Plasma as a Fluid

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Grew up in Maryland

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What is a fluid?

- What words do we typically use to describe a fluid?
- What makes a fluid unique from a solid?
- What makes a fluid unique from a hand-full of particles?

Particle distribution functions track particle information (Where are the particles, what are they doing?)

Number density of particles

$$f(x, v, t)$$

At a location x | **With a velocity v** | **At a time t**

Examples:

1D Beam, Steady State $f(x, v_x, t)$

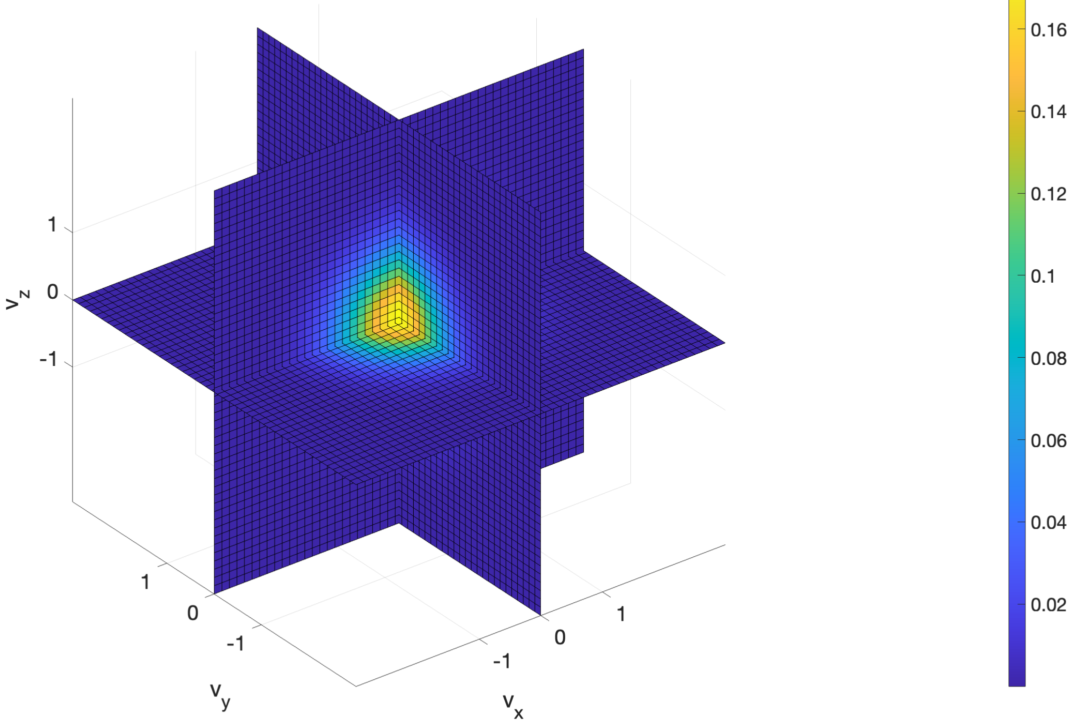
Glow Discharge $f(r, z, v_r, v_z, t)$

Tokamak Plasma Equilibrium $f(r, \theta, \phi, v_{\parallel}, v_{\perp})$

Reconnecting Plasma $f(x, y, z, v_x, v_y, v_z, t)$

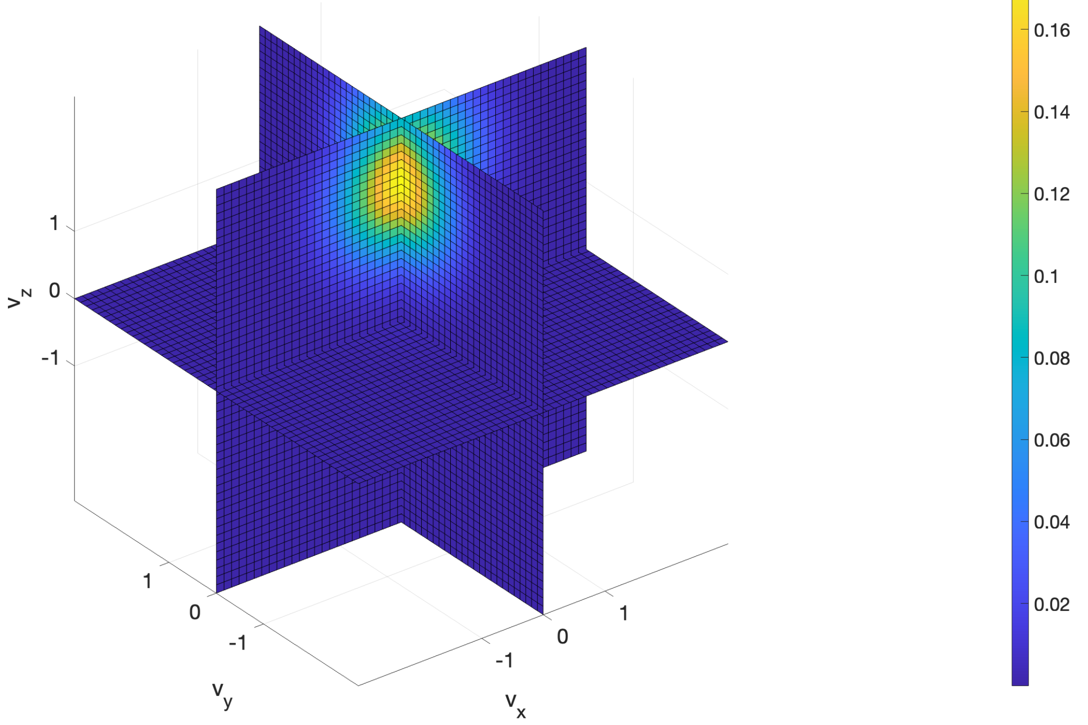
Example: 3D Maxwell-Boltzmann Velocity Distribution for a collection of particles in ***thermodynamic equilibrium***

$$f(v_x, v_y, v_z) = \left(\frac{m}{2\pi kT}\right)^{3/2} \exp\left(-\frac{m\left(v_x^2 + v_y^2 + v_z^2\right)}{2kT}\right)$$



Example: 3D Shifted Maxwell-Boltzmann Distribution (accelerated in the z-direction)

$$f(v_x, v_y, v_z) = \left(\frac{m}{2\pi kT}\right)^{3/2} \exp\left(-\frac{m\left(v_x^2 + v_y^2 + (v_z - 2)^2\right)}{2kT}\right)$$



Time-varying dynamics of a distribution function

$$\frac{d}{dt}f(t,x,v) = \frac{\partial f}{\partial t} + \frac{\partial f}{\partial x}\frac{dx}{dt} + \frac{\partial f}{\partial v}\frac{dv}{dt} = \frac{\partial f}{\partial t} + \frac{\partial f}{\partial x}v + \frac{\partial f}{\partial v}\frac{F}{m}$$

Velocity

Acceleration from *External* Forces

Interaction between particles can also lead to changes in the distribution function
e.g. thermal equilibration of a beam of particles interacting with each other

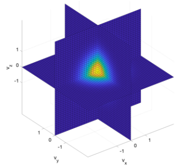
$$\frac{d}{dt}f(t,x,v) = \frac{\partial f}{\partial t} + v\frac{\partial f}{\partial x} + \frac{F}{m}\frac{\partial f}{\partial v} = \left(\frac{df}{dt}\right)_{coll}$$

Collision “Operator”
(*Internal* Forces)

Boltzmann Equation

With no collisions → Vlasov Equation

Calculating macroscopic properties from a distribution function



f(x,v,t) = Number density of particles

$$\frac{\partial f}{\partial t} + v\frac{\partial f}{\partial x} + \frac{F}{m}\frac{\partial f}{\partial v} = \left(\frac{df}{dt}\right)_{coll}$$

Velocity Moments

0th

Particle Density:

$$n(x,t) = \int 1\, dv\, f(x,v,t)$$

Conservation of Particles:

$$\int v^0 dv \left(\frac{\partial f}{\partial t} + v\frac{\partial f}{\partial x} + \frac{F}{m}\frac{\partial f}{\partial v}\right) = \int v^0 dv \left(\frac{df}{dt}\right)_{coll}$$

1st

Flow Velocity:

$$u(x,t) = \frac{1}{n}\int v^1\, dv\, f(x,v,t)$$

Conservation of Momentum:

$$\int v^1\, dv \left(\frac{\partial f}{\partial t} + v\frac{\partial f}{\partial x} + \frac{F}{m}\frac{\partial f}{\partial v}\right) = \int v^1\, dv \left(\frac{df}{dt}\right)_{coll}$$

2nd

Kinetic Energy:

$$Q(x,t) = \frac{1}{n}\frac{m}{2}\int v^2 dv\, f(x,v,t)$$

Conservation of Energy:

$$\frac{m}{2}\int v^2\, dv \left(\frac{\partial f}{\partial t} + v\frac{\partial f}{\partial x} + \frac{F}{m}\frac{\partial f}{\partial v}\right) = \frac{m}{2}\int v^2\, dv \left(\frac{df}{dt}\right)_{coll}$$

⋮

A phenomenological approach to conservation laws

- Pick an infinitesimally small volume of the fluid
- The time rate of change of quantity *X* in the volume must be equal to
- The net flux of *X* into the volume
- Plus the net sources of *X* inside the volume

$$\frac{\partial X}{\partial t} = -\nabla \cdot (X\,u) + (Volumetric\ Sources\ - Volumetric\ Sinks)$$

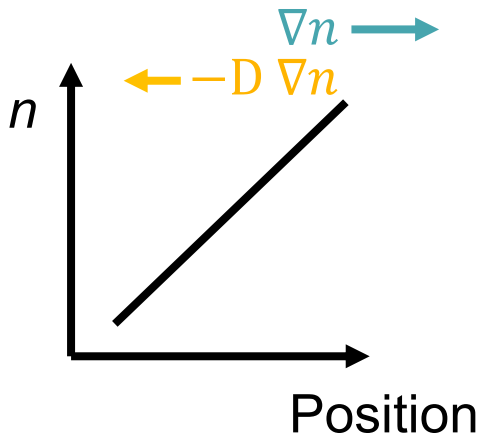
Diffusion coefficients describe flows induced by concentration gradients in the fluid

$$\frac{\partial X}{\partial t} = -\nabla \cdot \overset{\text{(Flux)}}{(X\,u)} + (Volumetric\ Sources\ - Volumetric\ Sinks)$$

$$X\,u = -\mathrm{D}\,\nabla X$$

Spatial Gradient in X

Diffusion Coefficient (Diffusivity)



Gradients in a fluid induce flows directed to suppress the gradient (e.g. density, velocity, temperature)

Conservation of Particles (Continuity Equation)

- For a given plasma volume, what sources/sinks of particles might exist?
  - Ionization
  - Chemical reactions
  - Ion recombination
  - Vacuum pumps (global sink)
  - etc.

Conservation of Momentum (Force Balance)

- For a given plasma volume, what sources/sinks of momentum might exist?
  - Electric field force
  - Magnetic field force
  - Gravitational force
  - Pressure gradient force
  - Friction force
  - etc.

Conservation of Energy

- For a given plasma volume, what sources/sinks of energy might exist?
  - Resistive heating power
  - Electromagnetic heating power
  - Particle beam heating power
  - Fusion reaction power
  - Radiated power (continuum, line)
  - etc.

How would you describe a hydrogen plasma as a fluid?

- Write out equations with words and/or symbols
- State any assumptions that you make
- Be as descriptive as you can!
- Hint: First write out what species are present (why?)
  - H_2, H_2^+, H^0, H^+, e^-

Free diffusion of a plasma: Part 1

- Suppose you have a plasma with a single ion species.
  - Derive an expression for the ratio between the velocity of an electron and an ion, assuming that you know their kinetic energies. (What equation do you need?)
    - **$E = (1/2)mv^2$ → $v_e/v_i = \mathrm{sqrt}(m_i/m_e) * \mathrm{sqrt}(E_e/E_i)$**
  - Calculate the value of this ratio when the ions and electrons have the same temperature (kinetic energy), and the ions are H^+.
    - **$E_e = E_i$ → $\mathrm{sqrt}(E_e/E_i) = 1$, $m_i/m_e \sim 1840$ → $\mathrm{sqrt}(m_i/m_e) \sim 43$, $v_e/v_i \sim 43$**
  - Calculate the value of this ratio for Ar^+ ions when $T_i = 0.025$ eV and $T_e = 5$ eV.
    - **$\mathrm{sqrt}(E_e/E_i) = 14$, $\mathrm{sqrt}(m_i/m_e) \sim 270$, $v_e/v_i \sim 3816$**
  - Calculate the value of this ratio in the limit $T_i << T_e$ ("cold ion" approximation).
    - **v_e/v_i → infinite!**
  - Describe what these tell you, in general, about the behavior of electrons vs. ions.

Free diffusion of a plasma: Part 2

- Suppose you drop a finite volume of plasma into an infinite space.
  - Describe what happens to the electrons versus the ions.
    - The electrons spread out quickly, leaving the ions behind.
  - Describe what happens as a result of this difference in behavior.
    - The charge separation between the fast electrons and the slow ions left produces an electric field.
  - Describe what must happen to reach a steady-state equilibrium.
    - The electric field will decelerate the electrons and accelerate the ions, until the rate of ions spreading out is equal to the rate of electrons spreading out.

Ambipolar diffusion

- Suppose there is no change in the momentum of the freely diffusing plasma

Electric Field Force; Pressure Gradient Force; Friction Force; Collision Frequency

$$mn\frac{du}{dt} = 0 = qnE \; - \mathrm{kT}\nabla n \; - mn\,\nu_m u$$

- The outward particle fluxes of ions and electrons can be written in terms of their particle "mobility" mu and "diffusivity" D

$$\Gamma = nu = \pm\mu nE \; - D\nabla n$$

$$\mu = \frac{|q|}{m\,\nu_m} \qquad D = \frac{kT}{m\,\nu_m}$$

Plasmas will self-organize to maintain quasi-neutrality!

- An electric field is generated, which slows down the electrons and speeds up the ions to achieve equal fluxes

$$\Gamma_i = \Gamma_e \qquad E = \frac{D_i - D_e}{\mu_i + \mu_e}\frac{\nabla n}{n}$$

- The effective diffusivity of ions and electrons is equal

$$\Gamma_i = \Gamma_e = \Gamma_a = -D_a\nabla n$$

$$D_i < D_a = \frac{\mu_i D_e + \mu_e D_i}{\mu_i + \mu_e} < D_e$$

Diffusion of a plasma across a toroidal magnetic field

Confinement Time ~ Diffusivity $^{-1}$

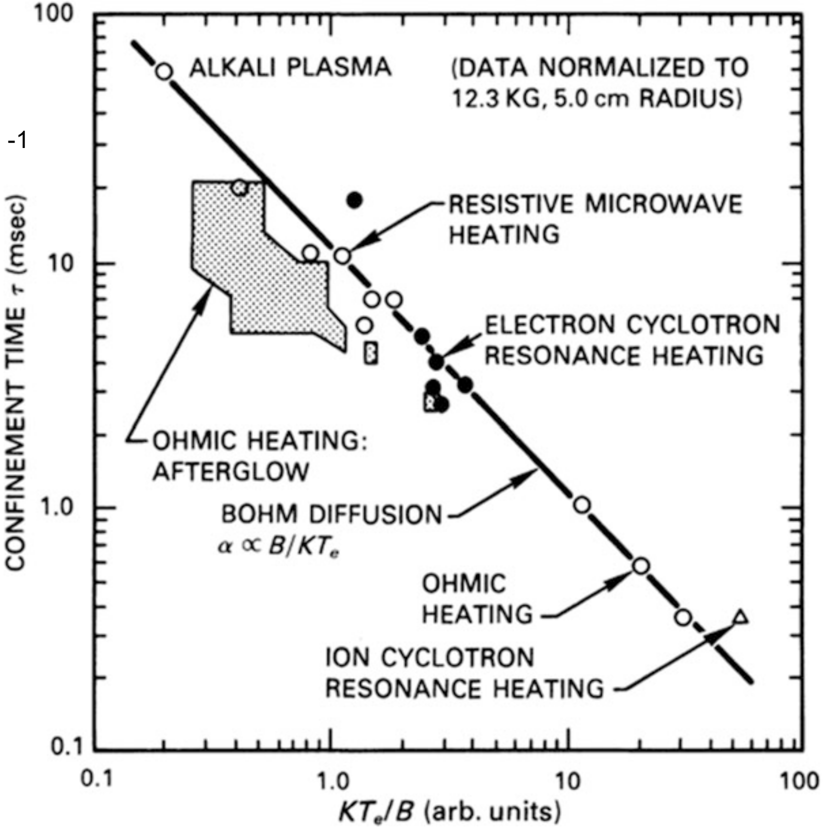


Fig. 5.20 from “Introduction to Plasma Physics and Controlled Fusion”, Chen

$T \uparrow \quad \tau \downarrow \quad D \uparrow$

$B \uparrow \quad \tau \uparrow \quad D \downarrow$

Diffusion mechanisms depend on plasma collisionality

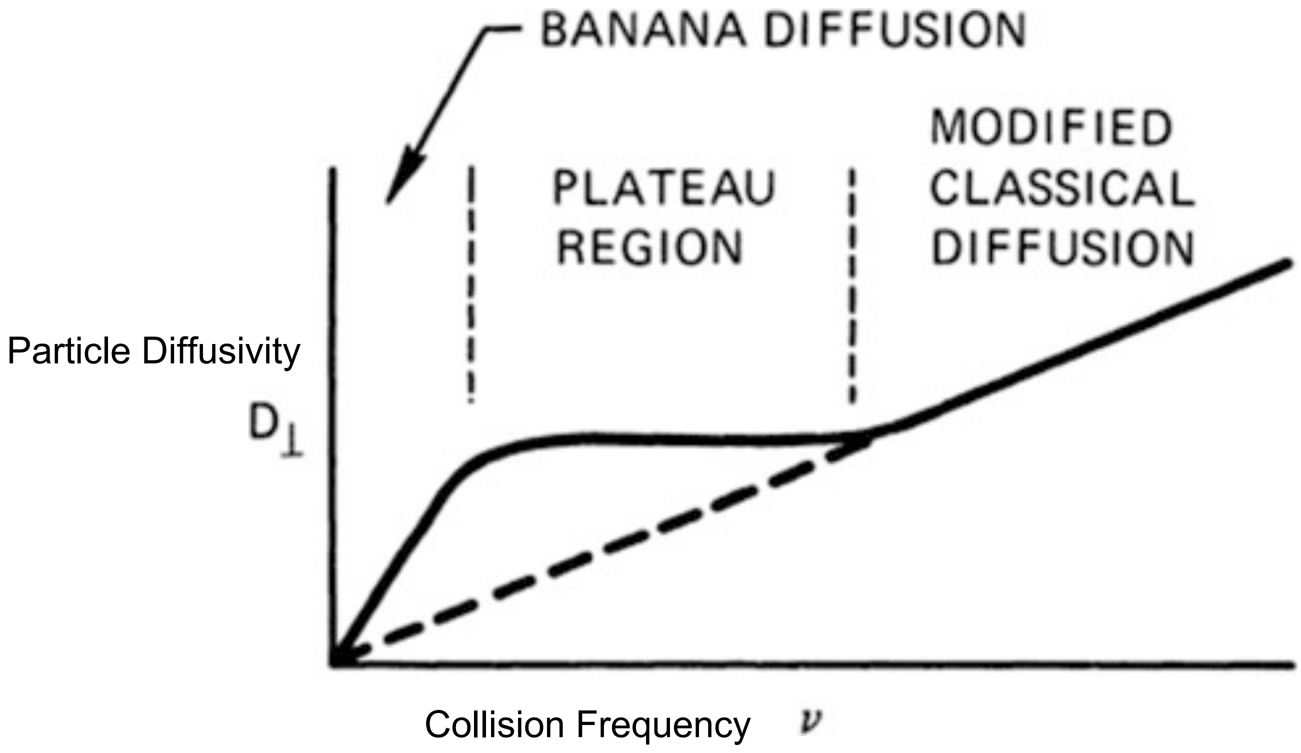
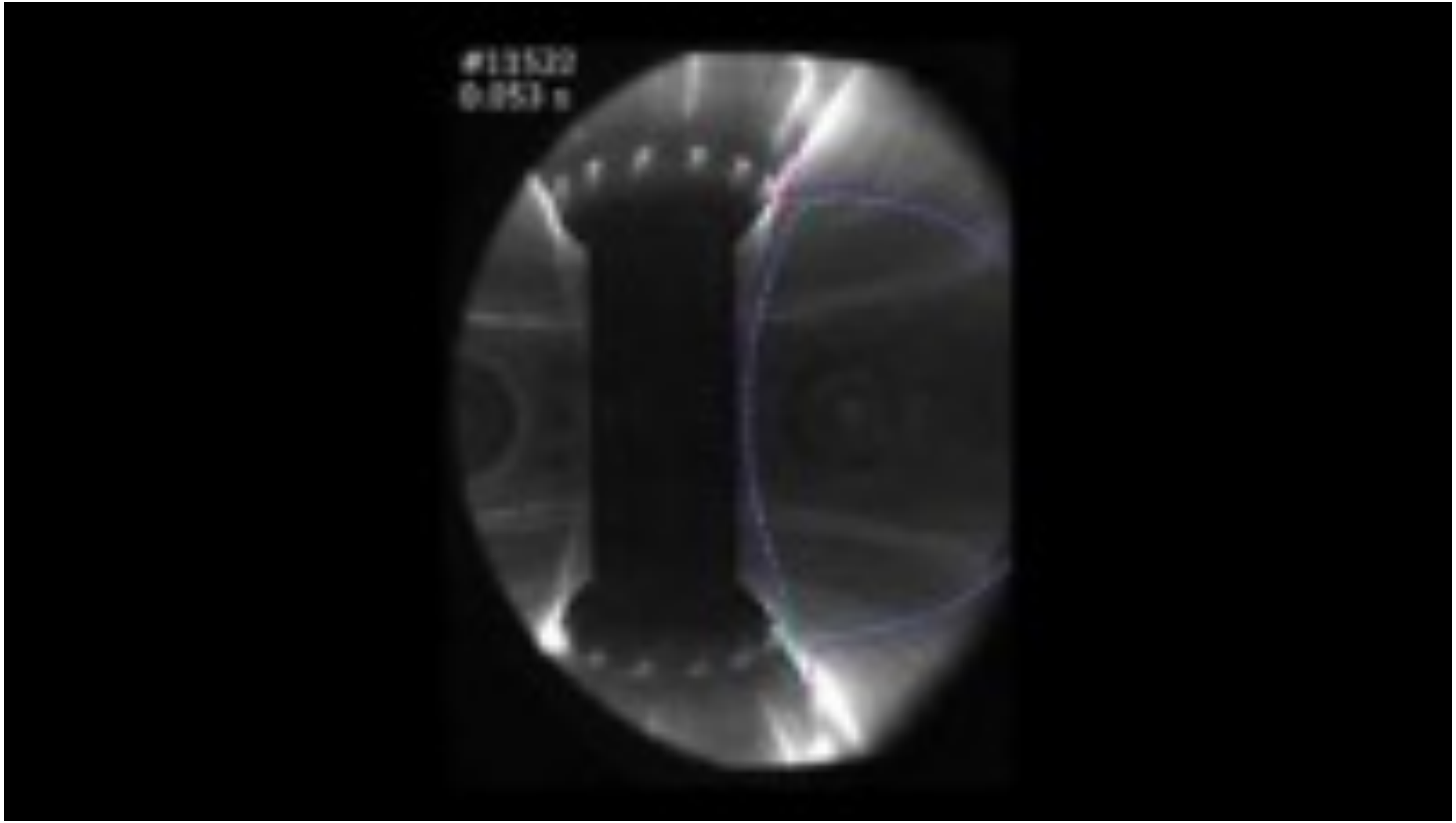


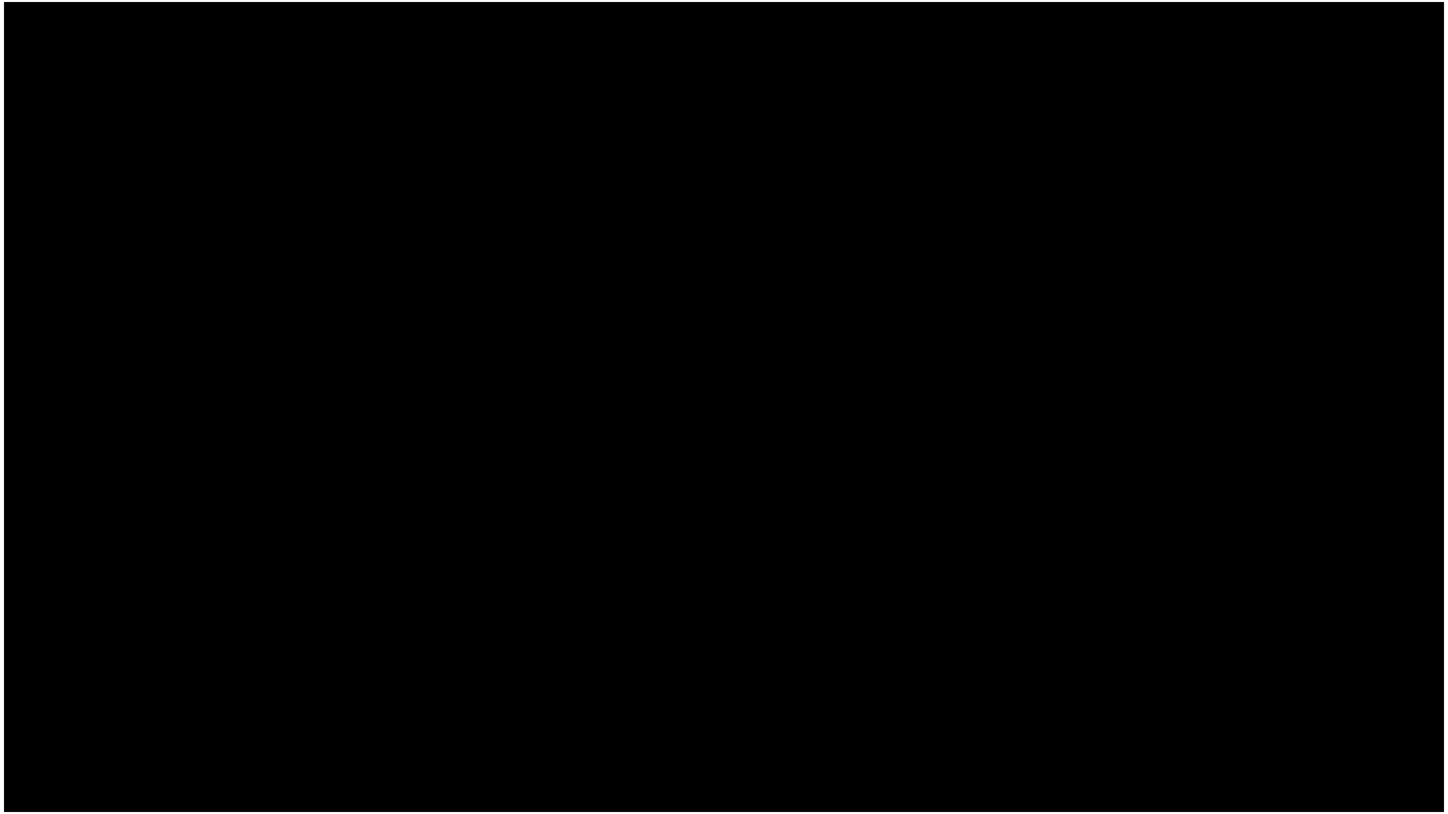
Fig. 5.22 from “Introduction to Plasma Physics and Controlled Fusion”, Chen

Examples of fluid dynamics in plasma/fusion research: Tokamak MagnetoHydroDynamic (MHD) Stability



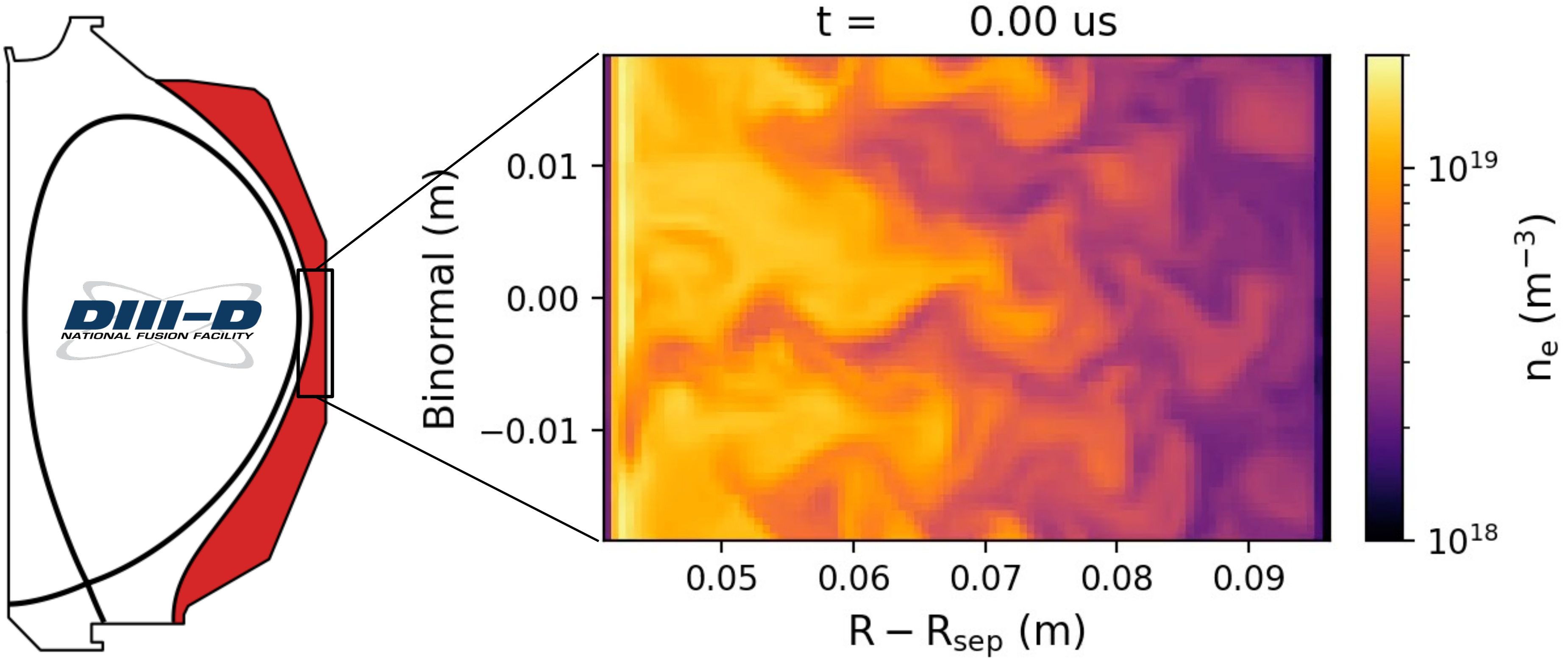
https://www.youtube.com/watch?v=PwknwUZdHWs

Examples of fluid dynamics in plasma/fusion research: Black Hole Accretion Disks



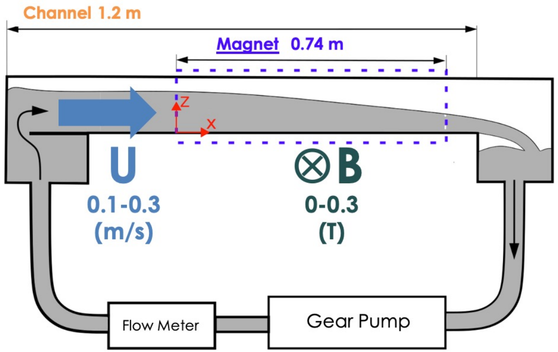
https://jila.colorado.edu/~pja/MRI_movies.html

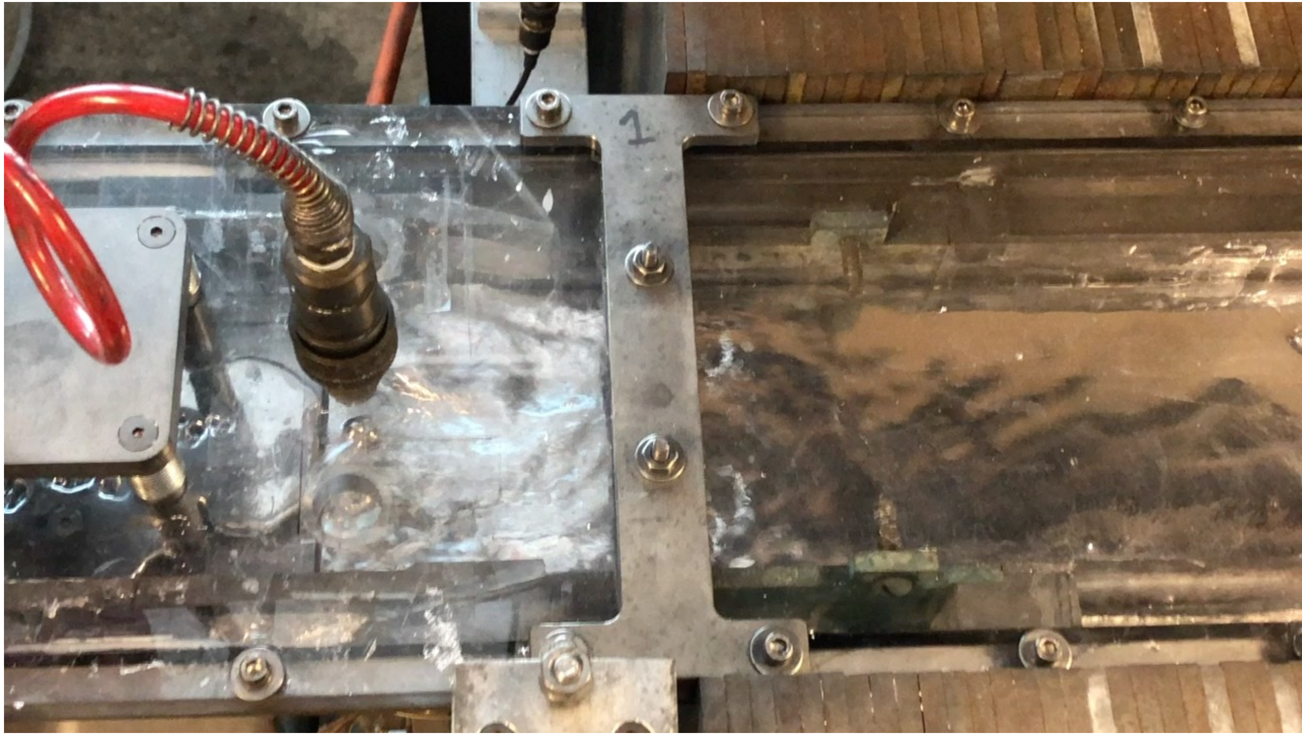
Examples of fluid dynamics in plasma/fusion research: Tokamak Scrape-off-Layer Transport



Courtesy of Shawn Zamperini, General Atomics

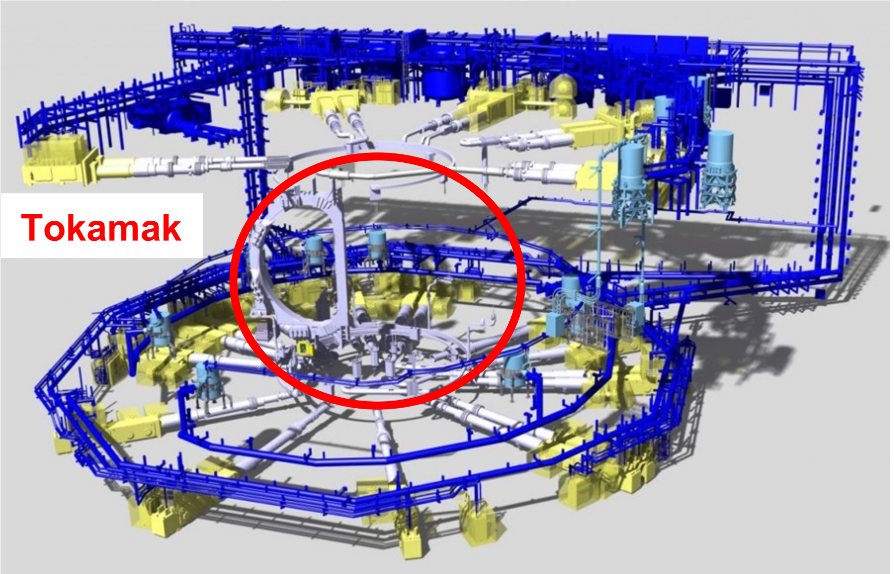
Examples of fluid dynamics in plasma/fusion research: Liquid Metal Plasma-Facing Surfaces

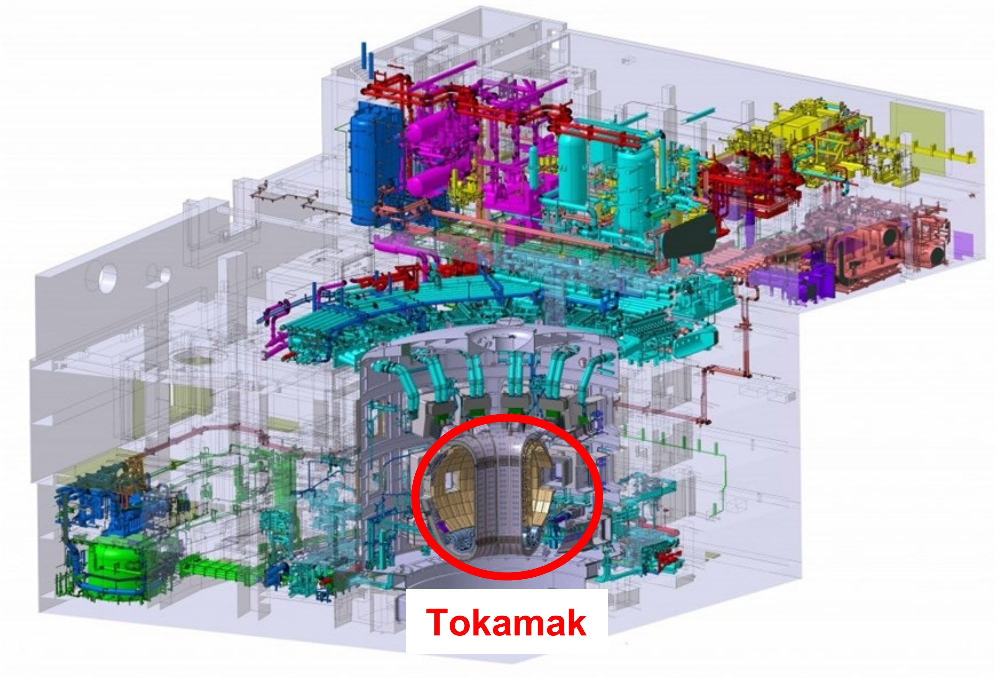




Courtesy of Egemen Kolemen, Princeton University

Examples of fluid dynamics in plasma/fusion research: Cryogenic and Water Cooling Systems for ITER





https://www.iter.org/mach/cryo

https://www.iter.org/mach/CoolingWater

For more on plasmas as fluids:

- Lieberman and Lichtenberg, Principles of Plasma Discharges and Materials Processing (2005)
- Stangeby, The Plasma Boundary of Magnetic Fusion Devices (2000)
- Freidberg , Plasma Physics and Fusion Energy (2007)
- Chen, Introduction to Plasma Physics and Controlled Fusion (2016)
- Hinton and Hazeltine, Reviews of Modern Physics, 48.2 (1976)