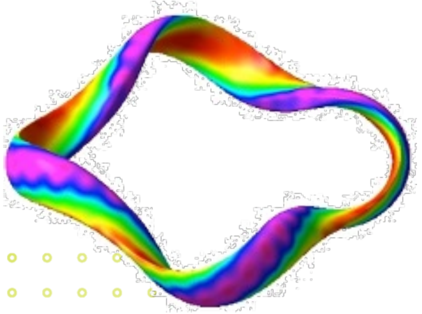
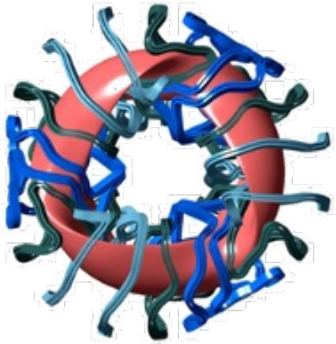
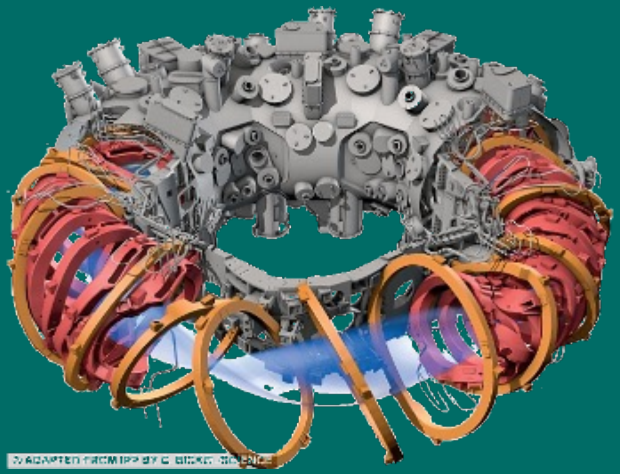


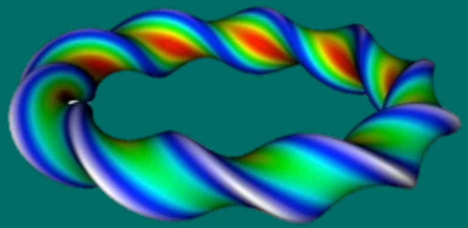


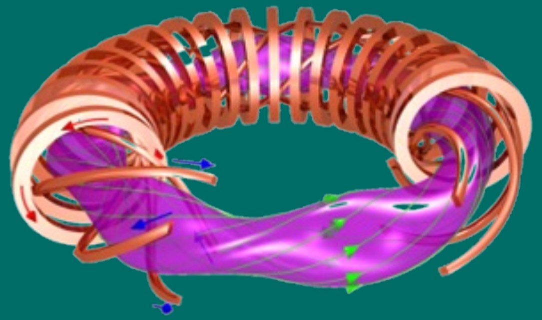
MAX-PLANCK-INSTITUT
FÜR PLASMAPHYSIK

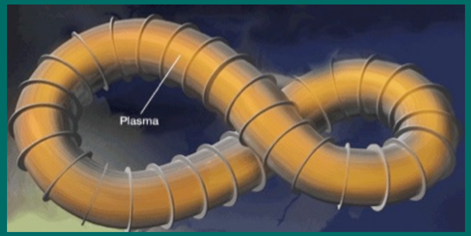
Introduction to stellarators

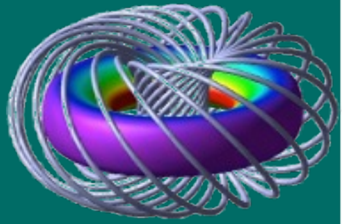


ADAPTED FROM IPP BY C. BICKEL/SCIENCE

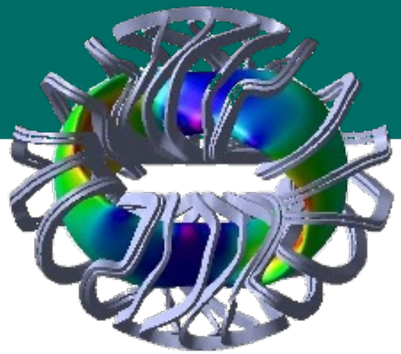






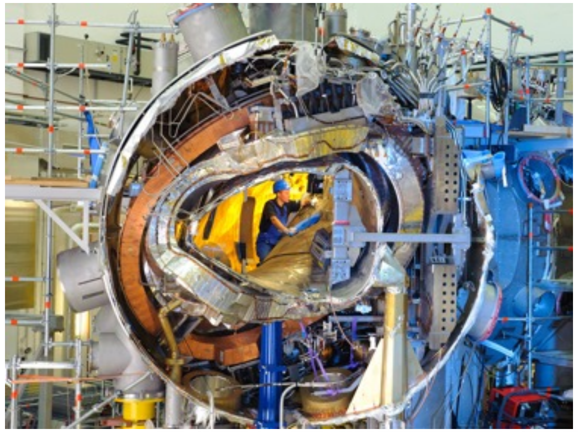


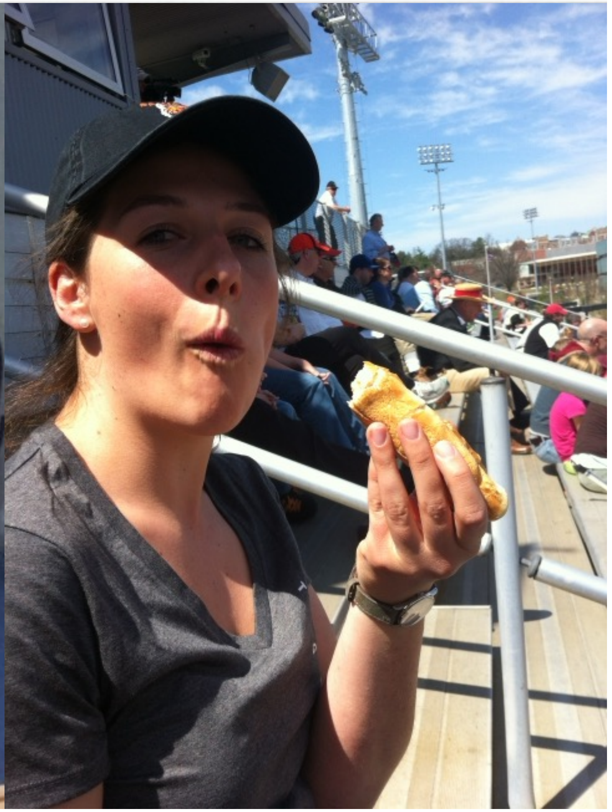
Josefine H.E. Proll – josefine.proll@ipp.mpg.de



About me

- 2003: visit to Max Planck Institute (IPP)
- 2004: school internship at IPP
- 2008: attended IPP summer university
- 2009: Bachelor thesis at IPP in stellarator theory
- 2010: Master thesis at Culham laboratory
- 2014: PhD at IPP in stellarator theory
- 2014/2015 PostDoc at IPP/PPPL
- 2017 professor Eindhoven University of Technology, The Netherlands
- 2024 Group leader stellarator modelling,IPP

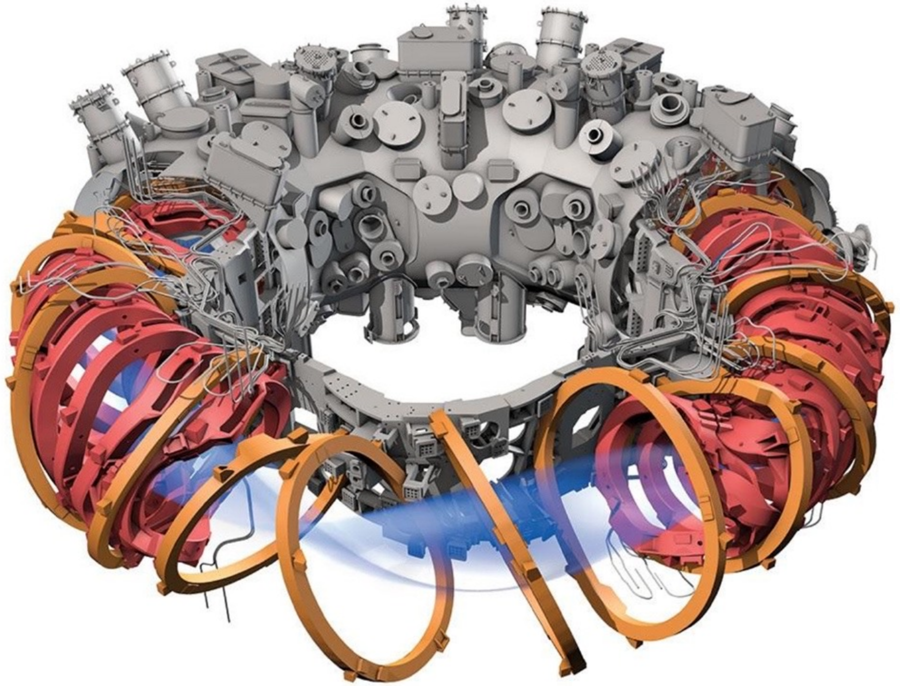




Join at menti.com | use code 3527 5822

What do you know about stellarators?

What is a stellarator?

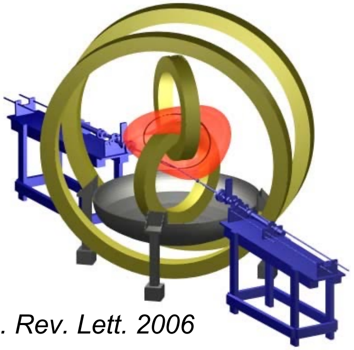


The Wendelstein 7-X Stellarator
Max Planck Institute for Plasma Physics, Greifswald, Germany
Image: T. Klinger et al., Nucl. Fusion 2019

- Toroidal magnetic plasma confinement device
- Magnetic field forms nested, closed flux surfaces
- Magnetic field is generated primarily by magnets external to the plasma
  - **Plasma current not required**

Why build a stellarator?

- Generate electricity
  - Inherently steady-state operation
  - Low/no plasma current required
    - Lower vulnerability to disruptions
    - Lower recirculating power required
- Perform basic plasma research
  - Single-species plasmas
  - Pair (positron/electron) plasmas



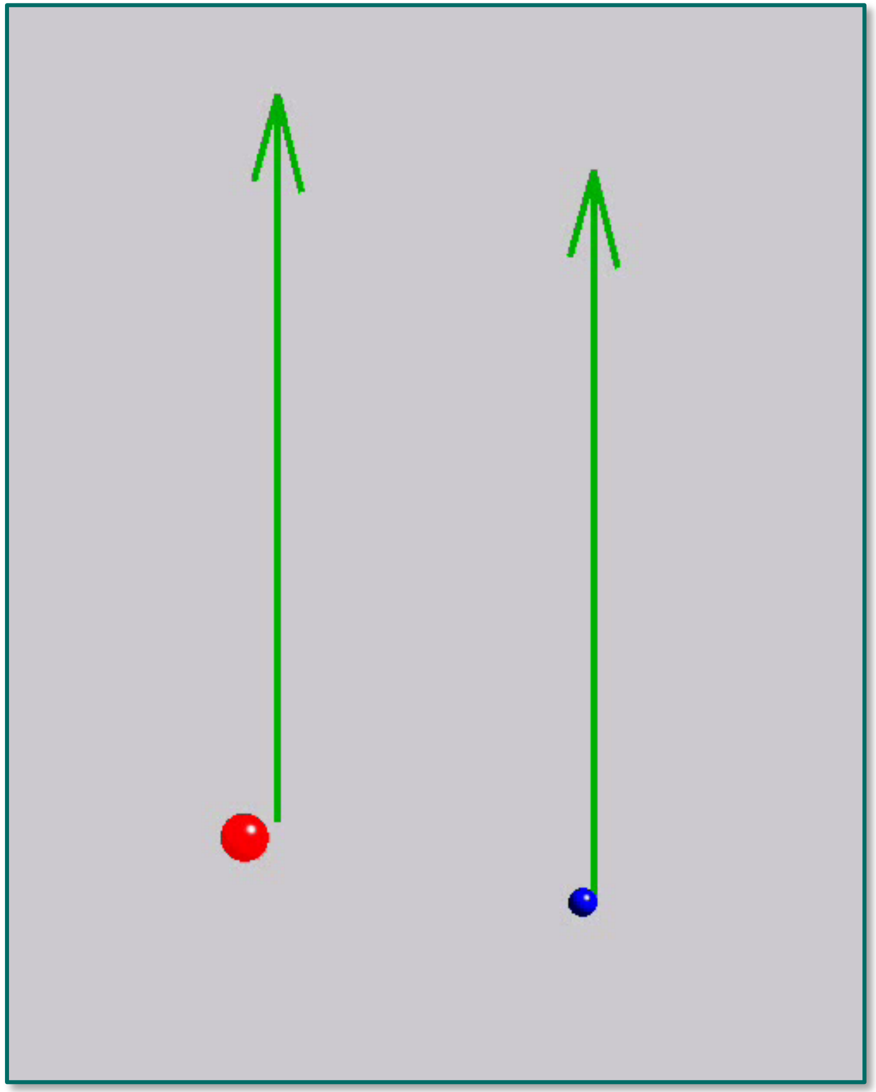
J. P. Kremer et al., Phys. Rev. Lett. 2006

S. Woodruff, Woodruff Scientific

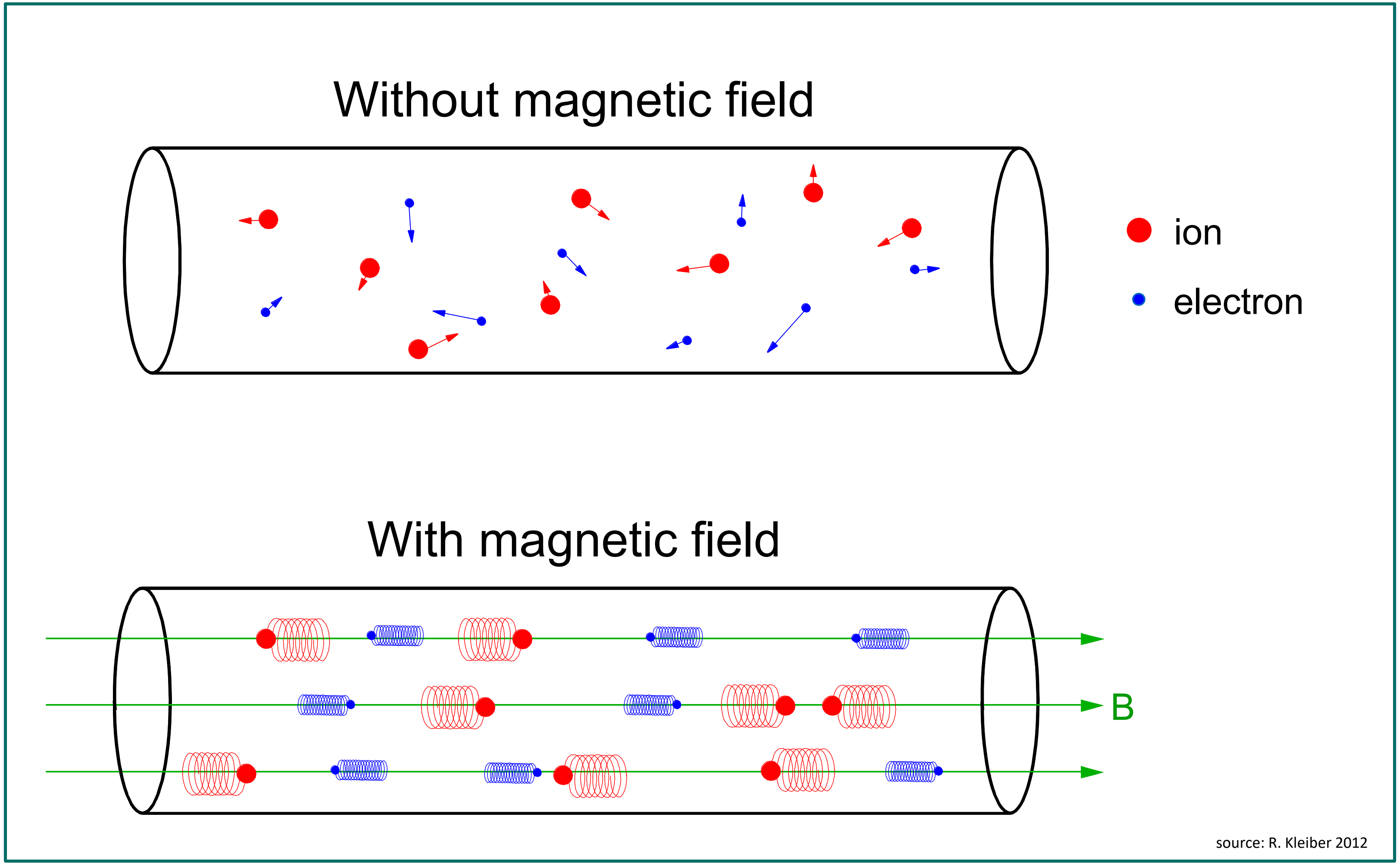
Magnetic Confinement

- A hot plasma cannot be confined by material walls.

  ➡ use magnetic field (Lorentz force)
- Magnetic field in fusion devices approx. 2.5 - 6 Tesla (earth magnetic field ≈ 50 μT)
- Gyration radius:
  - ions: 2 mm
  - electrons: 45 μm
- Gyration frequencies:
  - ions: 90 MHz
  - electrons: 170 GHz



Magnetic Confinement



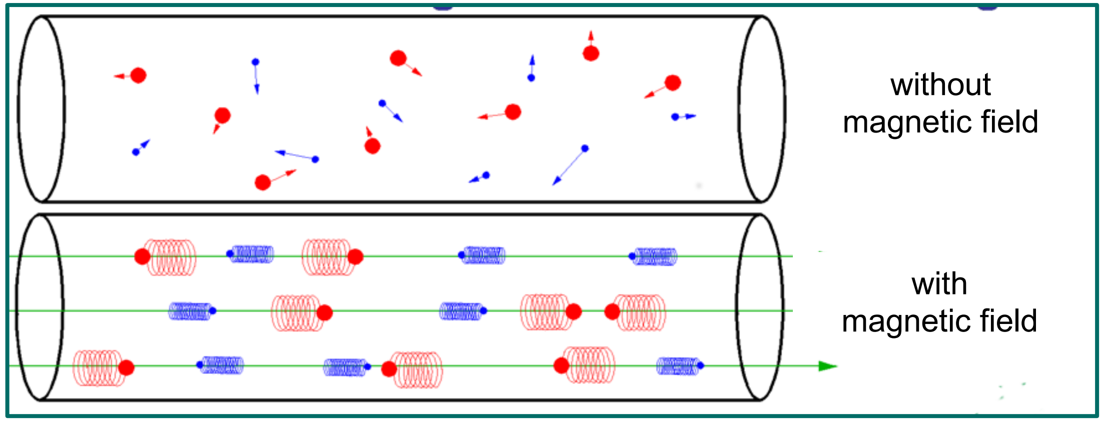
source: R. Kleiber 2012

Lorentz Force in Magnetic Field

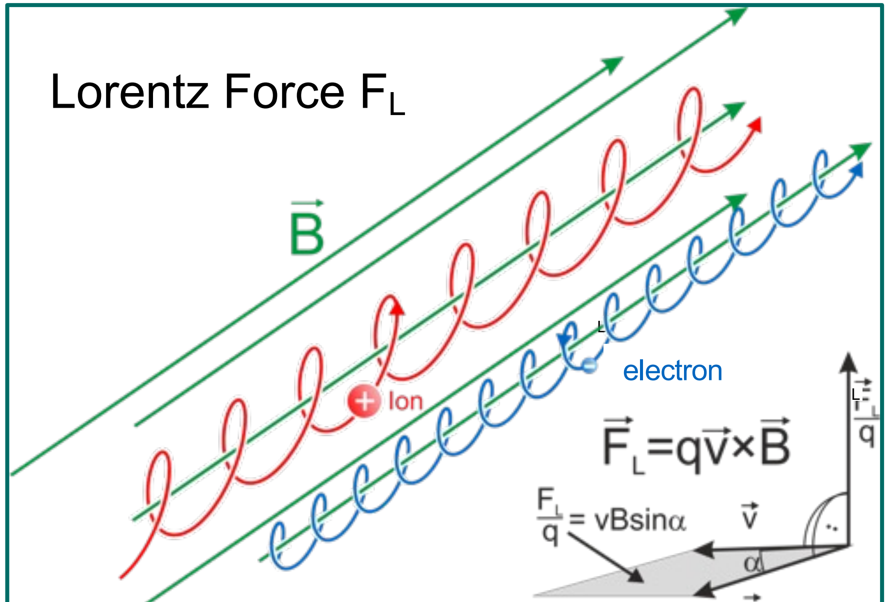
Hot plasmas cannot be confined by conventional material walls.

Force on electric charges in a magnetic field:

- Motion perpendicular to field is modified
- Particles forced on helical trajectories
- Transport properties are changed
- Fusion devices: approx. 2.5 - 6 Tesla (corresponds to about 10^5 times earth magnetic field strenght)

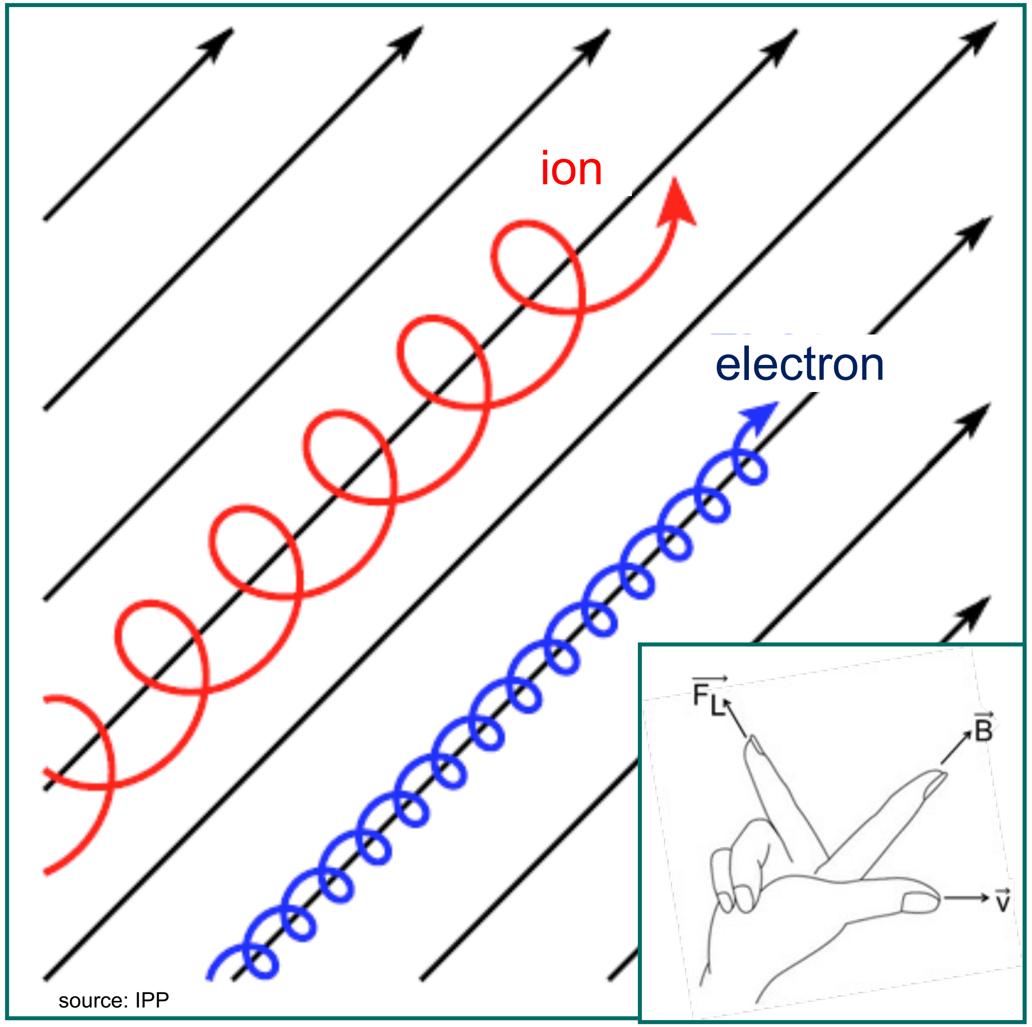


Source: R. Kleiber 2012



Source: R. Kleiber 2012

How to Confine a Plasma?

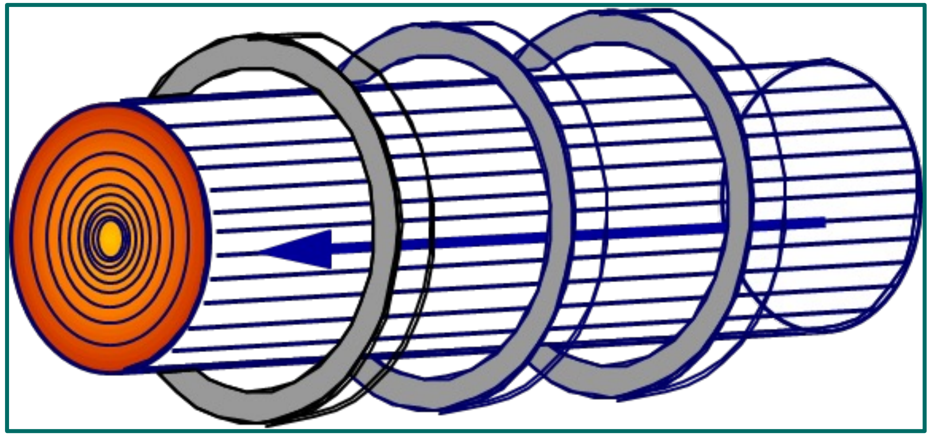


source: IPP

- Charged particles – ions and electrons – are tied to the magnetic field lines by **Lorentz Force.**
- Particles can move freely in the longitudinal direction of the lines.
- Torus of magnetic field lines

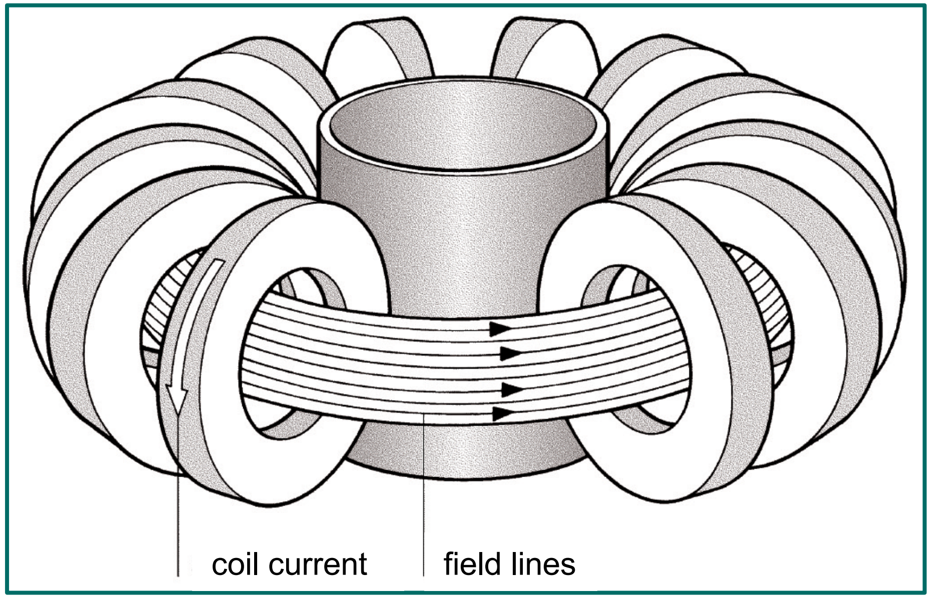
➡ keeps the plasma away from material walls

From a Cylinder Towards a Torus



Cylindrical homogeneous magnetic field

- confinement perpendicular to axis
- but: particles lost at the ends



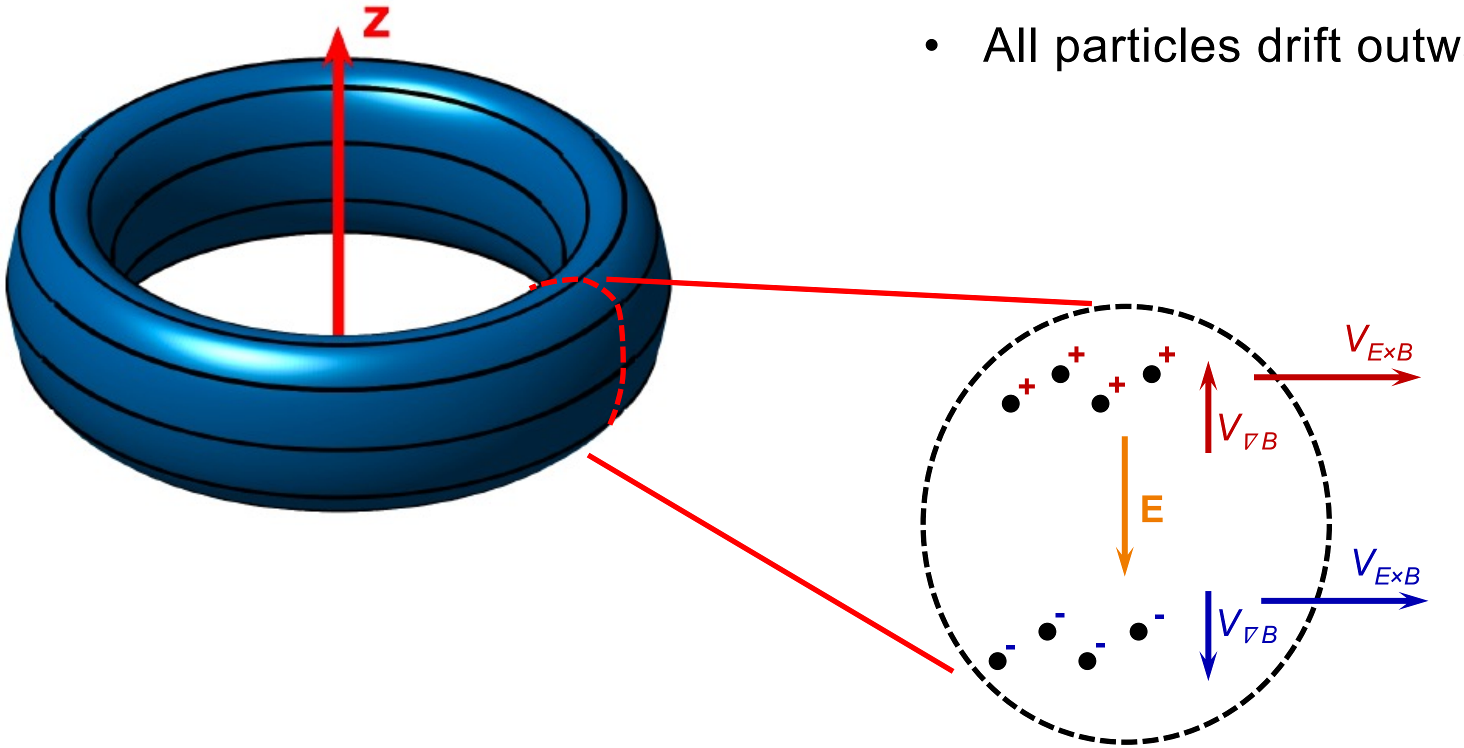
source: IPP

Closing the field toroidally

- quasi-endless configuration
- but: particle losses due to drift motion (reason: inhomogeneous field and curvature)

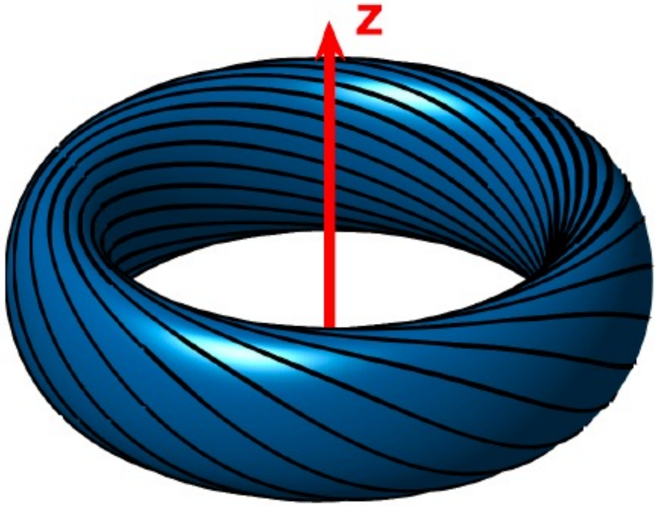
A purely toroidal field is not sufficient to confine a quasineutral plasma

- Curvature and gradient in magnetic field cause electrons, ions to drift in opposite directions
- Charge separation creates a vertical electric field
- All particles drift outward in E×B direction



L.-M. Imbert-Gérard et al., Introduction to Stellarators, 2020

To confine a plasma in a toroidal field, the field lines must twist helically



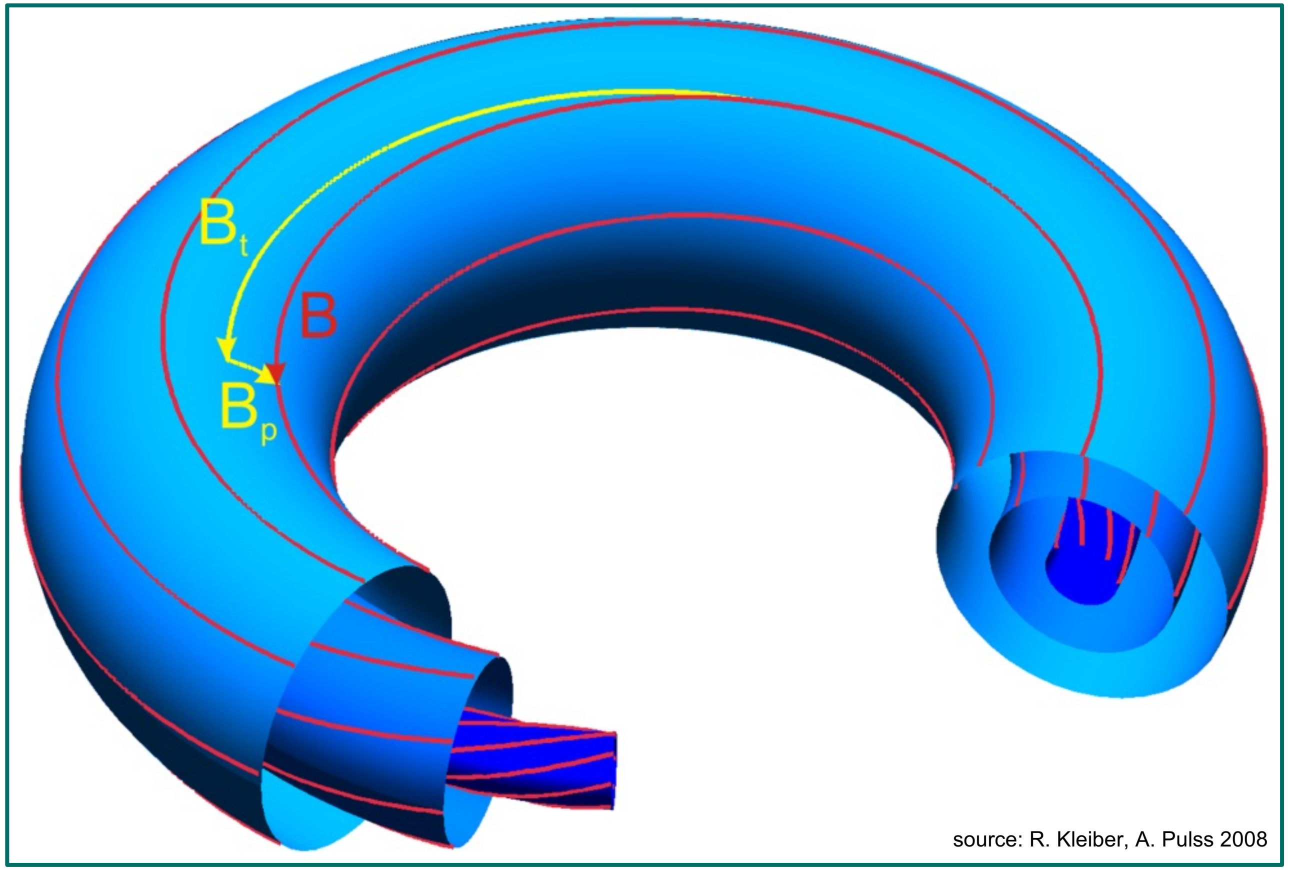
L.-M. Imbert-Gérard et al.

- Helical field lines sweep particles quickly from the top to the bottom of the torus and back
- Vertical drift motion persists, but alternates between inwards and outwards
- Sending the particles on helical paths is analogous to turning a honey dipper
  - Holding the dipper still: honey drips off
  - Rotating the dipper: honey remains confined

Twisted Field Lines (Rotational Transform)

Twisted magnetical field lead to confinement of particles.

- two field components
  - **B_t toroidal field**
  - **B_p poloidal field**
  - give rotational transform
- creation of magnetic flux surfaces
- rotational transform varies over radius

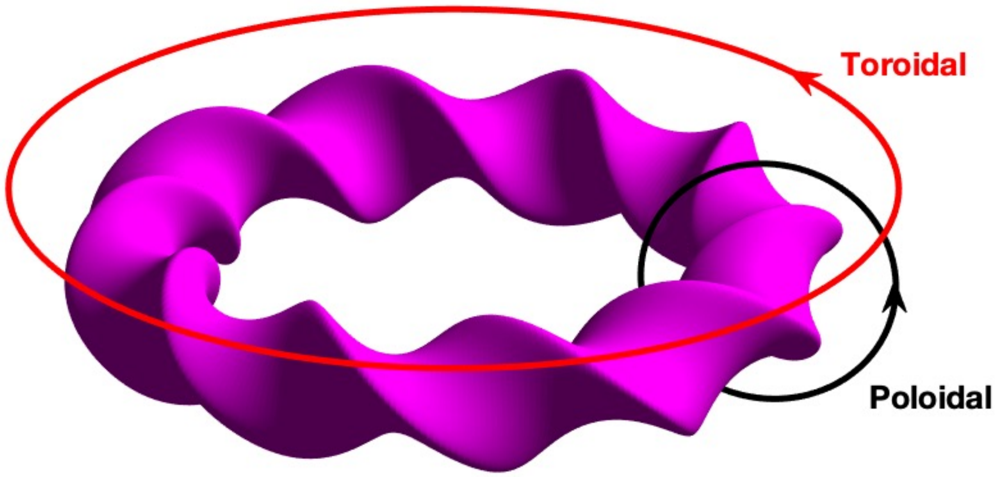


source: R. Kleiber, A. Pulss 2008

Two possible concepts to generate rotational transform:

tokamak & stellarator

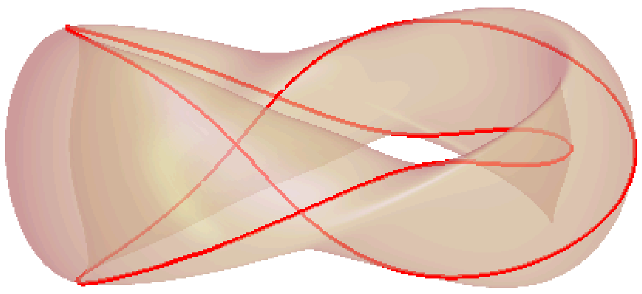
The solution: the twist, i.e. the rotational transform



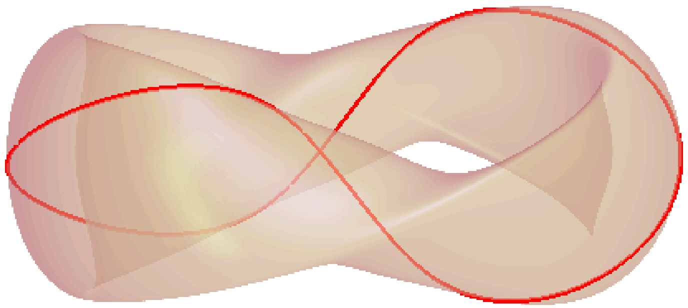
$$\iota = \frac{\#\ \text{poloidal turns}}{\#\ \text{toroidal turns}} = \frac{1}{q}$$

“rotational transform”

“safety factor”

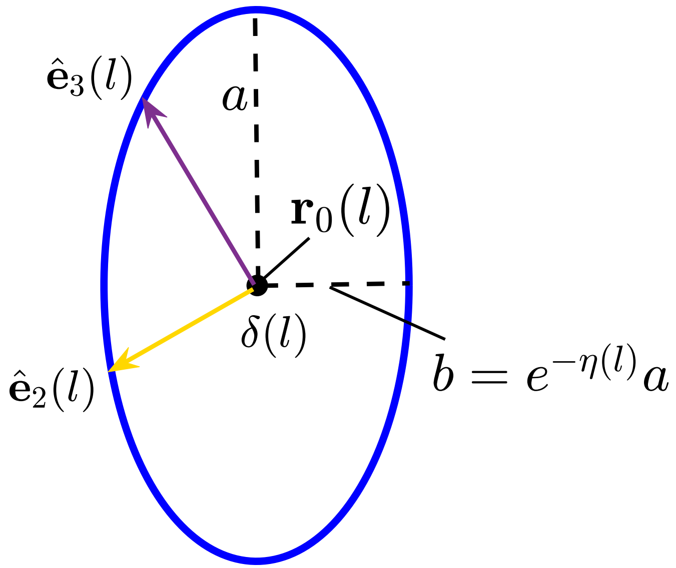


$\iota = 3/2$

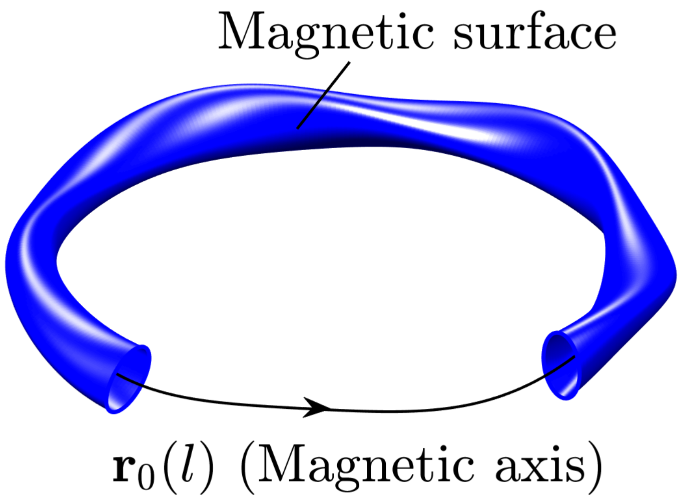


$\iota = 1$

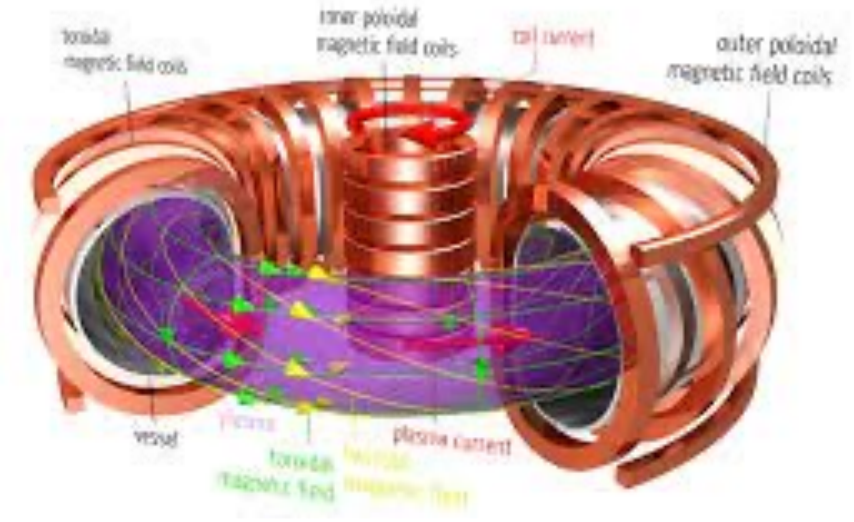
Rotational transform can be generated by symmetry-breaking

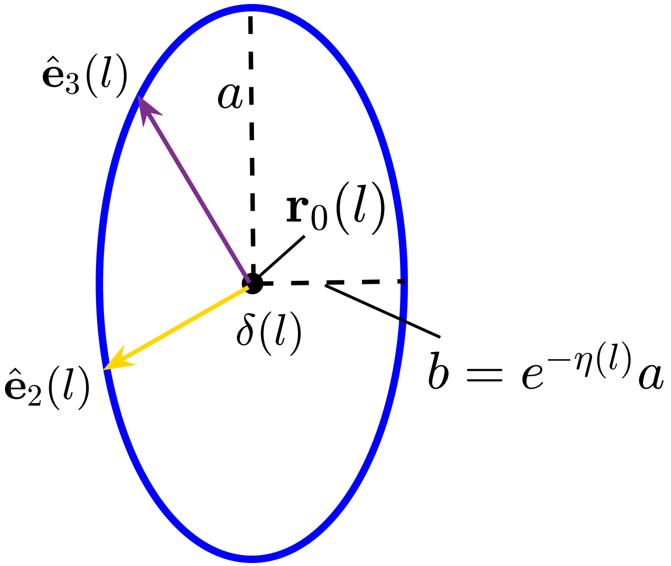


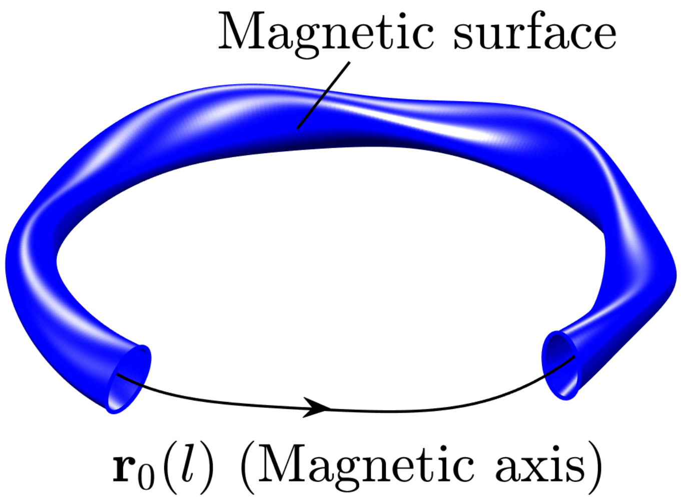
$\iota \approx$ current + rotating ellipticity + axis torsion



Rotational transform can be generated by symmetry-breaking



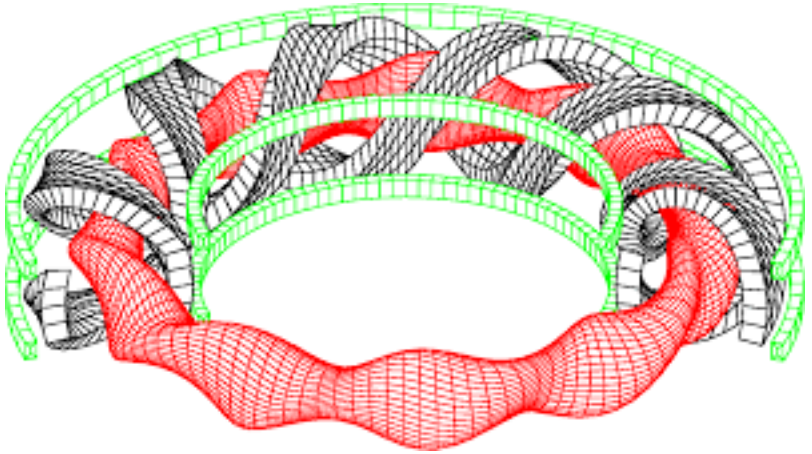


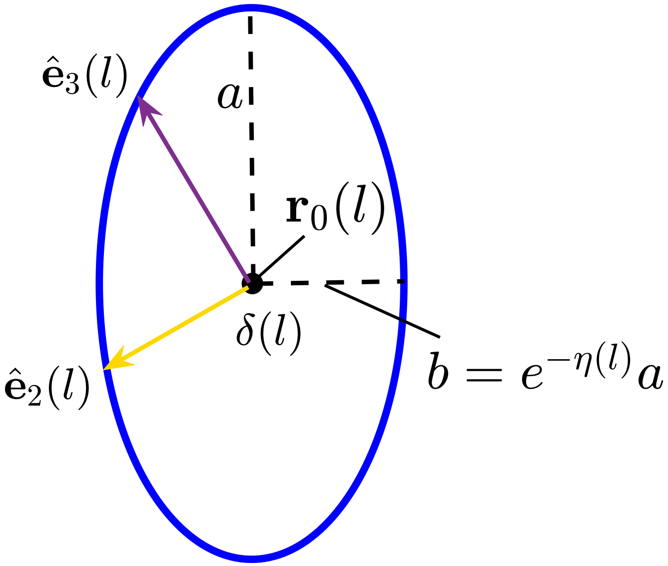


$\iota \approx$ current + rotating ellipticity + axis torsion

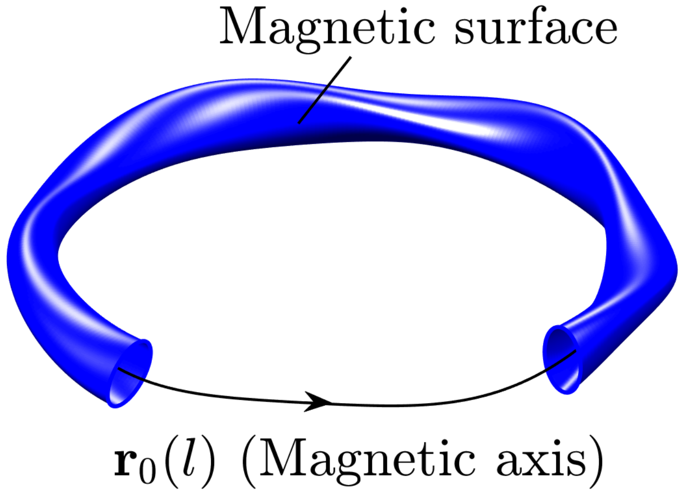
Rotational transform can be generated by symmetry-breaking

Large Helical Device



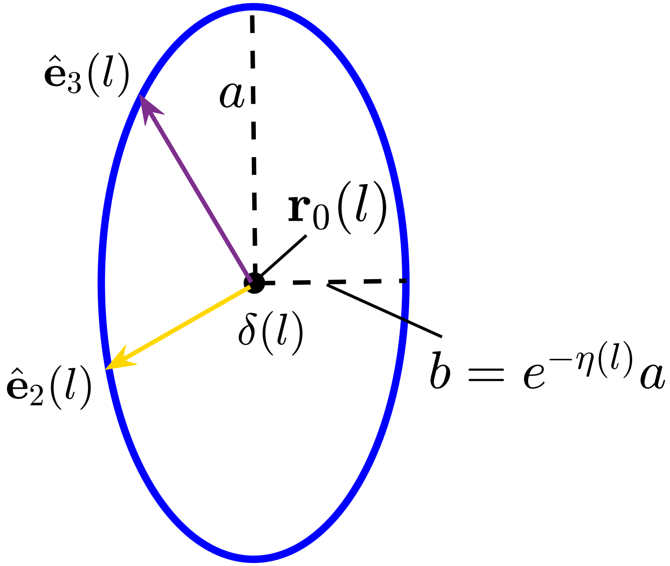


$\iota \approx$ current + rotating ellipticity + axis torsion

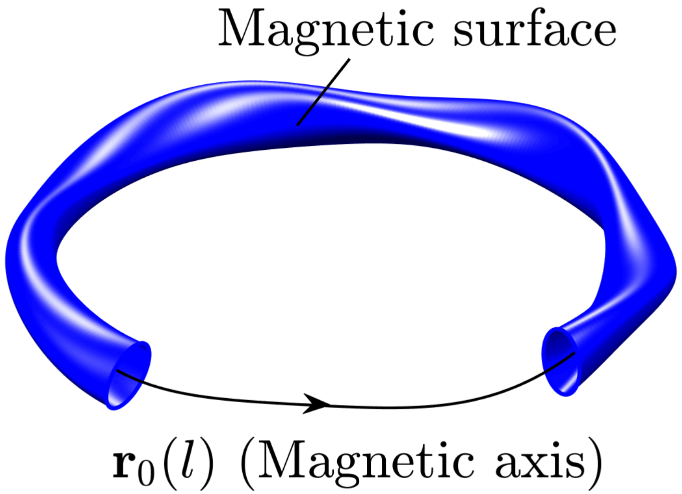


Rotational transform can be generated by symmetry-breaking

Spitzer's figure-eight

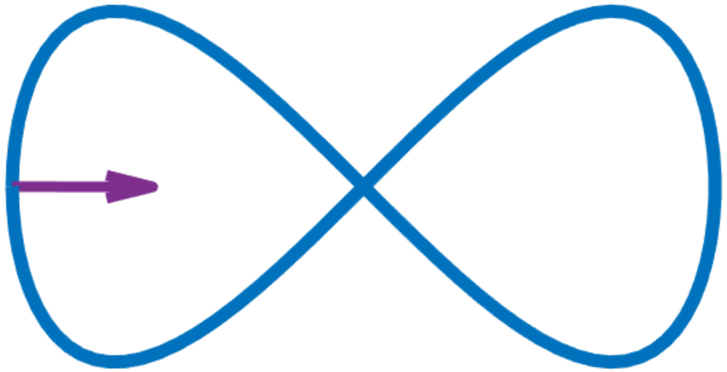


$\iota \approx$ current + rotating ellipticity + axis torsion

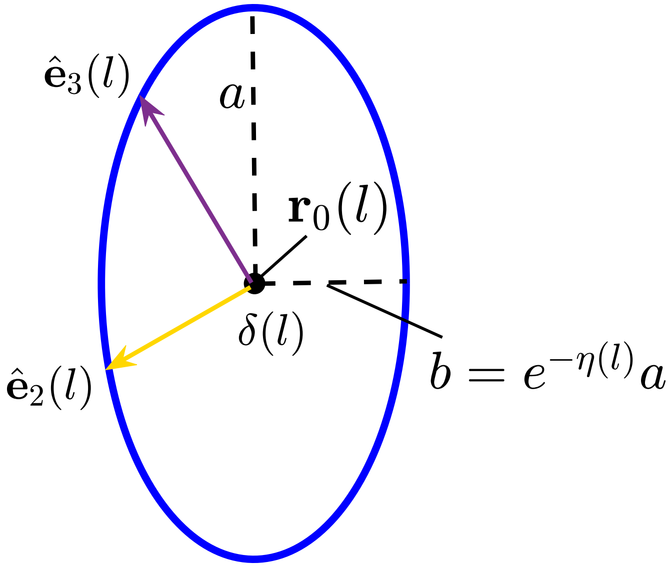


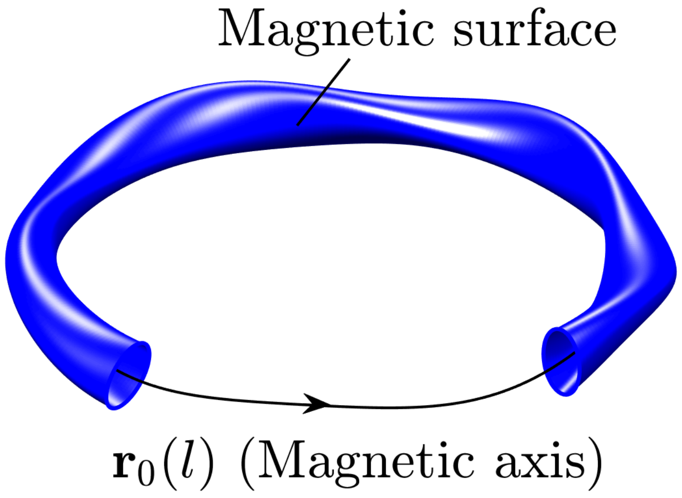
Rotational transform can be generated by symmetry-breaking

Top view



Side view

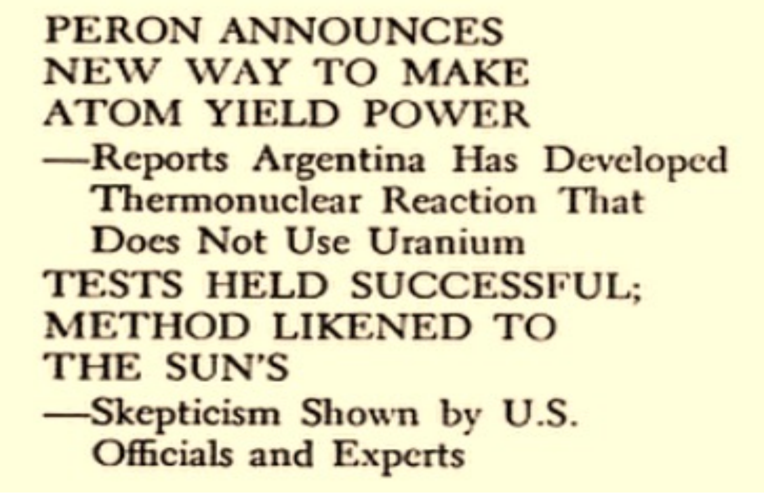




$\iota \approx$ current + rotating ellipticity + axis torsion

Lyman Spitzer – father of the stellarator

New York Times March 24, 1951:



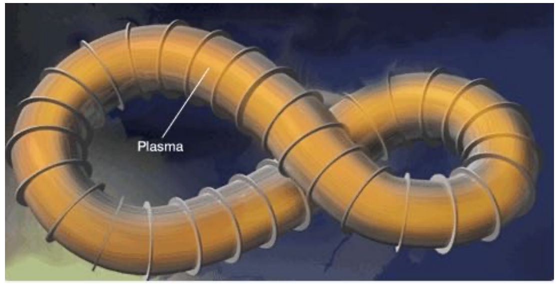
PERON ANNOUNCES NEW WAY TO MAKE ATOM YIELD POWER

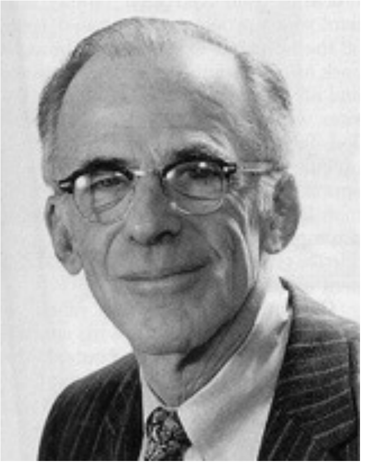
—Reports Argentina Has Developed Thermonuclear Reaction That Does Not Use Uranium

TESTS HELD SUCCESSFUL; METHOD LIKENED TO THE SUN'S

—Skepticism Shown by U.S. Officials and Experts

Spitzer, astrophysicist at Princeton University, working on Project Matterhorn



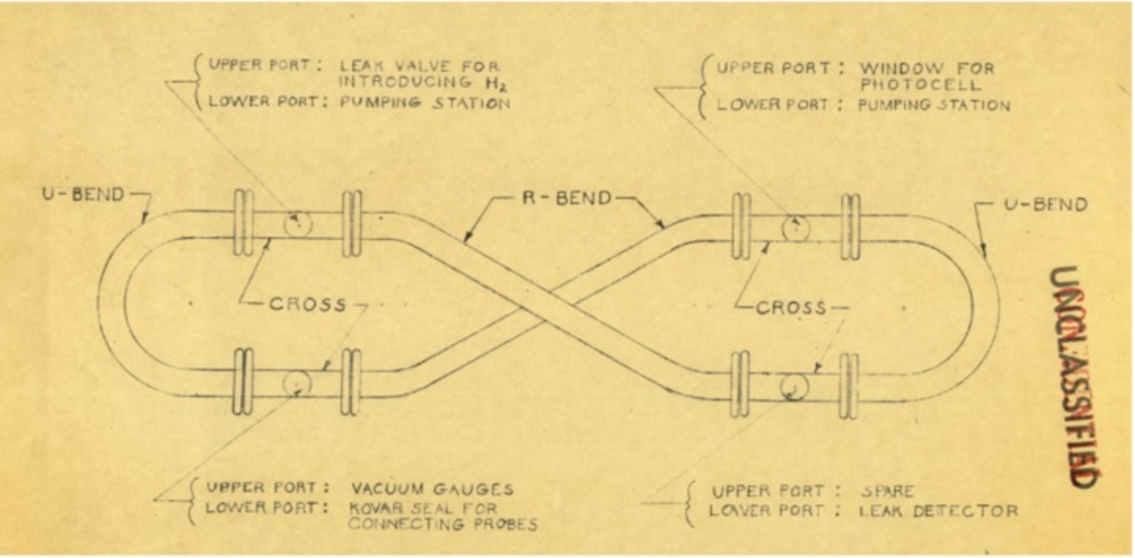


Spitzer came up with the figure-8 stellarator on the slow ski-lifts in Aspen CO

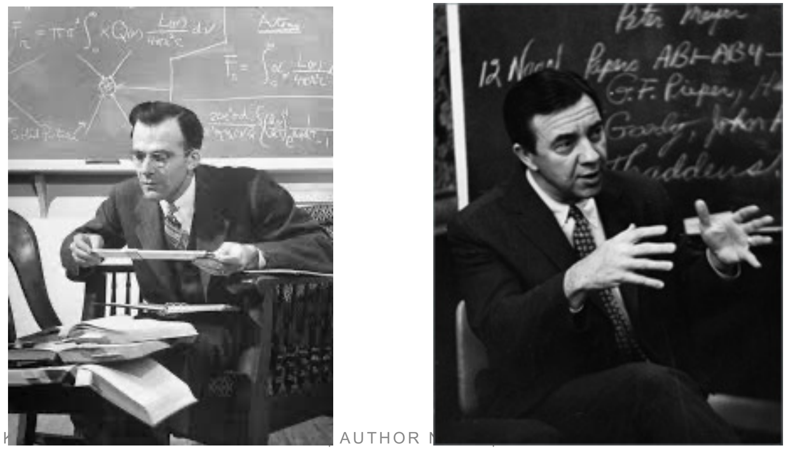
Stellarators: first introduced to the world at the 1958 Peaceful Uses of Atomic Energy Conference (US Nuclear Fusion Research declassified)

The first stellarator experiments

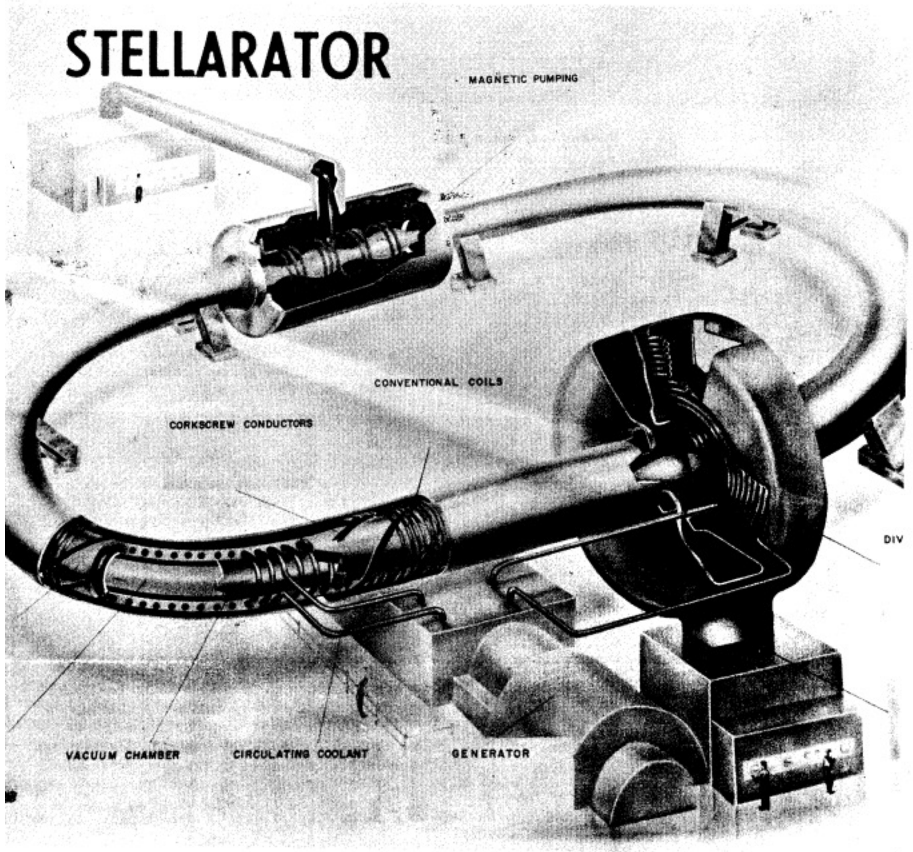
Figure-eight (Princeton Model A) – 1953-1958



C. H. Willis, *NJ Project Matterhorn* (1953).

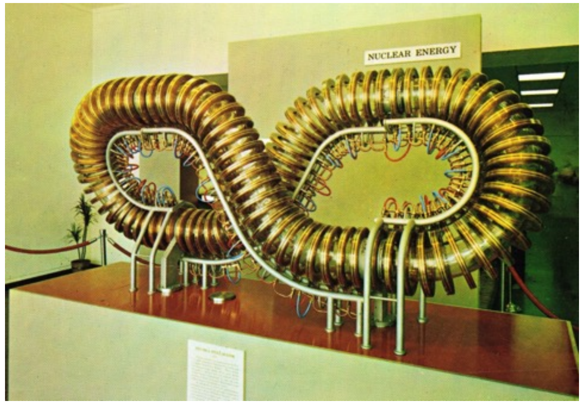


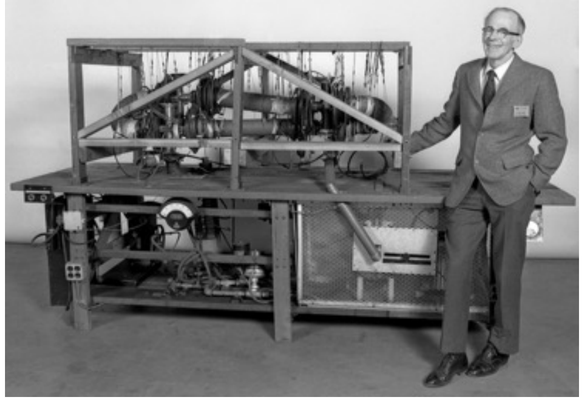
Racetrack (Princeton Model C) – 1962-1969

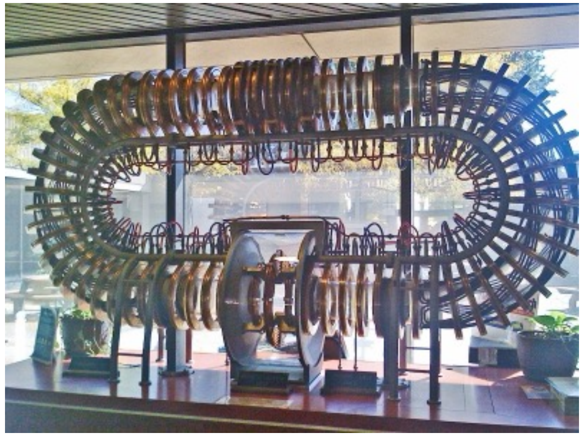


Stellarators at Princeton Plasma Physics Laboratory (PPPL)

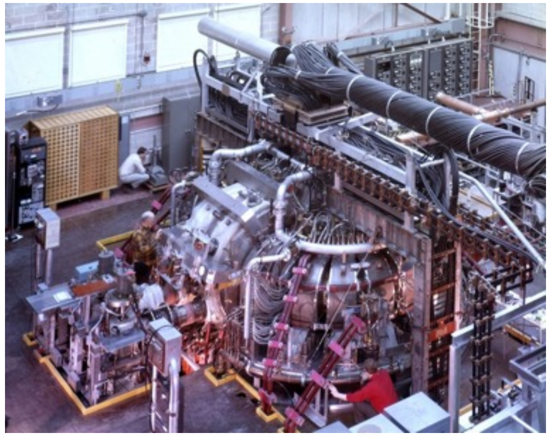
- 1951: Figure-8, racetracks
- Early optimism, models A, B, B-2, B-3, B-64/65/66
- First stellarator reactor design (model D) in 1955





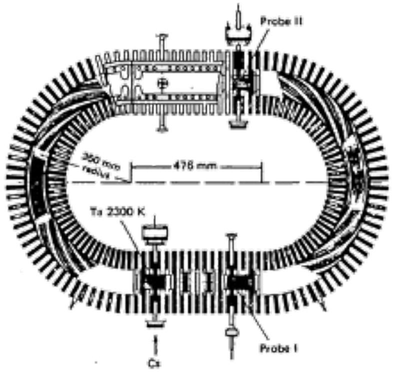


Model C: converted to a Tokamak in 1968



German stellarator experiments

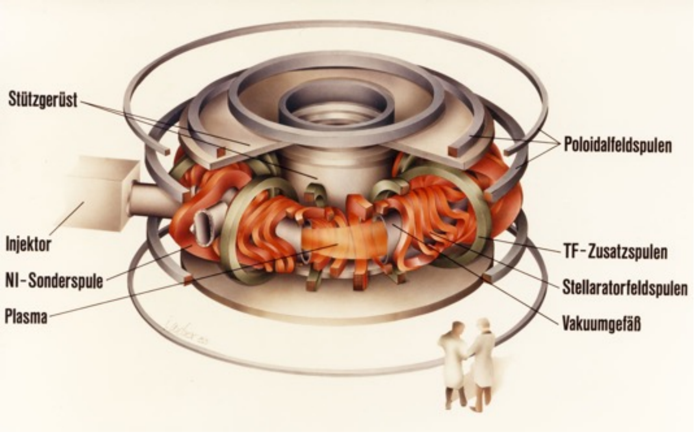
Wendelstein I (~1960)



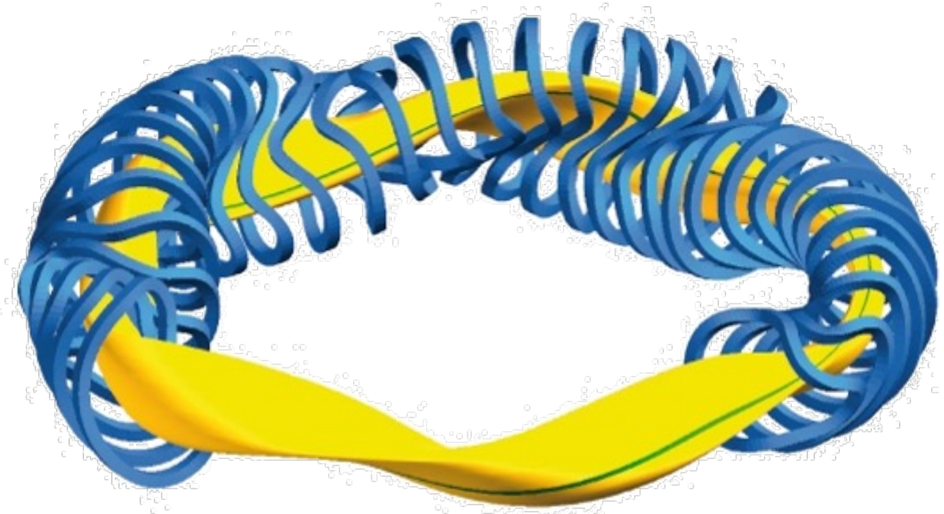
Wendelstein – a mountain in southern Germany

By Daniel Coral - Own work, CC BY 3.0, https://commons.wikimedia.org/w/index.php?curid=17211908

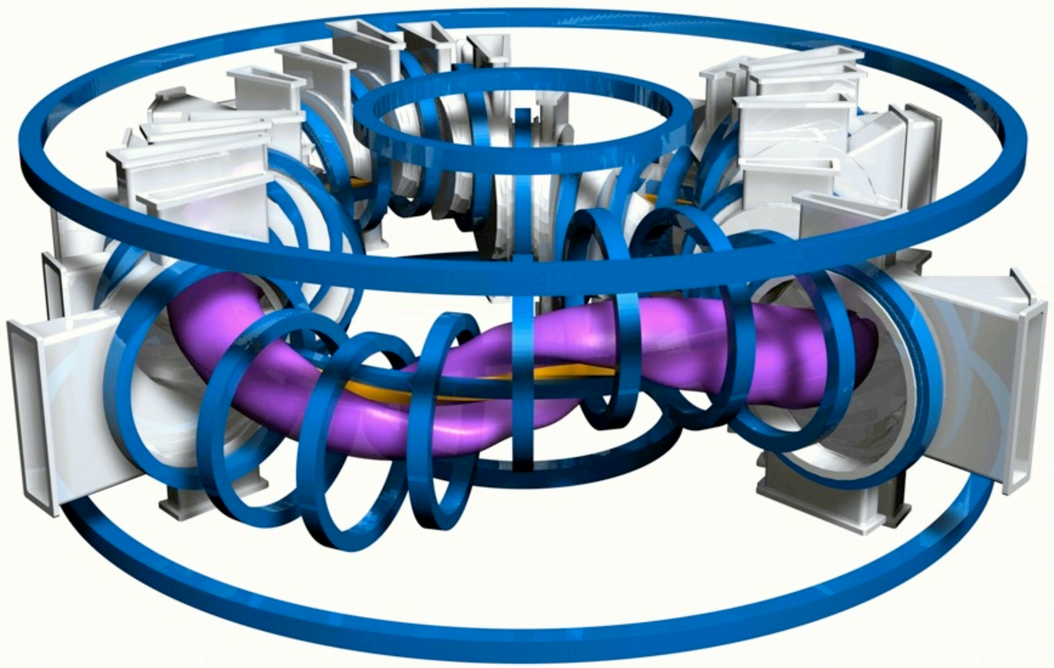
Wendelstein 7-AS



Wendelstein 7-X (2015 -)

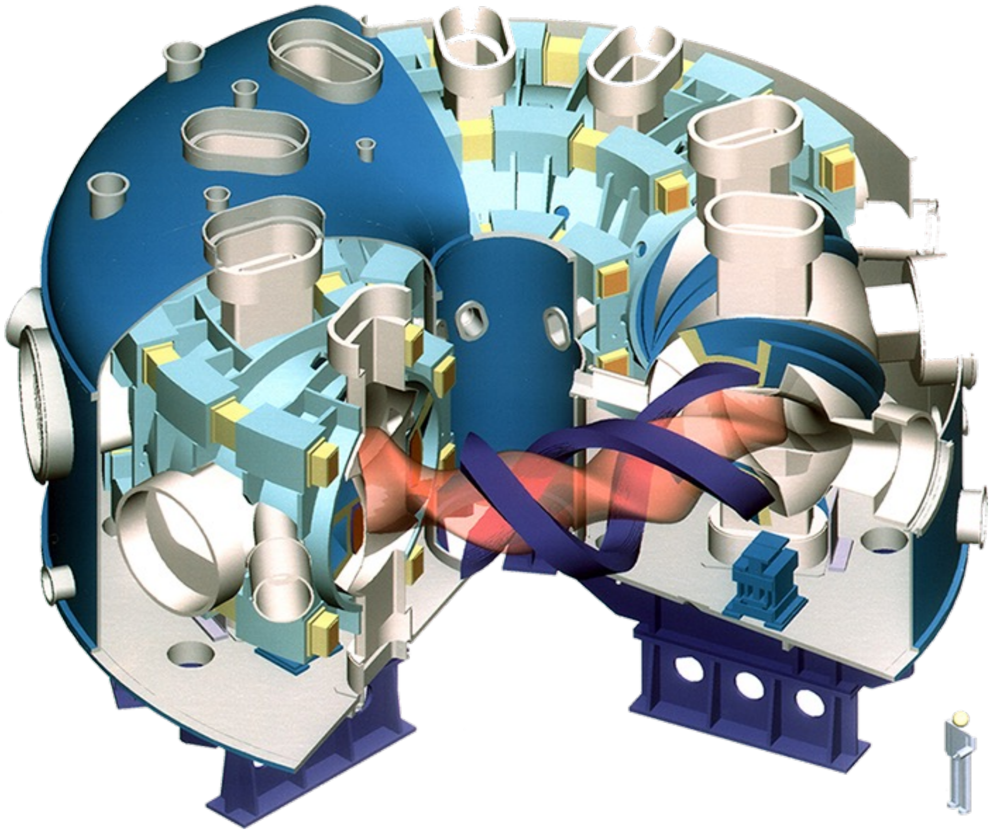


Can create stellarator fields with very different coils

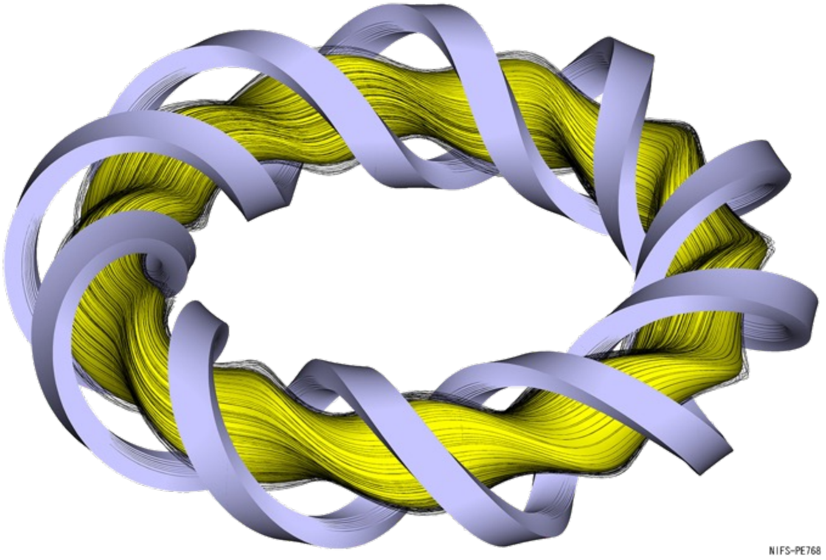


- TJ-II, CIEMAT, Madrid, Spain
  - “Heliac” configuration
  - Most coils are circular and planar

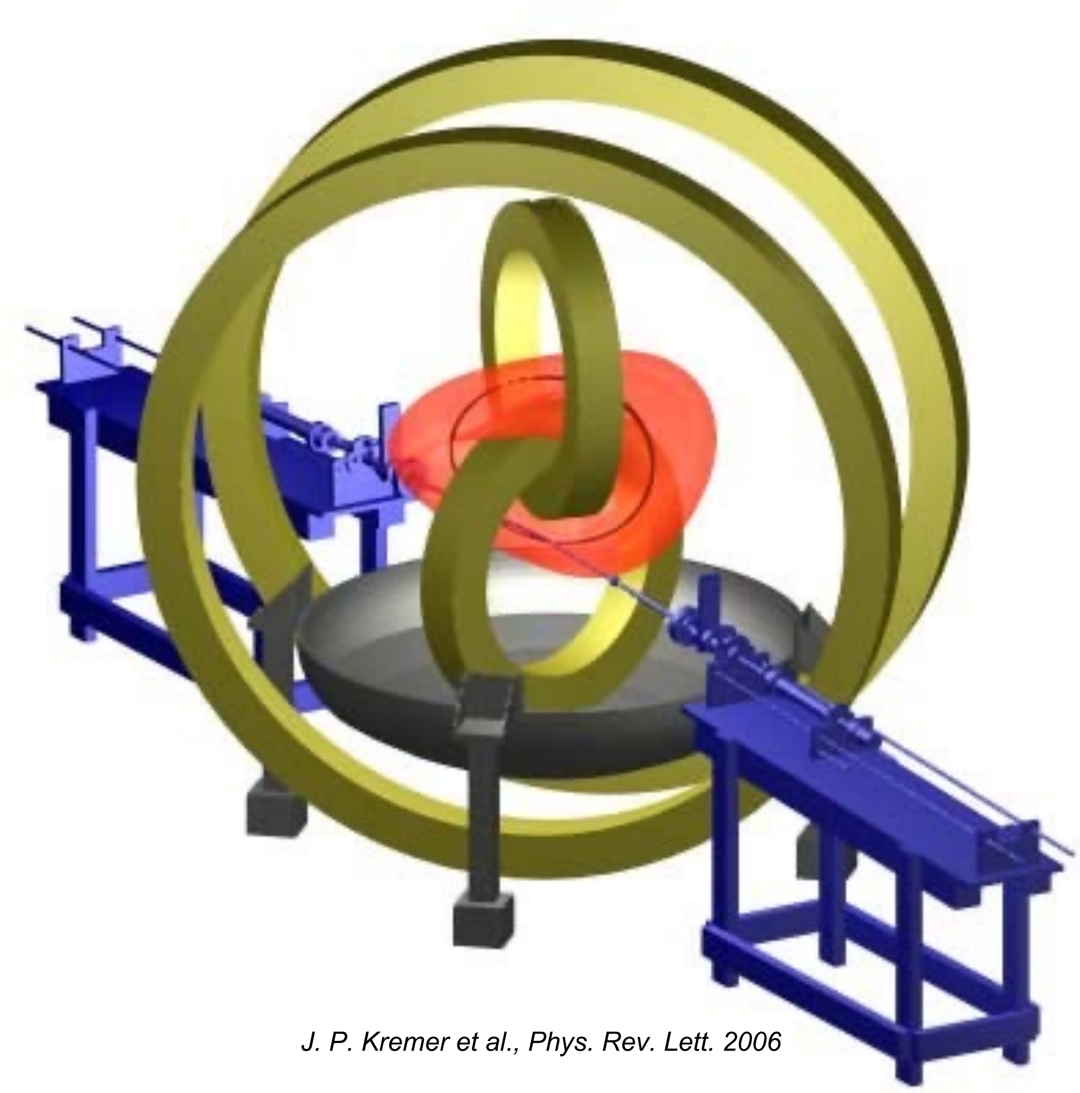
Can create stellarator fields with very different coils



- Large Helical Device (LHD), National Institute for Fusion Science, Toki, Japan
  - Two superconducting helical coils provide most of the magnetic field



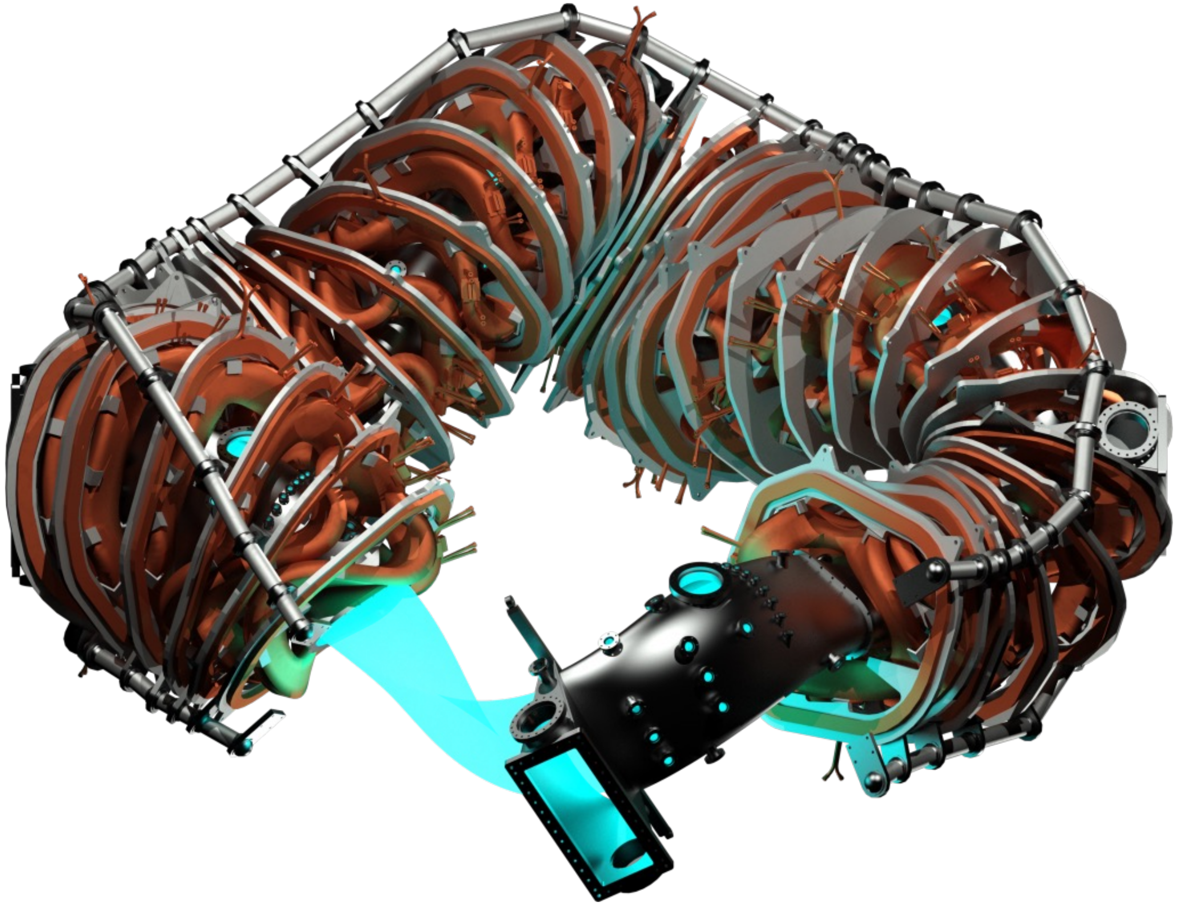
Can create stellarator fields with very different coils



J. P. Kremer et al., Phys. Rev. Lett. 2006

- CNT, Columbia University,
  New York, NY, USA
  - Arguably the simplest stellarator ever built
  - Four circular planar coils
  - Two coils are interlocked and tilted

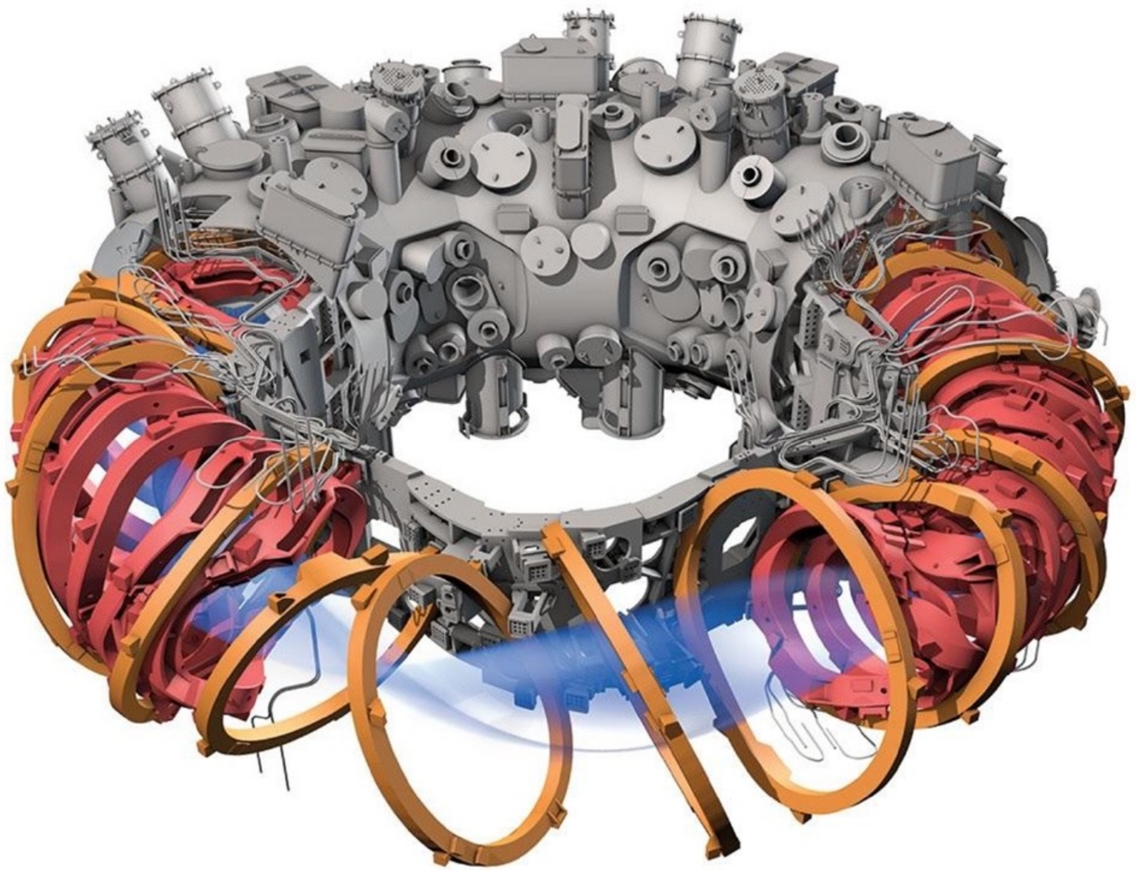
Can create stellarator fields with very different coils



HSX Group, University of Wisconsin

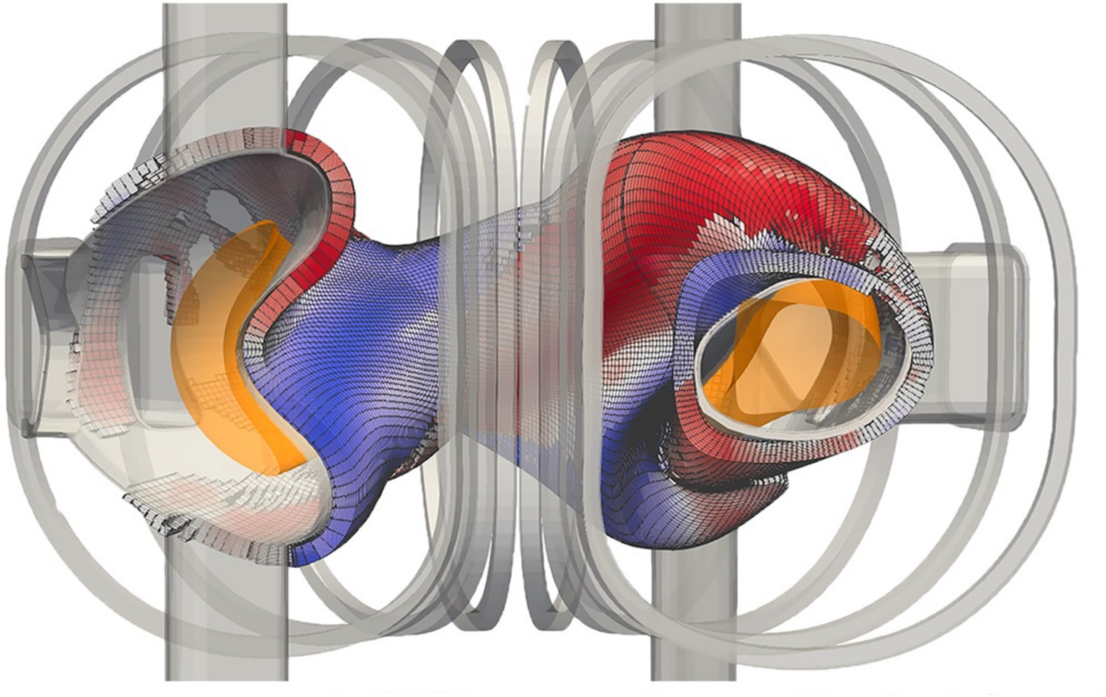
- Helically Symmetric Experiment (HSX), University of Wisconsin, Madison, WI, USA
  - Combination of Modular, “wiggly” coils and planar coils supply the main field
  - Modular coils optimized for good confinement

Can create stellarator fields with very different coils



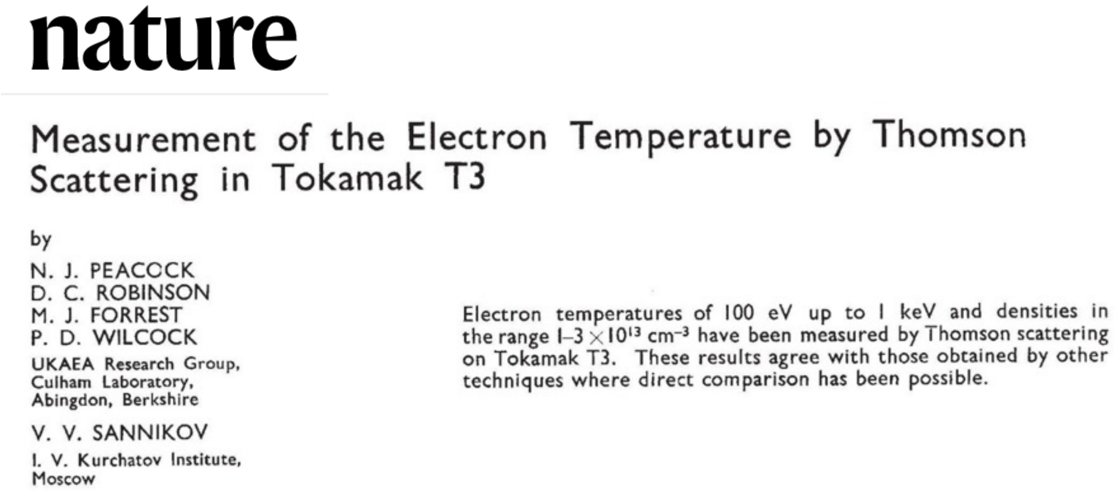
- Wendelstein 7-X (W7-X), Max Planck Institute for Plasma Physics, Greifswald, Germany
  - Most advanced stellarator built to date
  - Modular coils optimized for good confinement
  - Superconducting coils

Can create stellarator fields with very different coils – or permanent magnets



- MUSE, Princeton Plasma Physics Laboratory
  - Unique combination of simple coils and permanent magnets
  - Quasi-axisymmetric design

However … the tokamak initially outperformed the stellarator



nature

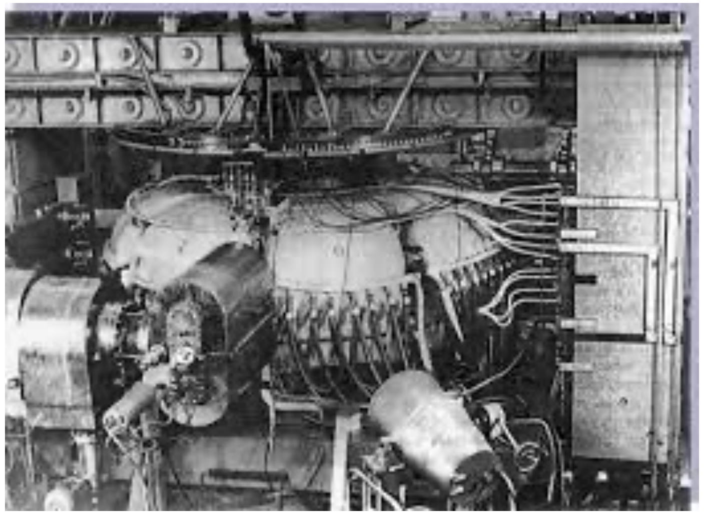
Measurement of the Electron Temperature by Thomson Scattering in Tokamak T3

by

N. J. PEACOCK
D. C. ROBINSON
M. J. FORREST
P. D. WILCOCK
UKAEA Research Group,
Culham Laboratory,
Abingdon, Berkshire

V. V. SANNIKOV
I. V. Kurchatov Institute,
Moscow

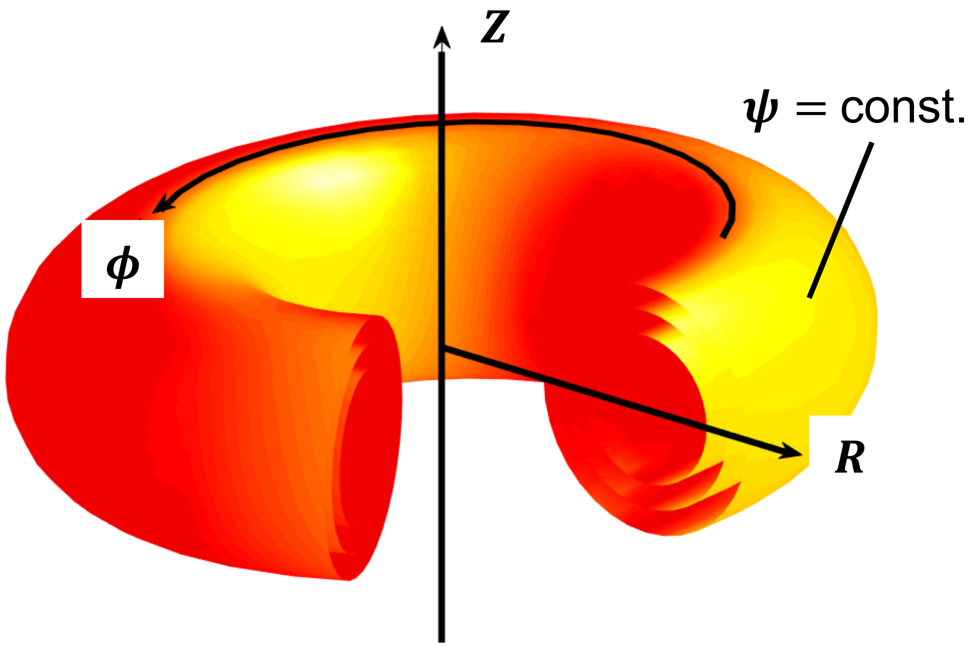
Electron temperatures of 100 eV up to 1 keV and densities in the range 1–3 × 10^{13} cm^{-3} have been measured by Thomson scattering on Tokamak T3. These results agree with those obtained by other techniques where direct comparison has been possible.



Continuous toroidal symmetry yields particle confinement

Single-particle Lagrangian

$$\mathcal{L}(\boldsymbol{x},\dot{\boldsymbol{x}}) = m\frac{|\dot{\boldsymbol{x}}|^2}{2} + q\boldsymbol{A}(\boldsymbol{x})\cdot\dot{\boldsymbol{x}}$$



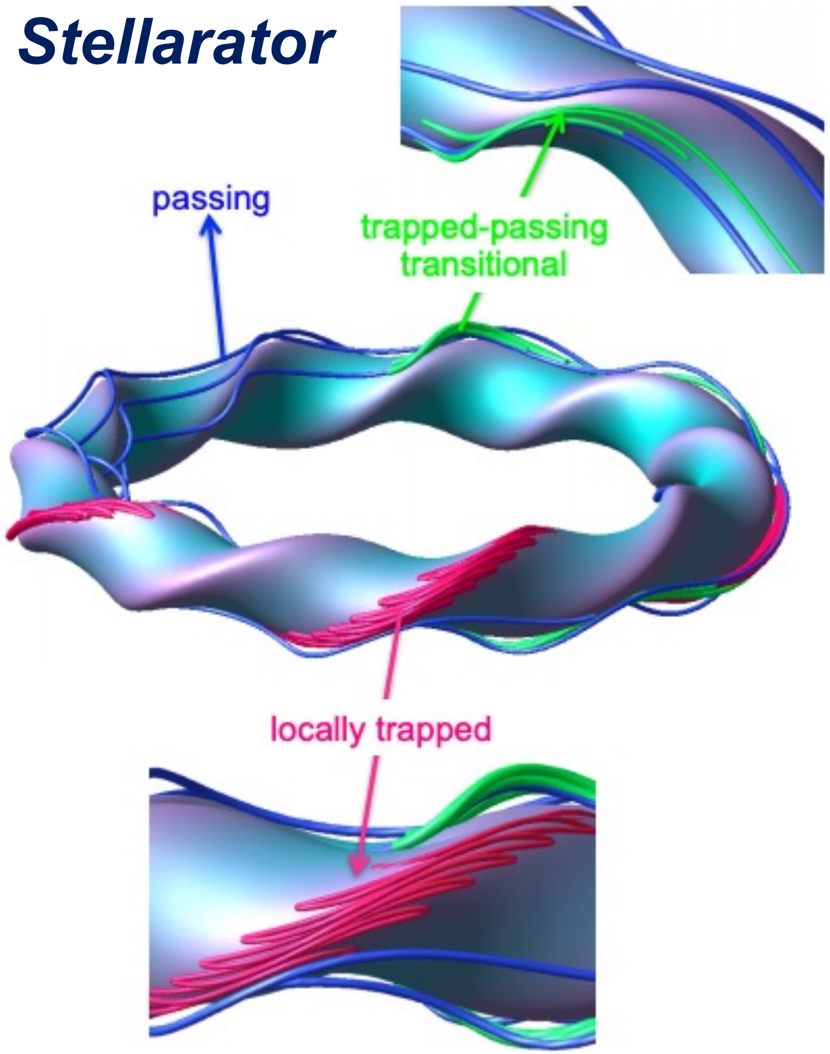
$$\frac{d}{dt}\underbrace{\left(\frac{\partial\mathcal{L}(R,\phi,Z,\dot{R},\dot{\phi},\dot{Z},t)}{\partial\dot{\phi}}\right)}_{p_\phi} = \cancel{\frac{\partial\mathcal{L}(R,\phi,Z,\dot{R},\dot{\phi},\dot{Z},t)}{\partial\phi}}$$

Toroidal symmetry

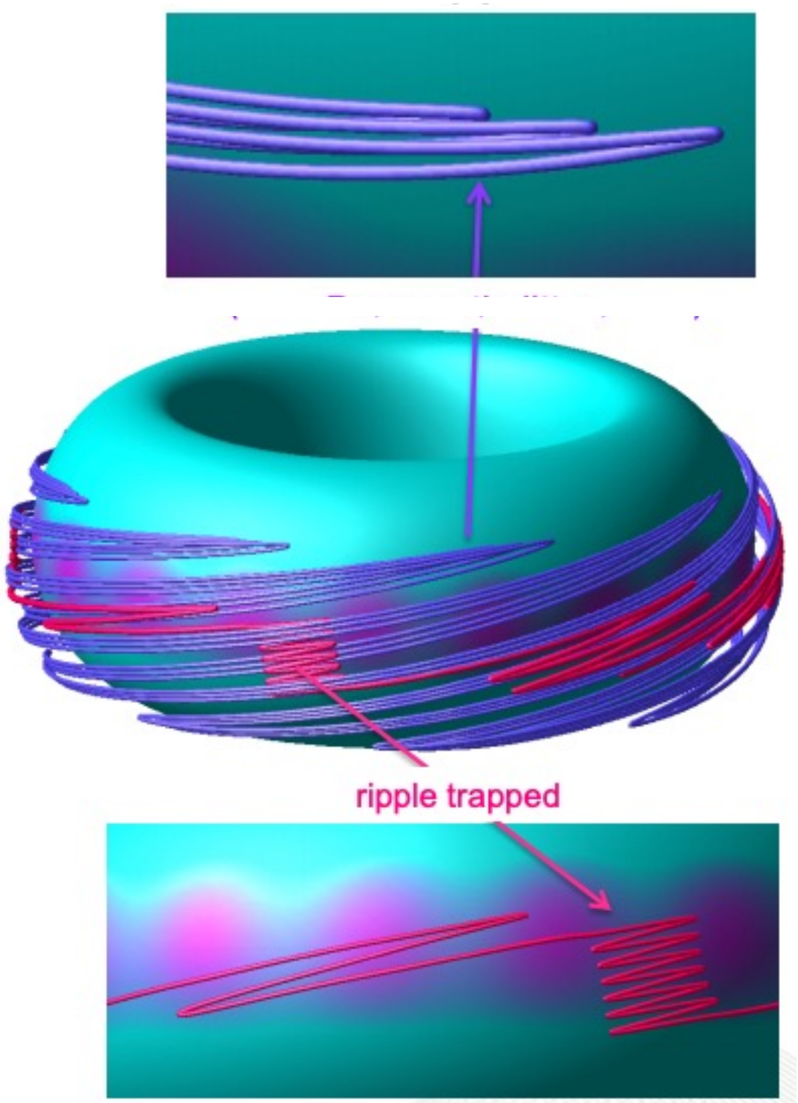
Confinement to ψ surfaces!

The zoology of particle orbits in 3D fields

Stellarator



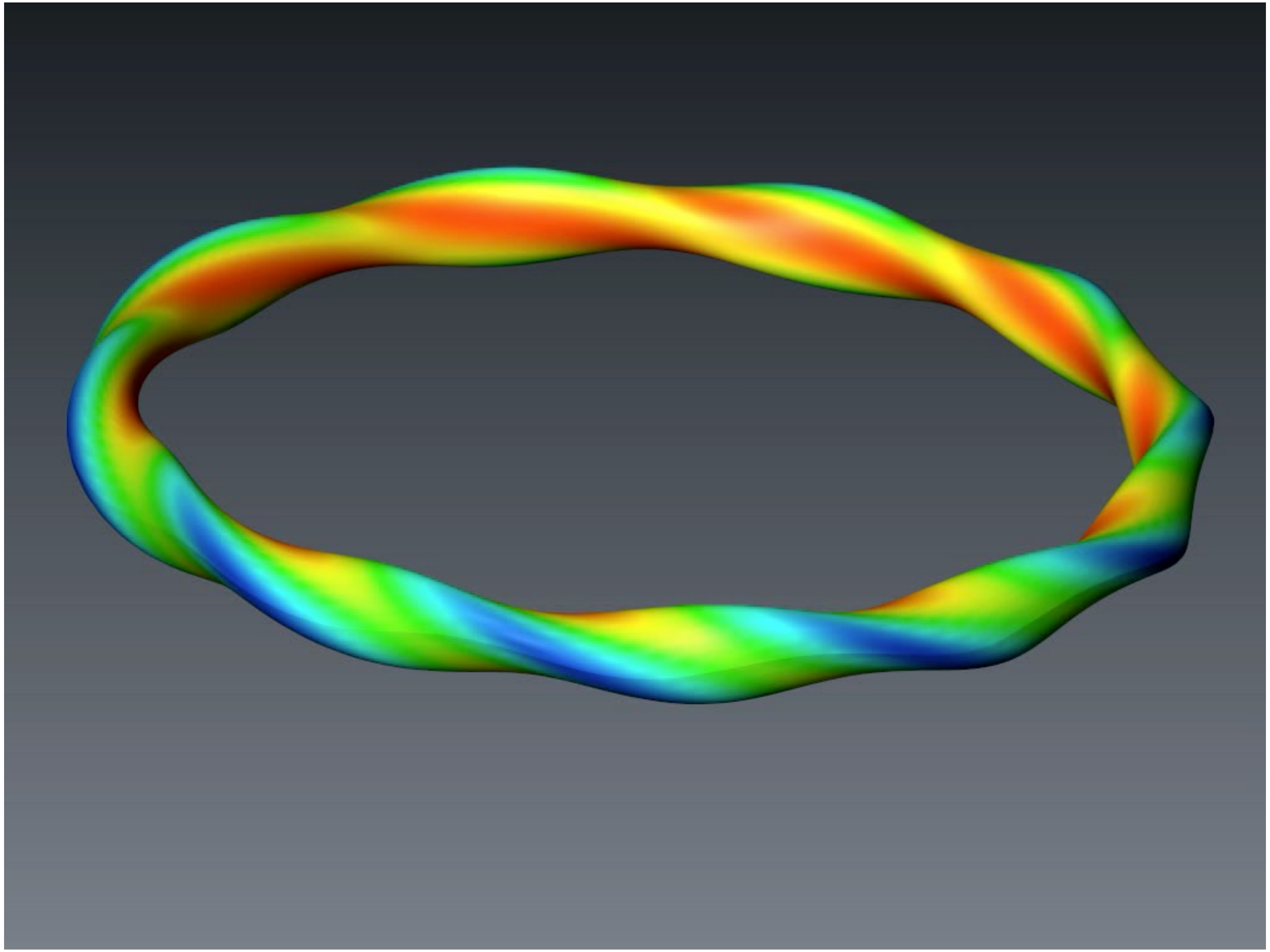
Perturbed tokamak



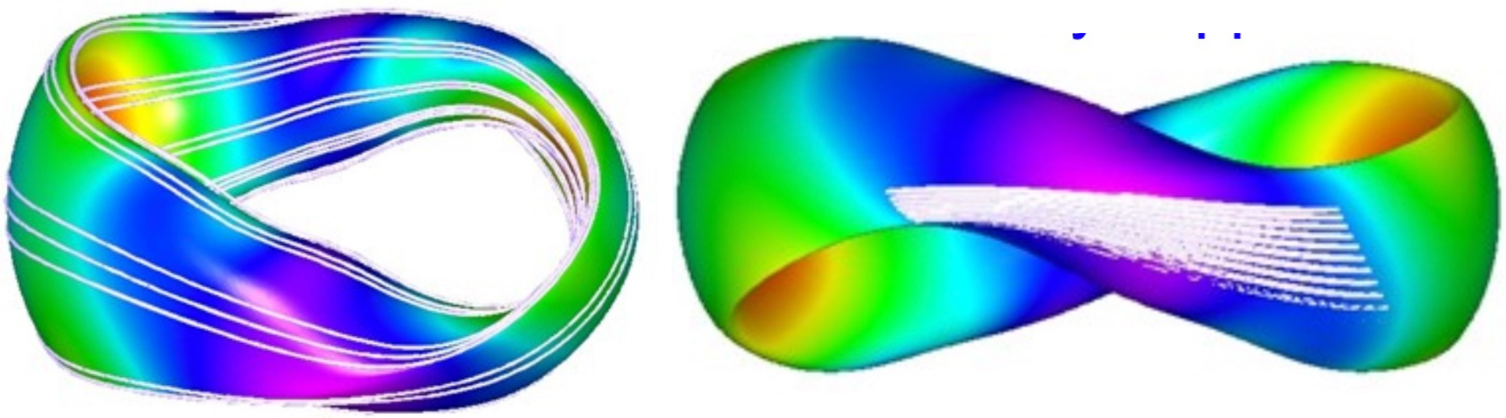
Can we confine them all?

D.A. Spong et al, *APS DPP* (2014).

Orbits in a stellarator with poor trapped-particle confinement

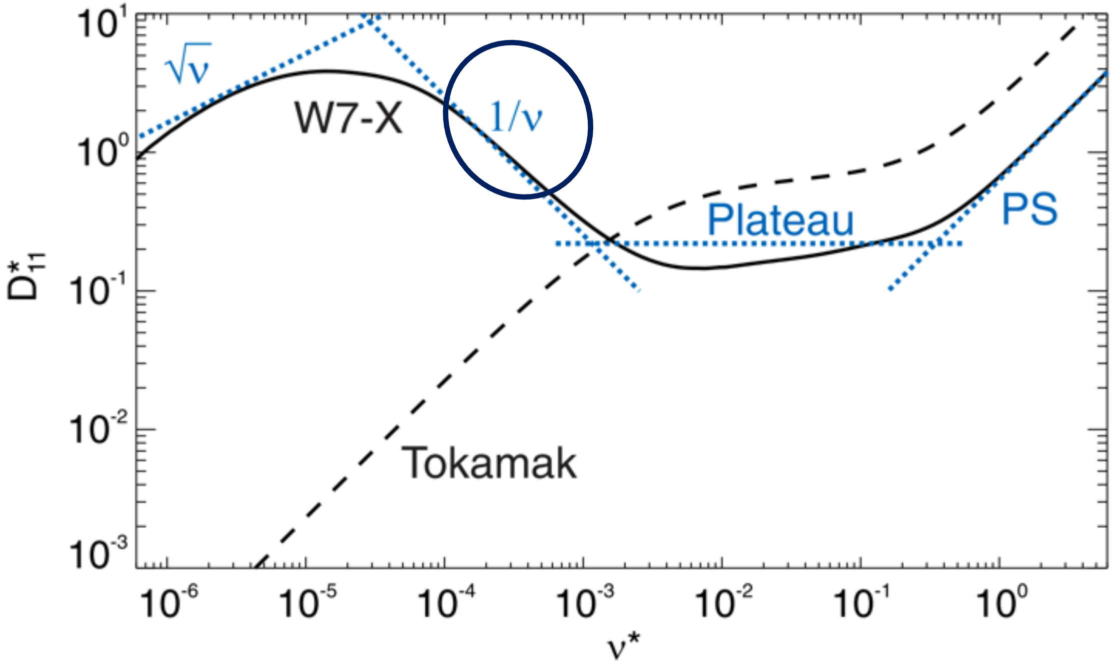


Collisional guiding center confinement



D.A. Spong et al, *IAEA* (2003).

P. Helander, *Rep. Prog. Phys.*, 77 (2014).



“drift kinetic equation”

$$\left(v_{||}\widehat{\boldsymbol{b}} + \boldsymbol{v}_{\mathrm{drift}}\right) \cdot \nabla f = C(f)$$

Guiding center motion **Collisions**

Symmetry of field strength yields particle confinement in 3D

$$\mathcal{L}(\boldsymbol{x}, \dot{\boldsymbol{x}}) = m\frac{|\dot{\boldsymbol{x}}|^2}{2} + q\boldsymbol{A}(\boldsymbol{x}) \cdot \dot{\boldsymbol{x}}$$

$$\Big\downarrow$$

Strongly magnetized

$$\mathcal{L}(\psi, \theta, \phi, \dot{\psi}, \dot{\theta}, \dot{\phi}) = \mathcal{L}(\psi, B(\psi, \theta, \phi), \dot{\psi}, \dot{\theta}, \dot{\phi})$$

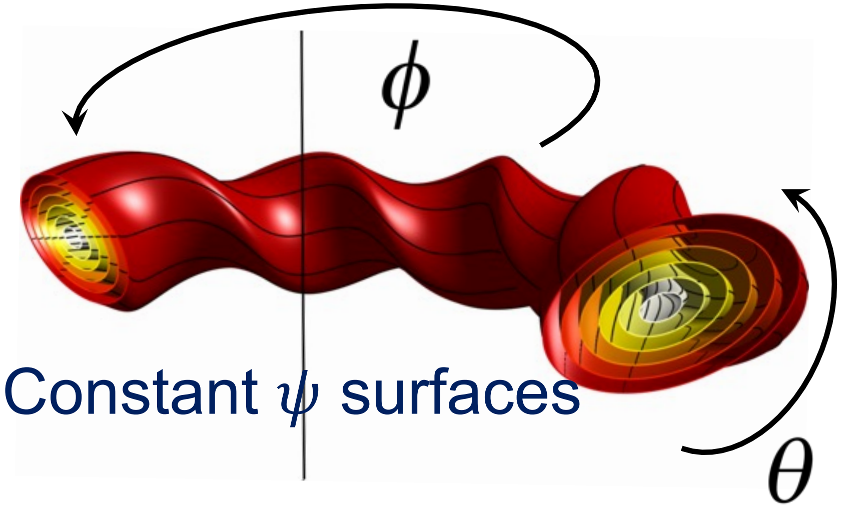
$$\frac{\partial B(\psi, \theta, \phi)}{\partial \phi} = 0 \longrightarrow \frac{dp_\phi}{dt} = 0$$

Ignorable coordinate

Conserved momentum

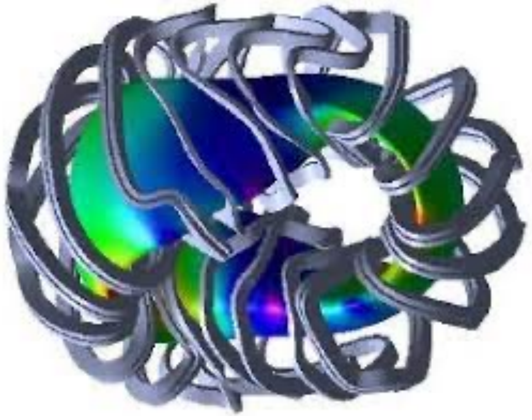
$$p_\phi = \cancel{\frac{mv_\phi}{qB}} + F(\psi)$$

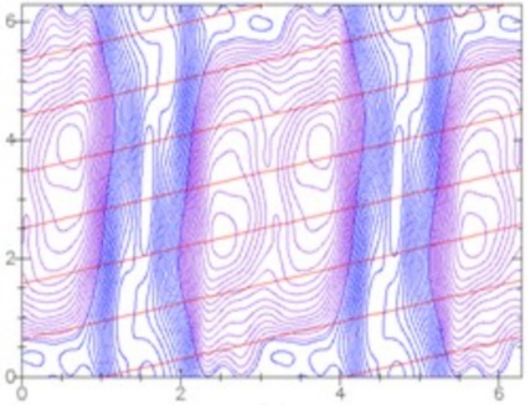
Particles stay confined to ψ surfaces



Quasisymmetry - a hidden symmetry of magnetic fields

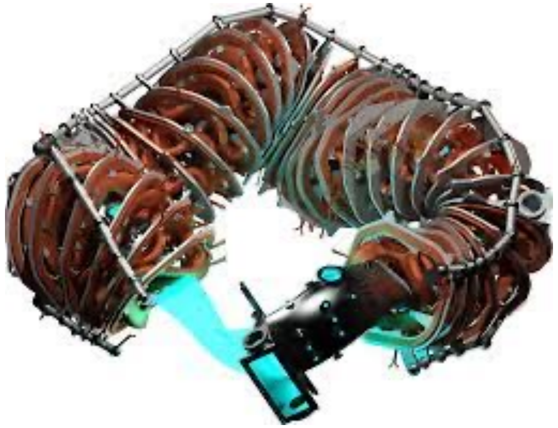
Quasi-poloidal symmetry

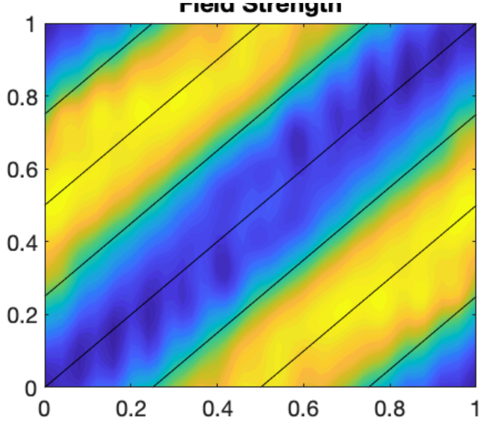




D. Strickler et al, *Fusion Eng. & Design*, 66 (2003).

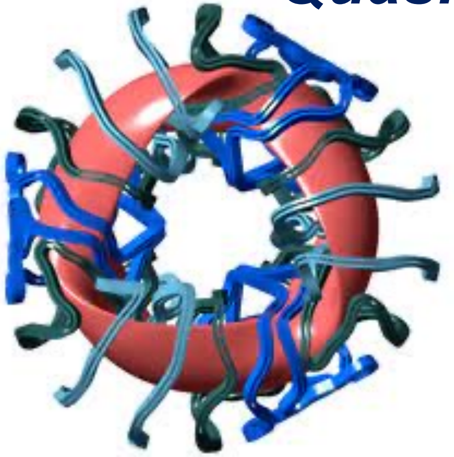
Quasi-helical symmetry

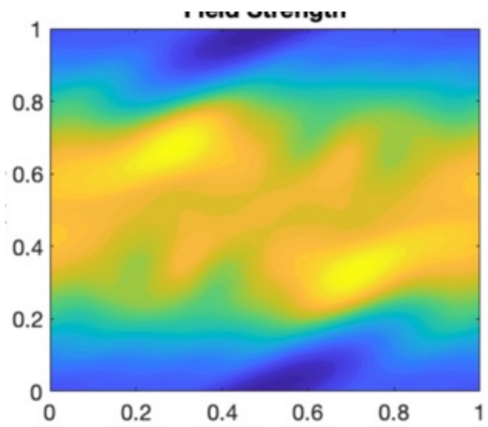




F. Anderson et al, *Fusion Tech.*, 27 (1994).

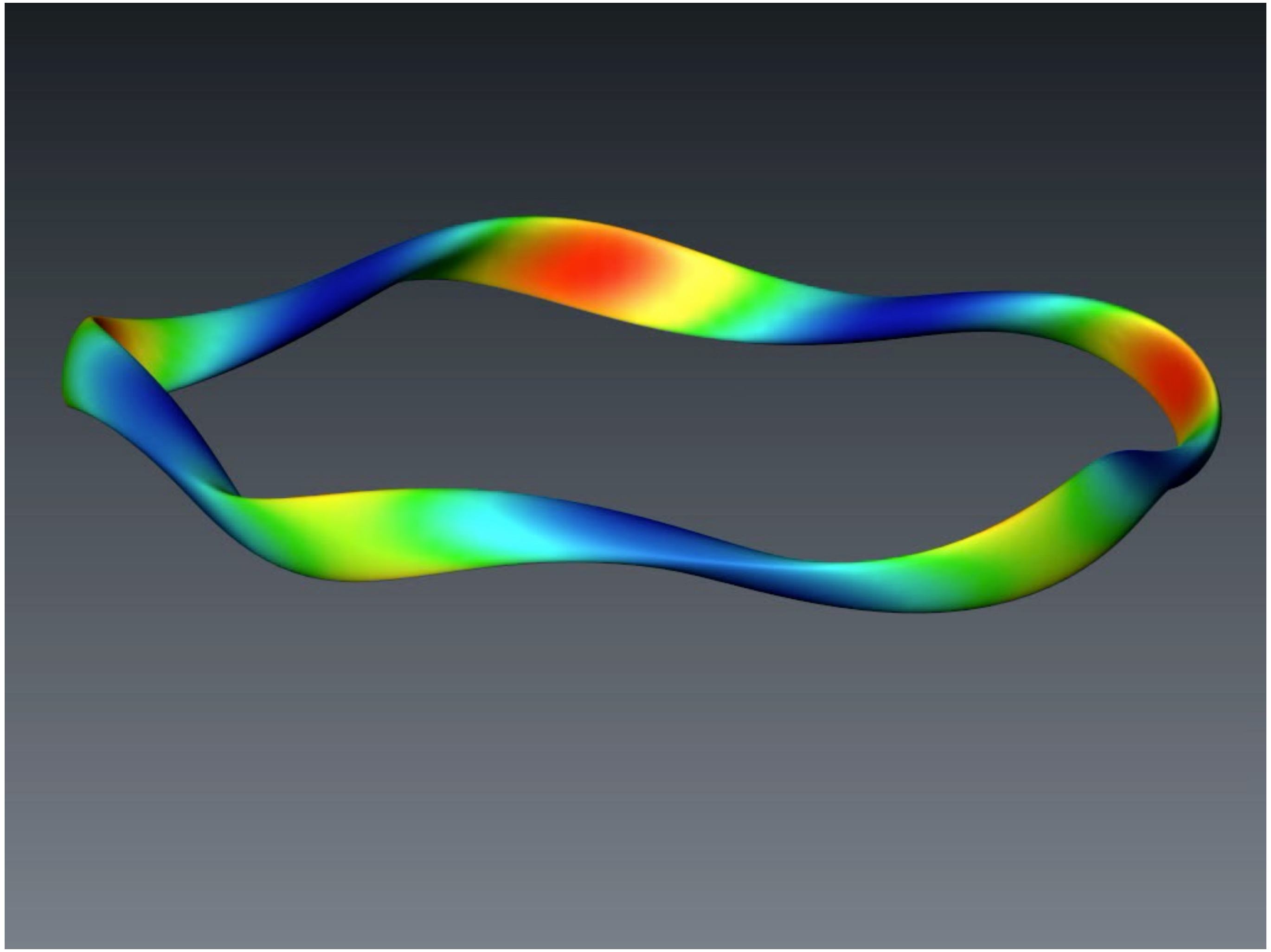
Quasi-axisymmetry



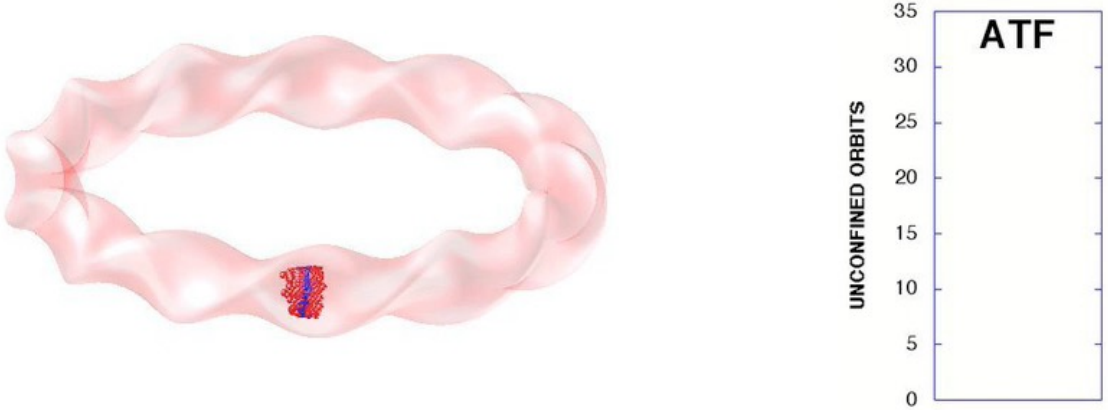


B. Nelson et al, *Fusion Eng. & Design*, 66 (2003).

With the right symmetries, stellarators can confine trapped particles!

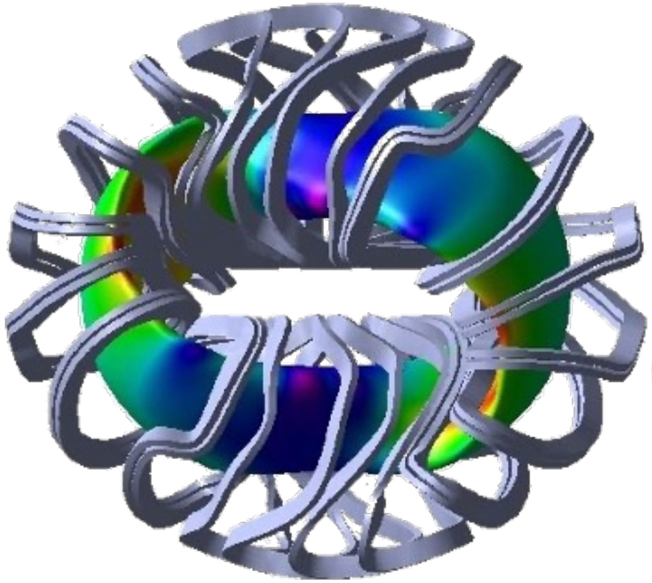


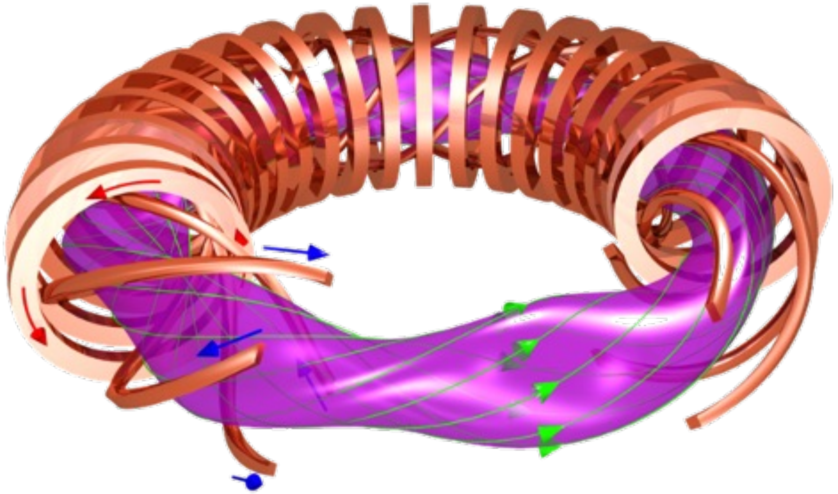
Fewer particles are drifting out and are lost in optimised stellarators (QPS) vs in non-optimised stellarators (ATF)

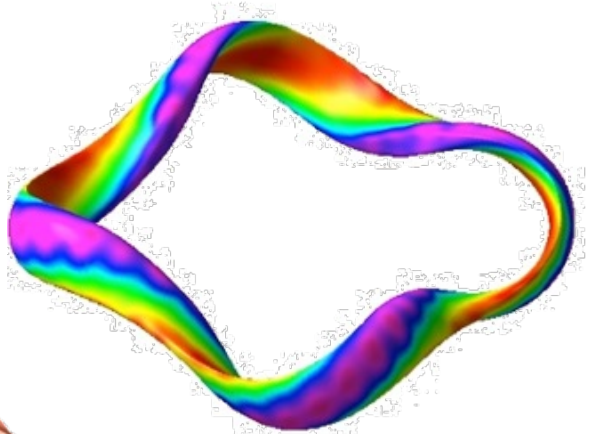


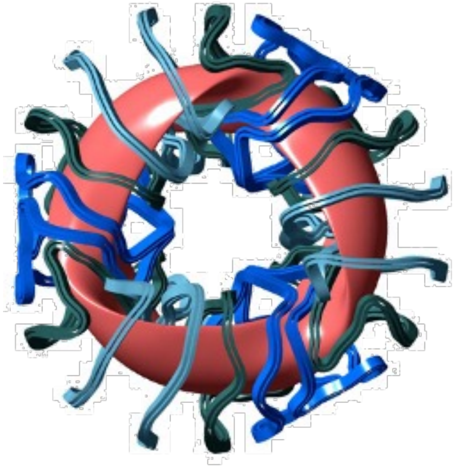
courtesy of D. Spong

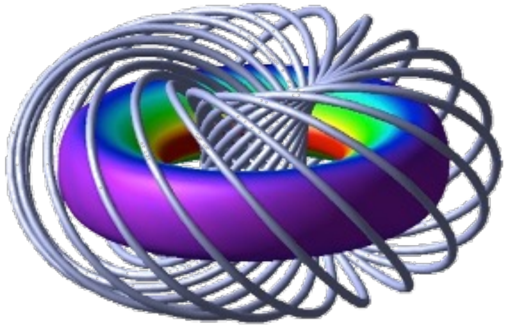
Which stellarator performs the best?

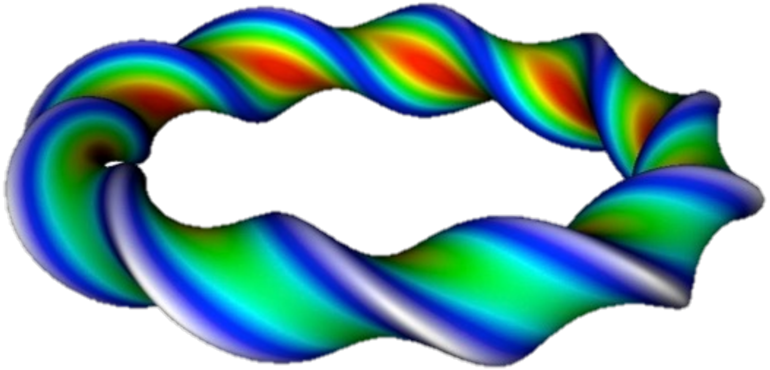


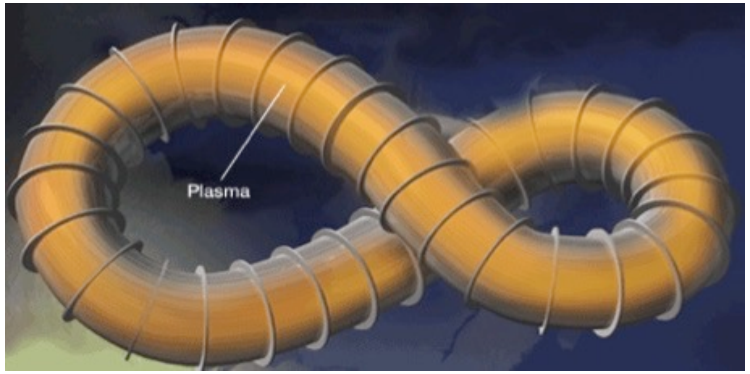


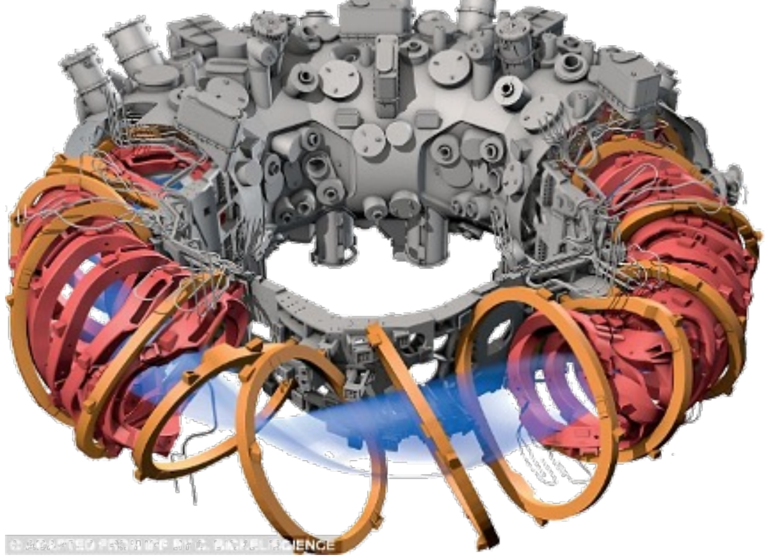




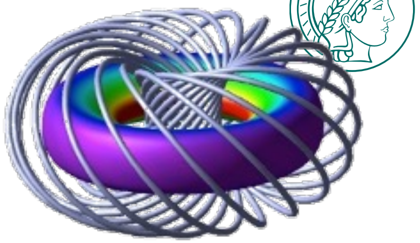


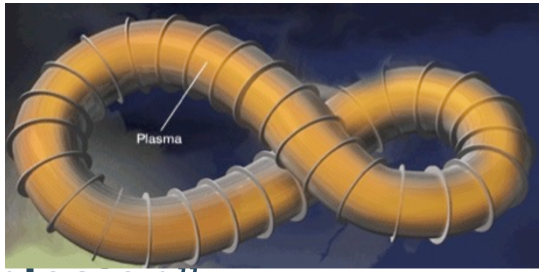




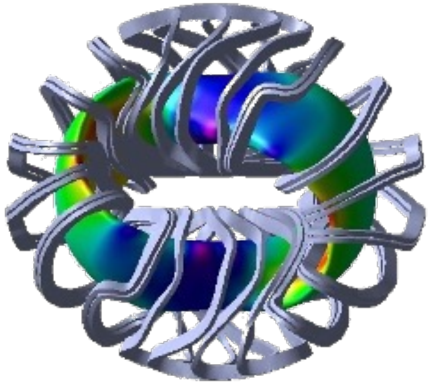

© ADAPTED FROM IPP BY C. BICKEL/SCIENCE

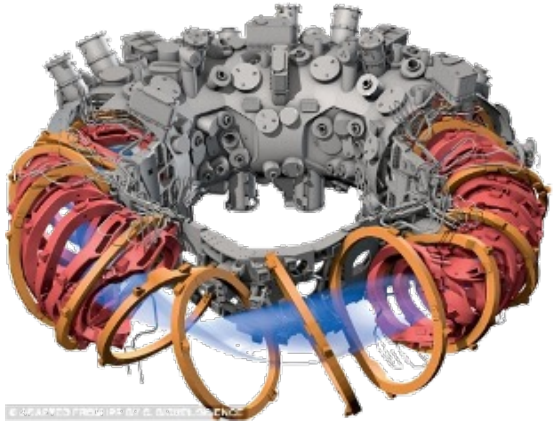
Why we can optimise stellarators

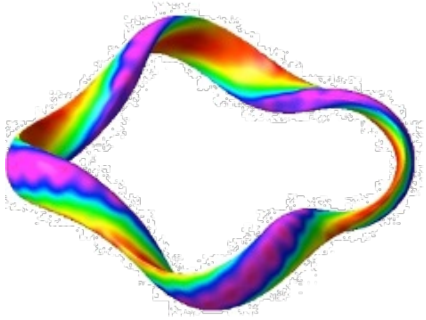


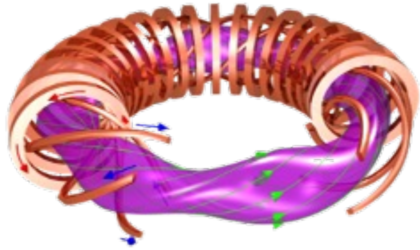


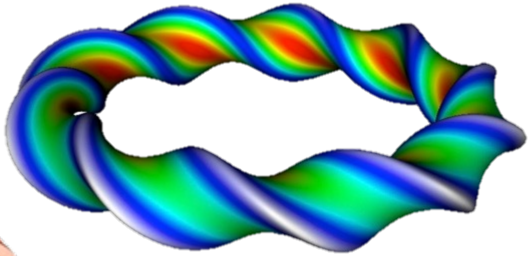
- 3D shapes open up very large design space: ~ 40 independent parameters (A. Boozer, L. P Ku, 2010) based on SVD analysis
- Axisymmetric tokamak shape parameters: $\boldsymbol{\varepsilon},\boldsymbol{\kappa},\boldsymbol{\delta}$
- Thought experiment: quantize shape parameters into 10 levels
  - **10^3 2D configurations vs. 10^{40} 3D configurations => “combinatorial explosion”**
  - **Other large numbers: 7×10^{22} visible stars, 6x1030 prokaryotes (bacteria) on earth’s surface**











Designing stellarators – different approaches

Building coils and see what happens, e.g.

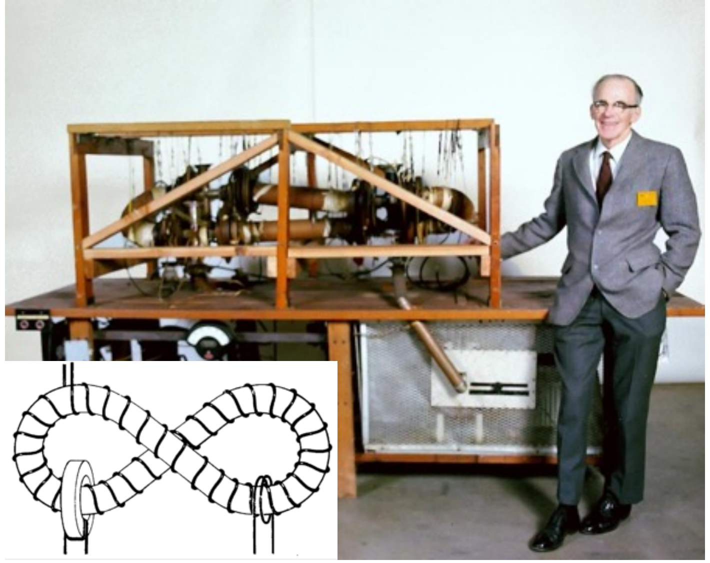
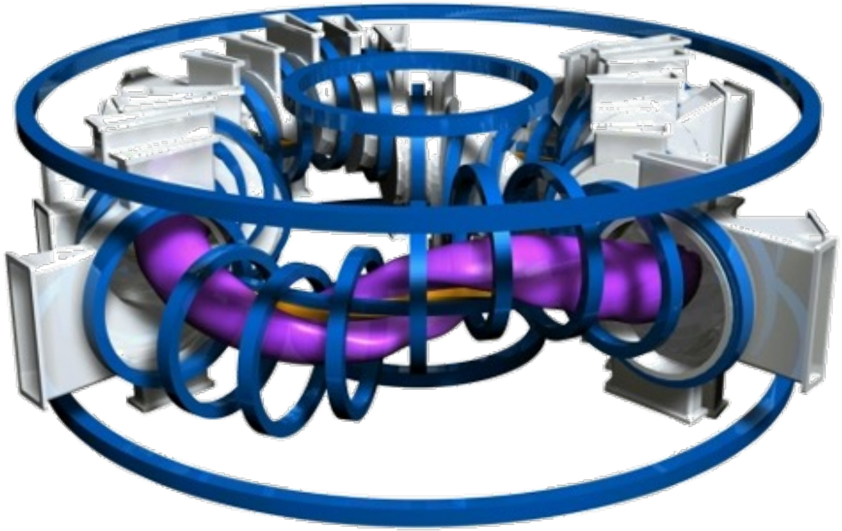
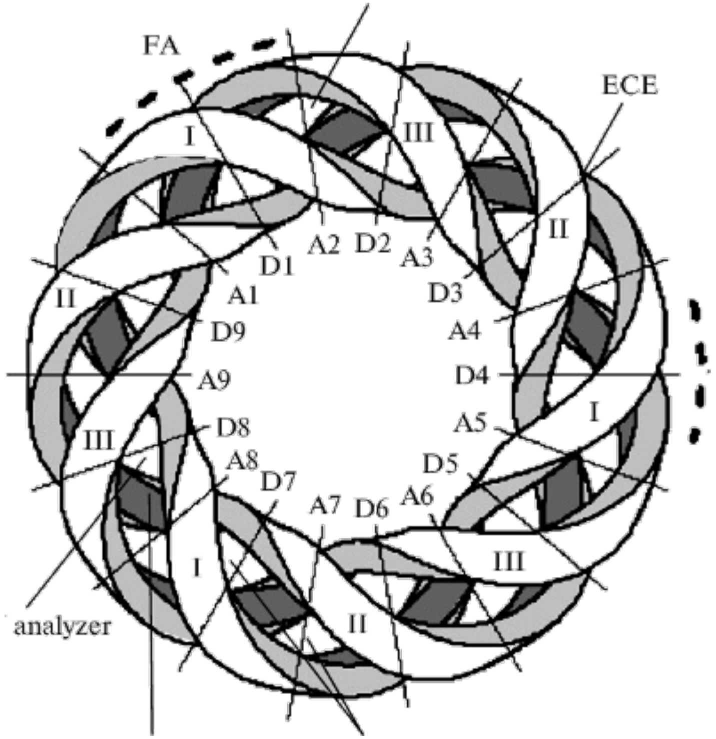


Figure 8-stellarator: first Stellarator designed by Lyman Spitzer



Heliac

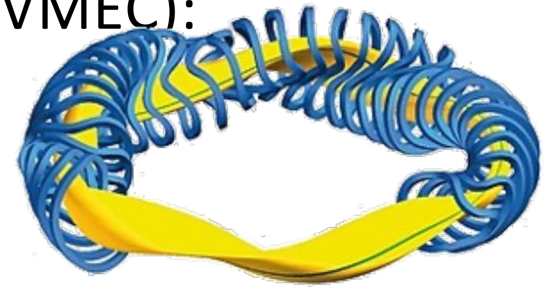


Torsatron

Standard stellarator-optimisation routine

Plasma-first optimisation with ideal MHD equilibrium code (namely VMEC):

1. **optimize plasma boundary**
2. **optimize coils to re-produce this plasma boundary**



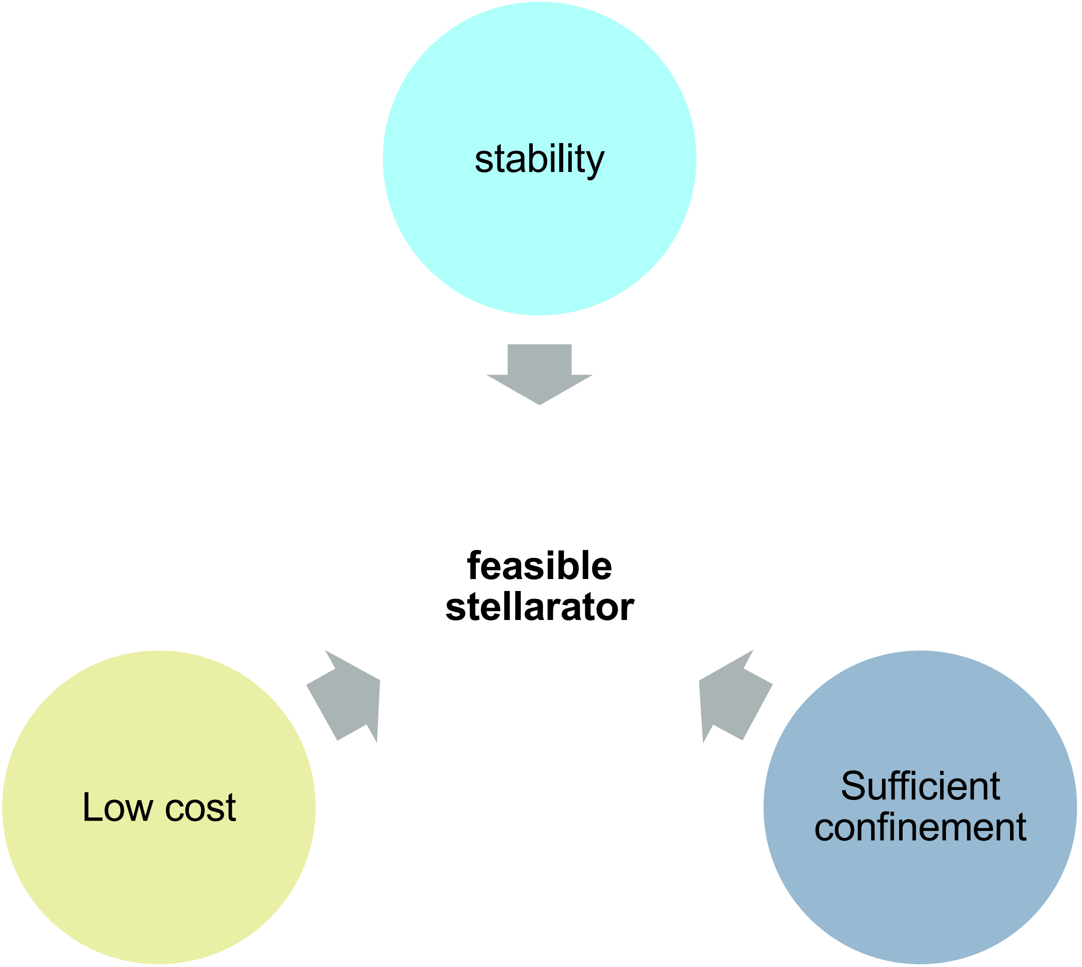
Advantages:

- One learns about plasma boundary effects on optimisation parameters
- Less numerically expensive than directly optimising the coils: one does not have to evaluate the magnetic field produced by the coils

Disadvantages:

- There might not be a set of finite filaments which can reproduce the magnetic field required
- Coil complexity can only be optimized with proxies or it becomes very numerically expensive
- One requires two optimisation procedures to obtain the set of coils
- Doesn't always account for islands and chaotic regions

Requirements for feasible stellarator



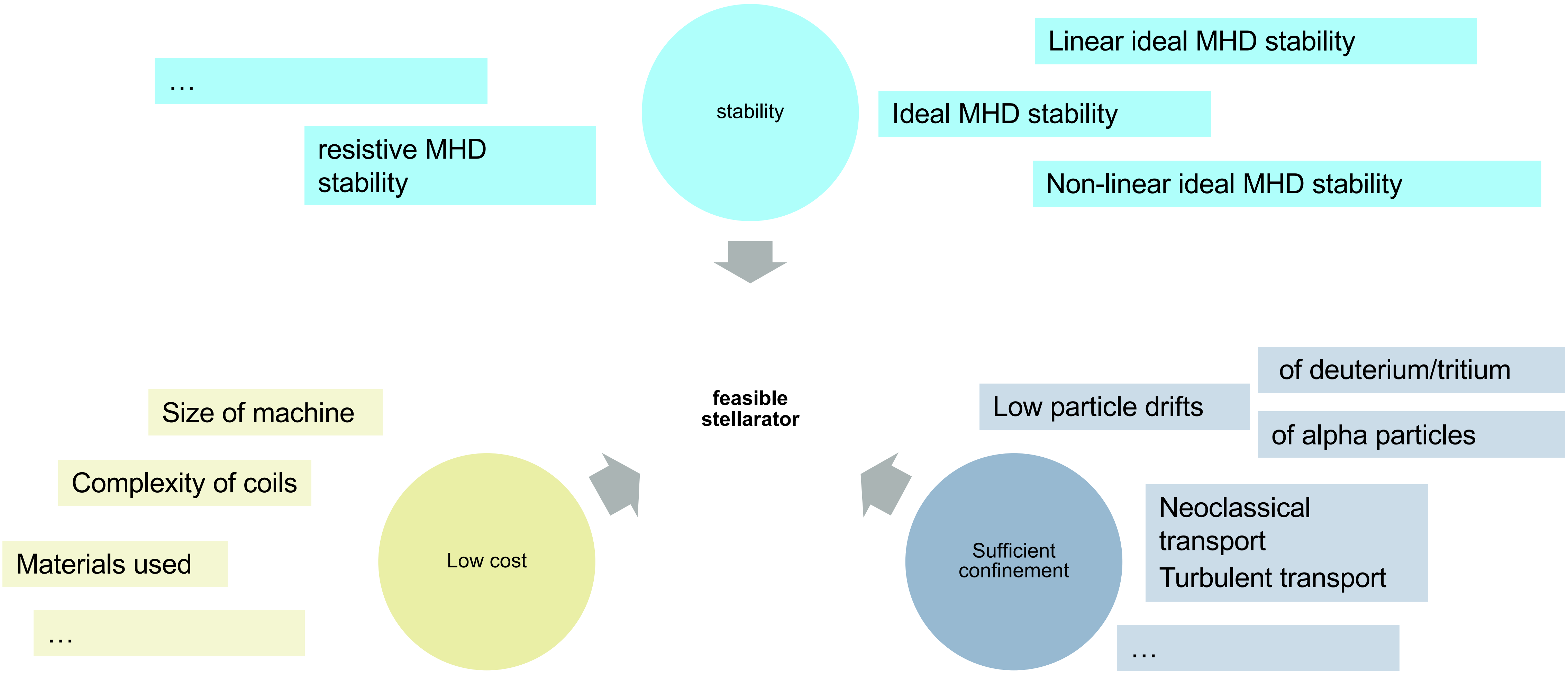
stability

feasible stellarator

Low cost

Sufficient confinement

Requirements for feasible stellarator



stability

…

resistive MHD stability

Linear ideal MHD stability

Ideal MHD stability

Non-linear ideal MHD stability

feasible stellarator

Low cost

Size of machine

Complexity of coils

Materials used

…

Sufficient confinement

Low particle drifts

of deuterium/tritium

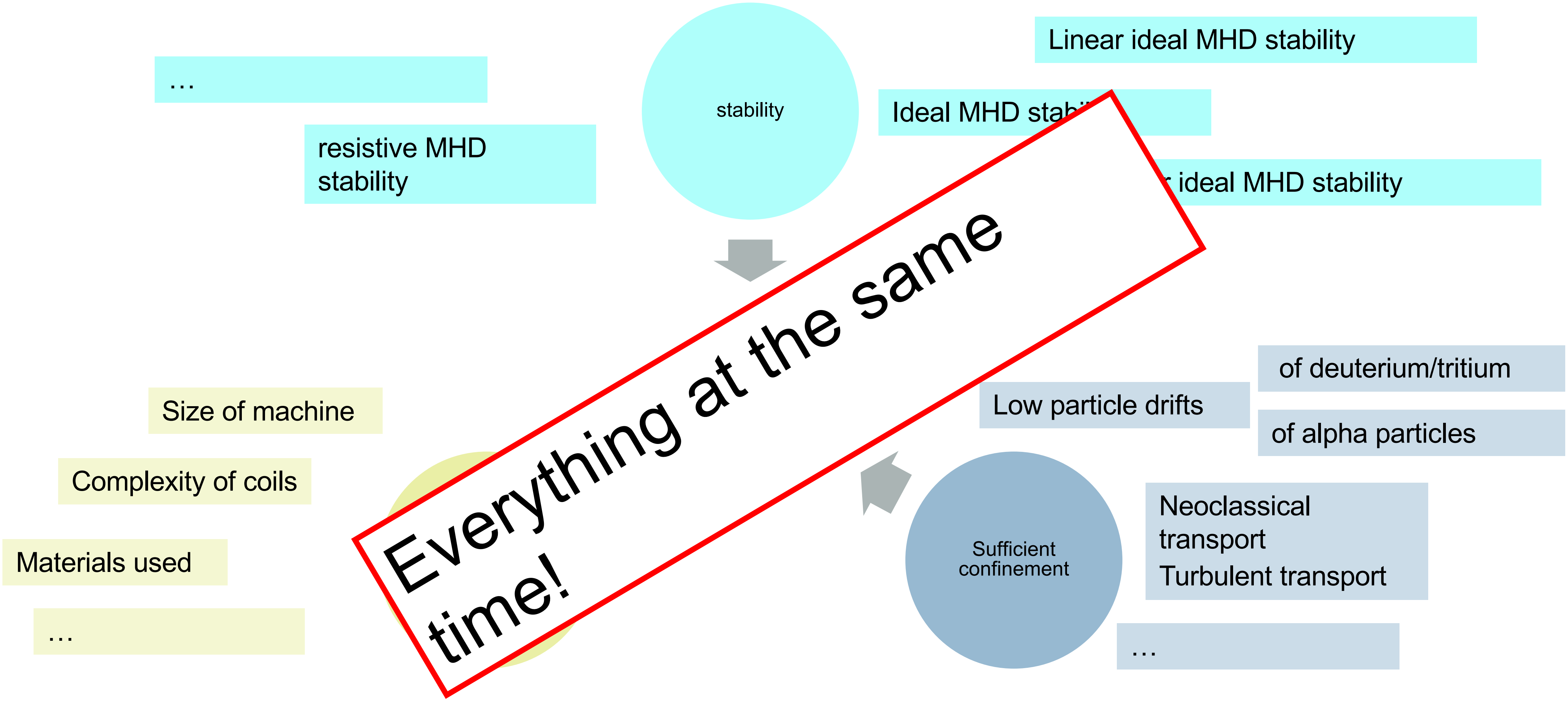
of alpha particles

Neoclassical transport

Turbulent transport

…

Requirements for feasible stellarator



stability

Linear ideal MHD stability

Ideal MHD stab

r ideal MHD stability

…

resistive MHD stability

Size of machine

Complexity of coils

Materials used

…

Sufficient confinement

Low particle drifts

of deuterium/tritium

of alpha particles

Neoclassical transport

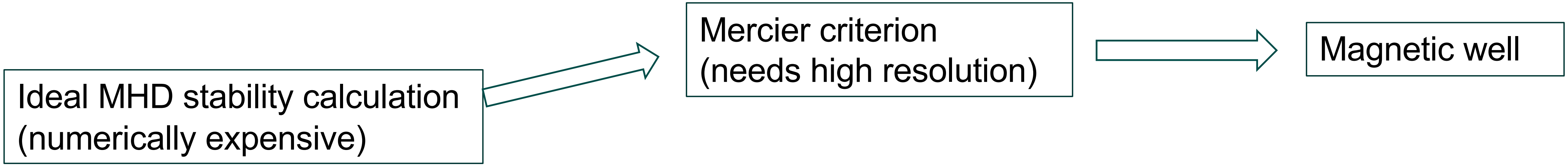
Turbulent transport

…

Everything at the same time!

Usage of proxies instead of expensive codes

- Mercier criterion [stability]
- Magnetic well [stability]



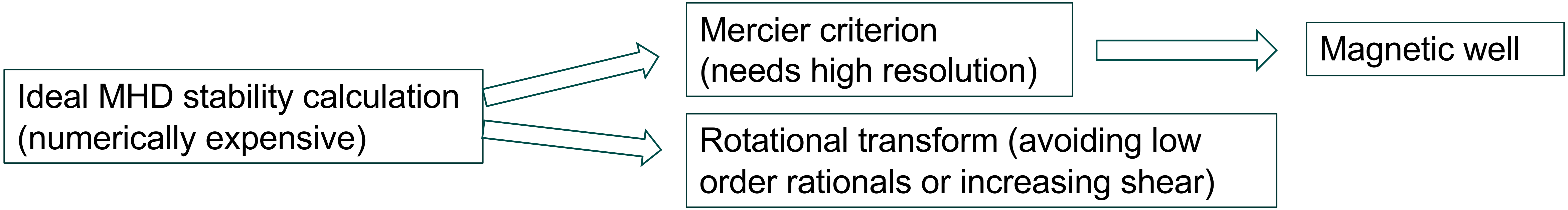
Ideal MHD stability calculation (numerically expensive)

Mercier criterion (needs high resolution)

Magnetic well

Usage of proxies instead of expensive codes

- Mercier criterion [stability]
- Magnetic well [stability]
- Rotational transform [stability & confinement]



Ideal MHD stability calculation
(numerically expensive)

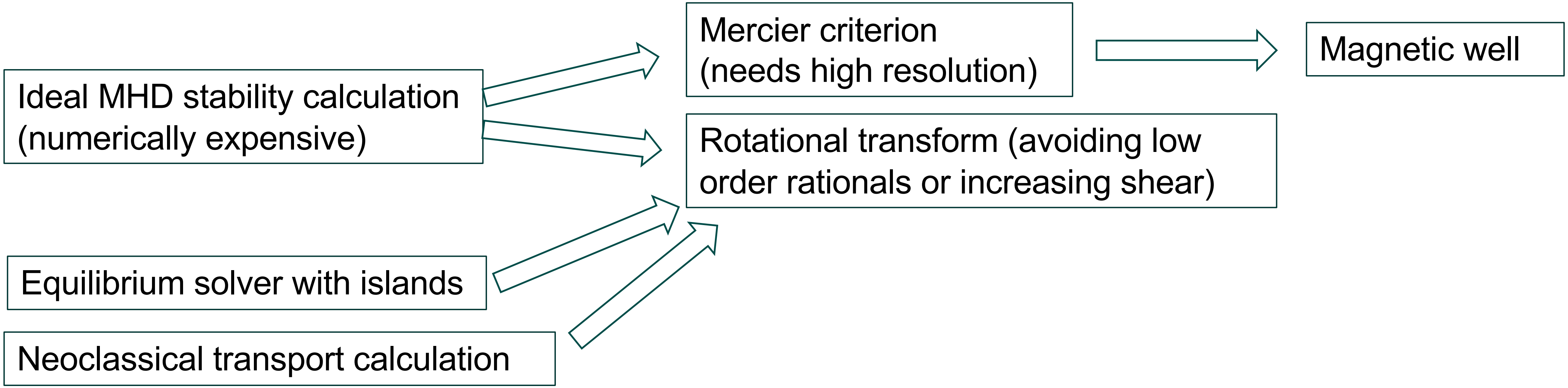
Mercier criterion
(needs high resolution)

Magnetic well

Rotational transform (avoiding low order rationals or increasing shear)

Usage of proxies instead of expensive codes

- Mercier criterion [stability]
- Magnetic well [stability]
- Rotational transform [stability & confinement]



Ideal MHD stability calculation (numerically expensive)

Equilibrium solver with islands

Neoclassical transport calculation

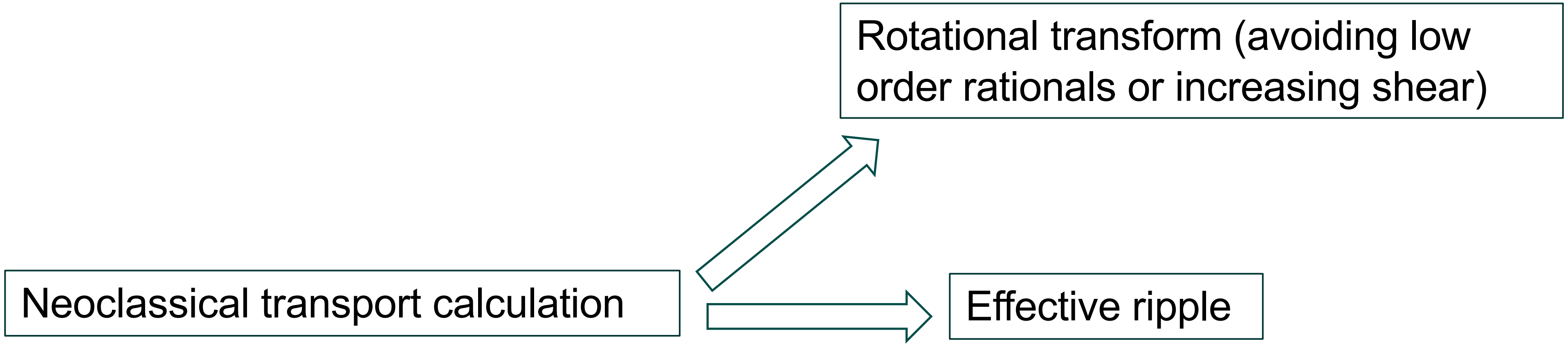
Mercier criterion (needs high resolution)

Rotational transform (avoiding low order rationals or increasing shear)

Magnetic well

Usage of proxies instead of expensive codes

- Mercier criterion [stability]
- Magnetic well [stability]
- Rotational transform [stability & confinement]
- Effective ripple [confinement]



Rotational transform (avoiding low order rationals or increasing shear)

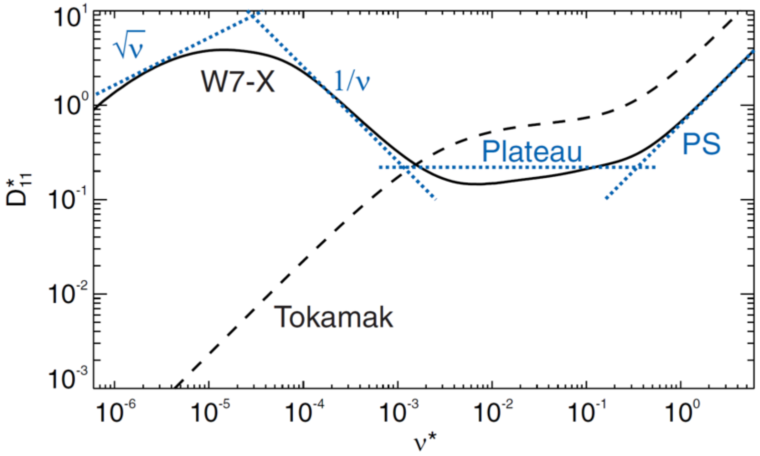
Neoclassical transport calculation

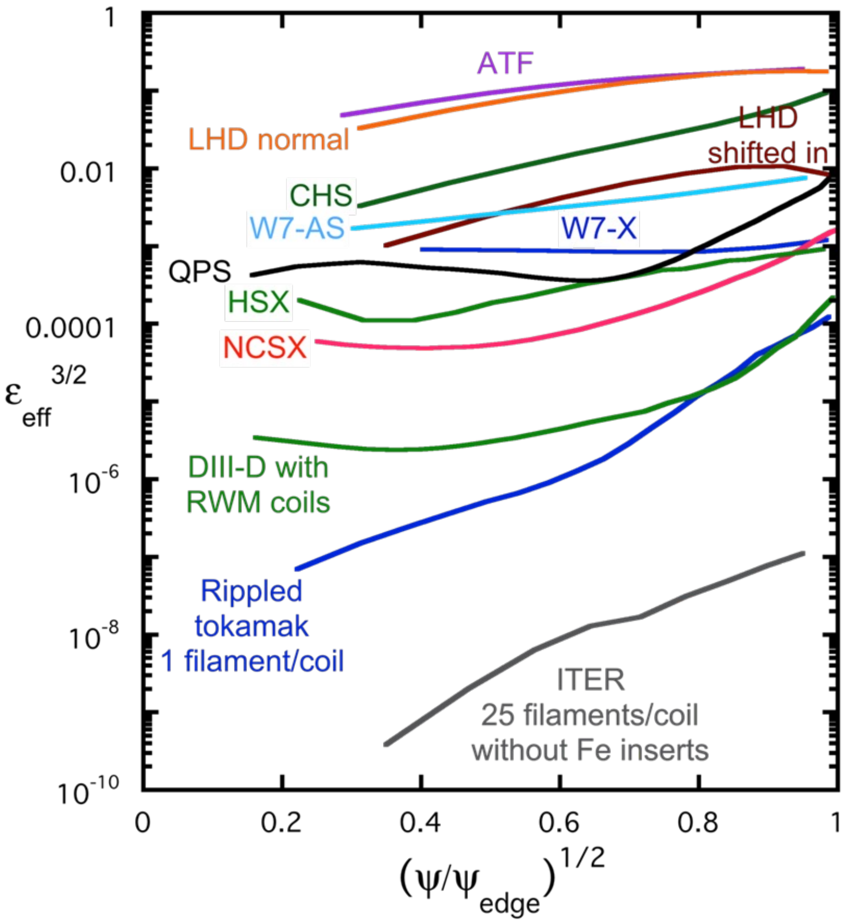
Effective ripple

Useful characterisation of ripple transport levels: effective ripple parameter

$$D_{1/\nu} / D_{plateau} = \left(\frac{4}{3\pi}\right)^2 \frac{(2\varepsilon_{eff})^{3/2}}{\nu^*}$$

- Nemov, Kasilov,Kernbichler (1999)
- e_{eff} = 0 for ideal tokamak, quasi-symmetry, or quasi-omnigeneity
- Simple measure of orbit deviations from ideal





Usage of proxies instead of expensive codes

- Mercier criterion [stability]
- Magnetic well [stability]
- Rotational transform [stability & confinement]
- Effective ripple [confinement]
- Curvature of plasma boundary [cost]
- Quasi-symmetry qa-error [confinement]
- ...

STELLOPT*/ROSE**/SIMSOPT***

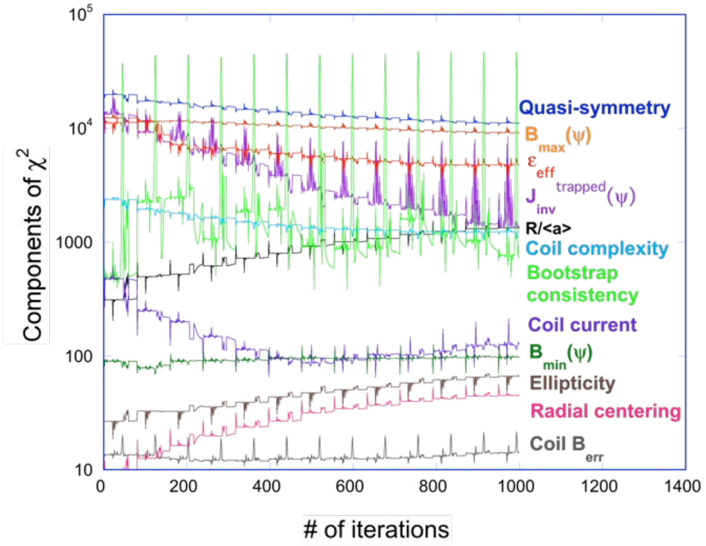
** D. Spong, S. Hirshman, et al. (, STELLOPT 1998)*
**Drevlak et al., Nuclear Fusion, 59, (2019), 016010
*** Landreman et al., (2021), 6(65), 3525

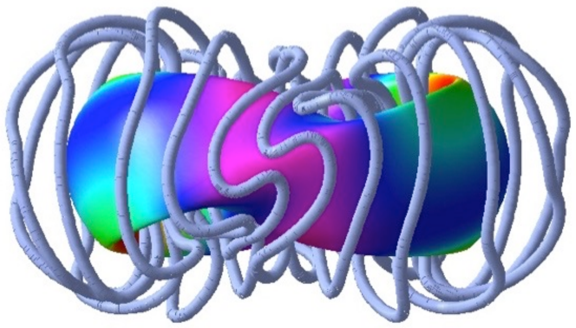
Adjust plasma boundary

Solve 3D Equilibrium (VMEC)

Calculate (physics + engr. Targets for evaluating target function

Optimizer
STELLOPT: e.g. Levenberg-Marquardt ROSE: e.g. BRENT





Target function: weighted sum optimisation

There are many criteria which one wishes to optimize simultaneously.

Easiest way to minimize all criteria at the same time is to create a scalar penalty function.

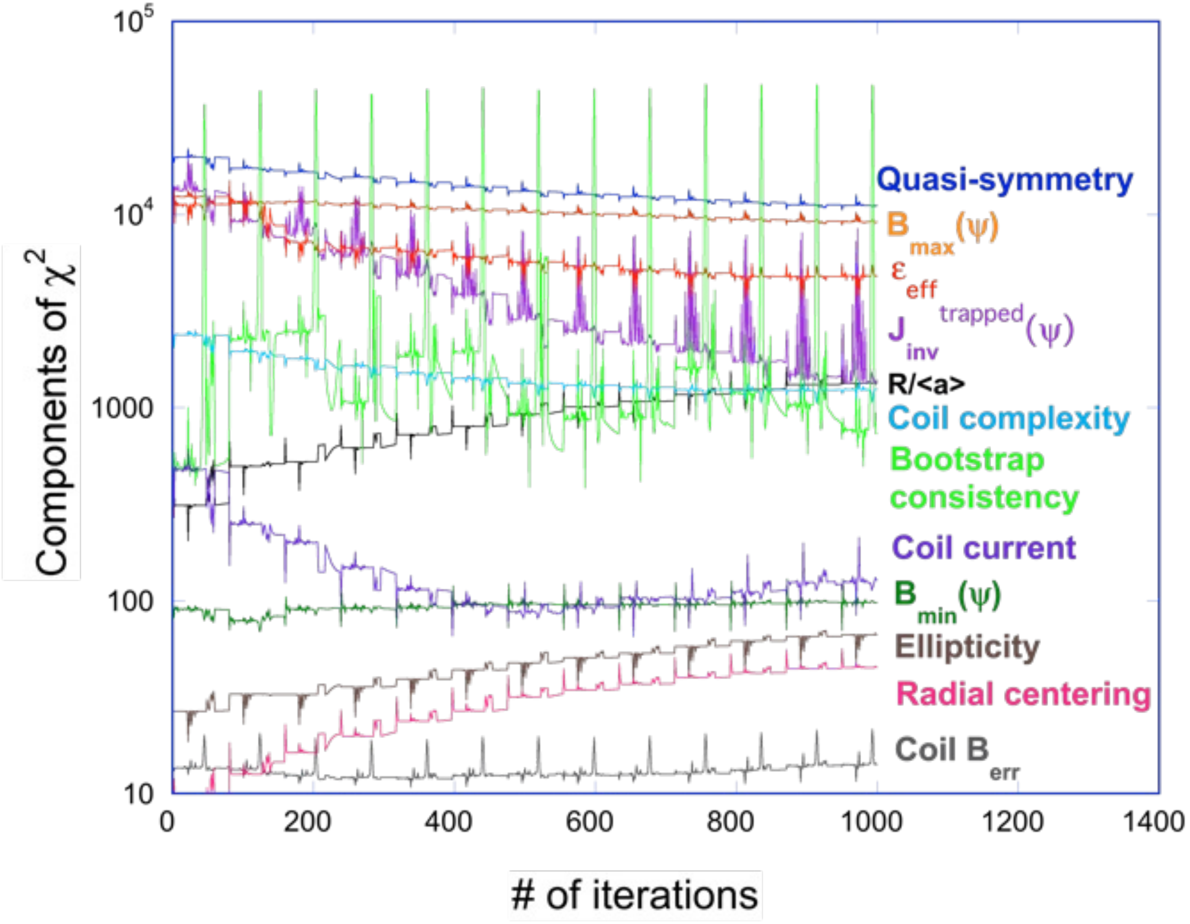
One way to construct the penalty function is called the weighted sum approach:

$$\boldsymbol{f} = \sum \boldsymbol{\omega_i} \left(\boldsymbol{F_i} - \widetilde{\boldsymbol{F_i}}\right)^2$$

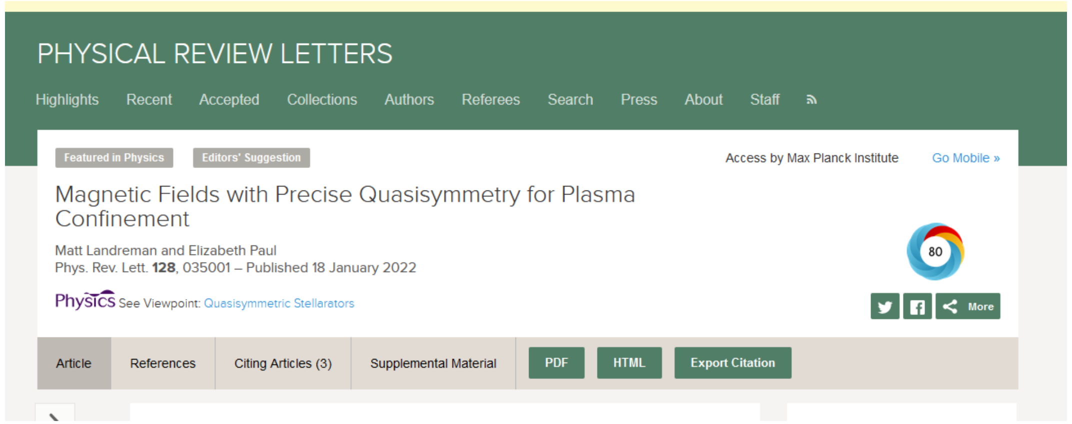
Where

- $\boldsymbol{\omega_i}$ are the weights which can be altered to obtain various optimal configurations,
- $\boldsymbol{F_i}$ is the value for the criterion i,
- and $\widetilde{\boldsymbol{F_i}}$ is the corresponding target value.

STELLOPT – example run



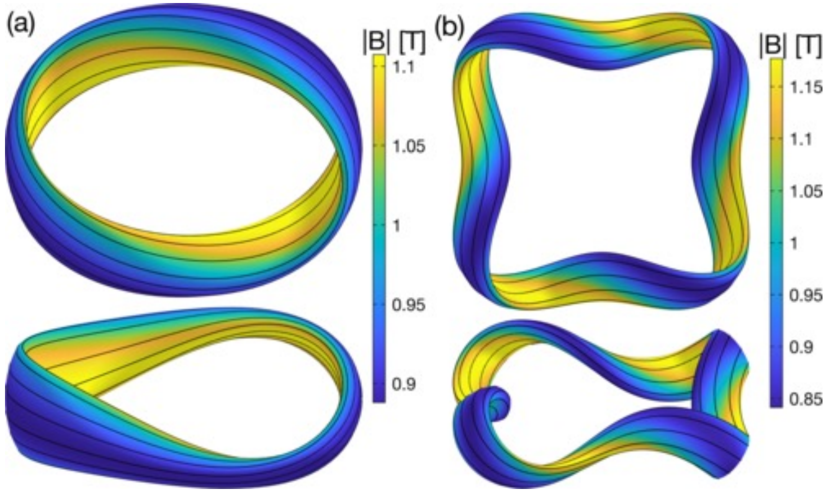
Exciting new achievements in stellarator optimisation:

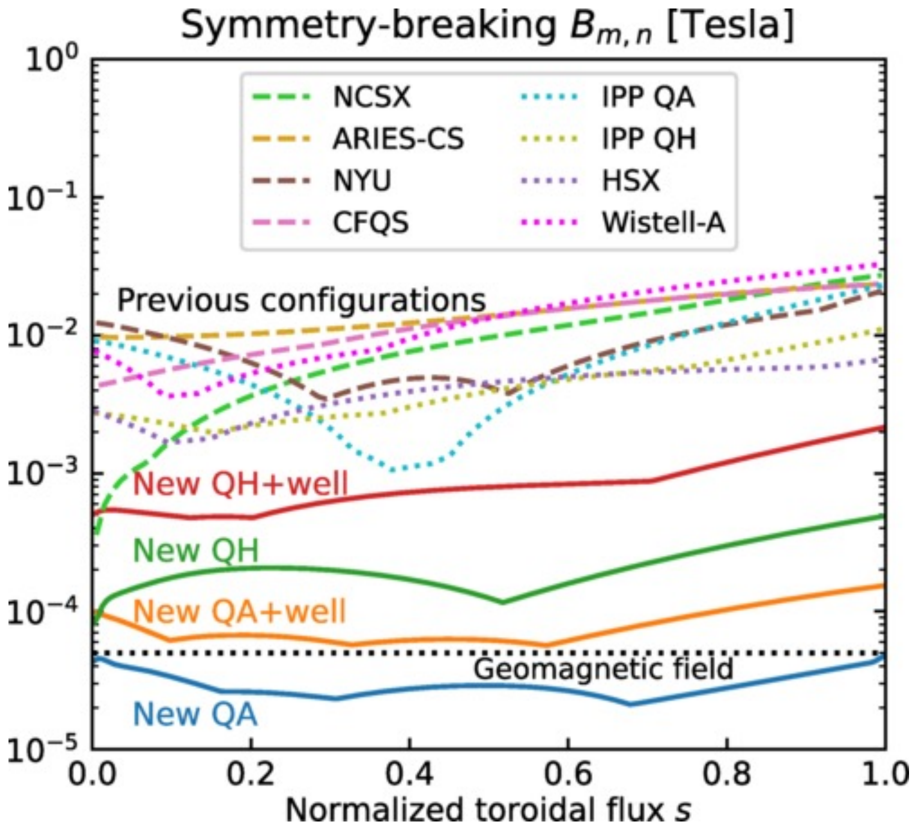


Magnetic Fields with Precise Quasisymmetry for Plasma Confinement

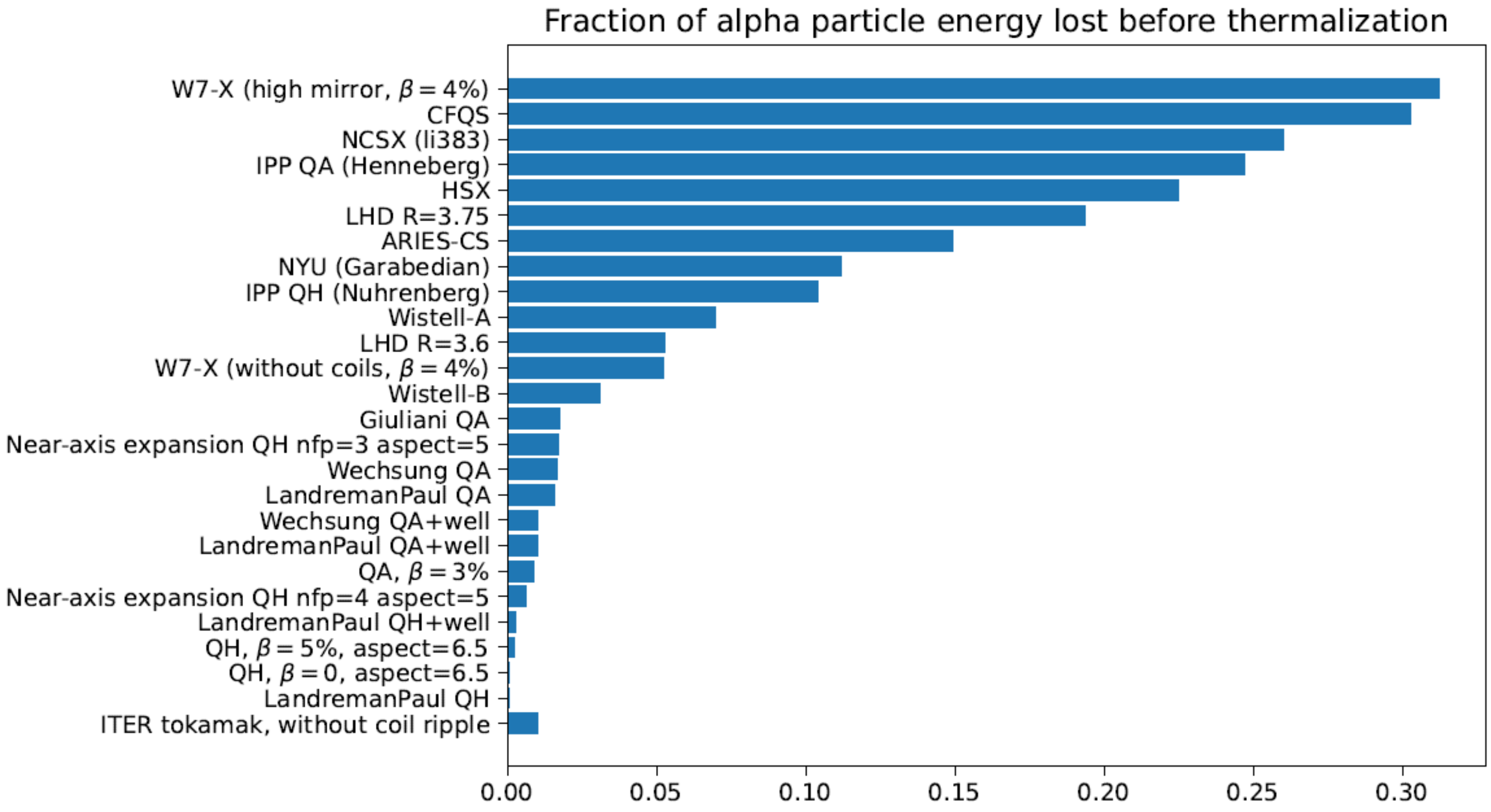
Matt Landreman and Elizabeth Paul
Phys. Rev. Lett. **128**, 035001 – Published 18 January 2022

Physics See Viewpoint: Quasisymmetric Stellarators

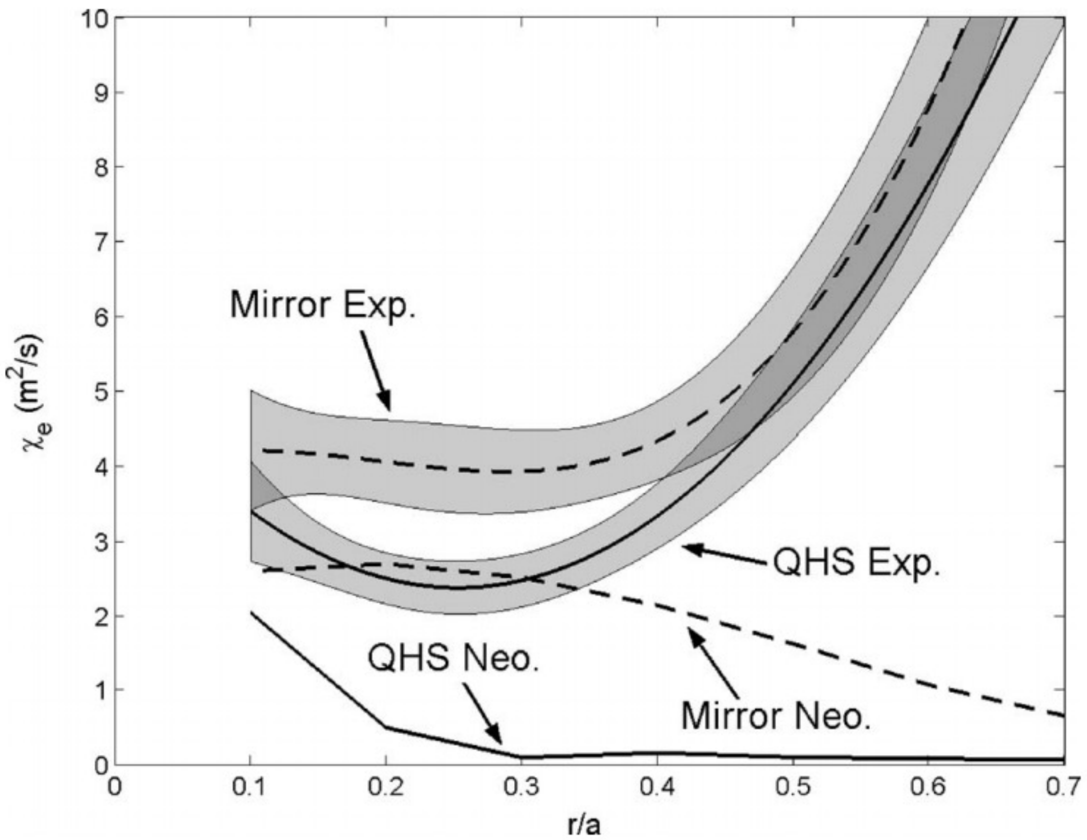




Exciting new achievements in stellarator optimisation:



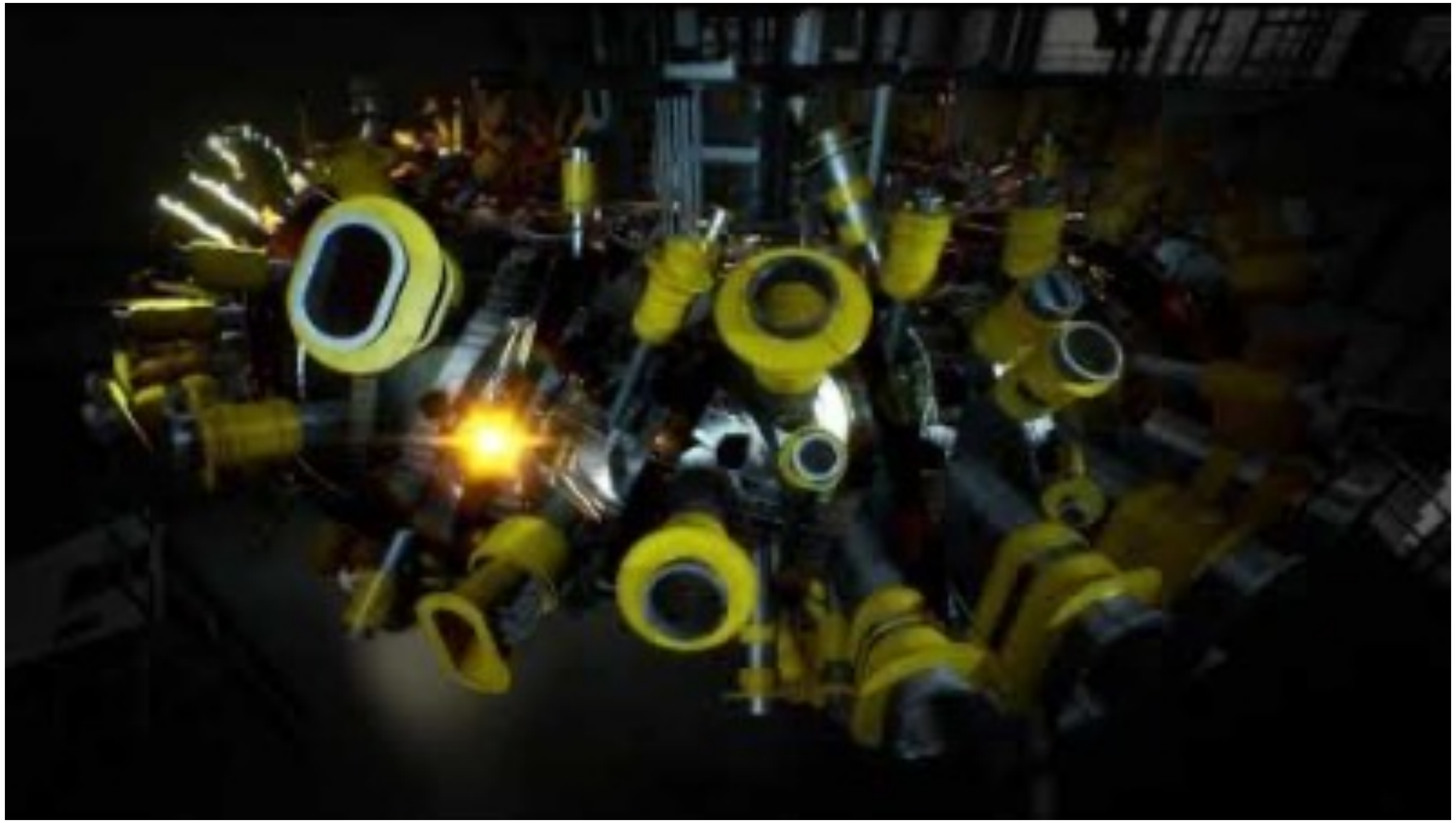
Stellarator optimization has been experimentally verified
Helically Symmetric Experiment (HSX)



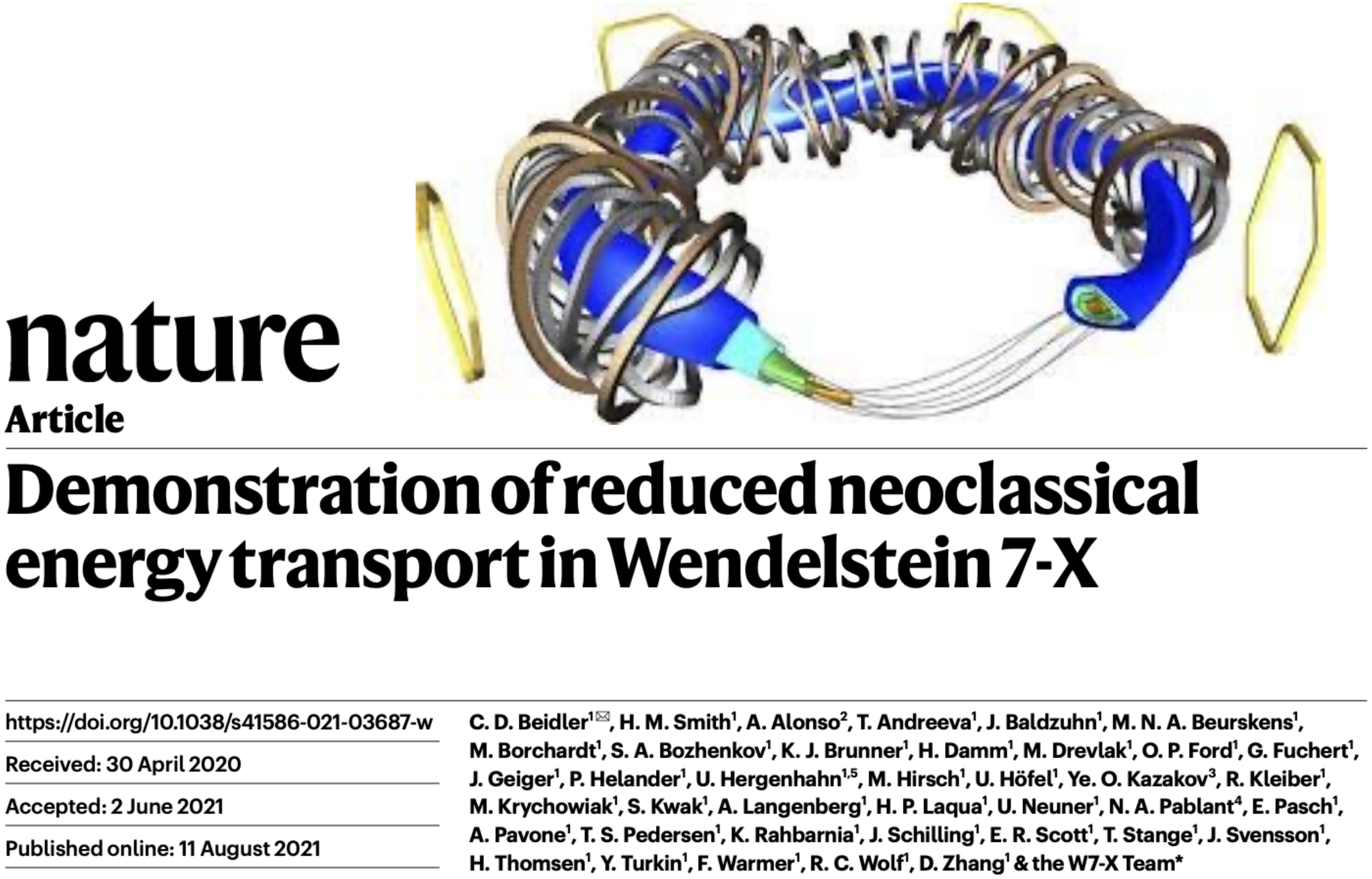
J. M. Canik et al, *Phys. Plasmas* 14, (2007).

Stellarator optimization at scale – Wendelstein 7-X

- ✓ Stable equilibria up to $\beta = \frac{p}{B^2/2\mu_0} = 5\%$
- ✓ Improved particle confinement
- ✓ Reduced plasma current



Stellarator optimization at scale – Wendelstein 7-X



nature

Article

Demonstration of reduced neoclassical energy transport in Wendelstein 7-X

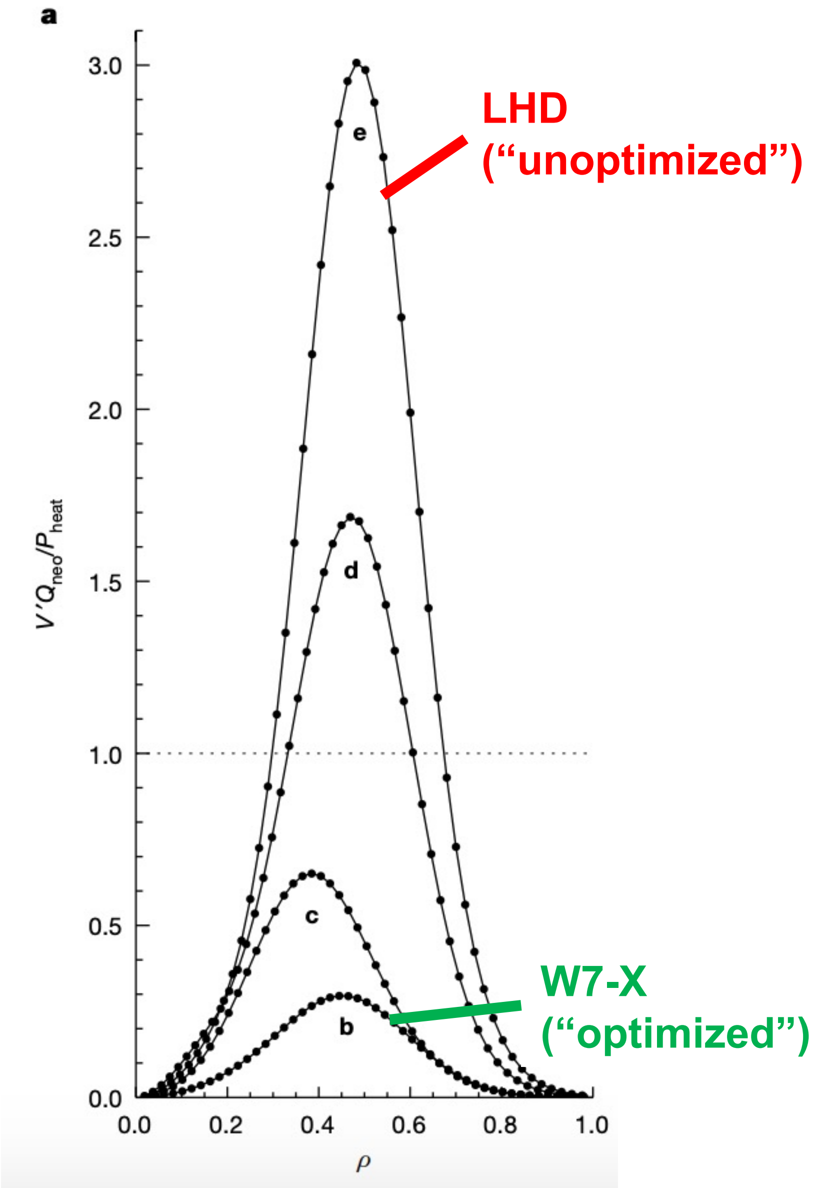
https://doi.org/10.1038/s41586-021-03687-w

Received: 30 April 2020

Accepted: 2 June 2021

Published online: 11 August 2021

C. D. Beidler[1]✉, H. M. Smith[1], A. Alonso[2], T. Andreeva[1], J. Baldzuhn[1], M. N. A. Beurskens[1], M. Borchardt[1], S. A. Bozhenkov[1], K. J. Brunner[1], H. Damm[1], M. Drevlak[1], O. P. Ford[1], G. Fuchert[1], J. Geiger[1], P. Helander[1], U. Hergenhahn[1,5], M. Hirsch[1], U. Höfel[1], Ye. O. Kazakov[3], R. Kleiber[1], M. Krychowiak[1], S. Kwak[1], A. Langenberg[1], H. P. Laqua[1], U. Neuner[1], N. A. Pablant[4], E. Pasch[1], A. Pavone[1], T. S. Pedersen[1], K. Rahbarnia[1], J. Schilling[1], E. R. Scott[1], T. Stange[1], J. Svensson[1], H. Thomsen[1], Y. Turkin[1], F. Warmer[1], R. C. Wolf[1], D. Zhang[1] & the W7-X Team*



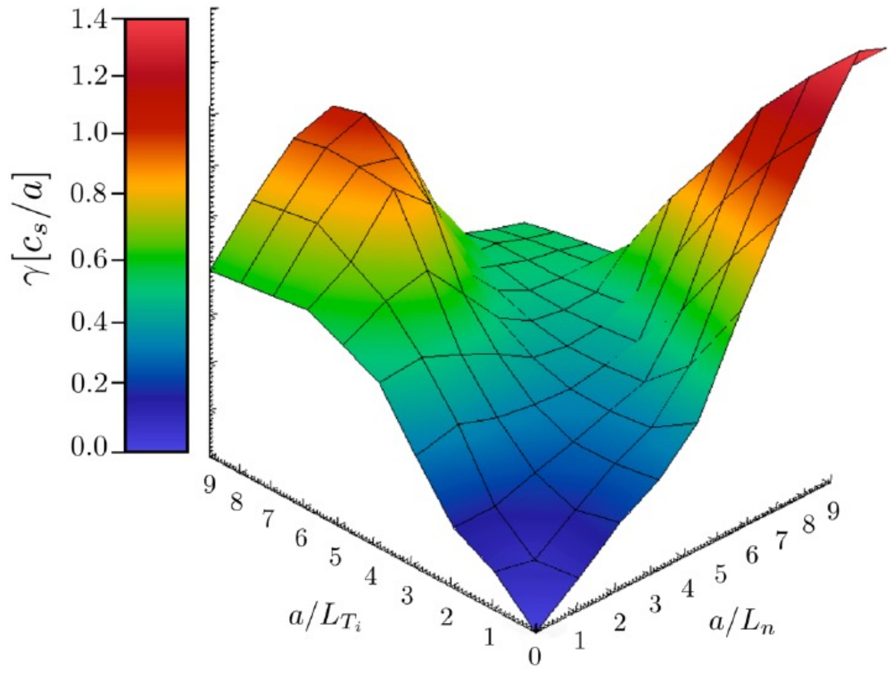
LHD
(“unoptimized”)

W7-X
(“optimized”)

Some uncertainties of stellarator(s) (optimisation)

- How to describe stability correctly?
  - **Magnetic well isn‘t always a good proxy (see LHD or TJ-II examples)**
- What exactly should we target to optimize confinement?
  - **Is neoclassical transport optimization sufficient? Or do we need turbulent transport optimization? (W7-X)**

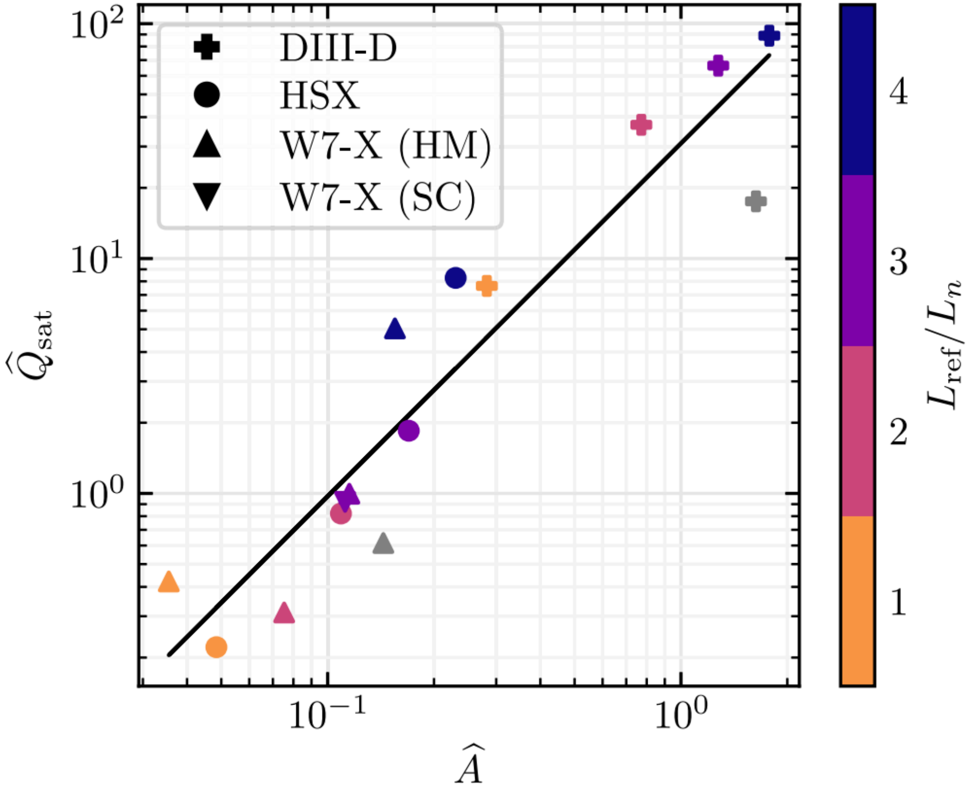
New approaches to optimise for reduced turbulence



W7-X can operate in a "stability valley"

J. Alucon et al, *PPCF* 62 (2020).

New theory to diagnose "available energy" for turbulence



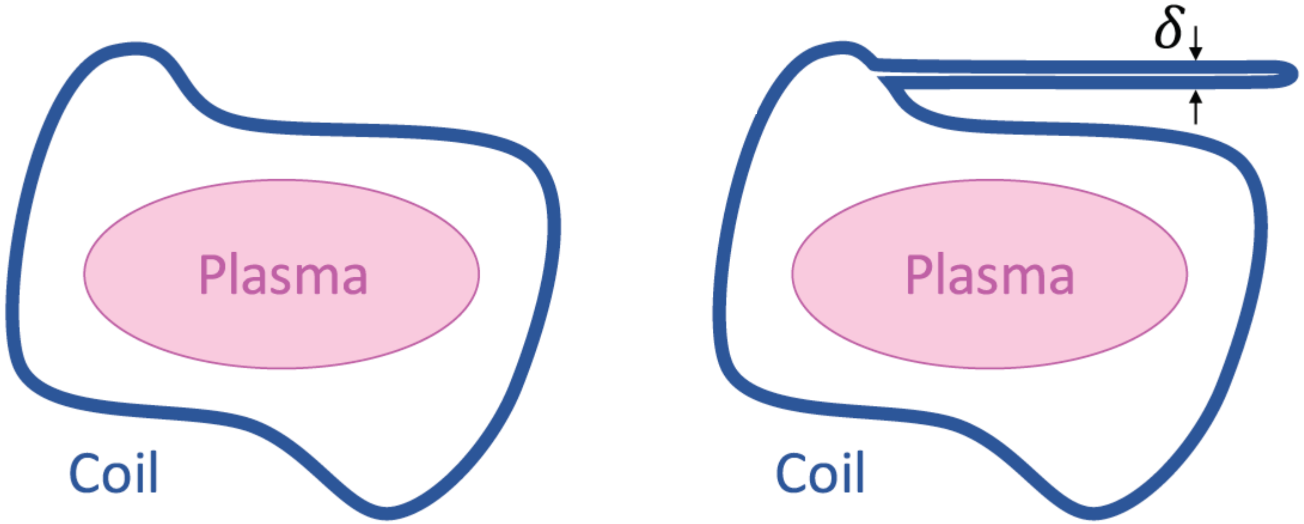
R. Mackenbach et al, *PRL 128* (2022).

Some uncertainties of stellarator(s) (optimisation)

- How to describe stability correctly?
  - **Magnetic well isn‘t always a good proxy (see LHD or TJ-II examples)**
- What exactly should we target to optimize confinement?
  - **Is neoclassical transport optimization sufficient? Or do we need turbulent transport optimization? (W7-X)**
  - **What is the best approach to optimize for turbulent transport?**
- Can we optimise the coils, make them simpler?

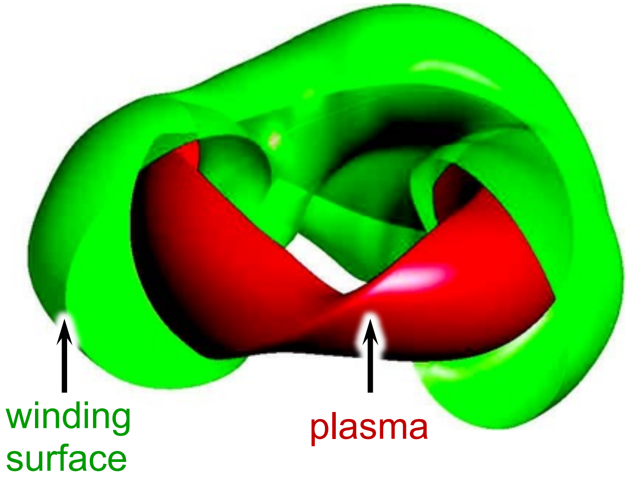
Modern stellarator coils are typically designed with an optimized plasma in mind

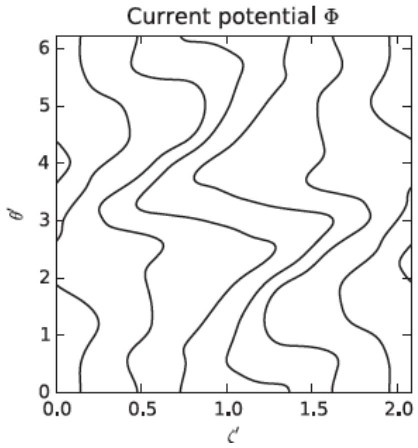
- Designing coils for a stellarator is an ill-posed problem: many nonunique solutions
- Designer must impose constraints



Merkel’s method has been used to define coils on many current stellarators

1. Define a winding surface outside the plasma
2. Calculate surface current distribution necessary to confine plasma
3. Discretize the surface current into curves
4. Design coils from the shapes of the curves

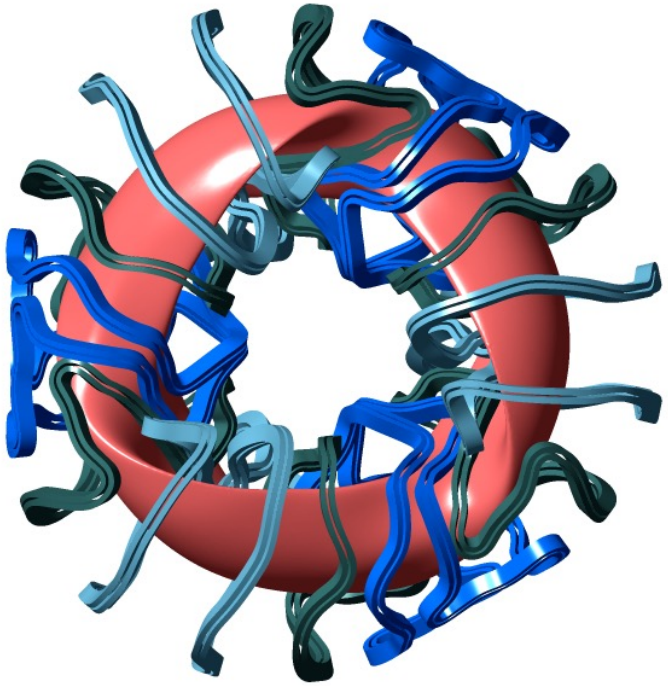




P. Merkel, Nucl. Fusion 1987
M. Landreman, Nucl. Fusion 2017

Coil simplification: background

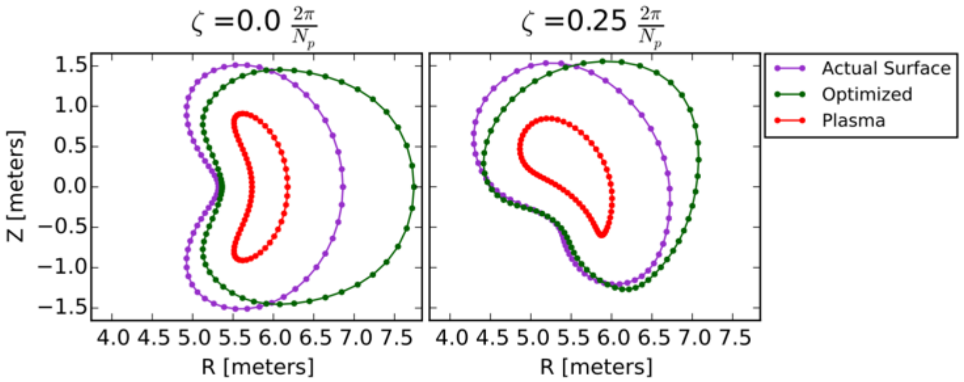
- Coils are one of the main cost drivers of current stellarators
  - Complex geometry
  - Tight tolerances
- Reducing complexity and/or increasing tolerances can reduce costs and make the stellarator more attractive as a reactor concept

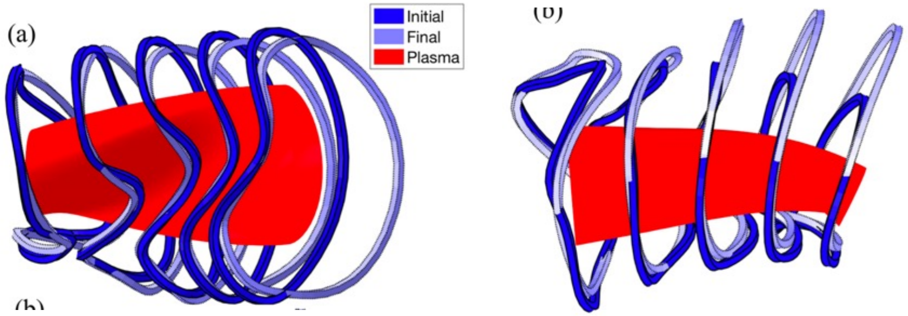


Coils and plasma for the NCSX stellarator, which was canceled due to delays and cost overruns

Coil simplification: optimization of the winding surface

- Initial choice of winding surface in Merkel’s method may not be the best one (or even a good one)
- Winding surface geometry can be optimized to improve:
  - Field accuracy
  - Current density (coil-coil separation)
  - Enclosed volume (more room for components)

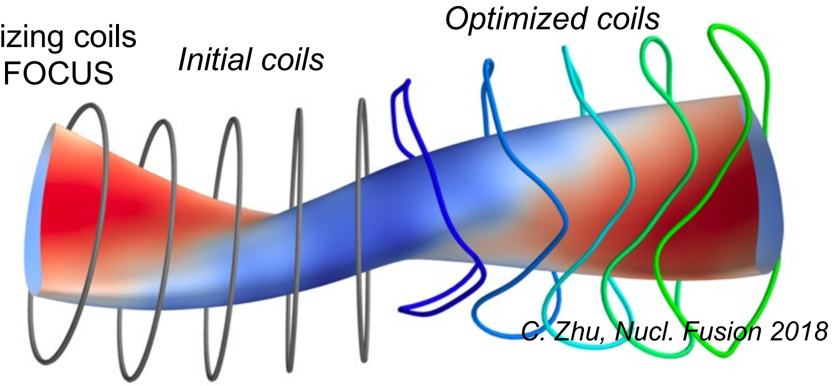




E. Paul, Nucl. Fusion 2018

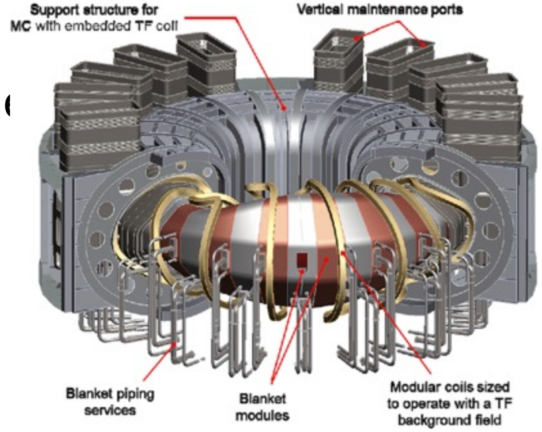
Coil simplification: direct optimization of coil shapes

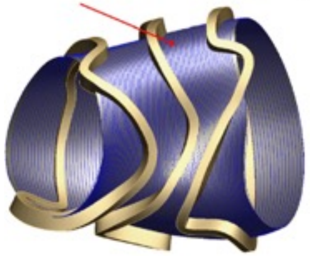
- Model each coil as a parametric curve
- Optimize the curve parameters for:
  - Field accuracy
  - Coil-coil separation
  - Curvature
  - etc.
- Constraints can be applied to curves, e.g. to enabl simpler maintenance
- Codes: FOCUS, COILOPT++



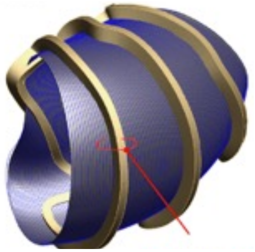
C. Zhu, Nucl. Fusion 2018

Reactor concept with straight-outer-leg coils





Inboard view

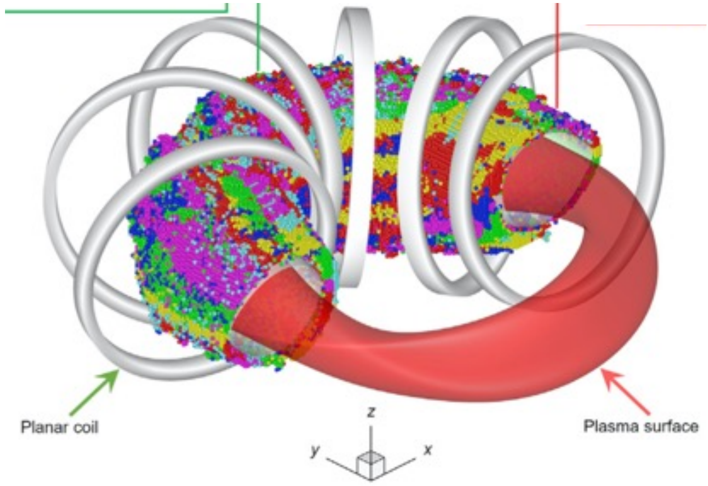


Outboard view

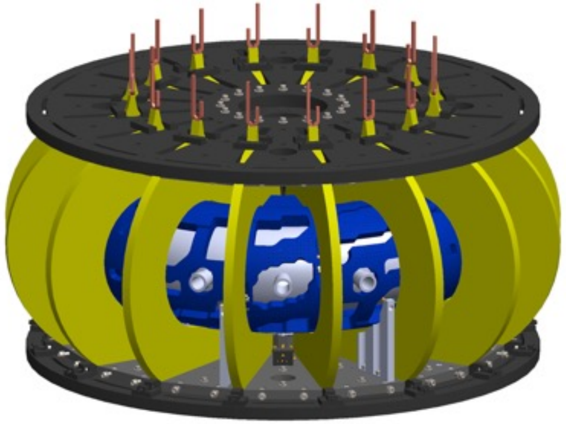
T. Brown et al., SOFE 2015

Coil simplification: use permanent magnets for shaping

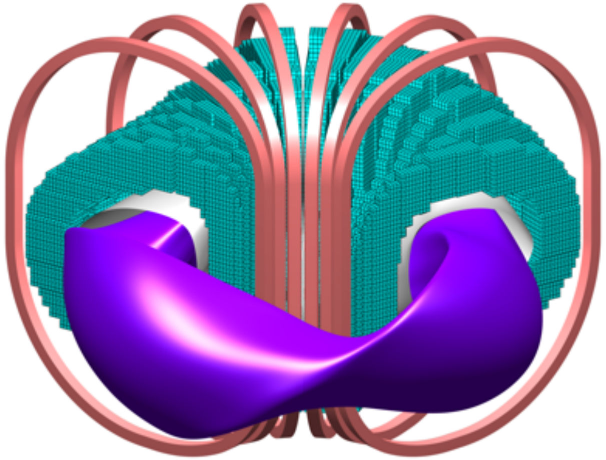
- External permanent magnets cannot create a toroidal magnetic field, but they can contribute to 3D shaping necessary for optimal plasma properties
- Recent designs combine planar coils with arrays of magnets



Z. Y. Lu et al., Cell Rep. Phys. Sci., 2022



MUSE: tabletop experiment at PPPL
T. Qian et al.



NCSX with scaled-down field
K. C. Hammond et al.

Stellarators are quite popular with fusion industry

US

TYPE ONE ENERGY

THEA ENERGY

STELLAREX

Europe

Proxima Fusion

GAUSS FUSION

RENAISSANCE FUSION

Join us in stellarator research at the Max Planck Institute for Plasma Physics, Greifswald, Germany – home to Wendelstein 7-X!

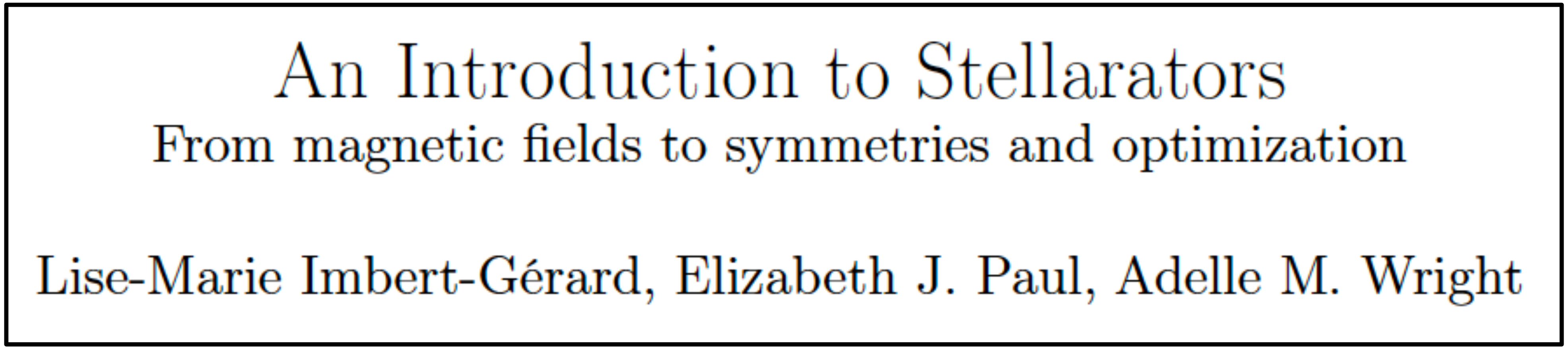
Join for 3-year PhD positions or internships!

Applications either through the graduate school HEPP (https://www.ipp.mpg.de/hepp) or drop me an email! (josefine.proll@ipp.mpg.de)

Summary

- Stellarators are toroidal magnetic plasma confinement devices
  - Three-dimensional, non-axisymmetric geometry
  - Magnetic field generated by external coils
  - Little to no plasma current required
- Stellarator coils can take on many forms
- Confining magnetic field must exhibit rotational transform and sufficient symmetry to confine trapped particles
- Numerical optimization is crucial element of modern stellarator design
- There is much more to be learned!

Further reading



An Introduction to Stellarators

From magnetic fields to symmetries and optimization

Lise-Marie Imbert-Gérard, Elizabeth J. Paul, Adelle M. Wright

[HTTPS://ARXIV.ORG/ABS/1908.05360](https://arxiv.org/abs/1908.05360)