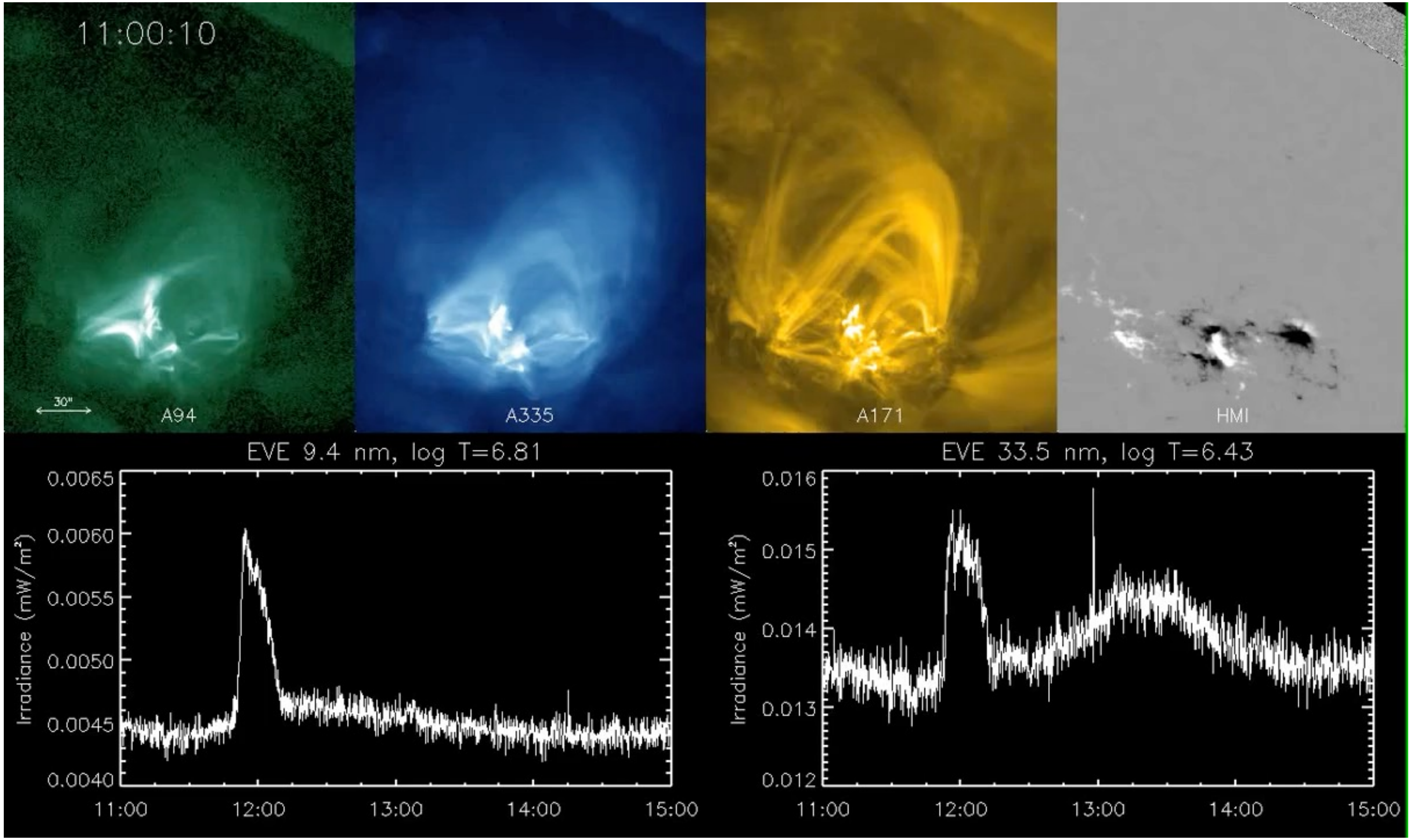
Magnetic reconnection

Introduction to Plasma and Fusion
PPPL June 2024

Muni Zhou (Institute for Advanced Study)

Reconnection related phenomena



Credit: NASA, SDO

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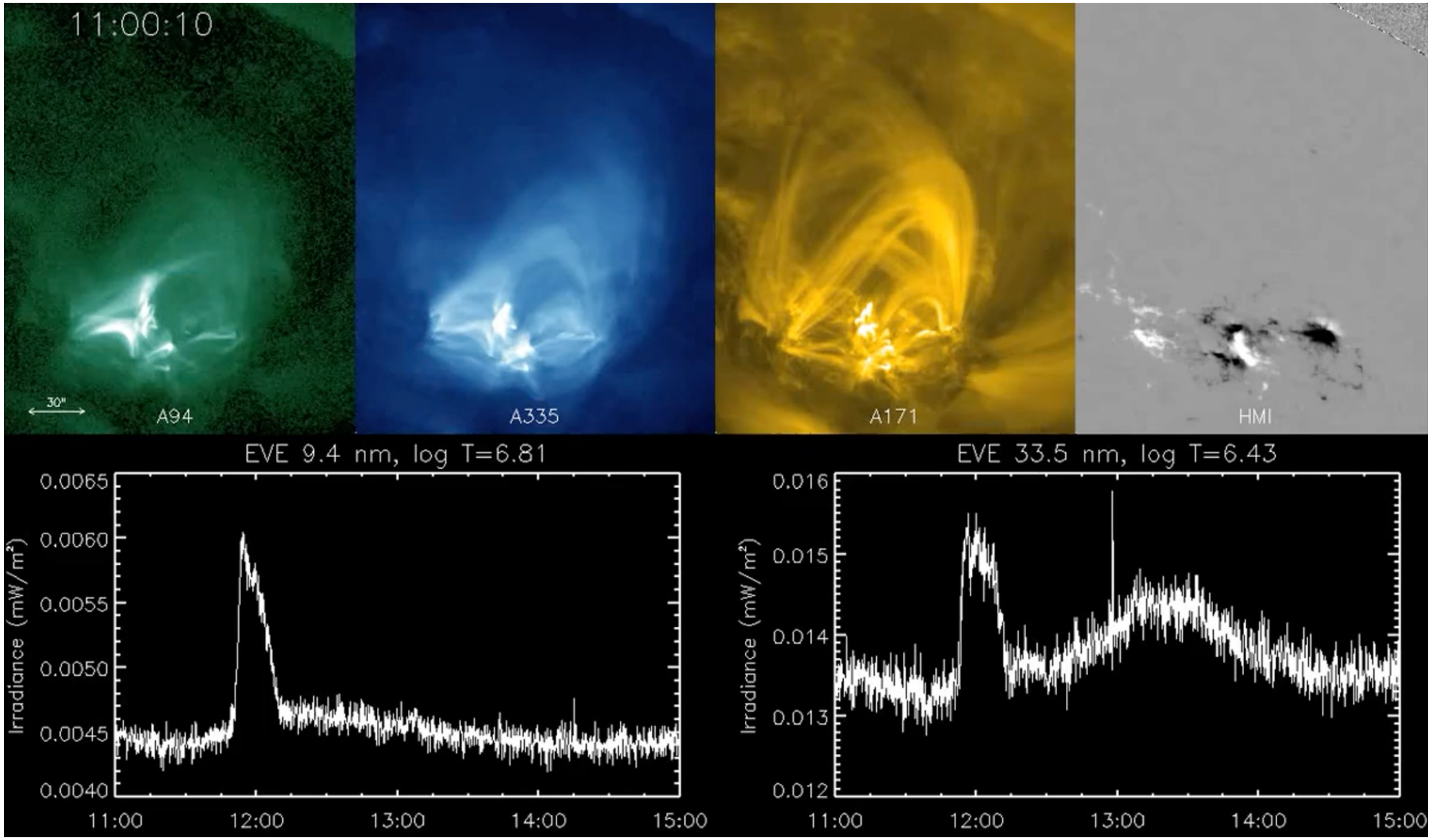

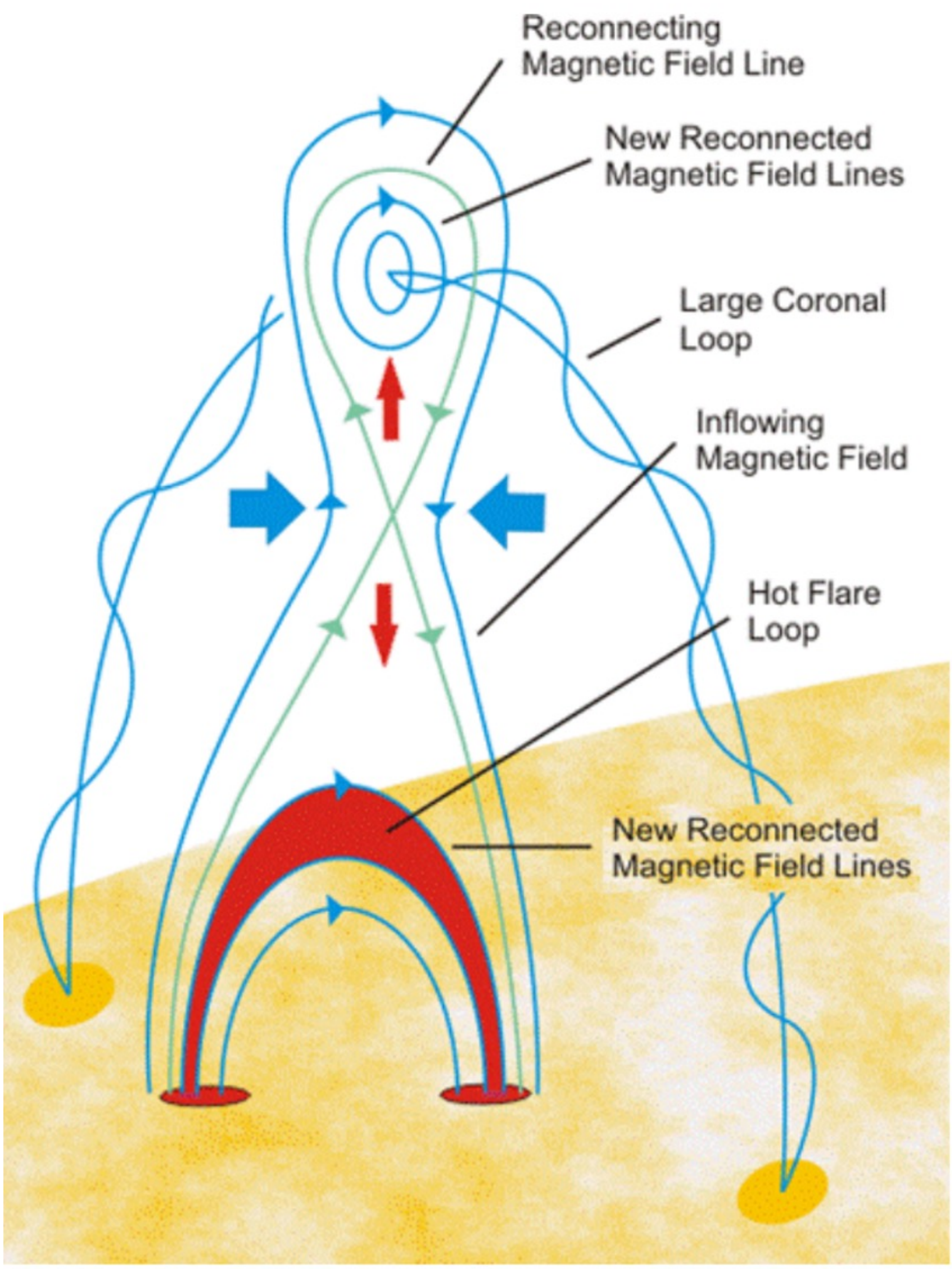

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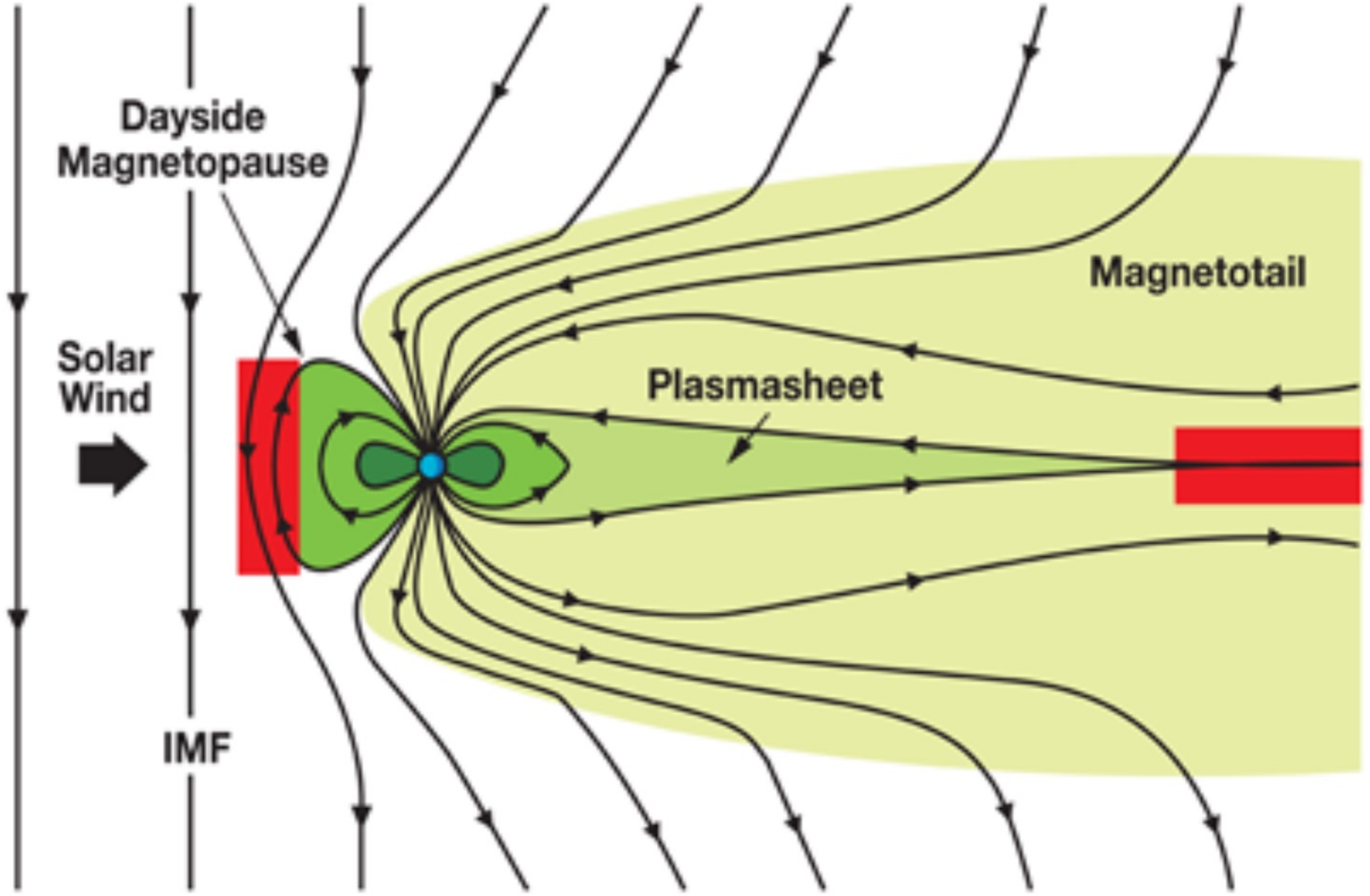
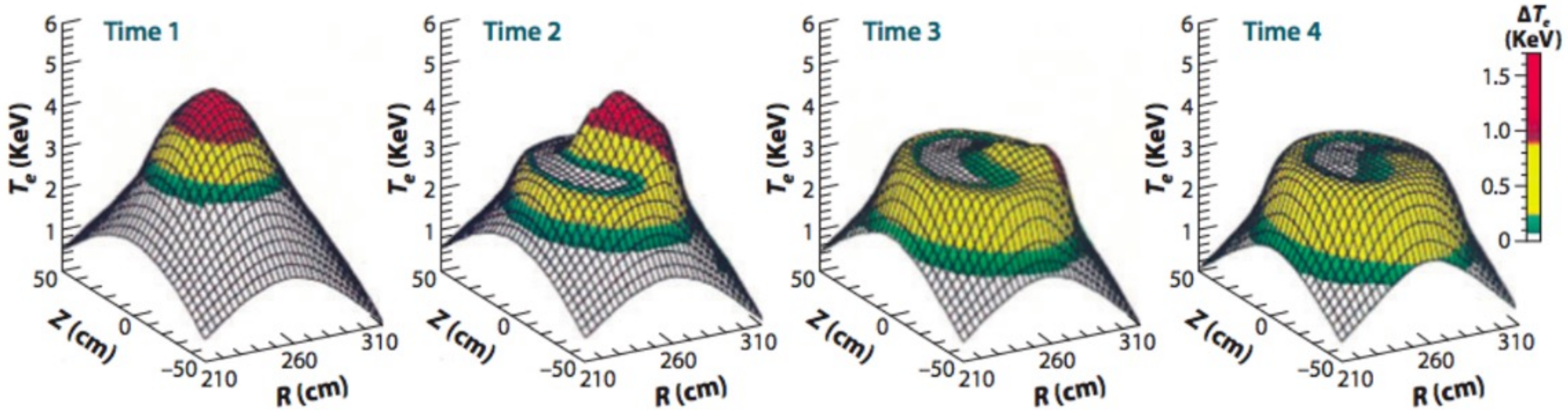


Reconnection around Earth's magnetosphere

Credit: NASA

Sawtooth instability in magnetically controlled fusion



e.g., Yamada+1994

Let's start from the MHD framework

Continuity: $$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \mathbf{u}) = 0$$

Momentum: $$\rho \left(\frac{\partial \mathbf{u}}{\partial t} + \mathbf{u} \cdot \nabla \mathbf{u} \right) = \frac{\mathbf{j} \times \mathbf{B}}{c} - \nabla p$$

Ohm's law: $$\mathbf{E} + \frac{\mathbf{u} \times \mathbf{B}}{c} = \eta \mathbf{j}$$

Equation of state: $$\frac{d}{dt} \left(\frac{p}{\rho^{5/3}} \right) = 0$$

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+

$$\nabla \times \mathbf{E} = -\frac{1}{c} \frac{\partial \mathbf{B}}{\partial t},$$
$$\nabla \times \mathbf{B} = \frac{4\pi}{c} \mathbf{j}.$$

(assuming incompressible flow)
Induction equation:

$$\frac{\partial \mathbf{B}}{\partial t} + \mathbf{u} \cdot \nabla \mathbf{B} = \mathbf{B} \cdot \nabla \mathbf{u} + \bar{\eta} \nabla^2 \mathbf{B}$$

$\bar{\eta} = \eta c^2 / 4\pi$

Plasmas (that we are mostly interested in) are good conductors -- resistivity is small.

Dimensionless numbers

Induction equation:

$$\frac{\partial \mathbf{B}}{\partial t} + \mathbf{u} \cdot \nabla \mathbf{B} = \mathbf{B} \cdot \nabla \mathbf{u} + \bar{\eta} \nabla^2 \mathbf{B}$$

Nonlinear term/Diffusion term

$$\frac{|\mathbf{B} \cdot \nabla \mathbf{u}|}{|\bar{\eta} \nabla^2 \mathbf{B}|} \sim \frac{Lu}{\bar{\eta}} \equiv R_m$$

magnetic Reynolds number

Dimensionless numbers

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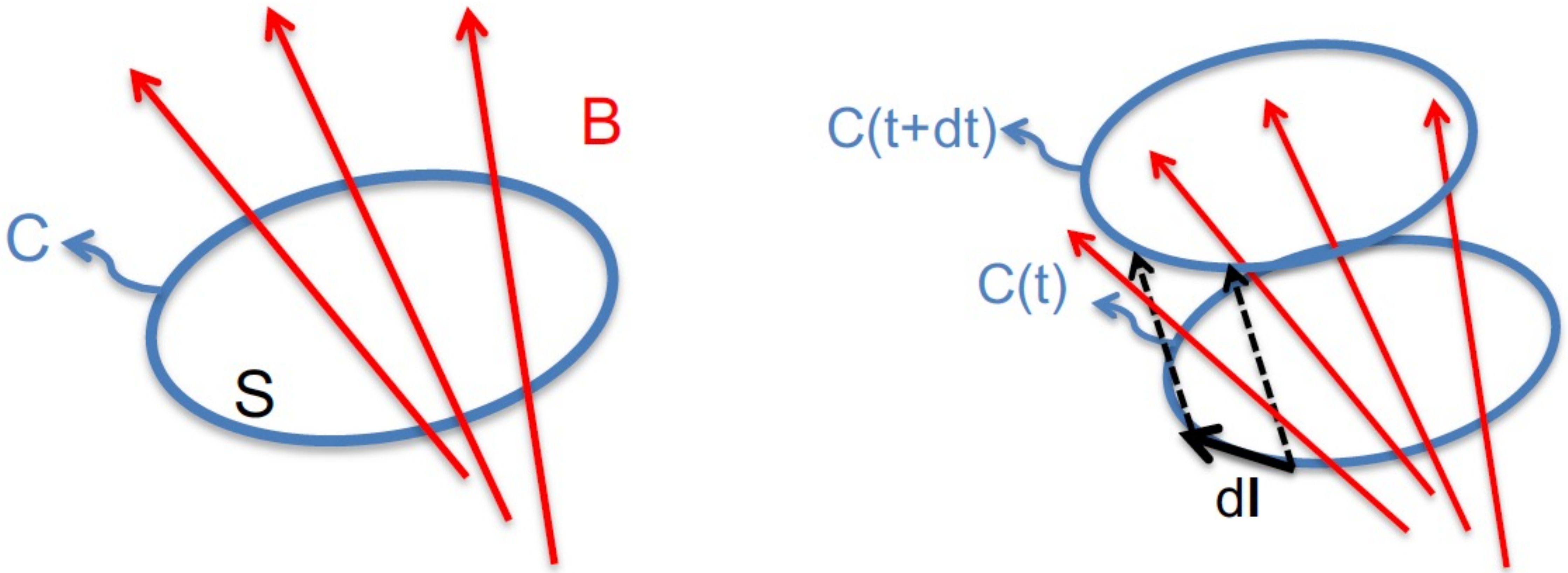
If estimating the relevant velocity as being the Alfven speed:

$$\frac{|\mathbf{B} \cdot \nabla \mathbf{u}|}{|\bar{\eta} \nabla^2 \mathbf{B}|} \sim \frac{LV_A}{\bar{\eta}} \equiv S$$ Lundquist number

Solar corona $S \sim 10^{13}$; ISM $S \sim 10^{18}$, typical tokamak $S \sim 10^{18}$.

At large (/system) scales, the diffusion of magnetic fields is negligible (compared to nonlinear effects)

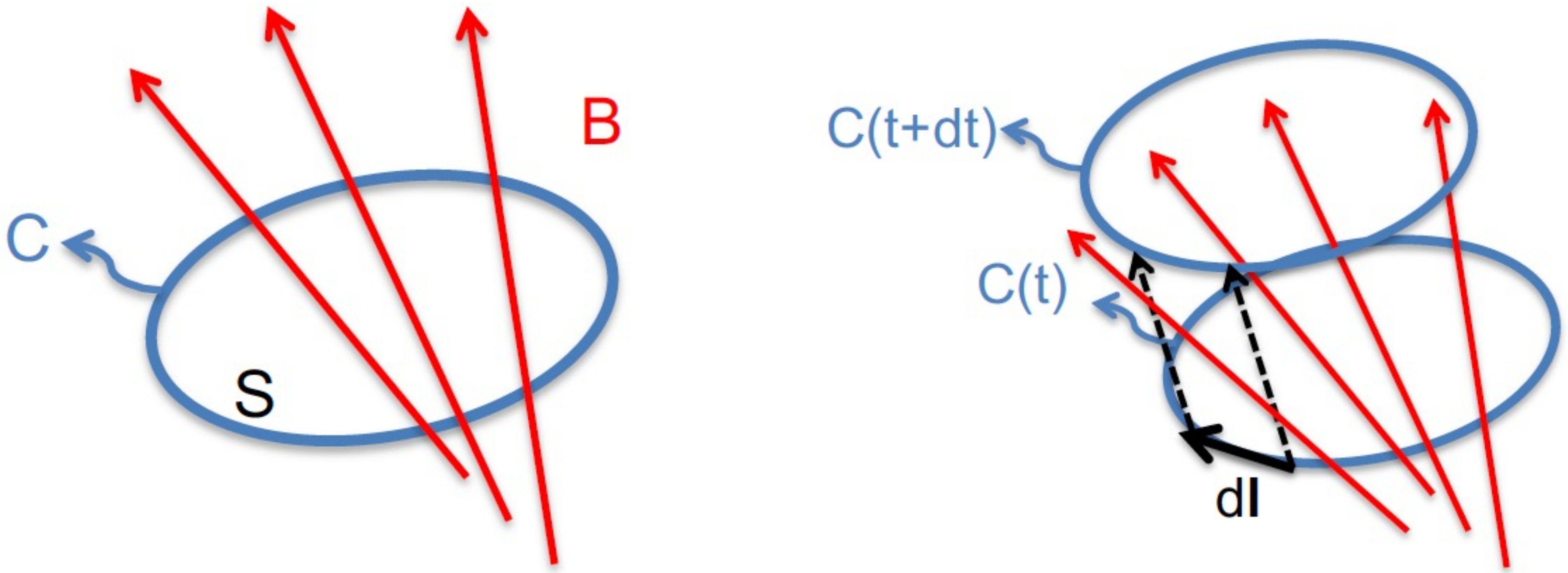
Frozen flux



Credit: N. Loureiro

Consider the magnetic flux through a surface S bounded by a curve C, $\Psi = \int_S B \cdot dS$.

Frozen flux



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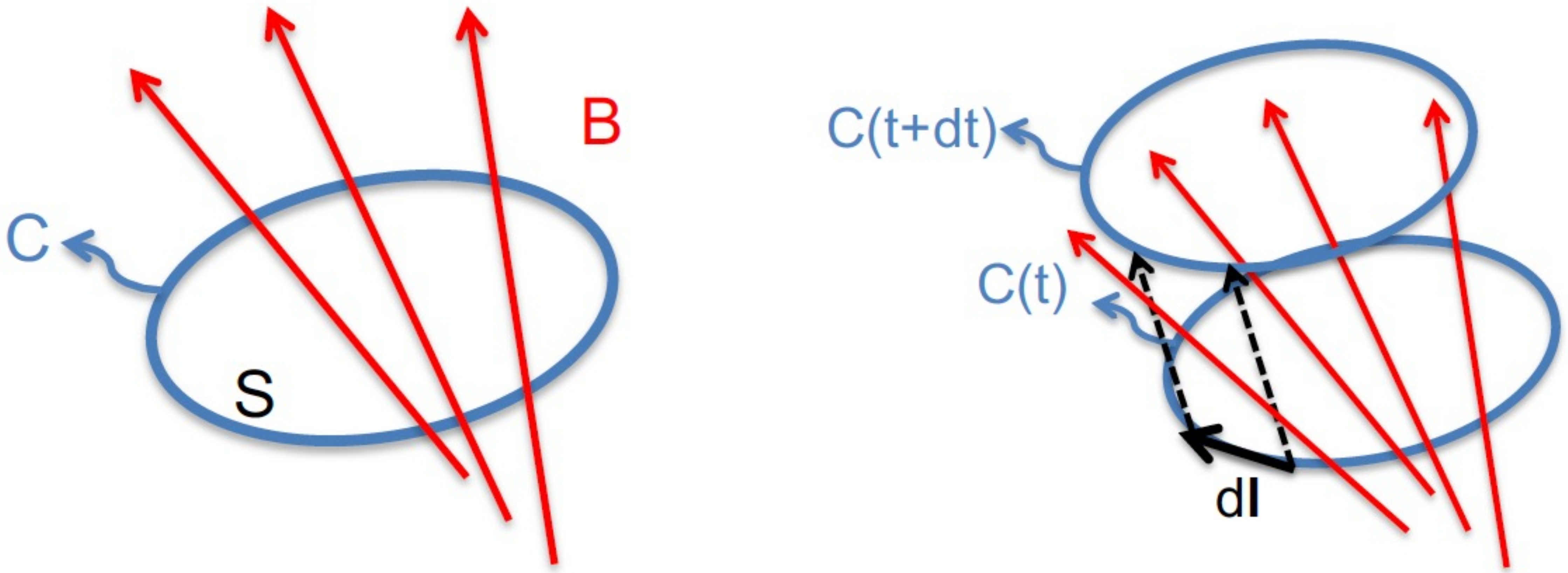
Let the surface move with the plasma fluid with velocity u, the change of flux is

Change of B Change of surface S

$$\frac{d\Psi}{dt} = \int_S \frac{\partial B}{\partial t} \cdot dS + \int_C B \cdot u \times dl$$

$$= -c \int_S \nabla\times E \, dS - \int_S \nabla\times(u\times B) dS$$

Frozen flux



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Recall the Ohm's law $E + \frac{u \times B}{c} = \eta J$

$$\frac{d\Psi}{dt} = -c\int_S \nabla \times (\eta J) dS$$

$\frac{d\Psi}{dt} = 0$ for ideal plasma, *flux is frozen in the flow*

How can resistivity be important?

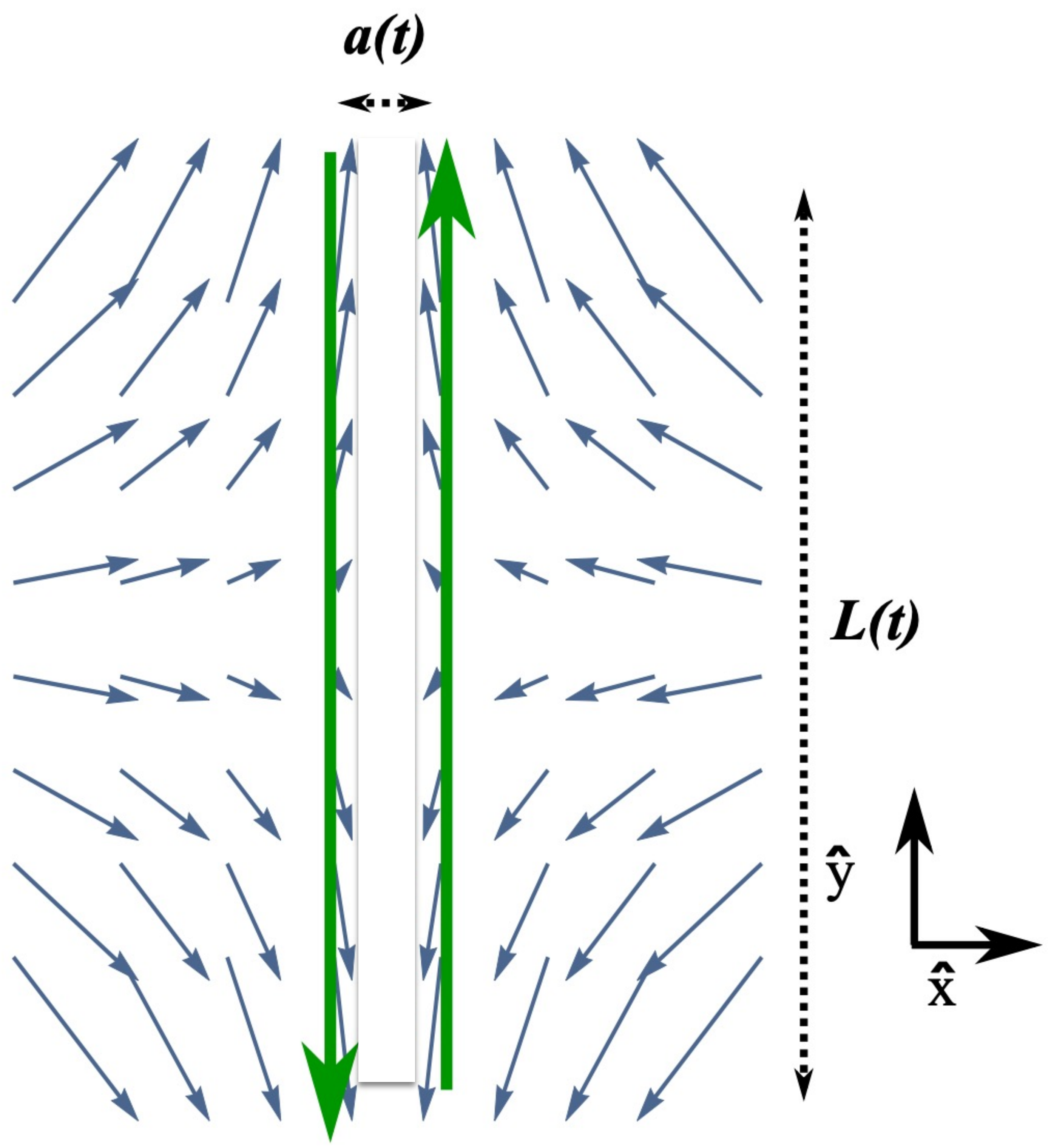
Momentum: $$\rho\left(\frac{\partial \mathbf{u}}{\partial t}+\mathbf{u}\cdot\nabla\mathbf{u}\right) = \frac{\mathbf{j}\times\mathbf{B}}{c}-\nabla p$$

Induction: $$\frac{\partial \mathbf{B}}{\partial t}+\mathbf{u}\cdot\nabla\mathbf{B}=\mathbf{B}\cdot\nabla\mathbf{u}+\bar{\eta}\nabla^2\mathbf{B}$$

Consider this configuration:

$$\mathbf{u}=\Gamma_0(-x,y,0)$$

$$\mathbf{B}=B_0\left(0,\frac{x}{a(t)},0\right)$$



Adapted from E. Tolman+ (2018) *JPP*

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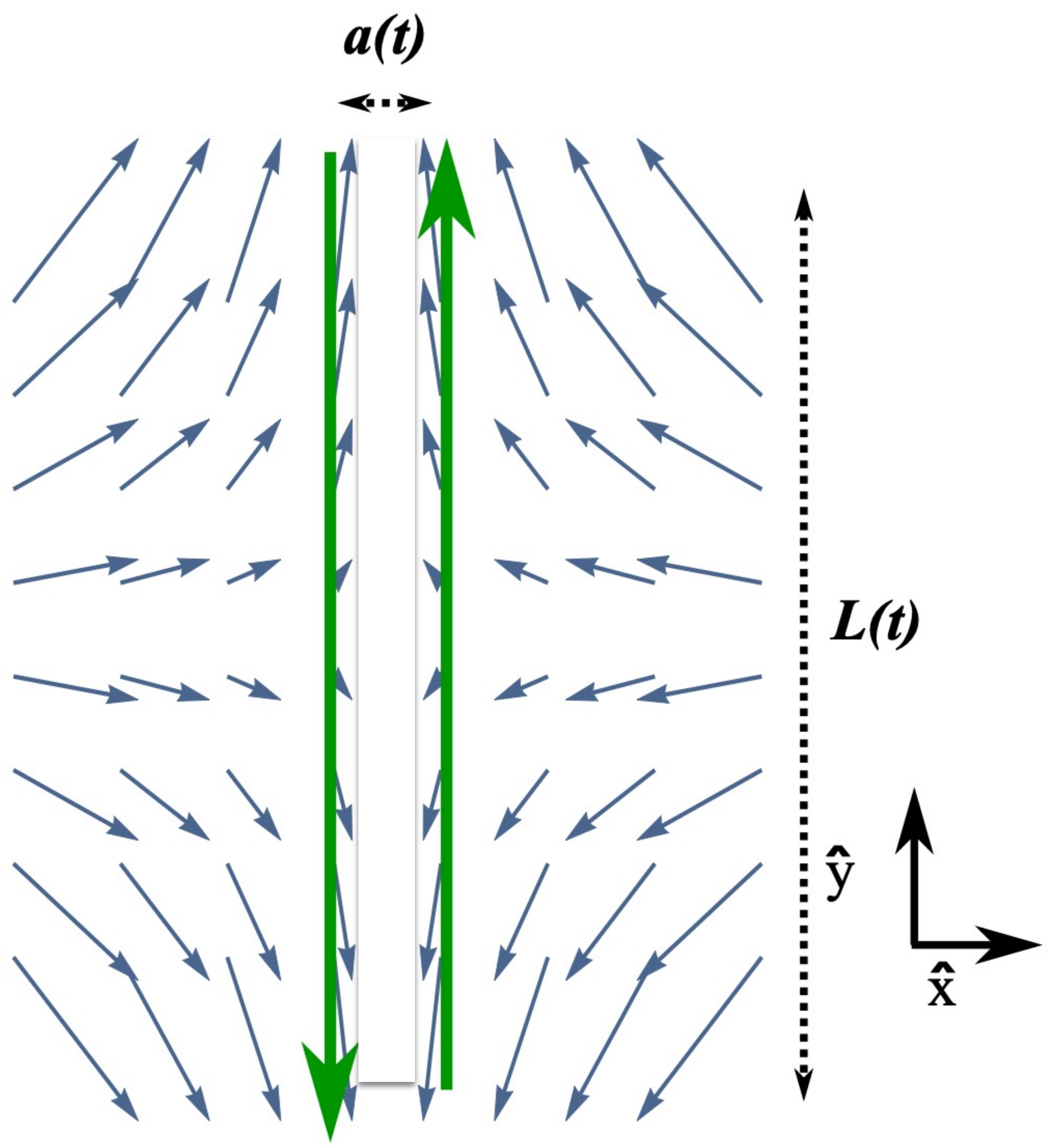

The equilibrium the limit of zero resistivity:

The configuration has a shrinking width $a(t) = a_0 e^{-2\Gamma_0 t}$
-- the exponential collapse

And thus an exponential growth of current density $\mathrm{j_z}(t) \propto e^{2\Gamma_0 t}$

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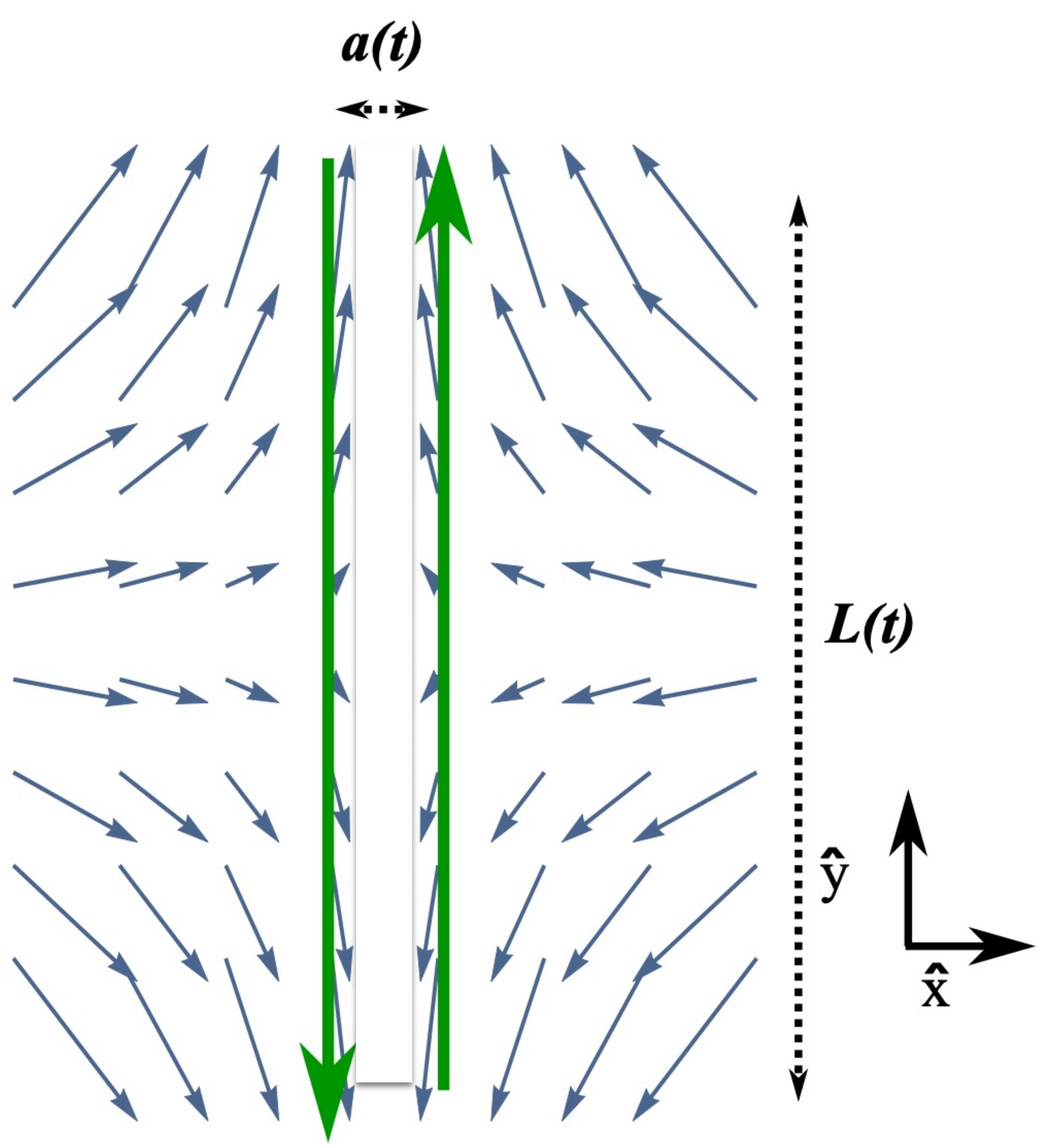

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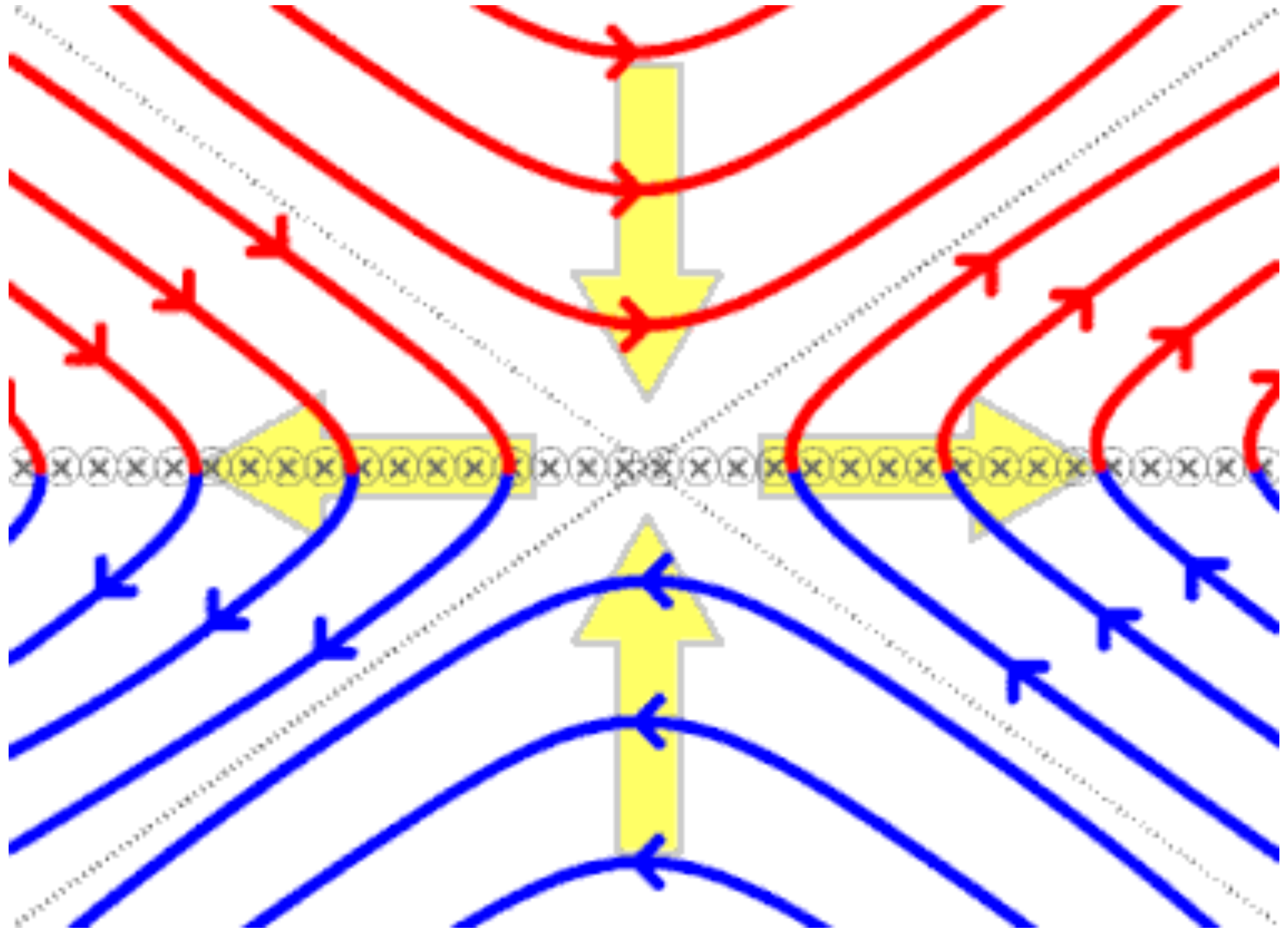
$\frac{d\Psi}{dt} = -c\int_S \nabla\times(\eta J)dS$ The resistive term becomes locally important in a boundary layer

Reconnection can happen due to the resistivity

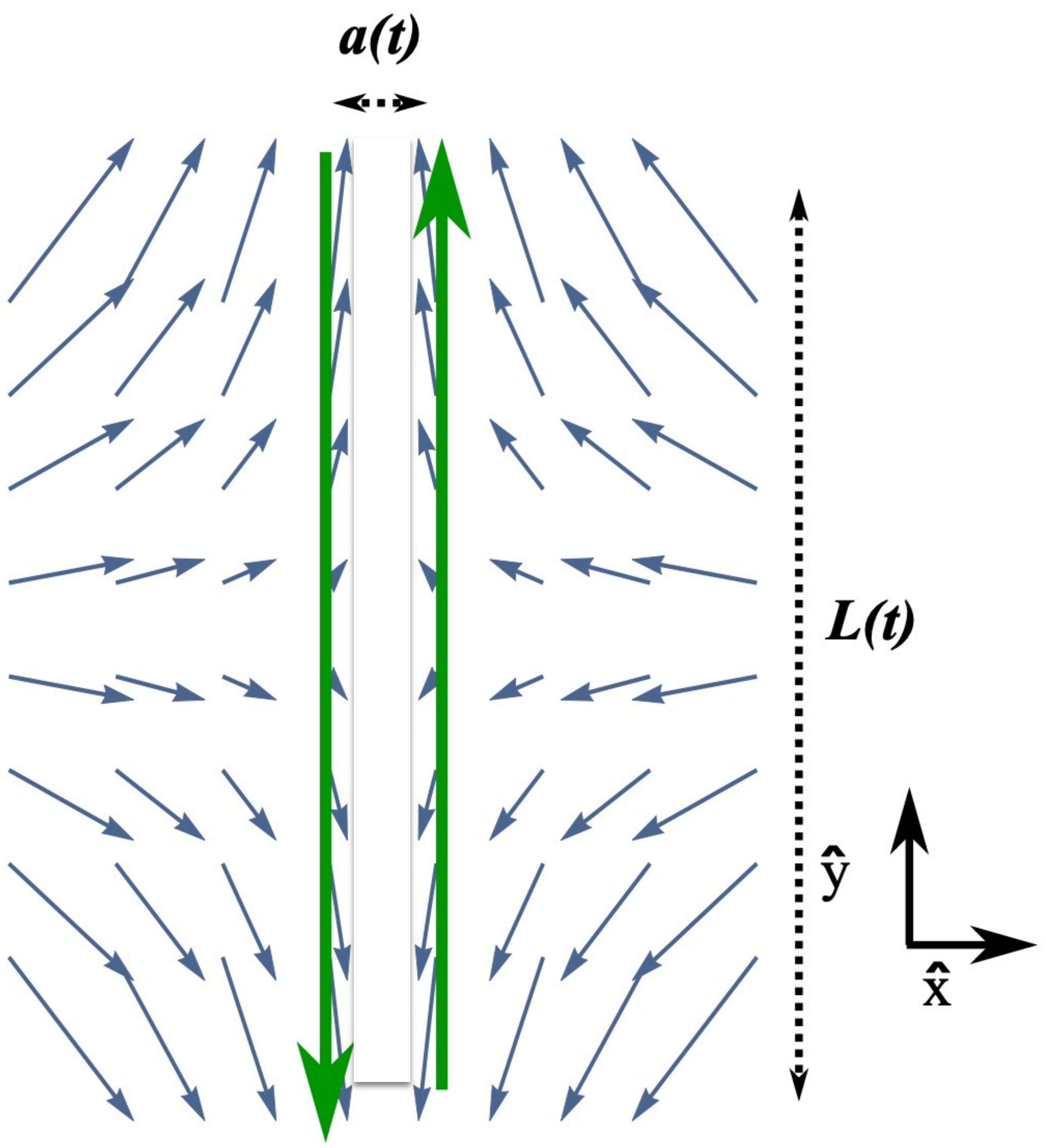
Resistivity η is usually small, need strong current for a significant change of flux.



Reconnection:

change of magnetic fields' topology

fast release of magnetic energy



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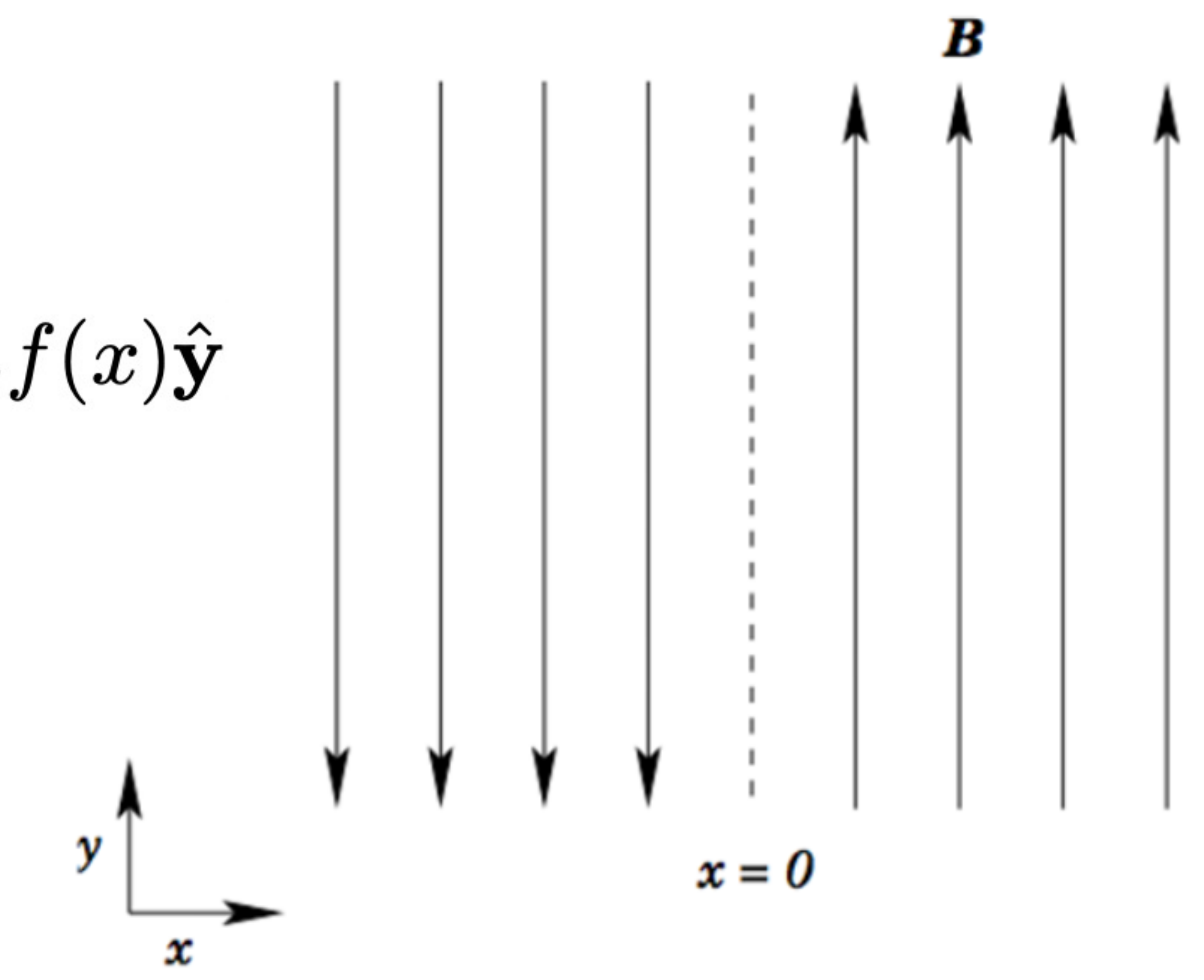
The resistive term becomes locally important in a boundary layer

How to initiate reconnection? Linear regime

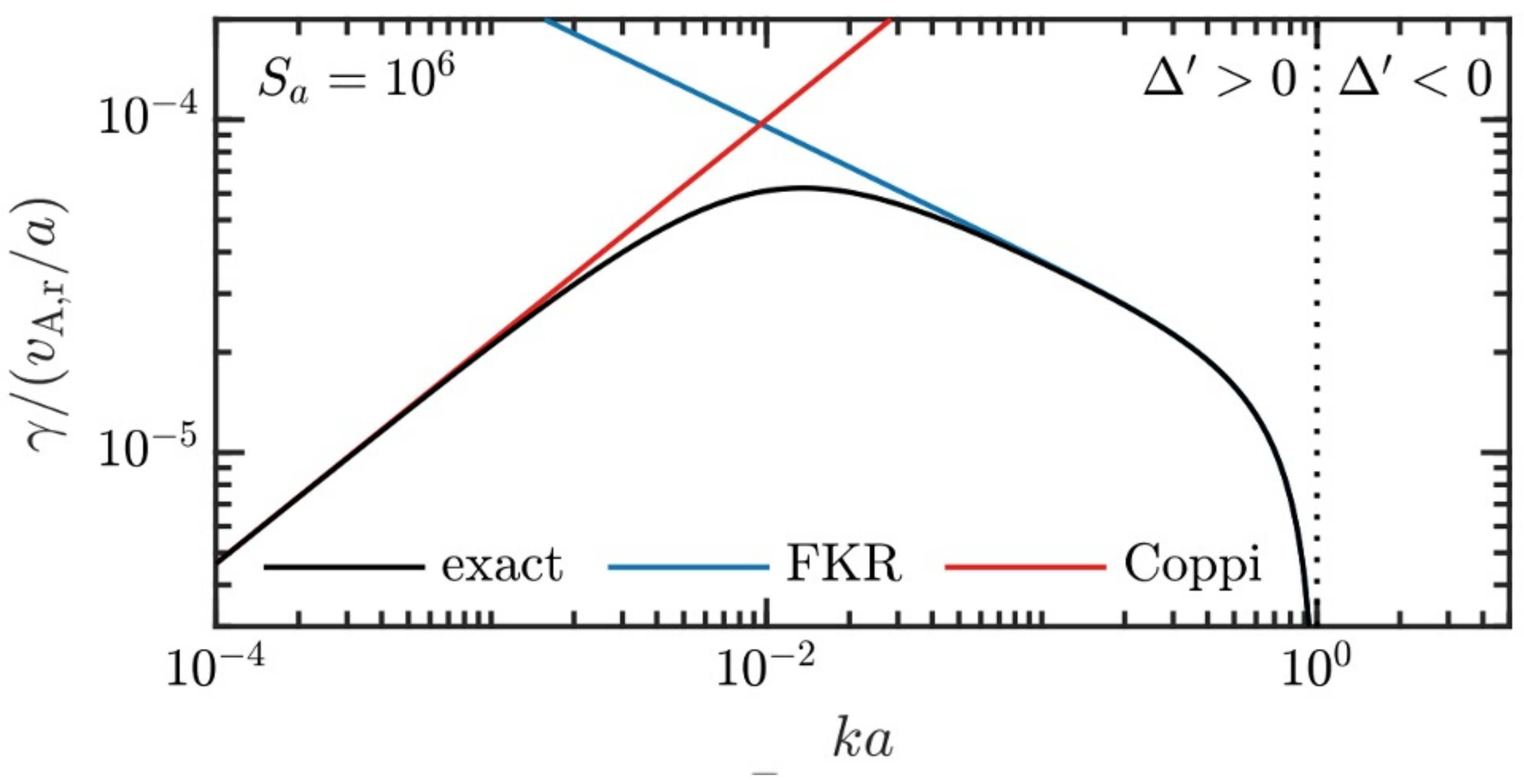
Momentum: $$\rho\left(\frac{\partial \mathbf{u}}{\partial t}+\mathbf{u}\cdot\nabla\mathbf{u}\right) = \frac{\mathbf{j}\times\mathbf{B}}{c}-\nabla p$$

Induction: $$\frac{\partial \mathbf{B}}{\partial t}+\mathbf{u}\cdot\nabla\mathbf{B}=\mathbf{B}\cdot\nabla\mathbf{u}+\bar{\eta}\nabla^2\mathbf{B}$$

$$\mathbf{B}_0 = B_{0y}f(x)\hat{\mathbf{y}}$$



Credit: Fitzpatrick



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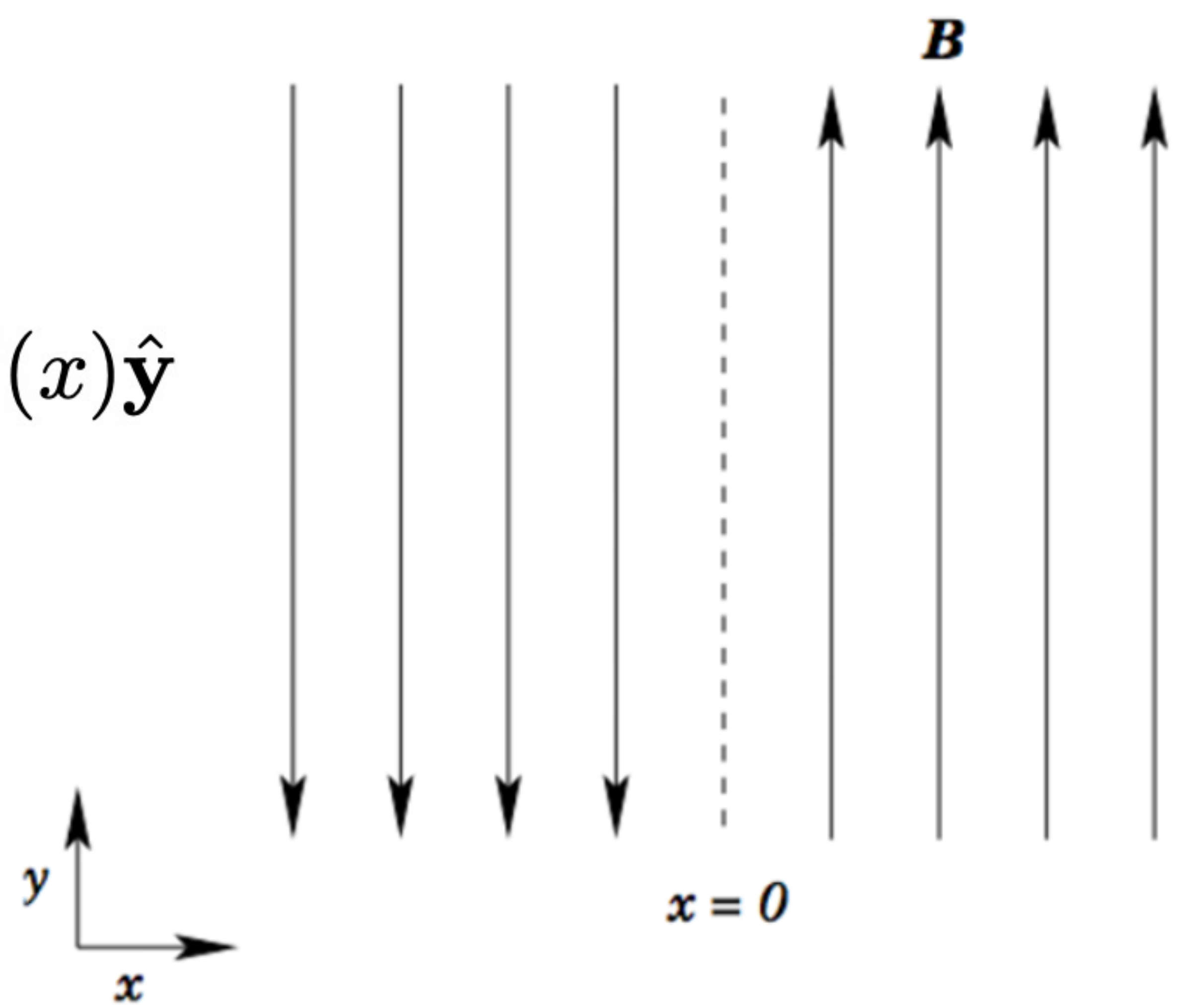

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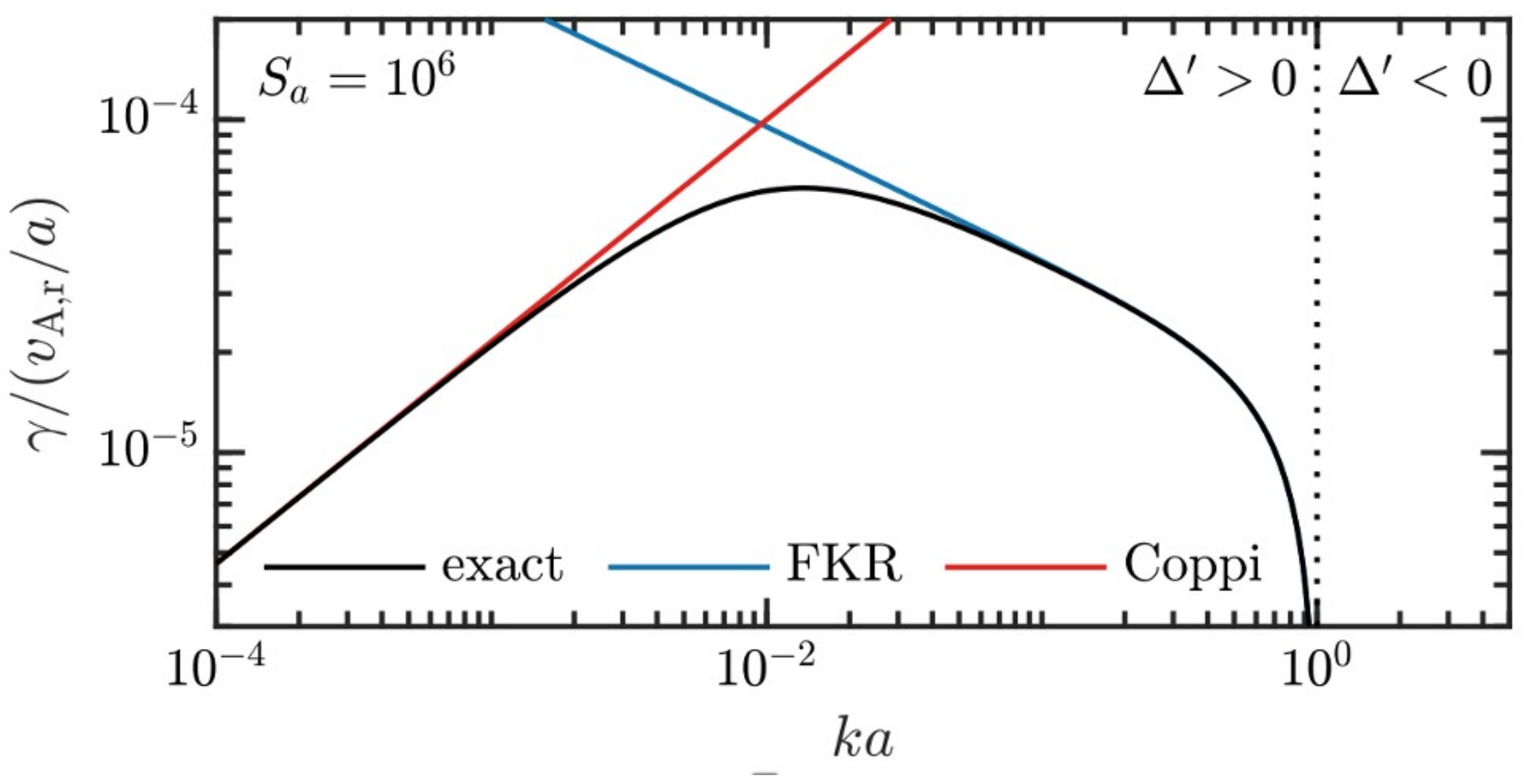
Performing linear perturbation analysis

Looking for the growth rate to be hybrid, i.e., intermediate between the resistive diffusion time scale (slow) and the ideal Alfven time scale (fast)

Assuming ideal MHD behavior away from the reconnection layer and diffusion/resistivity dominating in the reconnection layer

$$\gamma_{max} \sim S_a^{-1/2}/\tau_A \sim v_A^{1/2} a^{-3/2} \tilde{\eta}^{1/2}$$



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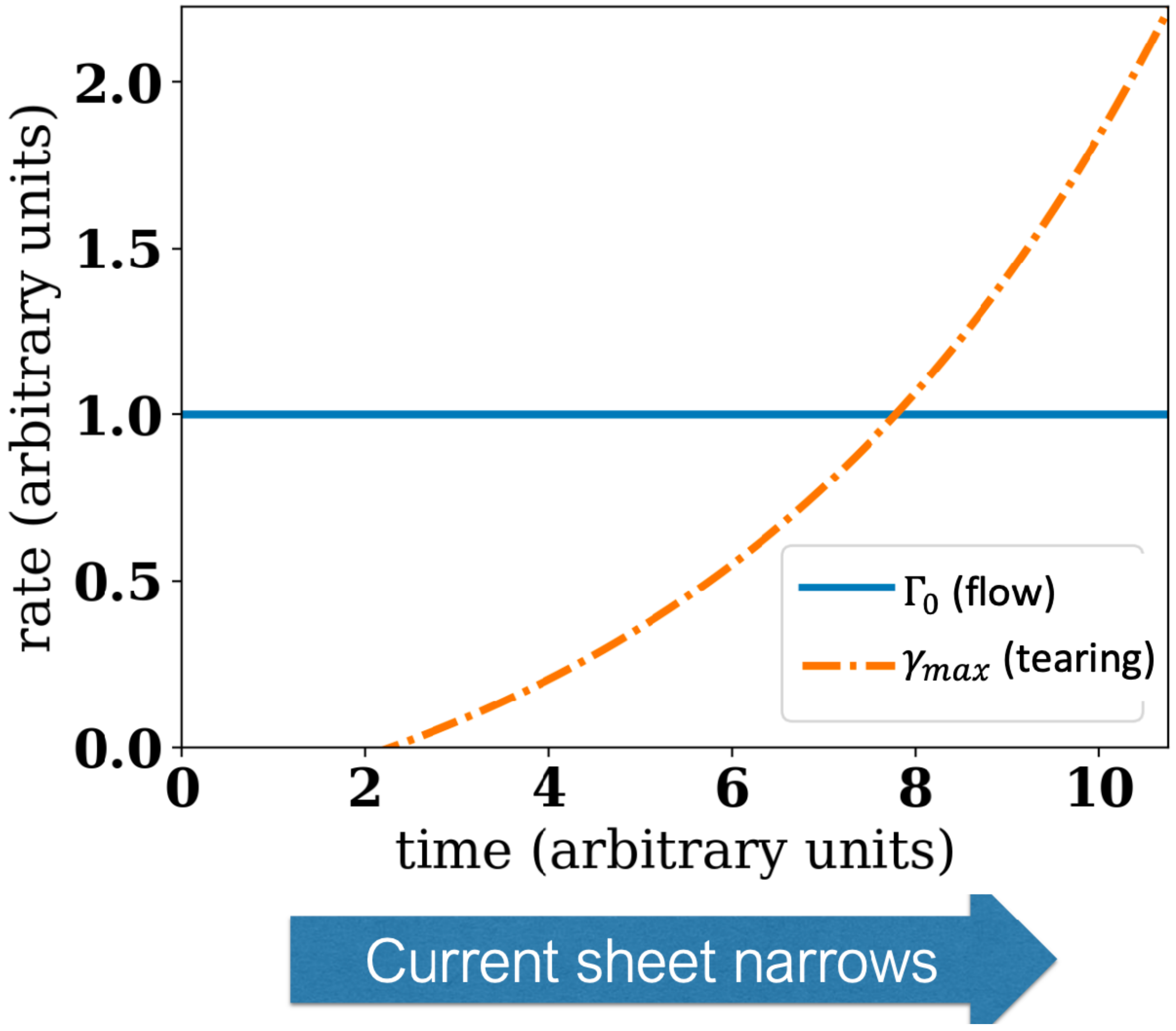
Onset of tearing

Stage dominated by ideal dynamics

Configuration is shrinking, forming the current layer.

Tearing mode is weak.



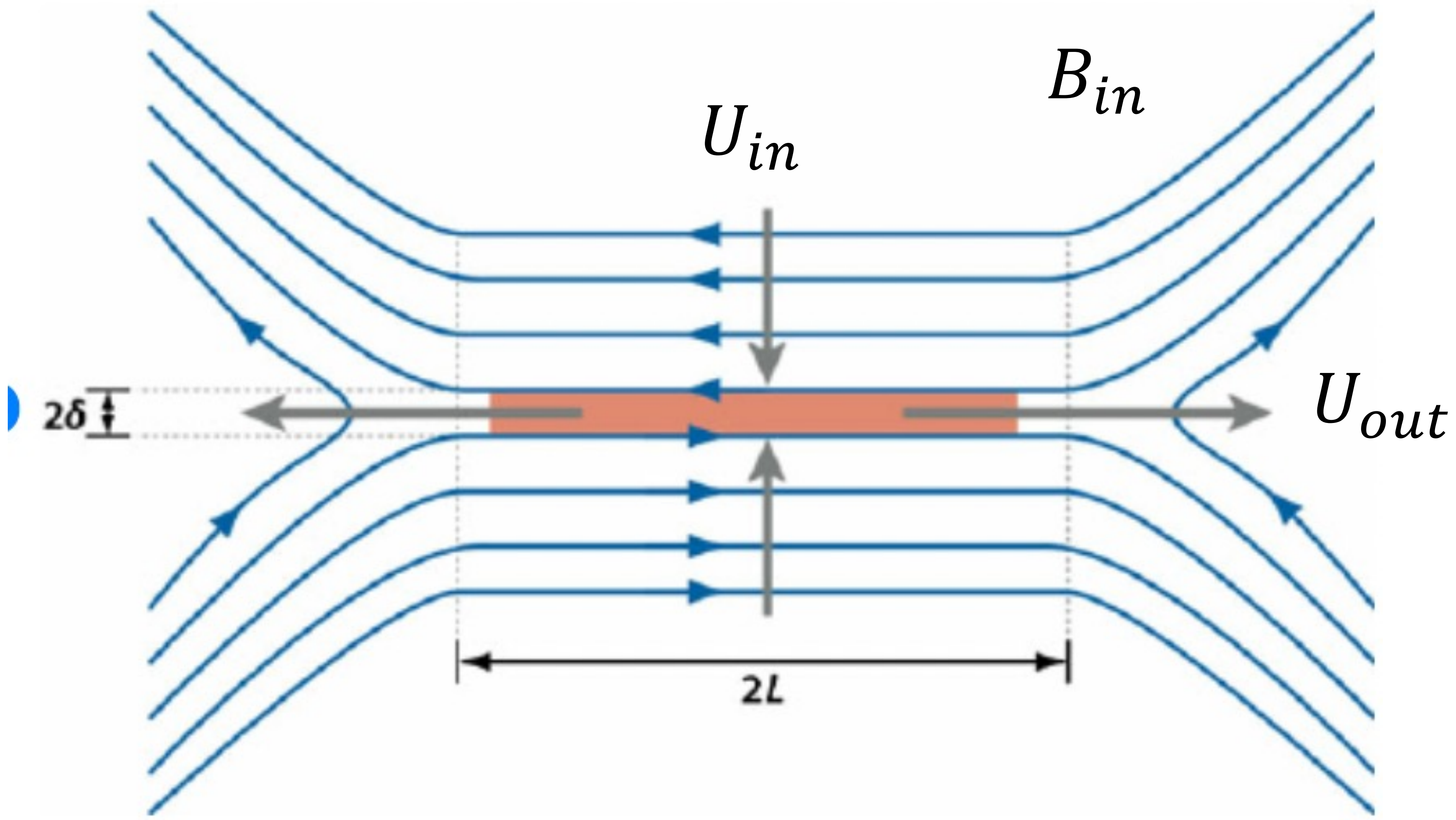
Credit: E. Tolman

Linear tearing stage:

The current layer becomes sufficiently thin that the growth of tearing mode surpass the evolution of the flow.

We will skip the discussion about the nonlinear evolution of tearing and X-point collapse. Please refer to Rutherford(1973), Militello&Porcelli (2004), Escande&Ottaviani (2004), Loureiro+(2005).

A classic nonlinear solution --- Sweet Parker Reconnection



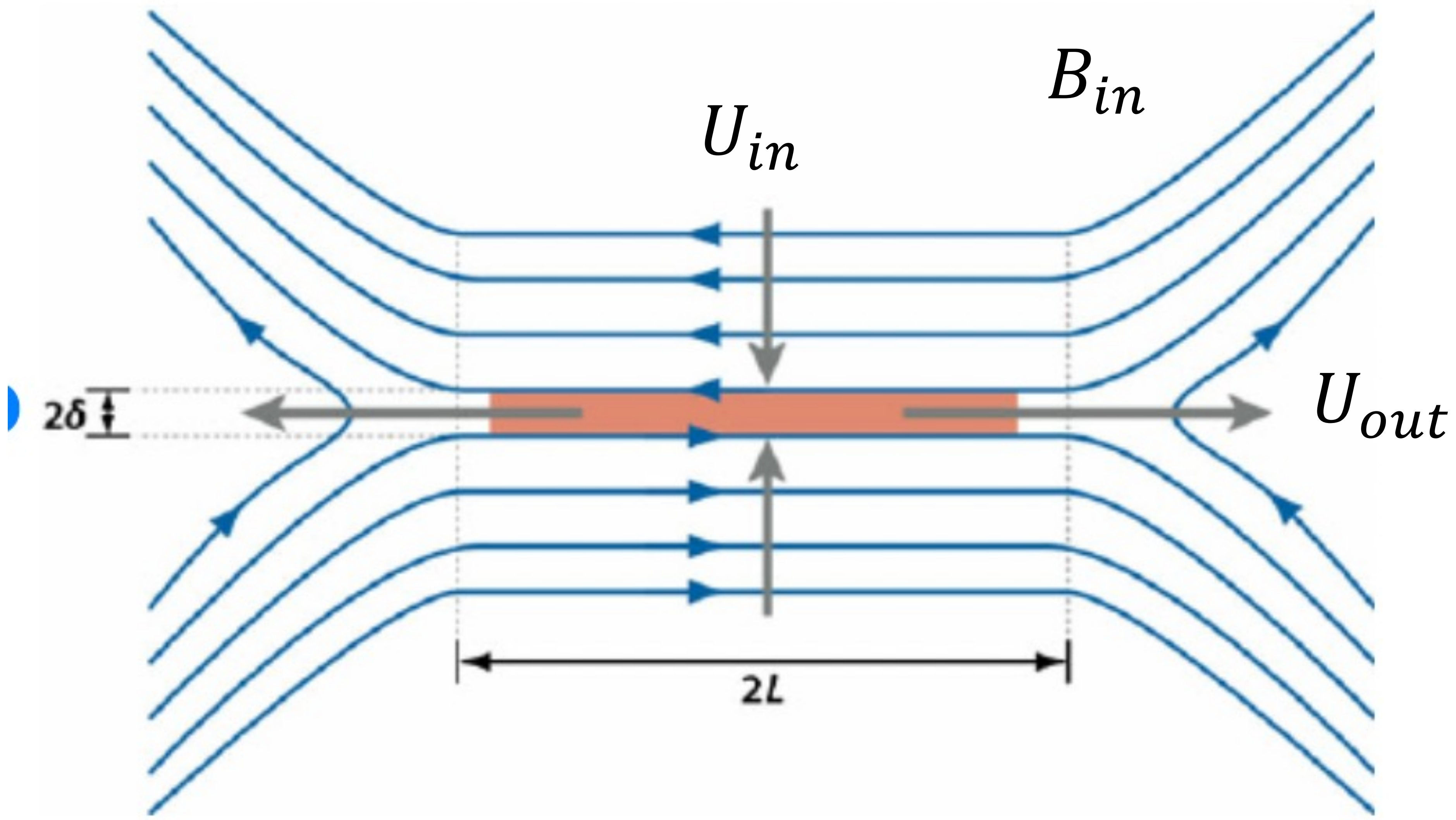
Set up: 2D; Current sheet aspect ratio $\frac{\delta}{L} \ll 1$;

steady state → uniform electric field ;

Outer regime: ideal plasma; $E_{away} \approx u_{in} B_{in}/c$.

Current sheet: resistivity dominates; $E_{CS} \approx \eta j \approx \eta \frac{c}{4\pi} \frac{B_{in}}{\delta}$

Classic model --- Sweet Parker Reconnection



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Lundquist number $S \equiv L\frac{v_A}{\eta} \sim \frac{|\mathrm{B}\cdot\nabla u|}{|\,\eta|\nabla^2 B|}$

Solar corona $S \sim 10^{12}$; Solar wind $S \sim 10^{15}$; ISM $S \sim 10^{18}$.

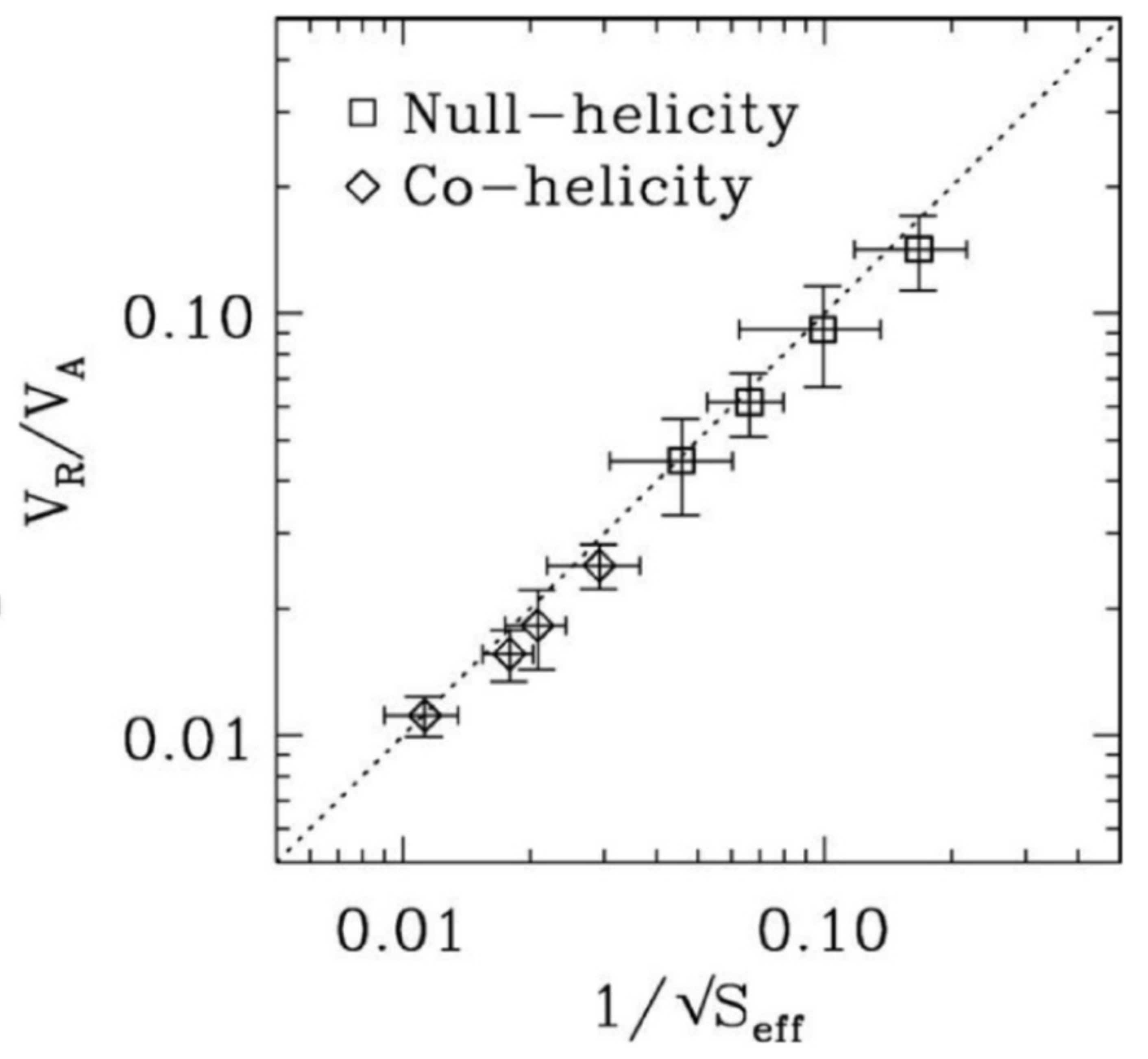
$$\frac{\delta}{L} \sim \frac{u_{in}}{u_{out}} \sim S^{-1/2}$$

The flux reconnects at the rate $\frac{d\psi}{dt} = cE_z$

Normalized reconnection rate $\epsilon_{rec} \equiv \frac{cE_z}{V_A B_{in}} \sim \frac{u_{in}}{V_A} \sim S^{-1/2}$

Classic model --- Sweet Parker Reconnection

Experiment confirmation by MRX



[Ji+1999]

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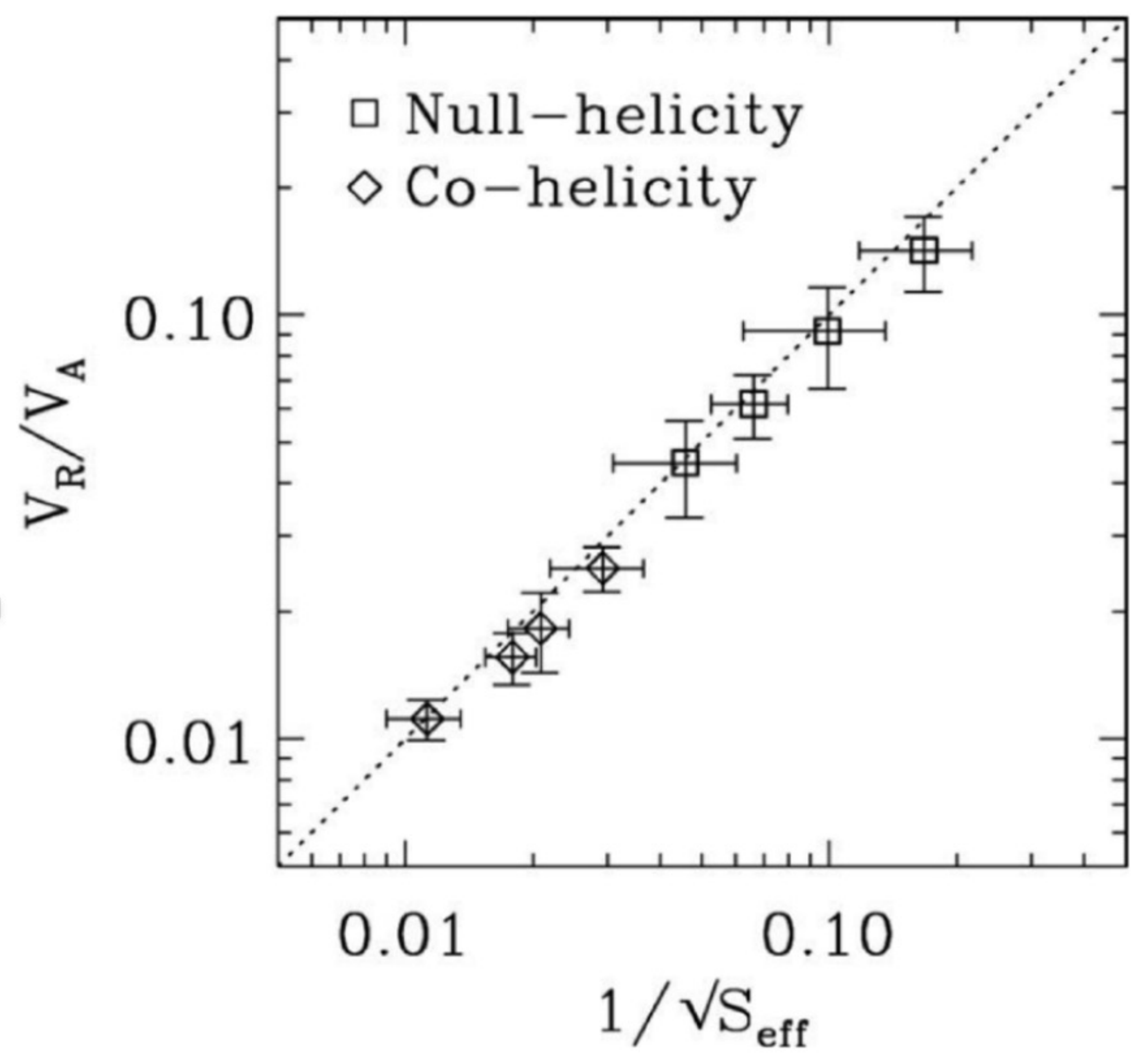
Does the Sweet Parker work? – Yes!

Experiment confirmation by MRX



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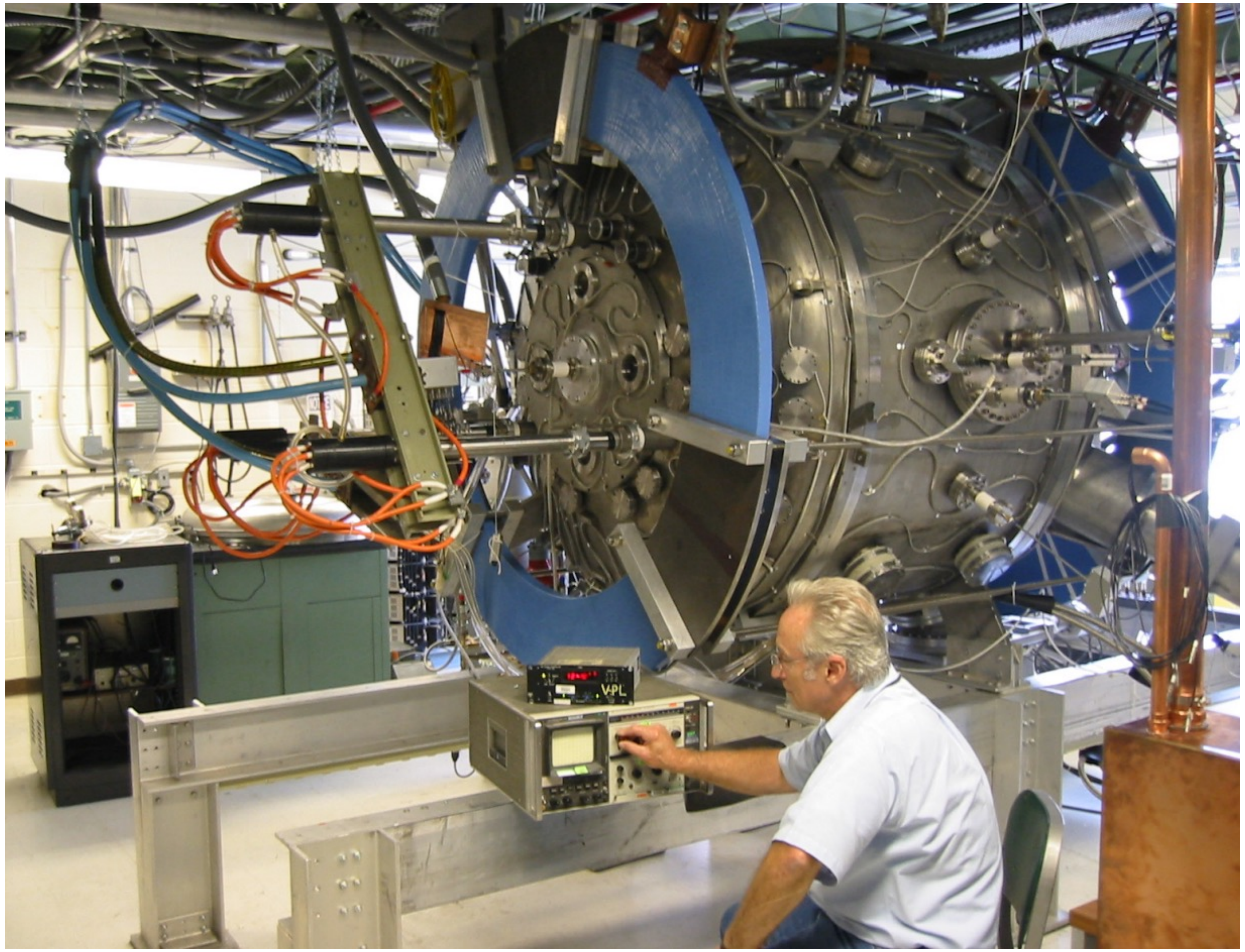
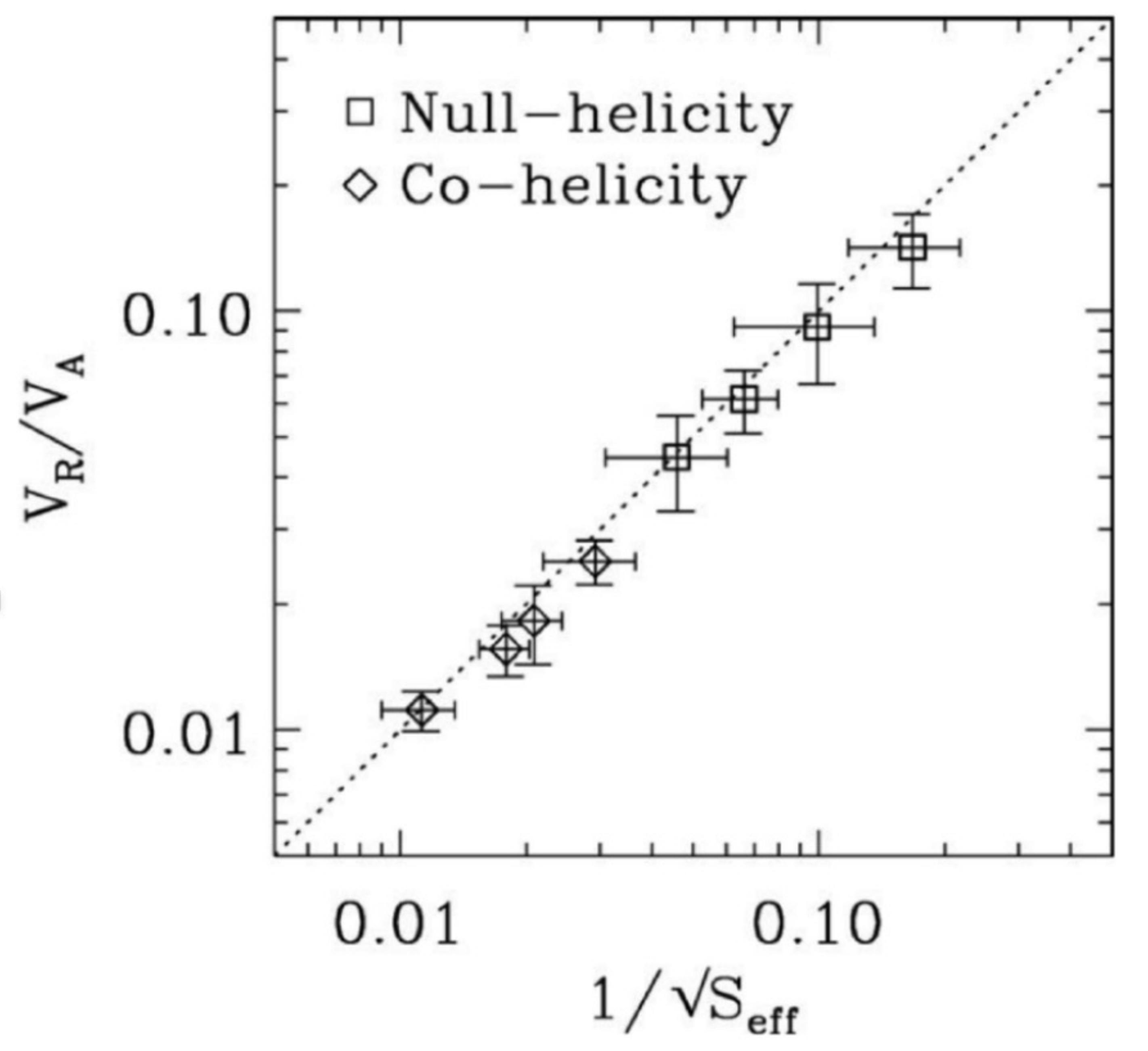


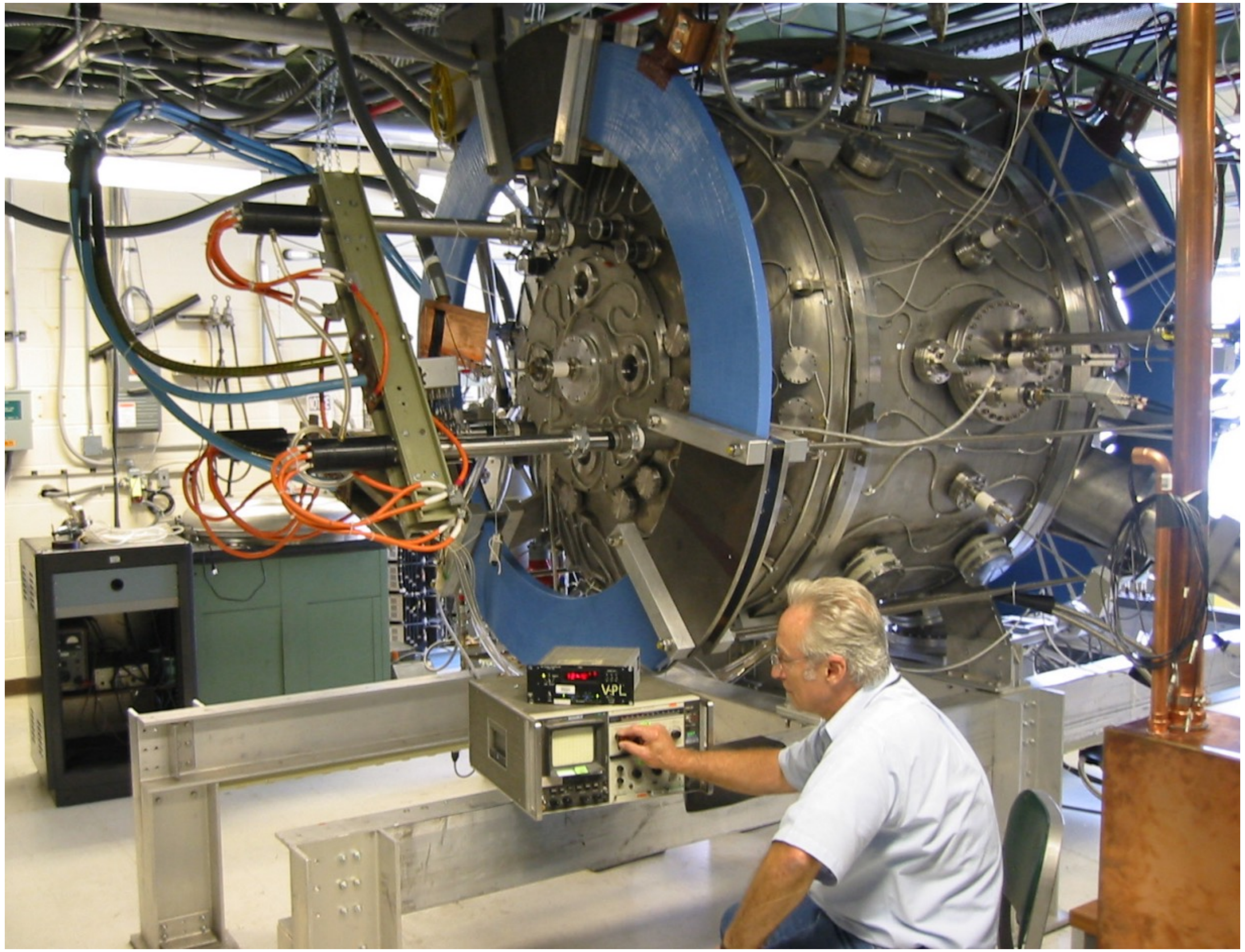
But…

Experiment confirmation by MRX



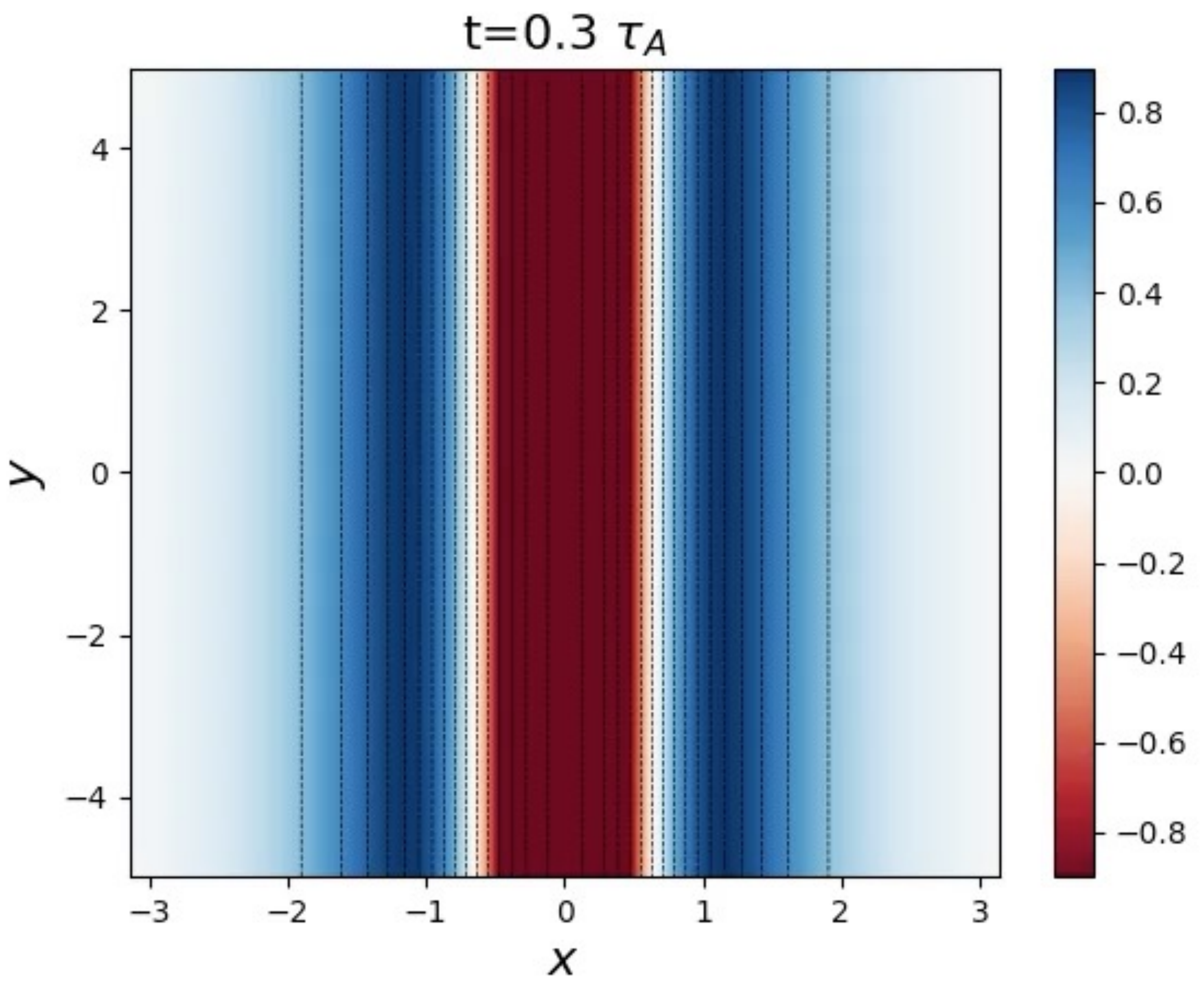
[Ji+1999]

In solar corona: $\tau_A = \frac{L}{v_A} \sim 1s, \tau_{sp} \sim 10^6 s \gg 10^2 s$ (solar flare). Too slow!!!



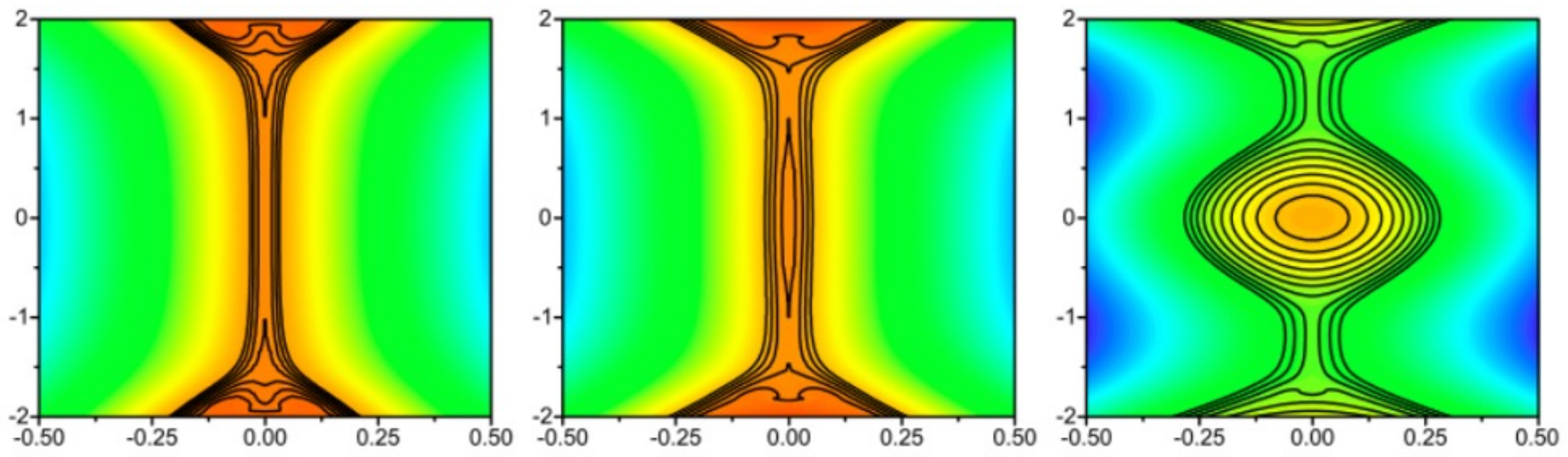
MRX at PPPL

Is the Sweet-Parker current sheet always stable?



At high S , the SP sheet become unstable to the ``plasmoid instability” --- a tearing-like instability which gives rise to multiple magnetic islands (plasmoids)

(Biskamp 1986, Shibata&Tanuma 2001, Loureiro et al. 2007)



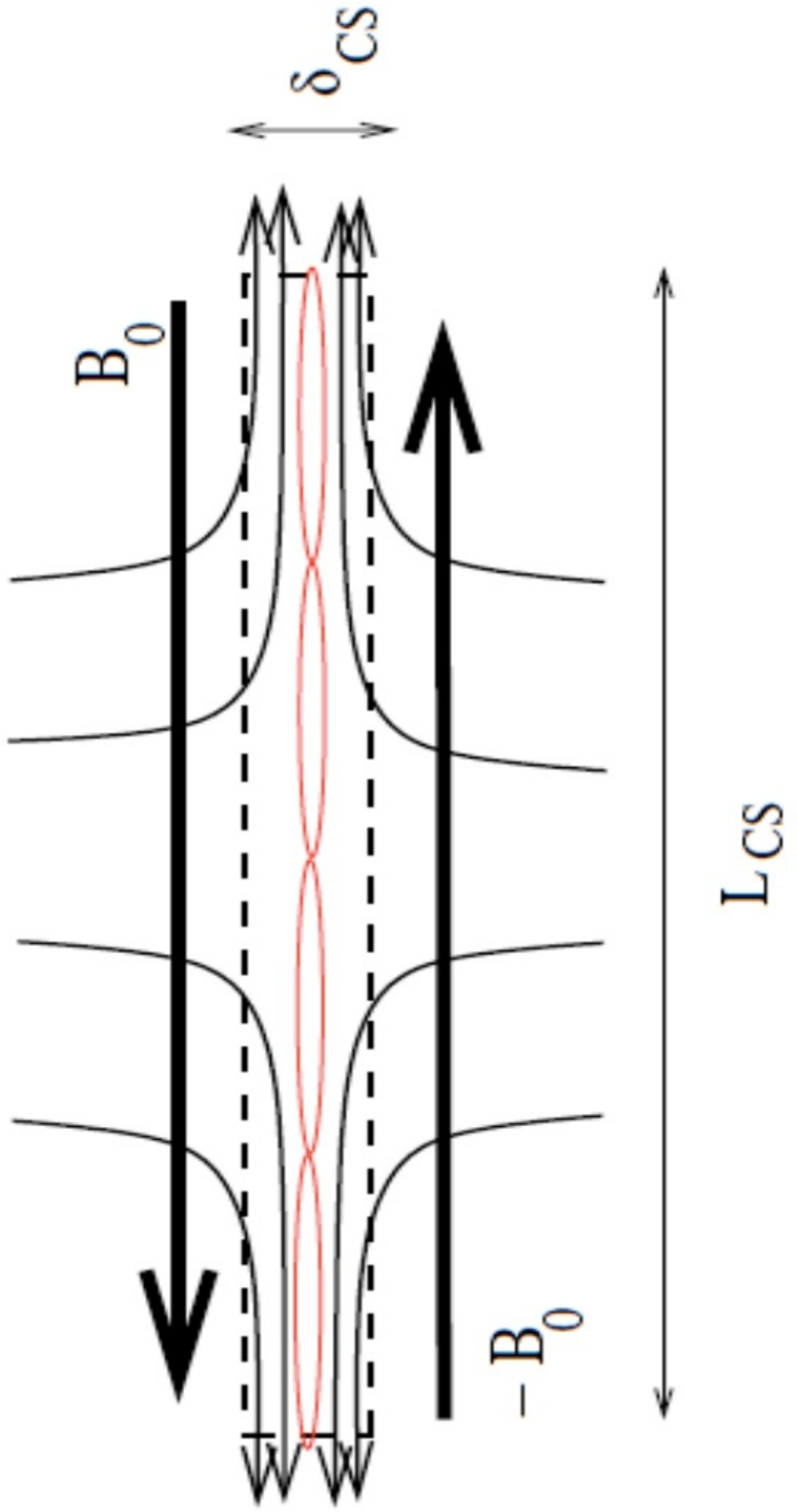
[Loureiro+ 2005]

Tearing instability on Sweet-Parker current sheets

Loureiro et al. 2007, 2013 performed the first calculation of tearing modes on a SP-current-type of configuration:

$$\gamma_{\max}\tau_A \sim S^{1/4}$$

$$k_{\max}L_{CS} \sim S^{3/8}$$



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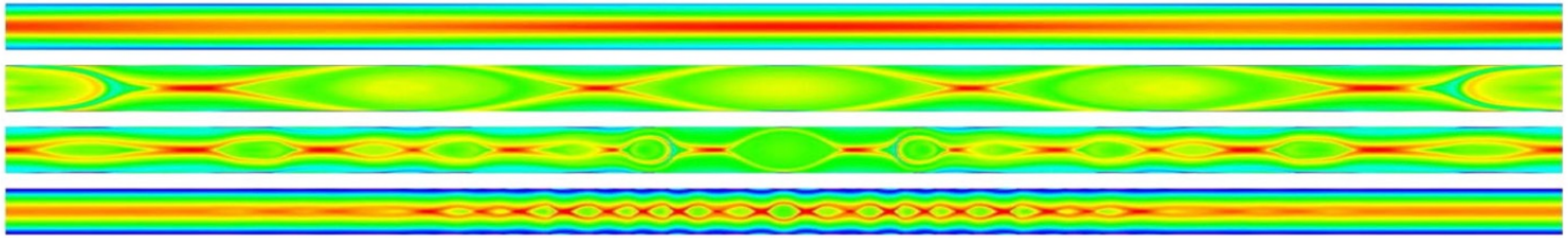
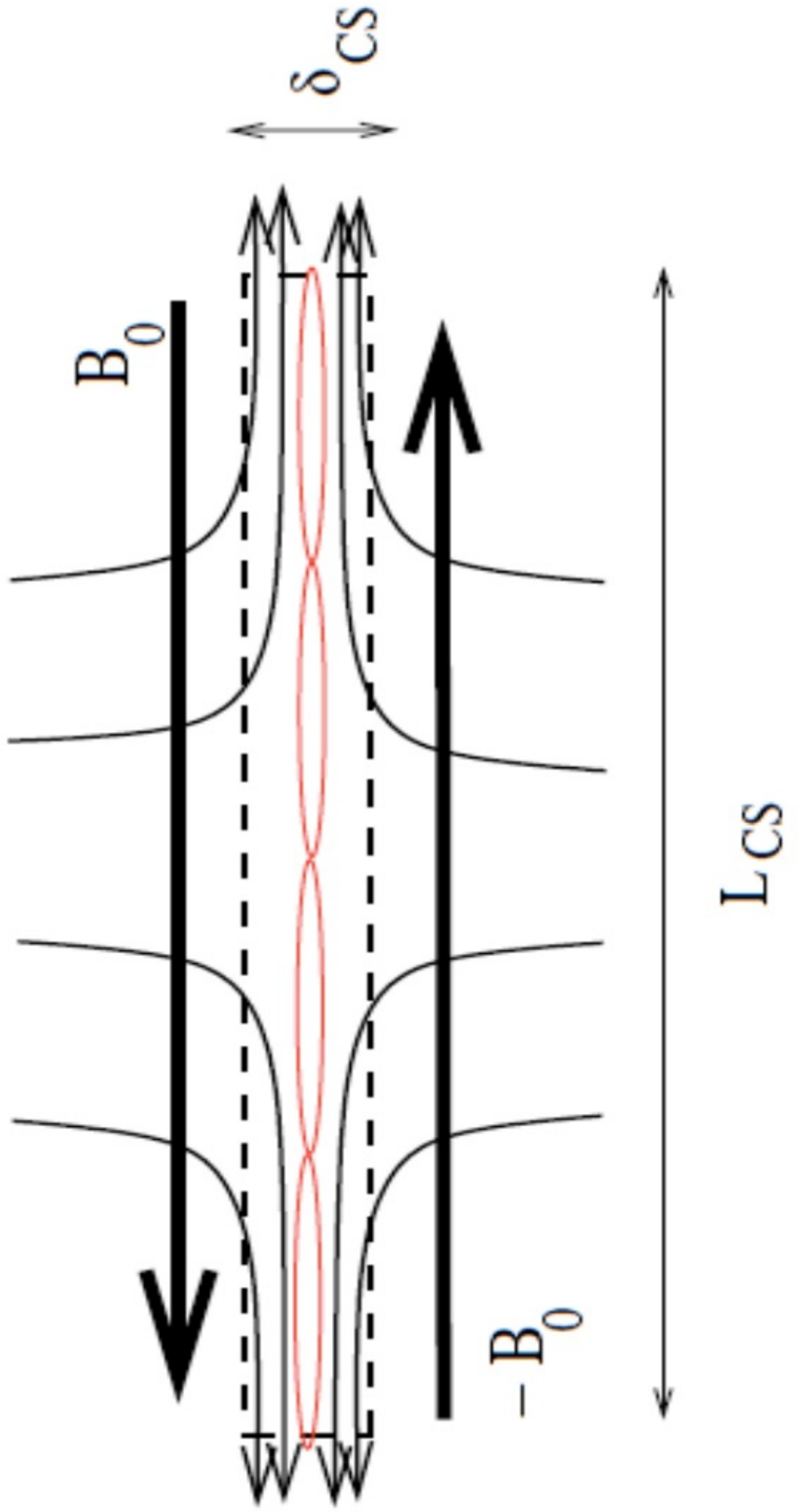
Threshold for the instability: $S \geq 10^4$



As S increases (from top to bottom, $S = 10^4, 10^5, 10^6, 10^7$), the number of plasmoids and their growth rate increase.

[Samtaney et al. 2009]



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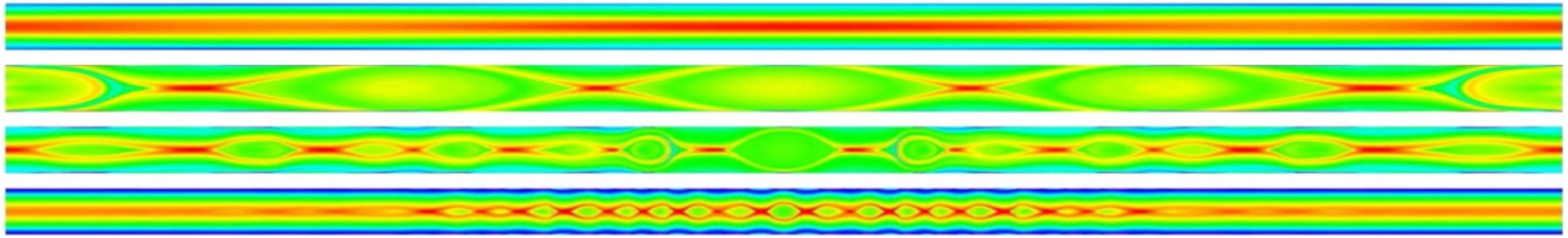

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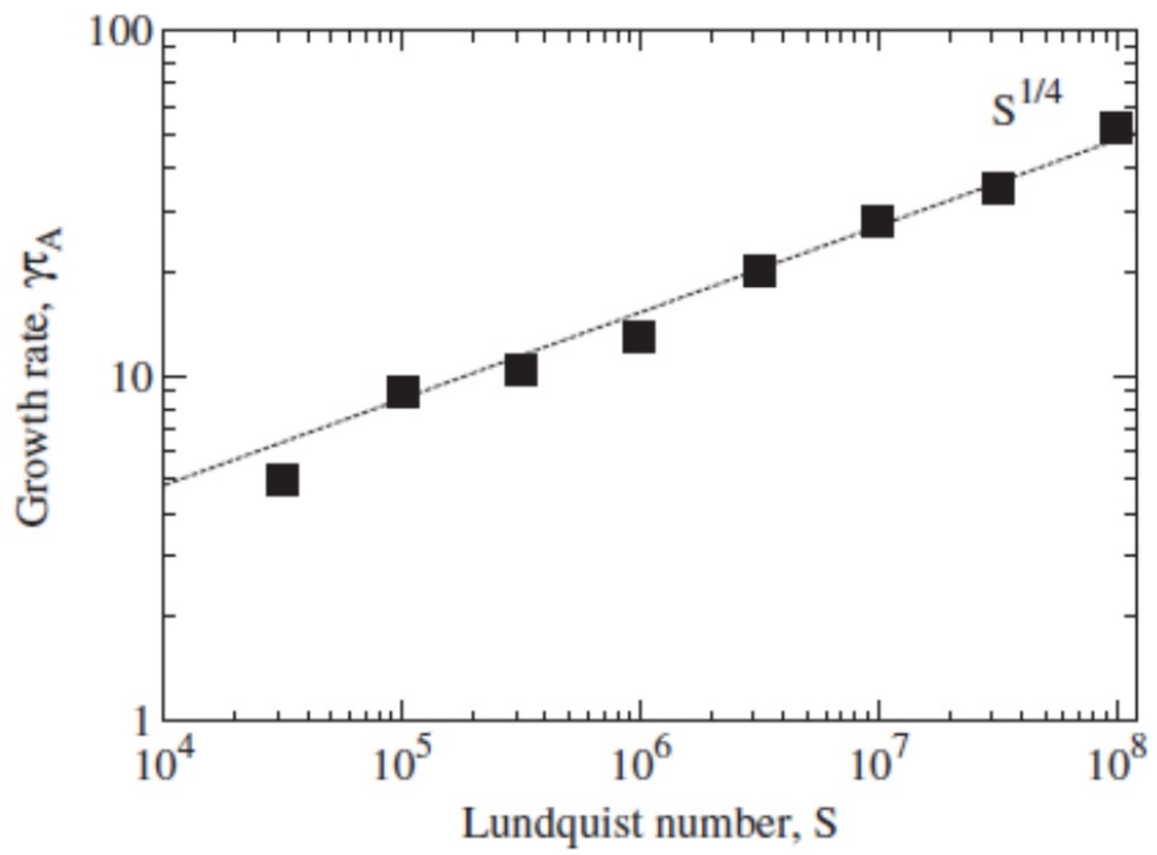

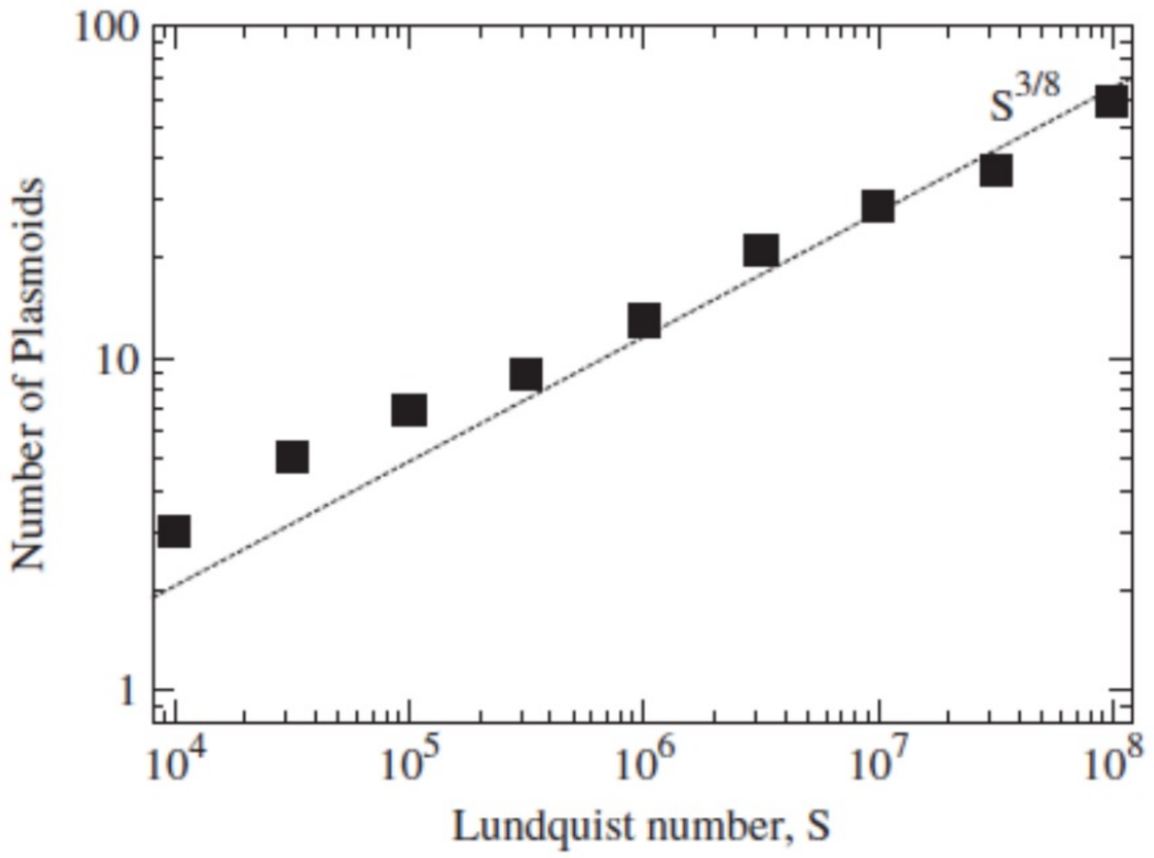

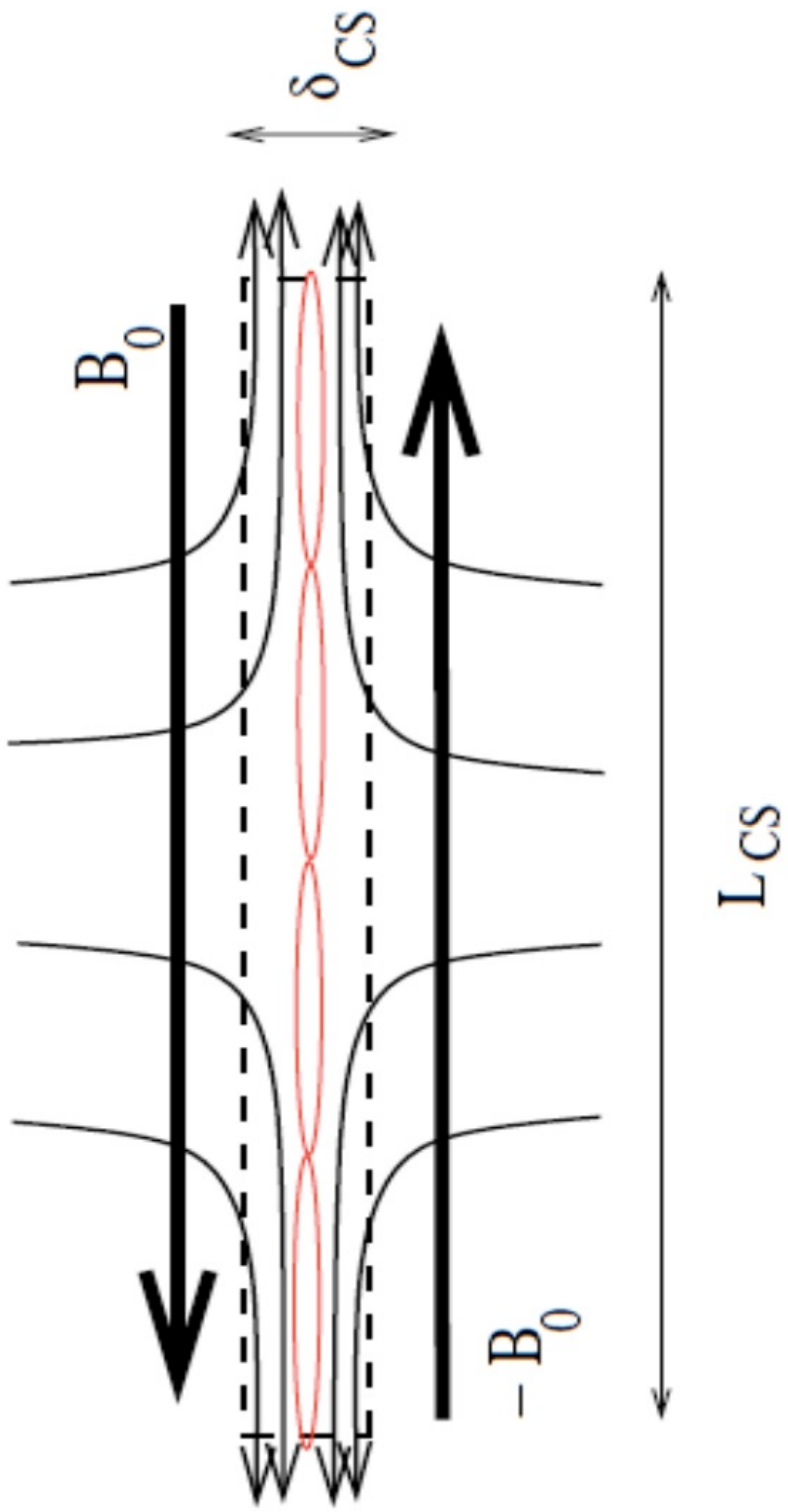

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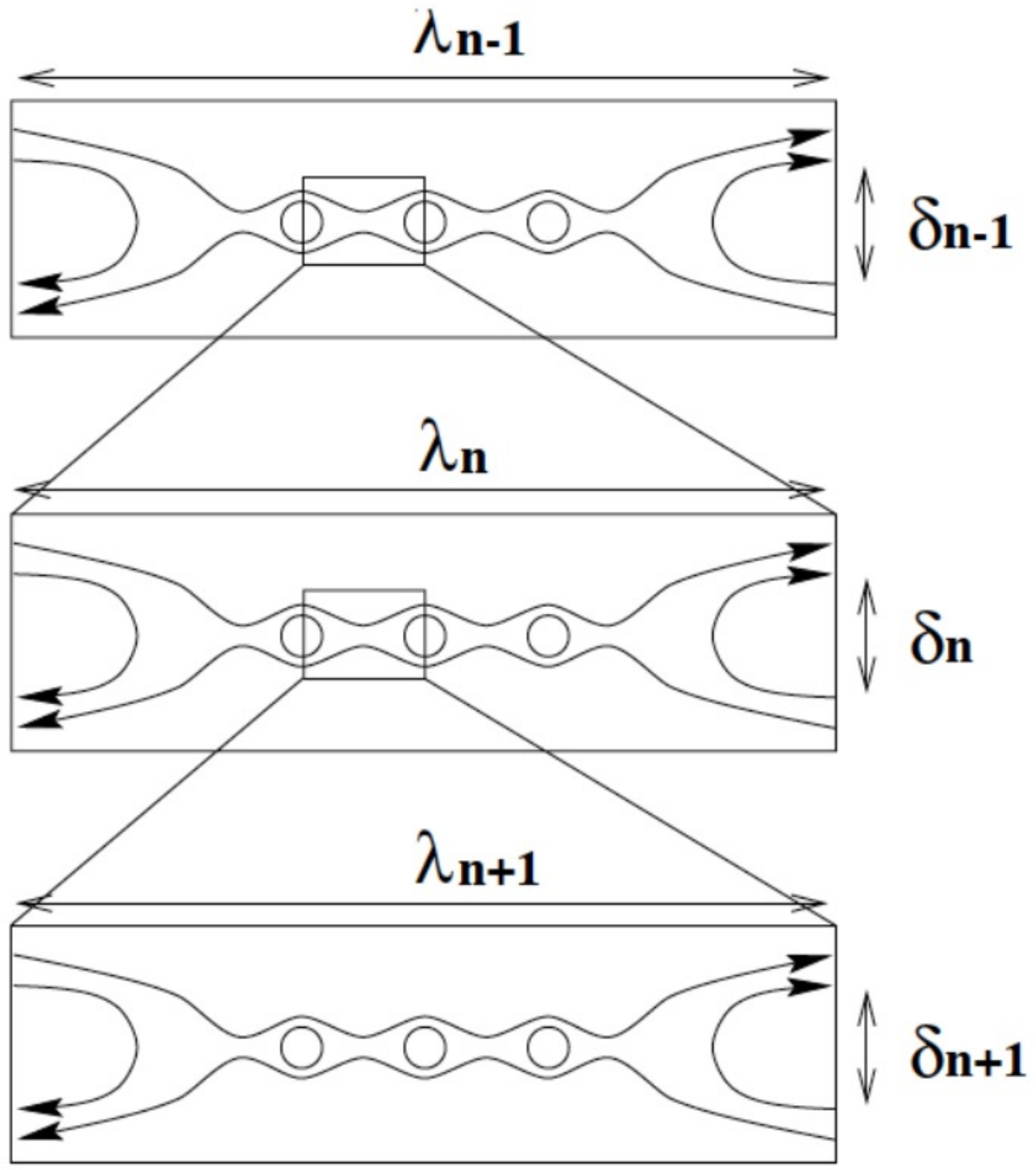
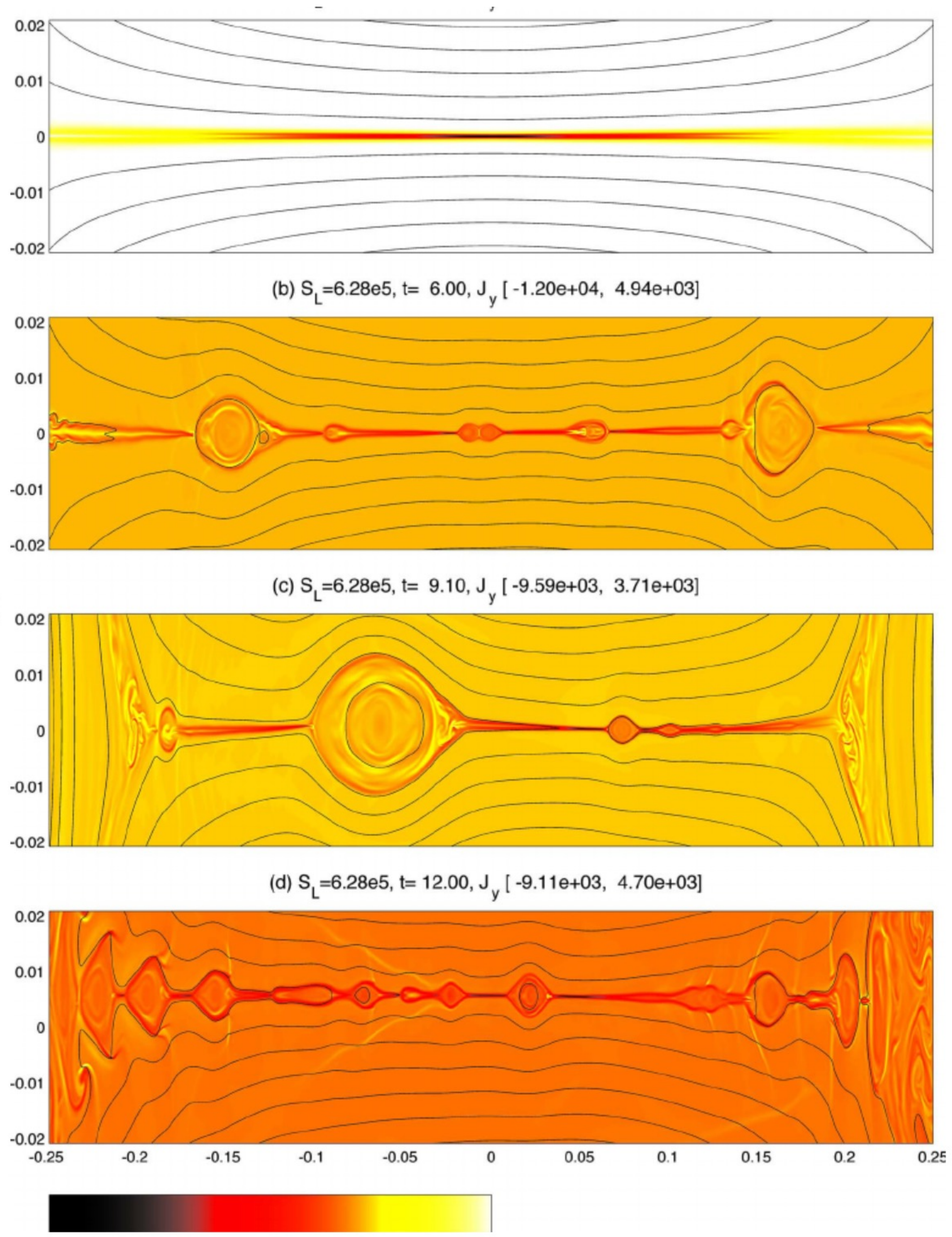
Nonlinear regime: hierarchical plasmoid chains



[Shibata & Tanuma 2001]



[Huang and Bhattacharje. 2012]

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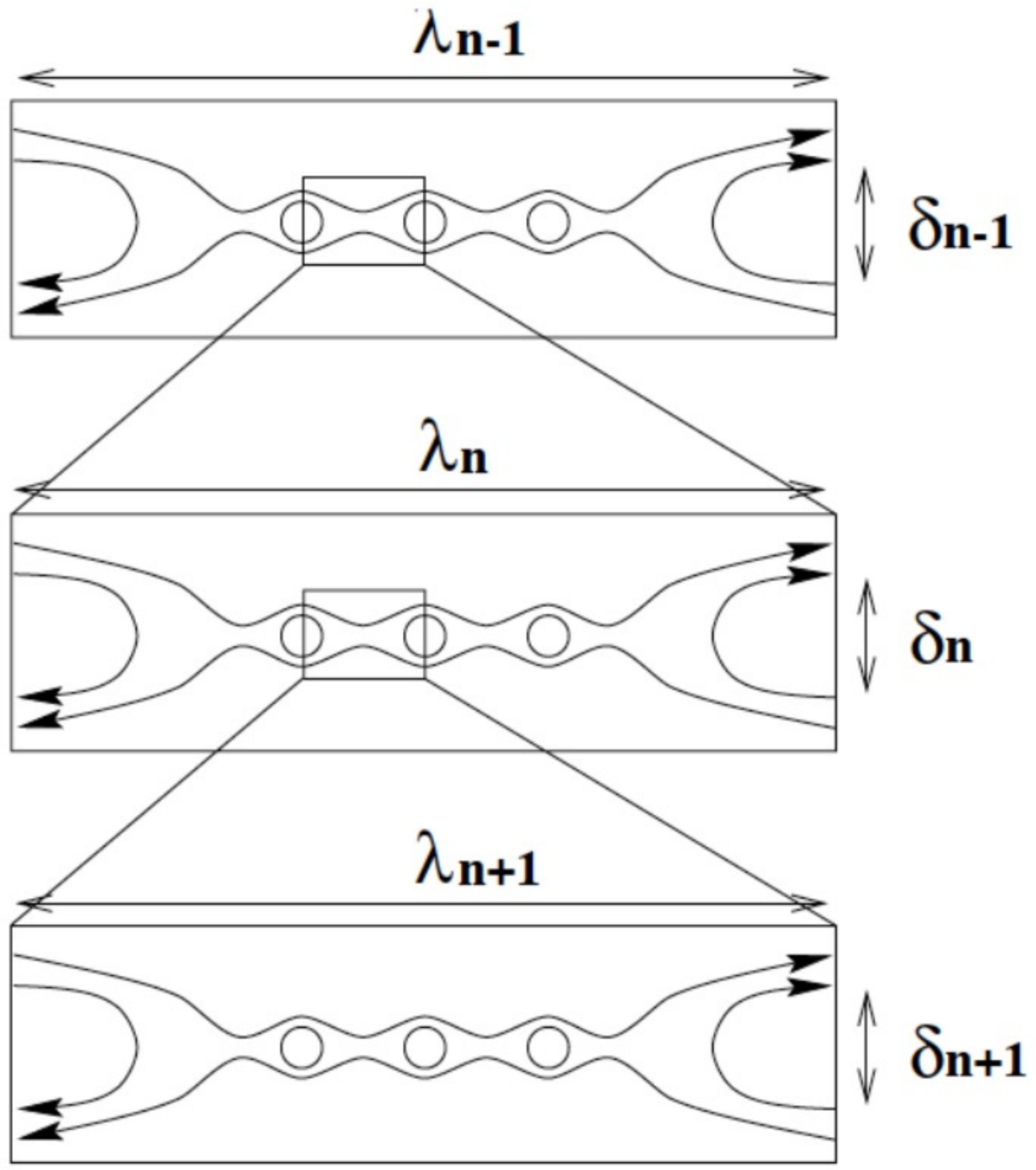

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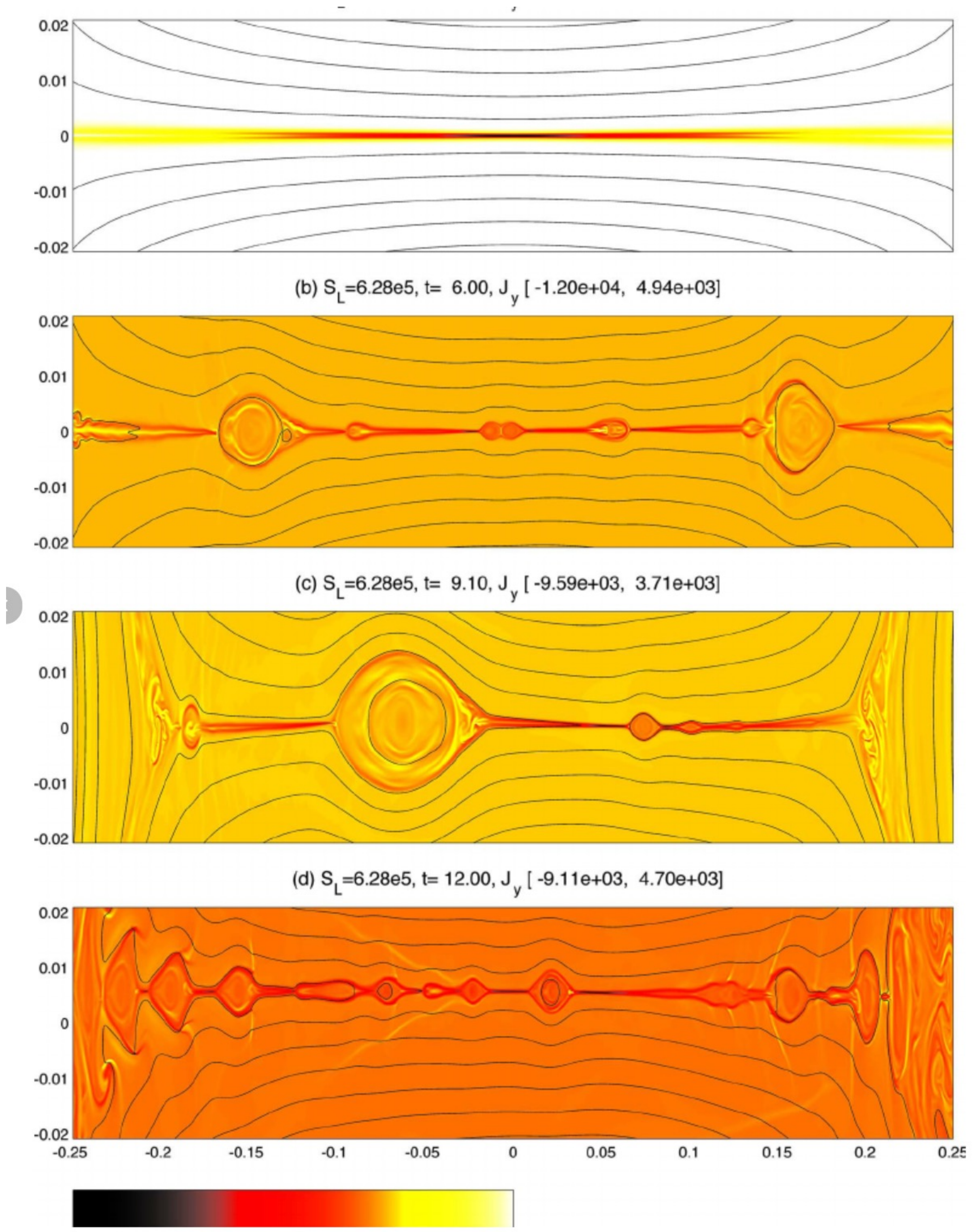
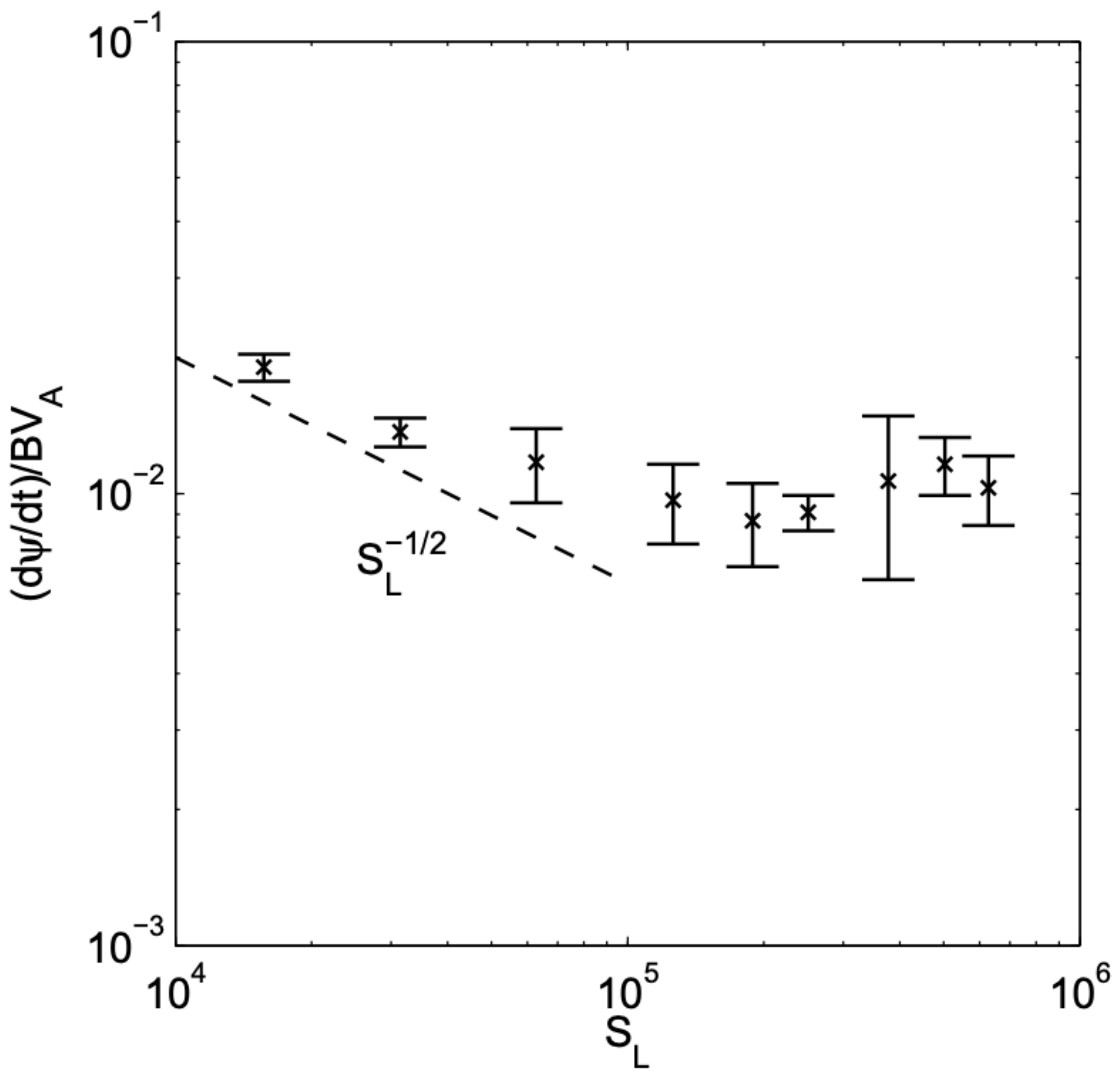


[Huang and Bhattacharje. 2012]

In the nonlinear statistical steady state, the reconnection rate is independent of Resistivity/Lundquist $\boldsymbol{\epsilon_{rec} \sim 0.01}$

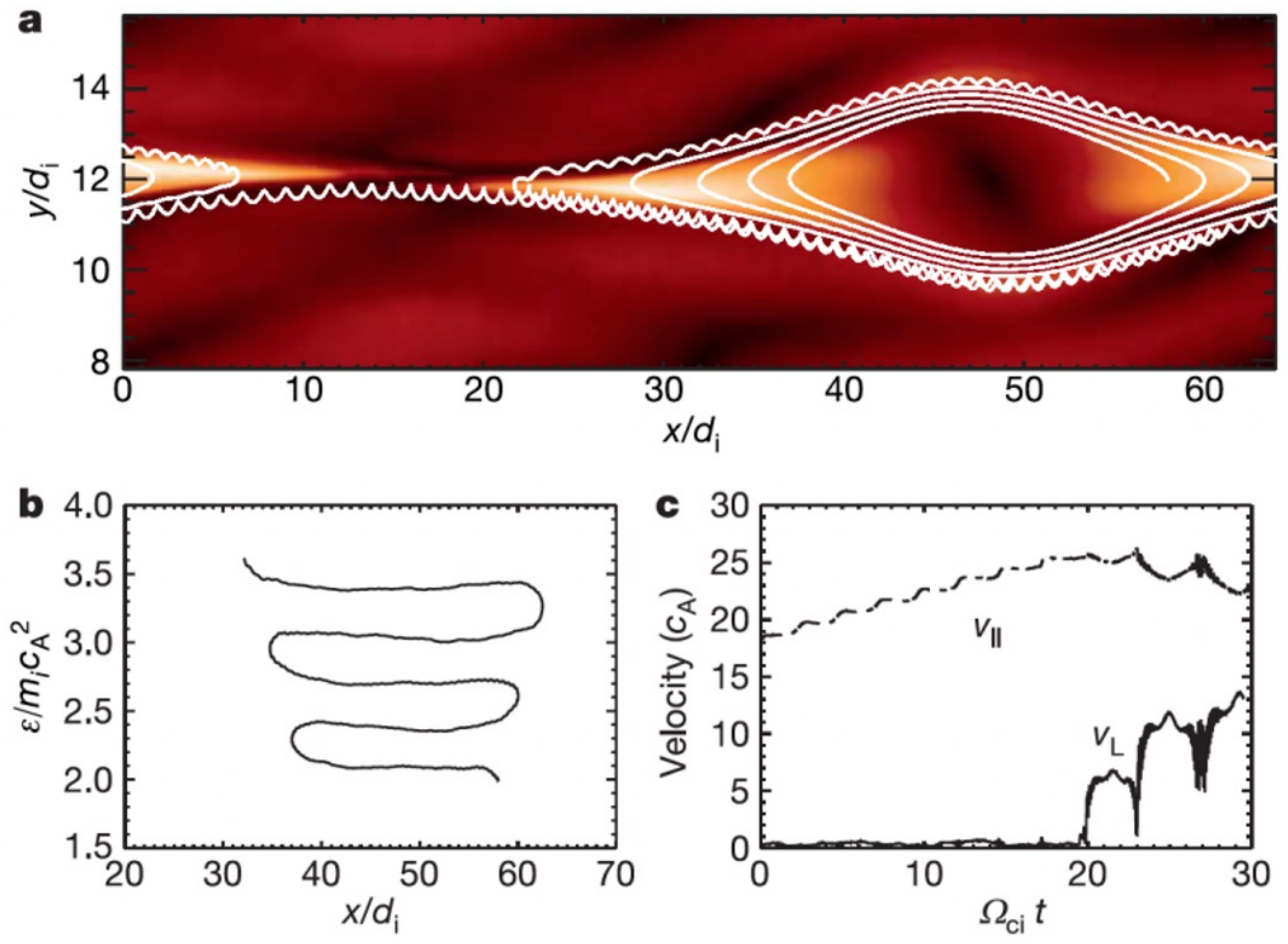
[Uzdensky et al 2010]



[Bhattacharjee et al. 2009]

Implication of the plasmoid instability

The formation of plasmoids changes the structure of magnetic fields and particle energization

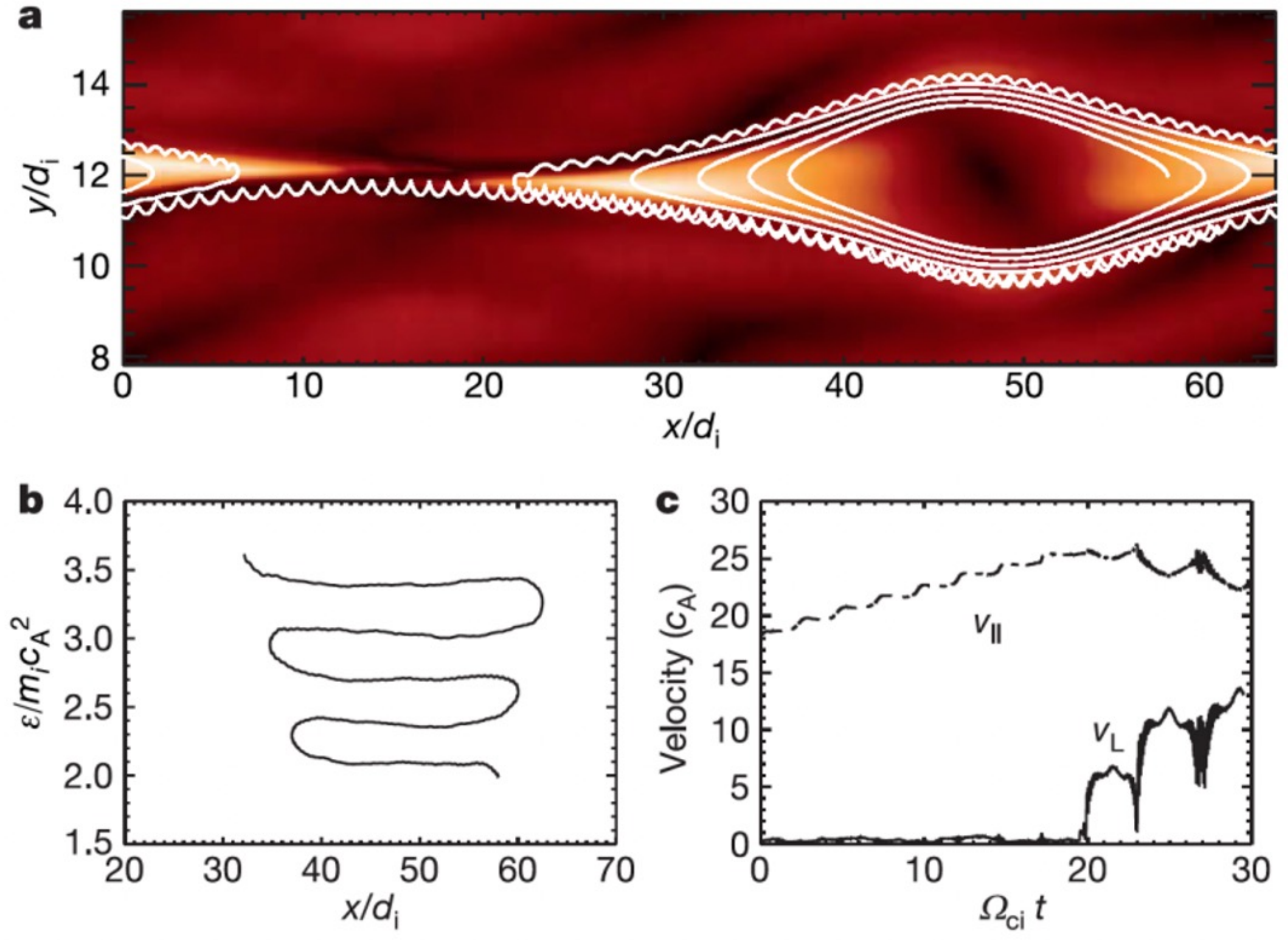


[Drake et al. 2006]

Particles are trapped and accelerated at/around plasmoids.

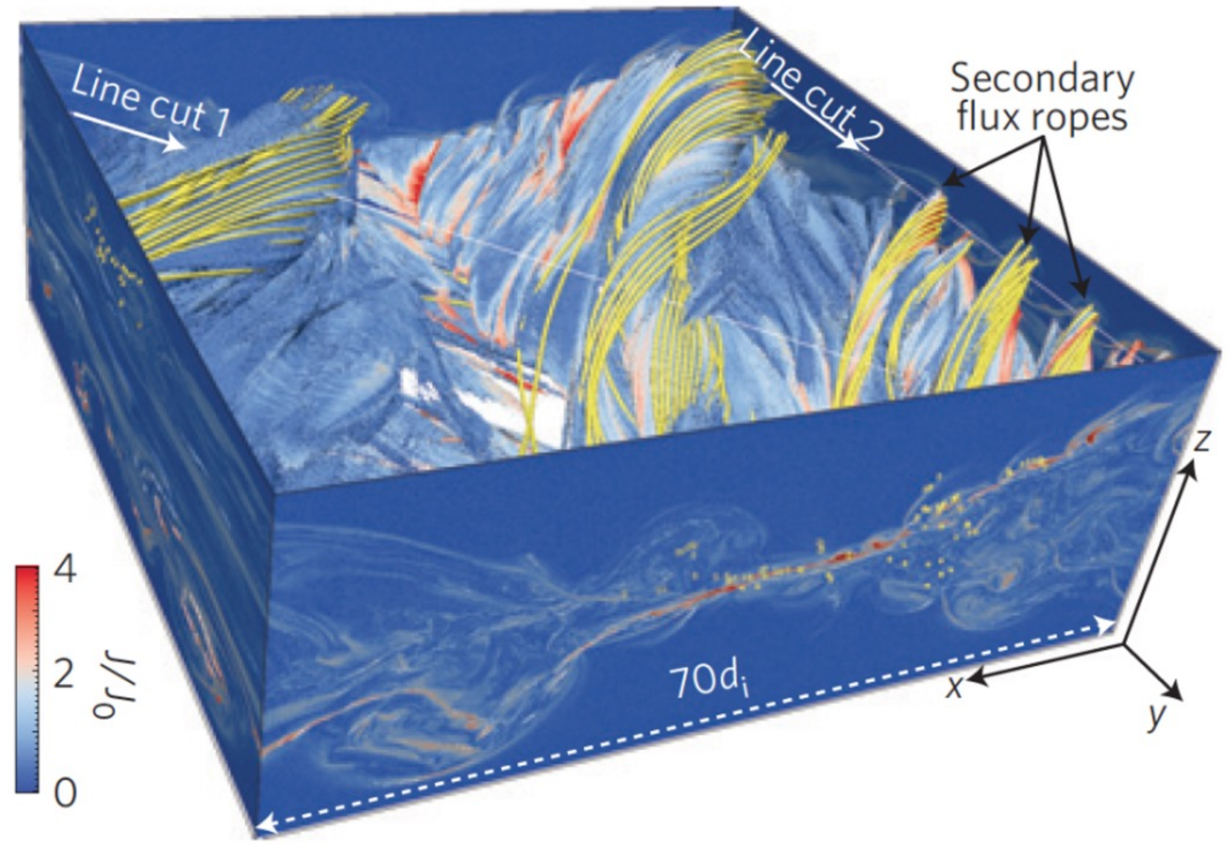
Implication of the plasmoid instability

The formation of plasmoids changes the structure of magnetic fields and particle energization



[Drake et al. 2006]

Particles are trapped and accelerated at/around plasmoids.

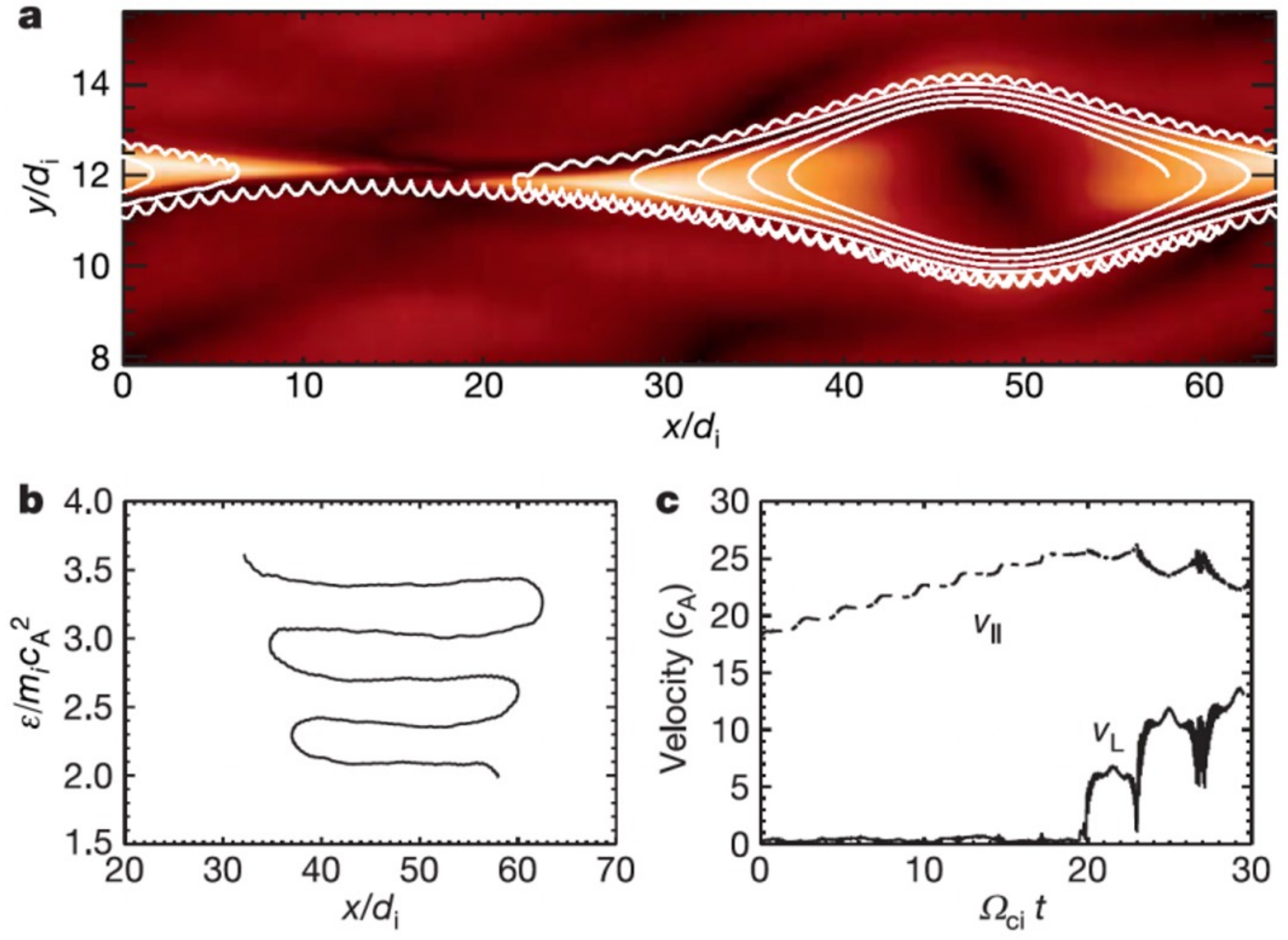


[Daughton et al. 2011]

Instabilities and complex dynamics of plasmoids drive turbulence.

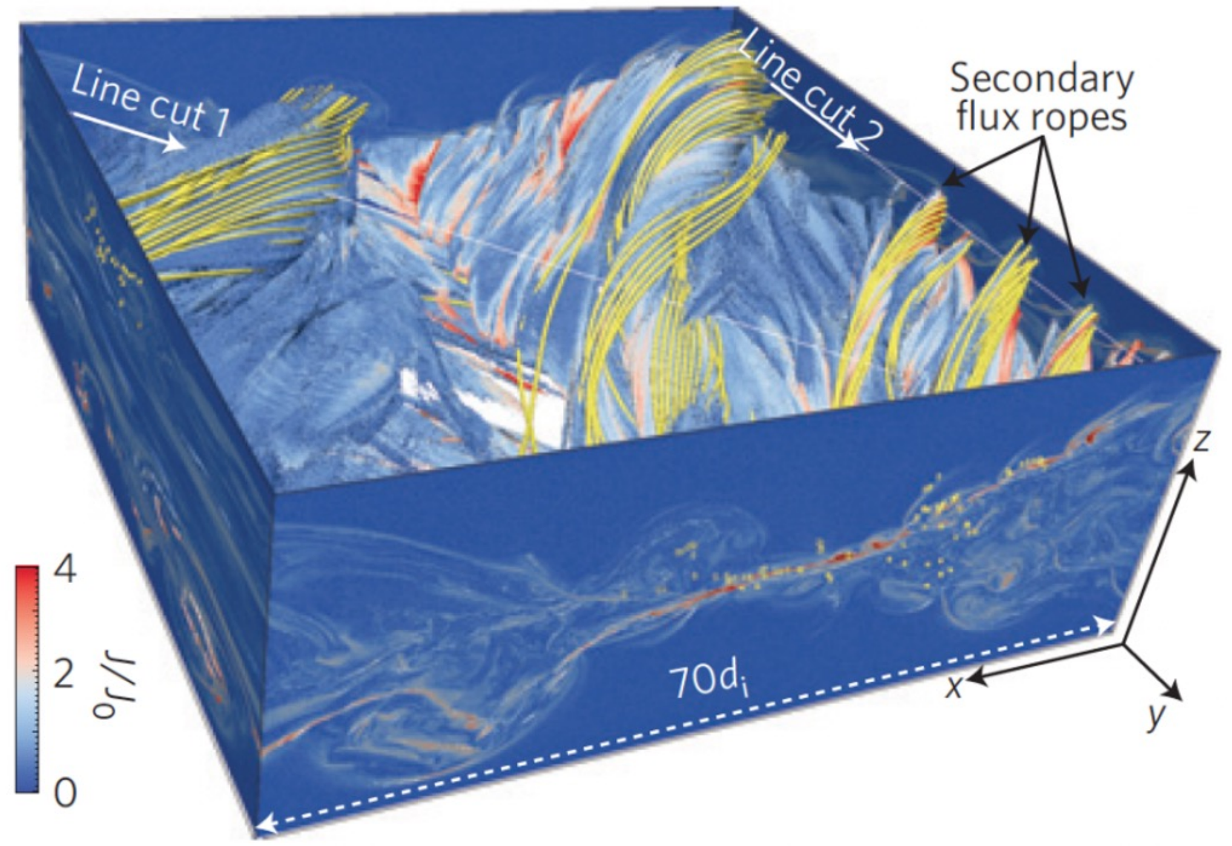
Implication of the plasmoid instability

The formation of plasmoids changes the structure of magnetic fields and particle energization



[Drake et al. 2006]

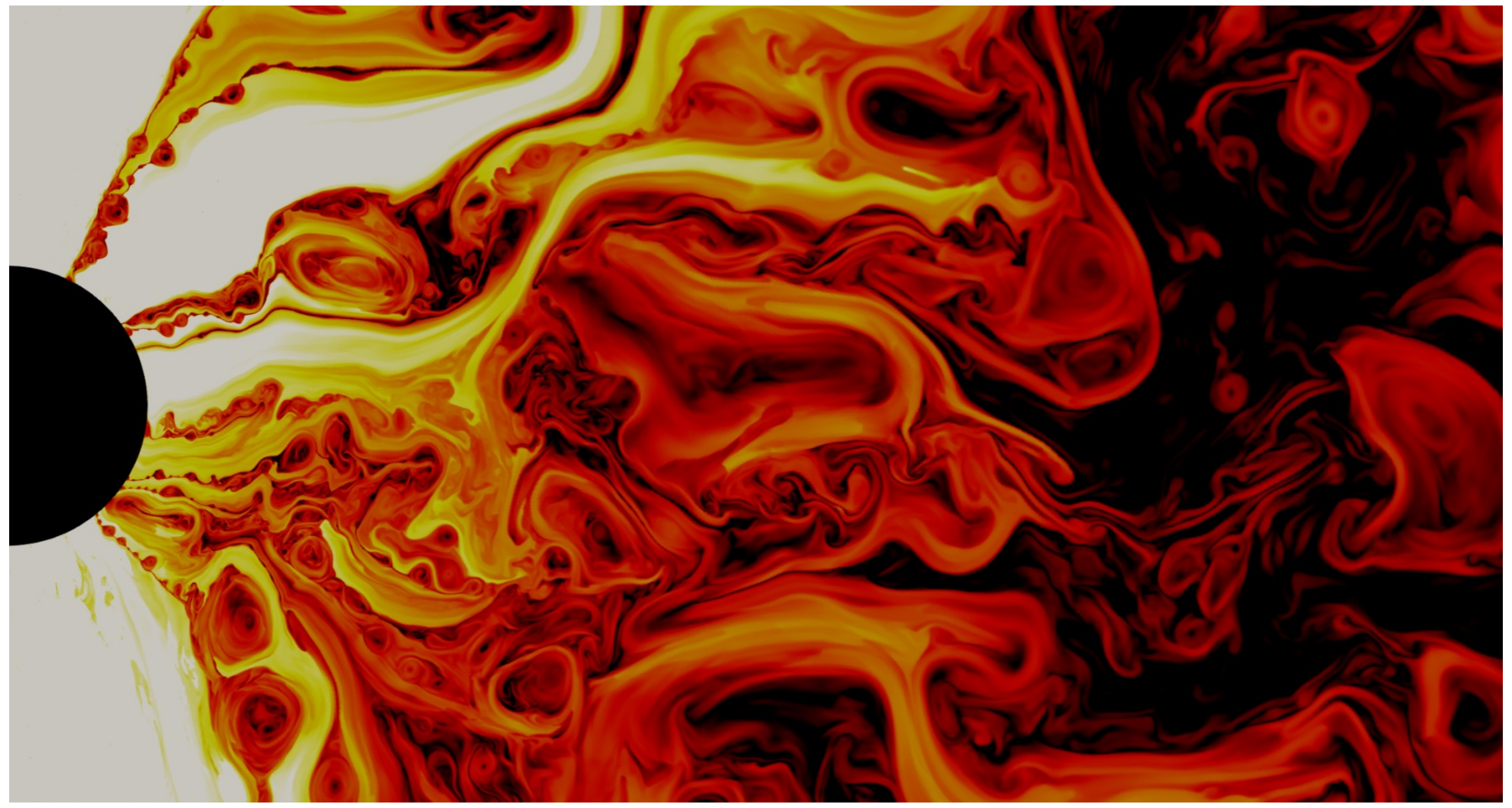
Particles are trapped and accelerated at/around plasmoids.



[Daughton et al. 2011]

Instabilities and complex dynamics of plasmoids drive turbulence.

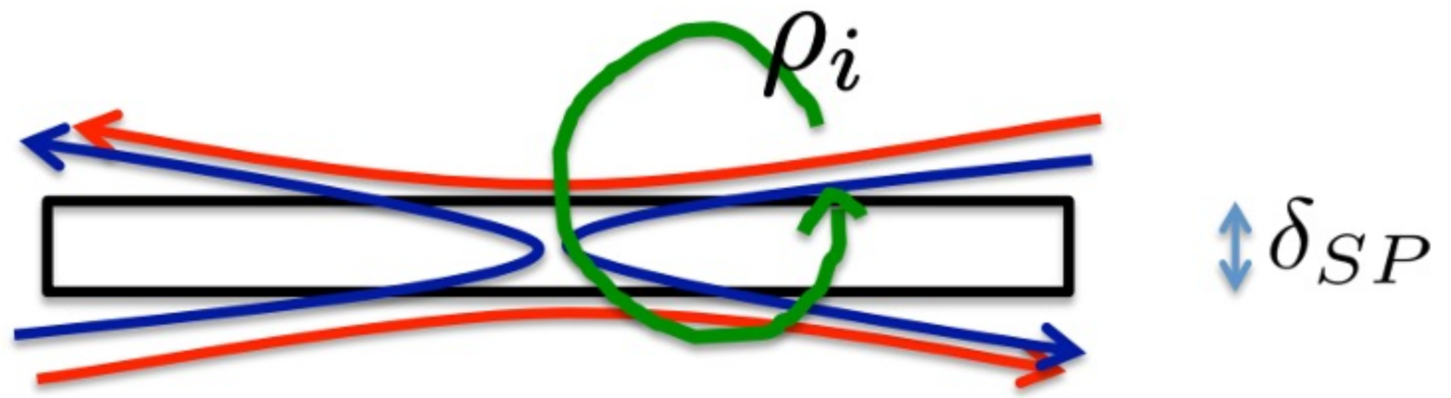
The formation of plasmoids changes the global dynamics of accretion disks



[Ripperda et al. 2019]

Collisionless reconnection

Non–ideal effects (hall effect, ambipolar diffusion, small-scale–turbulence, plasma instability) provide effective resistivity. Reconnection is fast with the rate $\epsilon_{rec} \sim 0.1$ → an ongoing popular research topic



Credit: N. Loureiro

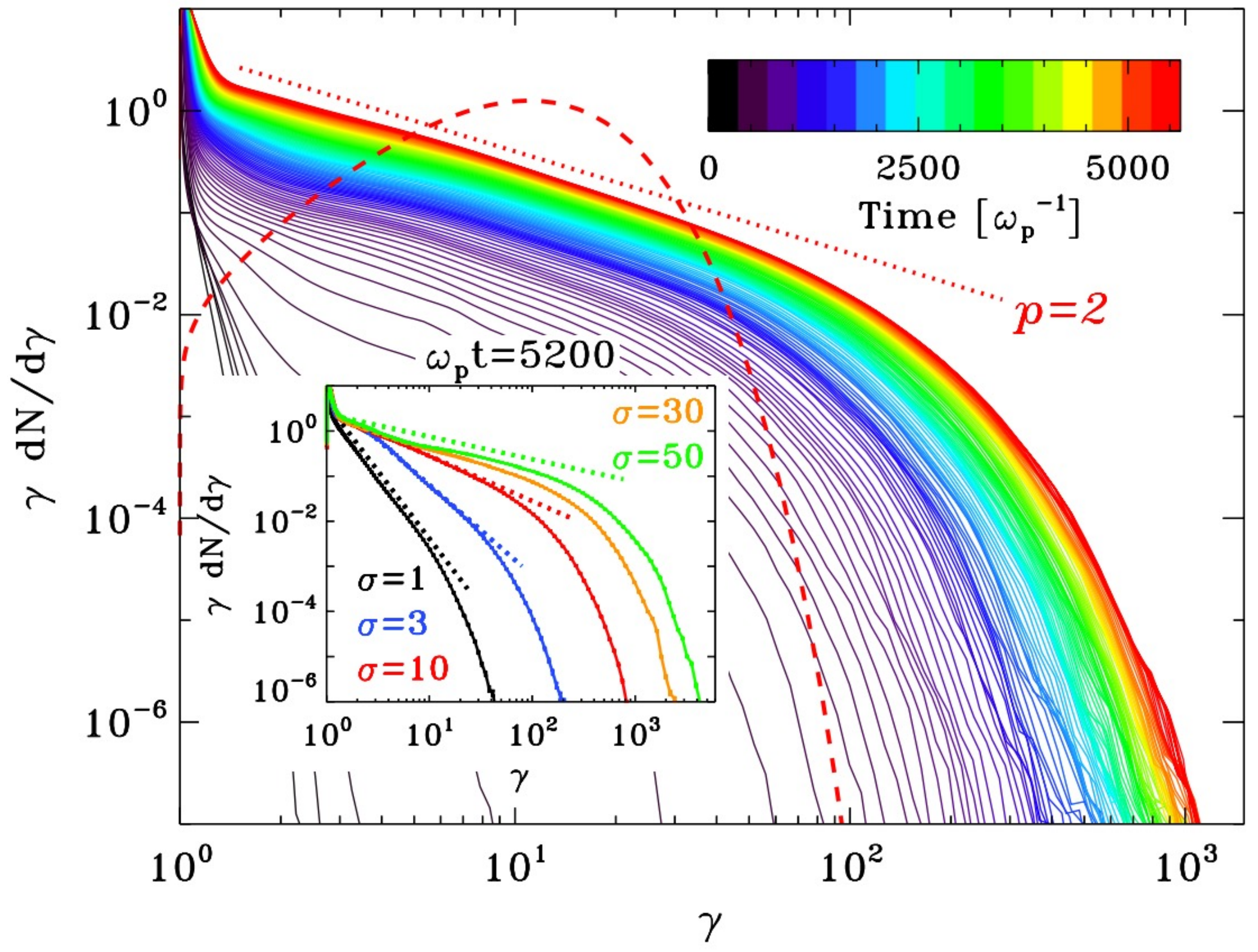
$$\mathbf{E} + \frac{\mathbf{v}_i \times \mathbf{B}}{c} = \eta \mathbf{j} + \frac{\mathbf{j} \times \mathbf{B}}{nec} - \frac{1}{ne} \nabla \cdot \mathbf{P}_e - \frac{m_e}{e} \frac{d\mathbf{v}_e}{dt}$$

Hall term; Whistler waves; c/ω_{pi} → does NOT break frozen flux

Electron pressure tensor; KAW; ρ_s → Break frozen-flux

Electron inertia; c/ω_{pe} → Break frozen-flux

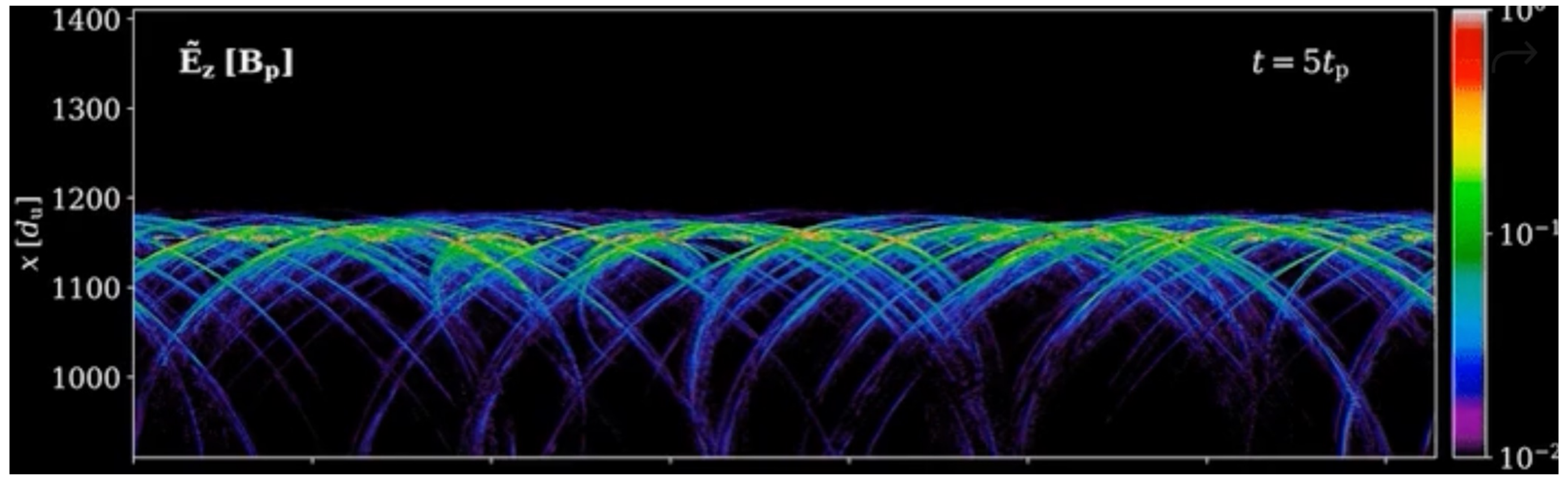
Reconnection in astrophysical phenomena



Reconnection is an efficient source of non-thermal particles.

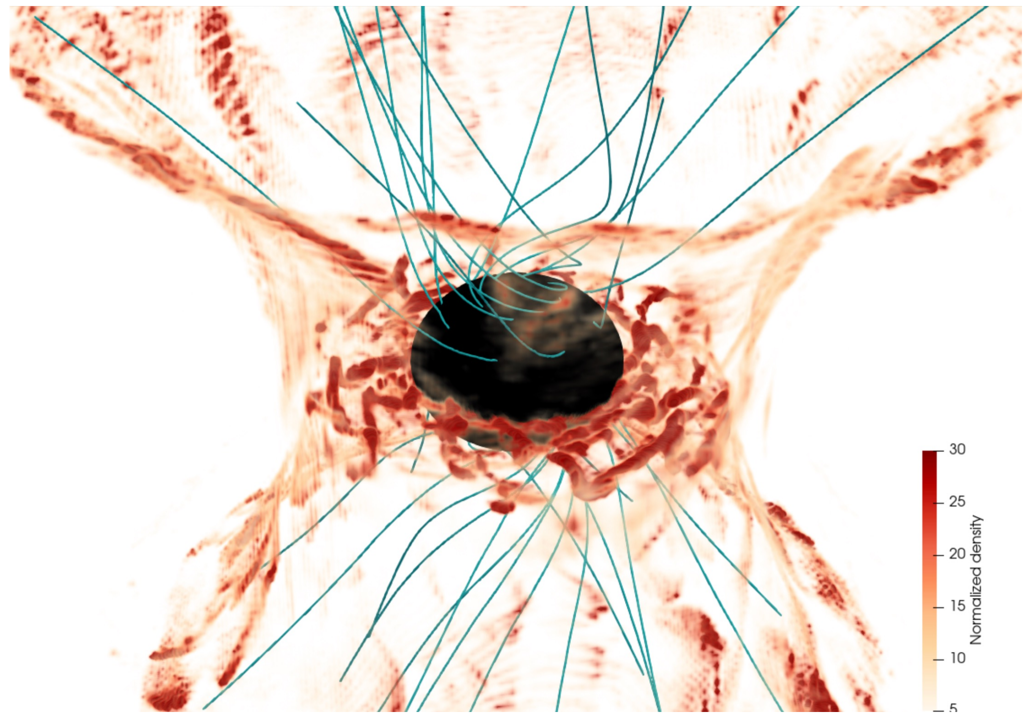
[e.g., Sironi et al. 2014]

Plasmoid-mediated production of Fast radio bursts (FRBs) Mahlmann+ 2022



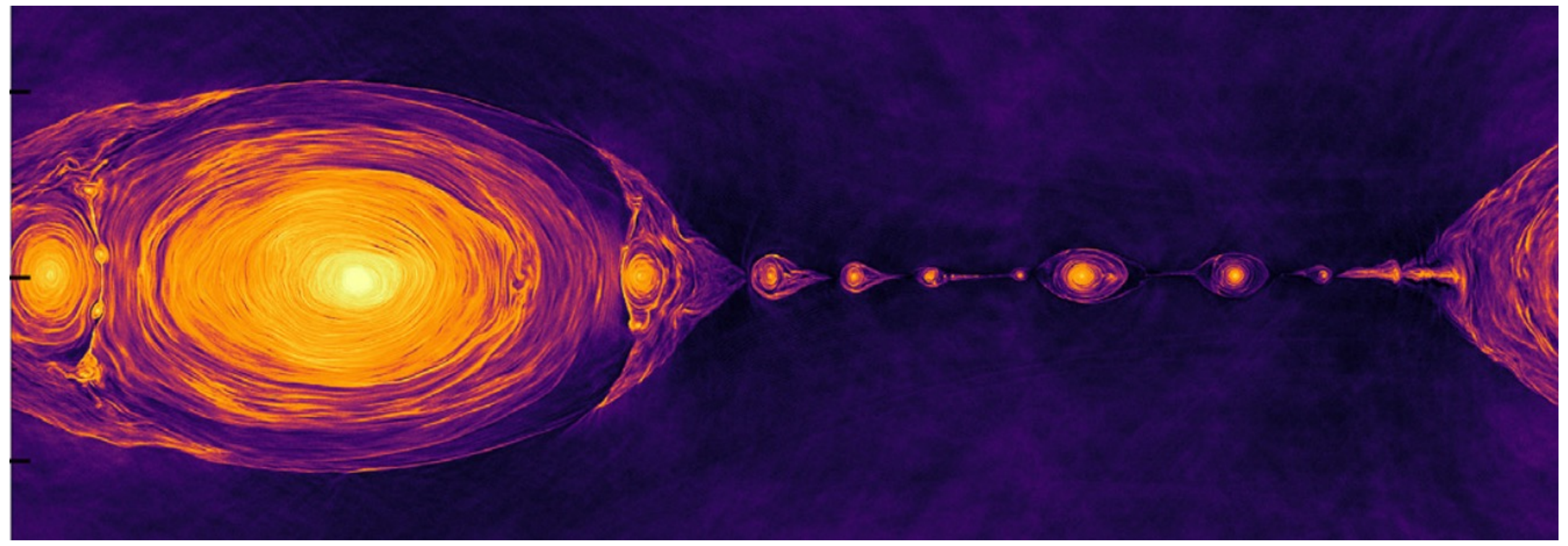
Reconnection around blackholes

Crinquand+2022

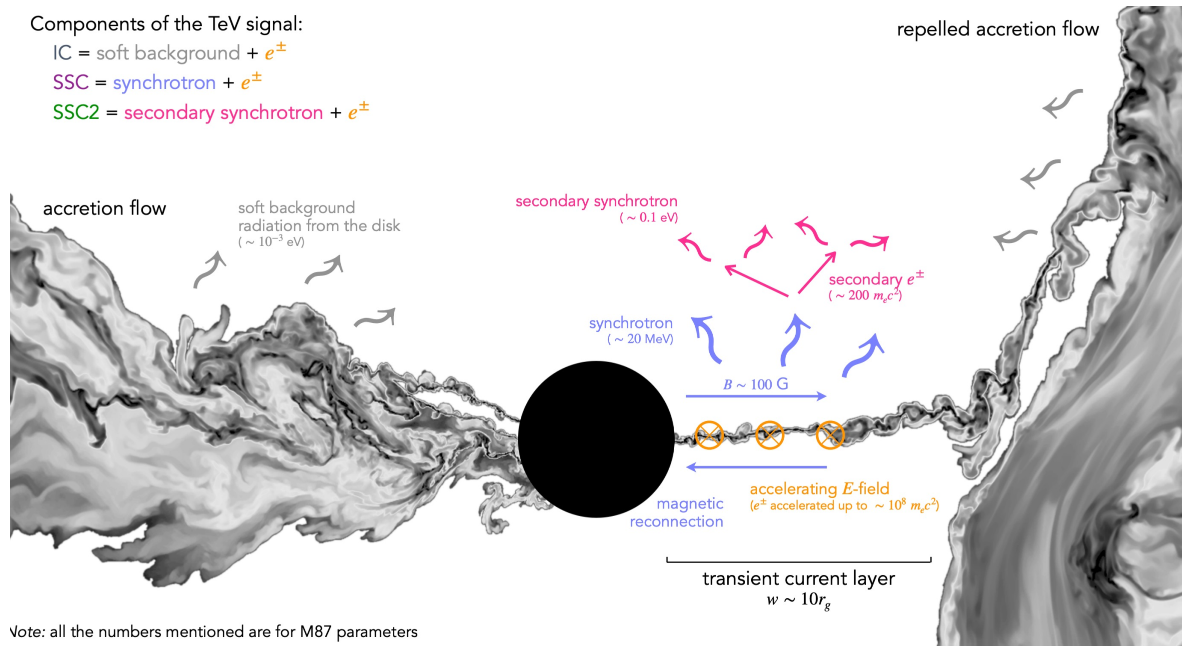


Radiative kinetic reconnection

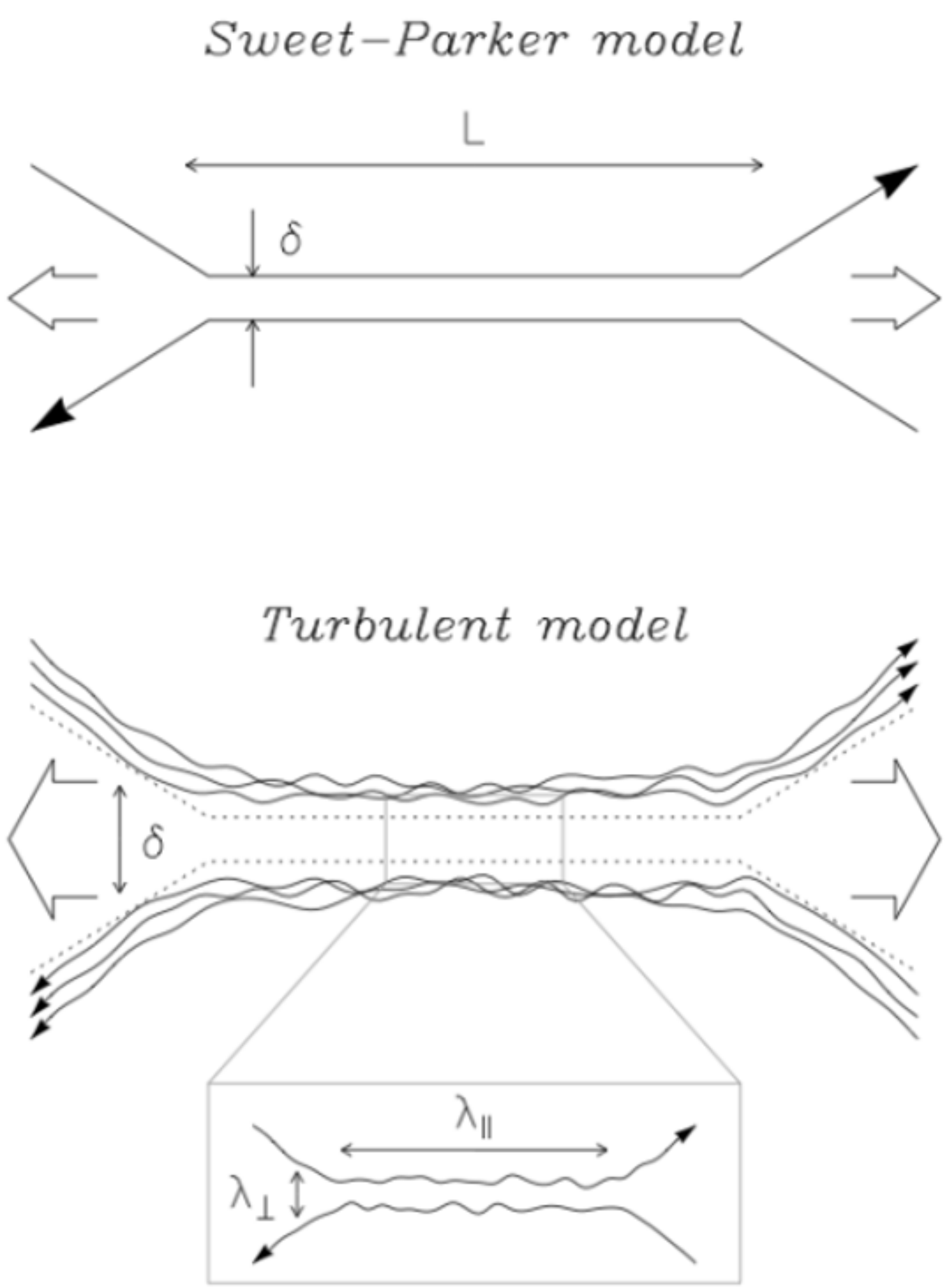
Werner+2018



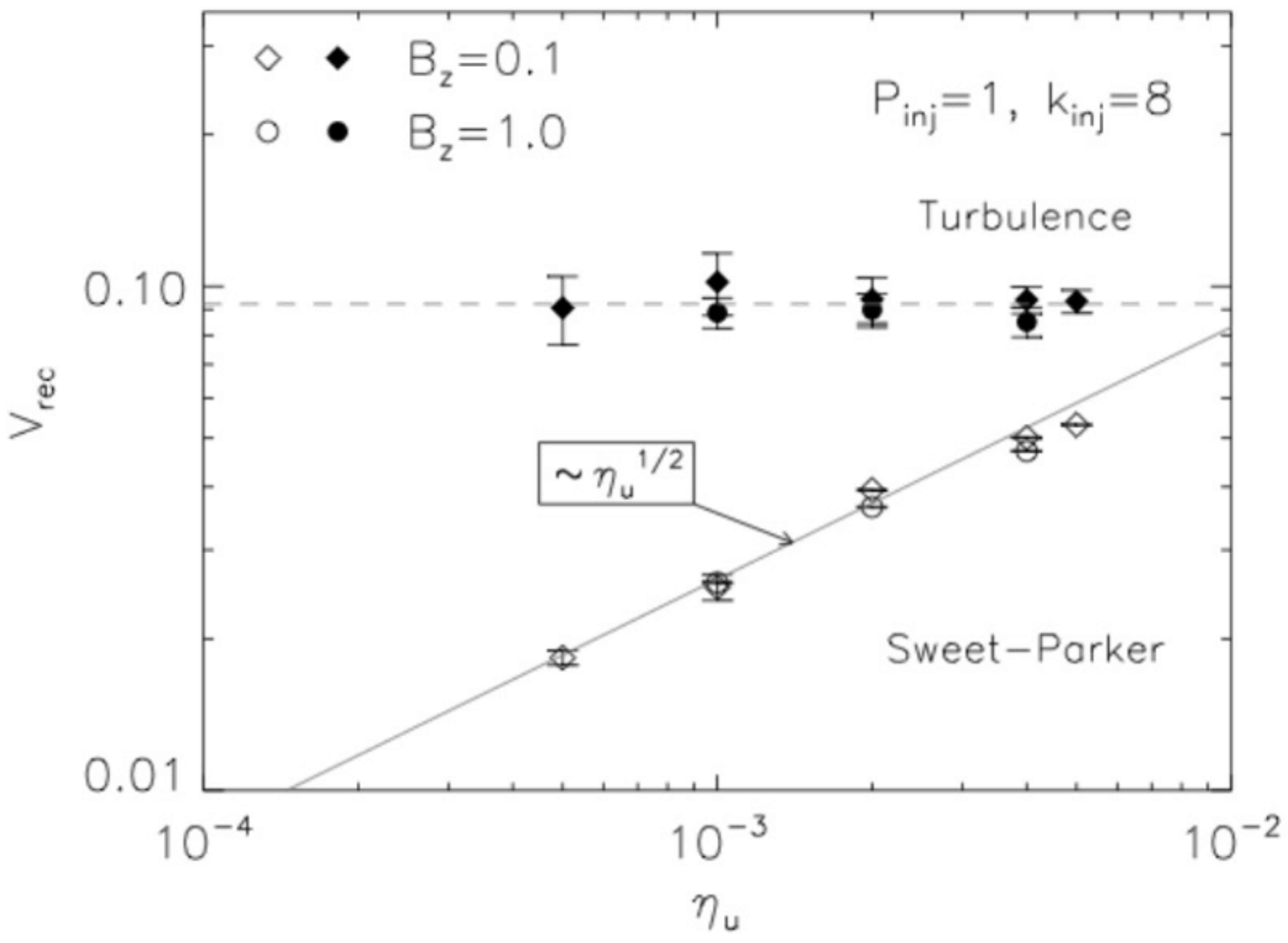
Reconnection-powered Tev flares around black holes



Interaction between reconnection and turbulence



Reconnection in a turbulent background



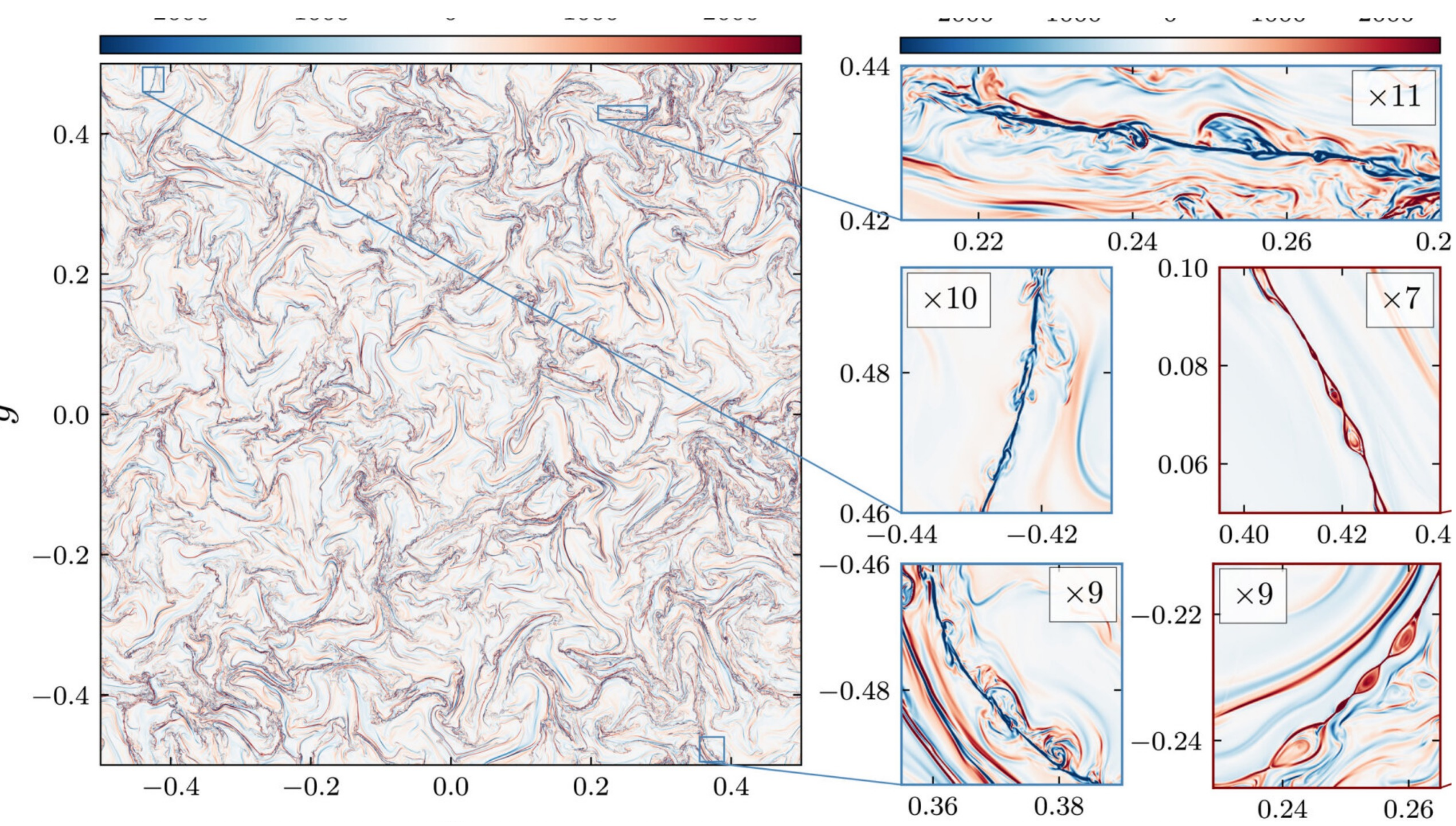
Thickness of the reconnection layer is determined by typical field line's wandering

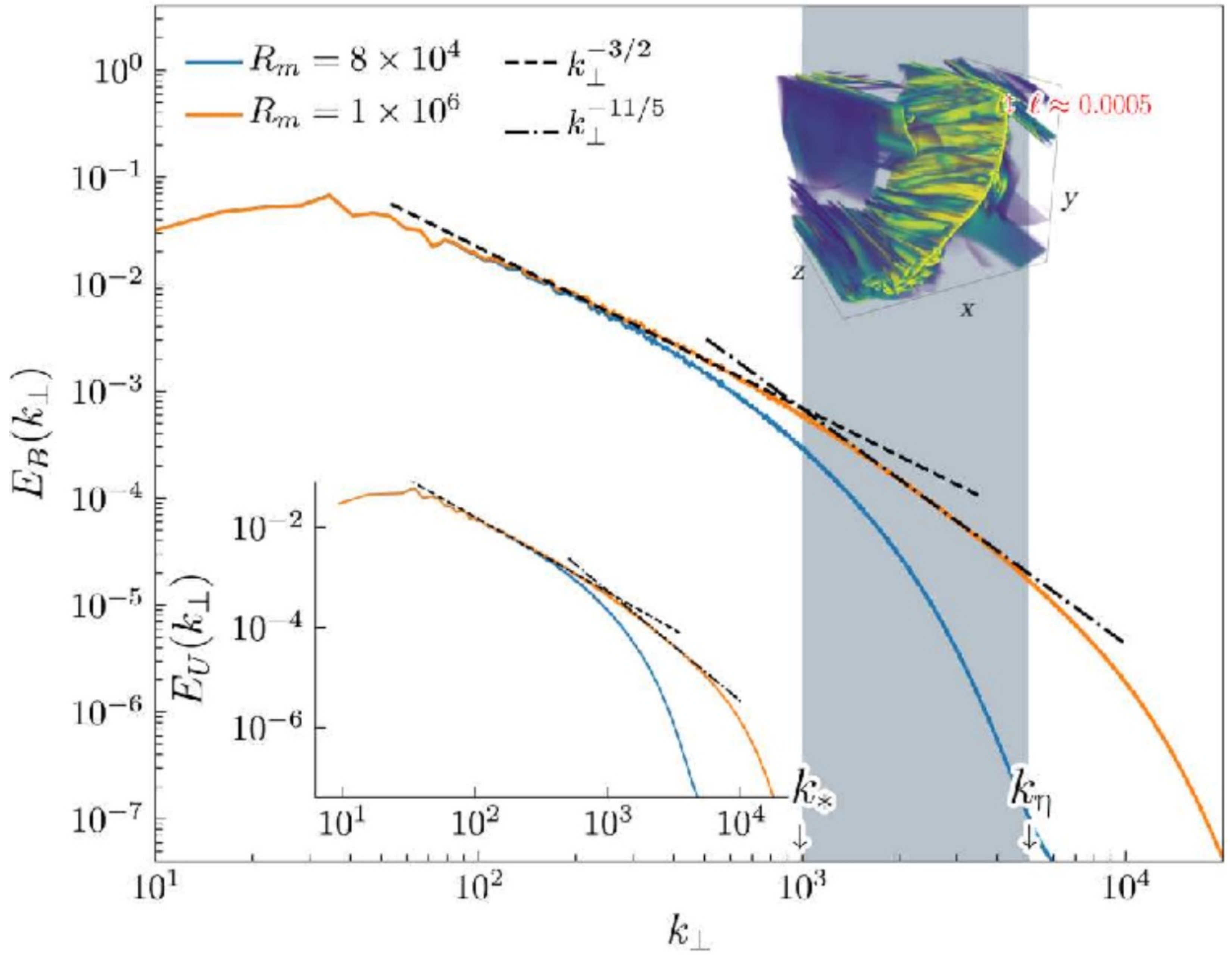
Lazarian & Vishniac 1999

Reconnection rate is close to 0.1 with the presence of the turbulent background

Kowal et al. 2009

Reconnection-mediated turbulence





Theory: Loureiro & Boldyrev 2017, 2018

Simulation: Dong+2018, 2023

Biggest simulation on turbulence confirms that the mediation of reconnection changes the turbulent spectrum

Some references

This presentation adapt materials available at:

Fitzpatrick's text book: https://farside.ph.utexas.edu/teaching/plasma/lectures/

Kunz's lecture notes: https://www.astro.princeton.edu/%7Ekunz/Site/AST521/AST521_lecture_notes_Kunz.pdf

Loureiro's presentation http://userpages.irap.omp.eu/%7Efrincon/houches/Loureiro.pdf

E. Tolman's presentation: https://elizabethtolman.com//wp-content/uploads/2020/11/aps-dpp-6.pdf

For further reading about magnetic reconnection:

Zweibel and Yamada (2009) https://ui.adsabs.harvard.edu/abs/2009ARA%26A..47..291Z/abstract

Yamada et al (2010) https://journals.aps.org/rmp/abstract/10.1103/RevModPhys.82.603

Zweibel and Yamada (2016) https://royalsocietypublishing.org/doi/10.1098/rspa.2016.0479