Stellarators

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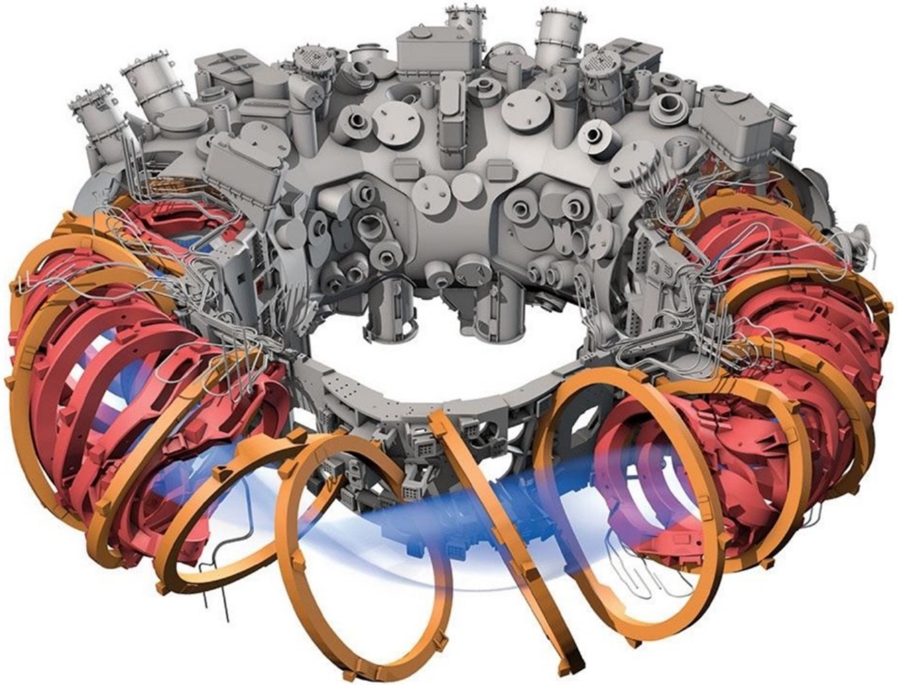
PPPL SULI Introduction to Plasma and Fusion Course, June 2026

About me

- Grew up in Saratoga Springs, NY
- Hobbies: track, cross-country, trombone
- Wanted (and still want) to do something about climate change and decarbonization
- 2008-2012: majored in physics at Harvard
    - 2011: SULI internship @ PPPL!
- 2012-2017: PhD in applied physics at Columbia University
- 2017-2019: postdoc at the Max Planck Inst. for Plasma Physics, Greifswald, Germany
- 2019-present: research physicist, PPPL
    - Still collaborating with IPP Greifswald
    - Also work on magnet design and optimization

Introduction

What is a stellarator?



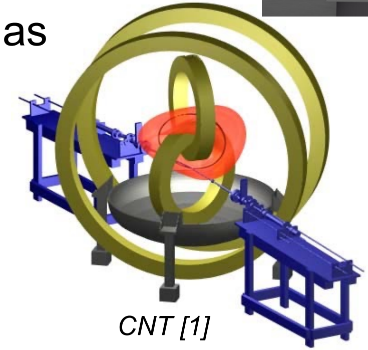
The Wendelstein 7-X Stellarator

Max Planck Institute for Plasma Physics, Greifswald, Germany
Image: T. Klinger et al., Nucl. Fusion 2019

- Toroidal magnetic plasma confinement device
- Magnetic field forms nested, closed flux surfaces
- Magnetic field is generated primarily by magnets external to the plasma
  - Plasma current not required

Why build a stellarator?

- Generate electricity
  - Inherently steady-state operation
  - Low/no plasma current required
    - Lower vulnerability to disruptions
    - Lower recirculating power required
- Perform basic plasma research
  - Single-species plasmas
  - Pair (positron/electron) plasmas



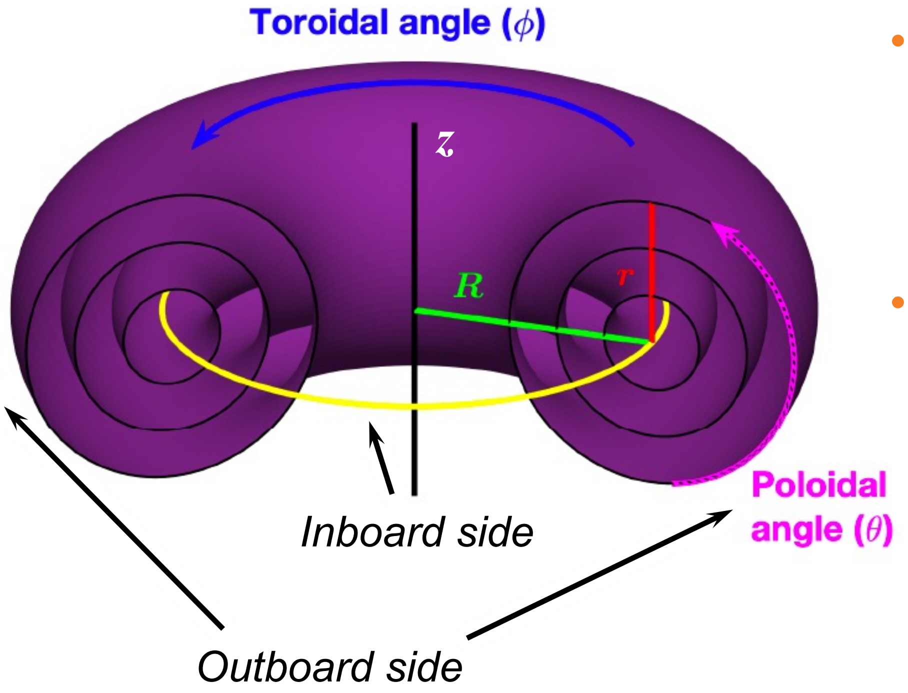
CNT [1]

S. Woodruff, Woodruff Scientific

[1] J. P. Kremer et al., *Phys. Rev. Lett.* **97**, 095003 (2006)

Toroidal geometry and magnetic fields

Basics of toroidal geometry

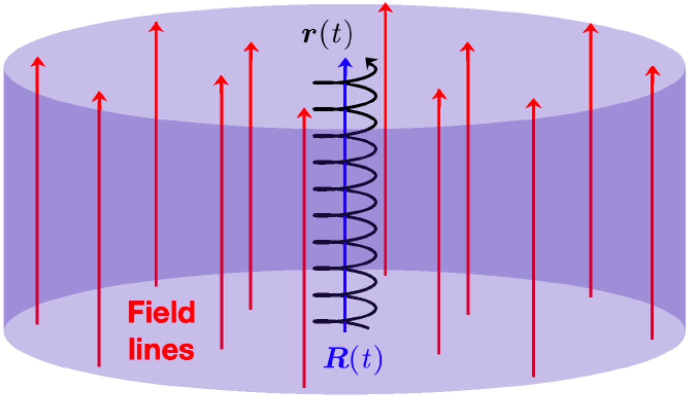


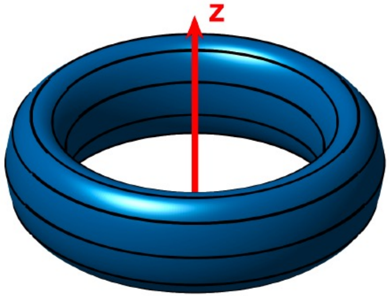
- Torus: a doughnut shape
- Cylindrical representation
  - *R*: radial dimension; distance from *z axis*
  - *z:* vertical dimension
  - ***ϕ***: azimuthal/toroidal dimension
- Toroidal coordinates
  - *r:* minor radius; distance from yellow curve
  - ***θ***: poloidal angle; short way around the torus
  - ***ϕ***: toroidal angle; long way around the torus

[2] L.-M. Imbert-Gérard et al., *Introduction to Stellarators*, SIAM (2025)

Toroidal magnetic confinement: motivation

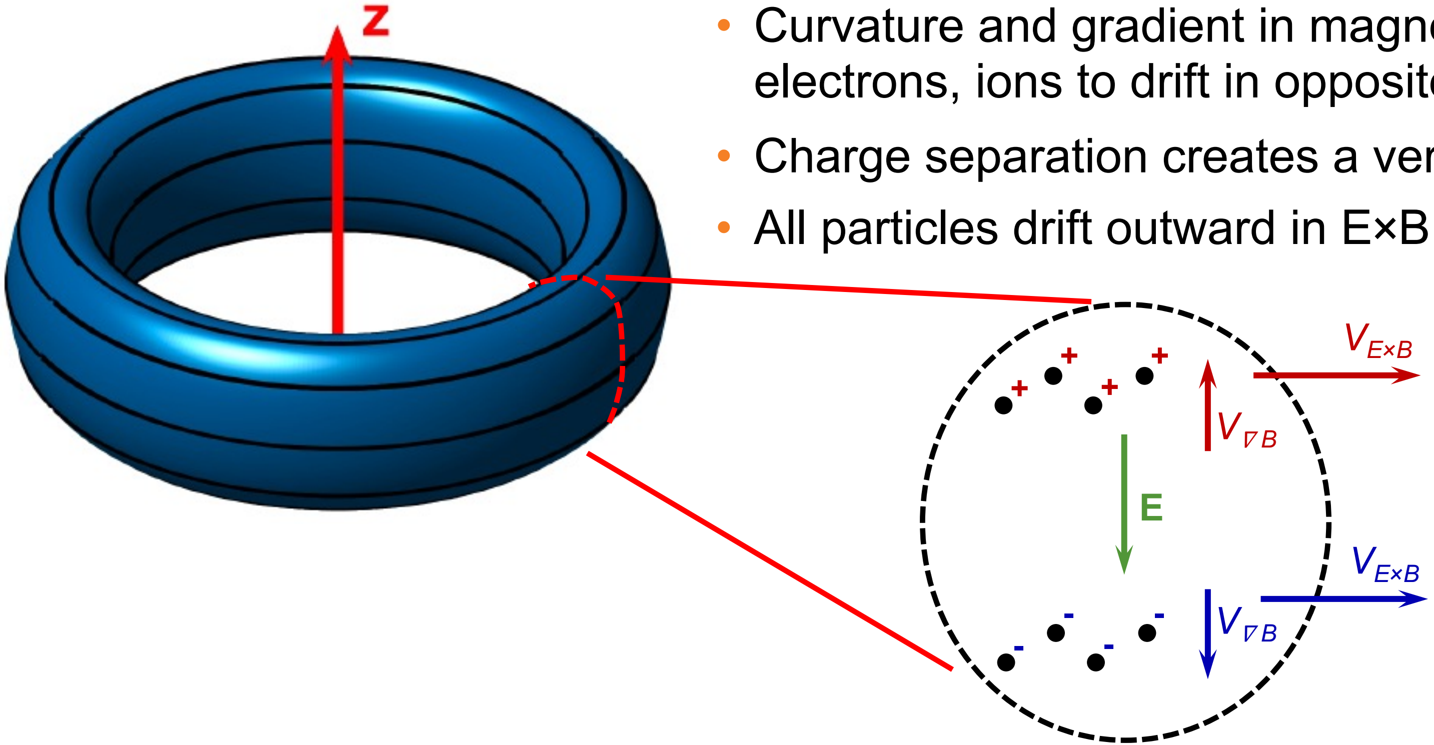
- Charged particles tend to gyrate around magnetic field lines
  ⟶ confined in the direction perpendicular the magnetic field **B**
- Particles can stream freely along magnetic field lines
  ⟶ not intrinsically confined parallel to **B**
- If the field lines wrap around form a torus
  - Particles stream quickly around the torus
  - Particles are slower to exit the torus
  - Enclosing wall is protected from the plasma (and vice versa)





[2] L.-M. Imbert-Gérard et al., *Introduction to Stellarators*, SIAM (2025)

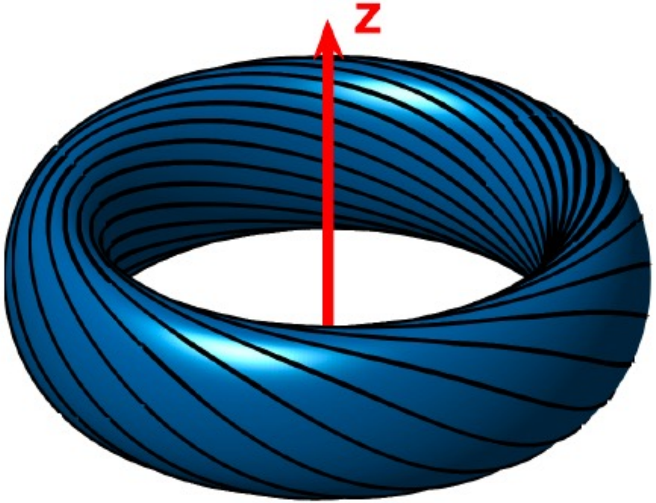
A purely toroidal field is not sufficient to confine a quasineutral plasma



- Curvature and gradient in magnetic field cause electrons, ions to drift in opposite directions
- Charge separation creates a vertical electric field
- All particles drift outward in E×B direction

[2] L.-M. Imbert-Gérard et al., *Introduction to Stellarators*, SIAM (2025)

To confine a plasma in a toroidal field, the field lines must twist helically



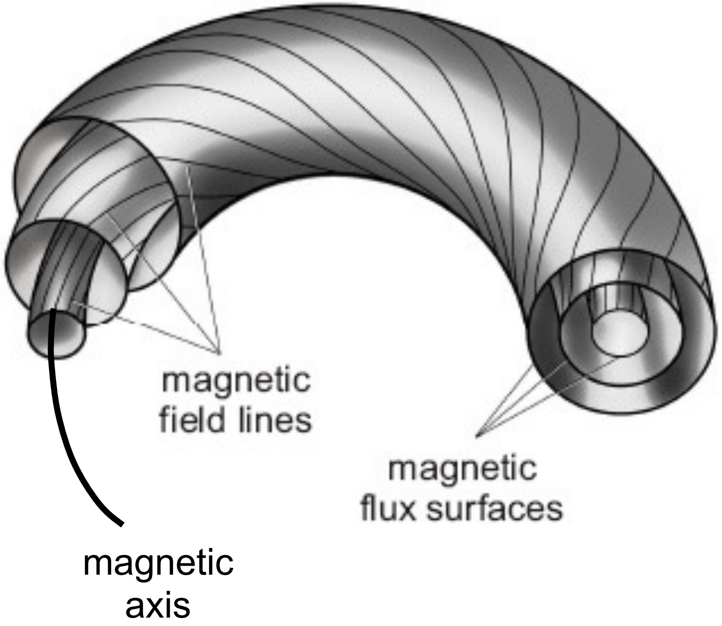
L.-M. Imbert-Gérard et al. [2]

iStockPhoto

- Helical field lines sweep particles quickly from the top to the bottom of the torus and back
- Vertical drift motion persists, but alternates between inwards and outwards
- Sending the particles on helical paths is analogous to turning a honey dipper
  - Holding the dipper still: honey drips off
  - Rotating the dipper: honey remains confined

[2] L.-M. Imbert-Gérard et al., *Introduction to Stellarators*, SIAM (2025)

Flux surfaces and rotational transform



M. Bernert [3]

- For good confinement, magnetic field lines should form nested *flux surfaces*
  - Surfaces surround a central curve: *magnetic axis*
- *Rotational transform, ι* is a measure of helical twistiness of the field on each flux surface
  - In tokamaks: the *safety factor q* is $1/\iota$
- Generating rotational transform
  - Tokamaks use a toroidal plasma current
  - Stellarators use external coils/magnets

[3] M. Bernert, PhD Thesis, LMU München (2013)

How to generate a stellarator magnetic field

Origins of the stellarator

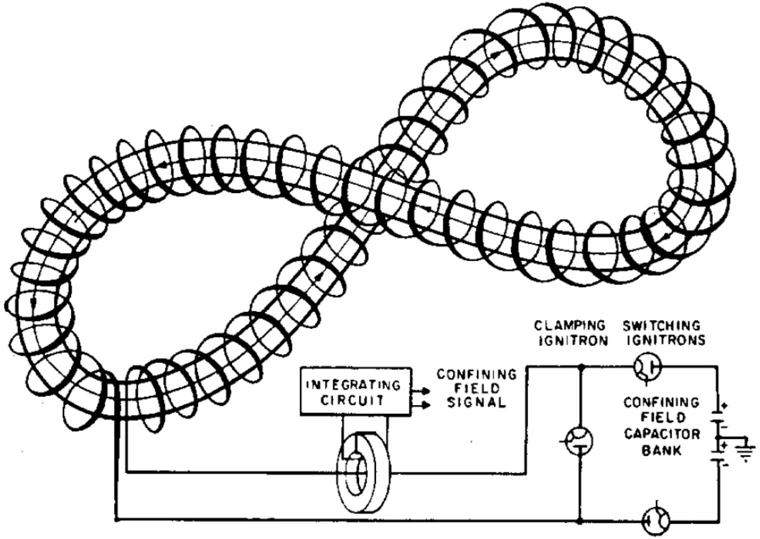
- Invented in the 1950s by Princeton astronomy professor Lyman Spitzer, Jr.
- Developed as part of Project Matterhorn, which eventually became the Princeton Plasma Physics Laboratory

Lyman Spitzer with the Model A stellarator

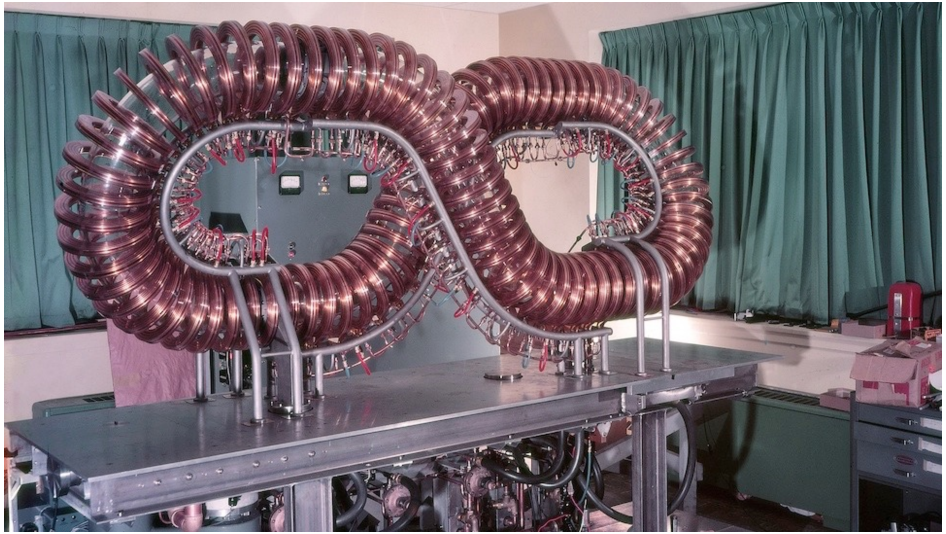
Image: PPPL

How can external coils generate rotational transform?

- Spitzer's approach: solenoidal tube twisted into a figure-8
  - Crucial feature: *torsion*, or the departure of the tube's axis from a single plane



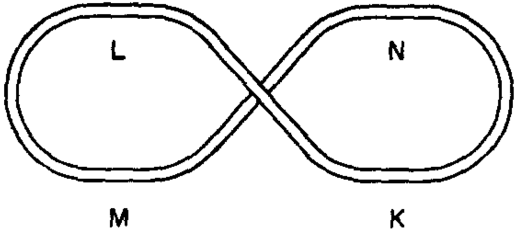
T. Coor et al. [4]



PPPL file photo

[4] T. Coor et al., *Phys. Fluids* **1**, 411 (1958)

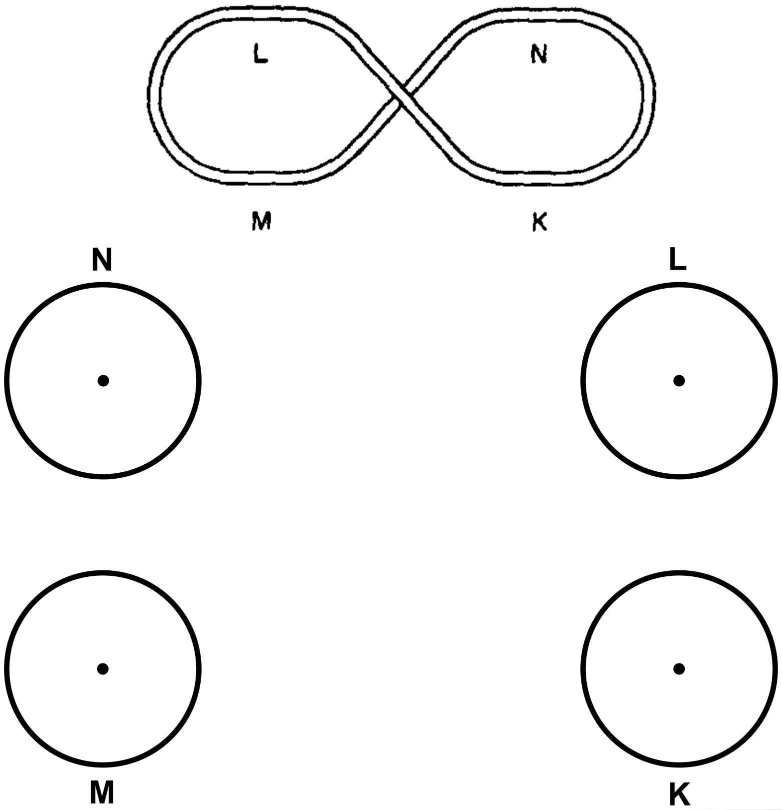
Rotational transform in a figure-8 solenoid [5]



- Magnetic field in the figure-8 is approximately *solenoidal*, i.e. everywhere parallel to the central axis of the tube

[5] L. Spitzer, *Phys. Fluids* **1**, 253 (1958)

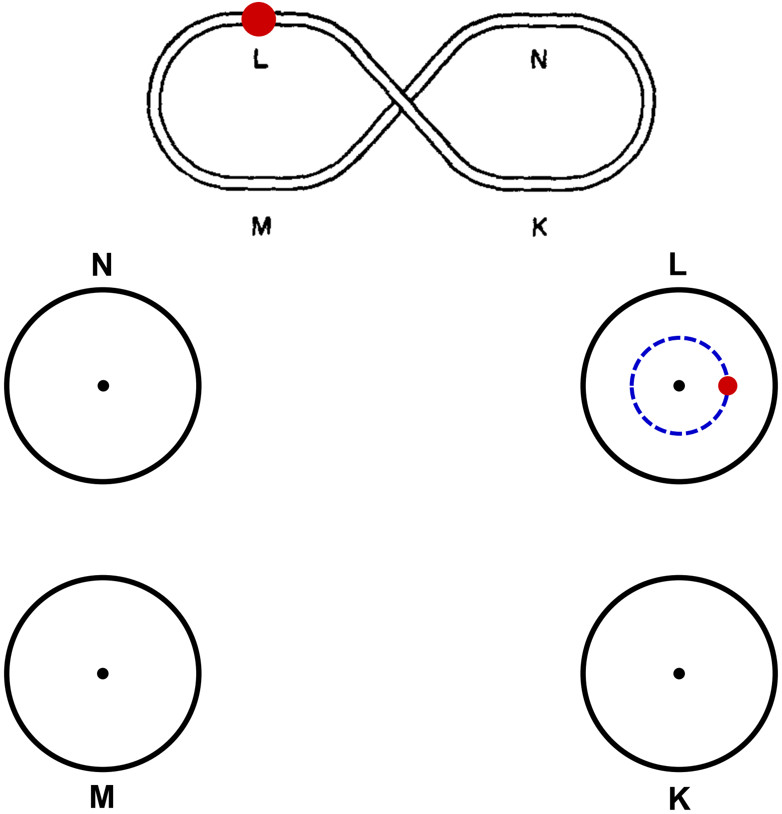
Rotational transform in a figure-8 solenoid



- Magnetic field in the figure-8 is approximately *solenoidal*, i.e. everywhere parallel to the central axis of the tube

[5] L. Spitzer, *Phys. Fluids* **1**, 253 (1958)

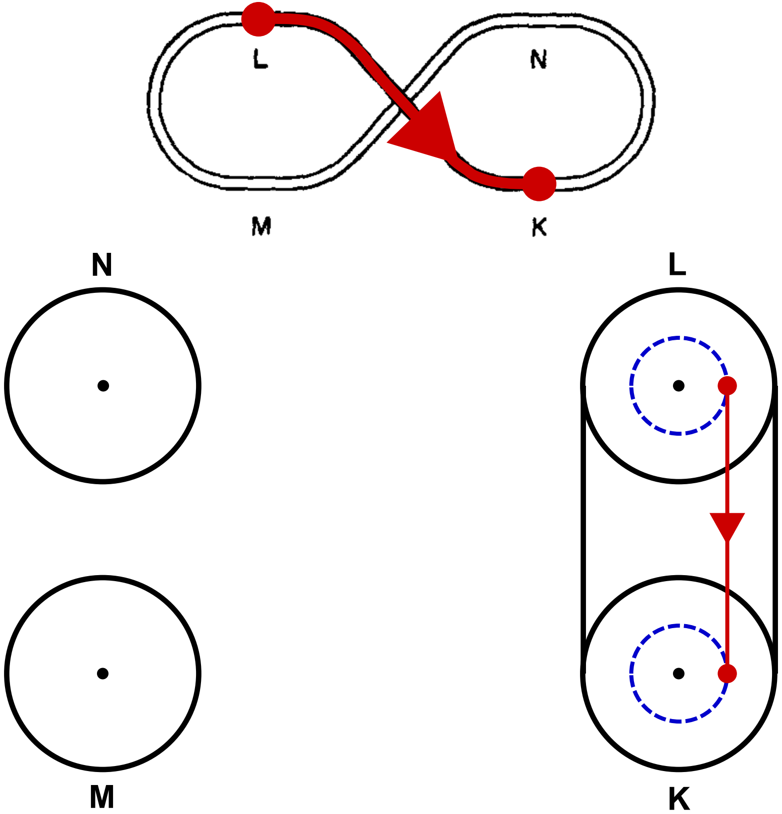
Tracing the field line around the track



- Initialize a field line at at point in cross-section *L* a distance *r* from the central axis

[5] L. Spitzer, *Phys. Fluids* **1**, 253 (1958)

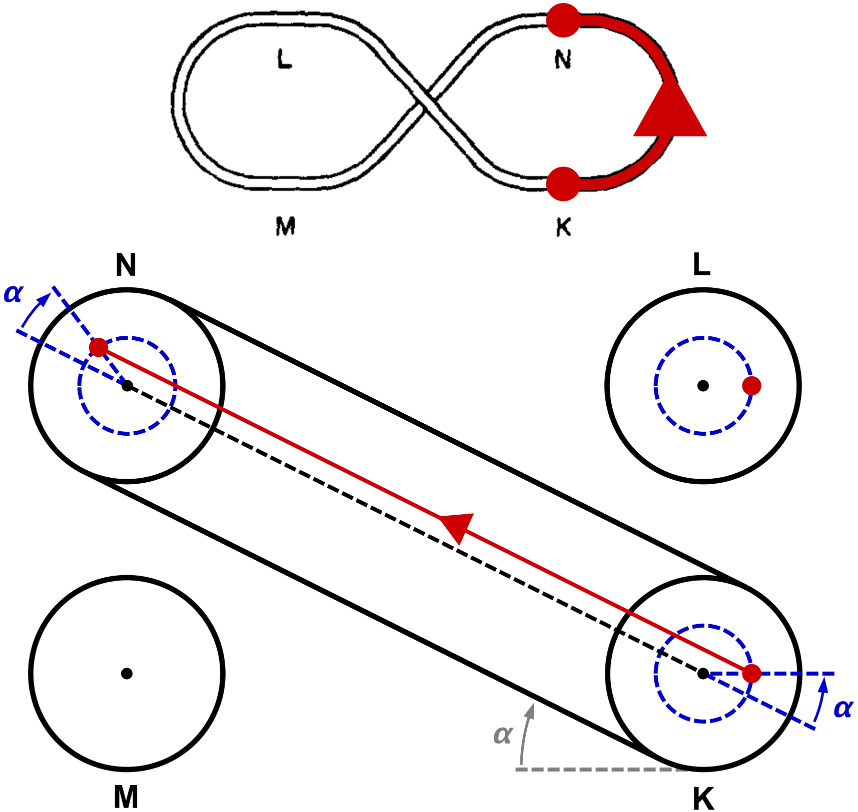
First “downhill” (*L*-*K*)



- Field line trajectory
  - Remains on vertical plane, at “3:00” with respect to magnetic axis

[5] L. Spitzer, *Phys. Fluids* **1**, 253 (1958)

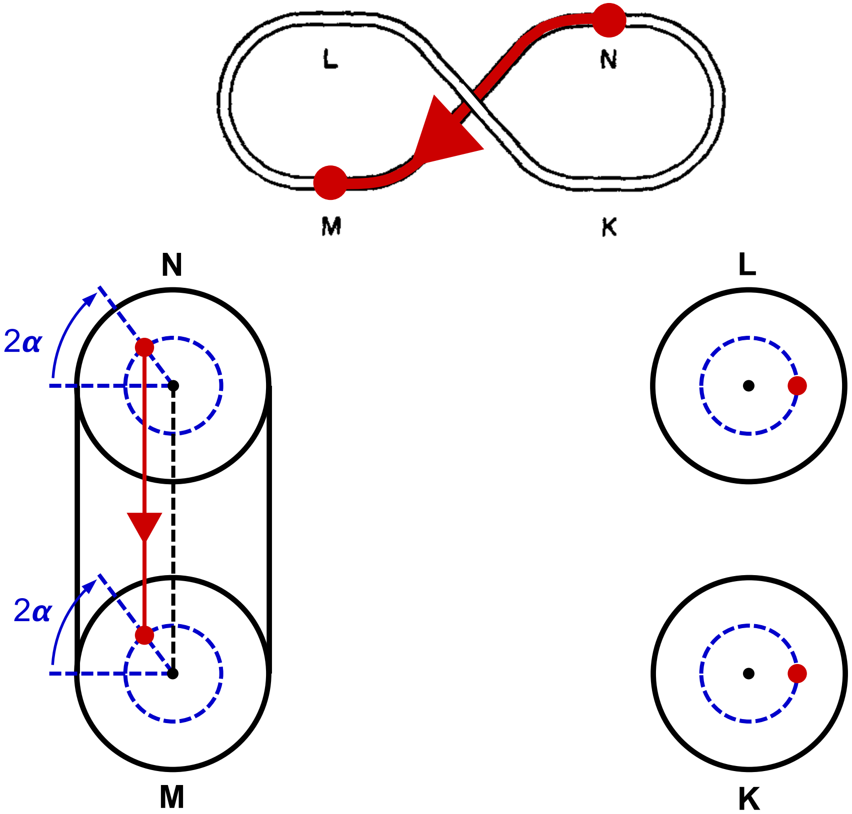
First upward semicircle bend (*K*-*N*)



- Field line trajectory
  - Travels parallel to the plane of curve (tilted an angle ***α*** to the horizontal)
  - Remains at an angle ***α*** above the plane of the curve relative to magnetic axis

[5] L. Spitzer, *Phys. Fluids* **1**, 253 (1958)

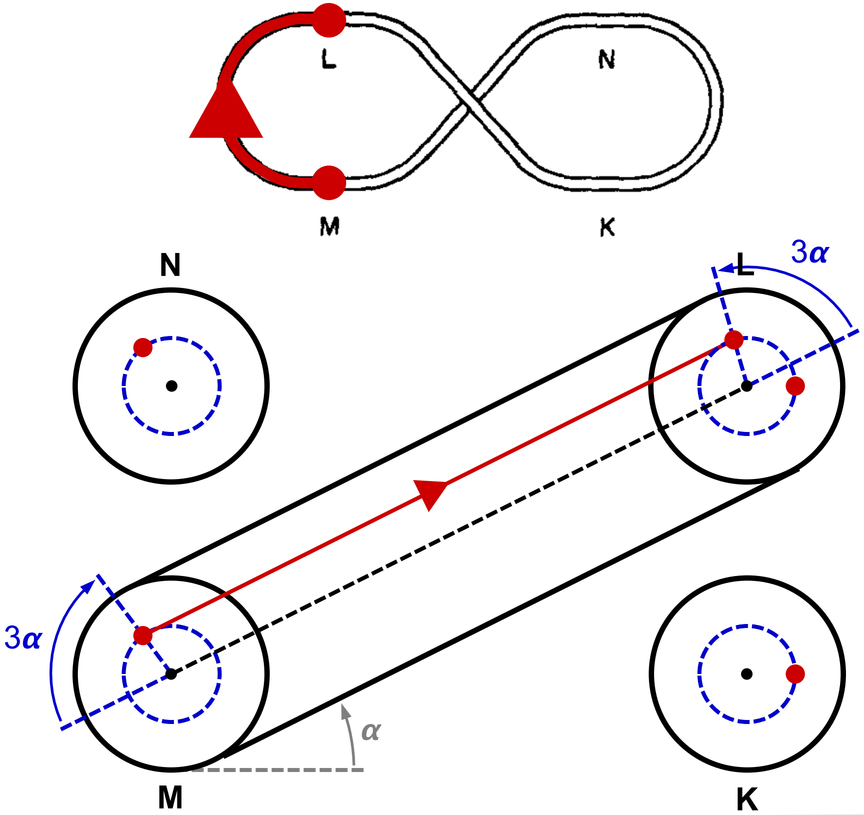
Second “downhill” (*N*-*M*)



- Field line trajectory
  - Remains on vertical plane at an angle 2***α*** above the horizontal plane about the magnetic axis

[5] L. Spitzer, *Phys. Fluids* **1**, 253 (1958)

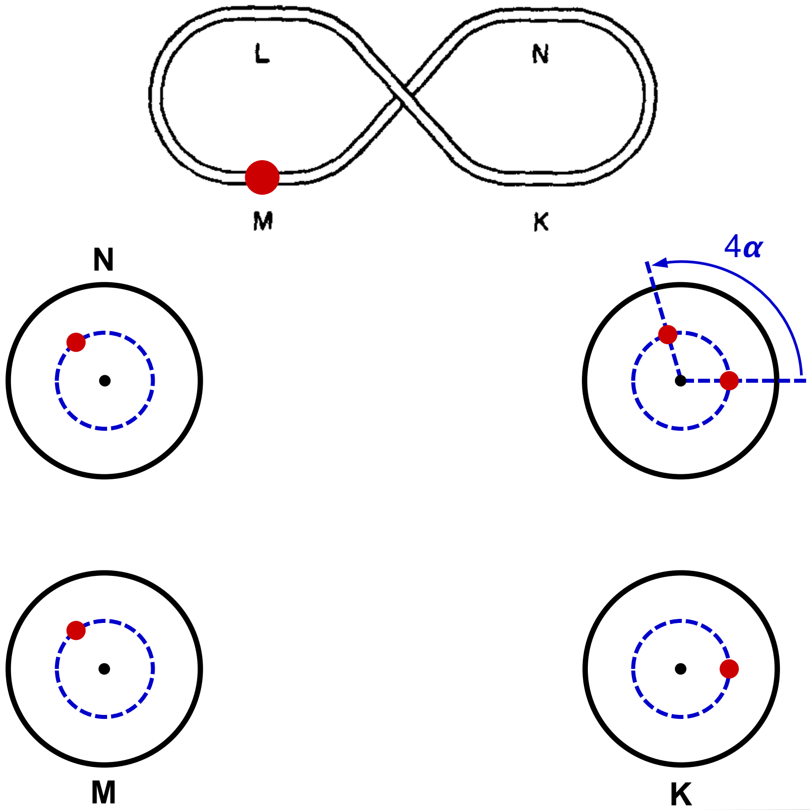
Second upward semicircular bend (*M*-*L*)



- Field line trajectory
  - Travels parallel to the plane of curve (tilted an angle α to the horizontal)
  - Remains at an angle 3α above the plane of the curve relative to the magnetic axis

[5] L. Spitzer, *Phys. Fluids* **1**, 253 (1958)

Field line has rotated poloidally upon its return to the starting plane!

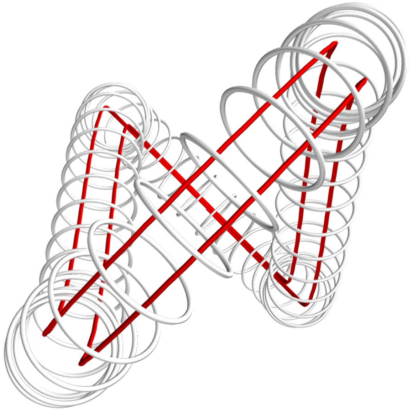


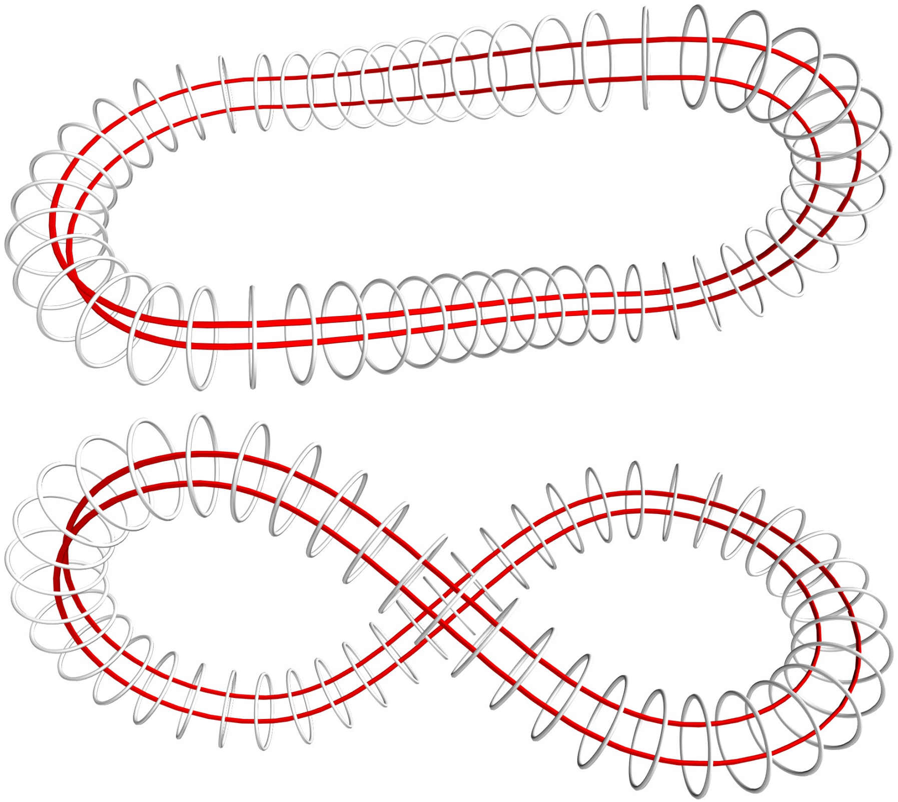
- "Puncture point" has rotated an angle of $4\boldsymbol{\alpha}$ from its starting location with respect to the central axis
- This corresponds to a *rotational transform* $\boldsymbol{\iota} = 4\boldsymbol{\alpha}/(2\boldsymbol{\pi})$
- Same argument applies for any radial distance from the central axis ⟶ every flux surface exhibits the same rotational transform

[5] L. Spitzer, *Phys. Fluids* **1**, 253 (1958)

Numerical model of figure-8 stellarator agrees very well with simple geometric predictions

- Model set up in SIMSOPT [6,7]
- Magnetic field calculated from 50 ideal circular coils in figure-8 pattern
- ODE solver used to trace field lines



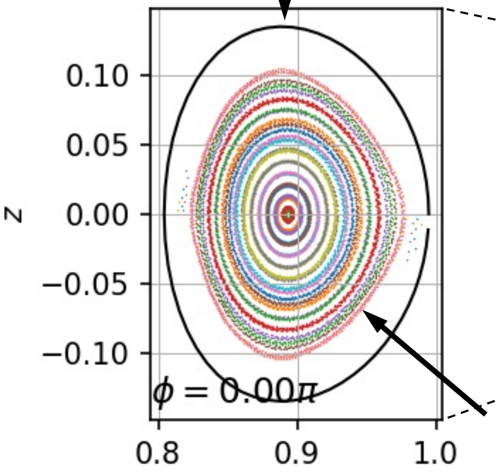


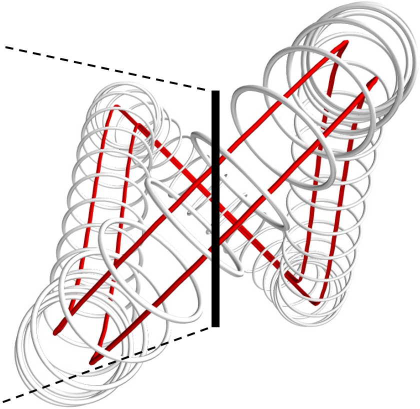
[6] M. Landreman et al., *J. Open Source Soft.* **6**, 3525 (2021)

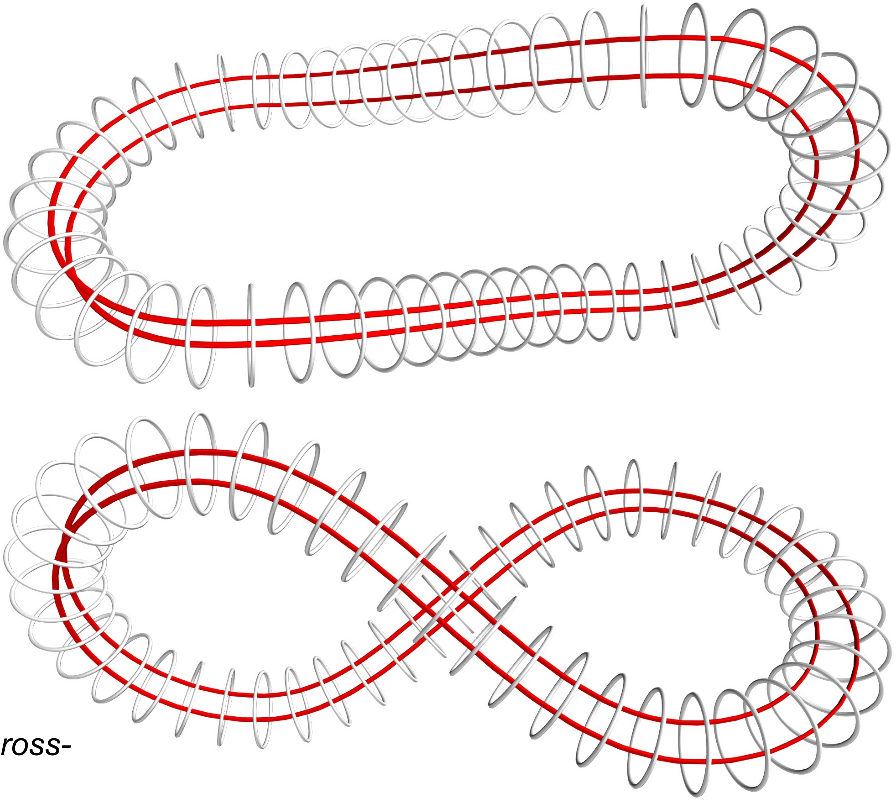
[7] https://github.com/hiddenSymmetries/simsopt

Numerical model of figure-8 stellarator agrees very well with simple geometric predictions

- $\boldsymbol{\alpha} = \boldsymbol{\pi}/4 \longrightarrow$ prediction: $\iota = 1/2$
  - 3D field line visualization: shows that field lines “bite their tails” after two transits →
  - *Poincaré plot*: shows that field lines are confined to flux surfaces

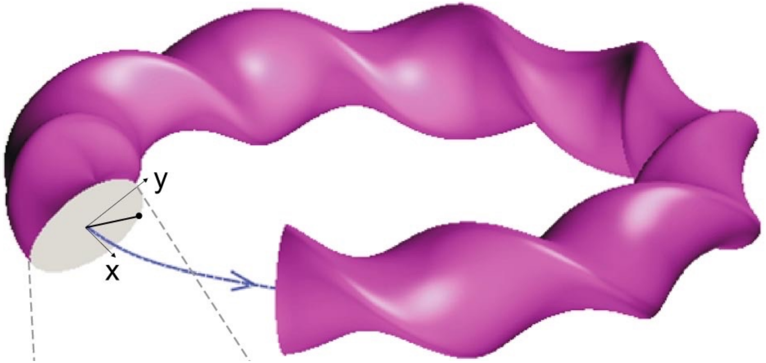






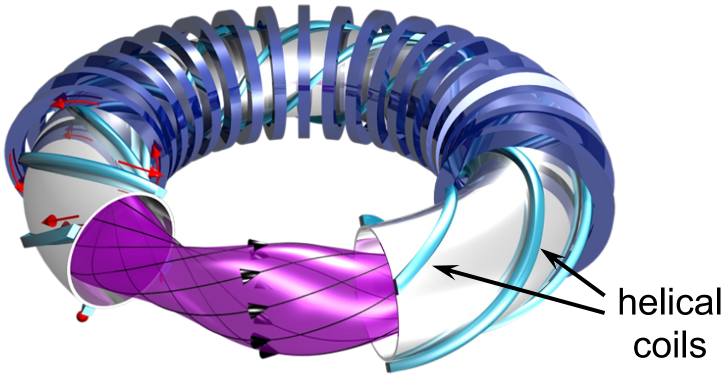
“Puncture points” of field lines in a cross-section after many toroidal transits

Rotational transform can also arise from a helically rotating poloidal cross-section



P. Helander, Rep. Prog. Mod. Phys., 2014

- This effect can be achieved with suitable coil shaping
  - Helical windings
  - Modular coils with non-planar shapes



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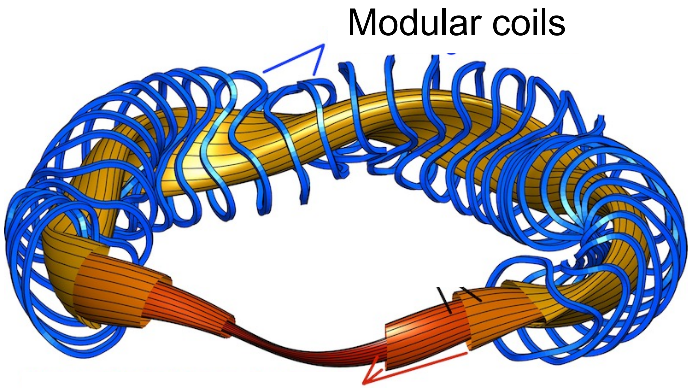
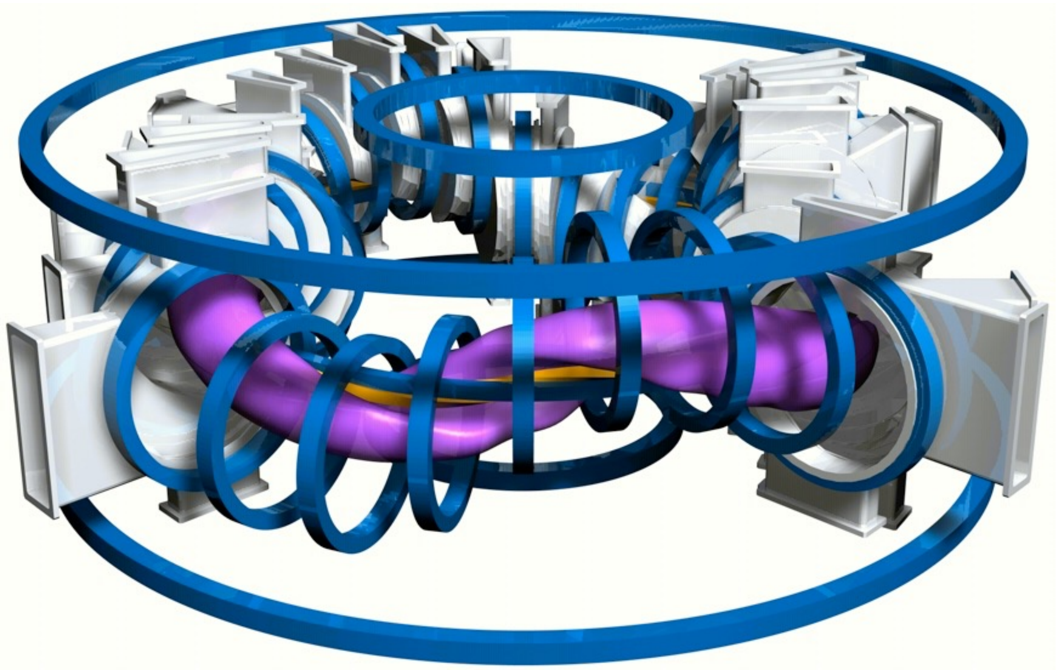


M. Landreman

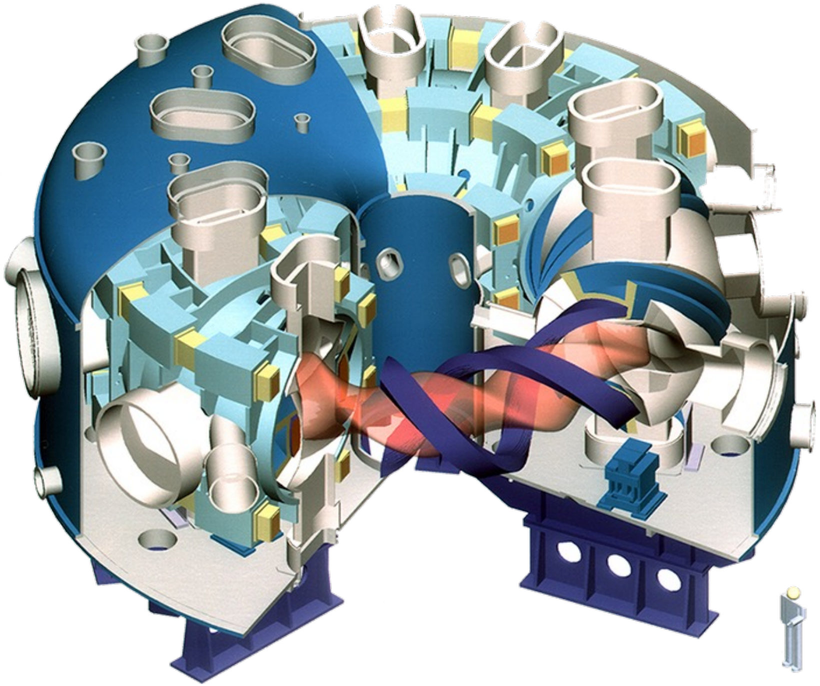
Stellarators can take many forms



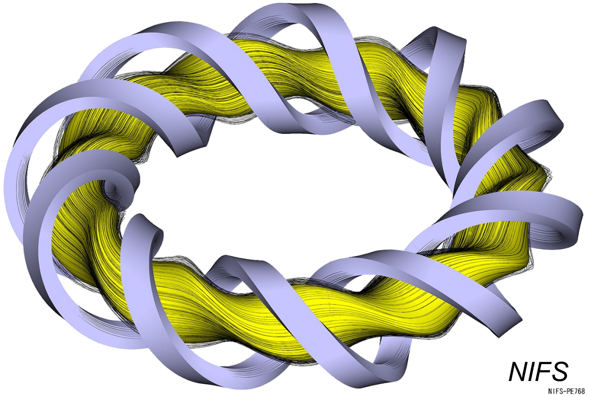
CIEMAT

- TJ-II, CIEMAT, Madrid, Spain
  - “Heliac” configuration
  - Most coils are circular and planar

Stellarators can take many forms



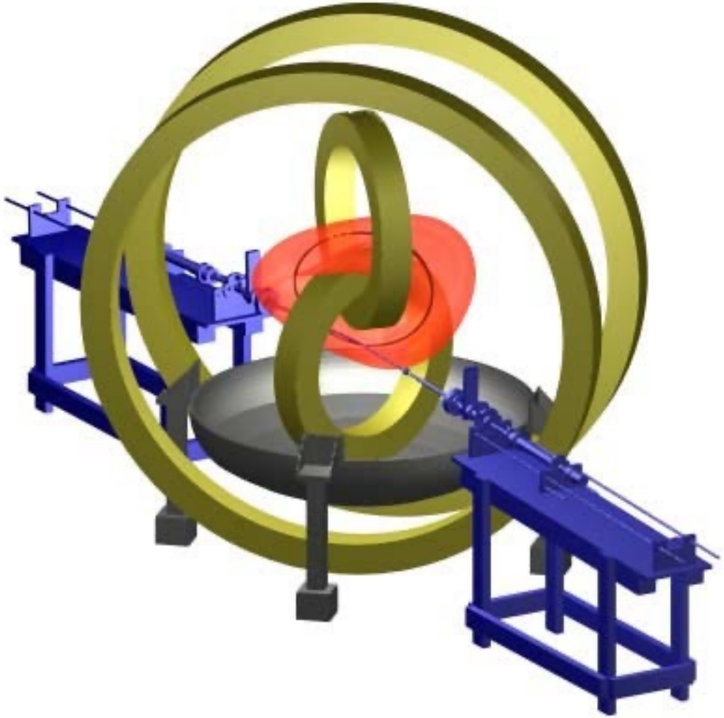
- Large Helical Device (LHD), National Institute for Fusion Science, Toki, Japan
  - Two superconducting helical coils provide most of the magnetic field
  - Concluded final campaign in December 2025



NIFS

NIFS-PE768

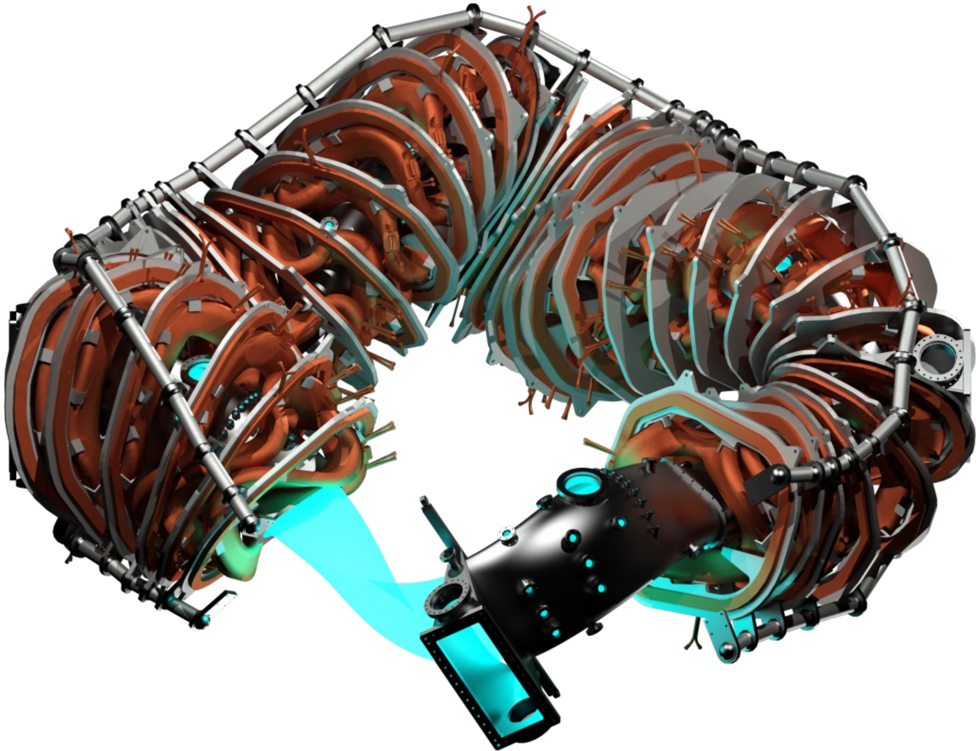
Stellarators can take many forms



J. P. Kremer et al., Phys. Rev. Lett. 2006

- CNT, Columbia University,
  New York, NY, USA
  - Four circular planar coils
  - Two coils are interlocked and tilted

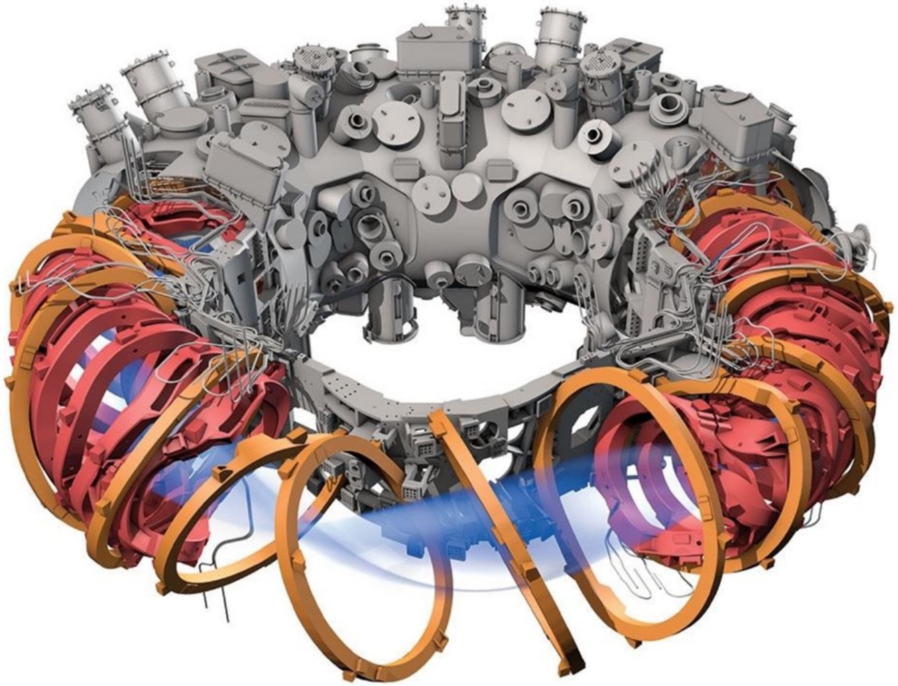
Stellarators can take many forms



HSX Group, University of Wisconsin

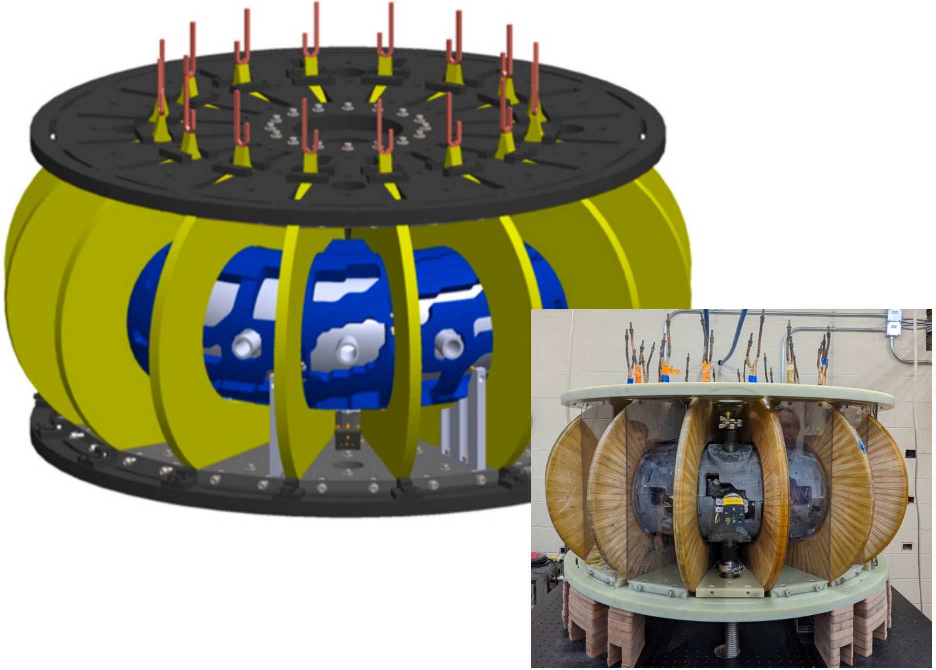
- Helically Symmetric Experiment (HSX), University of Wisconsin, Madison, WI, USA
  - Combination of Modular, “wiggly” coils and planar coils supply the main field
  - Modular coils optimized for good confinement

Stellarators can take many forms



- Wendelstein 7-X (W7-X), Max Planck Institute for Plasma Physics, Greifswald, Germany
  - Most advanced stellarator built to date
  - Modular coils optimized for good confinement
  - Superconducting coils

Stellarators can take many forms



T. Qian et al., *Nucl. Fusion* 2022
T. Qian et al., *Nucl. Fusion* 2023

- MUSE, PPPL
  - First stellarator that uses permanent magnets to produce 3D field shaping
    - Nearly 10,000 magnets altogether
  - Planar coils supply toroidal field
  - Assembled primarily by students
  - Quasi-axisymmetric

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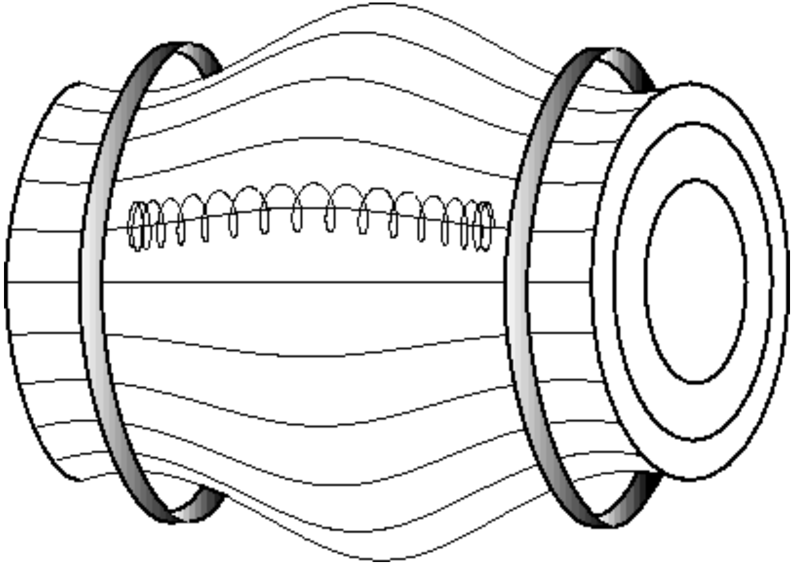
Keeping particle orbits confined

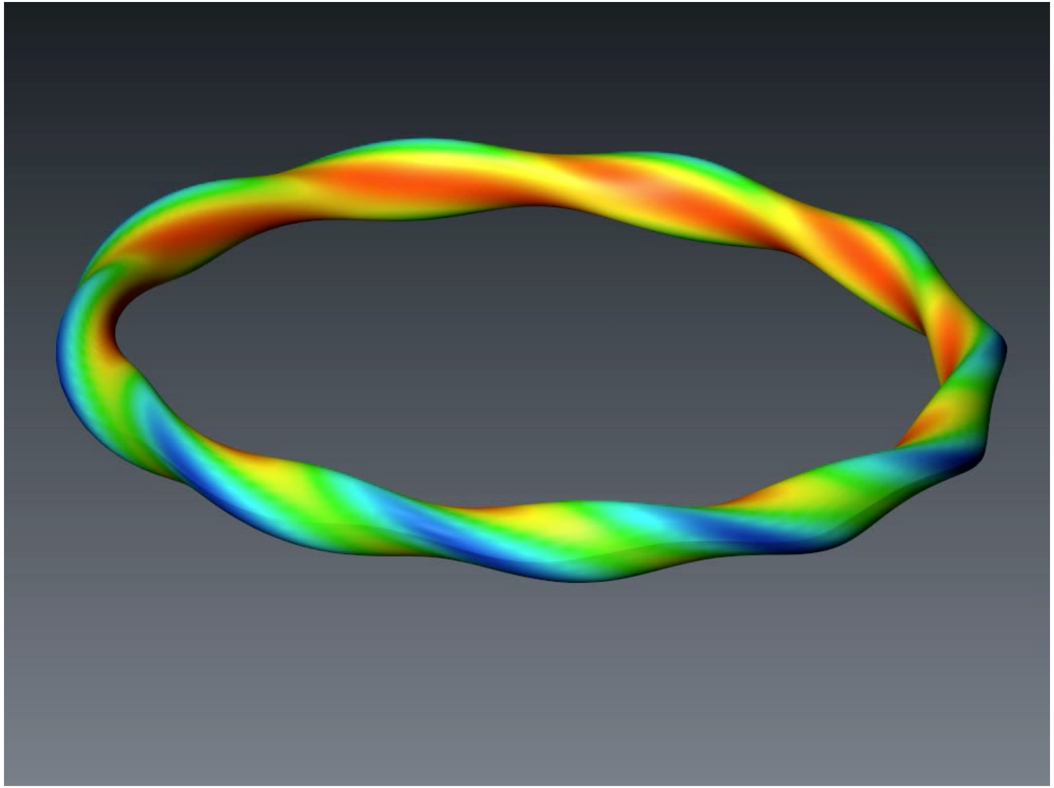
Rotational transform is necessary but not sufficient for good confinement

- Early stellarators exhibited poor energy and particle confinement despite producing flux surfaces with rotational transform
- Helical field lines generally confine *passing* particles well, but not necessarily *trapped* particles
  - *Trapped* particles bounce back and forth between points with high field strength due to magnetic mirror effect
  - *Passing* particles have a high enough $v_{\parallel}/v_{\perp}$ orbit toroidally without bouncing



R. Fitzpatrick

Orbits in a stellarator with poor trapped-particle confinement



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Particles trapped in a magnetic mirror exhibit constants of motion

- First adiabatic invariant: magnetic moment ***μ***

$$\mu = \frac{mv_\perp^2}{2B}$$

$v_\perp$ = *perpendicular velocity*

- Second adiabatic invariant: $\mathcal{J}$

$$\mathcal{J} = \int_{\ell_1}^{\ell_2} mv_{||} d\ell$$

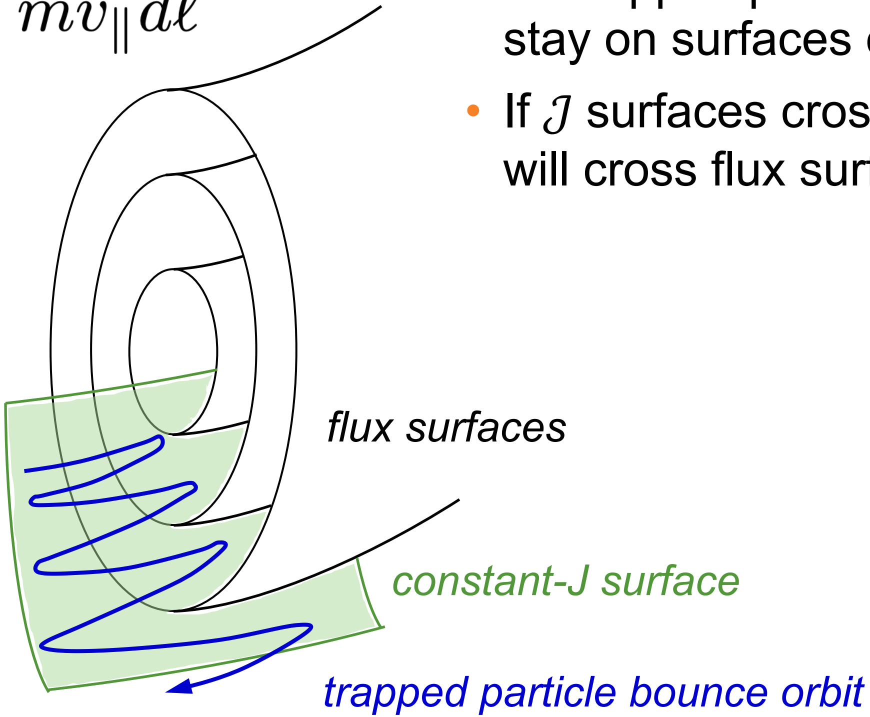
$v_{||}$ = *parallel velocity*

  - For a trapped particle in a magnetic mirror, $\mathcal{J}$ tends to remain constant for each particle as it bounces
  - When drifting (due to curvature, ∇B, etc.), particles follow trajectories that conserve $\mathcal{J}$

Conservation of $\mathcal{J}$ impacts how trapped particles drift among flux surfaces

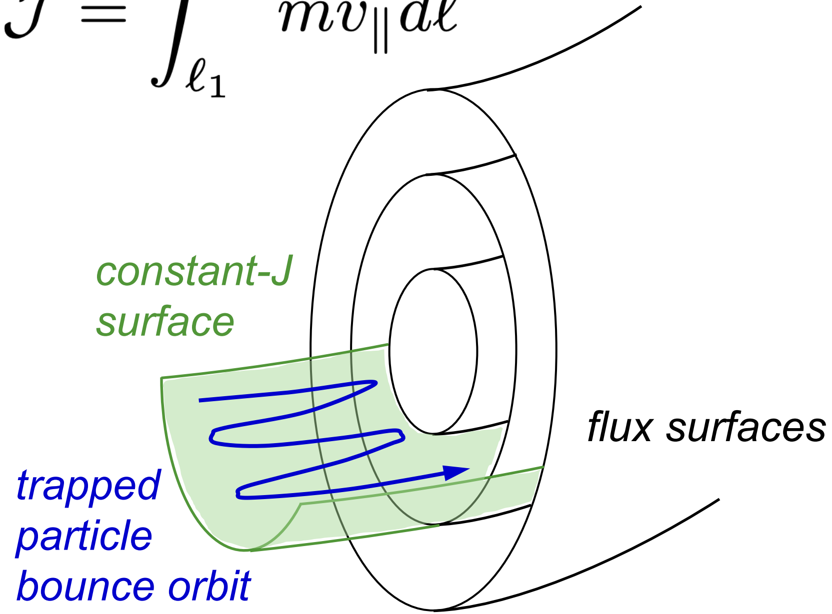
$$\mathcal{J} = \int_{\ell_1}^{\ell_2} m v_{\|} d\ell$$

- As trapped particles drift across field lines, they stay on surfaces of constant $\mathcal{J}$
- If $\mathcal{J}$ surfaces cross flux surfaces, trapped particles will cross flux surfaces (and eventually escape)



If constant-$\mathcal{J}$ surfaces coincide with flux surfaces, trapped particles remain confined

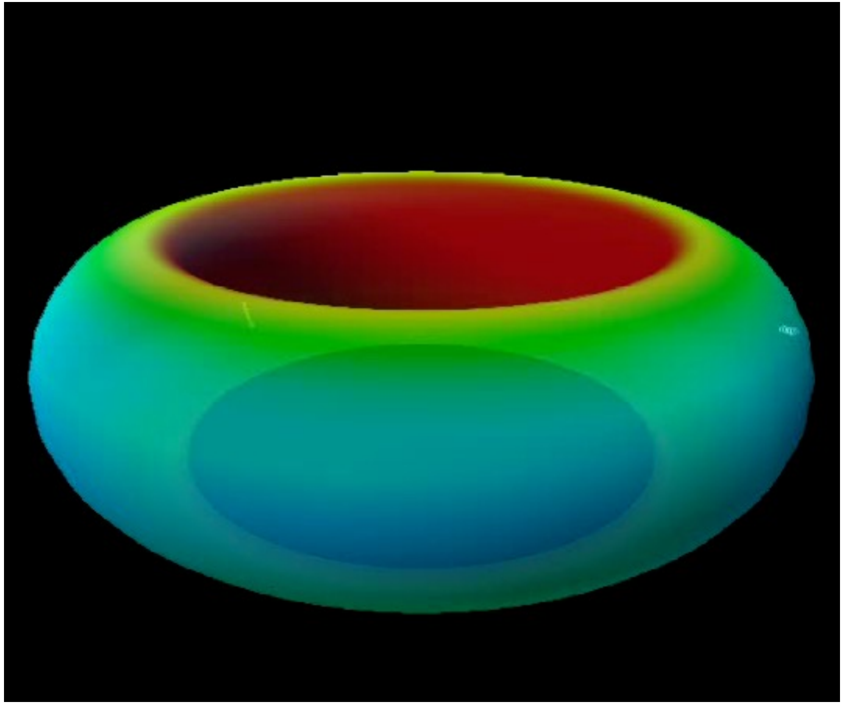
$$\mathcal{J} = \int_{\ell_1}^{\ell_2} m v_{\parallel} d\ell$$



- Stellarators exhibiting this property are referred to as *omnigenous*
- Equivalent definition: an omnigenous field is one with zero time-averaged radial magnetic drift

Axisymmetric devices are automatically omnigenous

- Field lines on a given flux surface are indistinguishable
- Trapped particles remain confined
- *But:* axisymmetric plasmas…
  - Do not exhibit torsion of the magnetic axis or helical rotation
  - Require a large plasma current to generate the helical field line twist
  - Are not stellarators 😭



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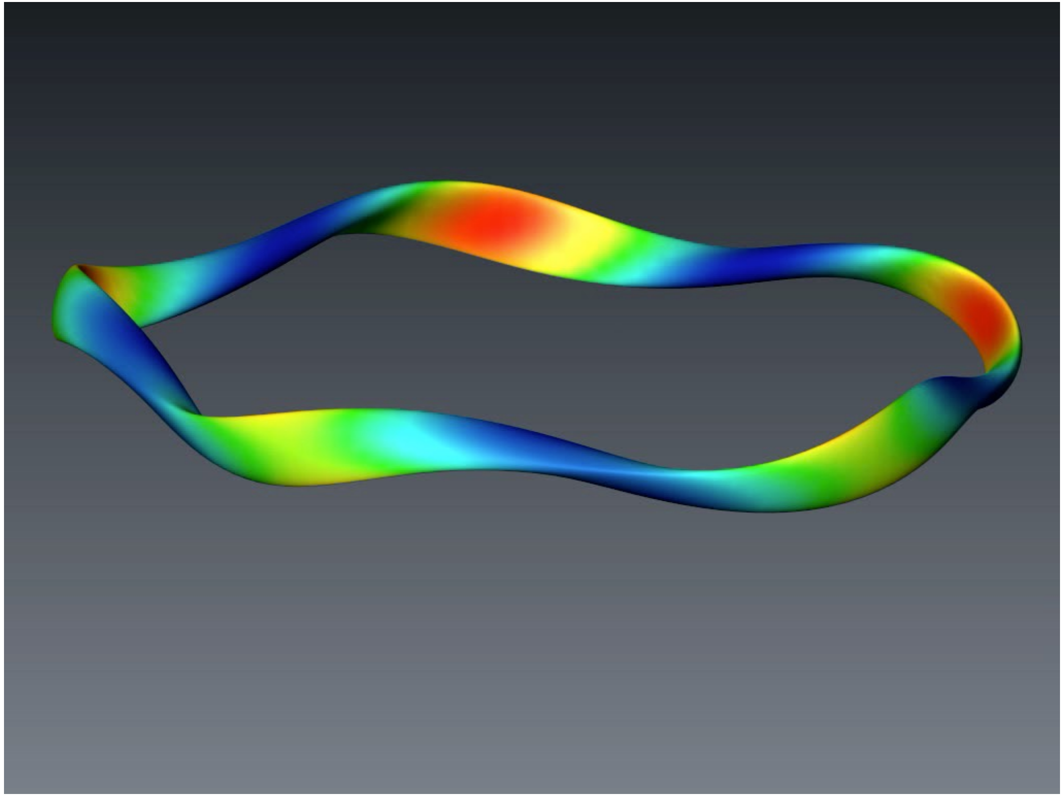
With the right shaping, stellarators can be omnigenous, too!



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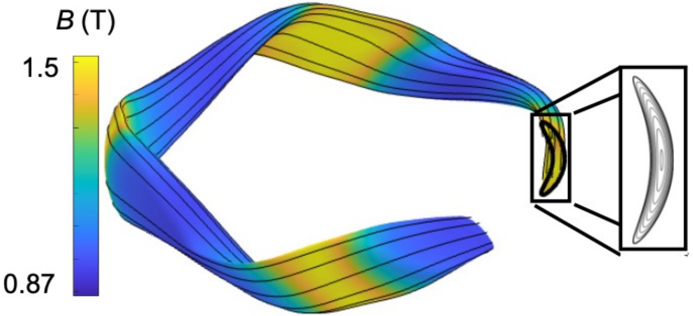
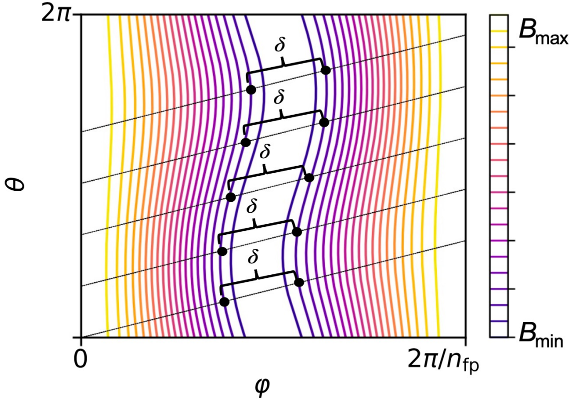
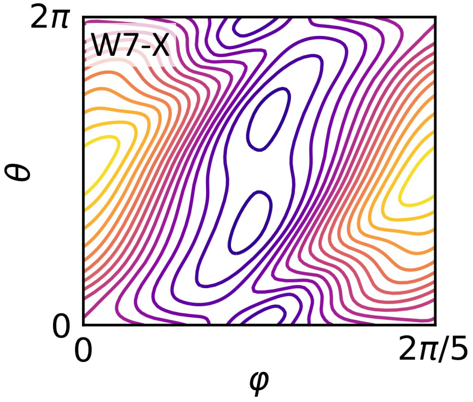
Popular types of omnigenous stellarators

Quasi-isodynamic (QI)

- Contours of |B| close poloidally around flux surfaces







A. Goodman et al [9]

- Example: Wendelstein 7-X
  - Many B contours not actually closed poloidally
  - Still exhibits substantially lower drift losses than in stellarators with no optimization

[9] A. Goodman et al., *PRX Energy* **3**, 023010 (2024)

Quasisymmetry (QS)

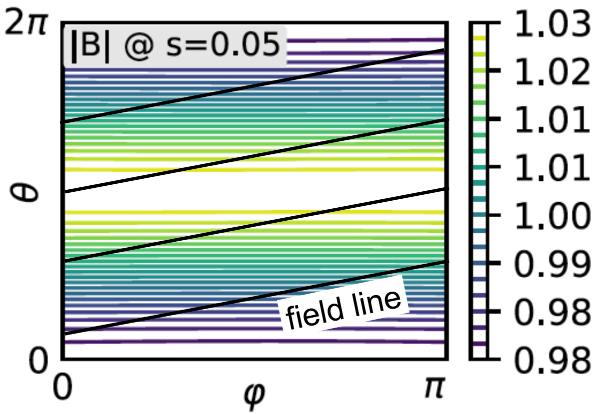
- Note that integrand for $\mathcal{J}$ can be expressed as a function of $B(\ell)$ only
  - $B(\ell)$ = field strength at position ℓ along field line

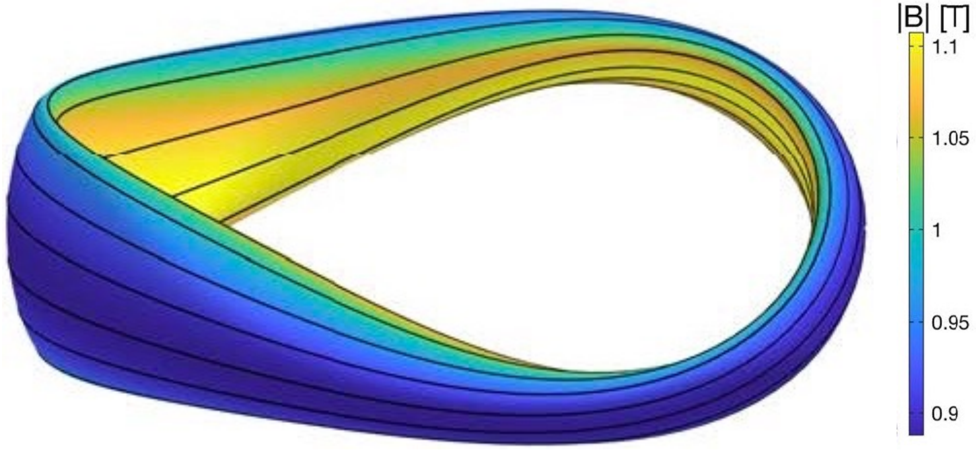
$$\mathcal{J} = \int_{\ell_1}^{\ell_2} m v_{||} d\ell = \sqrt{2\underset{\substack{\uparrow \\ \textit{magnetic} \\ \textit{moment}}}{\mu} m} \int_{\ell_1}^{\ell_2} \sqrt{\underset{\substack{\uparrow \\ \textit{field strength} \\ \textit{at bounce point}}}{B_*} - B(\ell)}\, d\ell$$

- Quasisymmetry: $B(\ell)$ is the same for any field line on a flux surface
  - $\mathcal{J}$ is automatically uniform across the flux surface
    - Satisfies the condition for omigeneity
  - Particles on a given flux surface can't "tell" which field line they are on

Quasi-axisymmetry

- Quasi-axisymmetry (QA): quasi-symmetry in which *B* contours are purely toroidal
- Non-axisymmetric analog to a tokamak



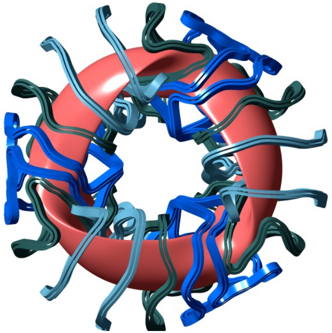


Equilibrium with near-perfect QA found in 2022 [10]

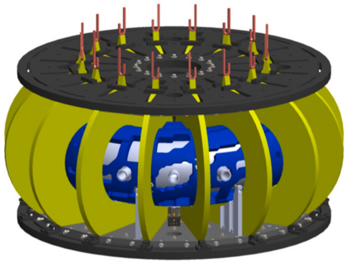
[10] M. Landreman and E. Paul, *Phys. Rev. Lett.* **128**, 035001, (2022)

Quasi-axisymmetry: examples

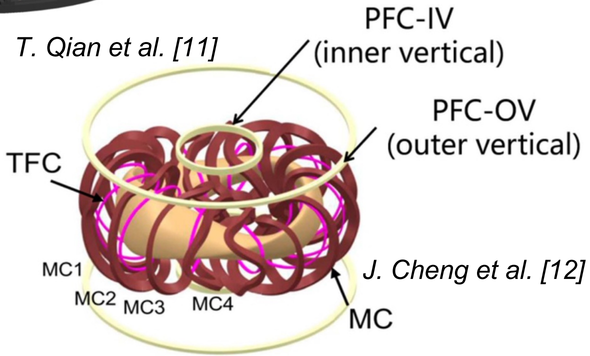
- NCSX (National Compact Stellarator Experiment) [10]
  - Planned for PPPL, partly constructed
  - Cancelled due to cost and schedule overruns
- MUSE [11]
  - First and only permanent magnet stellarator
  - Tabletop scale with B ~ 0.15 T
- CFQS (Chinese First Quasi-axisymmetric Stellarator) [12]
  - Modular coil device
  - Constructed and currently operating at low field (eventually will run at ~ 1 T)



PPPL



T. Qian et al. [11]

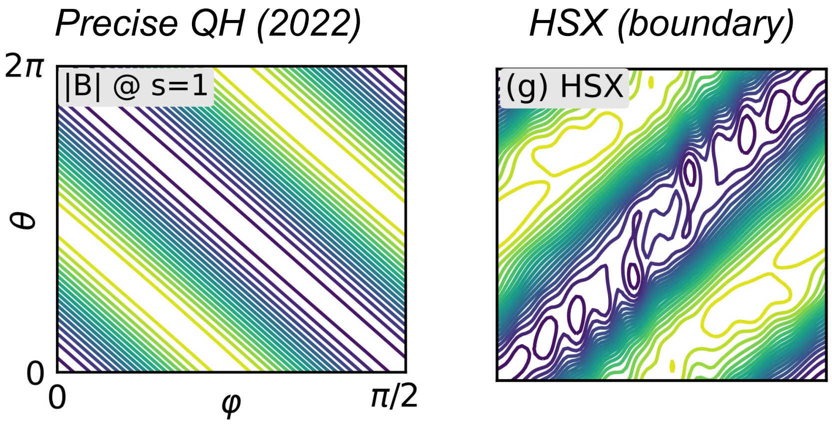


J. Cheng et al. [12]

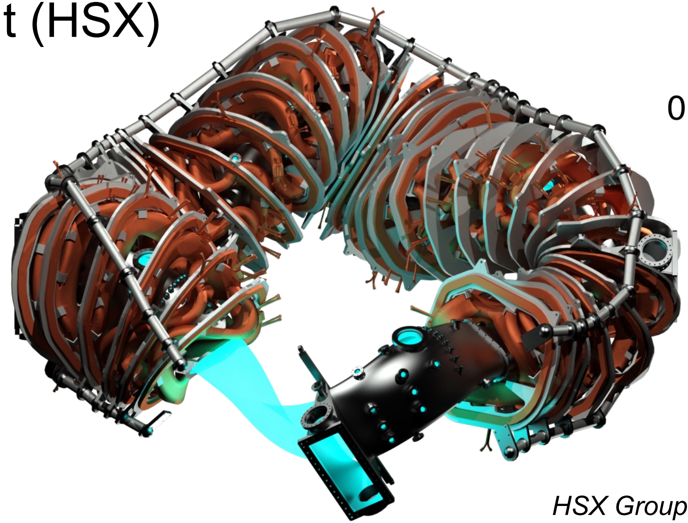
[10] M. Zarnstorff et al., *PPCF* **43**, A237 (2001)
[11] T. Qian et al., *J. Plasma Phys.* **89**, 955890502 (2023)
[12] J. Cheng et al., *PPCF* **67**, 105011 (2025)

Quasi-helical symmetry

- Quasi-helical symmetry (QH): quasi-symmetry in which *B* contours wrap both poloidally and toroidally
- Example: the Helically Symmetric Experiment (HSX)



M. Landreman and E. Paul, [10]

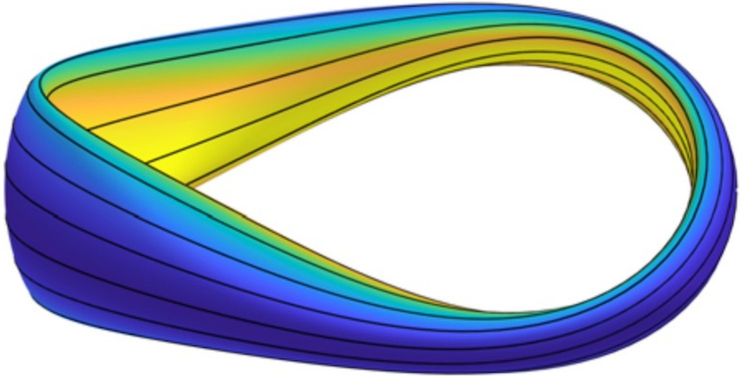


HSX Group, University of Wisconsin

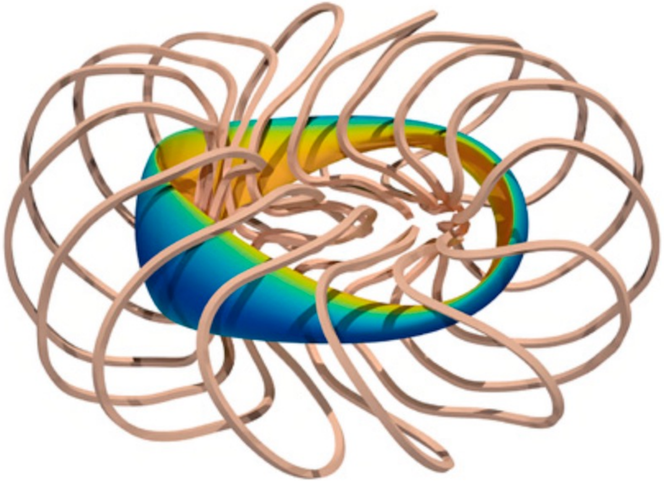
Designing a stellarator

Stellarator design is often done in two stages

1. Design the plasma for good physics properties
2. Design magnets that create the fields necessary to confine the plasma



Landreman and Paul 2022



Wechsung et al. 2022

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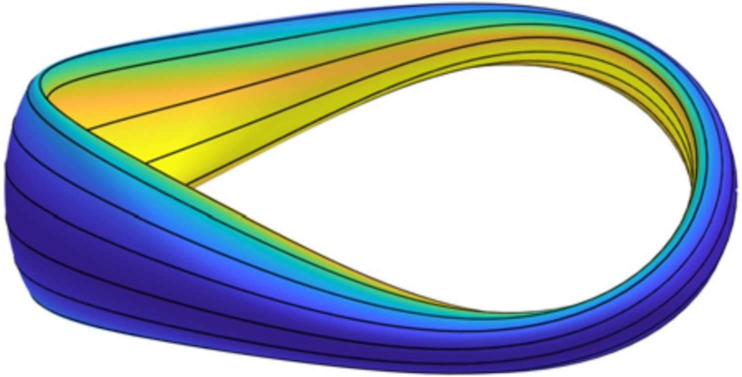

Landreman and Paul 2022

Plasmas can be designed with numerical optimization

- Decide on what you want to optimize for
  - Specific rotational transform profile
  - Quasisymmetry, quasi-isodynamicity, etc.
  - Fast particle confinement
  - MHD stability
  - Turbulence minimization
  - …

Plasmas can be designed with numerical optimization

- Formulate *objective functions* that penalize deviations from the desired attributes
  - $\mathbf{x}_{plasma}$ = parameters that specify the plasma
  - Individual objectives for each attribute:
    - $f_{quasisymmetry}(\mathbf{x}_{plasma})$ = deviation from quasisymmetry for plasma defined by $\mathbf{x}_{plasma}$
    - $f_{stability}(\mathbf{x}_{plasma})$ = level of fast particle losses for plasma defined by $\mathbf{x}_{plasma}$
    - $f_{turbulence}(\mathbf{x}_{plasma})$ = level of instability of plasma defined by $\mathbf{x}_{plasma}$
    - …
  - Total objective: weighted sum of of individual objectives
    - $f(x) = w_{quasisymmetry}\, f_{quasisymmetry}(\mathbf{x}_{plasma}) + w_{stability}\, f_{stability}(\mathbf{x}_{plasma}) + w_{turbulence}\, f_{turbulence}(\mathbf{x}_{plasma}) + \ldots$
    - Weights *w* determine which objectives get penalized the most

Plasmas can be designed with numerical optimization

- Use a numerical optimization algorithm to minimize the total objective
  - Supply an initial guess of the plasma ($\mathbf{x}_{plasma,0}$)
    - *Cold start:* simple, arbitrary plasma (e.g. an axisymmetric torus)
    - *Warm start:* pre-existing “ok” equilibrium that the optimizer will improve upon
  - Run an optimization code that adjusts $\mathbf{x}_{plasma}$ to minimize $f(\mathbf{x}_{plasma})$
    - SIMSOPT [6]
    - DESC [13]
    - STELLOPT [14]
    - ROSE [15]
    - …
  - Output: improved $\mathbf{x}_{plasma}$ specifying a plasma that better satisfies the objectives

[6] M. Landreman et al., *J. Open Source Soft.* **6**, 3525 (2021)
[13] D. W. Dudt et al., *J. Plasma Phys.* **89**, 955890201 (2023)
[14] S. Lazerson et al., *STELLOPT*, https://doi.org/10.11578/dc.20180627.6
[15] M. Drevlak et al., *Nucl. Fusion* **59**, 016010 (2018)

Stellarator design is often done in two stages

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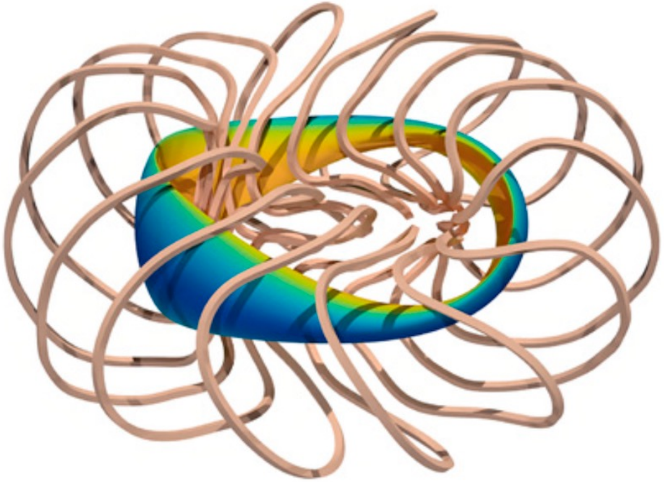
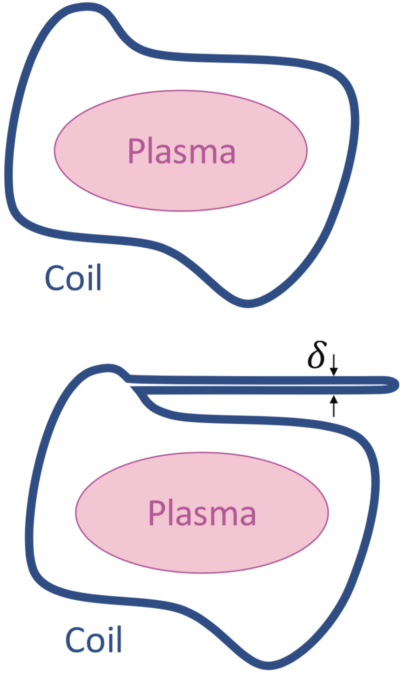


Wechsung et al. 2022

Coil optimization allows for many approaches and design schemes

- Goal of coil optimization: find external magnets that produce the desired field within the plasma volume
- This is an *ill-posed problem* [16]
  - Example: adding an out-and-back current path to a coil will result in a different magnet shape but the same field in the plasma
- This can actually be a good thing
  - No one right way to design magnets
  - Lots of flexibility in how to define and parametrize the solution space
- On the other hand…
  - Difficult to know for sure if a given solution is optimal or if a better one exists

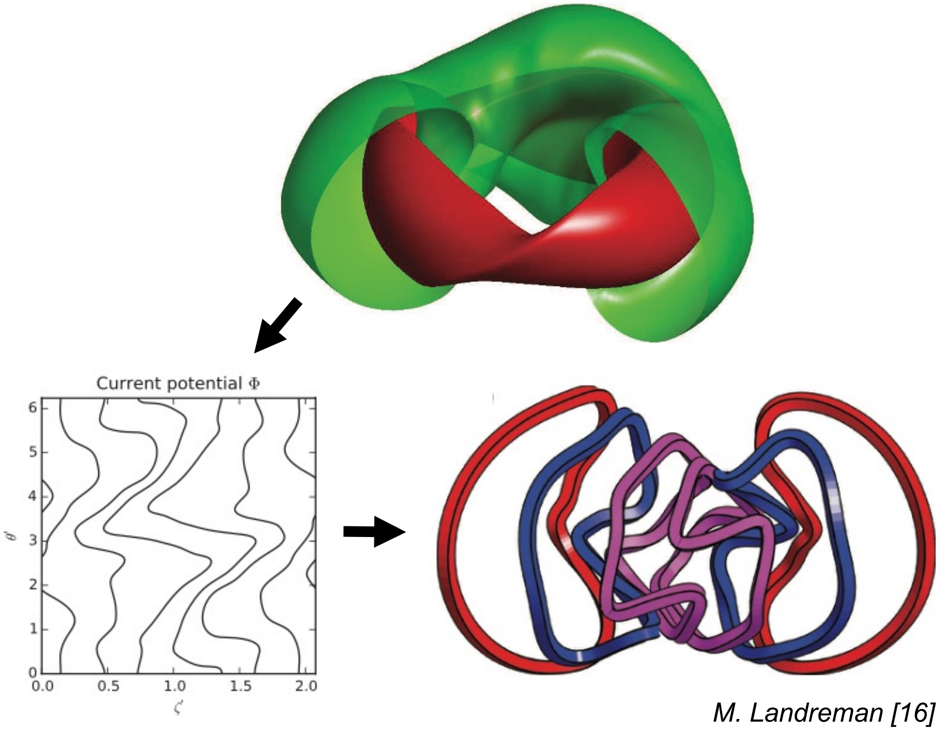


M. Landreman [5]

[16] M. Landreman, *Nucl. Fusion* **57**, 046003 (2017)

Winding surface approach / Merkel’s method [17]

- Define a toroidal *winding surface* enclosing the plasma
- Find an optimal current distribution on the winding surface that creates the necessary field on the plasma
- Discretize the current distribution to determine coil shapes
  - Calculate streamlines of surface current
  - Define filamentary coils along those streamlines

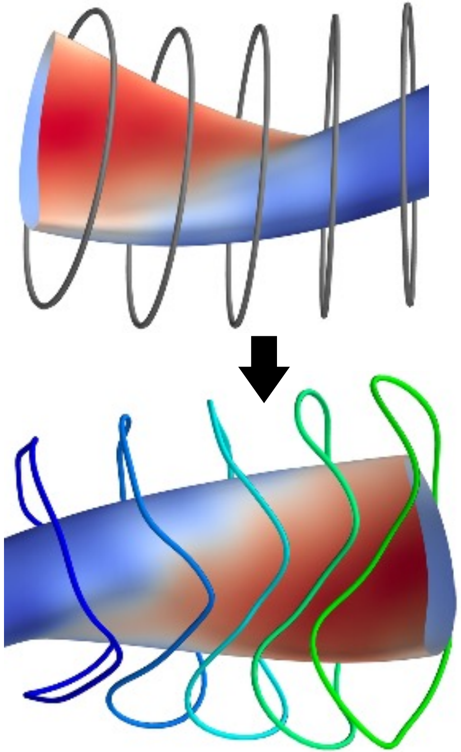


M. Landreman [16]

[17] P. Merkel, *J. Comp. Phys.* **66**, 83 (1986)

Space curve optimization

- Initialize a specific number of coils around the plasma
    - Define geometric parameters $\mathbf{x}_{coils}$ specifying the shape of each coil
    - Simple initial guesses (i.e. planar circles) are typically fine
- Minimize an objective function $f(\mathbf{x}_{\mathrm{coils}})$ that penalizes field error and engineering properties
    - $f(\mathbf{x}_{coils}) = f_{accuracy}(\mathbf{x}_{coils}) + f_{curvature}(\mathbf{x}_{coils}) + f_{separation}(\mathbf{x}_{coils}) + \ldots$
- Optimized shapes ($\mathbf{x}_{coils}$) form the basis for the design of the coils

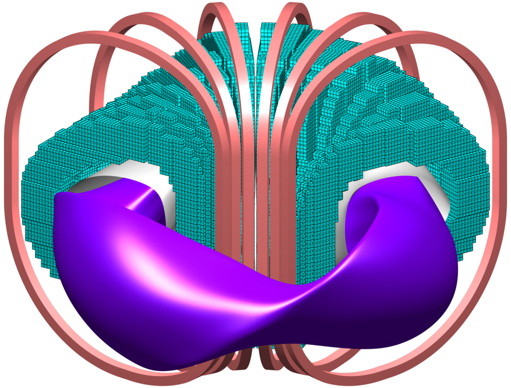


C. Zhu et al. [18]

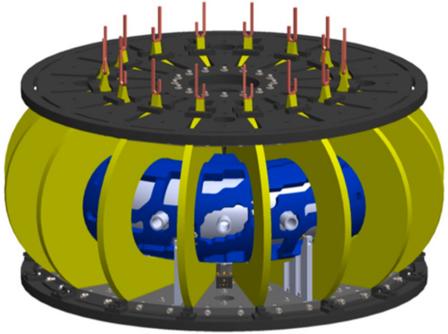
[18] C. Zhu et al., *Nucl. Fusion* **58**, 016008 (2018)

Permanent magnet optimization

- Permanent magnets can be used to simplify coils
  - Cannot confine a plasma alone – some coils are needed to generate a toroidal field
  - Permanent magnets can generate 3D shaping so that coils can be less complicated
- Optimize dipole moments of magnets at selected positions around plasma



K. C. Hammond et al [19]

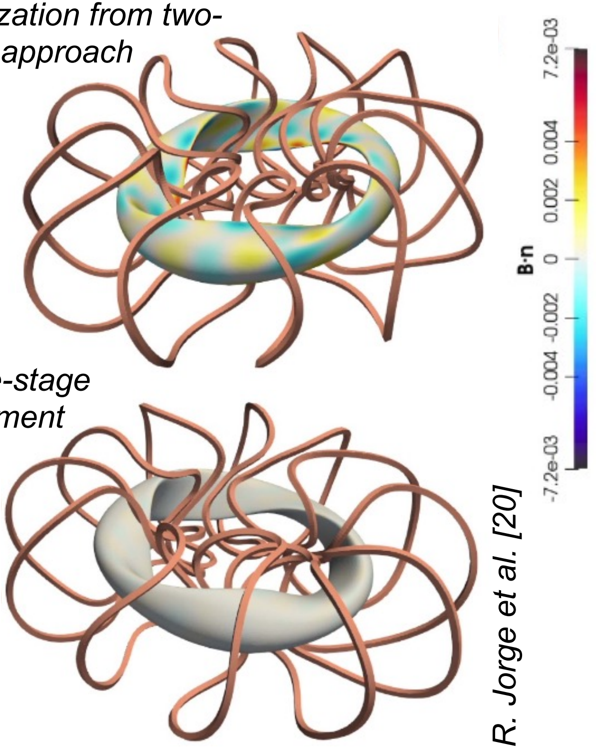


T. Qian et al. [11]

[11] T. Qian et al., *J. Plasma Phys.* **89**, 955890502 (2023)
[19] K. C. Hammond et al., *Nucl. Fusion* **62**, 126065 (2022)

Single stage optimization

- Optimize plasma and coils at the same time
  - Minimize a combined objective: $f(\mathbf{x}_{plasma}, \mathbf{x}_{coils})$
- Advantage: steers solution toward plasmas that are feasible to build coils for
- Disadvantage: more computationally expensive than separate stage-1 and stage-2 optimizations
- Tends to be more successful when done from a warm start with plasmas and coils that are already "pretty good"



Initialization from two-stage approach

Single-stage refinement

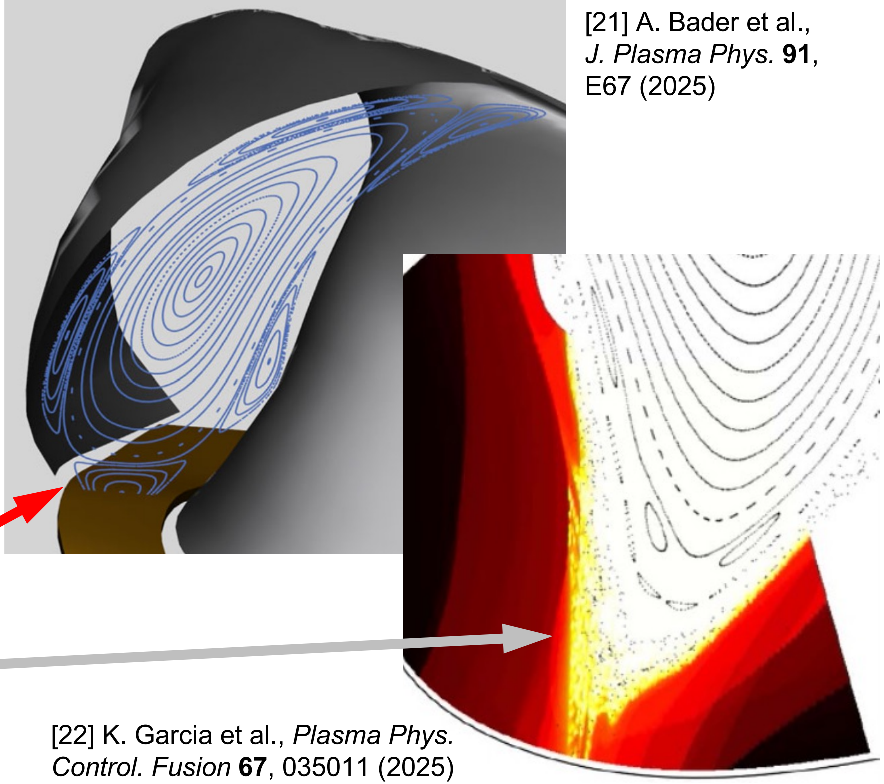
R. Jorge et al. [20]

[20] R. Jorge et al., *Plasma Phys. Control. Fusion* 65, 074003 (2023)

Some current research areas

Handling heat and particle exhaust

- Fusion devices need to exhaust He ash efficiently without overheating
- Divertor:
  - Receives majority of plasma flux
  - Directs exhaust gas to pumps
- Stellarator divertors deal with (or take advantage of) complexity of 3D magnetic fields
  - *Island divertors:* “magnetic islands” isolate plasma from targets (used in W7-X)
  - *Non-resonant divertors:* chaotic field lines isolate plasma from targets (not extensively tested)

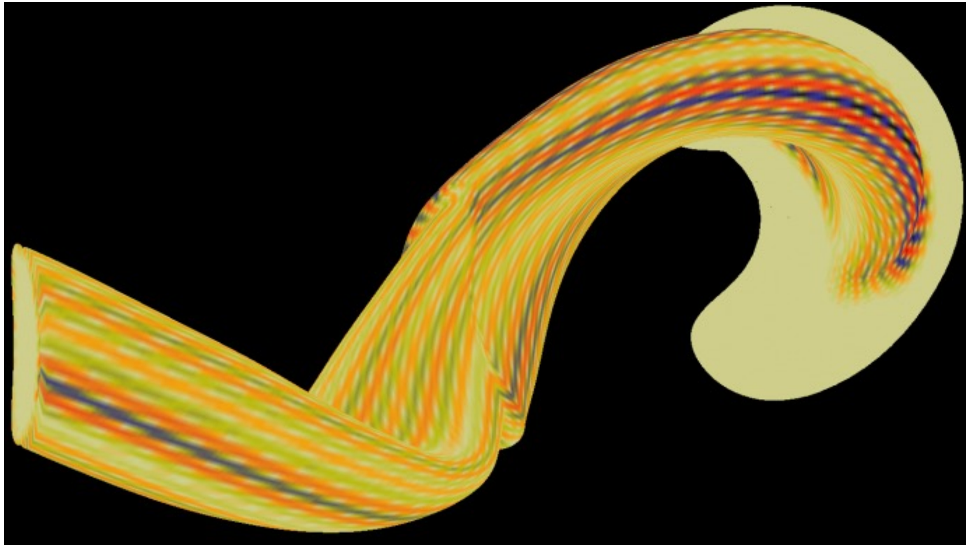


[21] A. Bader et al., *J. Plasma Phys.* **91**, E67 (2025)

[22] K. Garcia et al., *Plasma Phys. Control. Fusion* **67**, 035011 (2025)

Reducing turbulent transport

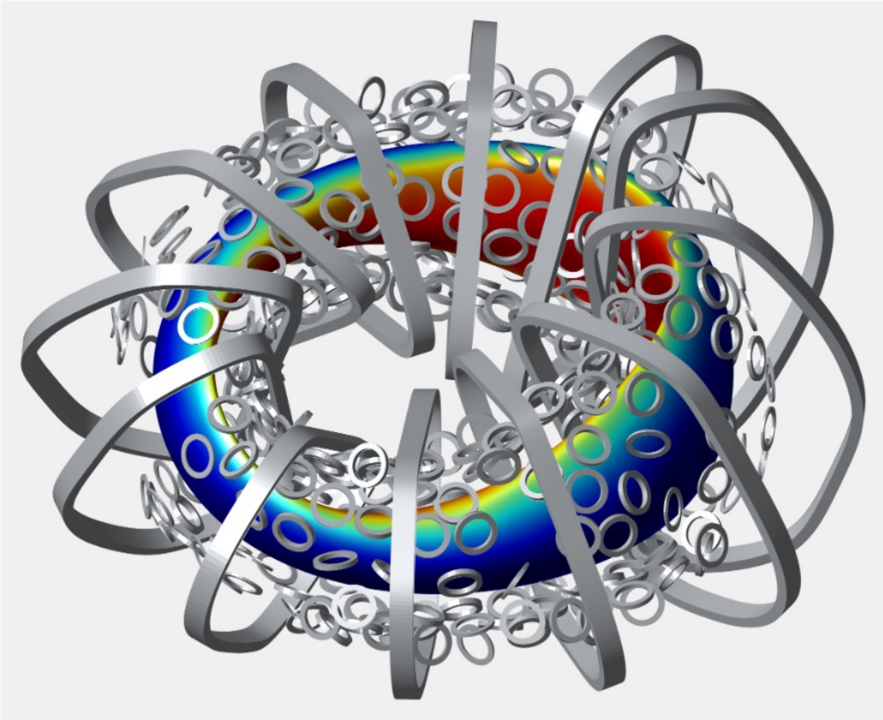
- Turbulence: small-scale fluctuations that can push heat and particles out of the plasma
- Strategies for mitigating turbulent transport
  - Optimize the magnetic geometry to reduce susceptibility to micro-instabilities that cause turbulence
  - Optimize plasma heating and fueling to achieve density and temperature gradients that stabilize micro-instabilities



I. Calvo et al., https://sites.fusion.ciemat.es/multitranstell/

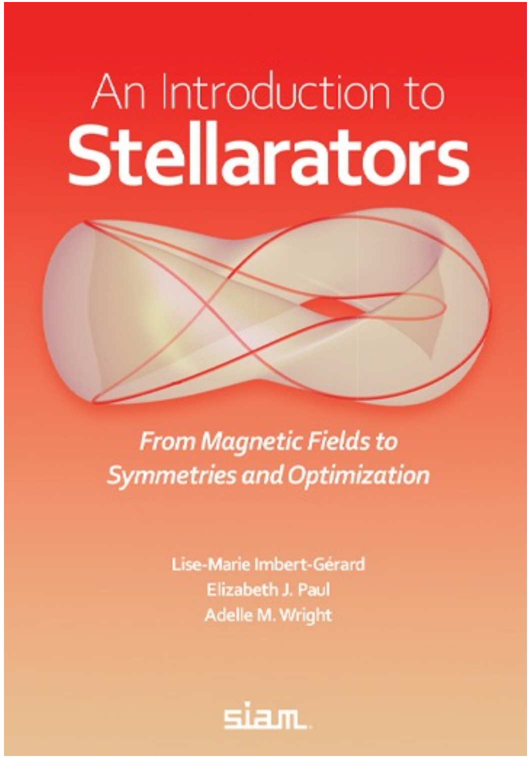
Simplifying the coils

- Coils are a major cost driver of modern stellarators
  - Complex shapes
  - Tight engineering tolerances (~10^{-3}-10^{-4})
- Ideas for simplifying coils in the future
  - Stochastic optimization to reduce sensitivity to misalignments
  - Designs with large arrays of simple coils (e.g. Thea Energy)



[27] C. P. S. Swanson et al., https://arxiv.org/abs/2512.08027

Recommended reading



An Introduction to Stellarators

From Magnetic Fields to
Symmetries and Optimization

Lise-Marie Imbert Gérard, Elizabeth J. Paul,
and Adelle M. Wright

https://doi.org/10.1137/1.9781611978223

ArXiV preprint: https://doi.org/10.48550/arXiv.1908.05360

PRINCETON
PLASMA PHYSICS
LABORATORY

Stellarators

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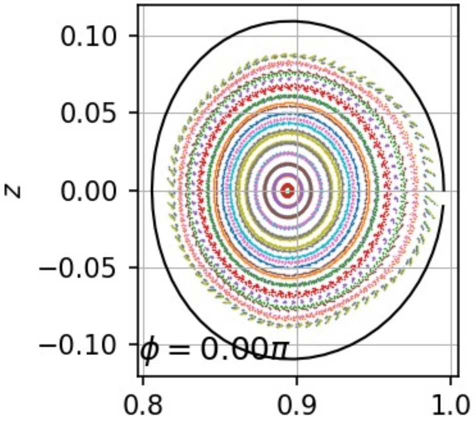
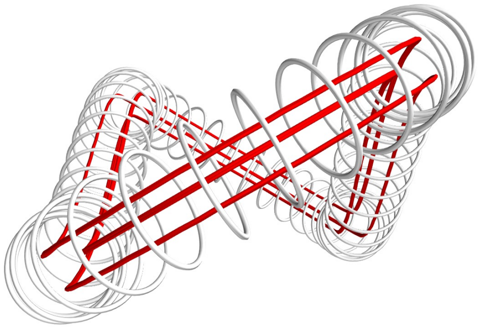
PPPL SULI Introduction to Plasma and Fusion Course, June 2026

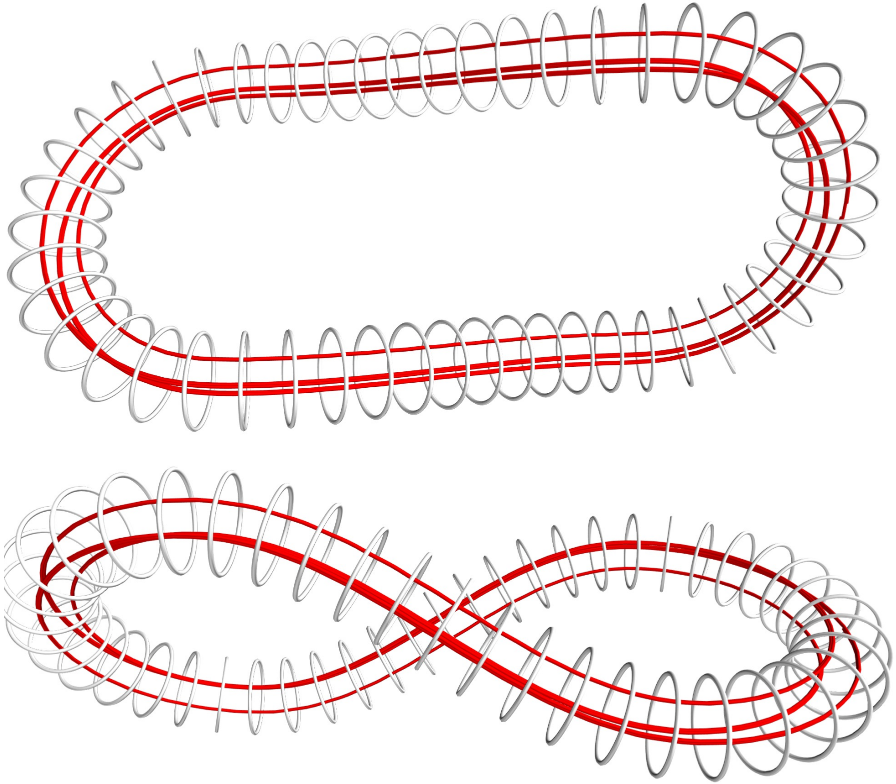
Back-up slides

Numerical model of figure-8 stellarator agrees very well with simple geometric predictions

- $\boldsymbol{\alpha} = \boldsymbol{\pi}/6 \longrightarrow$ prediction: $\boldsymbol{\iota} = 1/3$
  - Field lines “bite their tails” after three transits

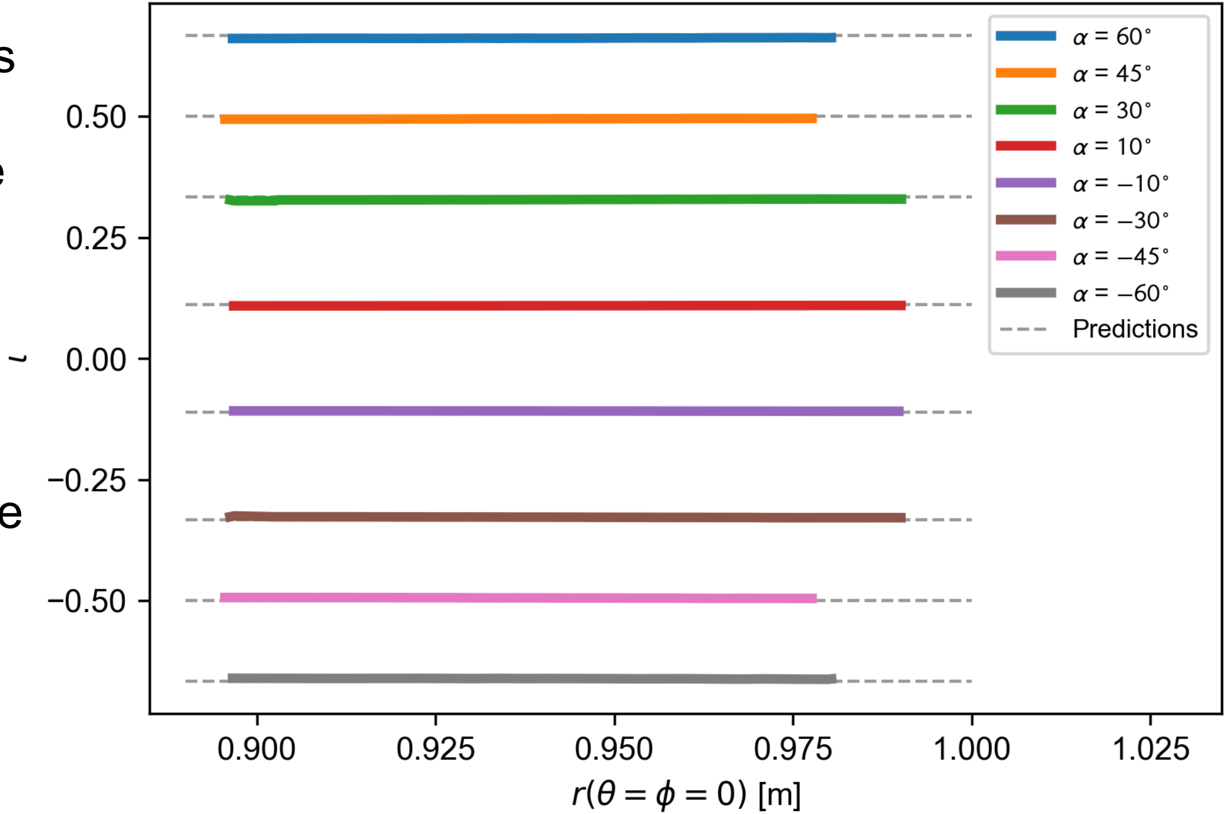




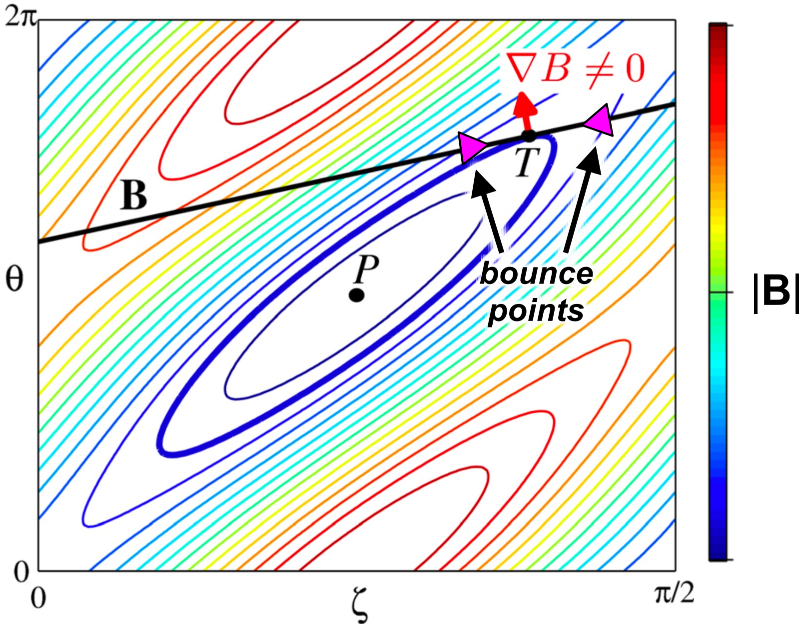


Numerical model of figure-8 stellarator agrees very well with simple geometric predictions

- Rotational transform profiles agree to within 3% of predictions for a wide range of tilt angles
  - *Profile:* dependence of quantity on minor radius (distance from core/axis)
- Flatness of profiles (zero *shear*) affirms independence of ι on minor radius



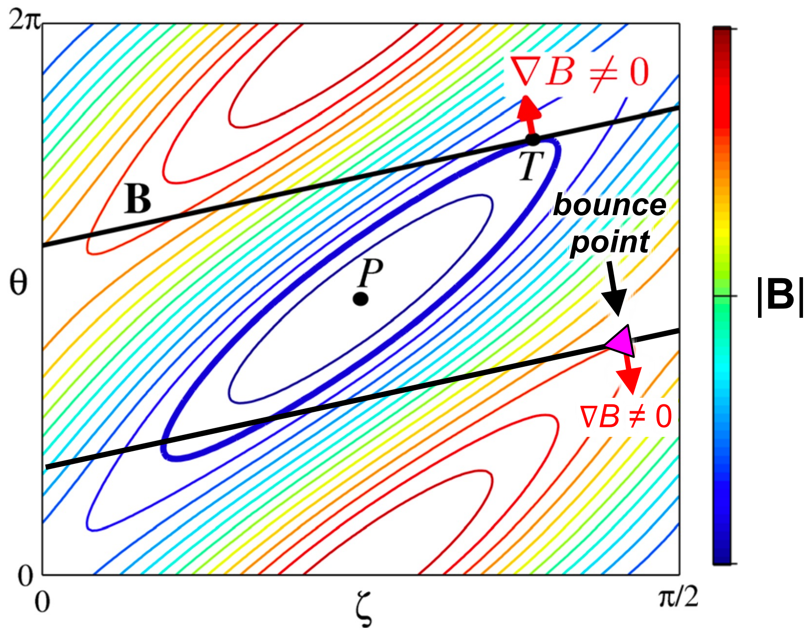
Key property of omnigenous stellarators: no local point *minima* of |B| on flux surfaces [28]



- Some contours of B = |**B**| will be tangent to field lines
- If a particle is deeply trapped in the vicinity of the tangency point:
  - **B** × **∇**B has the same sign on both sides of the minimum
  - Radial drift velocity is in the same direction for entire bounce trajectory
  - Particle exhibits nonzero mean radial drift: violation of omnigenity

[28] M. Landreman and P. Catto, *Phys. Plasmas* **19**, 056103 (2012)

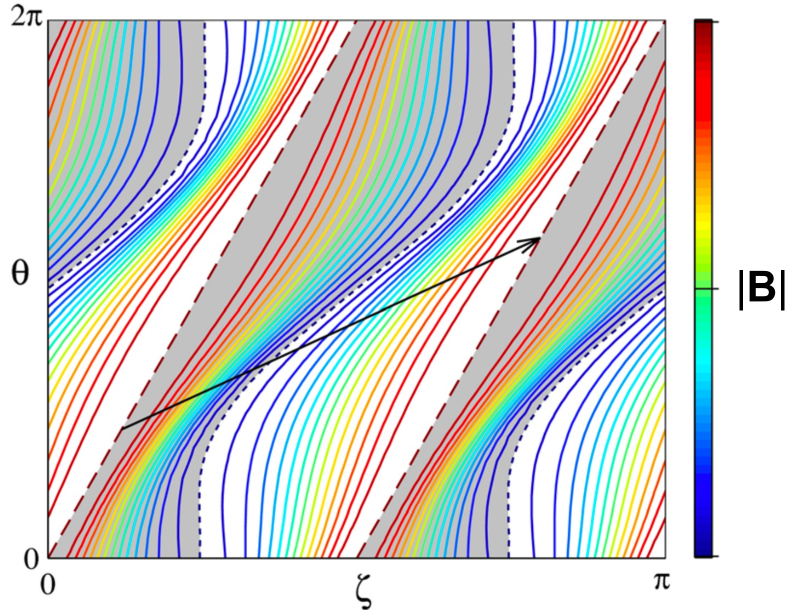
Key property of omnigenous stellarators: no local point *maxima* of |B| on flux surfaces



- Drift velocity is nonzero at the tangency point
- Barely trapped particles stay near the bounce point for a long time
  - May be long enough to drift out of the plasma

[28] M. Landreman and P. Catto, *Phys. Plasmas* **19**, 056103 (2012)

Therefore: for omnigenity, all |B| contours must wrap toroidally and/or poloidally



- Drift velocity is in the opposite direction on each side of the minimum *B*
- Drift velocity is zero at local maxima in *B* along field lines ⟶ weakly trapped particles do not escape
- Additional properties
  - Distances along field lines between points of equal *B* are the same across the flux surface
  - Contours of the maximum *B* are straight in Boozer coordinates

[28] M. Landreman and P. Catto, *Phys. Plasmas* **19**, 056103 (2012)