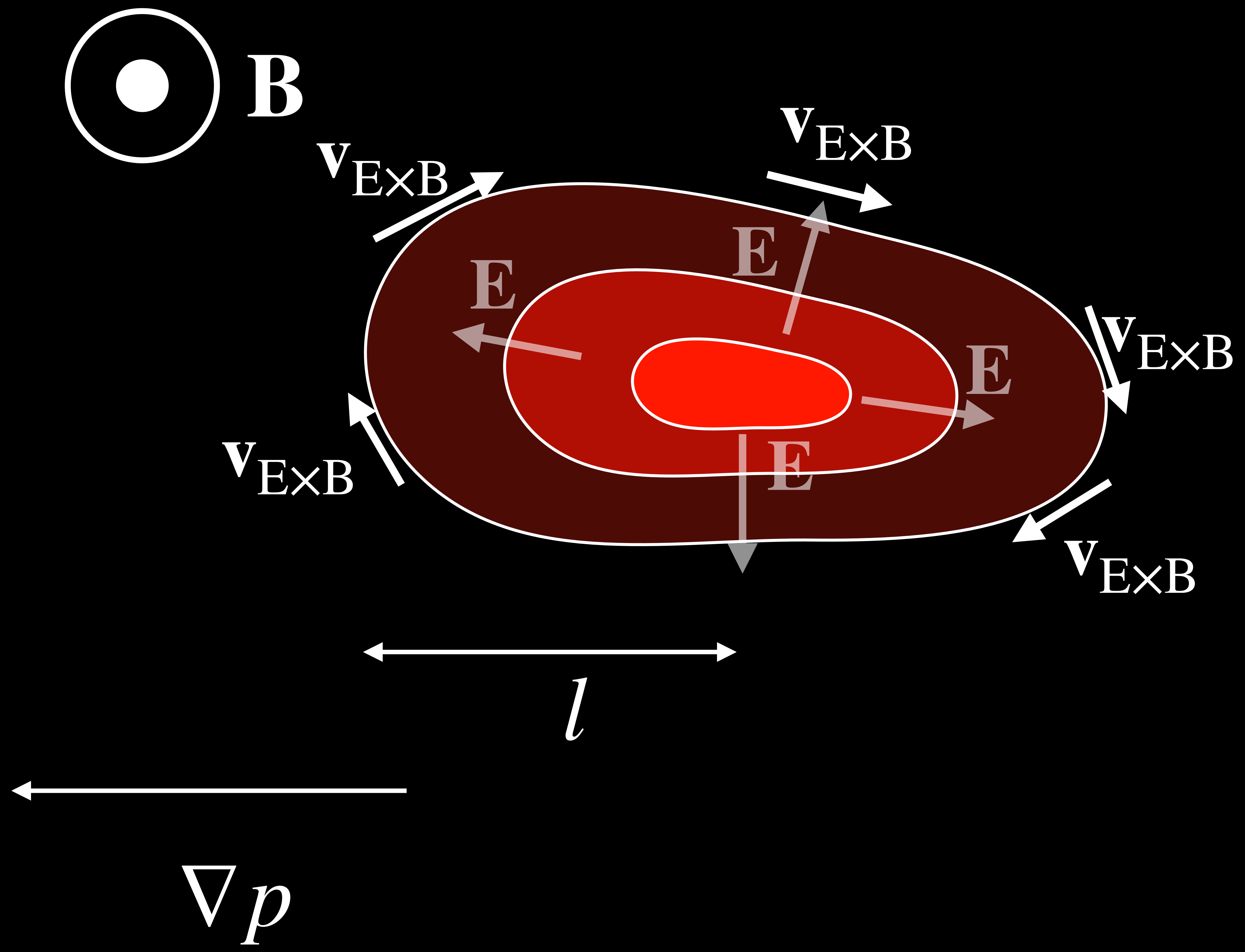

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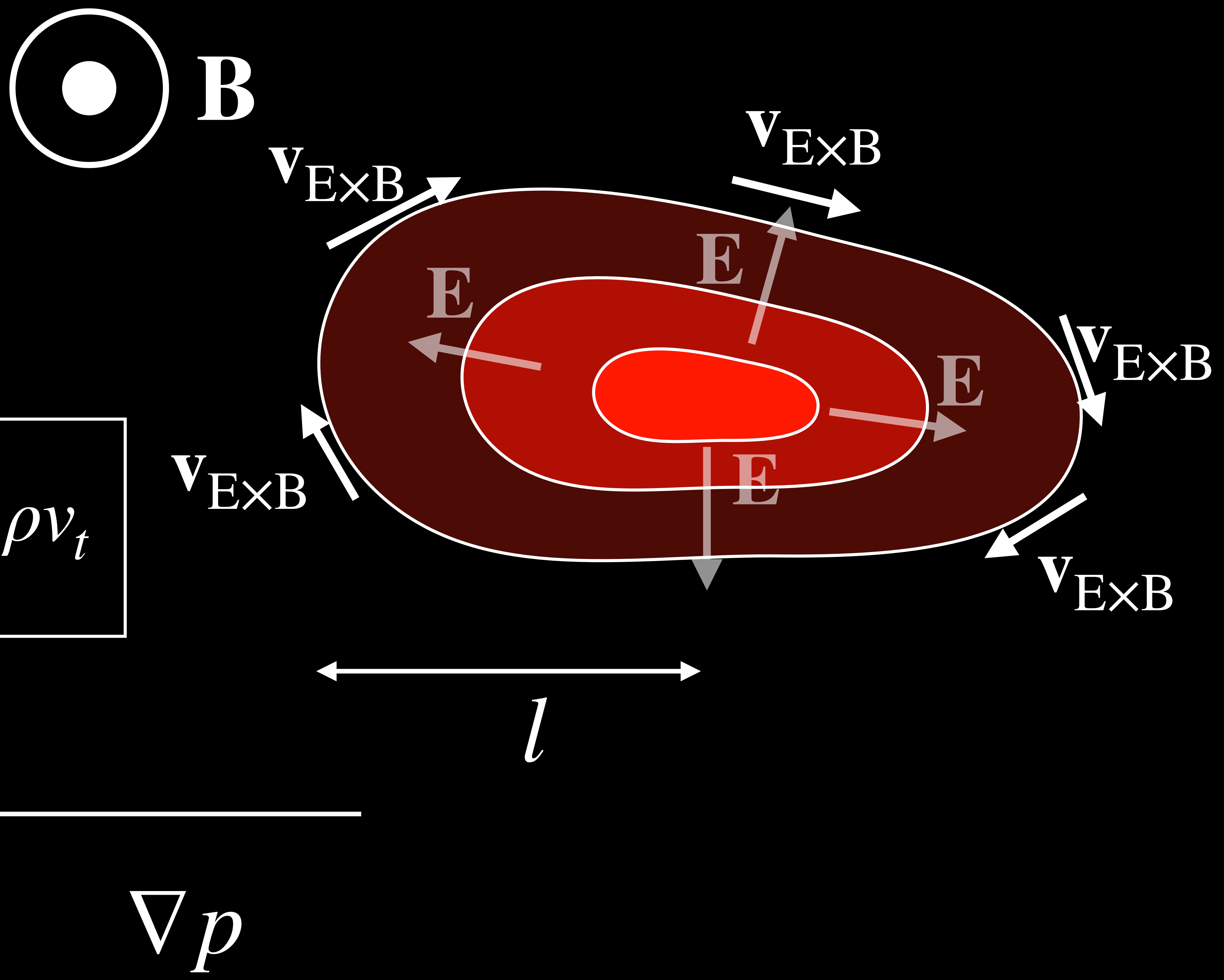
- diffusion coefficient

$$D_{\mathrm{turb}} \sim \frac{l^2}{\tau_{\mathrm{nl}}} \sim \frac{l}{L}\rho v_t \sim \frac{l}{\rho} D_{\mathrm{gB}}$$

$$\boxed{D_{\mathrm{gB}} \equiv \frac{\rho}{L}\rho v_t}$$

gyroBohm diffusivity



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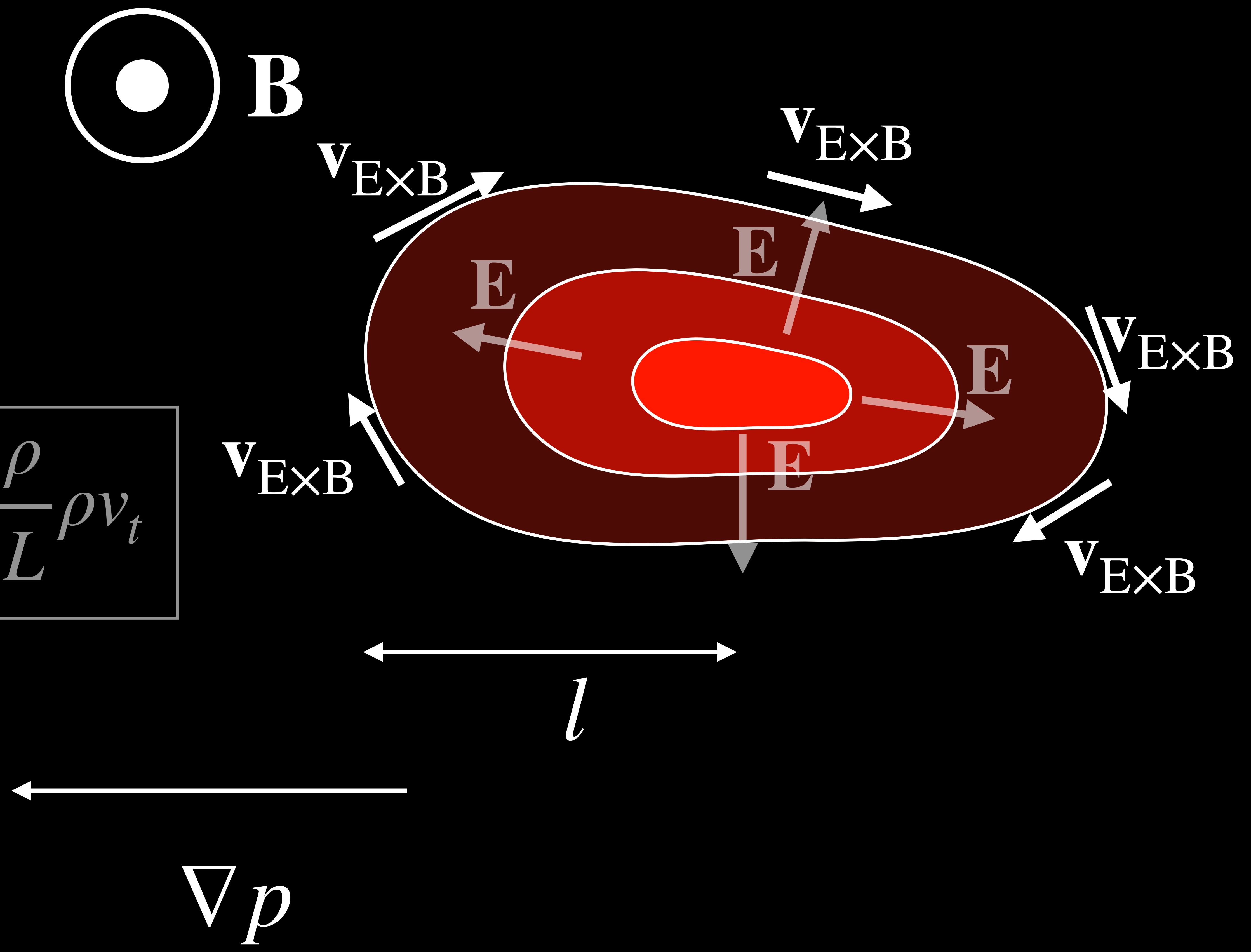
- If $D_{\mathrm{turb}} = D_{\mathrm{gB}}$, eddies with gyroradius size ρ transport heat



Turbulence and Transport in Magnetic Confinement Fusion

GyroBohm Transport

- Energy confinement time

  $$\tau_E \sim L^2/D \sim L^3 \frac{1}{l\rho v_t}$$

$$\tau_E \equiv \frac{\text{plasma stored energy}}{\text{plasma heating power}}$$

Turbulence and Transport in Magnetic Confinement Fusion

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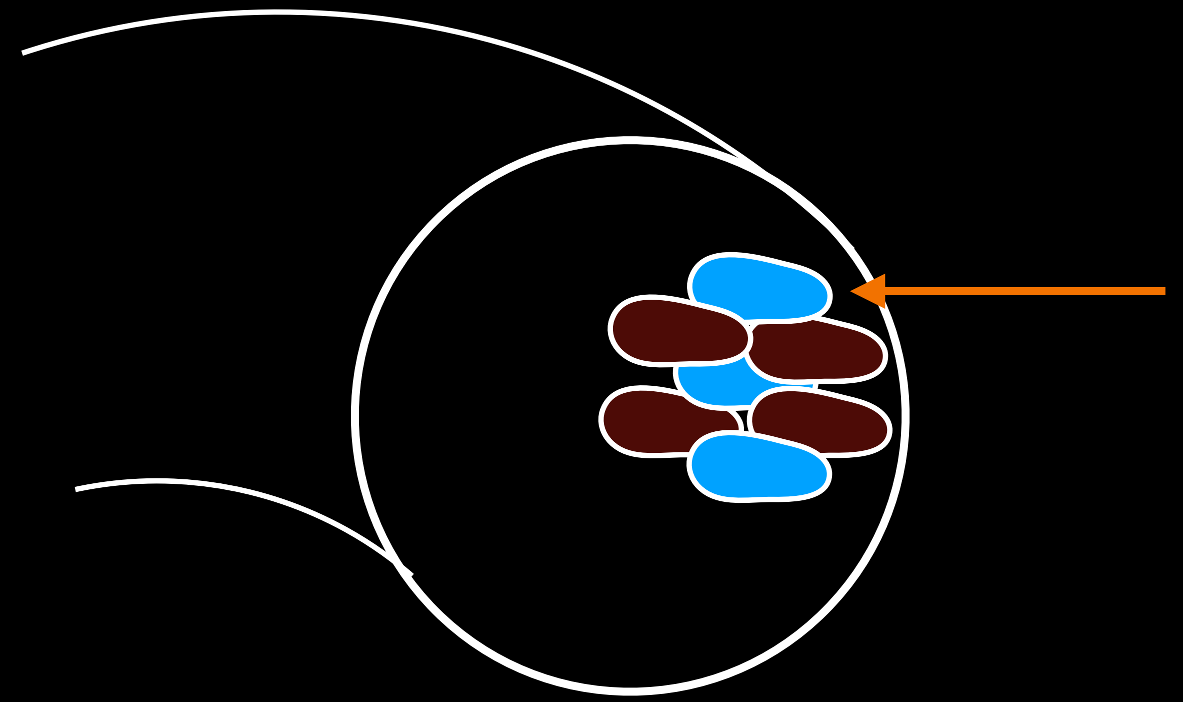
For better confinement, make device bigger by increasing L and decreasing $\rho_* \ll 1$

Turbulence and Transport in Magnetic Confinement Fusion

GyroBohm Transport


Turbulent eddies

- Energy confinement time

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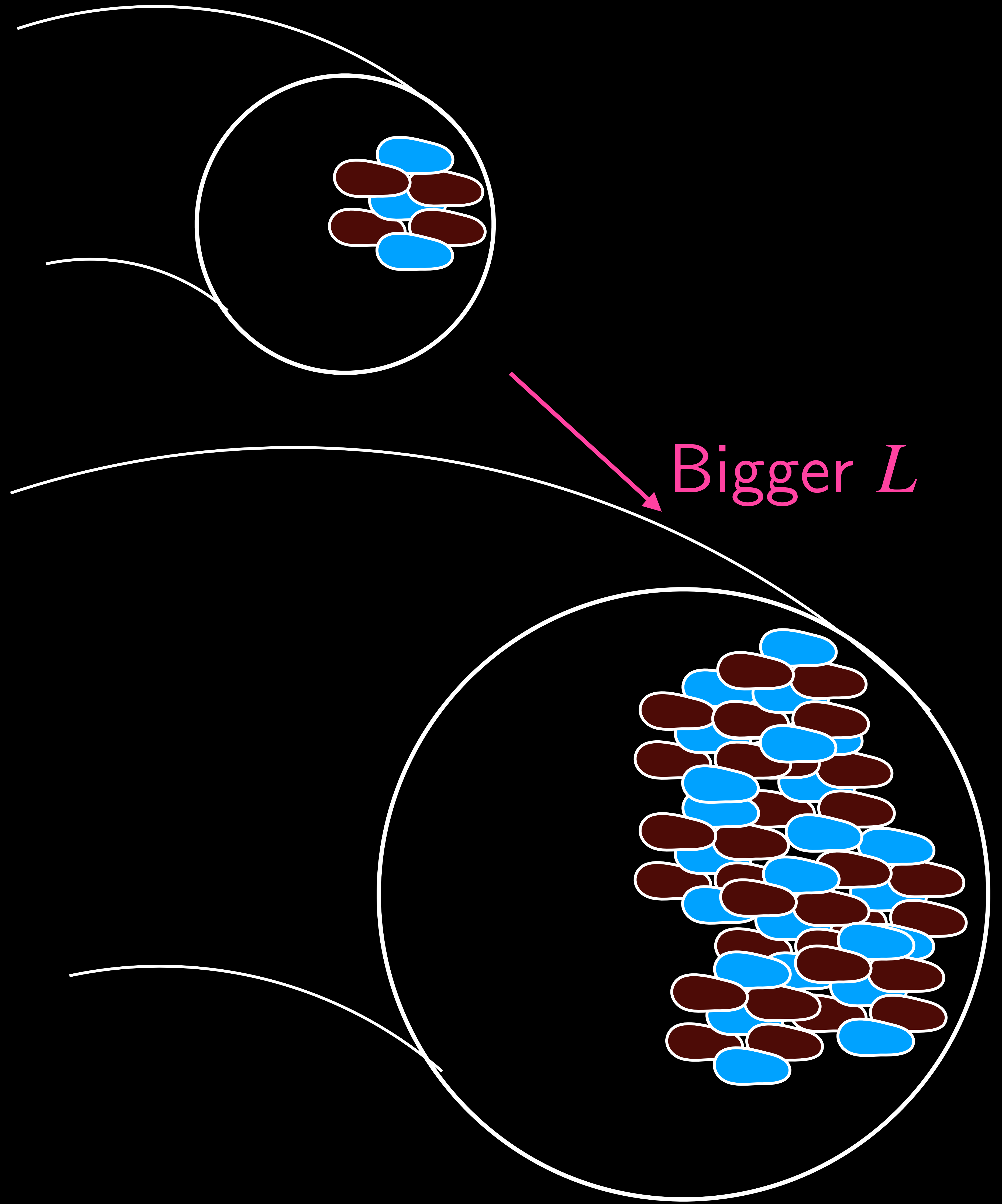

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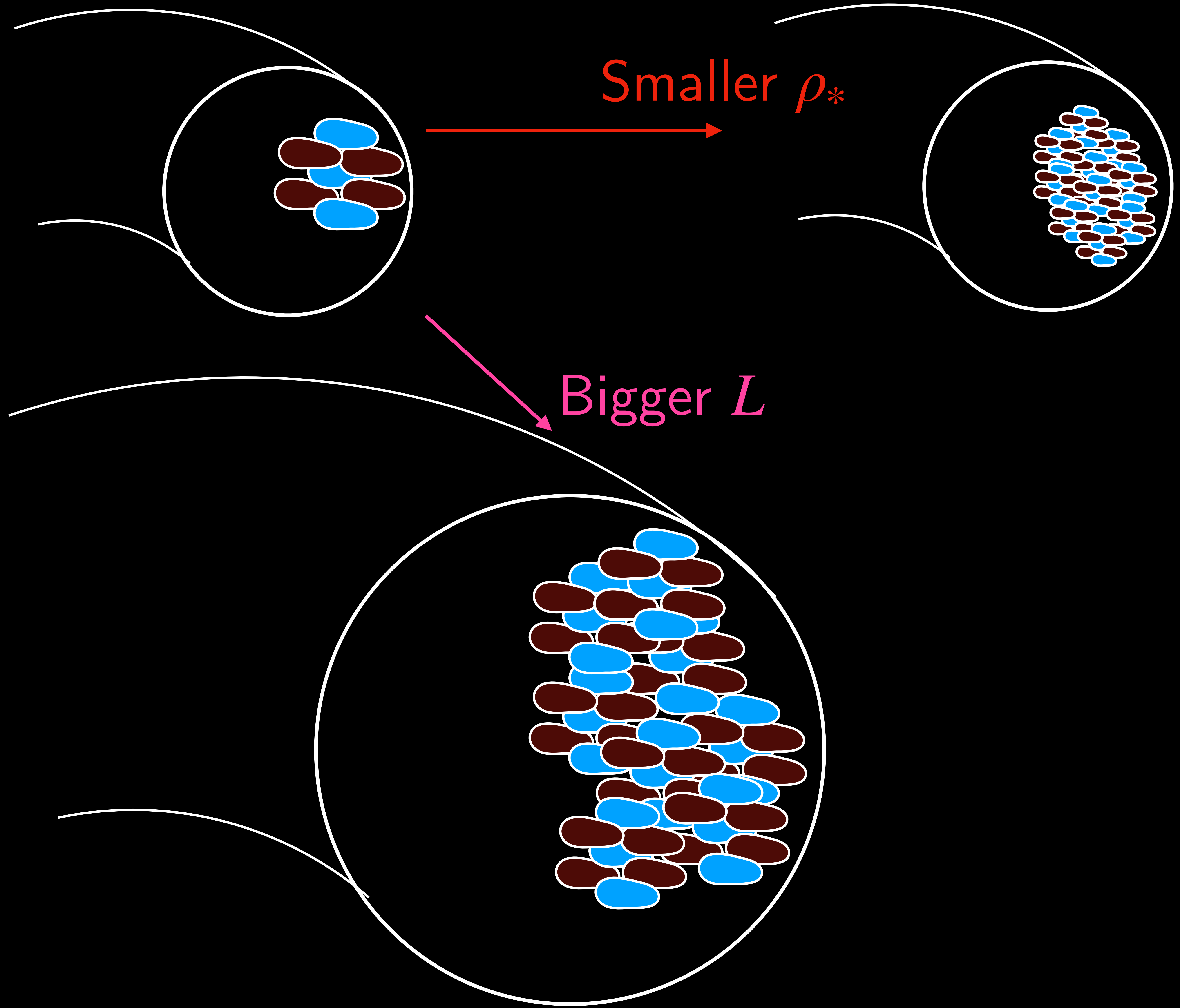

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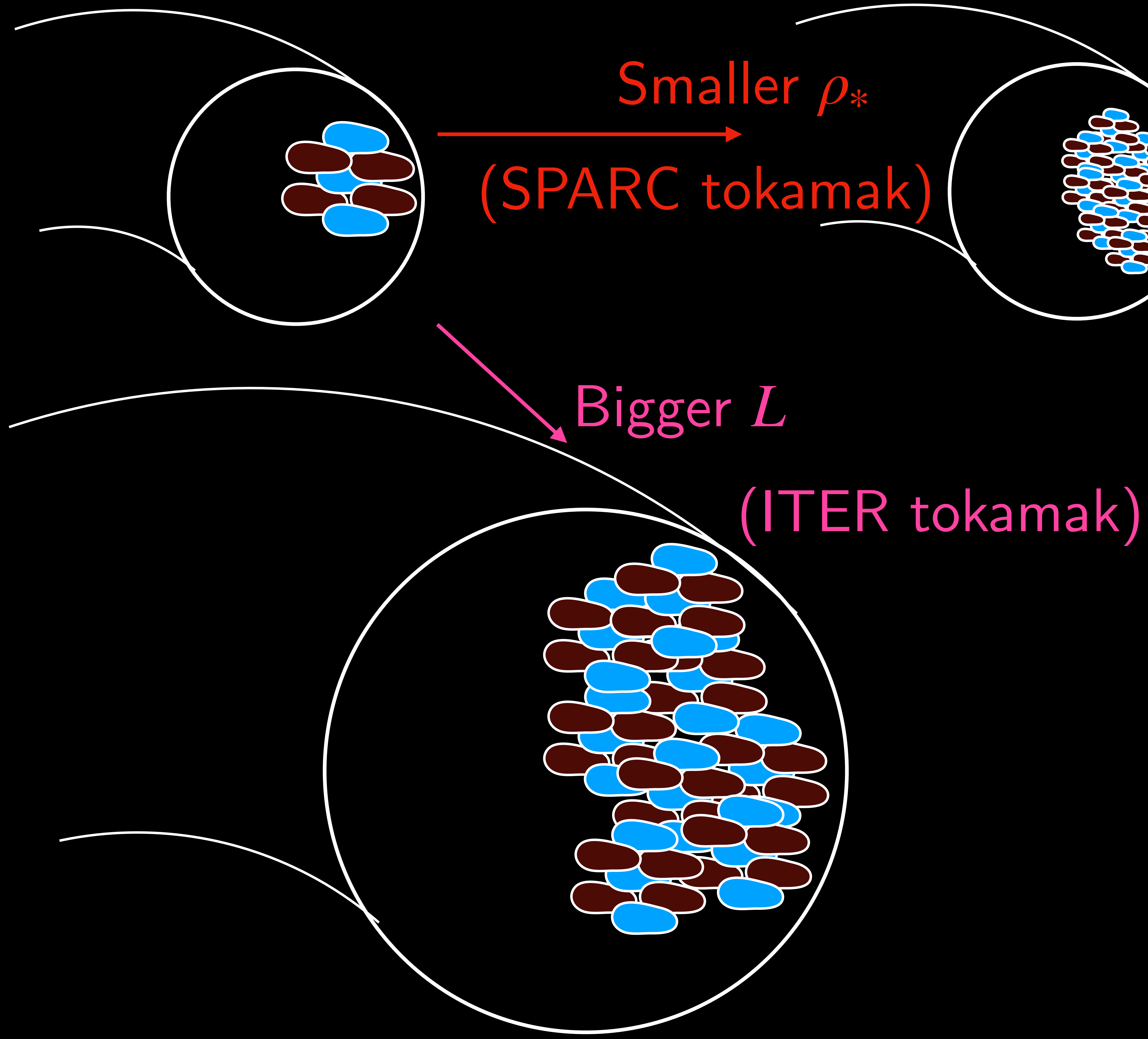

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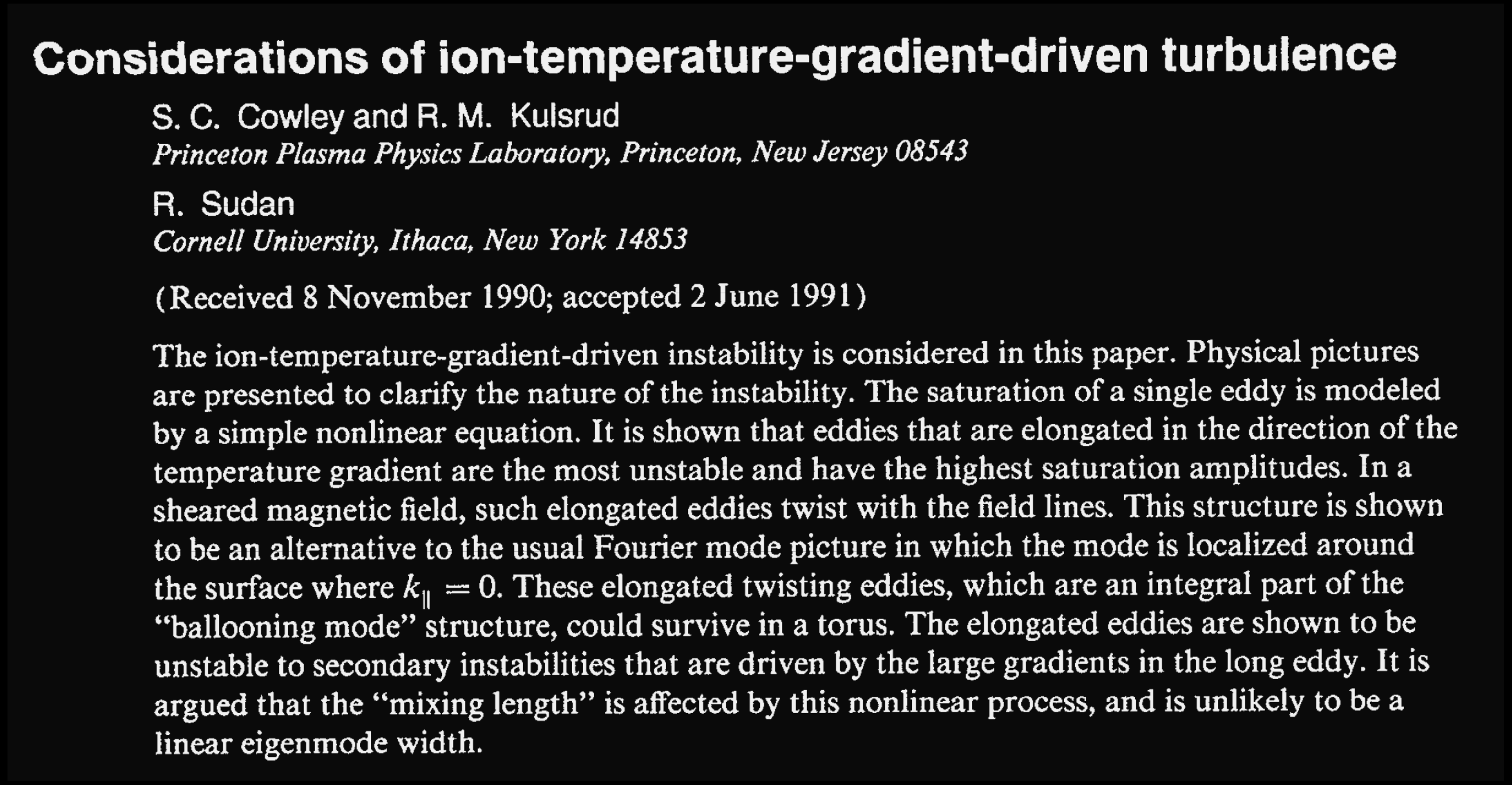
Fusion history snippet*



Considerations of ion-temperature-gradient-driven turbulence

S. C. Cowley and R. M. Kulsrud
Princeton Plasma Physics Laboratory, Princeton, New Jersey 08543
R. Sudan
Cornell University, Ithaca, New York 14853

(Received 8 November 1990; accepted 2 June 1991)

The ion-temperature-gradient-driven instability is considered in this paper. Physical pictures are presented to clarify the nature of the instability. The saturation of a single eddy is modeled by a simple nonlinear equation. It is shown that eddies that are elongated in the direction of the temperature gradient are the most unstable and have the highest saturation amplitudes. In a sheared magnetic field, such elongated eddies twist with the field lines. This structure is shown to be an alternative to the usual Fourier mode picture in which the mode is localized around the surface where $k_\parallel = 0$. These elongated twisting eddies, which are an integral part of the "ballooning mode" structure, could survive in a torus. The elongated eddies are shown to be unstable to secondary instabilities that are driven by the large gradients in the long eddy. It is argued that the "mixing length" is affected by this nonlinear process, and is unlikely to be a linear eigenmode width.

Paper predicted ion turbulence might cause disastrously high transport due to super large eddies $l \gg \rho$

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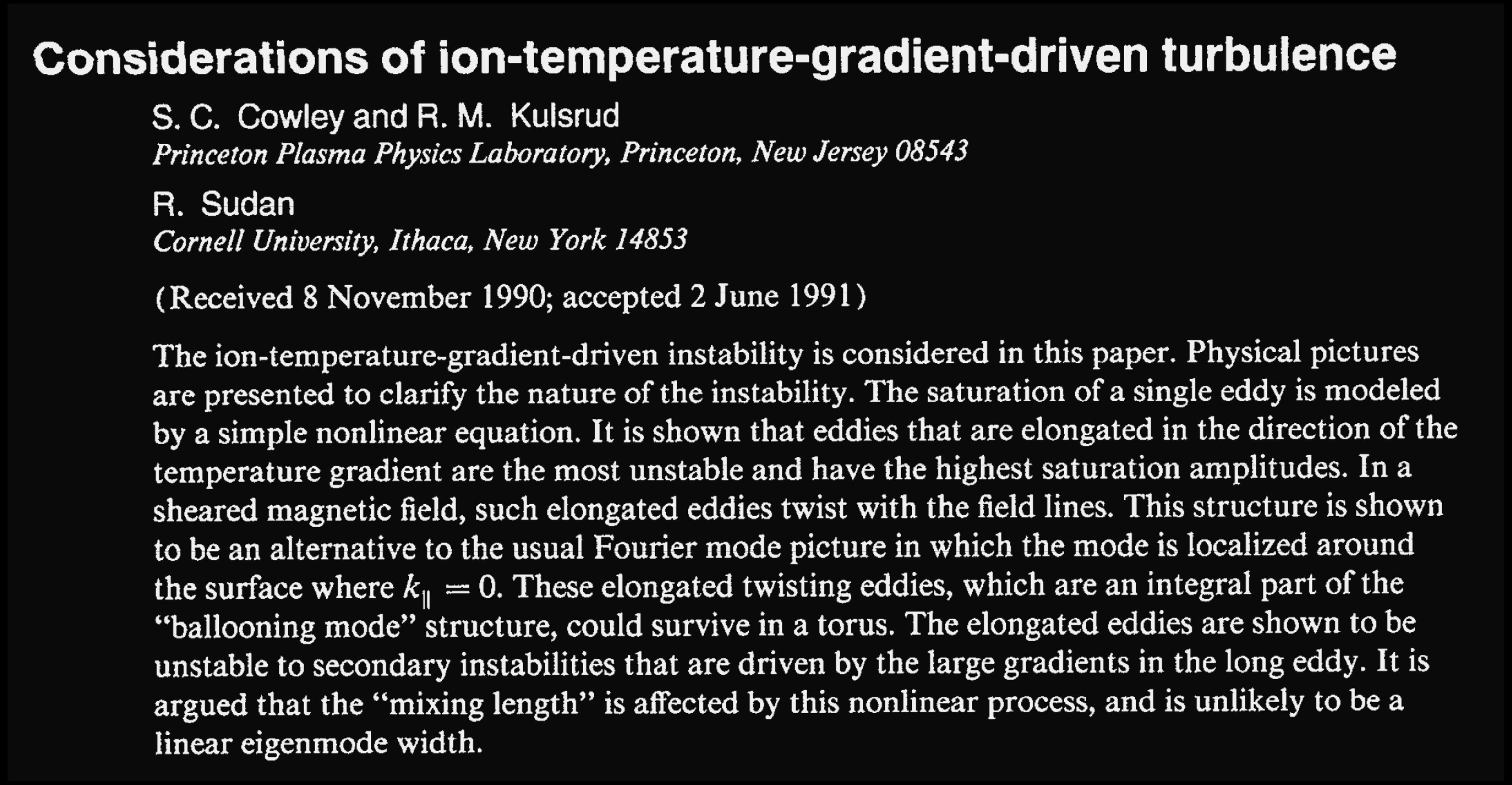

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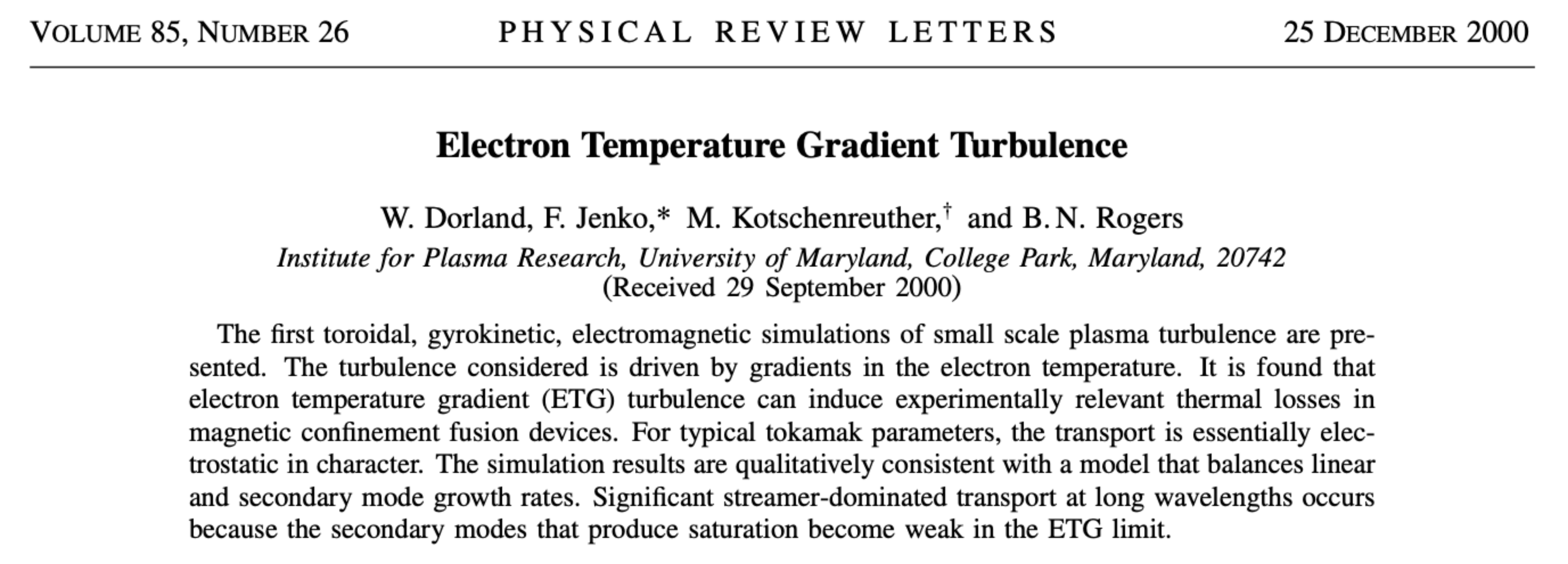


VOLUME 85, NUMBER 26 PHYSICAL REVIEW LETTERS 25 DECEMBER 2000

Electron Temperature Gradient Turbulence

W. Dorland, F. Jenko,* M. Kotschenreuther,† and B. N. Rogers

Institute for Plasma Research, University of Maryland, College Park, Maryland, 20742

(Received 29 September 2000)

The first toroidal, gyrokinetic, electromagnetic simulations of small scale plasma turbulence are presented. The turbulence considered is driven by gradients in the electron temperature. It is found that electron temperature gradient (ETG) turbulence can induce experimentally relevant thermal losses in magnetic confinement fusion devices. For typical tokamak parameters, the transport is essentially electrostatic in character. The simulation results are qualitatively consistent with a model that balances linear and secondary mode growth rates. Significant streamer-dominated transport at long wavelengths occurs because the secondary modes that produce saturation become weak in the ETG limit.

Fortunately, this does not happen for ion turbulence, but often happens for electron turbulence

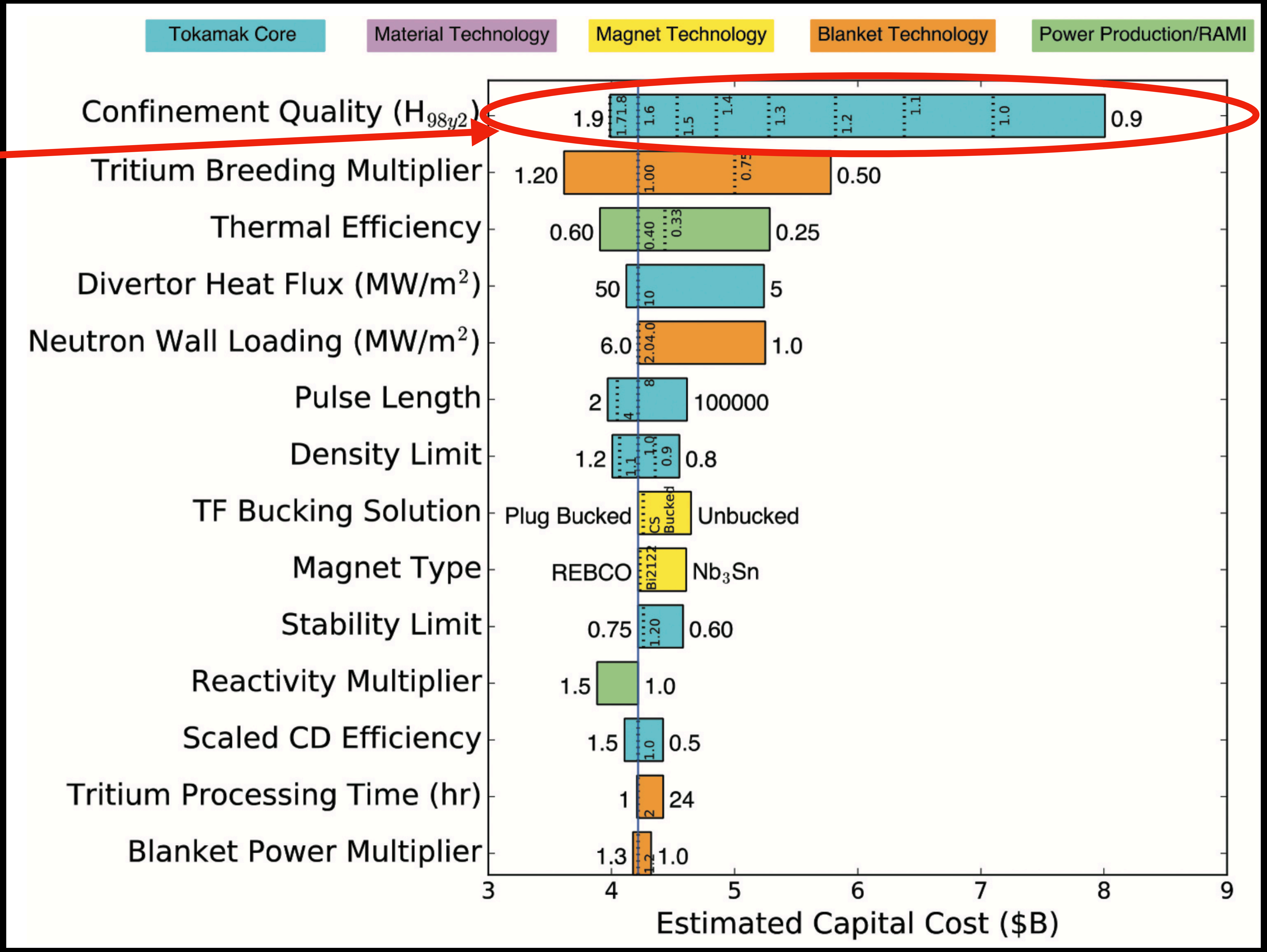
Turbulence and Transport in Magnetic Confinement Fusion

Sensitivity of power plant cost on confinement

- Capital cost most sensitive to confinement quality H, which is proportional to τ_E

⟶

Need all-out effort to boost confinement



Wade, M. R., and J. A. Leuer. "Cost drivers for a tokamak-based compact pilot plant." *Fusion Science and Technology* 77.2 (2021): 119-143.

Turbulence and Transport in Magnetic Confinement Fusion

Why did we get plasma turbulence in the first place?

Turbulence and Transport in Magnetic Confinement Fusion

Why did we get plasma turbulence in the first place?

Plasma instabilities driven by pressure gradients grow and give rise to turbulence!

Turbulence and Transport in Magnetic Confinement Fusion

Plasma instabilities → turbulent eddies

Turbulence and Transport in Magnetic Confinement Fusion

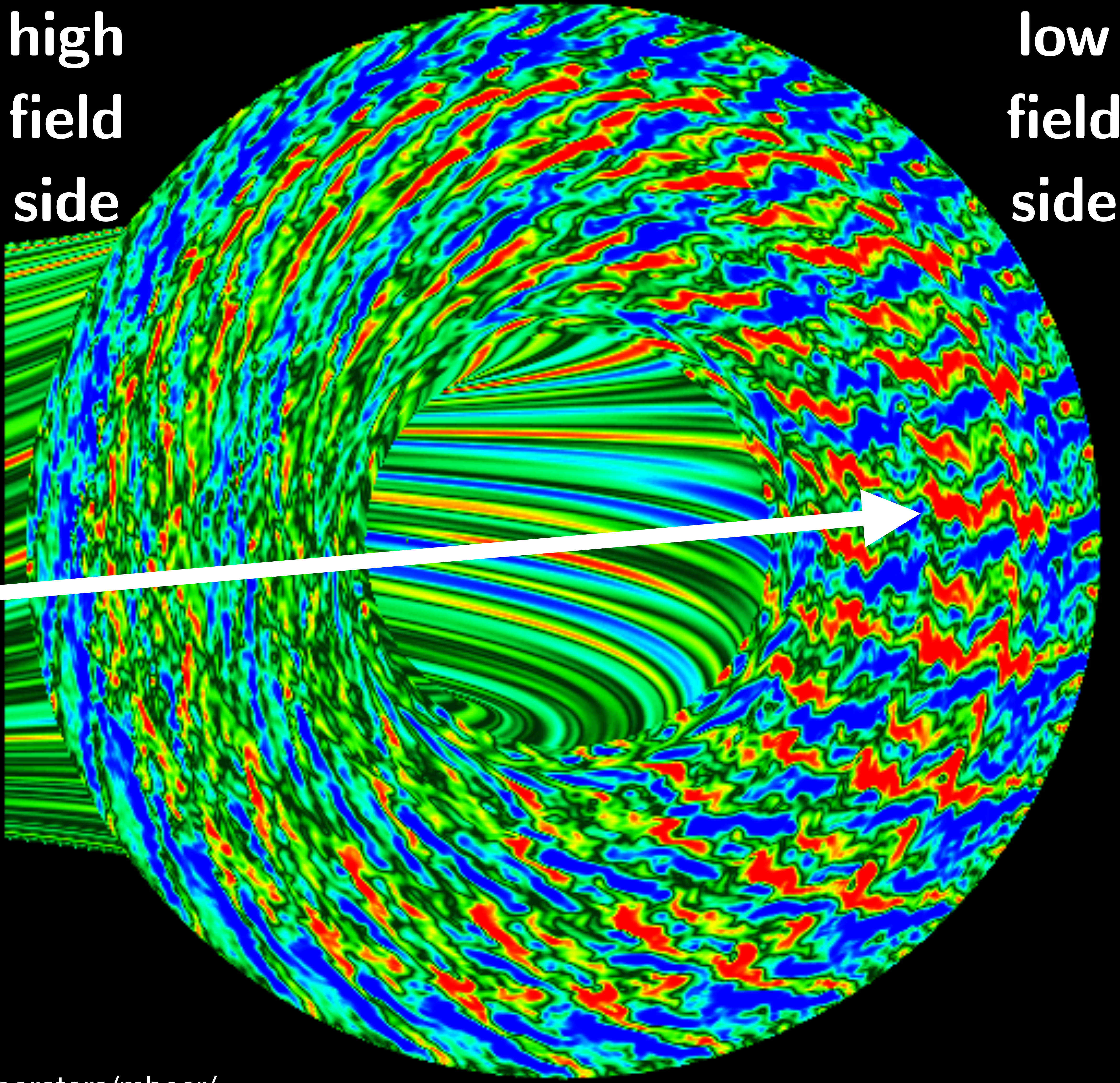
Plasma instabilities → turbulent eddies

- Quintessential tokamak instability is electron-temperature-gradient (ETG) instability

Turbulence and Transport in Magnetic Confinement Fusion

Plasma instabilities → turbulent eddies

- Quintessential tokamak instability is electron-temperature-gradient (ETG) instability
- Instability strongest at **tokamak low field side**, explaining why we usually see stronger turbulence there



Credit:
https://w3.pppl.gov/~hammett/collaborators/mbeer/

Electron-temperature-gradient (ETG) instability physical picture

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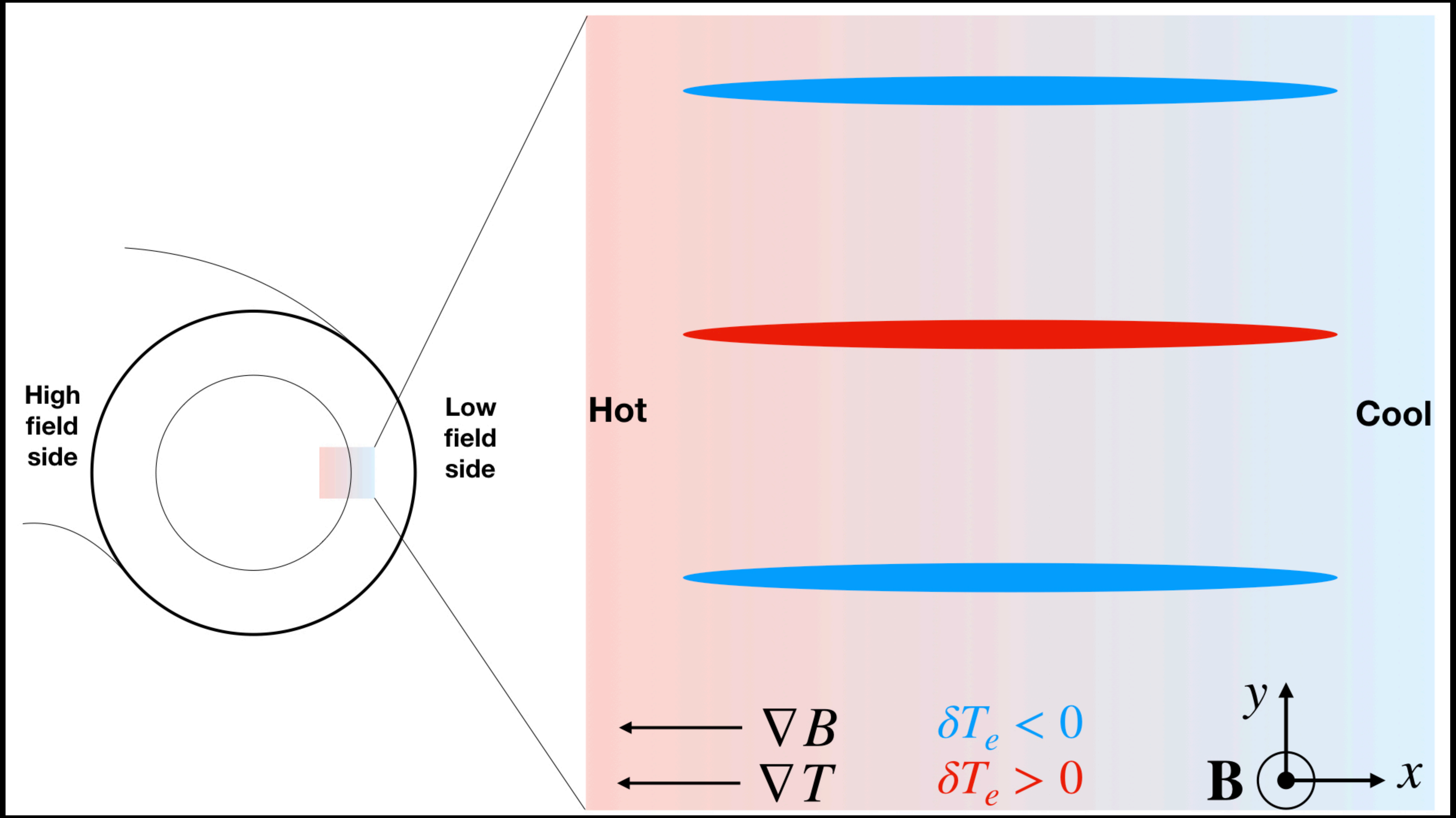

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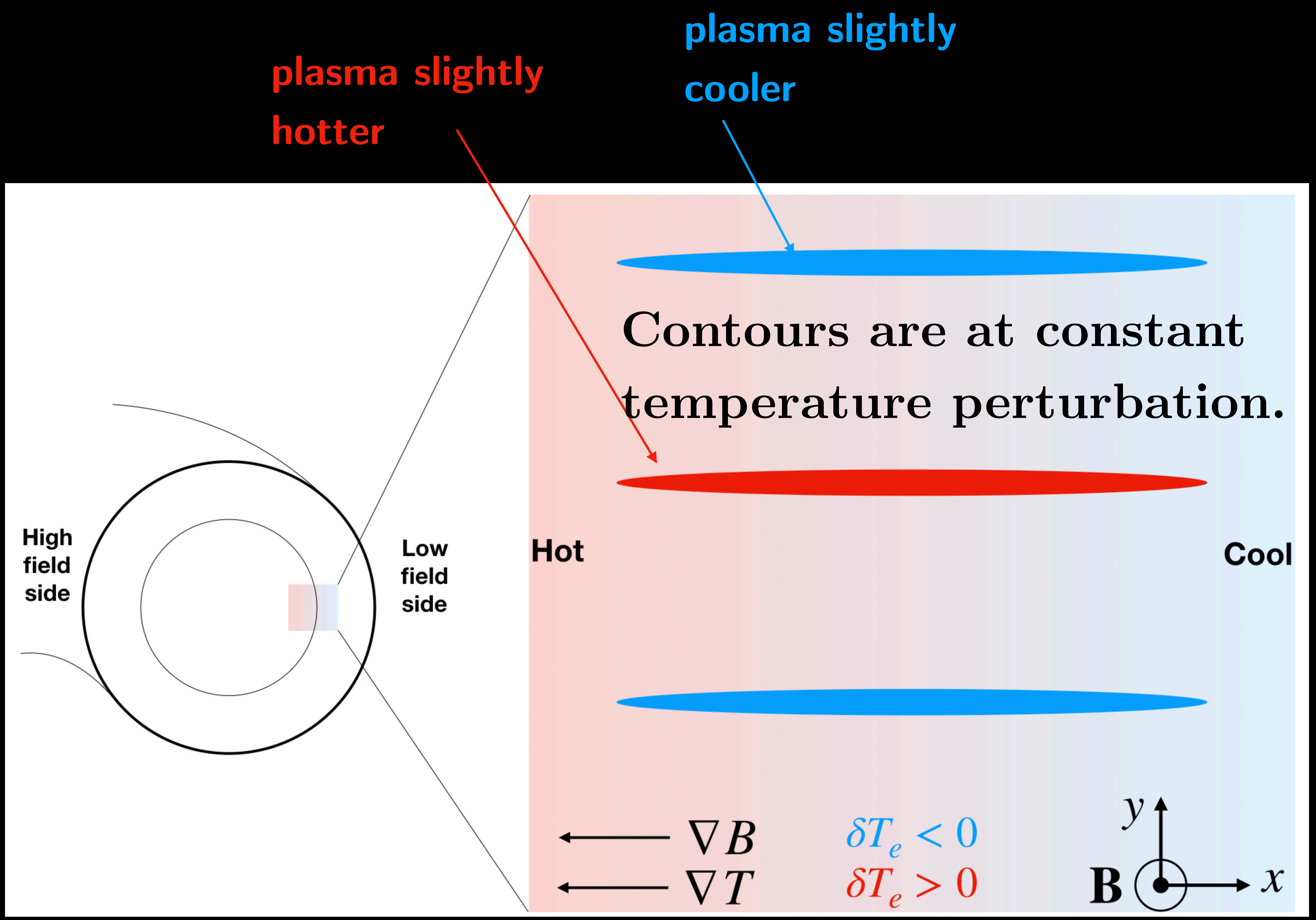


plasma slightly hotter

plasma slightly cooler

Contours are at constant temperature perturbation.

Electron-temperature-gradient (ETG) instability physical picture

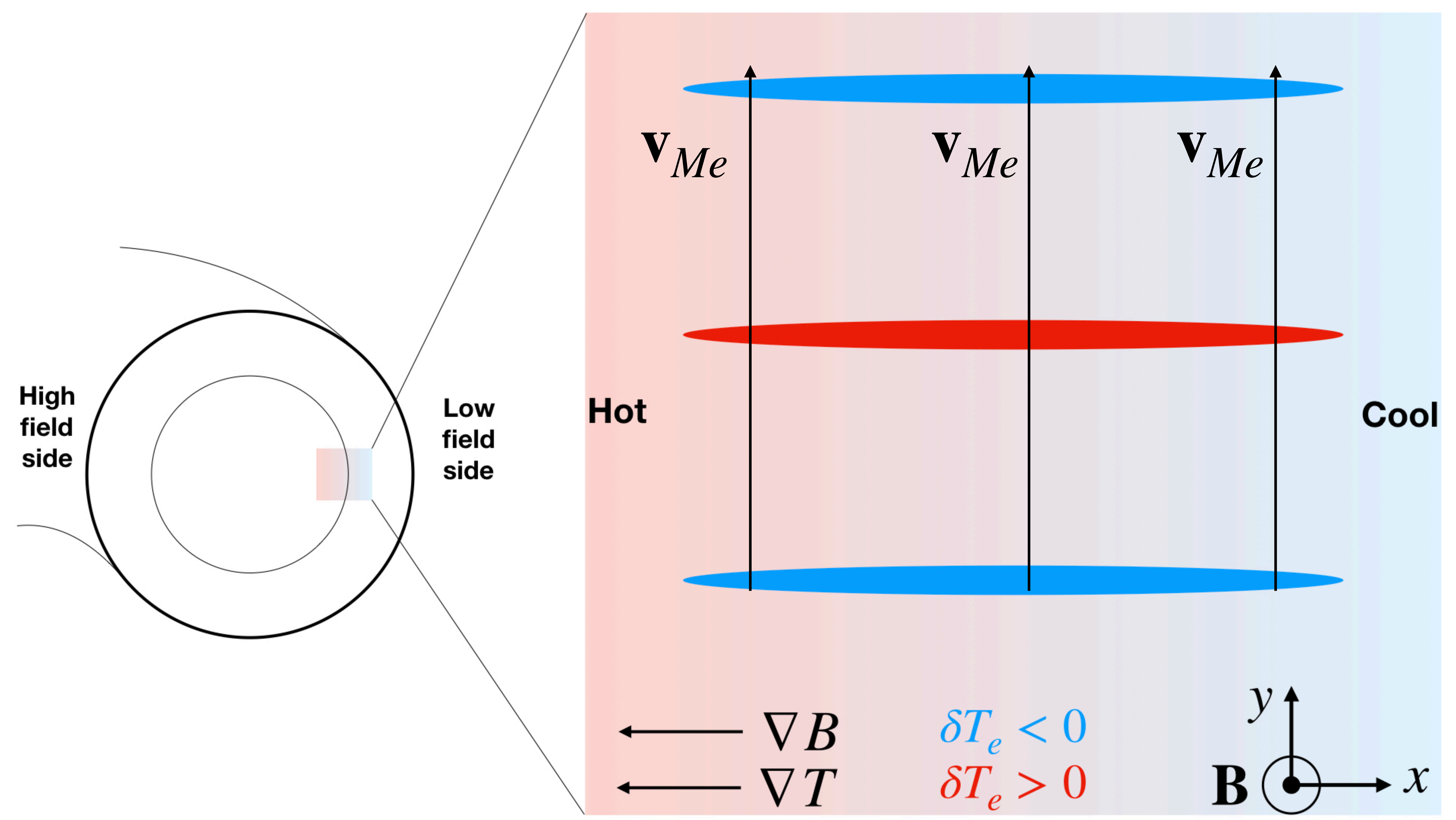
- Toroidal instability strongest at tokamak low field side.

Electrons undergo 'magnetic' drift due to magnetic field gradient ∇B

$$\mathbf{v}_{Me} \sim \frac{v_t^2}{\Omega_e} \mathbf{B} \times \nabla \mathbf{B}$$

Electron gyrofrequency

$\Omega_e = -eB/m_e$



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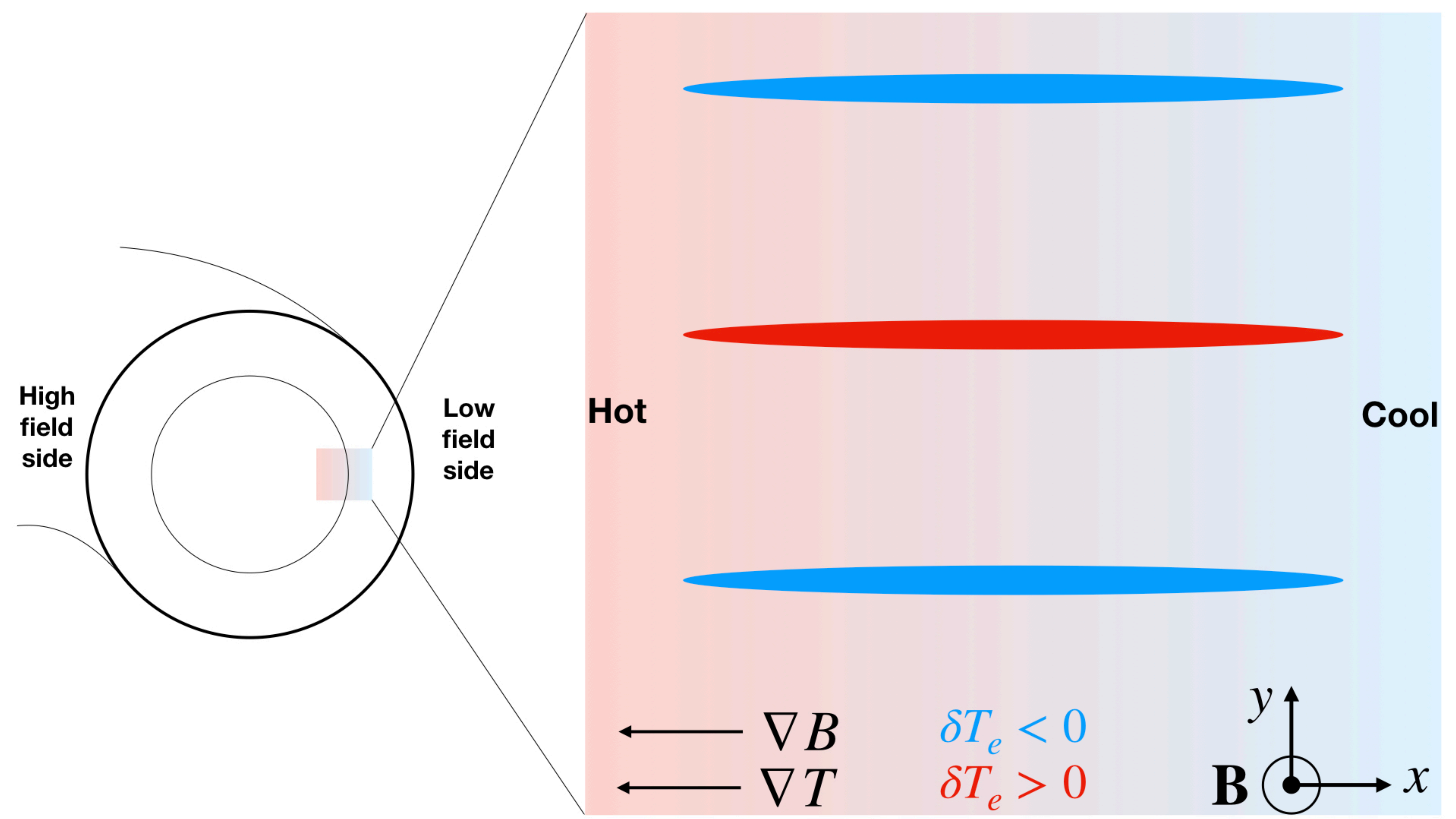
$\delta T_e \rightarrow$

perturbed electron magnetic drifts $\delta \mathbf{v}_{Me}$



Electron-temperature-gradient (ETG) instability physical picture

$\delta T_e \rightarrow$

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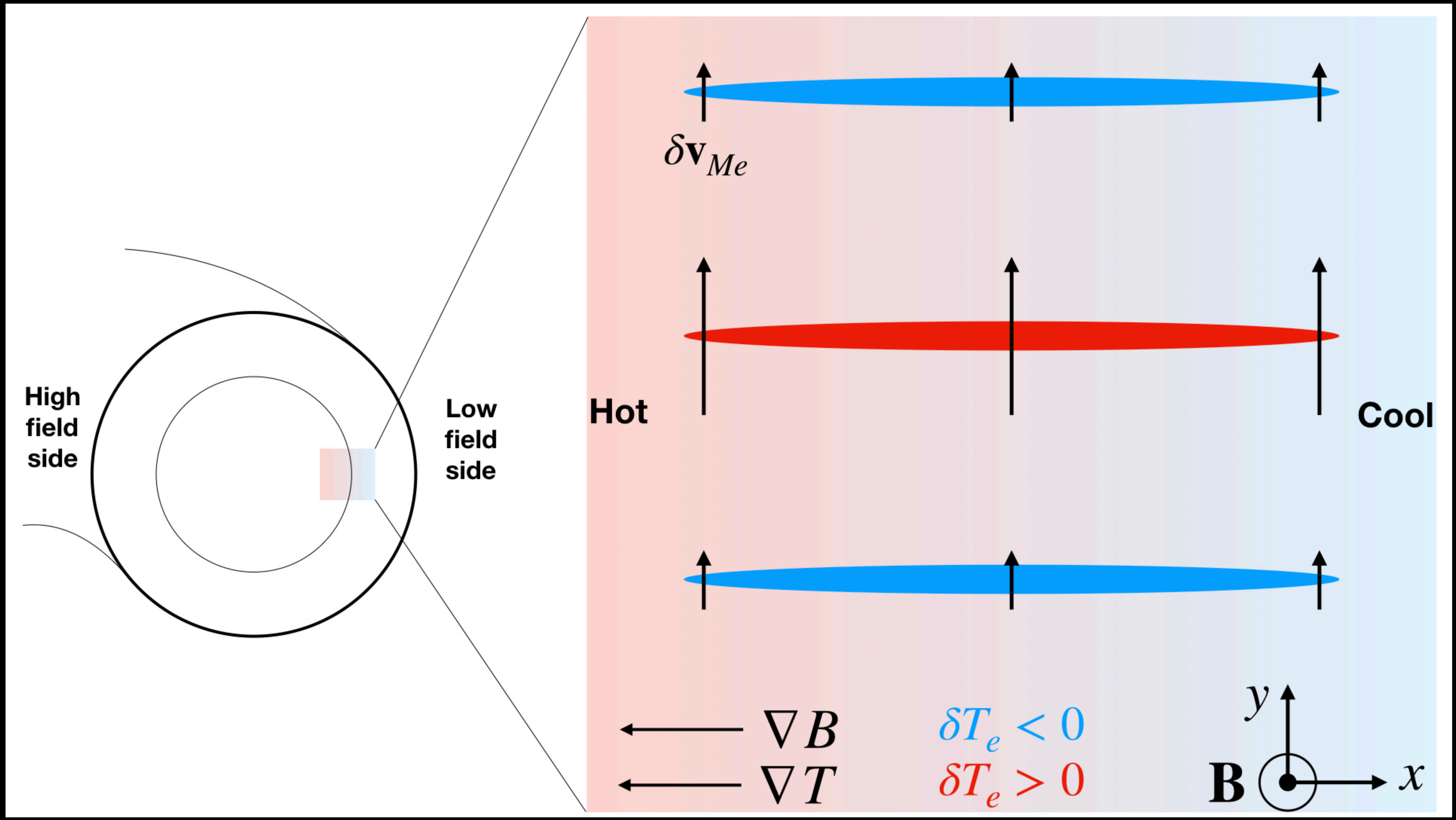
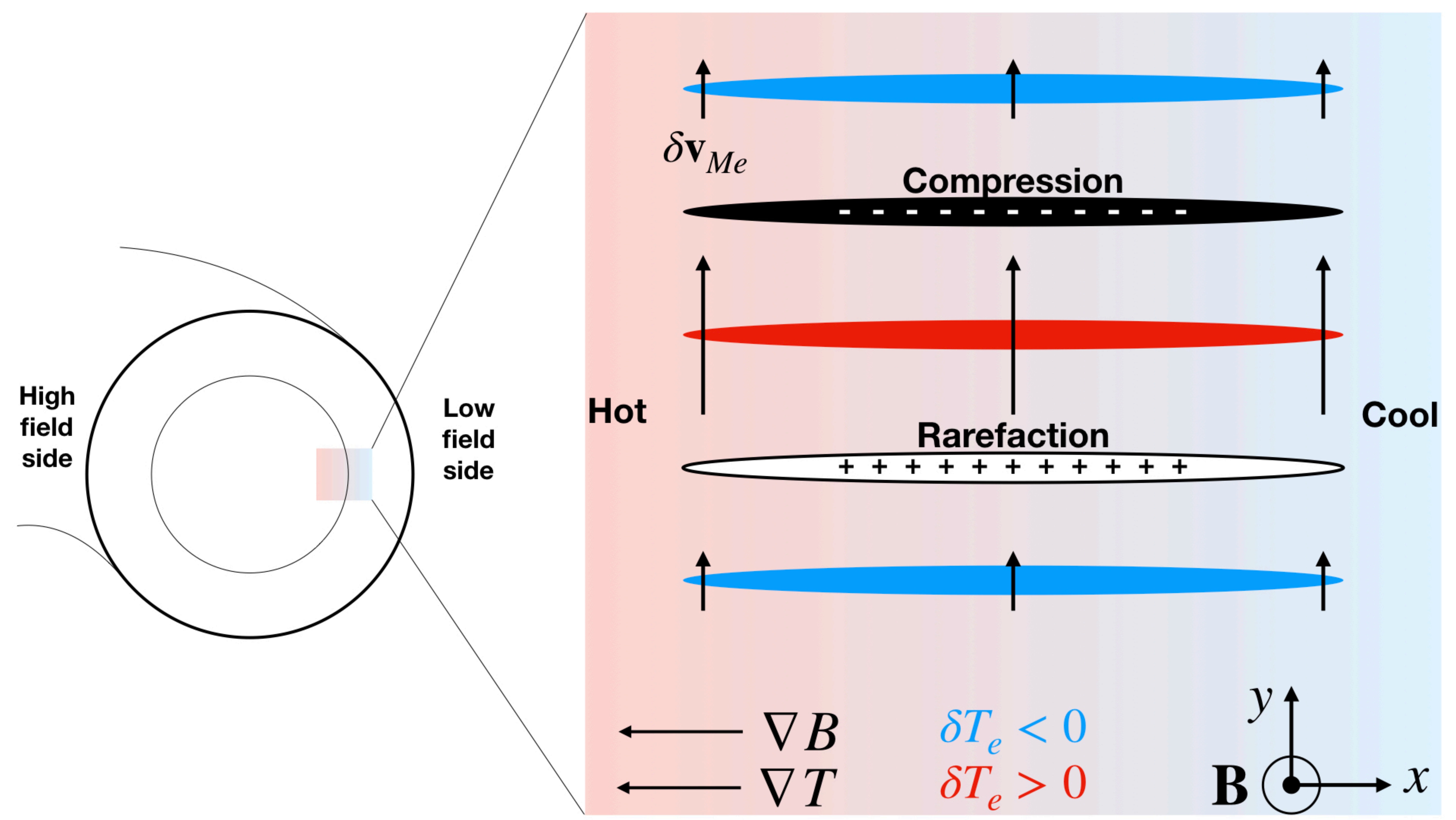


Electron-temperature-gradient (ETG) instability physical picture

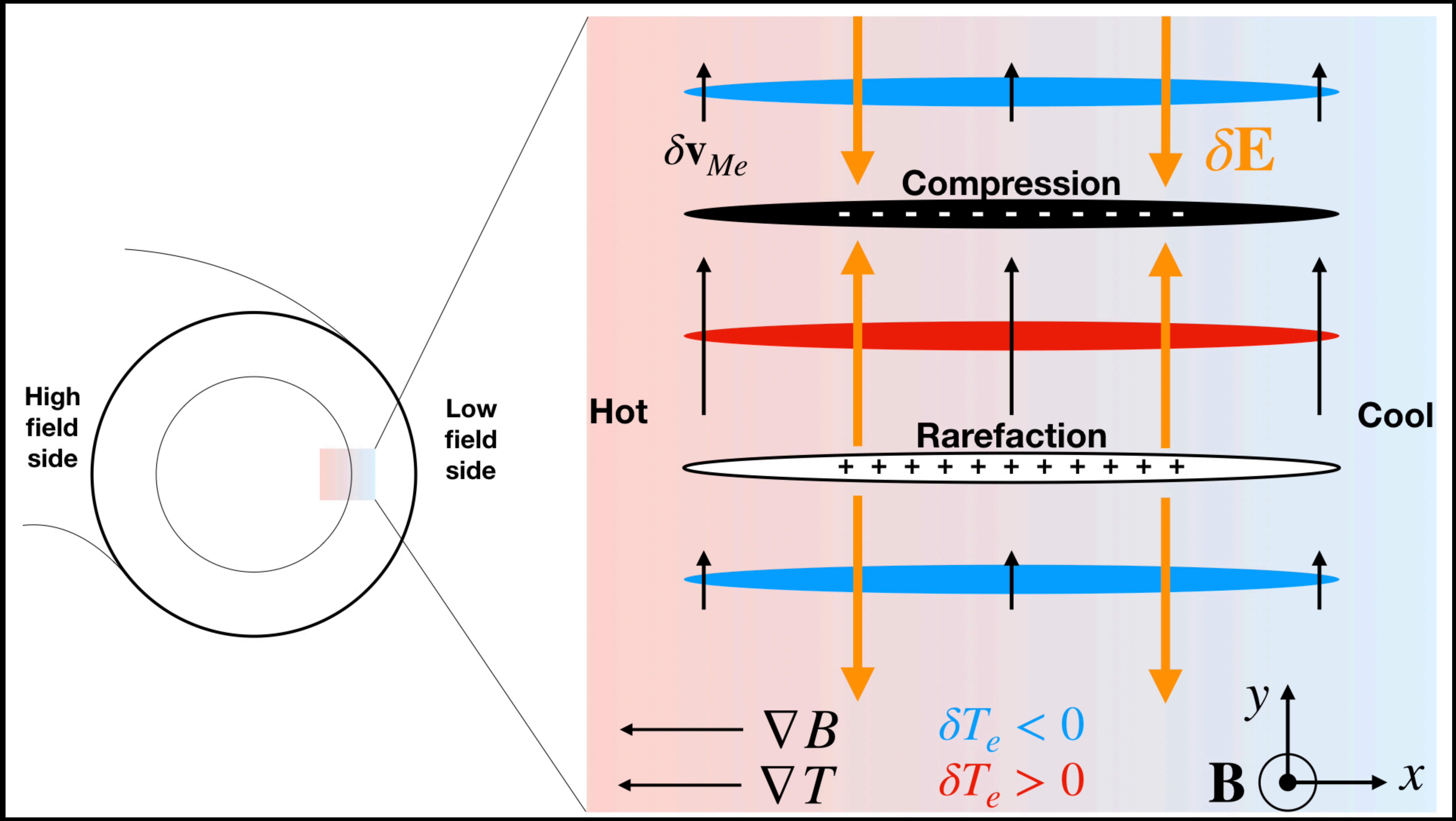
→ compression and rarefaction

→ charge accumulation



Electron-temperature-gradient (ETG) instability physical picture

$\rightarrow$ induce perturbed electric field $\delta\mathbf{E}$

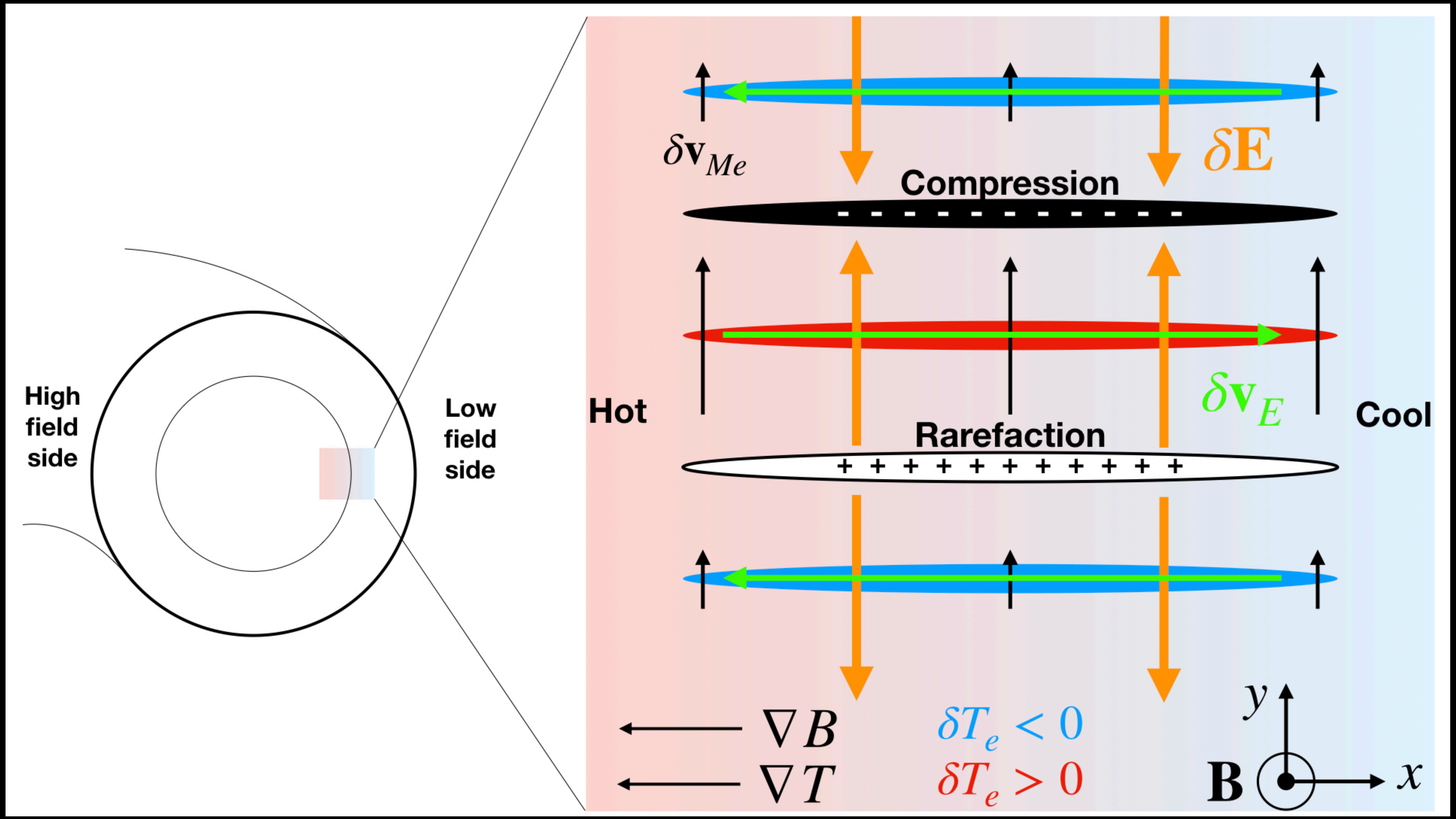


Electron-temperature-gradient (ETG) instability physical picture

→ cause $\delta \mathbf{v}_E$ (ExB drift).

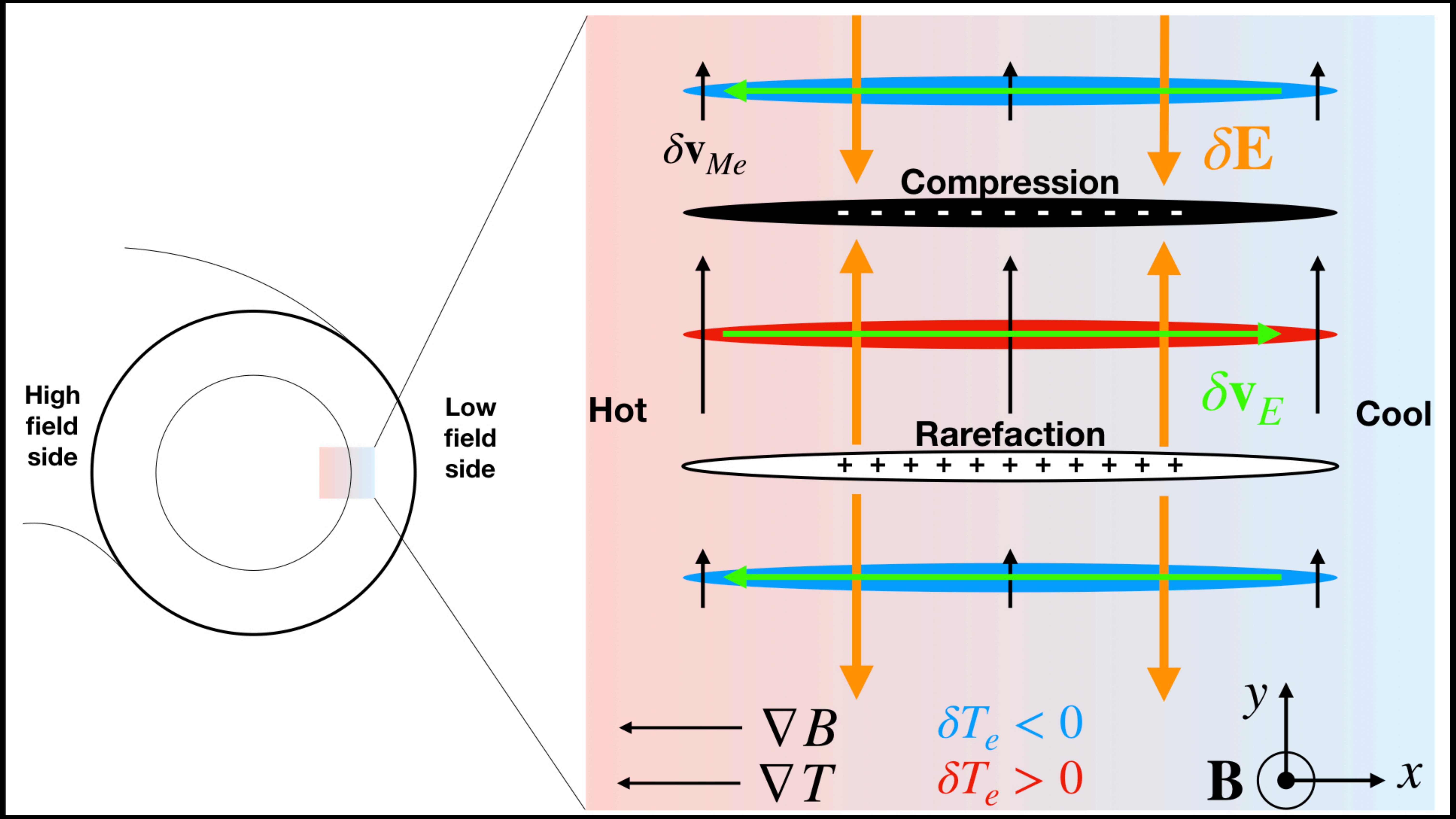
$\delta \mathbf{v}_E$ sucks hot plasma into $\delta T_e > 0$,

cold plasma into $\delta T_e < 0$,



Electron-temperature-gradient (ETG) instability physical picture

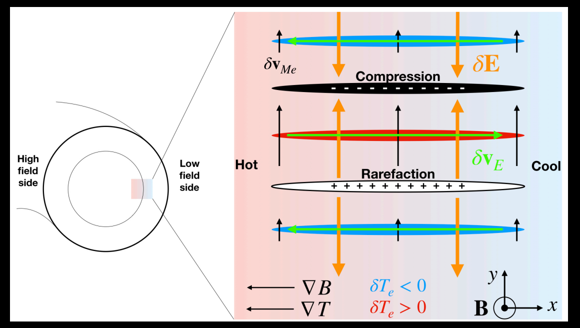
$\delta\mathbf{v}_E$ reinforces temperature perturbation, which causes a feedback loop —>**instability!**



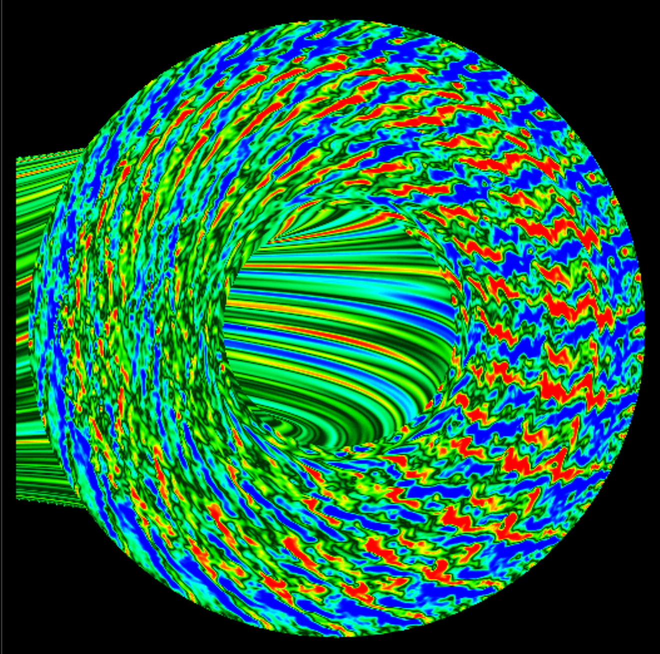
Turbulence and Transport in Magnetic Confinement Fusion

Recap

Plasma instabilities such as electron-temperature-gradient modes give rise to turbulence



→



Credit: https://w3.pppl.gov/~hammett/collaborators/mbeer/

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Recap

Plasma instabilities such as electron-temperature-gradient modes give rise to turbulence

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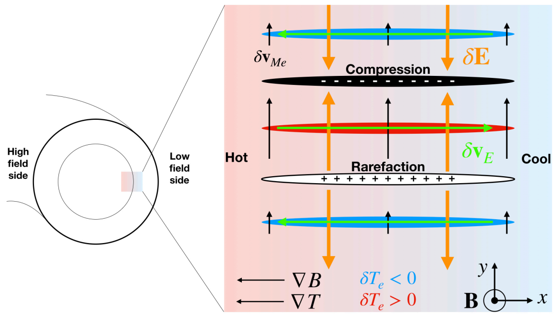
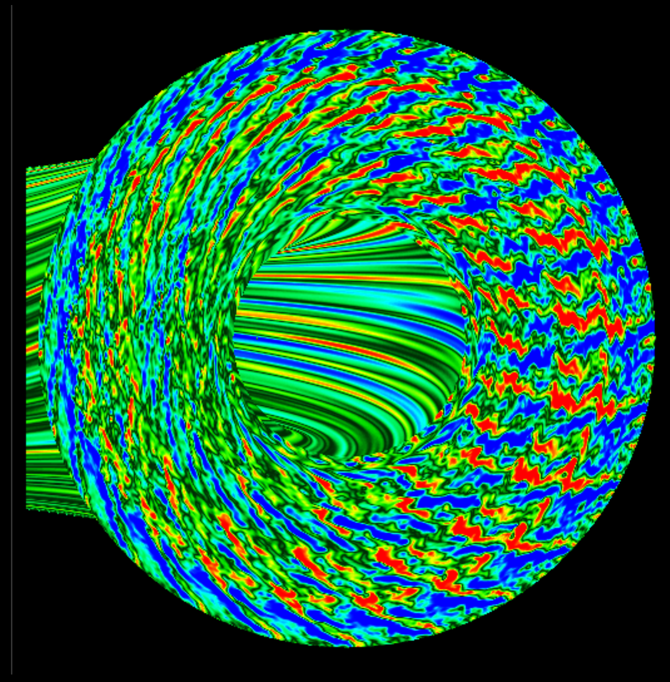
Turbulent eddies cause high heat transport, resulting in shorter plasma confinement times and lower core temperature





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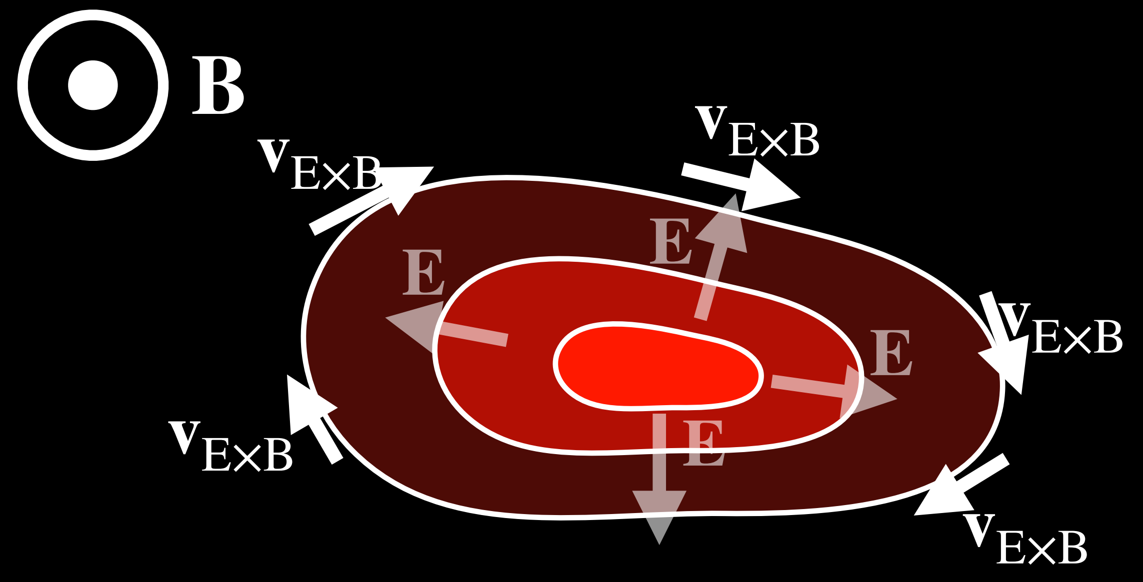

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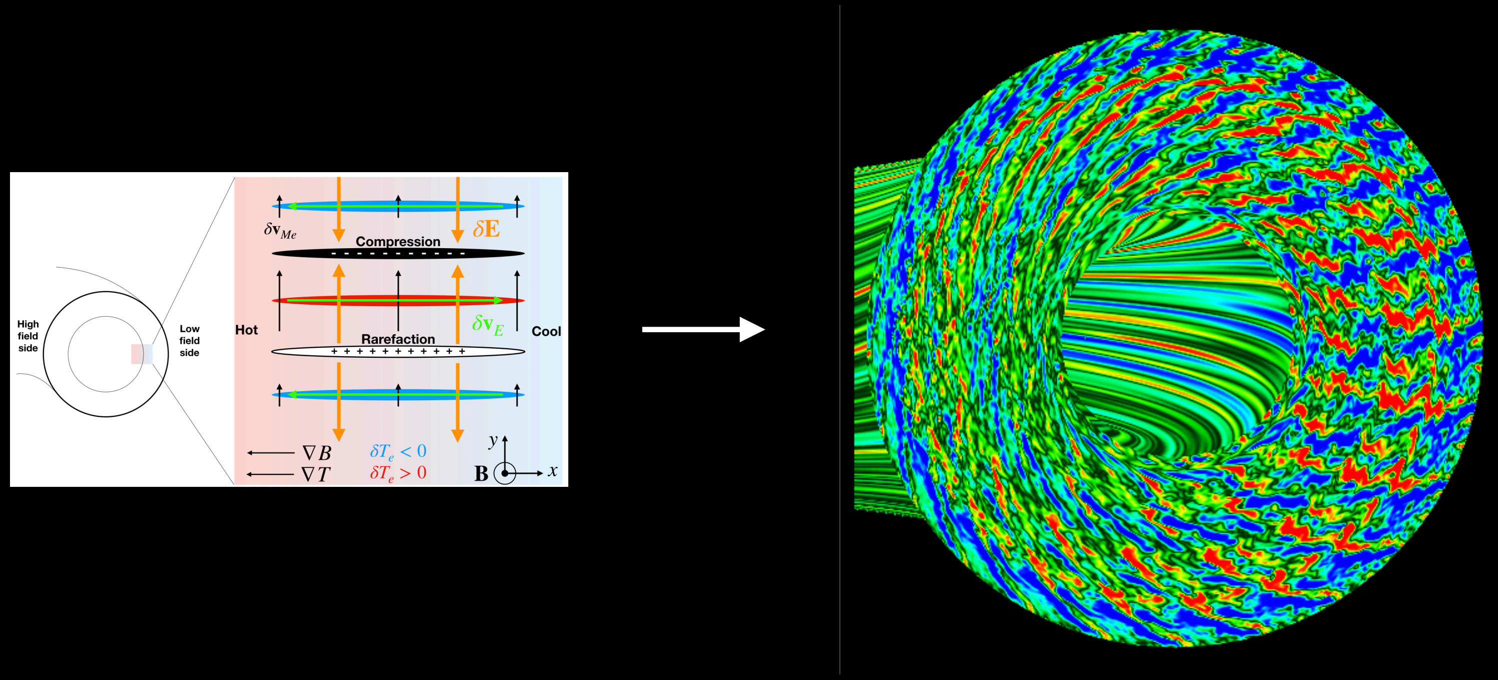

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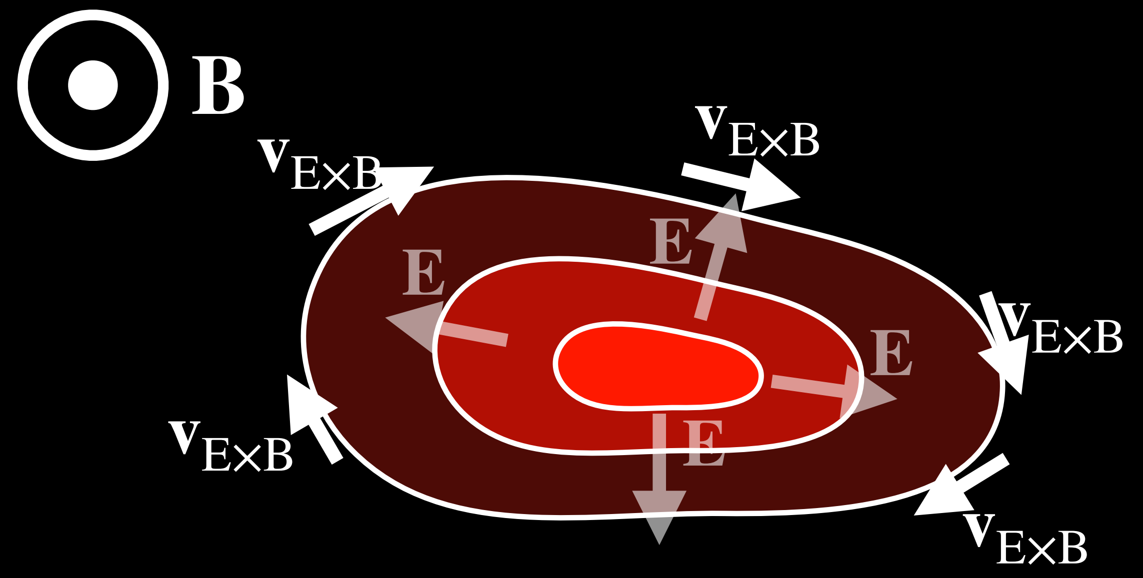
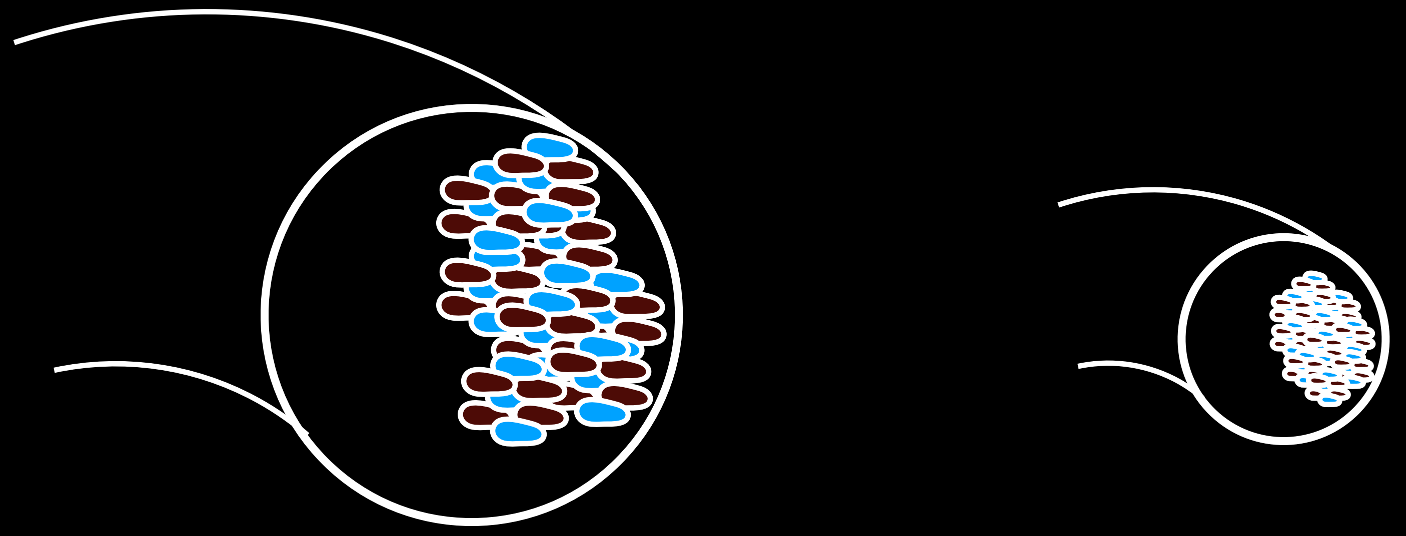


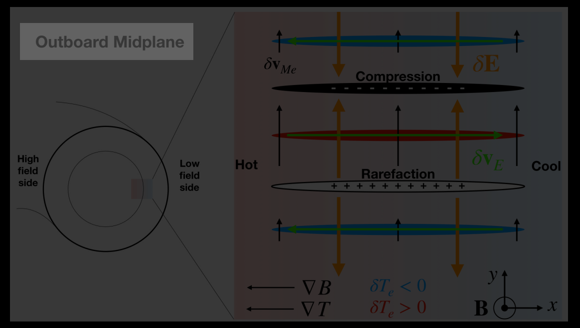
Fusion experiments need to be larger, or reduce $\rho_* = \rho/L$

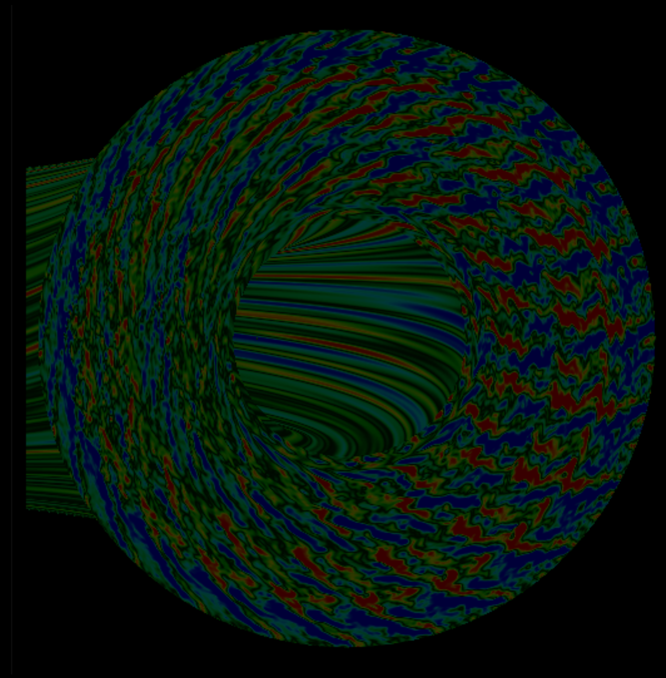


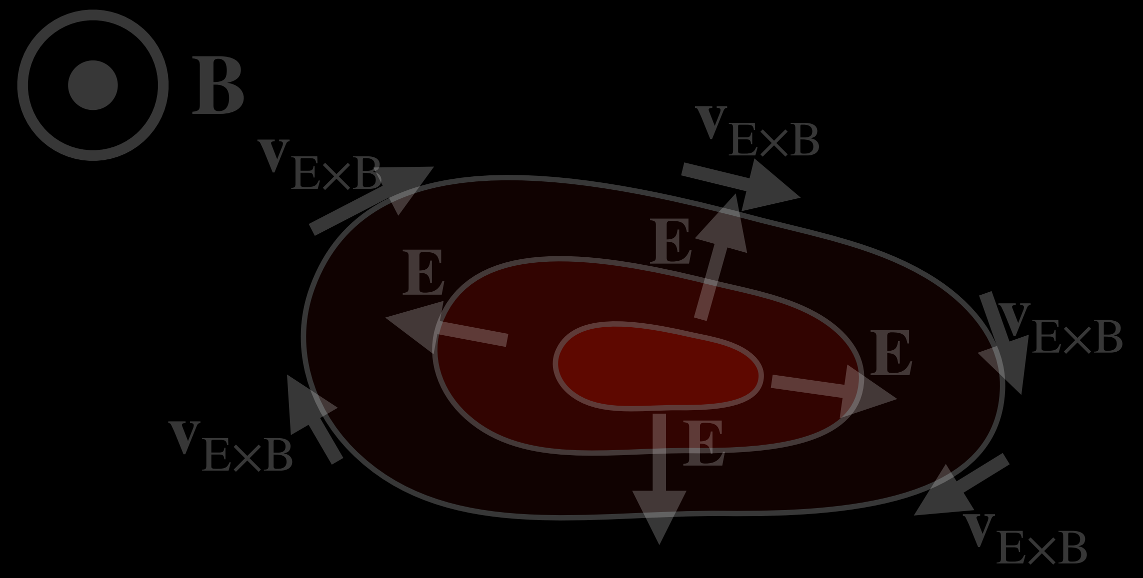
Bigger L Smaller ρ_*

Turbulence and Transport in Magnetic Confinement Fusion

Recap







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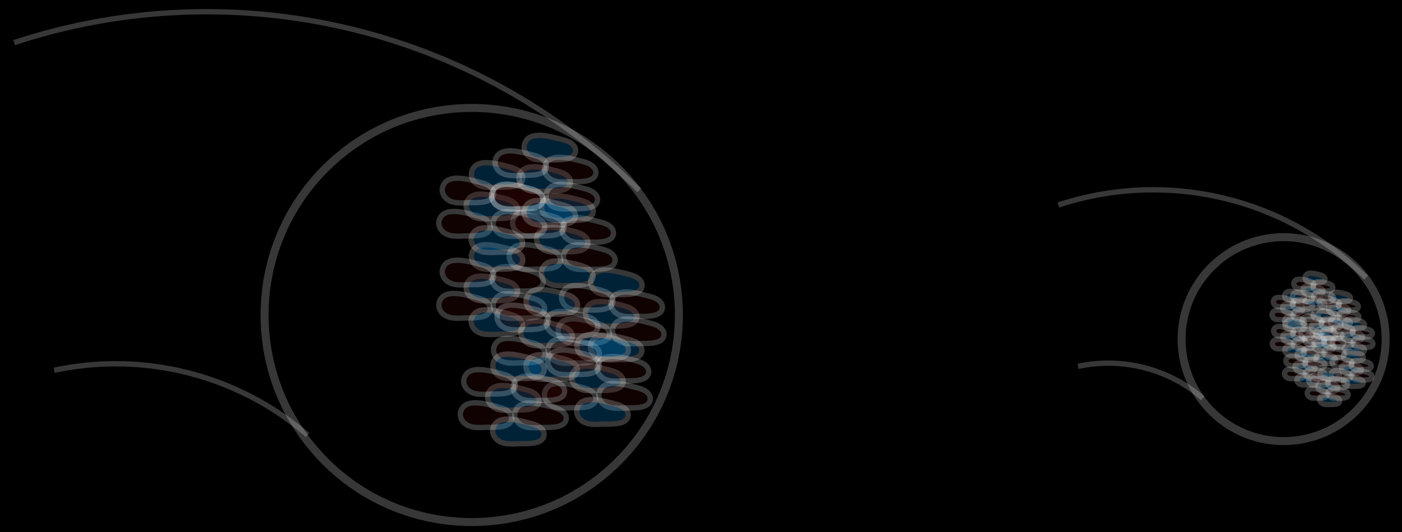

‡… or, try other tricks to stabilize the plasma!

Turbulence and Transport in Magnetic Confinement Fusion

Tricks for reducing turbulence!

- **Rotational shear:** spatial shear in plasma rotation can shear apart eddies, reducing transport
- **Magnetic geometry:** plasma shaping has significant impact on transport coefficients
- **Turbulence-generated flows:** turbulent flows self-generated by plasma can often improve transport
- **H-mode:** plasma phase transition from a low confinement mode (L-mode) to high confinement mode (H-mode)

Turbulence and Transport in Magnetic Confinement Fusion

Other important topics in turbulence and transport

- Existence and smoothness of Navier-Stokes equations
- Astrophysical turbulence
- Fusion turbulence diagnostics and measurements
- Wide range of plasma instabilities!
- Turbulent particle transport, coupling with neoclassical effects, fast particle modes
- Beneficial effects of turbulence?

Summary

- Turbulence theory describes how energy cascades from large injection scale to small dissipative scale
- Turbulent heat transport in fusion experiments driven by plasma instabilities
- Many observations still unexplained
- Reason for optimism: small improvements in plasma physics confinement → significant improvements in power plant performance!
- Turbulence might not all be bad