Plasma Turbulence

2025 Introduction to Fusion Energy and Plasma Physics

Maria Pia Valdivia

Center for Energy Research,
University of California San Diego

How did I get here?

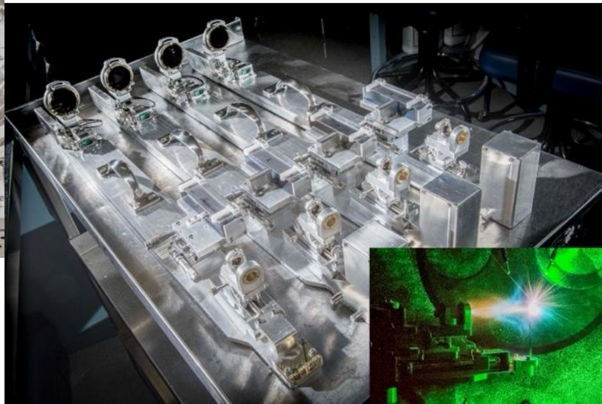
- Undergrad and PhD in Physics:
- Pontificia Universidad Católica de Chile


Conseil Européen pour la Recherche Nucléaire (CERN)
Geneva, Switzerland

Advanced Light Source (ALS):
Lawrence Berkeley National Lab (LBNL)
University of California Berkeley

Advanced Photon Source (APS):
Argon National Lab (ANL)
Northwestern University

Laboratoire pour l'Utilisation des Lasers Intenses (LULI)
École Polytechnique, Paris, France

Laboratory for Laser Energetics (LLE)
University of Rochester, NY

It takes a village!

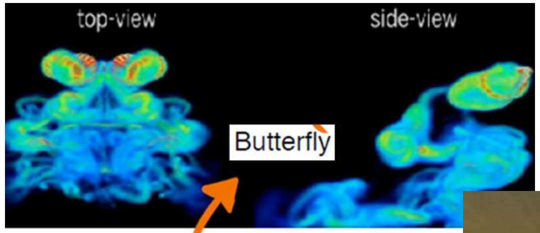
39th and 40th birthday party (a.k.a. Beamtime at LCLS-MEC, SLAC, Stanford)

L-10110 on-site team (left to right): Pawel Kozlowski (LANL), Simon Bott-Suzuki (UCSD), Bob Nagler (LCLS-MEC), Hae Ja Lee (LCLS-MEC), Milenko Vescovi (HZDR), Luisa Izquierdo (U. Catolica de Chile), Dimitri Khaghani (LCLS-MEC), Arianna Gleason (SLAC/Stanford), Gabriel Perez-Callejo (U. de Valladolid), Pia Valdivia (UCSD), Mikako Makita (Eu-XFEL), Claudia Parisuana (Stanford). Not pictured: Ann Truong (UCSD), Alexis Casner (CEA), Andrew Leong (LANL), Nik Christiansen (LANL)

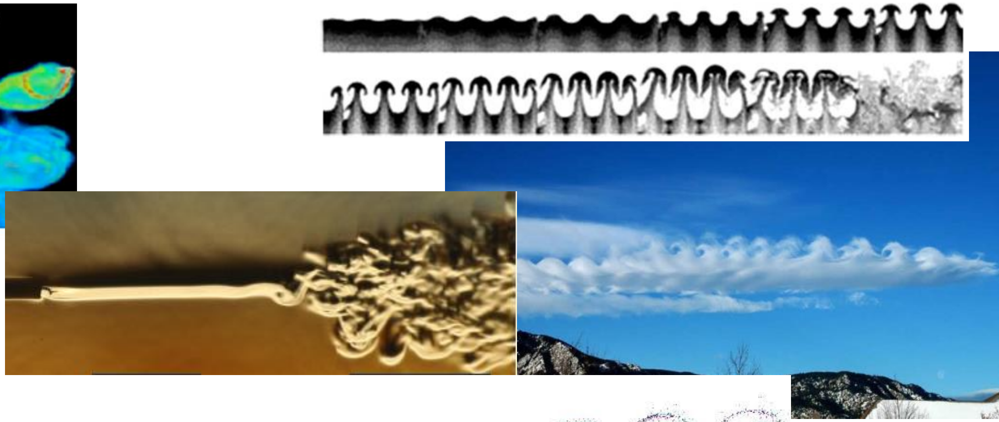
https://www.instagram.com/stories/highlights/17965892239021779/

Outline -> a.k.a “by the end of the lesson, students will be able to”

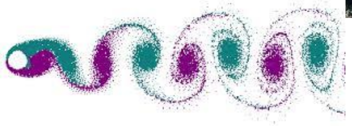
- Describe Turbulence: Fluids / Plasmas
    - Kolmogorov theory of fluid turbulence
    - Reynolds number: viscosity and convection



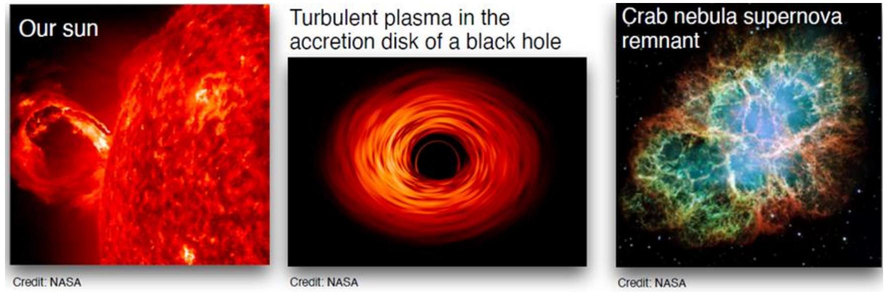
Fei, Yang, PRE 93, 033124 (2016)



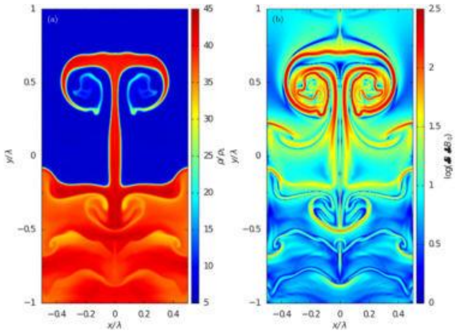
- Understand the role of instabilities in turbulent plasmas
    - Rayleigh–Taylor (RTI), Richtmyer–Meshkov (RMI), Kelvin – Helmholtz, Electro-thermal (ETI)



- Identify mechanisms that drive (or suppress) turbulence
    - Turbulent energy spectrum
    - Energy transport



- Evaluate methods to characterize turbulent states in plasmas:
    - Diagnostics: What parameters do we need to know and what ***can*** we probe?
    - Simulations: Theory, models, codes
        - Machine Learning methods discussed in 2022 lecture -> highly recommend!

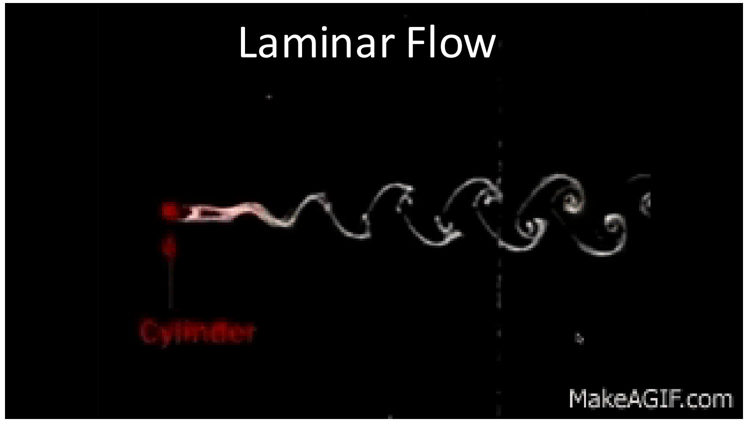


What is Turbulence?

Fluid dynamics definition:

Fluid motion pattern characterized by **chaotic changes** in pressure and flow velocity[1]. In contrast to a **laminar flow** regime, where fluid flows in **parallel layers**, with **no disruption** between those layers.

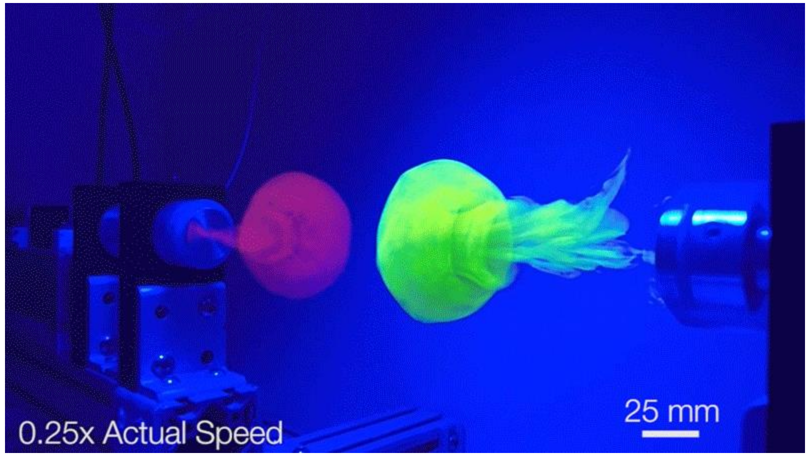
[1] G. Batchelor (2000). Introduction to Fluid Mechanics



*Better video https://www.youtube.com/watch?v=QzuzbwJWlYs

Turbulence ->

- Breaking of symmetry *(Saskia Mordijck - 2018, 2020)*
- Not a property of the fluid, but a feature of the flow *(Steffi Diem - 2019)*
- Swirly *(Manaure Francisquez - 2022 ; Galina Avdeeva - 2023)*

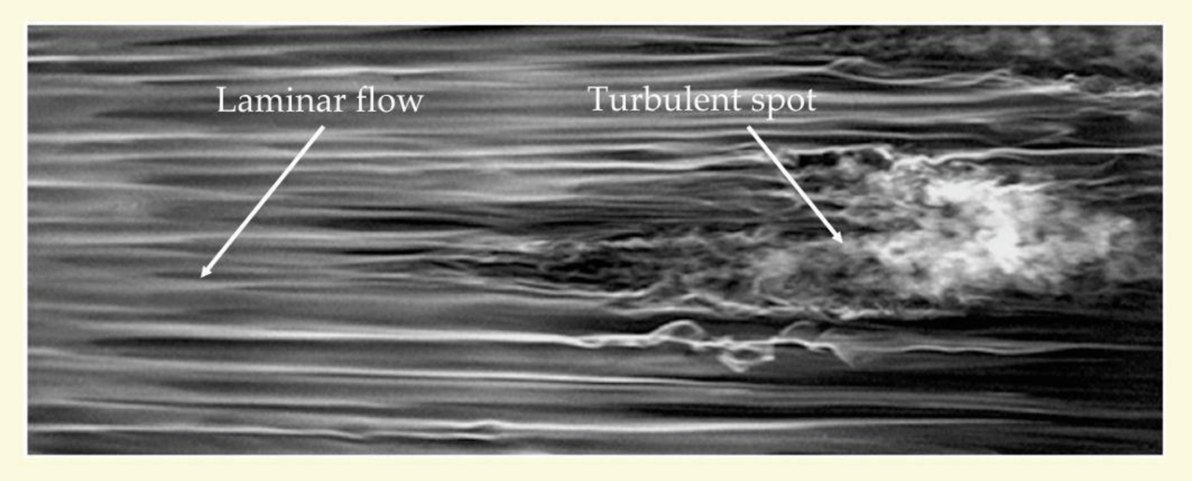


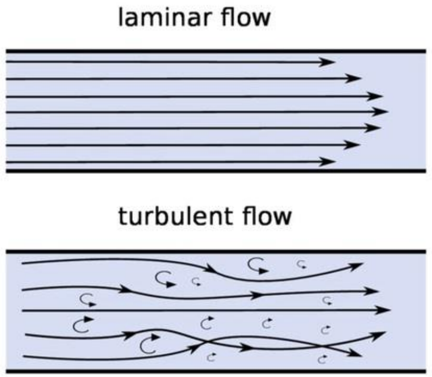
Vortex cannons fire in a 75-gallon aquarium. Each dyed a different color to observe how they interact. (Video courtesy of Harvard SEAS)

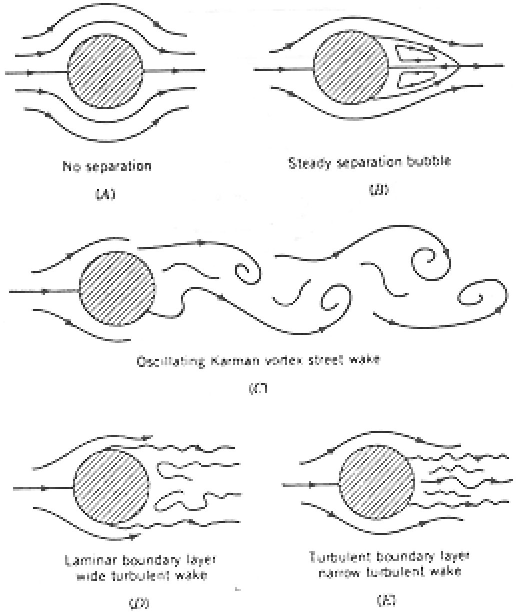
Visually we can say turbulence is:

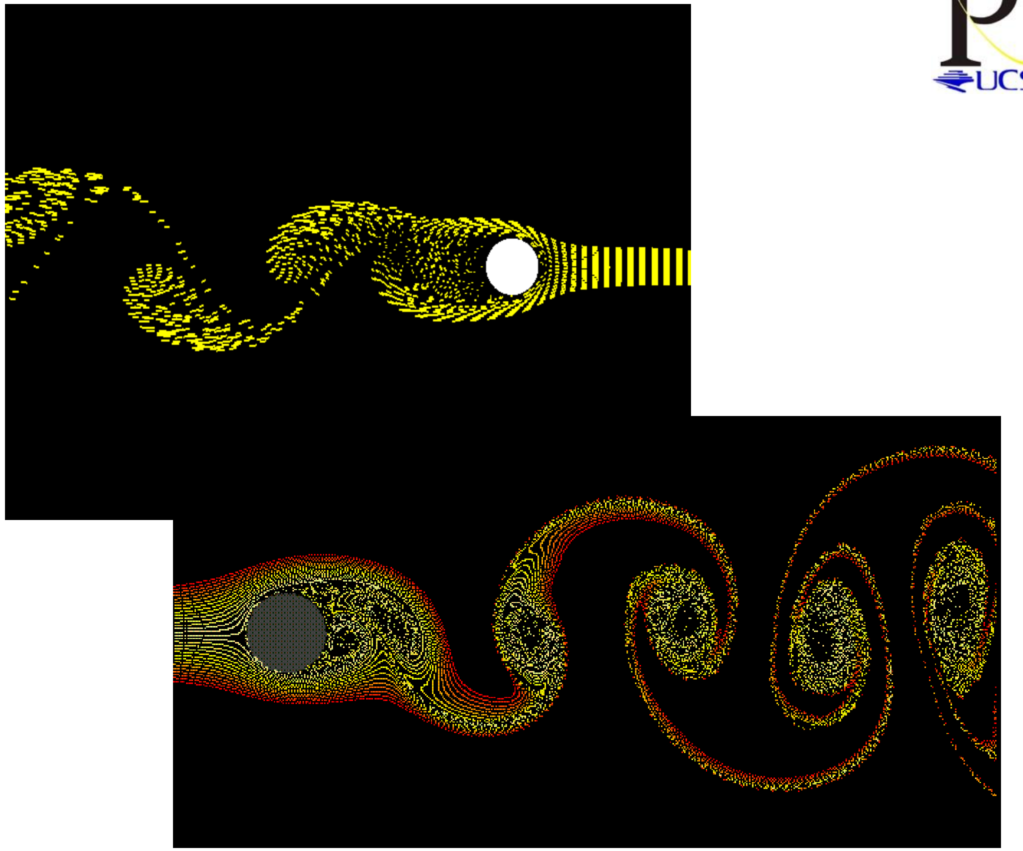
- Complex (irregular, unsteady, unstable, chaotic)
- Diffusive, mixing, ergodic
- Hard to predict -> *we know it will happen. Precise details, not so much…*

A few examples:

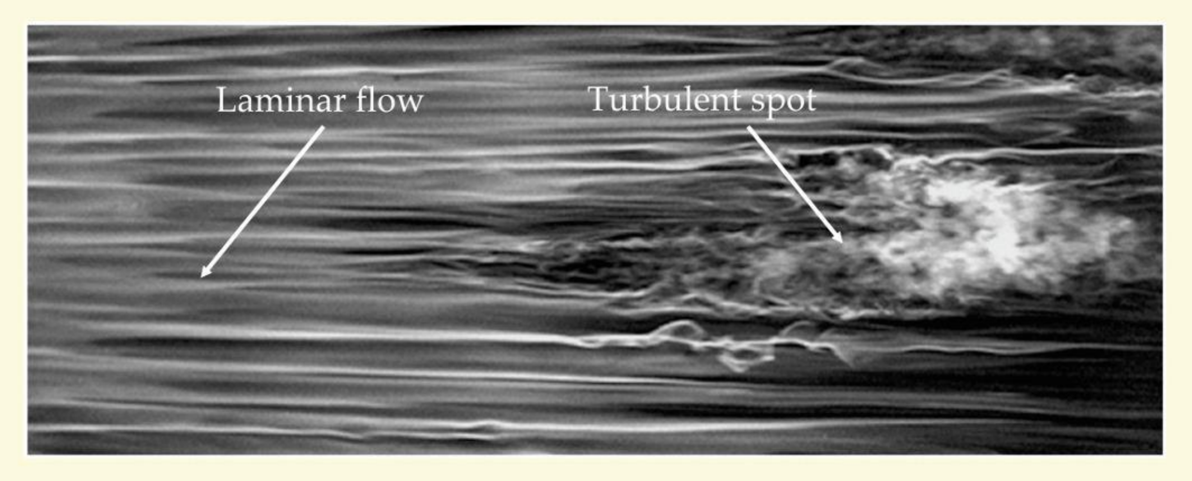


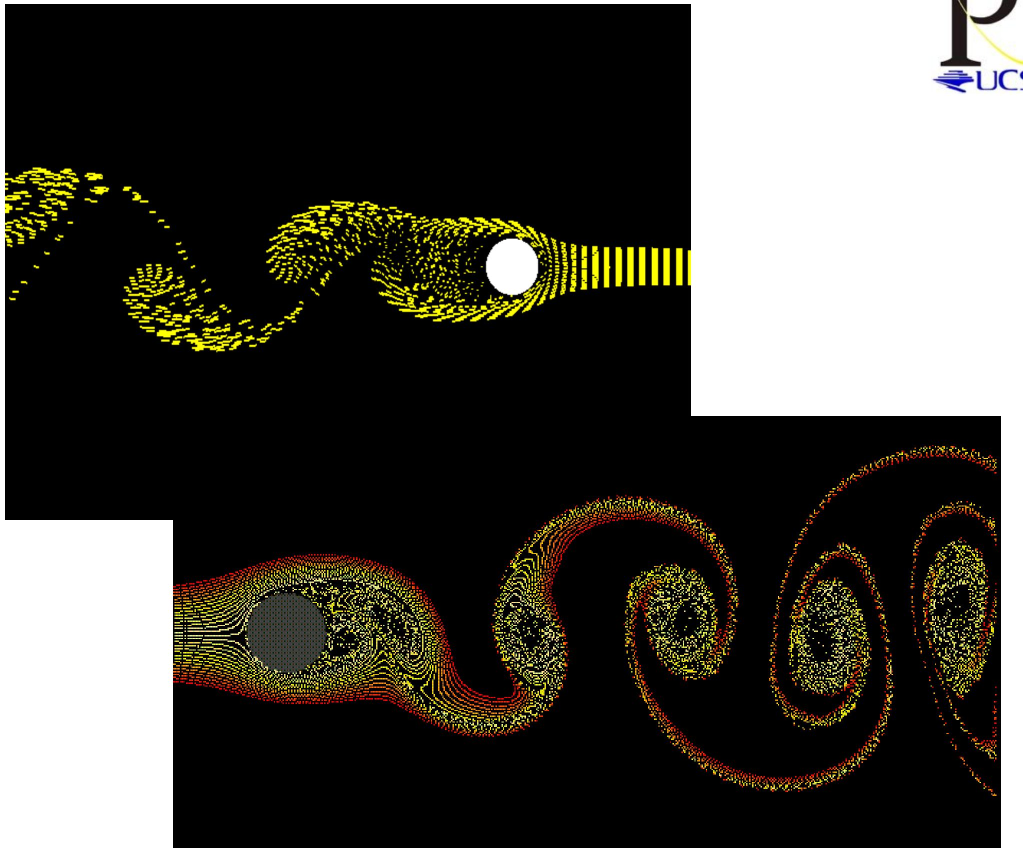


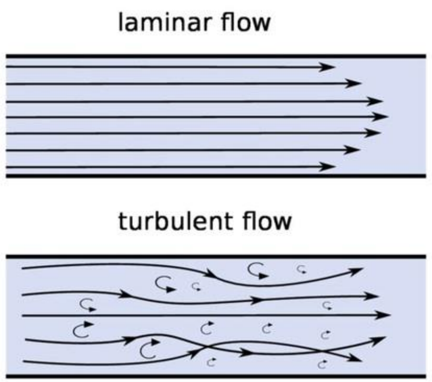


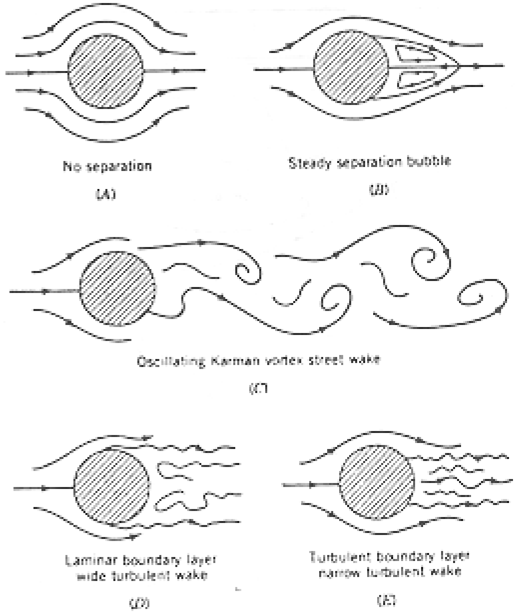


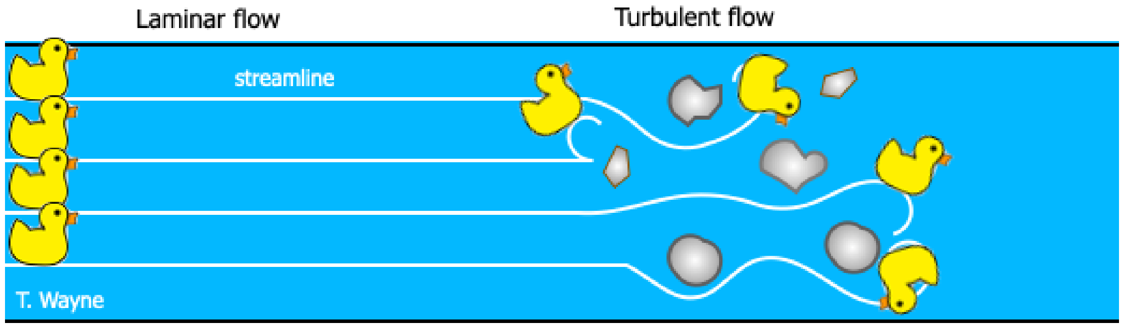
A few examples:











A recipe for Fusion – the Lawson Criterion

There are three main conditions are necessary for controlled nuclear fusion:

The temperature (T_{fuel}) (which is related to the kinetic energy of reactants) must be hot enough to allow the ions of deuterium and tritium to have enough kinetic energy to overcome the Coulomb barrier and fuse together.

The ions must be confined together in close proximity to allow them to fuse. A suitable high ion **density (n_{fuel})** is necessary.

The ions must be held together in close proximity at high temperature with a **confinement time ($\tau_{confinement}$)** long enough to avoid cooling.

The **Lawson criterion** quantifies the conditions for a fusion reactor in order to obtain a net yield of energy from DT fusion reactions. The quality of a fusion device can be measured in terms of the so called "fusion triple product”

$$n_{fuel}\, T_{fuel}\, \tau_{confinement} > 1\text{x}10^{21}\ \text{keV s / m}^3$$

(1 eV = 11,000 kelvin, so 1 keV is 11 million degrees centigrade)

You can bake fusion in (at least) 3 different ways

Magnetic Confinement Fusion (MCF): Compressing the fuel with magnetic fields

Magnetic Confinement Fusion is based on the fact that ions and electrons cannot travel across a magnetic field easily. Hot plasma can be confined by strong magnetic fields but **densities are limited** by magneto-hydro-dynamic effects. With these densities, the energy confinement time required is in the range of a few seconds:

n_{fuel} ~ 10^{20} m^{-3} $\tau_{confinement}$ ~ 10 s T_{fuel} ~ 10 keV   = $1e^{22}$

Inertial Confinement Fusion (ICF): Compressing the fuel with fast moving mass

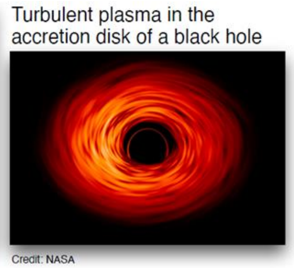
Inertial Confinement Fusion is based on maximizing density by rapid compression and heating of a small solid DT pellet through the use of lasers or particle beams. The energy confinement time is extremely short:

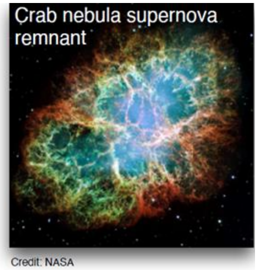
n_{fuel} ~ 10^{31} m^{-3} $\tau_{confinement}$ ~ $1^{\wedge -9}$ s T_{fuel} ~ 10 keV   = $1e^{23}$

Gravitational Confinement Fusion: Compressing the fuel under large gravitational field

The sun fuses hydrogen, by keeping it hot and together (i.e. at high density) for long times under gravity.

n_{fuel} ~ 10^{29} m^{-3} $\tau_{confinement}$ > $1e^{12}$ s T_{fuel} ~ 2 keV   > $1e^{40}$



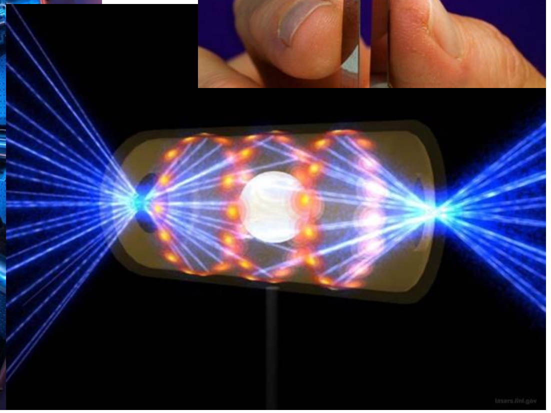


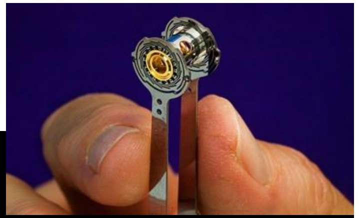
Large scale problems require large scale experiments

Drivers for Inertial Confinement Fusion

High Intensity Lasers

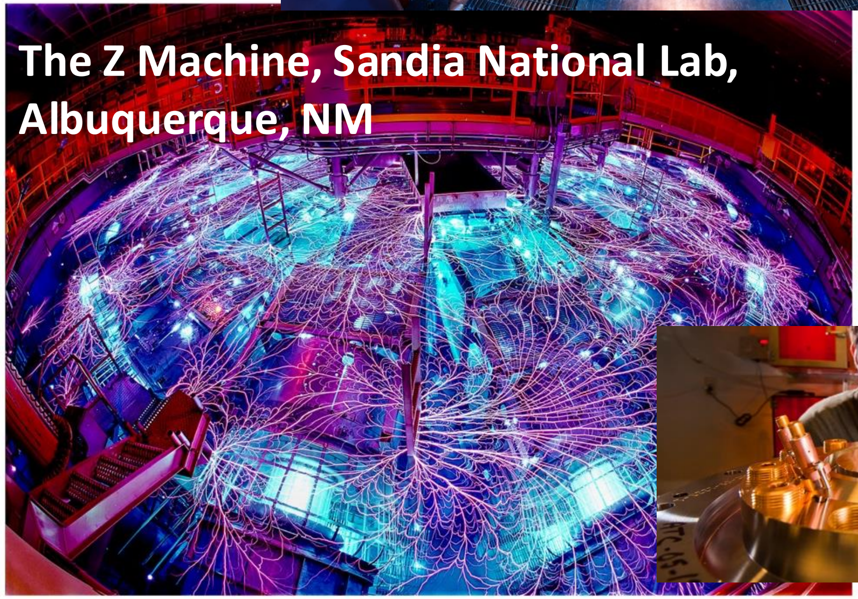
- Direct Drive
- Indirect Drive
- Shock fusion
- Fast ignition

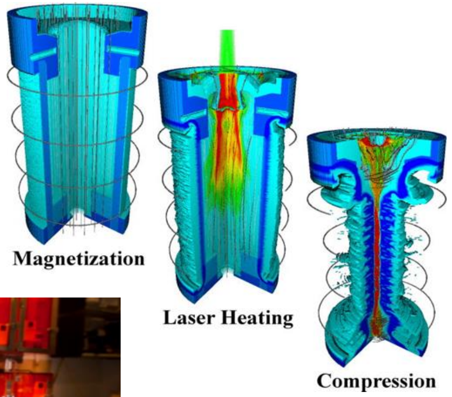




Pulsed Power

- Liner fusion
- Plasma Focus
- Indirect drive

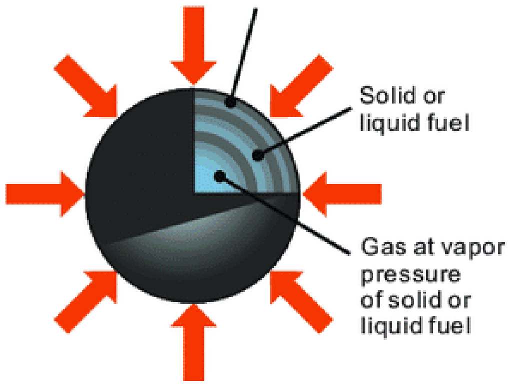
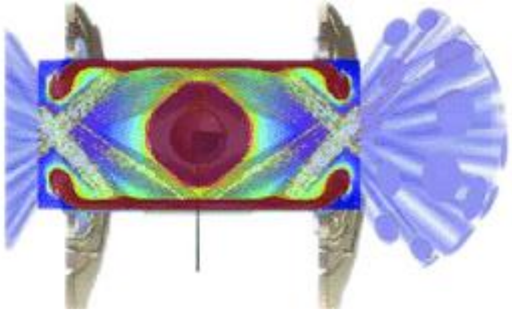
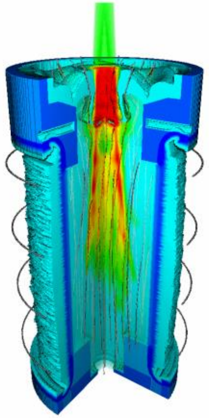




Heavy Ion Beams

- Direct Drive
- Indirect drive

Main Inertial Confinement Fusion (ICF) approaches

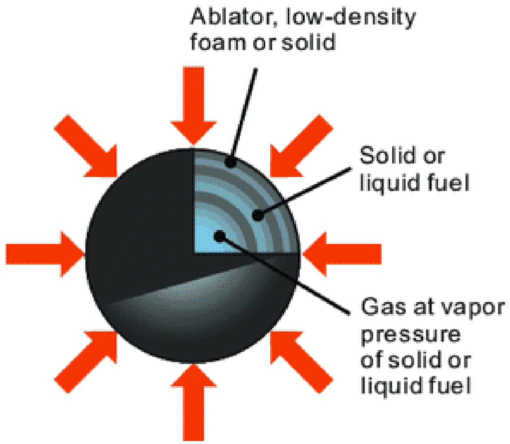
		Why?	Main target issues for getting more energy out (Gain >1)
Direct driver laser fusion 60 lasers hit the outside of the spherical target filled with fuel -> OMEGA 60 laser		Direct use of laser beams make best use of their energy	• Still not 100% symmetric • Hydrodynamic instabilities • Lasers are very inefficient (2%)
Indirect driver laser fusion* 192 lasers hit the inside of a metal can. It heats up and radiates x-rays that heat a fuel-filled spherical target -> National Ignition Facility (NIF)		Using can (hohlraum) makes symmetrically heating the target much easier	• Hydrodynamic instabilities • Even more inefficient than direct drive
Magnetized liner fusion Pulsed power machine crushes a cylindrical can filled with fuel with magnetic field	 	Pulsed power (capacitors) is much more efficient than lasers (20%) Cylindrical target makes more sense to drive a high current	• Magneto-Hydrodynamic instabilities • Electrodes delivering the current are destroyed every time

Main Inertial Confinement Fusion (ICF) approaches

Main target issues for getting more energy out (Gain >1)

Why?

Direct driver laser fusion
60 lasers hit the outside of the spherical target filled with fuel -> OMEGA 60 laser



Direct use of laser beams make best use of their energy

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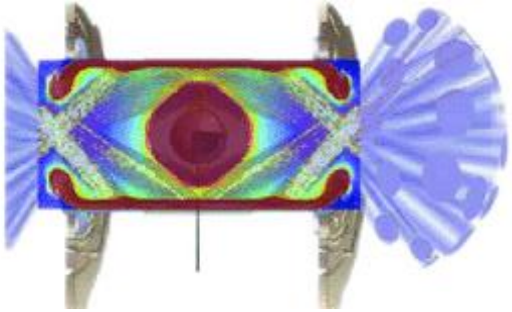

Using can (hohlraum) makes symmetrically heating the target much easier

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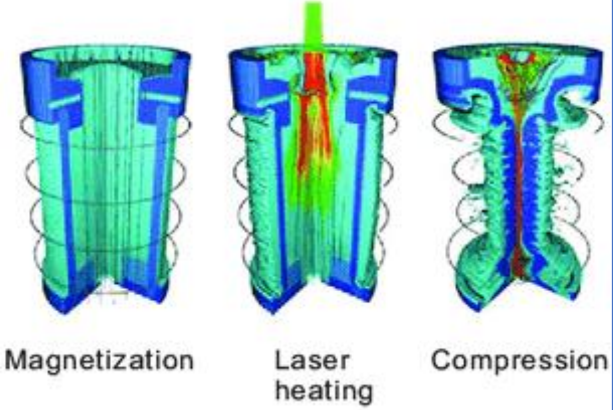
- Even more inefficient than direct drive

Magnetized liner fusion
Pulsed power machine crushes a cylindrical can filled with fuel with magnetic field



Pulsed power (capacitors) is much more efficient than lasers (20%)

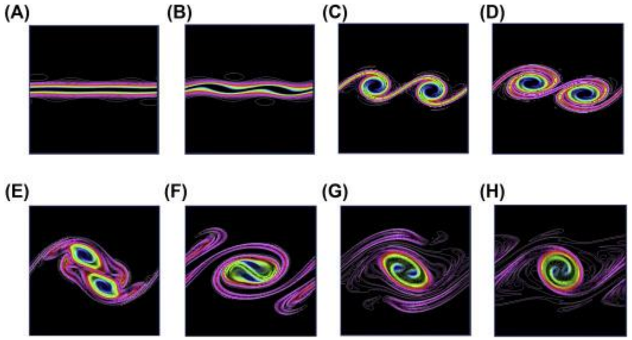
Cylindrical target makes more sense to drive a high current

- Magneto-Hydrodynamic instabilities
- Electrodes delivering the current are destroyed every time

Features of turbulence:

- Eddies -> *many different length scales*

Swirly



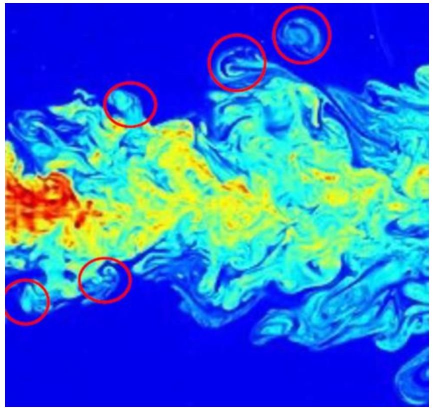
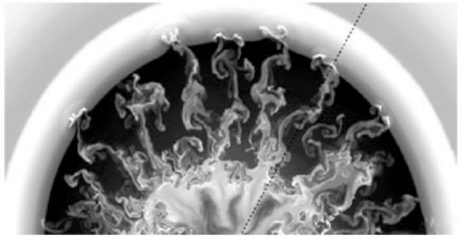
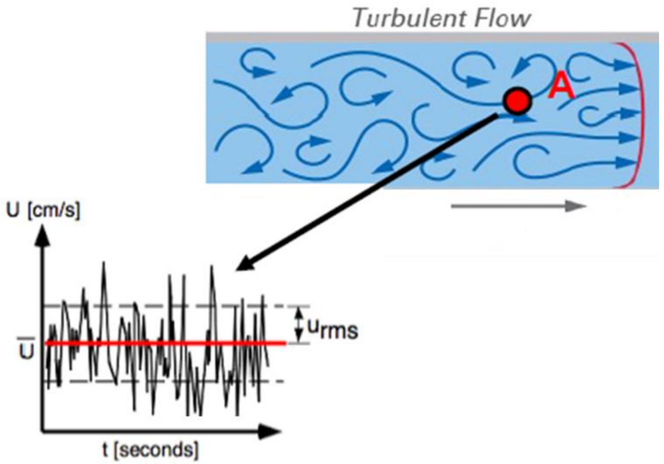


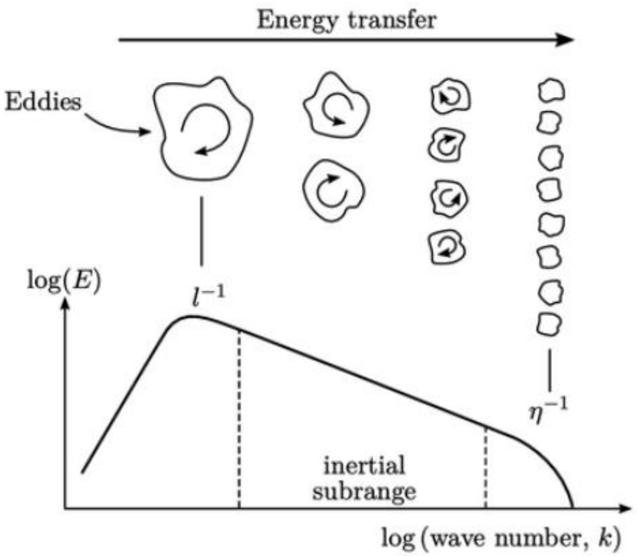
Fig. 4.27 Result of a simulation of the explosion of SN 1987A. The forward and reverse shocks are evident near the outer edges. The dramatic structures are discussed in Chap. 5. Credit: Kifonidis et al. (2003)



- Irregularity -> *should be treated statistically*



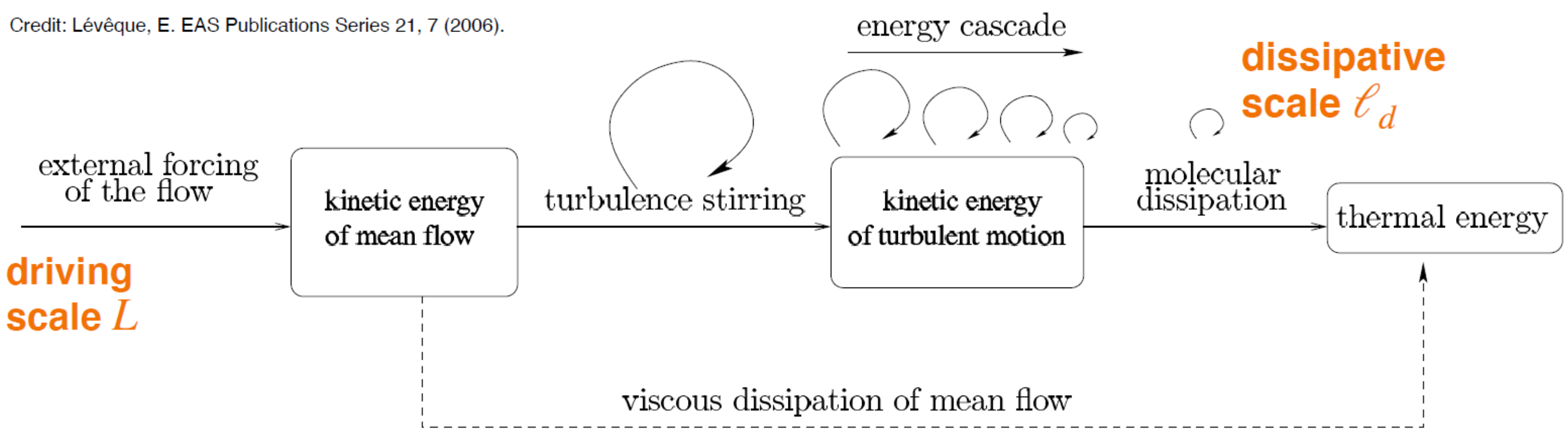
- Energy dissipation / energy transfer-> *so-called "energy cascade"*



Energy transfer in turbulent flow

- Turbulence evolution -> ***Energy cascade***
  - Energy is transferred from large scale structures giving way to smaller (and smaller) eddies
  - Eddies heat up the fluid through molecular dissipation -> kinetic to thermal energy conversion

Credit: Lévêque, E. EAS Publications Series 21, 7 (2006).



- Kolmogorov determined the spectrum of the energy as a function of the size of the eddies[1].
  - Differences in velocity between two points in stationary, homogeneous and ***isotropic*** *turbulence*

$$\delta \mathbf{u}(\mathbf{x}, \mathbf{r}, t) = \mathbf{u}(\mathbf{x}+\mathbf{r}, t) - \mathbf{u}(\mathbf{x}, t)$$

[1] Kolmogorov, A. N. Dokl. Akad. Nauk SSSR, 32, 19-21 (1941).

Transfer of energy between scales

- Equation for $\mathbf{u}(\mathbf{x}, t)$

$$\rho\frac{\partial \mathbf{u}}{\partial t} + \rho\mathbf{u}\cdot\nabla\mathbf{u} = \text{pressure} + \text{viscosity}$$

- For $\mathrm{Re} >> 1$, only possible typical time given by

$$\Delta t \sim \frac{1}{\mathbf{u}\cdot\nabla} \sim \frac{\Delta x}{\Delta u}$$

  - For each scale Δx, there is a corresponding velocity Δu

- Only viscosity dissipates energy $\Rightarrow$ energy flow ε to small scales must be constant within inertial range

$$\varepsilon = \frac{\text{energy}/\text{volume}}{\text{time}} \sim \frac{\rho\Delta u^2}{\Delta t} \sim \frac{\rho\Delta u^3}{\Delta x}$$

***Parra - 2021**

Velocity increment

- Need to determine the velocity Δu for each scale Δx
  - Average velocity of a patch of size Δx is irrelevant, so we need the <u>velocity increment</u>

$$\Delta\mathbf{u}(\Delta\mathbf{x}, \mathbf{x}, t) = \mathbf{u}(\mathbf{x}+\Delta\mathbf{x}, t) - \mathbf{u}(\mathbf{x}, t)$$

- $\Delta\mathbf{u}$ fluctuates $\Rightarrow$ use <u>structure function</u> $\langle \Delta u^2\rangle_{V,t}$
- $\mathbf{u}(\mathbf{x}+\Delta\mathbf{x}, t) \simeq \mathbf{u}(\mathbf{x}, t) + \Delta\mathbf{x}\cdot\nabla\mathbf{u}(\mathbf{x}, t) \Rightarrow \langle \Delta u^2\rangle_{V,t} \propto \Delta x^2$ for small Δx, BUT in the inertial range of turbulence

$$\langle \Delta u^2\rangle_{V,t} \propto \Delta x^{\beta}$$

  - Constant energy flow ε from one scale to another:

$$\varepsilon \sim \frac{\rho\Delta u^3}{\Delta x} \Rightarrow \Delta u \sim \left(\frac{\varepsilon}{\rho}\Delta x\right)^{1/3} \Rightarrow \langle \Delta u^2\rangle_{V,t} \propto \Delta x^{2/3} \Rightarrow \boxed{\beta = \frac{2}{3}}$$

Velocity increment and spectrum

*Parra - 2021

- Expand square inside structure function

$$\langle \Delta u^2 \rangle_{V,t} = \underbrace{\langle u^2(\mathbf{x}+\Delta\mathbf{x})\rangle_{V,t}}_{=\langle u^2\rangle_{V,t}} + \underbrace{\langle u^2(\mathbf{x})\rangle_{V,t}}_{=\langle u^2\rangle_{V,t}} - 2\underbrace{\langle \mathbf{u}(\mathbf{x}+\Delta\mathbf{x})\cdot\mathbf{u}(\mathbf{x})\rangle_{V,t}}_{=\langle u^2\rangle_{V,t} C(\Delta\mathbf{x})}$$

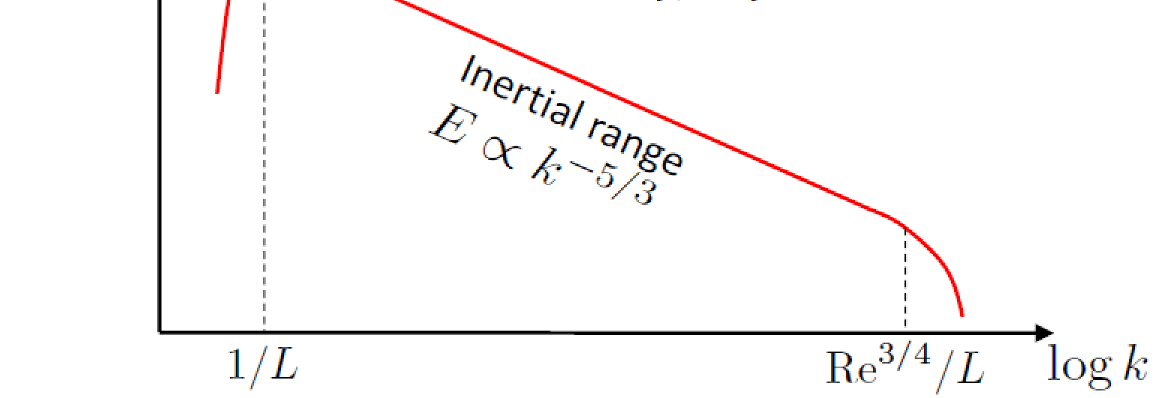
$$\Rightarrow \langle \Delta u^2 \rangle_{V,t} = 2\langle u^2\rangle_{V,t}[1 - C(\Delta\mathbf{x})] = \boxed{\frac{2(2\pi)^3}{V}\int \langle U^2\rangle_t \,[1 - \exp(i\mathbf{k}\cdot\Delta\mathbf{x})]\; d^3k}$$

- Effectively, integral is between
  - Lower limit: $k \sim 1/\Delta x$ ($1 - \exp(i\mathbf{k}\cdot\Delta\mathbf{x})$ is small for smaller k)
  - Upper limit: the dissipation scale
- For Δx in inertial range,

$$\langle \Delta u^2 \rangle_{V,t} \propto \int_{1/\Delta x}^{\text{dissipation}} k^{-\alpha}\, dk \simeq \frac{\Delta x^{\alpha-1}}{\alpha - 1} \Rightarrow \boxed{\beta = \alpha - 1}$$

Kolmogorov's spectrum

- $\langle \Delta u^2 \rangle_{V,t} \propto \Delta x^{2/3} \Rightarrow \beta = 2/3 \Rightarrow \alpha = 5/3$
- Dissipation: $\underbrace{\varepsilon}_{\sim \rho u^3/L} \sim \underbrace{\rho\nu \left\langle \sum_i \left| \frac{\partial \mathbf{u}}{\partial x_i} \right|^2 \right\rangle_{V,t}}_{\sim \rho\nu \Delta u_\nu^2/\Delta x_\nu^2} \Rightarrow \Delta x_\nu \sim \frac{L}{\mathrm{Re}^{3/4}}$



More on cascades

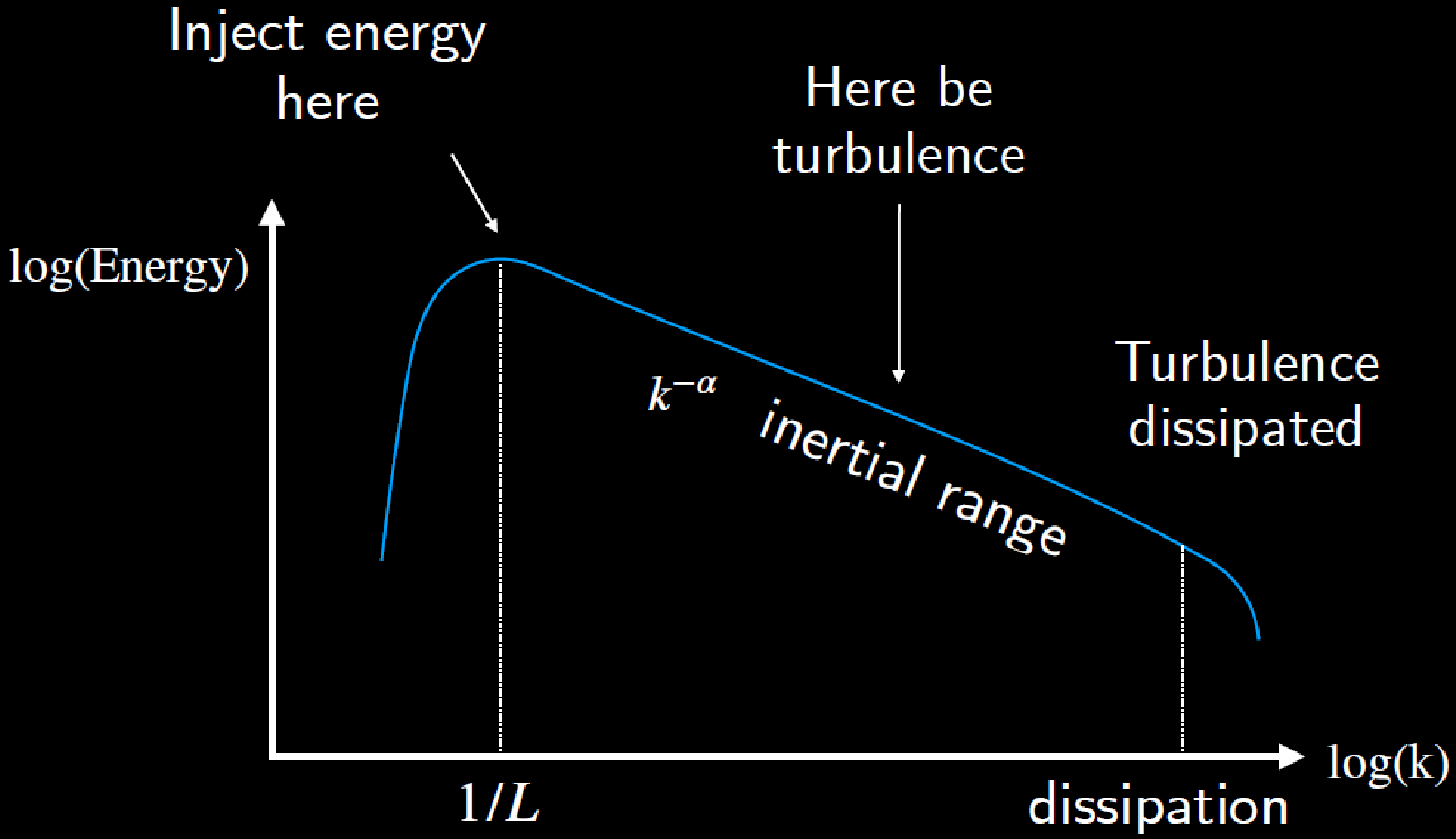
- Far more rigorous treatment in U. Frisch's book *Turbulence: The legacy of A.N. Kolmogorov*

Turbulent cascade

Turbulence cascades energy from injection scale to dissipative scale

Injection scale L

***Parisi - 2024**



Plot energy versus wavenumber k

Energy transfer between scales: The Kolmogorov spectrum[1,2]

The two hypotheses were:

1. At $r \ll L$ the distributions of $\delta\mathbf{u}(\mathbf{x},\mathbf{r},t)$ are "universal", i.e. independent of the type of external force or driving mechanism, & only a function of ν (viscosity) and the mean energy dissipation rate per unit mass ($\bar{\epsilon}$).

   Units $\left.\begin{array}{l}[\nu] = m^2/s \\ [\bar{\epsilon}] = J/(kg\, s) = m^2/s^3\end{array}\right\}$ $\langle|\delta\mathbf{u}(\mathbf{x},\mathbf{r},t)|^2\rangle = \sqrt{\nu\bar{\epsilon}}\,\Phi(r/\ell_d)$

   can also infer the dissipation length scale $\ell_d = \left(\nu^3/\bar{\epsilon}\right)^{1/4}$

$\langle\ldots\rangle$: ensemble average

Φ : Universal dimensionless function

2. At $\ell_d \ll r \ll L$ the distributions of $\delta\mathbf{u}(\mathbf{x},\mathbf{r},t)$ are independent of ν.

$$\langle|\delta\mathbf{u}(\mathbf{x},\mathbf{r},t)|^2\rangle = b\,(\bar{\epsilon}\,r)^{2/3} = b\,\bar{\epsilon}^{2/3}\,k^{-2/3}$$

k : wavenumber

From this last equation deduce the energy spectrum

$$\boxed{\mathcal{E}(k) = C\,\bar{\epsilon}^{2/3}\,k^{-5/3}}$$

Kolmogorov spectrum

since

$$\langle|\delta\mathbf{u}(\mathbf{x},\mathbf{r},t)|^2\rangle \propto E = \int_0^\infty \mathcal{E}(k)\;\mathrm{d}k$$

[1] Kolmogorov, A. N. Dokl. Akad. Nauk SSSR, 32, 19-21 (1941).
[2] Lévêque, E. EAS Publications Series 21, 7 (2006).

PRINCETON PLASMA PHYSICS LABORATORY

- Turbulent energy transfer is typically from large to small scales, but here's an example of **inverse energy *cascade***

Jupiter's turbulent atmosphere forms organized "jet streams"

Somehow energy is transferred from small scales to these large scale flows.

*Francisquez – 2022

Van Gogh's *La Nuit étoilée*

"the probability distribution function (PDF) of luminance fluctuations in some impassioned van Gogh paintings, painted at times close to periods of prolonged psychotic agitation of this artist, compares notable well with the PDF of the velocity differences in a turbulent flow as predicted by the statistical theory of Kolmogorov" [1]

[1] J. L. Aragón, et al. J. of Mathematical Imaging & Vision 30, 275 (2008).

The determination of the energy transfer rate between scales is one of the most consistent results in turbulence research

How did we get here?

Transition to Turbulence

laminar flow —> .... —> turbulent flow

- Turbulent flow transition at a critical flow velocity, u_{crit}, or length scale L_{crit}
- Described by conservation of momentum:

viscosity

$$\rho \underbrace{\left(\frac{\partial \mathbf{u}}{\partial t} + \mathbf{u}\cdot\nabla\mathbf{u}\right)}_{\text{derivative moving with fluid particle}} + \nabla p = \rho\nu\nabla^2\mathbf{u}$$

density

pressure

flow velocity

- Balancing convective derivative with viscosity gives Reynolds number:

$$\boxed{\mathrm{Re} \equiv \frac{uL}{\nu}}$$

$$\frac{\rho\mathbf{u}\cdot\nabla\mathbf{u}}{\rho\nu\nabla^2\mathbf{u}} \sim \frac{u^2/L}{\nu u/L^2} = \frac{uL}{\nu}$$

*Parisi – 2024

Reynolds number

- Transition between laminar and turbulent flow can be understood by comparing terms in momentum conservation equation

density

$$\rho \frac{\mathrm{d}\mathbf{u}}{\mathrm{d}t} = -\nabla p + \rho\nu\nabla^2\mathbf{u}$$

Material time derivative

Pressure

Viscosity

$$\nabla^2 = \frac{\partial^2}{\partial x_1^2} + \frac{\partial^2}{\partial x_2^2} + \frac{\partial^2}{\partial x_3^2}$$

$$\frac{\mathrm{d}}{\mathrm{d}t} = \frac{\partial}{\partial t} + \boxed{\mathbf{u}\cdot\nabla}$$

$$= \frac{\partial}{\partial t} + \boxed{u_1\frac{\partial}{\partial x_1} + u_2\frac{\partial}{\partial x_2} + u_3\frac{\partial}{\partial x_3}}$$

Convection

- Ignore pressure p: for water, only ensures ρ = constant
- Compare the other two terms $\Rightarrow$ Reynolds number

$$\frac{|\mathrm{d}\mathbf{u}/\mathrm{d}t|}{\nu|\nabla^2\mathbf{u}|} \sim \frac{|\mathbf{u}\cdot\nabla\mathbf{u}|}{\nu|\nabla^2\mathbf{u}|} \sim \frac{u^2/L}{\nu u/L^2} = \frac{uL}{\nu} = \mathrm{Re}$$

- Transition to turbulence at $\mathrm{Re} \sim 10^4 \Rightarrow u_{\mathrm{crit}} \propto 1/L$

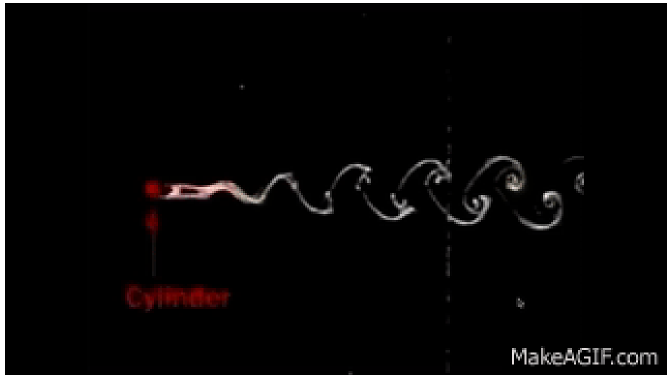
*Parra - 2021

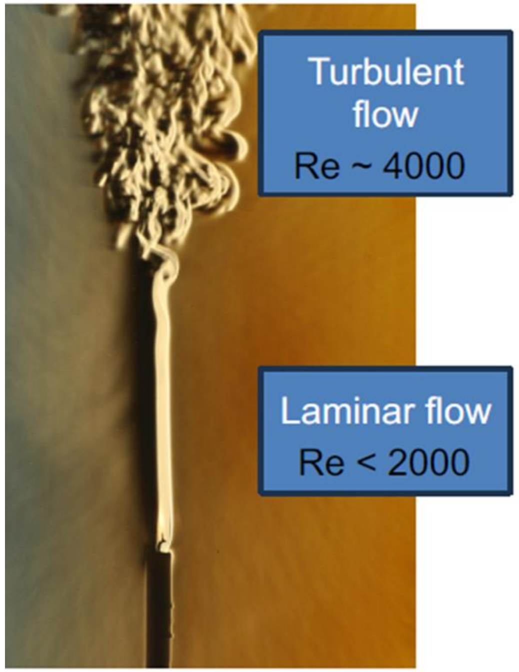
- Measure between kinetic energy and viscous damping:

$$\boldsymbol{R_e} = \frac{\boldsymbol{ud}}{\boldsymbol{v}}$$

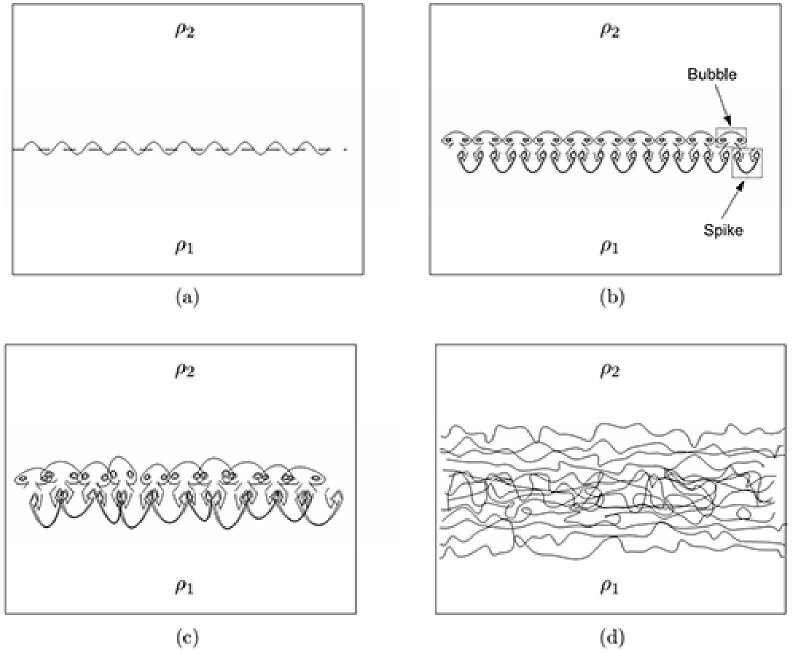
u = macroscopic fluid velocity
d = characteristic length
ν = kinematic viscosity of the fluid

*Avdeeva - 2023





Evolution into turbulent mixing

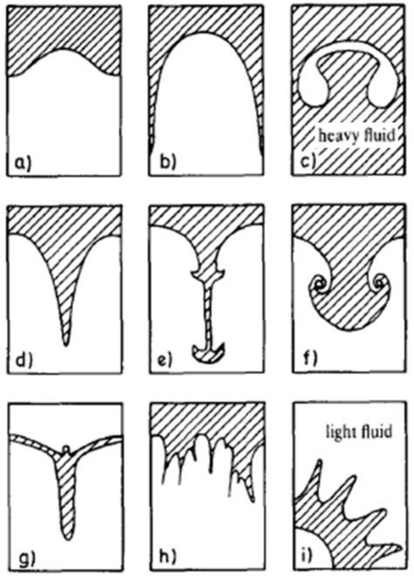


(a): sinusoidal initial perturbation. Non-linear effects begin to appear -> formation of the ubiquitous mushroom-shaped spikes (heavy fluid growing into light fluid) and bubbles (light into heavy)

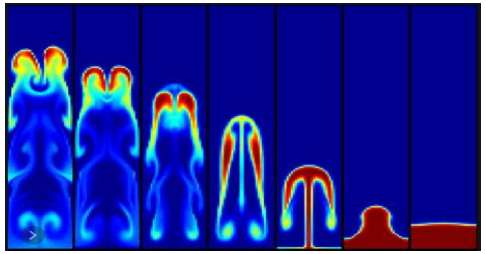
(b): Mushroom-like structures -> growth rate approximately constant in time

(c): Spikes and bubbles begin to interact with one another. Bubble merging

(d): Development of turbulent mixing



acceleration directed vertically upward in (a–h) and radially inward in (i). (a) Normal mode perturbation, (b) free-surface bubble, (c) deformation of a spherical gas bubble, (d) falling spike in a medium with negligible density, (e) falling spike in a medium with finite density, (f) vortices between intermixing fluids with nearly equal densities, (g) anti-spike at the rear side of a thin foil, (h) advanced stage of intermixing, (i) RT instability of a spherical shell.



Turbulence driven by instabilities

Atwood number (A): Ratio of the difference and the sum of fluid densities

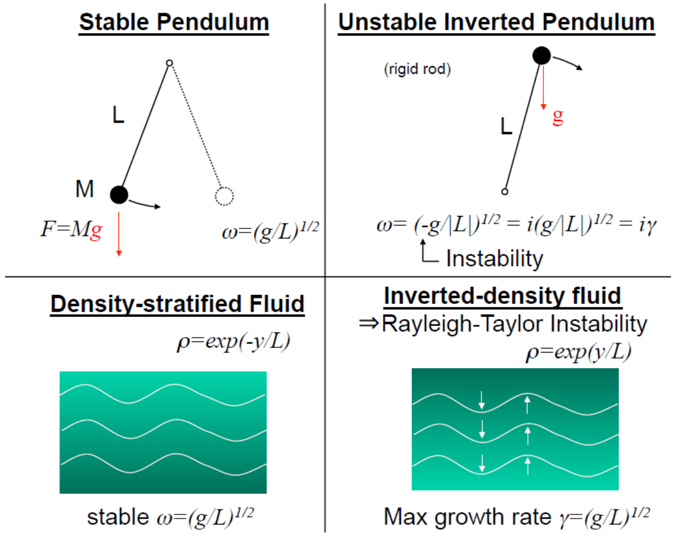
A ~ 1 -> a much lighter fluid below a heavier fluid takes the form of large bubble-like plumes.

A ~ 0 -> Unstable flow takes the form of symmetry fingers of fluid (**RT instability**).

$$A = \frac{\rho_1 - \rho_2}{\rho_1 + \rho_2}$$

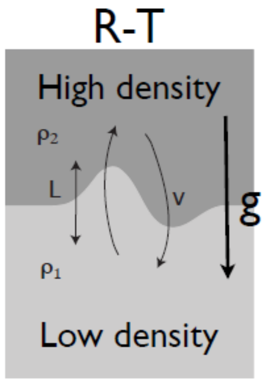
Turbulence arises from unstable equilibrium

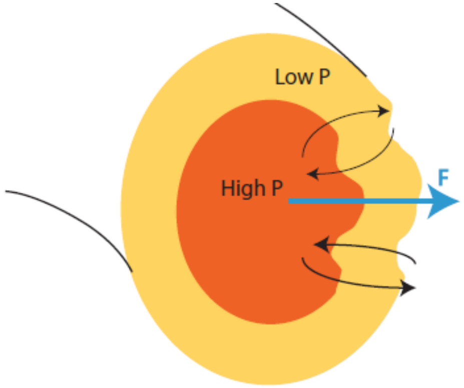
- Perturbations seed instability -> features grow over time



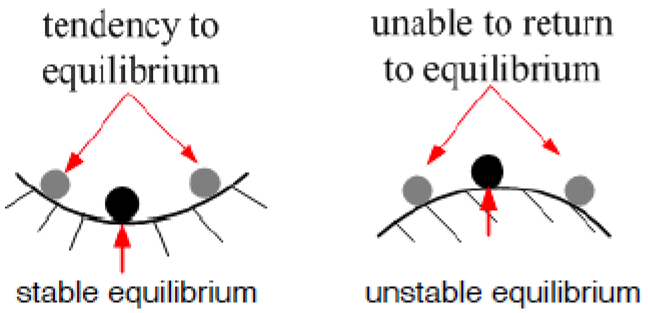
Instabilities -> thermal energy gradients…

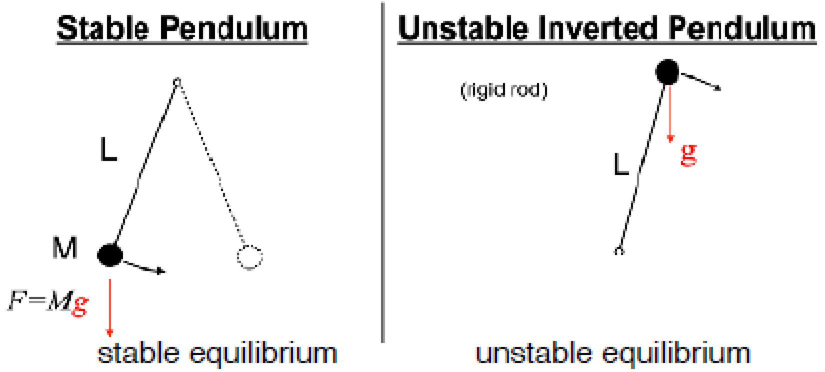
- Perturbations that mix hot core plasma and cold edge plasma can release free energy (and therefore grow)
- Interchange drive is important (analogous to Rayleigh- Taylor). In tokamak, effective gravity provided by magnetic field gradient/curvature





Instability is key: Turbulence arises in systems in unstable equilibrium, where perturbations can grow and lead to instability





- Using Newtonian mechanics ($ma = F$) one can show the frequency of these motions

$$\omega = \sqrt{\frac{g}{L}} \qquad \omega = \sqrt{-\frac{g}{L}} = i\sqrt{\frac{g}{L}} = i\gamma$$

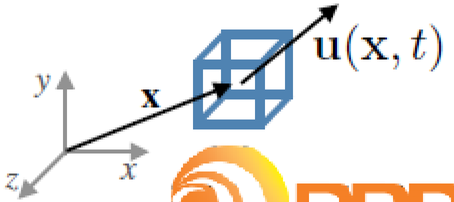
Recall the position of the pendulum: $x(t) \propto A\,e^{i\omega t}$

- Fluid mechanics turbulence is often studied via the Navier-Stokes equation. For incompressible fluids:

$$\underbrace{\rho\frac{D\mathbf{u}}{Dt} = \rho\left(\frac{\partial}{\partial t} + \mathbf{v}\cdot\nabla\right)\mathbf{v}}_{\text{inertia (like } ma \text{ in } F=ma)} = \underbrace{-\nabla p}_{\text{pressure force}} + \underbrace{\nu\rho\nabla^2\mathbf{u}}_{\text{viscous force}} + \underbrace{\rho\mathbf{g}}_{\text{gravitational force}}$$

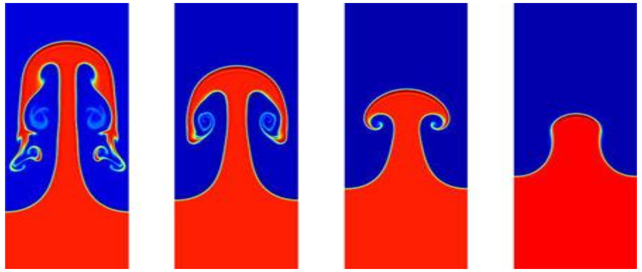
p : pressure
ν : viscosity
$\mathbf{g}$: gravitational acceleration

A statement of Newton's 2nd law for an infinitesimal element at coordinate $\mathbf{x}$

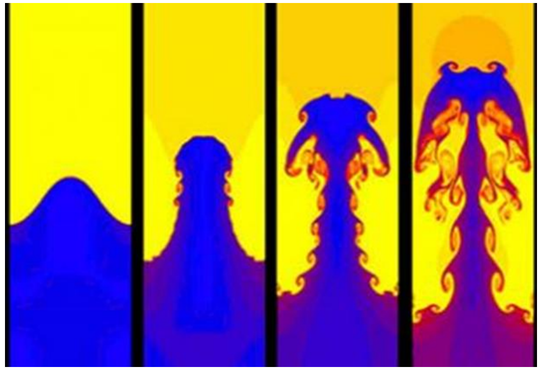


Instabilities

Rayleigh Taylor instability (RTI): Observed at the interface of two fluids -> lighter fluid pushes the denser fluid.



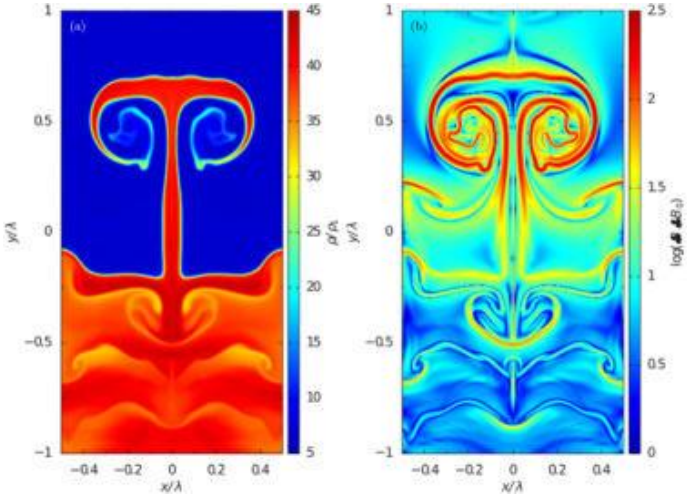
When the heavier fluid seeps in, the lighter fluid is displaced upwards -> equilibrium lost. Perturbation growth goes from linear to non-linear, "plumes" flowing upwards and "spikes" falling downwards.



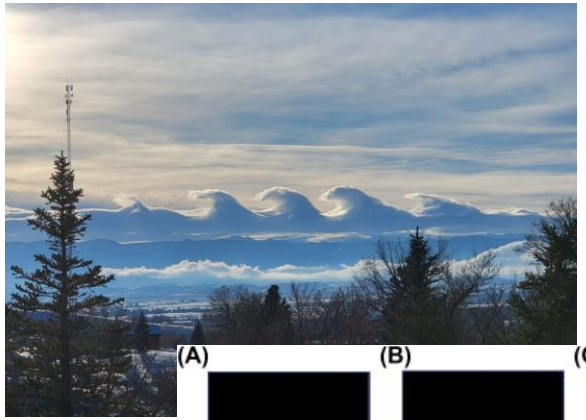
Richtmyer-Meshkov instability (RMI): In this case, two fluids of different density are impulsively accelerated (e.g., passage of a shock wave). Initially, small amplitude perturbations grow linearly in time.

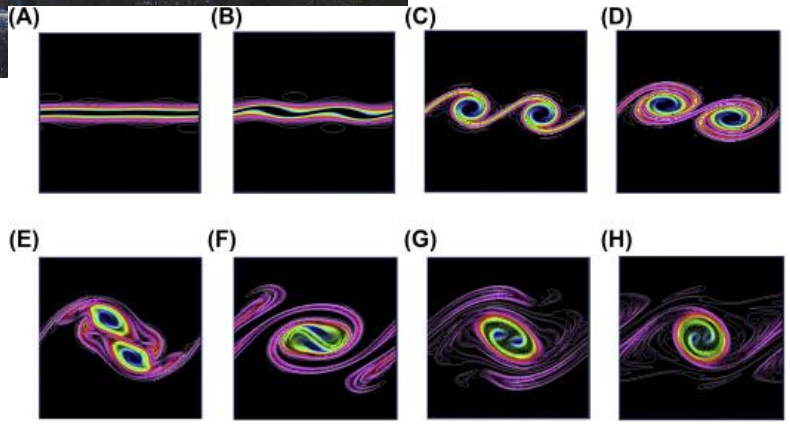
Non-linear regime follows. Light fluid penetrates the heavy fluid -> bubbles appear; spikes appear with heavy to light fluid penetration.

A chaotic regime eventually is reached and the two fluids mix.
RMI can be considered the <u>impulsive-acceleration limit</u> of RTI.



Instabilities





Kevin Helmholtz instability typically arises with:

- Velocity difference across the interface of two fluids. The lighter fluid tries to push the denser fluid.
- Velocity shear in a single continuous fluid induces vorticity at the interface

- Fluid becomes unstable -> vortex sheet rolls up into a spiral.

Many more instabilities in fusion plasmas:

- Alfvén eigenmodes
- Ballooning instability
- Drift wave instability
- Edge Localized Modes
- Electron Temperature Gradient Flute instability
- Geodesic Acoustic Mode (GAM)
- Interchange instability
- Ion Temperature Gradient
- Kink instability
- Sausage instability
- Tearing mode instability, see also Magnetic island
- Whistler mode

Instabilities in High Energy Density Physics

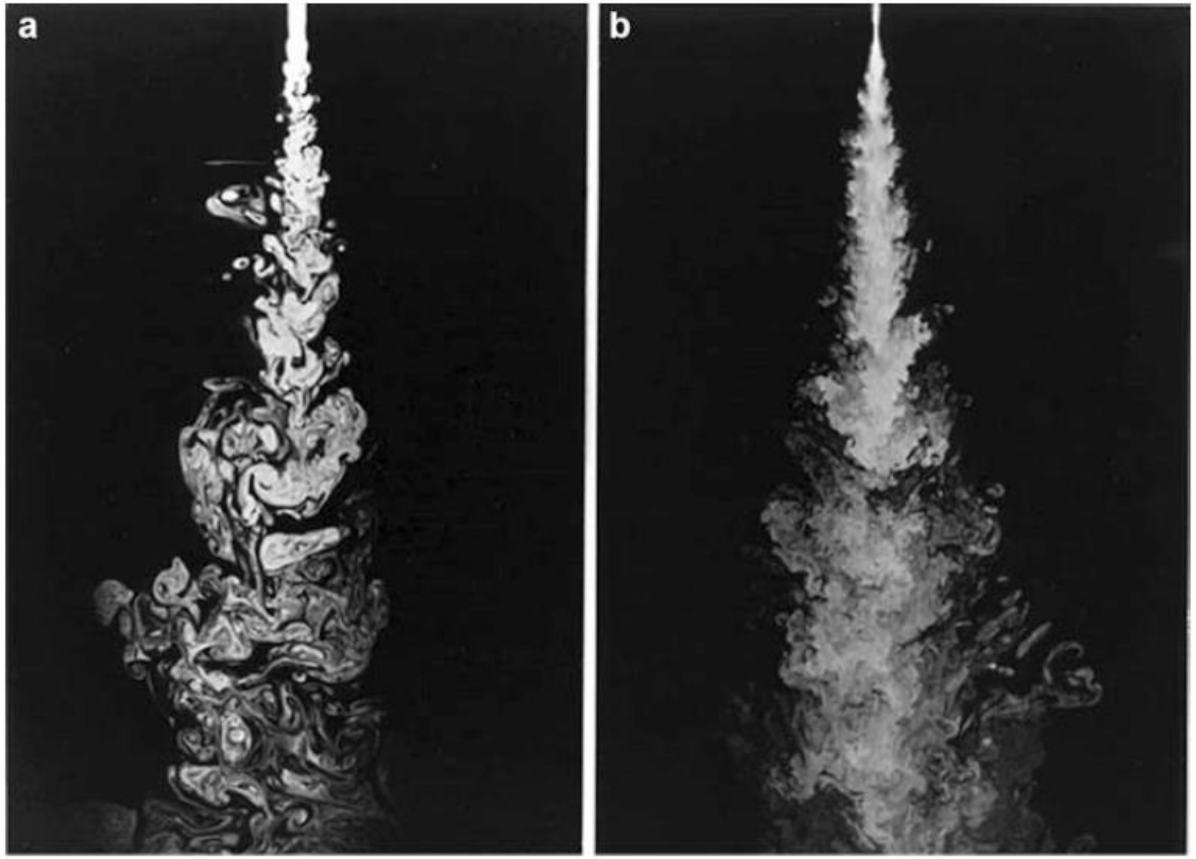


Fig. 5.21 Images of a slice through round turbulent jets in liquids, illuminated by a laser (Dimotakis 2005). (**a**) $Re \approx 2500$. (**b**) $Re \approx 10{,}000$

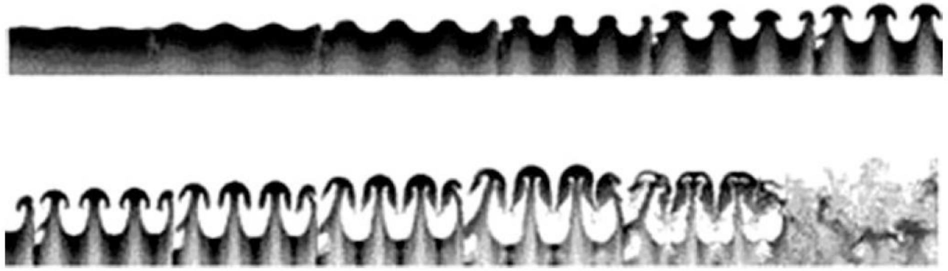
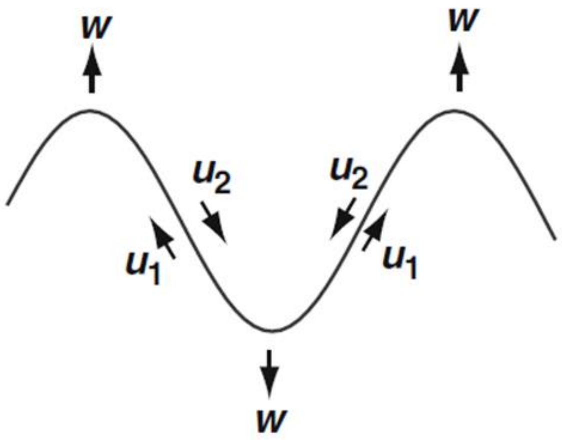


Fig. 5.1 Growth and saturation of the Rayleigh–Taylor instability, observed by acceleration of two fluids in a test facility. Credit: University of Arizona, Jeff Jacobs

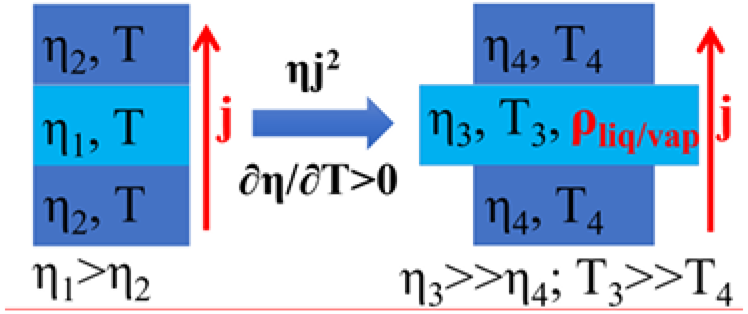
Fig. 5.2 Shear flow induced by Rayleigh-Taylor. The arrows show the location and direction of the maxima in the velocity perpendicular to the interface and along the interface. The amplitude shown is nonlinearly large. In the linear limit, $\boldsymbol{u}$ is horizontal to first order



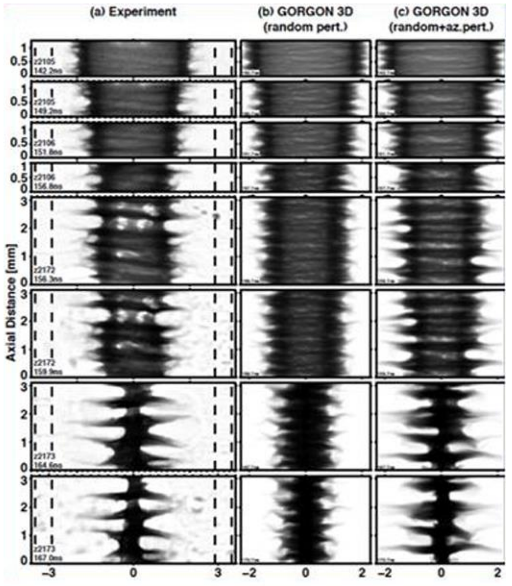
R. Paul Drake. "Introduction to high-energy-density physics."
High-Energy-Density Physics: Foundation of Inertial Fusion and Experimental Astrophysics (2018): 1-20.

Electro-Thermal Instability

- ETI is a Joule heating-driven instability.
- In a metal, the dominant mode manifests as hot/cold bands“striations”
- In a plasma, the dominant mode manifests as plasma filaments
- ETI can seed the Magneto Rayleigh Taylor (MRT) instability



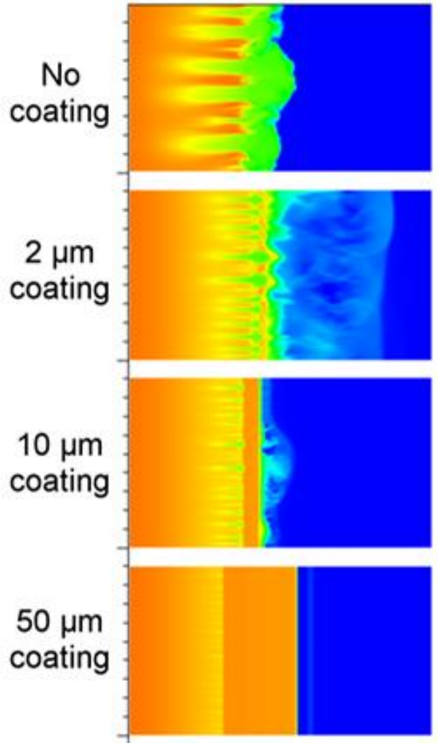
Coated Uncoated Dielectric coating carries current & implodes with liner MRT amplitude reduced by 10X for coated liner



McBride *et al.*, PRL **109**, 135004 (2012)

- If MRT is seeded by ETI, simulations suggest that the ETI-driven density perturbation can be mitigated!
- Thick (>10 μm) insulating coatings mitigate effects of ETI and reduce seed for MRT growth
- No ETI growth in the dielectric (no η,T feedback) Nonlinear mass redistribution from ETI is significantly tamped by the coating
- Reduces seed for MRT growth

*K.J. Peterson, T.J. Awe, et al., PRL 112, 135002 (2014)



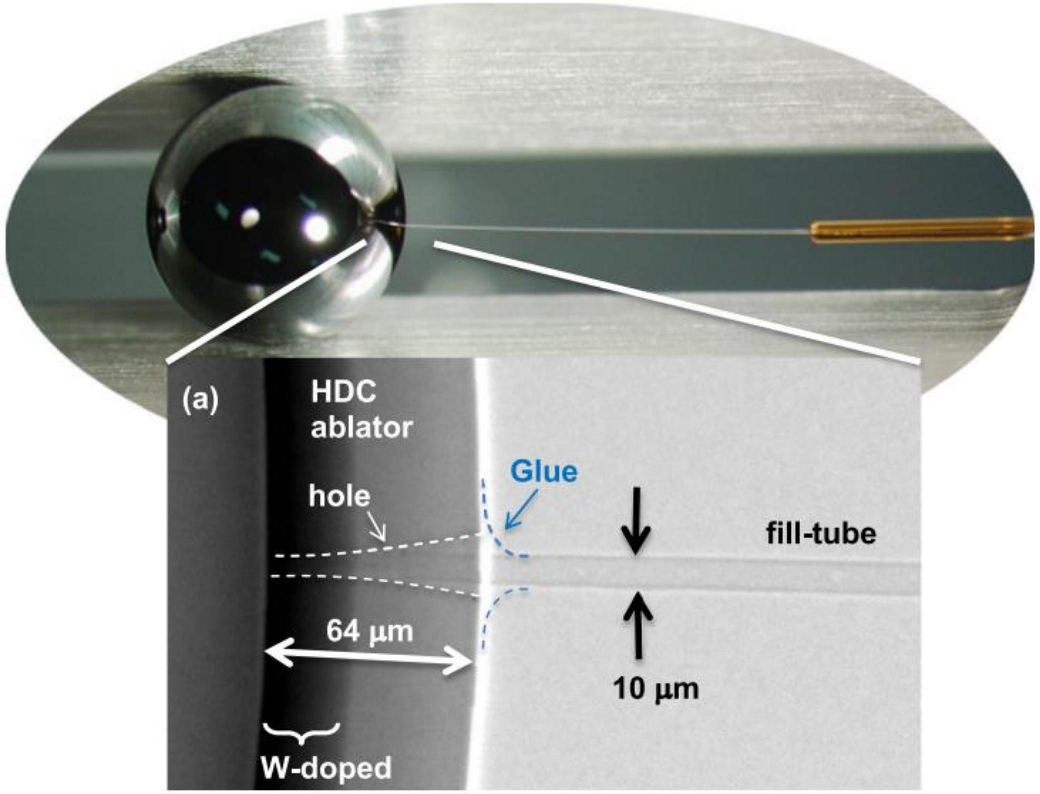
Adding a 70-micron-thick dielectric surface coating greatly enhances the stability of imploding liners!

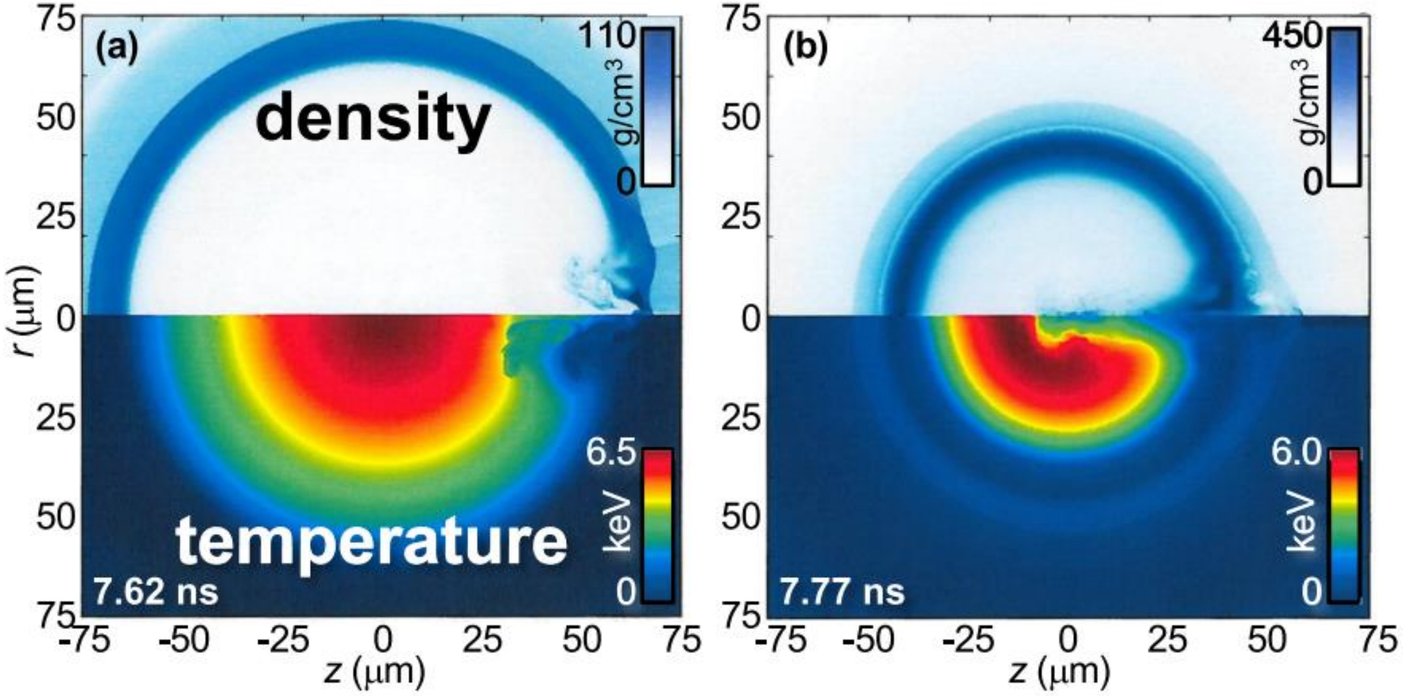
T.J. Awe et al., Phys. Rev. Lett. 116, 065001 (2016)

Why do we care about instabilities?

Example: RMI observed due to fill tube used for ICF capsules -> seeds jetting and instabilities through material mismatch and/or X-ray shadowing

Shrinking fill tube to 2µm significantly helped improve implosion symmetries, but the problem remains…





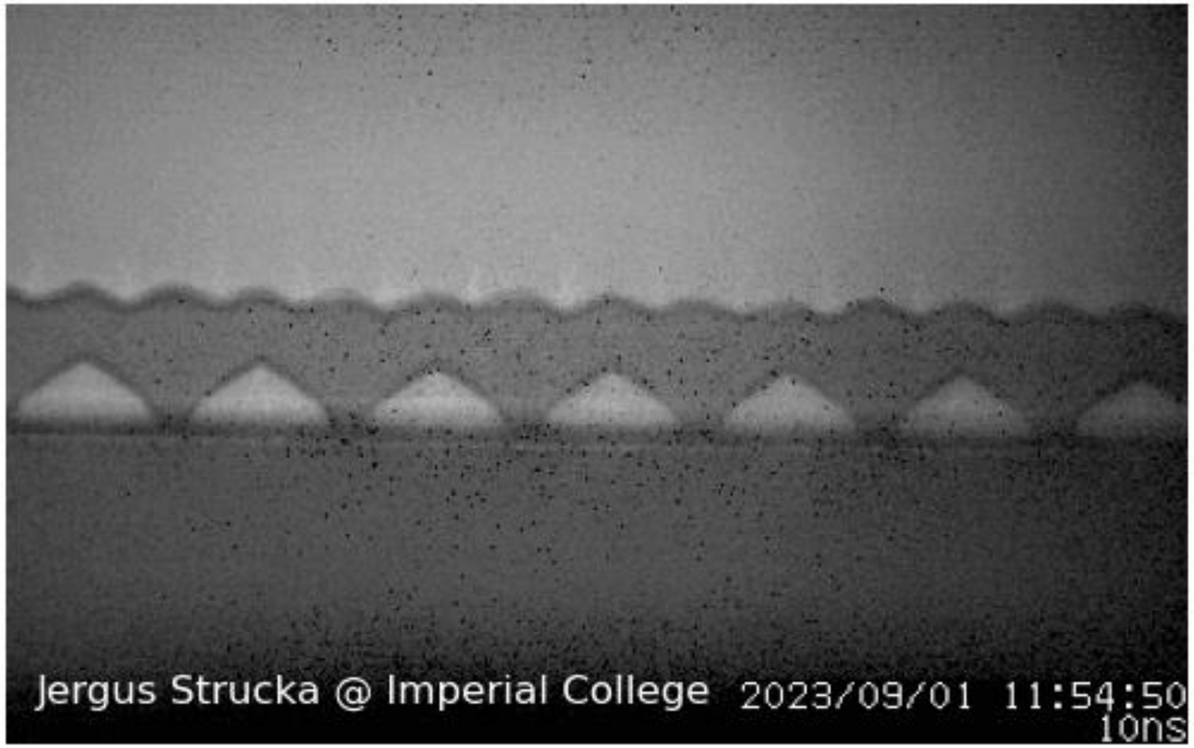
C. R. Weber et al, Phys. Plasmas 27 (2020)

Can we platform test different ideas for mitigating instabilities / jetting?

Yes! Suppression of RMI

Machine learning assisted hydrodynamics simulations at Livermore suggested introducing shaped voids in material could counter growth of RMI

Control



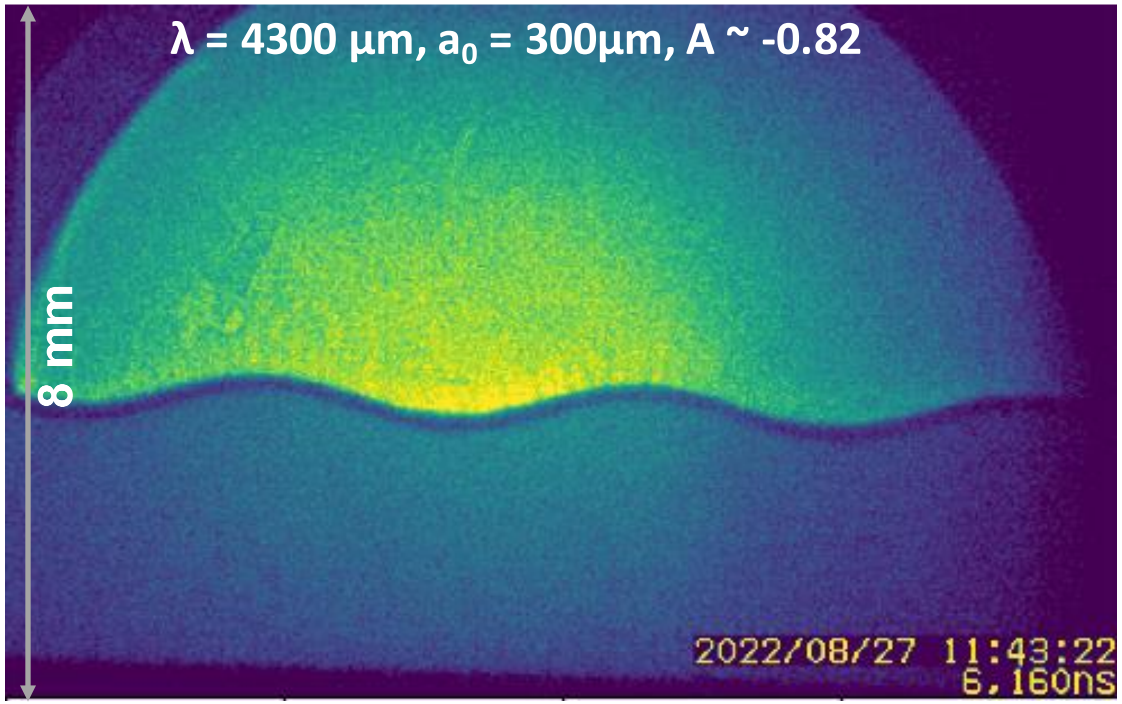
Suppressed

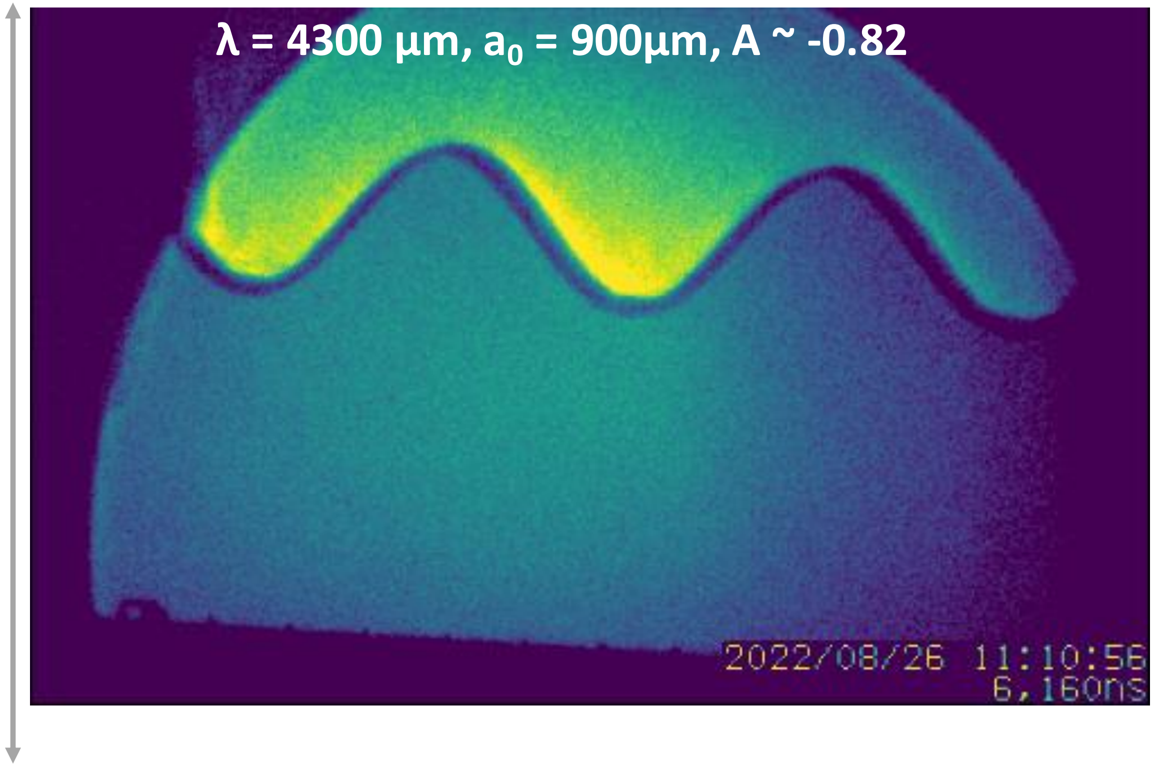
Tested in planar geometries at ESRF

Imperial College London

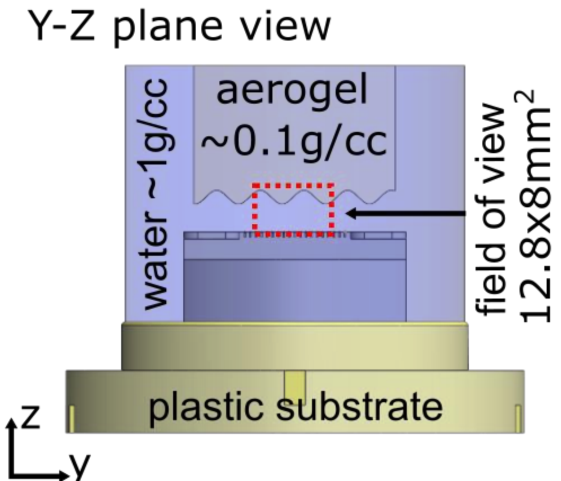
Magnetic Acceleration, Compression, & Heating (MACH)
Stewardship Science Academic Alliances Centers of Excellence

Linear and non-linear RMI growth





False colour X-ray image



Imperial College London

- Non-linear growth when amplitude comparable to wavelength.
- Rapid development of RMI + cavitation in water and KH instabilities

Simulations: FLASH

The mission:

Image instabilities evolution high resolution (better than achieved to date!)

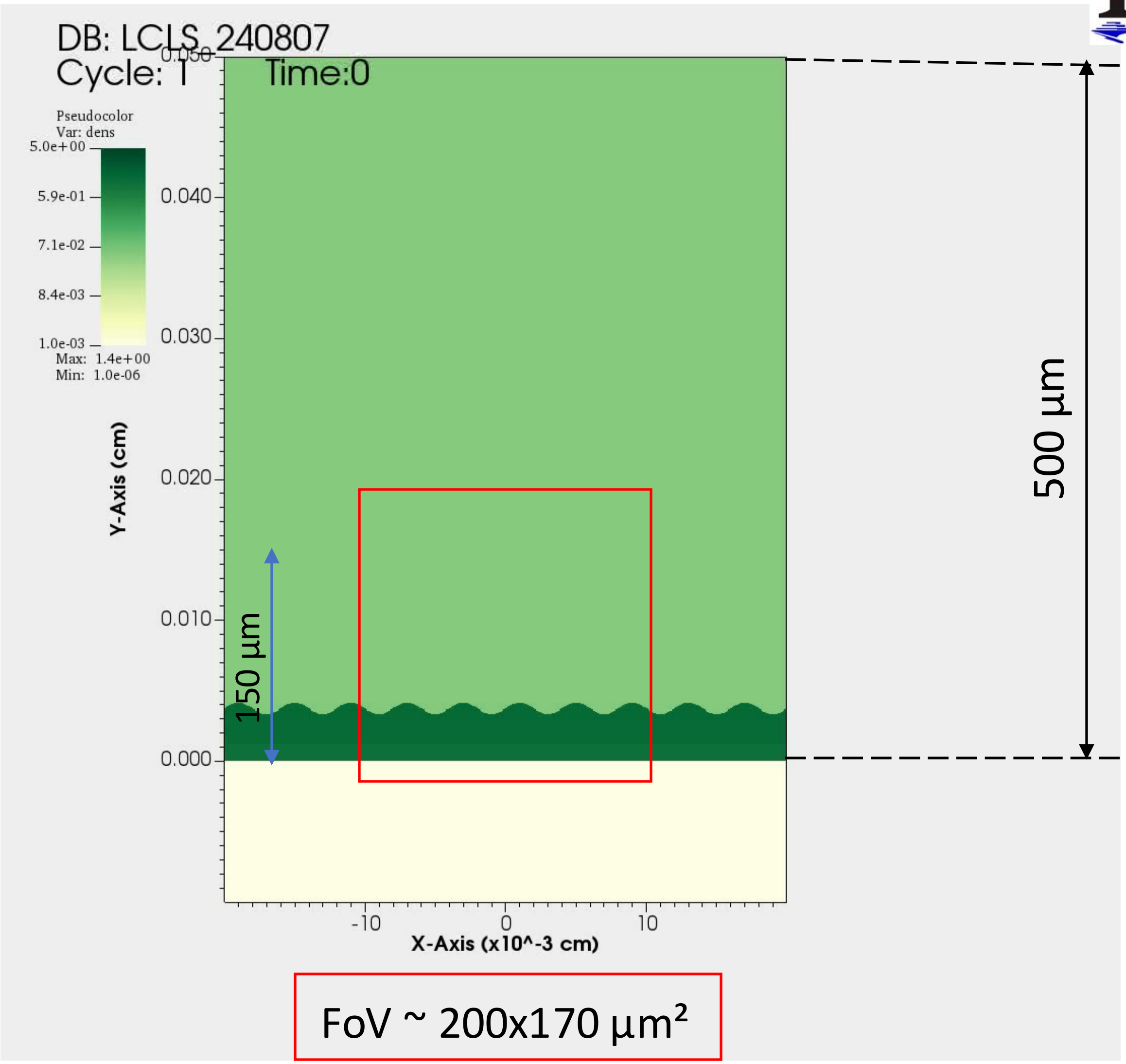
50mg/cc foam; 50J, 5ns laser driver

The challenge:

Detector area is limited -> we need to know shock location!

**Even before the experiment -> we need to design it!*

Platform: Laser driver, target design
Diagnostics: Probing energy (x-ray images), shock speed

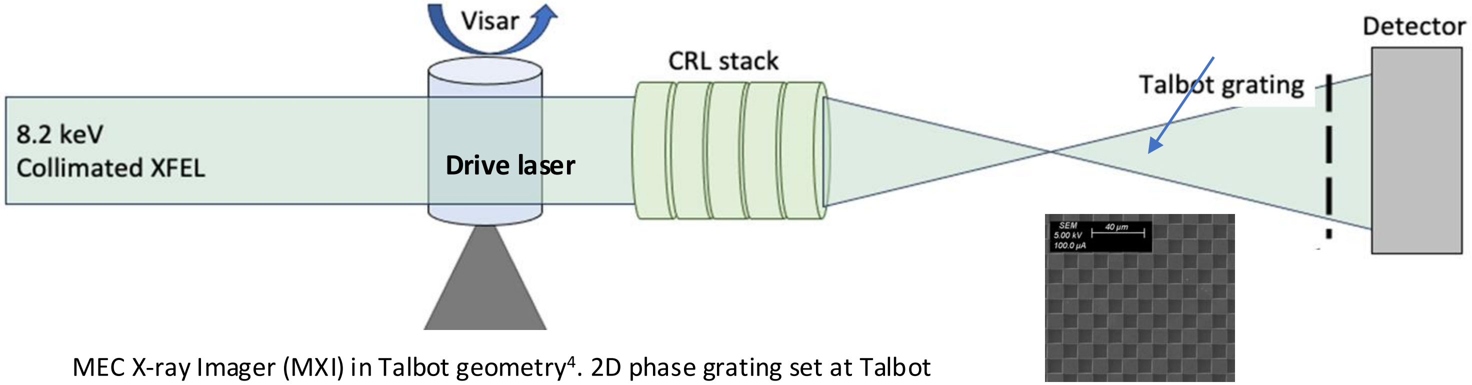


*Simulation G. Rigon, Ecole Polyechnique
**FLASH Center, University of Rochester

How do we know what's going on? -> Diagnostics!

High-resolution imaging of plasma dynamics

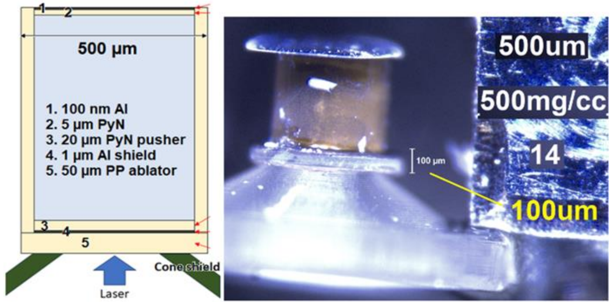
- Shocked foam platform -> Explore target parameters:
  - Polymers: Aerogel, 3D printed
  - Density: ~17-500 mg/cc
  - Structure:
    - Ablator/pusher/foam interface: Flat and modulated
    - Lattice configuration



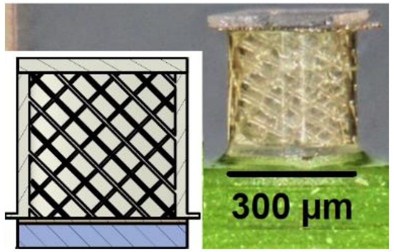
MEC X-ray Imager (MXI) in Talbot geometry[4]. 2D phase grating set at Talbot distance from detector → simultaneous retrieval of refraction and absorption

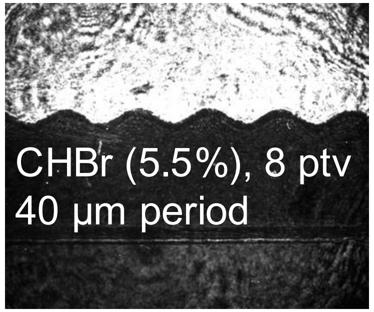
- Supporting simulations:
  - Model validation through standard code: e.g., xRAGE, FLASH

- Development of advanced data analysis tools and phase contrast imaging techniques



2PP targets

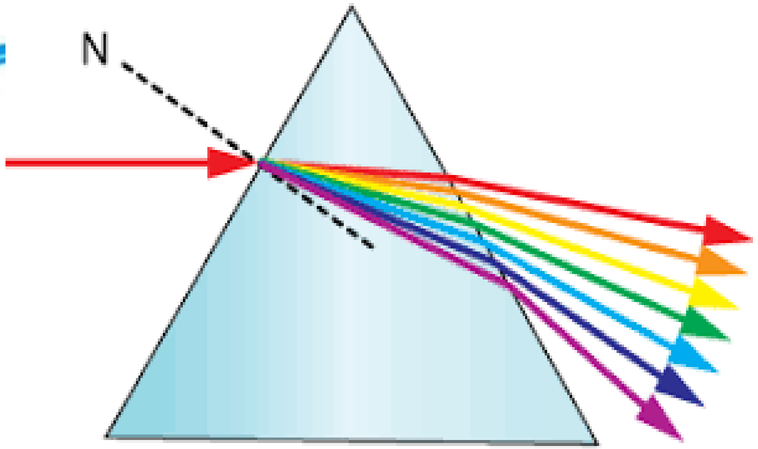


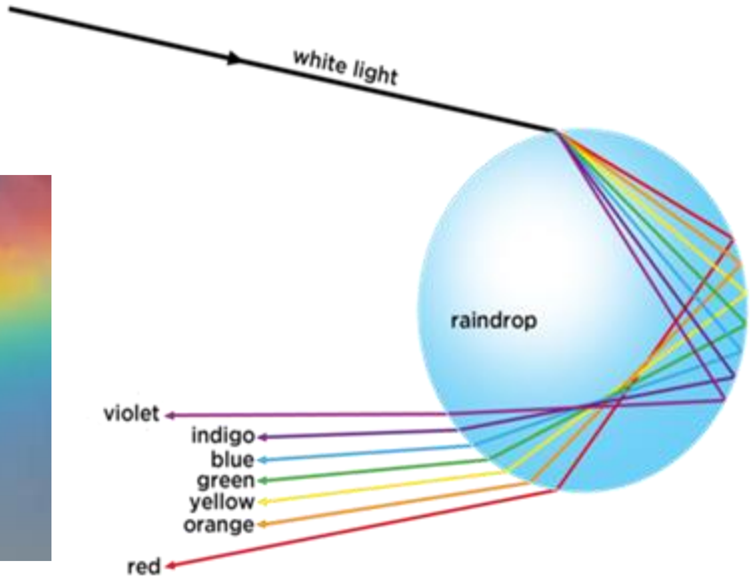


Polymer targets -> shock tubes: Left: Aerogel foams. Density = 20, 50, 100, 250, and 500 mg/cc. Center: LANL targets -> 3D printed lattices. ~17-105 mg/cc. Right: Aerogel foams and doped ablator with modulated interface.

[4] X-ray microscopy and talbot imaging with the matter in extreme conditions X-ray imager at LCLS, E. Galtier, et al., Sci Rep 15, 7588 (2025)

Refraction

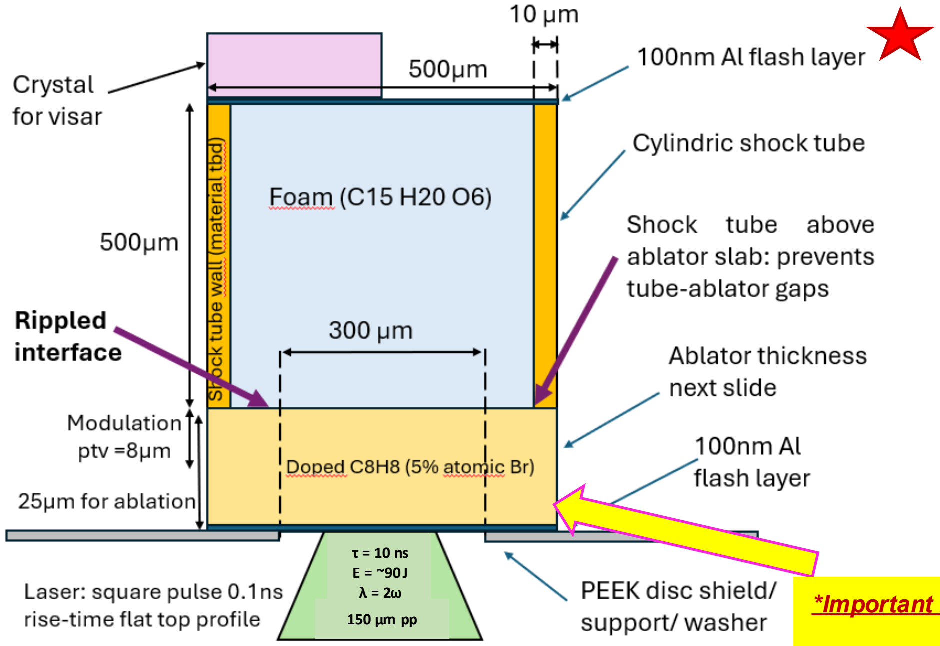




The Harvard Computers were a team of women at the Harvard College Observatory who processed astronomical data to classify stars.

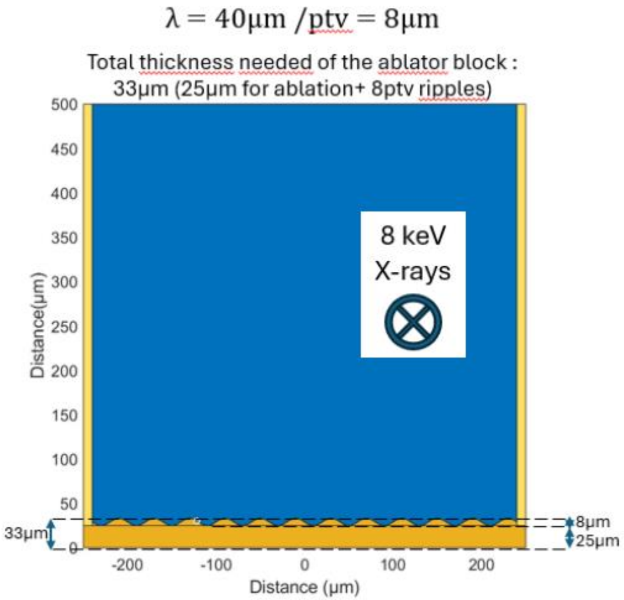
(1877-1919) -> remember who was allowed to be a “scientist “

New SciTech targets: designed for L-10361



Original design:

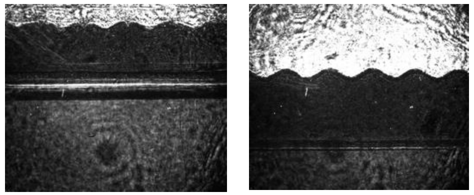
pusher interfaces



Important for simulations:

Br actually 5.5% (ρ = 1.46 g/cc)

Extra Al coating (~300 nm thickness) added on-site -> confirm which targets!



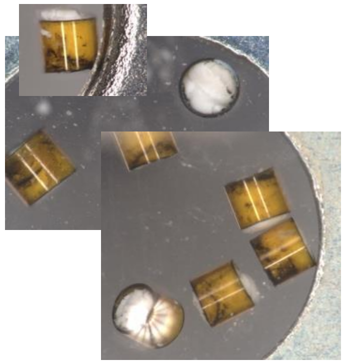
Zyla images of modulated ablator of new SciTech shock tubes (50, 100, 250 mg/cc):

Modulation: ~**8** - 10 µm ptv (i.e. sinewave A ~**4** - 5 µm)
~40 µm period

Ablator thickness: ~ 72 - 74 µm measured from bottom of CHBr layer to bottom of ripples (sine valley)

Flat **ablator thickness:** 49.4 ± 3.7 µm

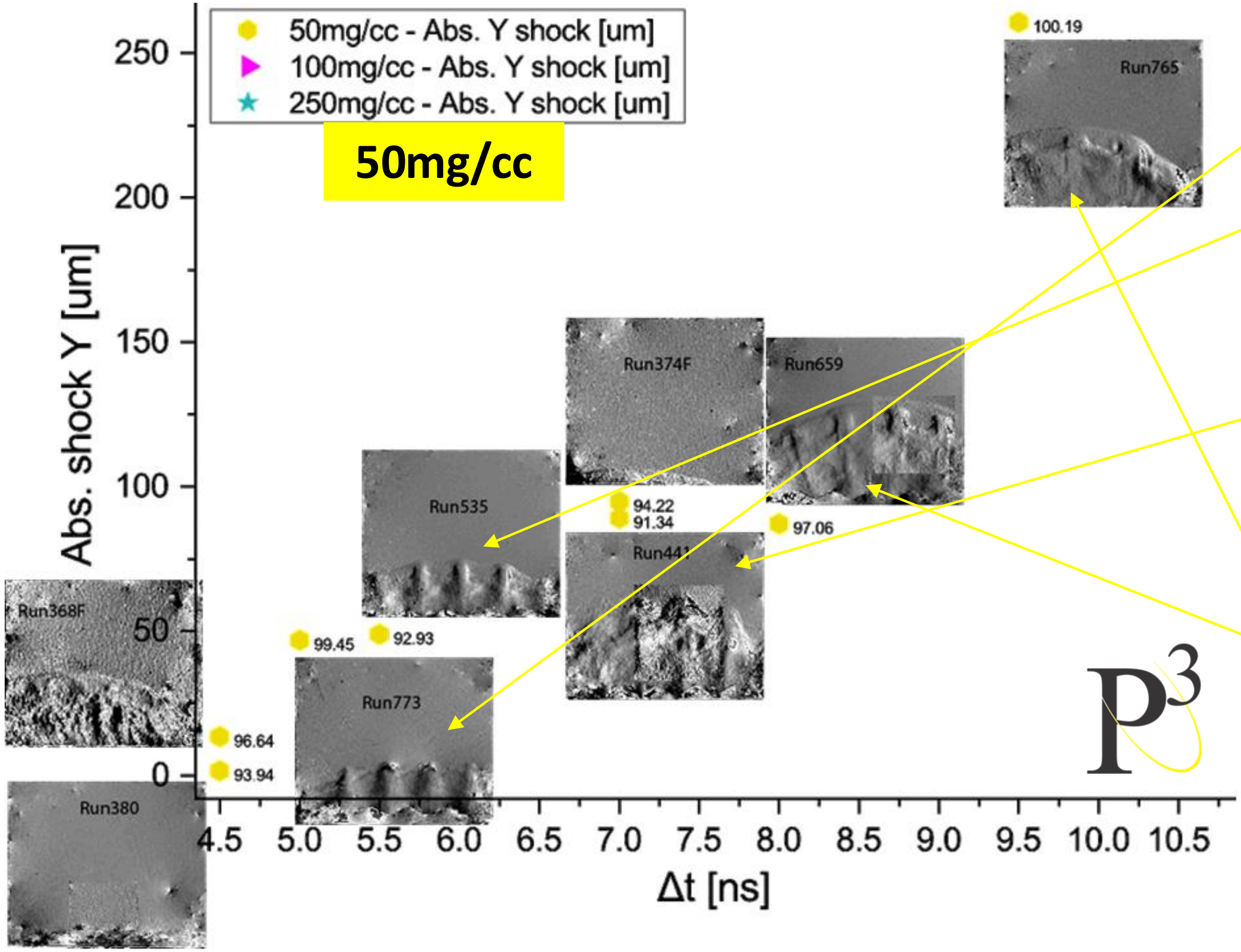
New SciTech shock tubes: Microscope images →

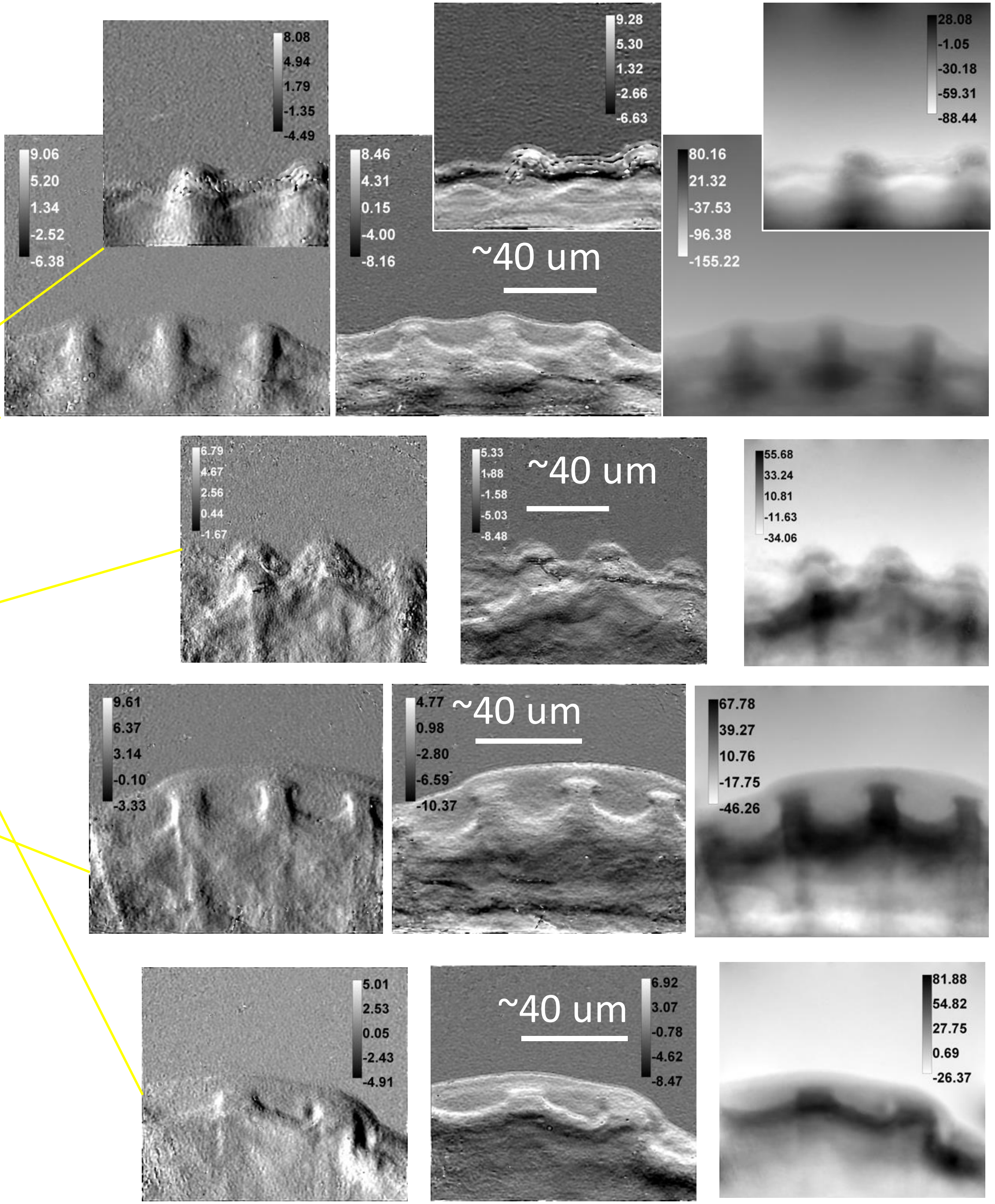


*Al flash coating on top of foam (black areas) for VISAR. Sallee added proper VISAR windows to some (most?) targets on site

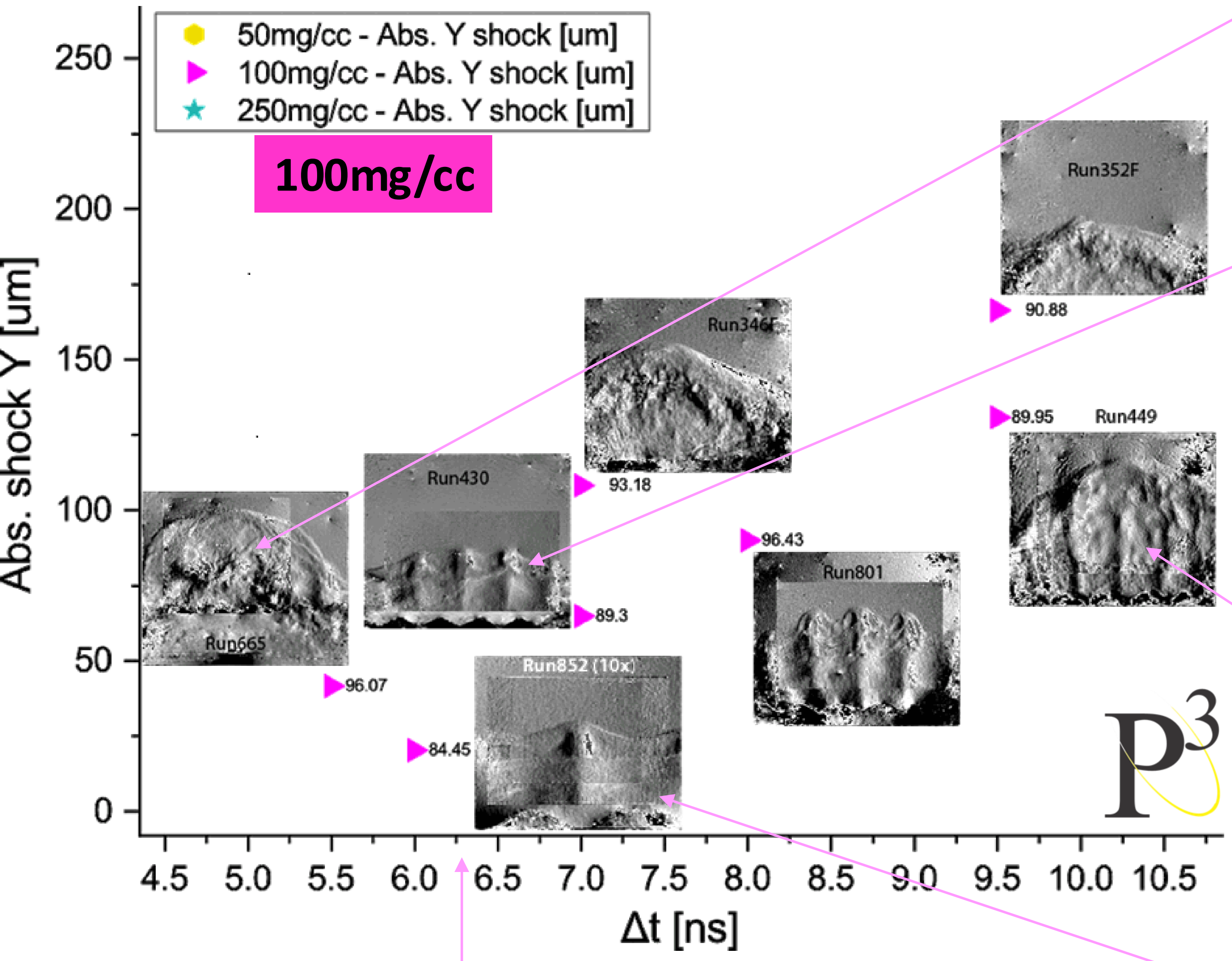
*Notice how foam protrudes in some places and there's gaps in other places. Sallee did her best to gently remove/push/relocate foam when placing ablator layer (flat or rippled), considering this, I wonder if some of these may change "local" density slightly… -> Pia will check data and try to correlate with Sallee's notes on tube quality

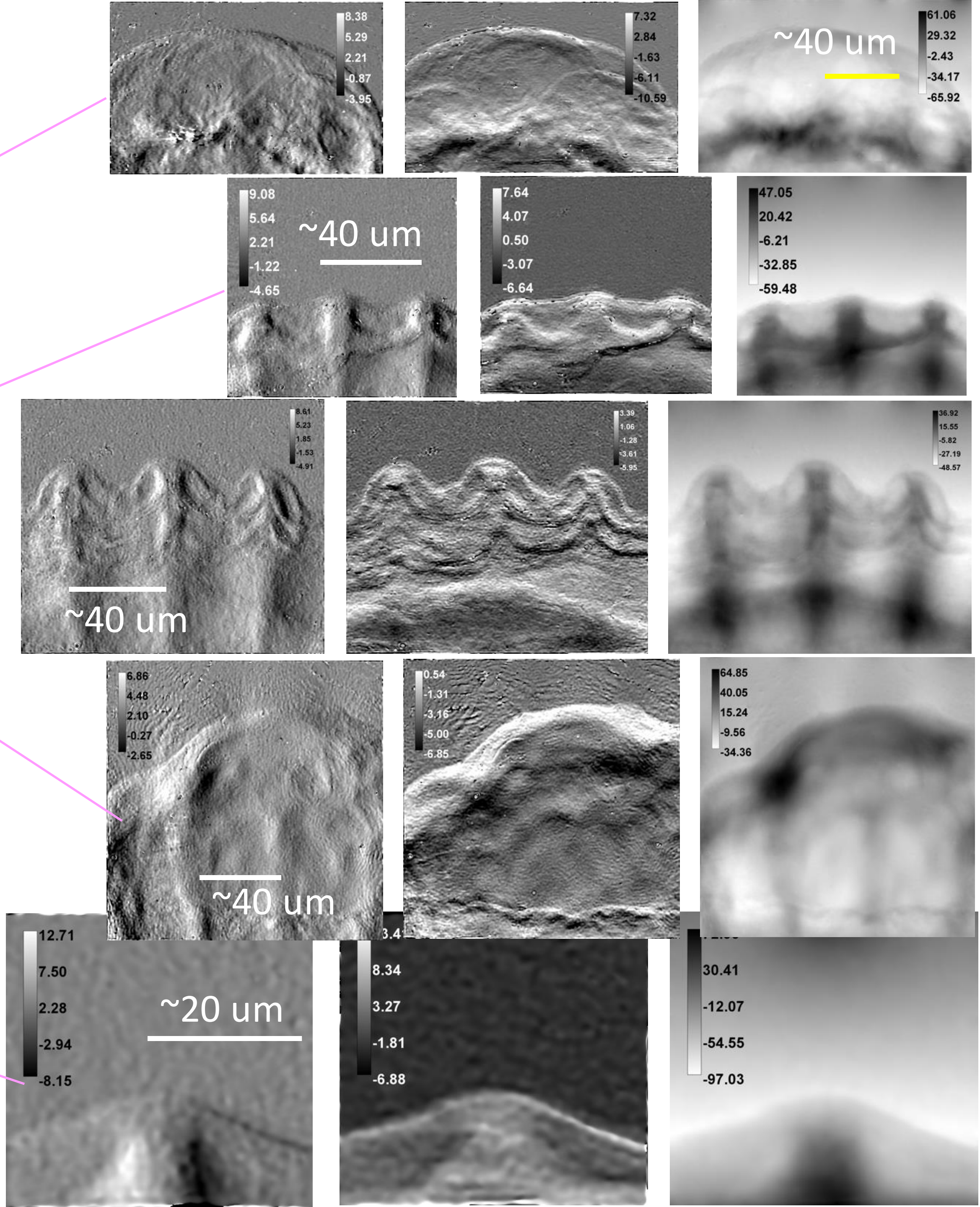
Modulated targets: hydrodynamic instabilities





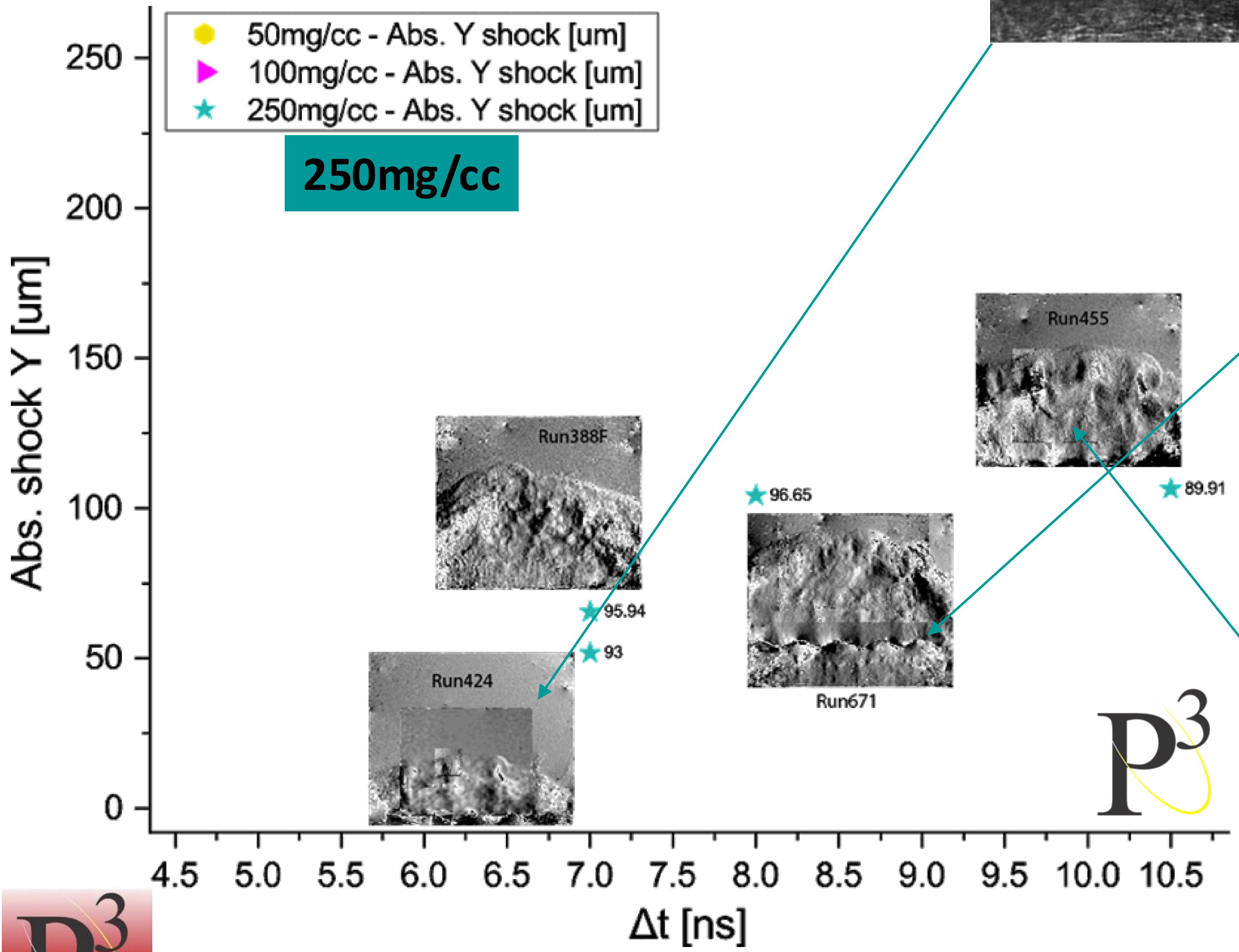
Modulated targets: hydrodynamic instabilities

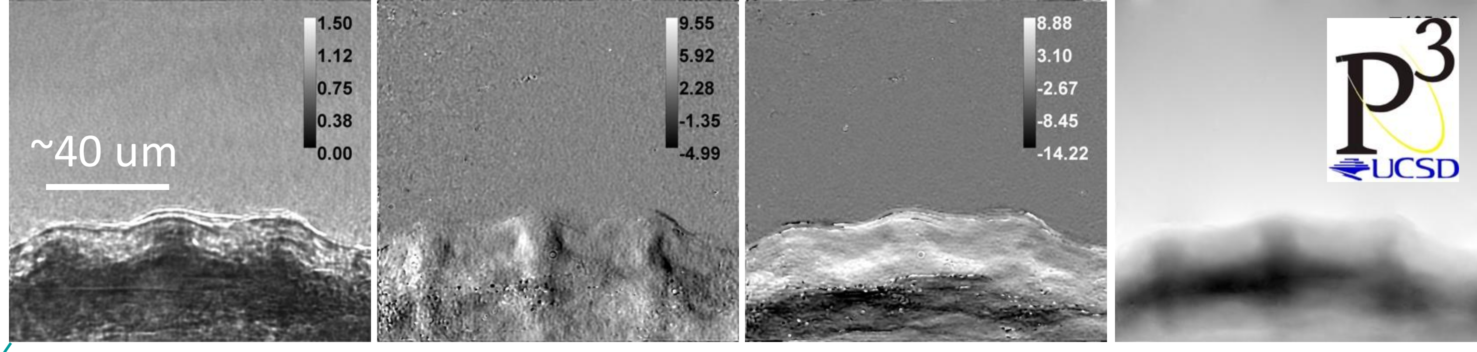


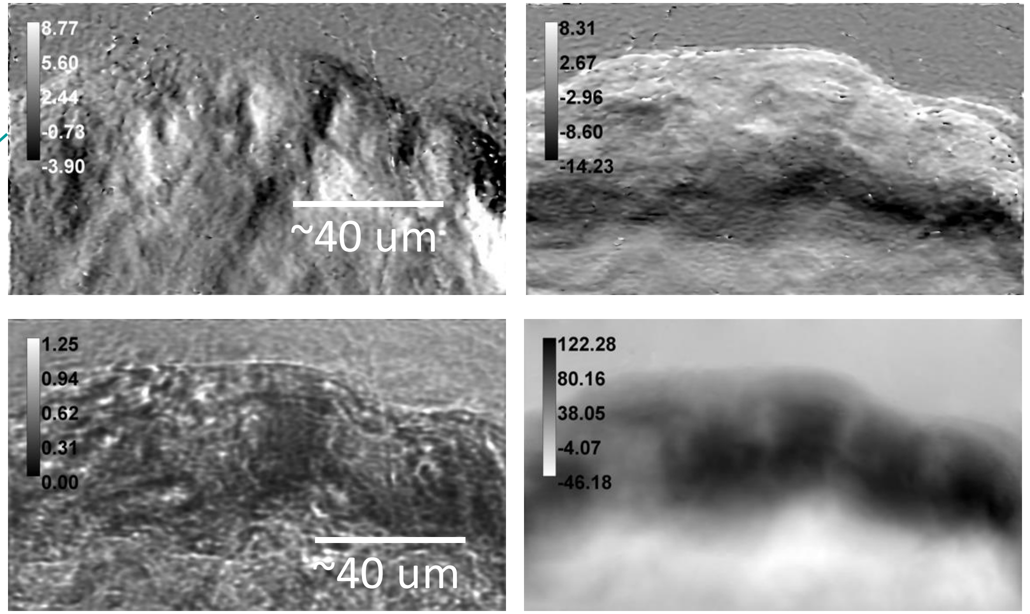


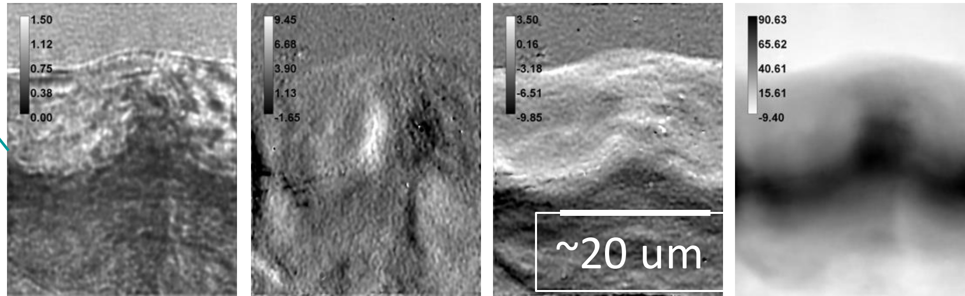
Additional 2.5x magnification

Modulated targets: hydrodynamic instabilities

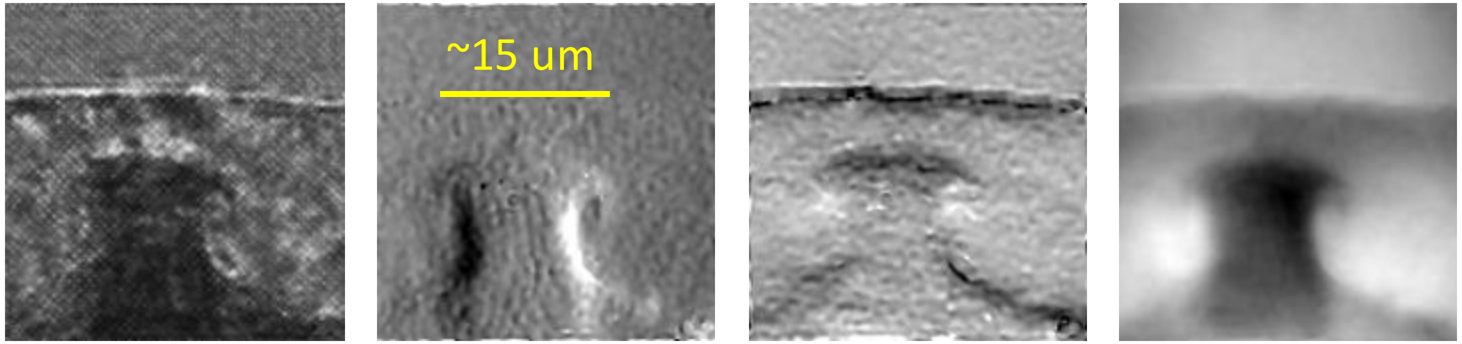


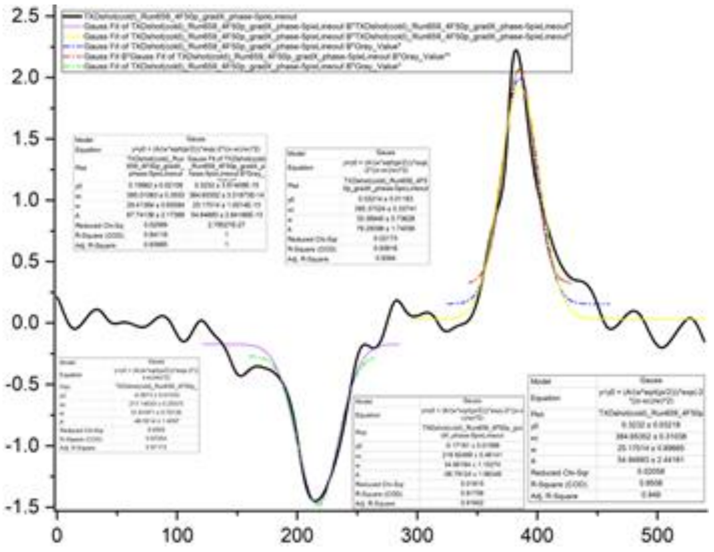






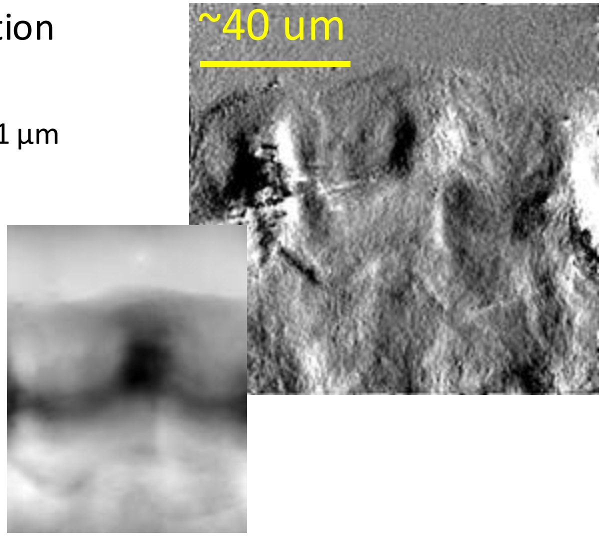
TXD: diagnostic sensitivity



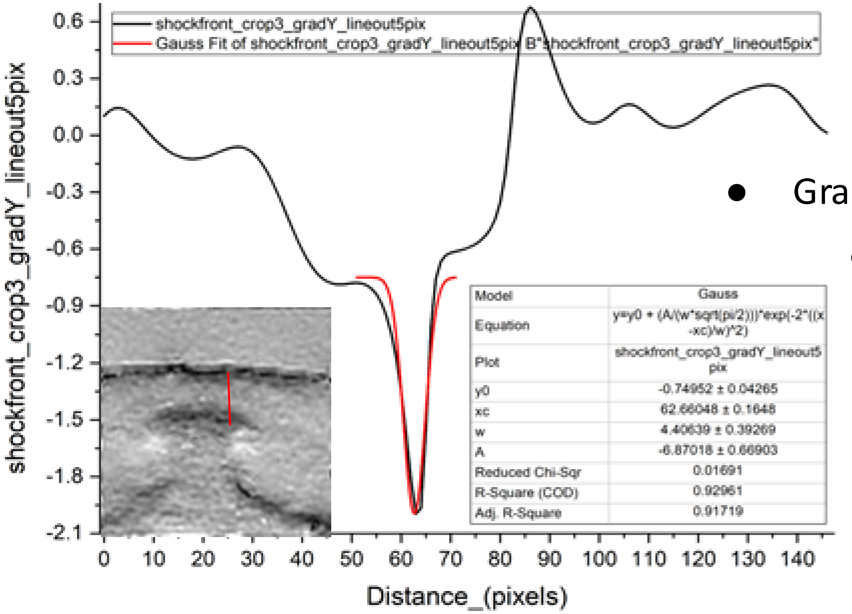


- GradX: determine density, structure size / resolution
  - Stem width ~ 12.99 um

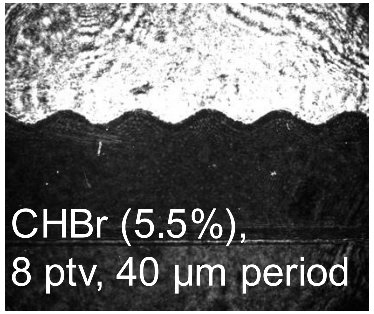
    ⇒ measured <3.396 rads/um, detected within ~3.11 μm

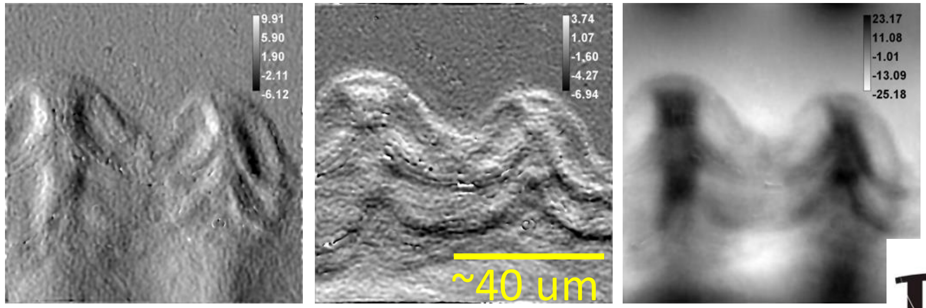


- Ripple circumference, shock layers, etc.



- GradY: determine min. ref. angle measured
  - "Sheath" width ~ 3.48 um





A few take aways

- Turbulence is:
  - Ubiquitous and often driven by a system in unstable equilibrium
  - Large range of scales with an energy cascade across -> characterized by the Kolmogorov spectrum
  - Driven by instabilities such as the Rayleigh-Taylor instability -> Swirly!
- To achieve fusion: we must mitigate turbulence -> control instabilities
  - Sophisticated computer codes exist -> model turbulent plasmas
  - Diagnostics are key! -> model validation to better describe turbulent plasmas
    - Fully characterize plasma systems: temperature, density, ionization, speed, etc…
    - Determine impact of instabilities and test approaches for mitigation
  - **Understand, Predict, Control!**

Additional references:

- Neutral fluid turbulence:
- • U. Frisch, *Turbulence: The legacy of A.N. Kolmogorov* (1995).
- • S. Pope, *Turbulent flows* (2000).

- Plasma turbulence:
- • P. H. Diamond, *Modern plasma physics: Volume 1, Physical Kinetics of Turbulence Plasmas* (2010).
- • R. J. Goldston & P. H. Rutherford, *Introduction to Plasma Physics* (1995).
- • J. Krommes, *The gyrokinetic description of microturbulence in plasmas.* Annual Review of Fluid Mechanics 44, 175 (2012).

- Also, if you are interested in data-mining turbulence simulations, check out the John Hopkins Turbulence Database: https://turbulence.idies.jhu.edu/home

• Turbulence – References

• See previous lectures by Saskia Mordjick and Troy Carter - http://suli.pppl.gov

• Greg Hammett has a lot of great introductory material to fusion, tokamaks, drift waves, ITG turbulence, gyrokinetics, etc… (*w3.pppl.gov/~hammett)*

• Greg Hammett & Walter Guttenfelder gave five 90 minute lectures on turbulence at the 2018 Graduate Summer School (*gss.pppl.gov*)

• Transport & Turbulence reviews:

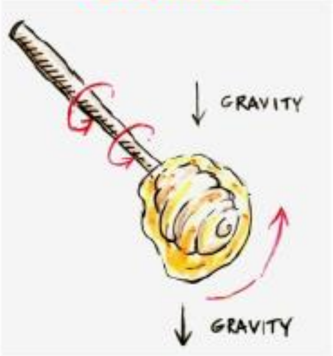
- Liewer, Nuclear Fusion (1985)
- Wootton, Phys. Fluids B (1990)
- Carreras, IEEE Trans. Plasma Science (1997)
- Wolf, PPCF (2003)
- Tynan, PPCF (2009)
- ITER Physics Basis (IPB), Nuclear Fusion (1999)
- Progress in ITER Physics Basis (PIPB), Nuclear Fusion (2007)

References from W. Guttenfelder / slide from S. Diem

The Secret for Stabilizing Bad-Curvature Instabilities

Twist in ***B*** carries plasma from bad curvature region to good curvature region:

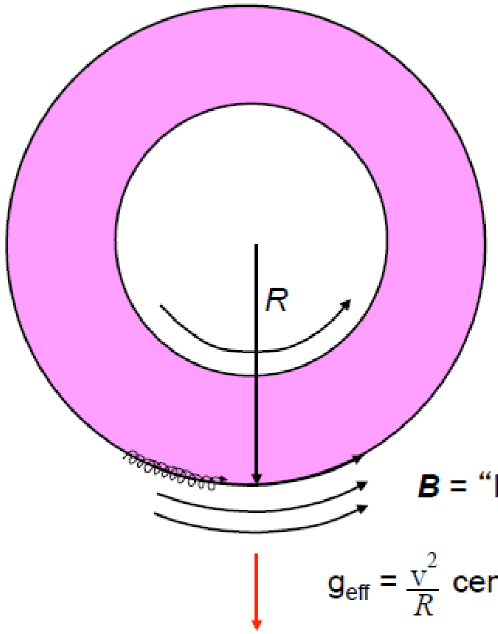
Unstable **Hammet** Stable



Similar to how twirling a honey dipper can prevent honey from dripping.

"Bad Curvature" instability in plasmas ≈ Inverted Pendulum / Rayleigh-Taylor Instability

Top view of toroidal plasma:



plasma = heavy fluid

B = "light fluid"

$g_{eff} = \frac{v^2}{R}$ centrifugal force

Growth rate:

$$\gamma = \sqrt{\frac{g_{eff}}{L}} = \sqrt{\frac{V_t^2}{RL}} = \frac{V_t}{\sqrt{RL}}$$

Similar instability mechanism in MHD & drift/microinstabilities

$1/L = |\nabla p|/p$ in MHD,
$\propto$ combination of ∇n & ∇T in microinstabilities.

5.6 The Kelvin–Helmholtz Instability

The Kelvin–Helmholtz (KH) instability, like Rayleigh-Taylor, is seen frequently in many disparate physical systems. Whenever two fluid regions flow past one another, with a sufficiently narrow transition region at their mutual boundary, fluctuations at the boundary are unstable and will grow. The transition region where the velocity changes quickly in magnitude but remains along the same axis is known as a *shear layer*. Figure 5.7 shows an example of modulations caused by a KH instability.

Modulations driven by KH can routinely be seen in clouds, in flowing water, and in the ripples in the sand at the beach. They are also observed at shear layers in the magnetosphere. Throughout astrophysics, there are many systems that produce shear layers, anytime a flow of material from one object or region passes through or around another object or region. In addition, the characteristic mushroom shape that develops at the spike tips in the RT instability is produced by the same sort of lift force that drives KH. (See, for example, the simulation results shown in Fig. 4.27.) One can see that this process is so prevalent that it is worthwhile to understand.

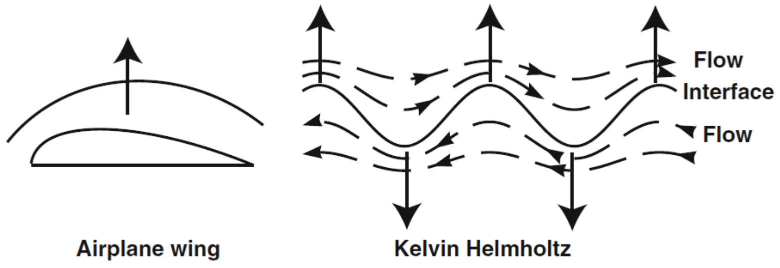


Fig. 5.8 Lift on airplane wings and in fluid flows. On the right, the solid curve is a rippled interface across which the velocity changes (a shear layer). The dashed curves show streamlines in the flow, which close when the surface extends into the fluid and spread out when the surface recedes

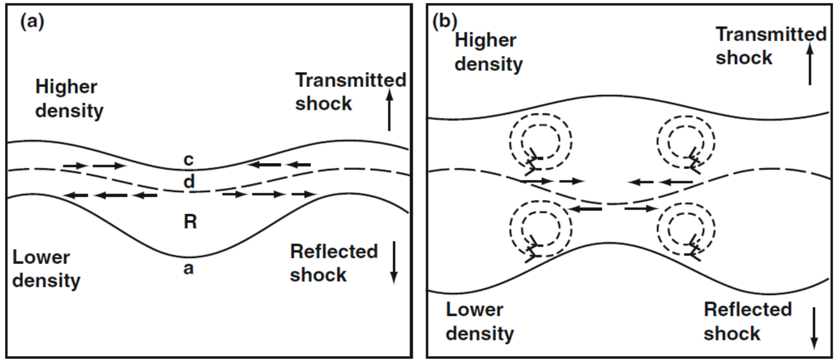


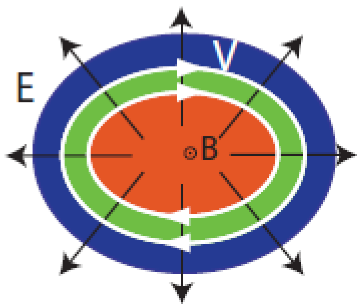
Fig. 5.17 (**a**) Lateral velocities after a shock interacts with an interface where the density increases. This schematic depicts the horizontal velocity components just after shock passage, and does not include any post-shock evolution. (**b**) Schematic of structure after shocks have each inverted and modulation at interface has grown slightly. Additional vortical flows near edges are not shown. In later evolution, the modulation of the interface will continue to grow while the shocks oscillate and damp. The letters show the regions as defined in Sect. 4.6.6

Kolmogorov’s cascade

- • At the injection scale, instabilities or active stirring create structures of size L
- • For $\mathrm{Re} >> 1$, viscosity is too small to dissipate energy at the injection scale
- • Energy must go somewhere, and if it goes to smaller scales (larger $\mathbf{k}$), it can be dissipated by viscosity
- ⇒ cascade = big turbulent structures (= eddies) break up into smaller eddies that in turn break into smaller eddies and so on

Turbulent Transport by "Eddies"

- Turbulent diffusion: random walk by eddy decorrelation



$L_c \sim 10\rho_s$

$$D \sim \frac{(\Delta x)^2}{\Delta t} \sim \frac{L_c^2}{\tau_c}$$

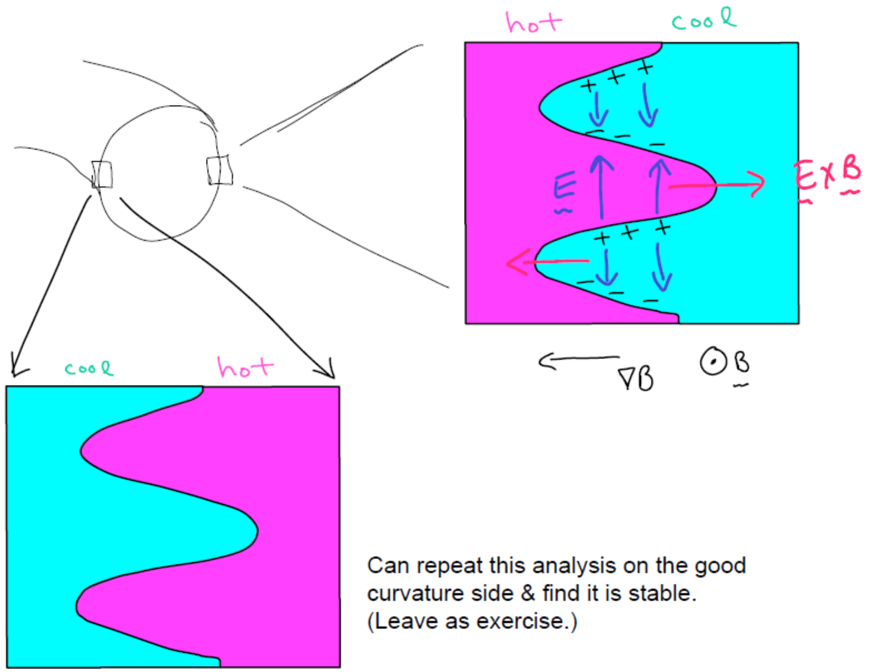
L_c^2 ← Eddy size

τ_c ← Eddy "turnover" time

$$\tau_c \sim \frac{L_c}{v} \qquad v \sim \frac{E}{B} \sim \frac{\phi}{L_c}\frac{1}{B}$$

$$\boxed{D \sim \frac{\phi}{B} \sim \frac{T}{B}}$$

Bohm diffusion



Can repeat this analysis on the good curvature side & find it is stable. (Leave as exercise.)

Rosenbluth-Longmire picture

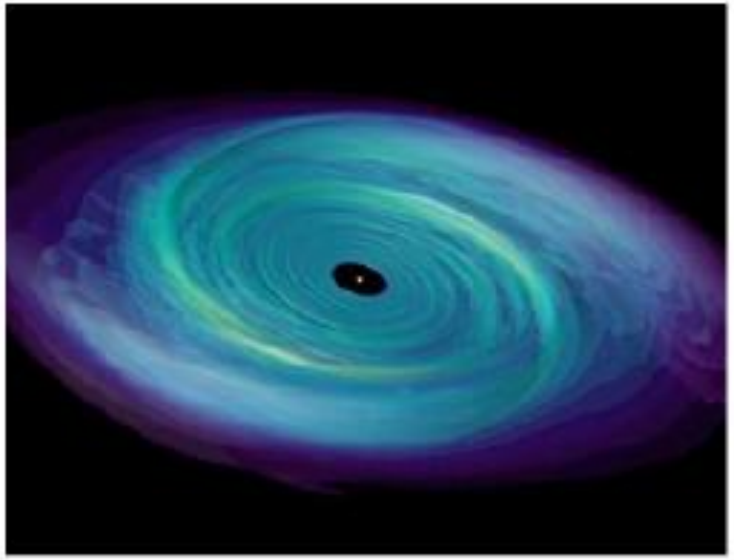
Classical diffusion: $D_{\text{class}} \sim \rho^2\nu \sim T^{-1/2} \quad (\nu \sim T^{-3/2})$

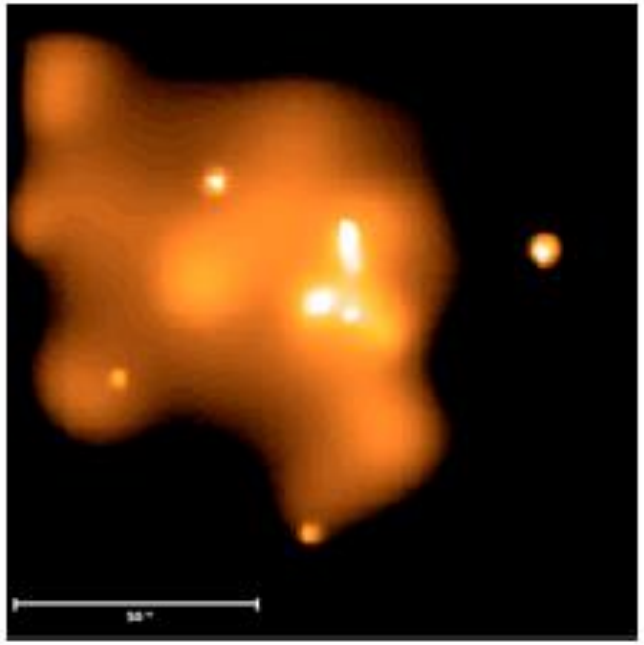
Collisional diffusion weaker as plasma gets hotter (hot plasmas are "collisionless")

- Turbulent diffusion coefficient orders of magnitude larger than classical (not shown here)
- More importantly: scaling with T is opposite. As T goes up (more heating power is added) confinement degrades. Consistent with so-called "low-confinement" mode or L-mode in experiments.

Accretion disks: feeders for forming/growing stars, planets, black holes

- Matter will orbit the central object, will not fall in unless there is friction/drag (conservation of angular momentum)
- By falling into object, tremendous amounts of energy can be released (of order 10x mc2 for black hole disk) (can lead to large luminosity, e.g. AGNs)
- Classical disks: collisions between protons and electrons causes friction; electrons are heated leading to strong radiation that keeps disk cool (relevant to protostar, planetary, some BH disks)





Problem with “hot” disks: collisions too infrequent to explain observed accretion rates

- Radiatively inefficient disks are often observed: not enough radiation to cool disk as matter accretes, energy gets stored in
thermal energy, get puffed-up, thick disk

- Because plasma is very hot, collisions are too infrequent to explain observed rates of accretion!

- Turbulence to the rescue? Problem: disks are hydrodynamically stable (no “linear” instability in Keplerian flow of neutral gas)

➡ However, if you acknowledge this “gas” is a plasma, and that magnetic fields can be present, there is an instability: Magnetorotational Instability (MRI) [Velikhov, Chandrasekhar, Balbus, Hawley]

Presence of weak magnetic field allows instability: angular momentum transported outward, matter inward
- Instability provides “anomalous” viscosity, accretion can occur
- Energy released in accretion gets taken up by turbulent magnetic fields which grow as part of the instability: where does this energy go and why isn't it radiated away?