Introduction to Plasmas II: Single Particle Motion

PPPL PRINCETON PLASMA PHYSICS LABORATORY
Arturo Dominguez
Head, Science Education Department

Today's presentation owes a lot to many!

- This presentation is 95% the study of Lorentz Force:

$$\vec{F} = q\left(\vec{v} \times \vec{B} + \vec{E}\right)$$

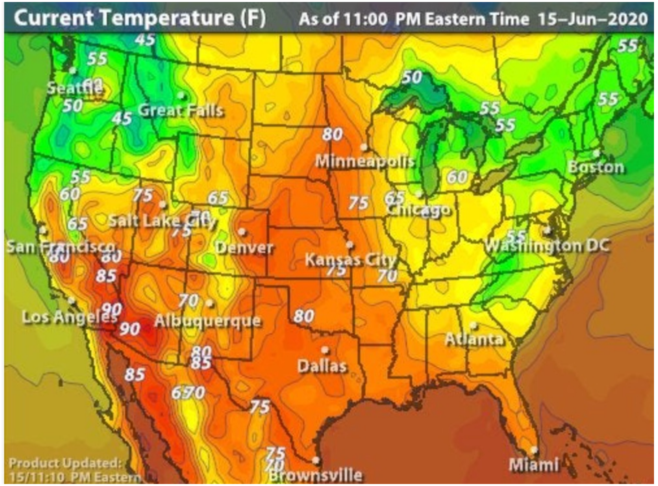
- Many of the slides and derivations today are from previous presentations, particularly Dr. Cami Collins' 2019 lecture on single particle motion.

Dr. Cami Collins
Group Lead, Advanced Tokamak Physics
Interim Section Head, Burning Plasmas
Oak Ridge National Laboratory

Super quick math review: fields

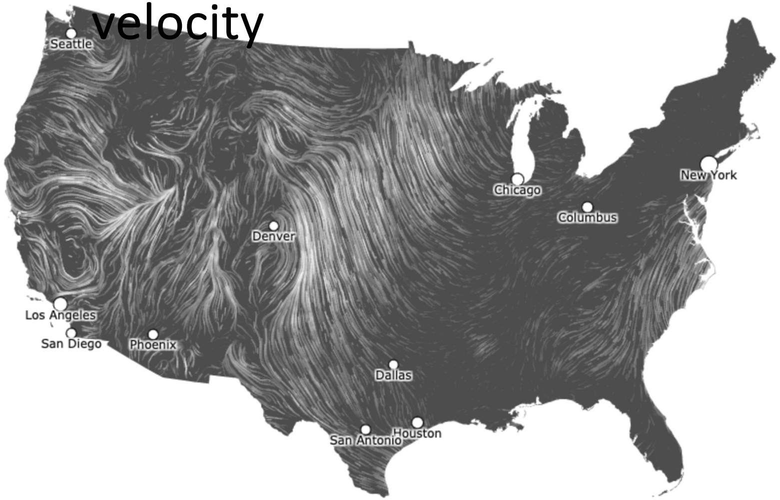
- Scalar fields: Every point in space is associated with a given value.

For example: Temperature



- Vector fields: Every point in space is associated with a given vector.

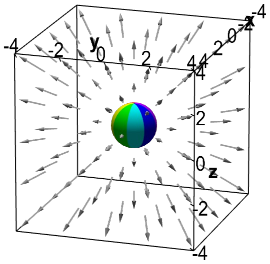
For example: Wind velocity



Super quick math review: Vector calculus

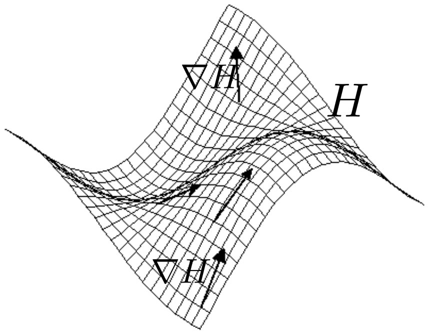
Divergence

$$\nabla \cdot \vec{V} > 0$$



$$\nabla H$$

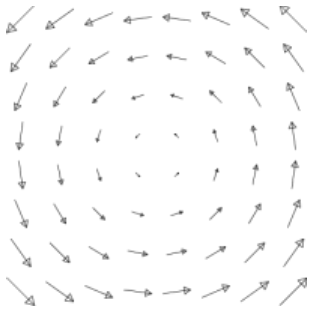
Gradient

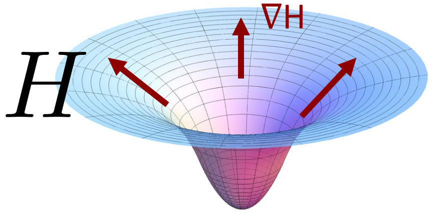


Curl

$$\nabla \times \vec{V} \neq 0$$

$$\odot$$



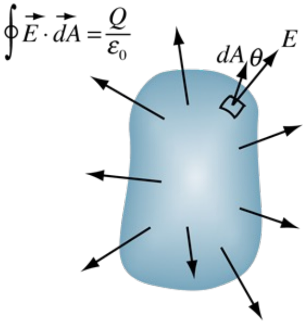


Laplacian

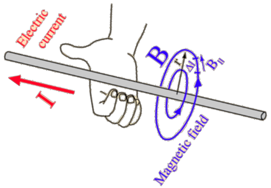
$$\nabla \cdot (\nabla H) = \nabla^2 H > 0$$

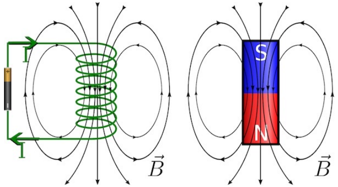
Gospel according to Maxwell

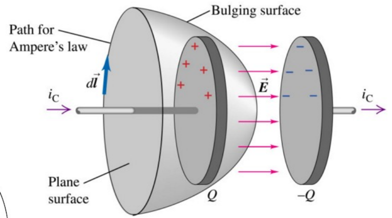
$$\nabla \cdot \vec{E} = \frac{\rho}{\epsilon_0}$$



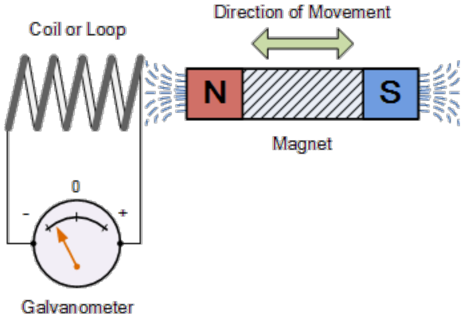
$$\nabla \times \vec{B} = \mu_0 \vec{J} + \frac{1}{c^2}\frac{d\vec{E}}{dt}$$



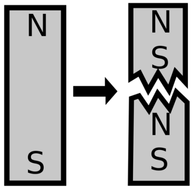


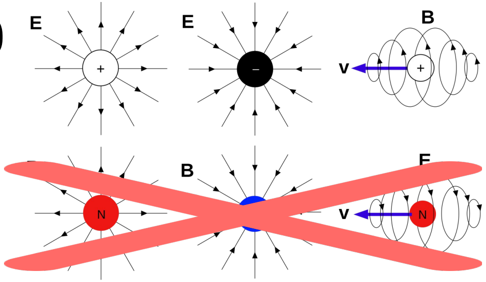


$$\nabla \times \vec{E} = -\frac{d\vec{B}}{dt}$$



$$\nabla \cdot \vec{B} = 0$$





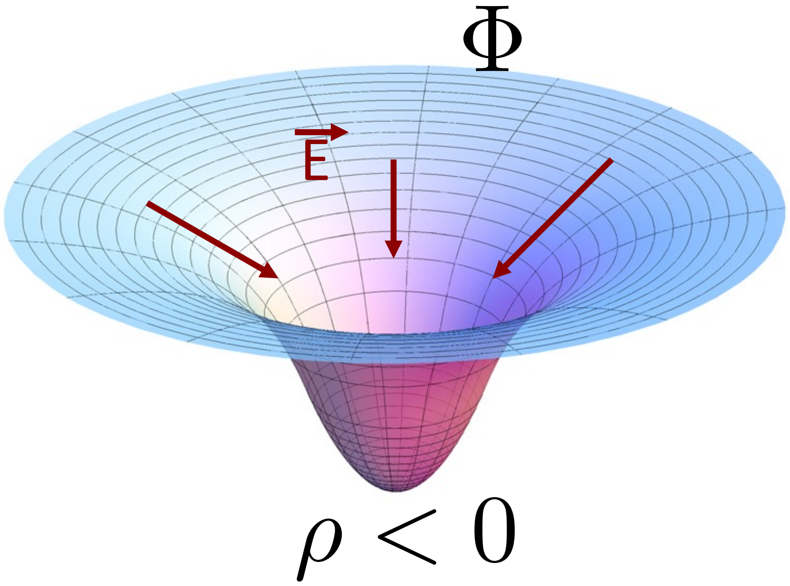
Electric potential and Poisson's equation

$$\vec{E} = -\nabla\Phi$$

$$\nabla \cdot \vec{E} = \frac{\rho}{\epsilon_0}$$

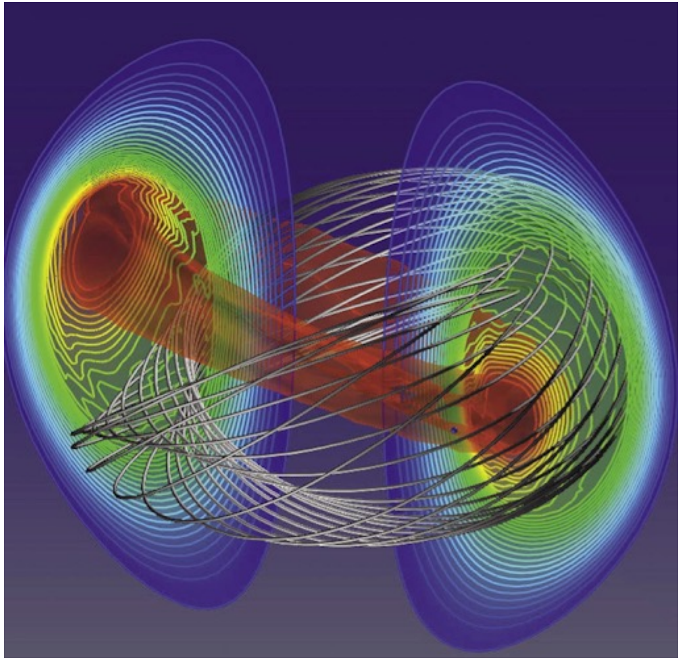
$$\nabla \cdot (-\nabla\Phi) = \frac{\rho}{\epsilon_0}$$

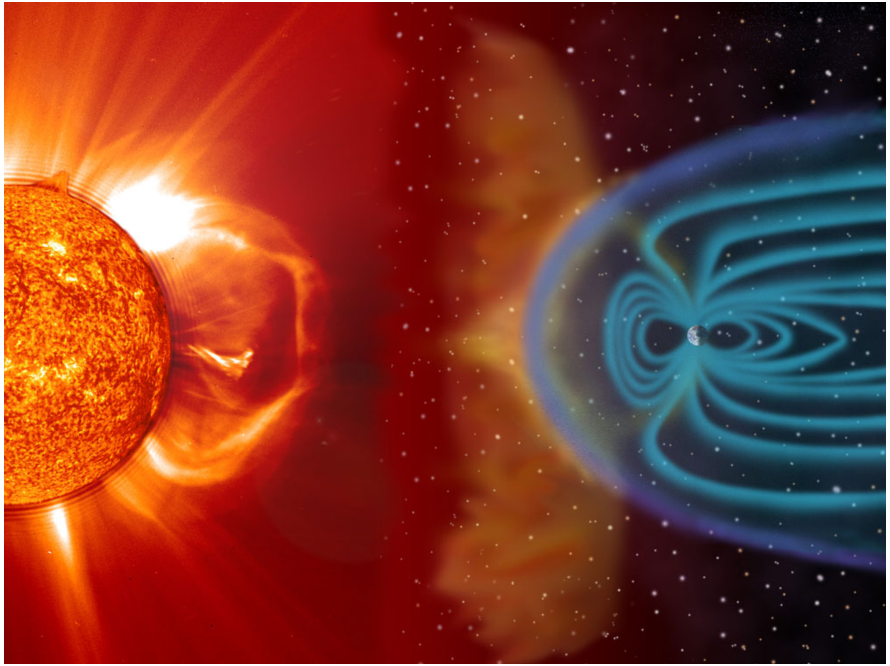
$$\nabla^2\Phi = -\frac{\rho}{\epsilon_0}$$



Potential energy of a charge in an electric potential = $q\Phi$

Single Particle Motion -> Magnetic Confinement





Roadmap

- Plasma in a Magnetic Field
- Magnetic Mirrors
- Plasma in a Magnetic + Electric field
- Generalized drift
- Drifts in a curved geometry
- Toroidal confinement
- Tokamaks and Stellarators
- …and Beyond

Roadmap

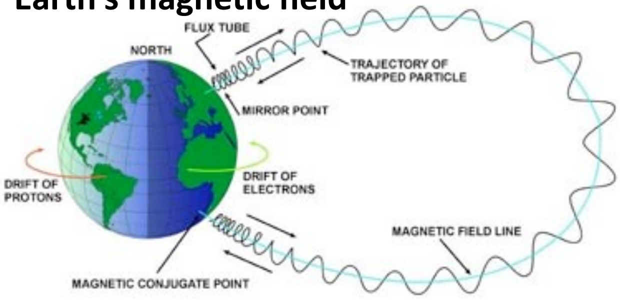
- Plasma in a Magnetic Field
- Magnetic Mirrors
- Plasma in a Magnetic + Electric field
- Generalized drift
- Drifts in a curved geometry
- Toroidal confinement
- Tokamaks and Stellarators
- …and Beyond

Roadmap

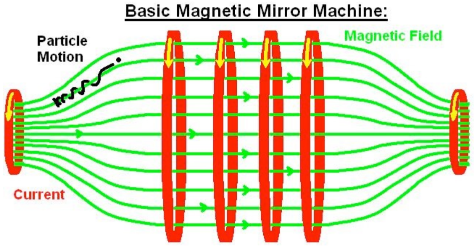
- Plasma in a Magnetic Field
- Magnetic Mirrors
- Plasma in a Magnetic + Electric field
- Generalized drift
- Drifts in a curved geometry
- Toroidal confinement
- Tokamaks and Stellarators
- …and Beyond

Examples of Mirrors

Charged particles can be trapped by Earth's magnetic field

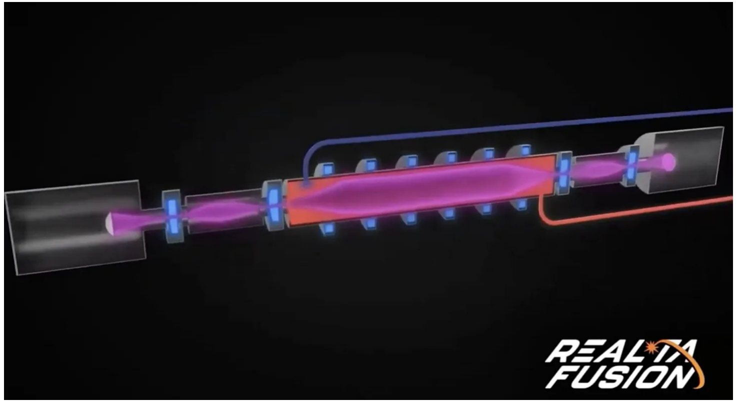


Early Fusion Experiments
Tandem Mirror Experiment (LLNL,1980's)



Particles with enough $v_{||}$ can still escape

Realta Fusion is trying to make fusion energy using a modified mirror



Realta uses HTS magnets and tandem mirrors to maximize confinement

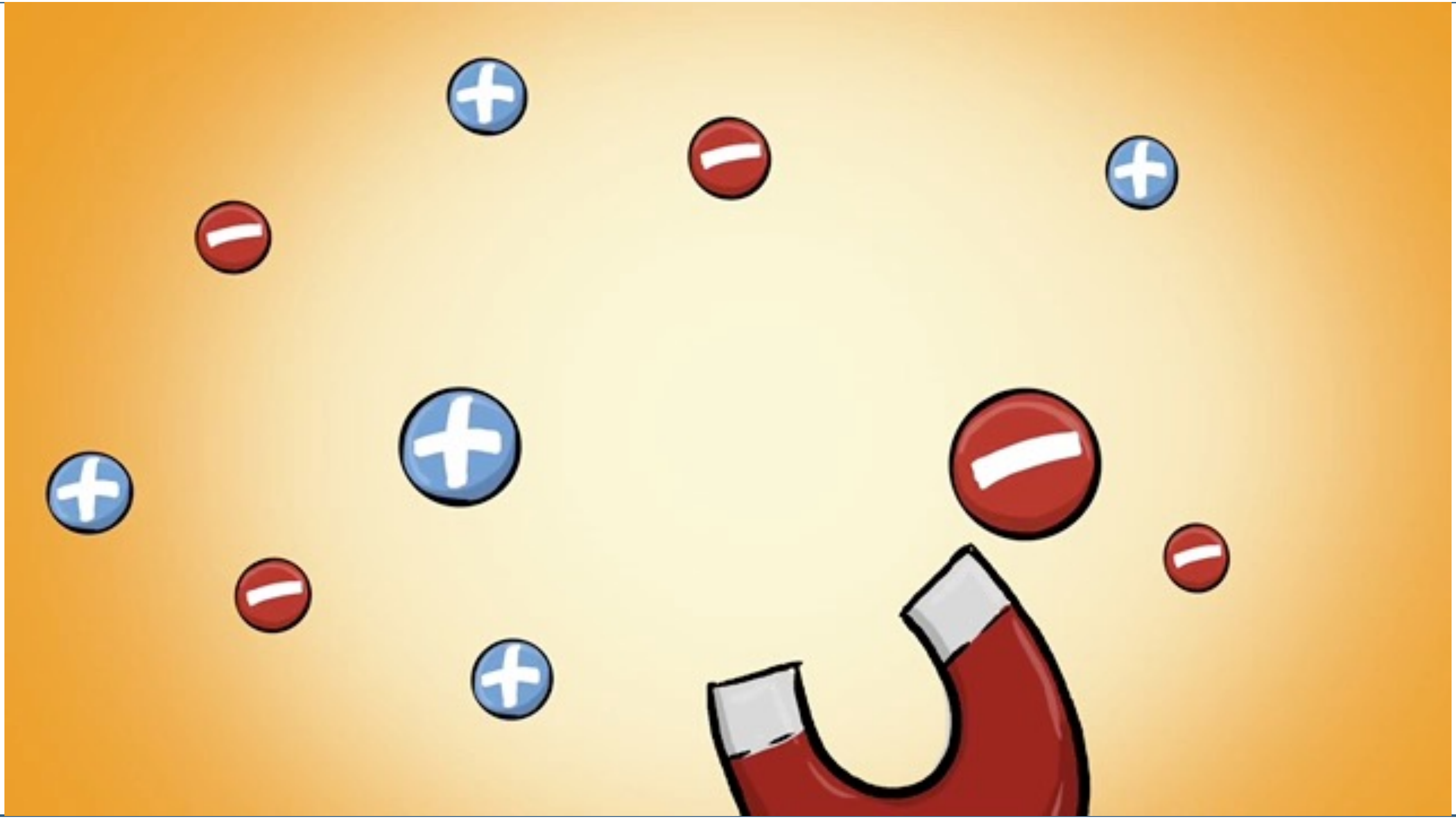
Roadmap

- Plasma in a Magnetic Field
- Magnetic Mirrors
- Plasma in a Magnetic + Electric field
- Generalized drift
- Drifts in a curved geometry
- Toroidal confinement
- Tokamaks and Stellarators
- …and Beyond

Roadmap

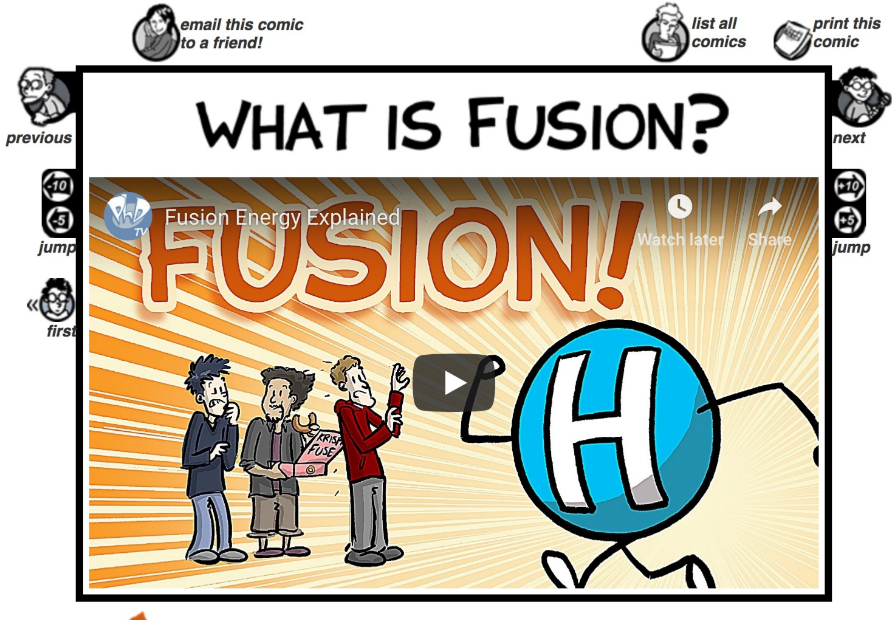
- Plasma in a Magnetic Field
- Magnetic Mirrors
- Plasma in a Magnetic + Electric field
- Generalized drift
- Drifts in a curved geometry
- Toroidal confinement
- Tokamaks and Stellarators
- …and Beyond

How about a B that bites its own tail?



How about a B that bites its own tail?

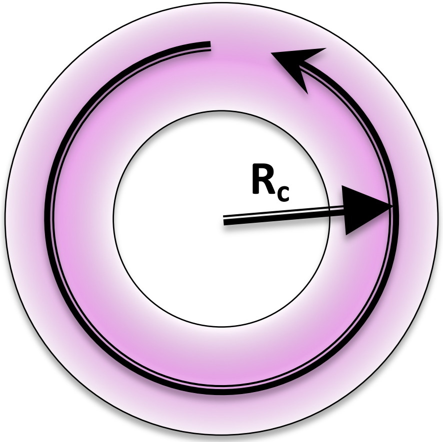
BTW: I'M A CARTOON CHARACTER!!



Roadmap

- Plasma in a Magnetic Field
- Magnetic Mirrors
- Plasma in a Magnetic + Electric field
- Generalized drift
- Drifts in a curved geometry
- Toroidal confinement
- Tokamaks and Stellarators
- …and Beyond

Particles feel a "centrifugal" force in their frame of reference

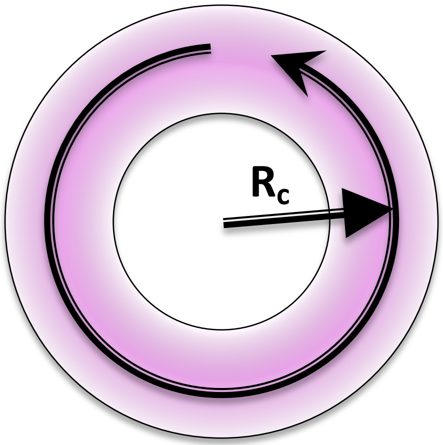


- Bend the magnetic field into a donut shape
- No end losses because the field lines go around and close on themselves
- BUT a particle following a toroidal magnetic field would experience $\mathbf{F}_{\mathbf{cf}}$

$$\mathbf{F}_{\mathbf{cf}} = \frac{m v_{\parallel}^2}{R_c}\hat{\mathbf{r}}$$

centrifugal

Particles feel a "centrifugal" force in their frame of reference



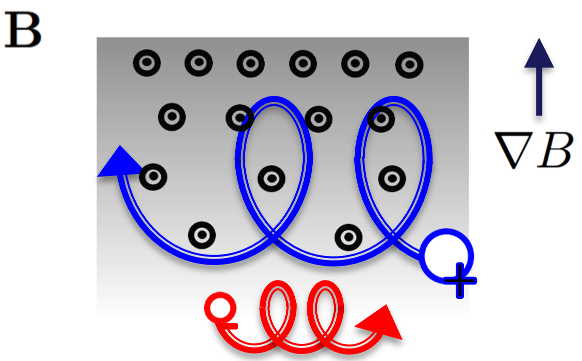
$$\mathbf{F_{cf}} = \frac{mv_\parallel^2}{R_c}\hat{\mathbf{r}}$$

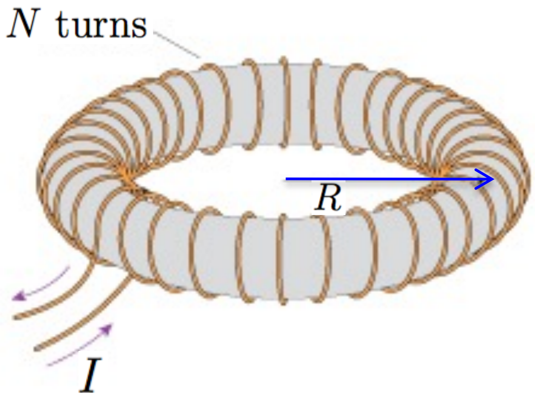
centrifugal

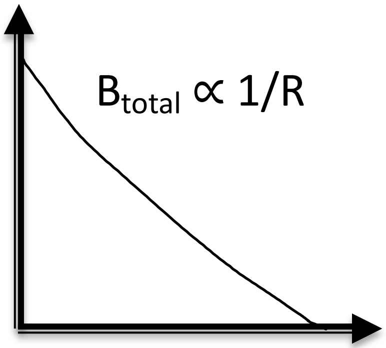
$$\mathbf{V_R} = \frac{mv_\parallel^2}{qB^2}\frac{\mathbf{R_c} \times \mathbf{B}}{R_c^2}$$

As compared to ExB, this drift is sign dependent

Another source of drift is the varying magnetic field





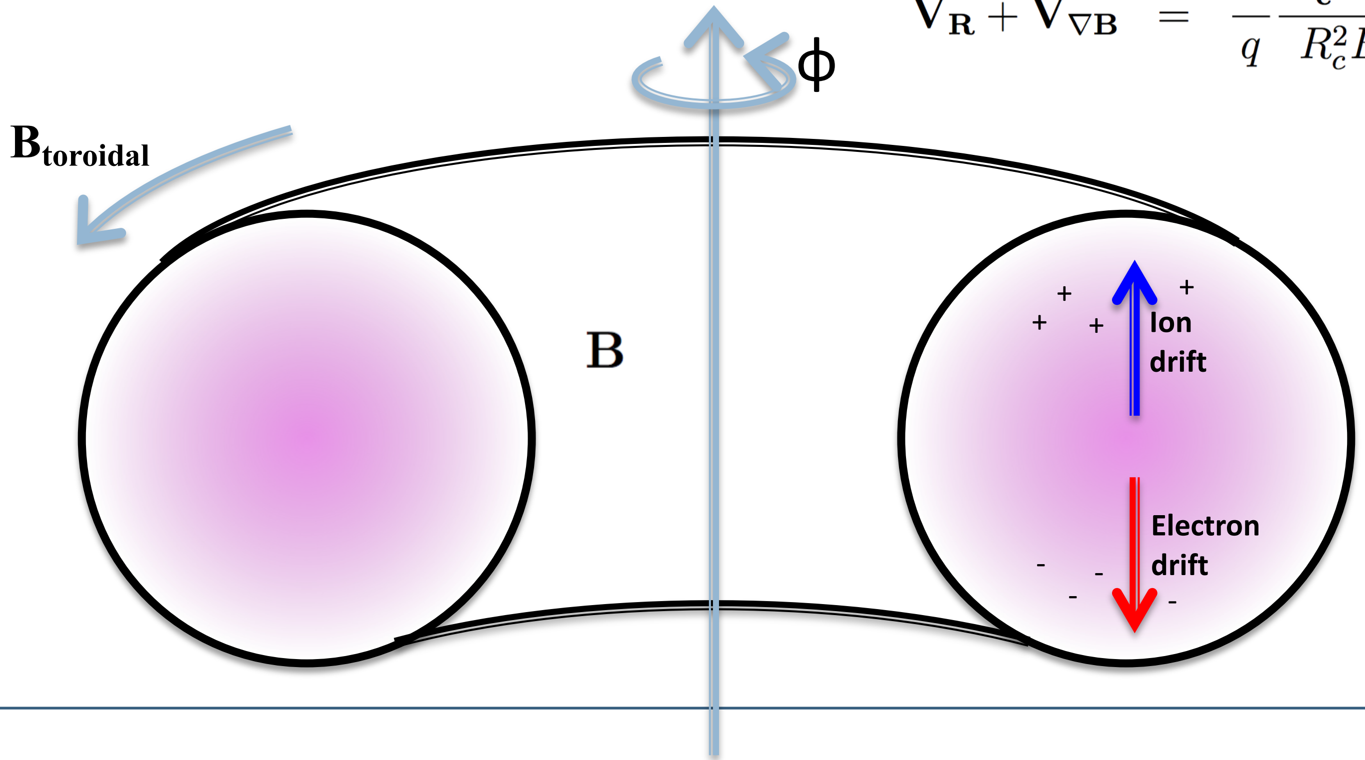


- **The gyro-radius will be larger where the field is weaker and smaller where the field is stronger**
- **The resulting drift velocity is described by:** $\mathbf{V}_{\nabla \mathbf{B}} = \frac{m v_\perp^2}{2qB} \frac{\mathbf{B} \times \nabla B}{B^2}$

In a toroidal configuration we're in trouble

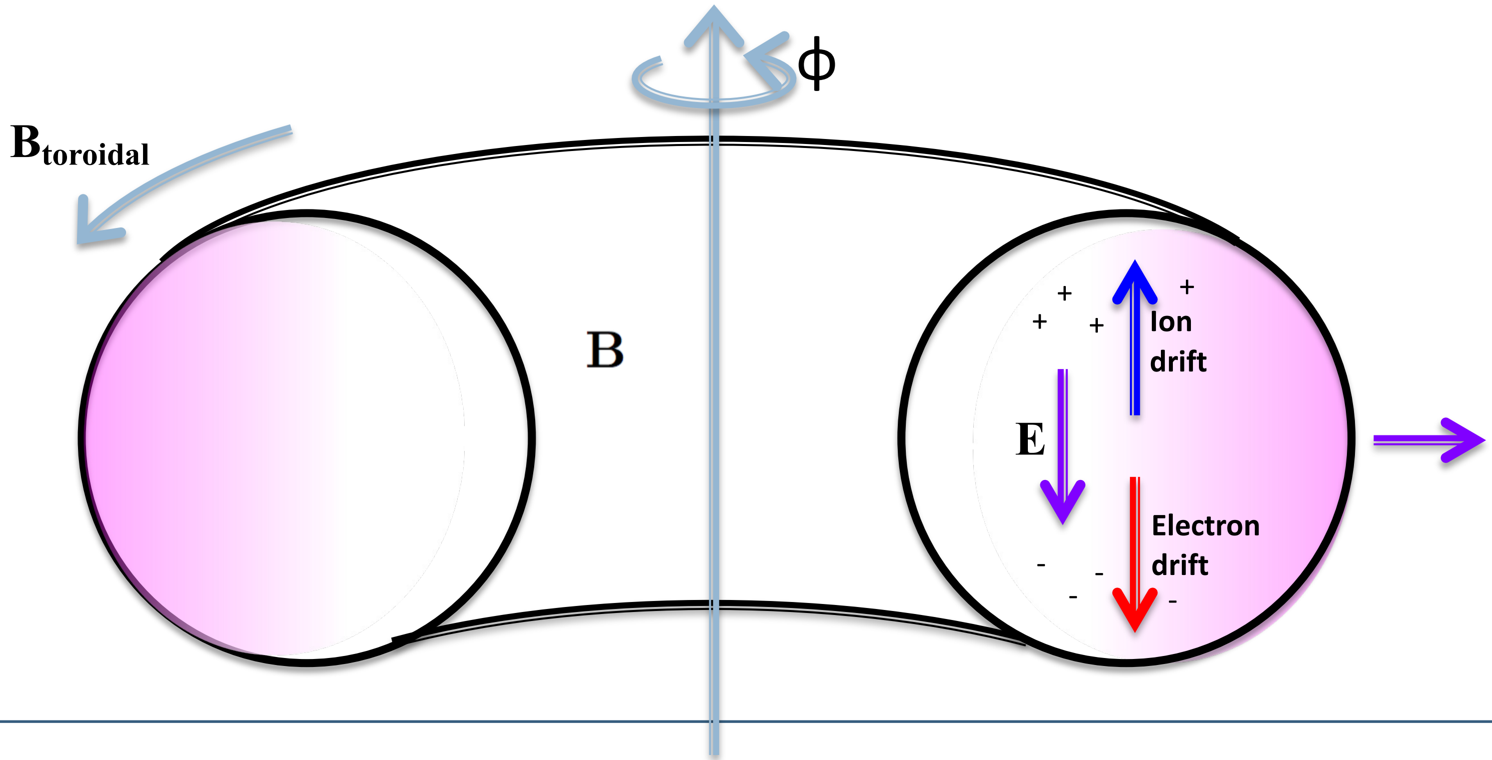
- **Curvature and Grad B drifts add in a toroidal configuration:**

$$\mathbf{V_R} + \mathbf{V_{\nabla B}} = \frac{m}{q}\frac{\mathbf{R_c} \times \mathbf{B}}{R_c^2 B^2}\left(v_\parallel^2 + \frac{1}{2}v_\perp^2\right)$$



In a toroidal configuration we're in trouble

- **THIS EFFECT COMBINES TO EJECT THE PLASMA**

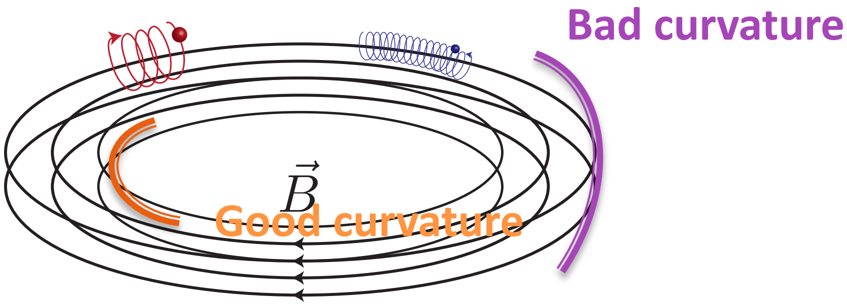


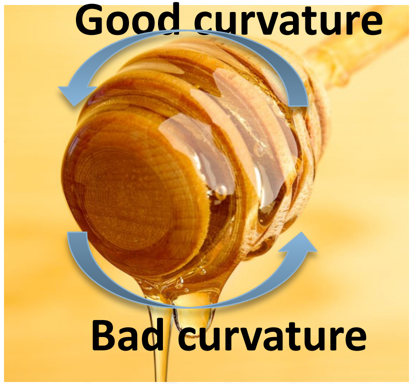
$$\mathbf{v_E} = \frac{\mathbf{E} \times \mathbf{B}}{B^2}$$

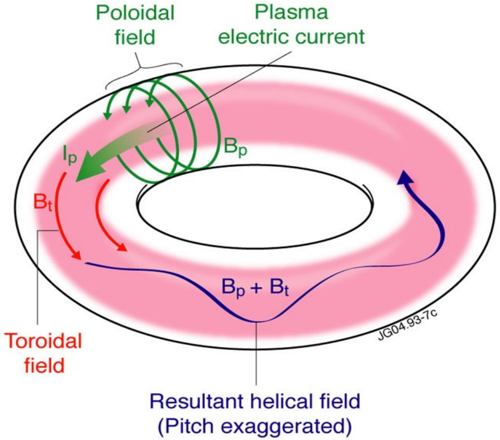
Roadmap

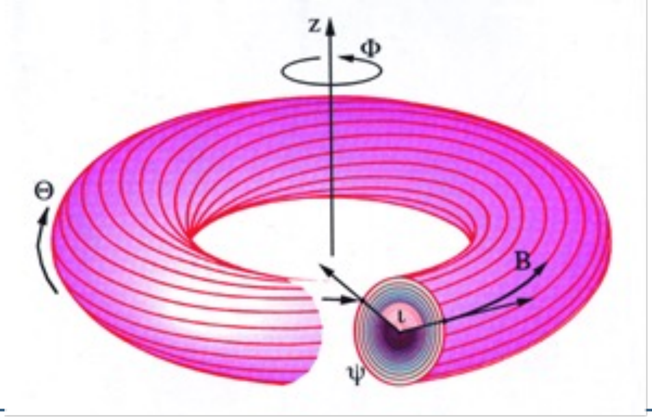
- Plasma in a Magnetic Field
- Magnetic Mirrors
- Plasma in a Magnetic + Electric field
- Generalized drift
- Drifts in a curved geometry
- Toroidal confinement
- Tokamaks and Stellarators
- …and Beyond

How do we fix this?

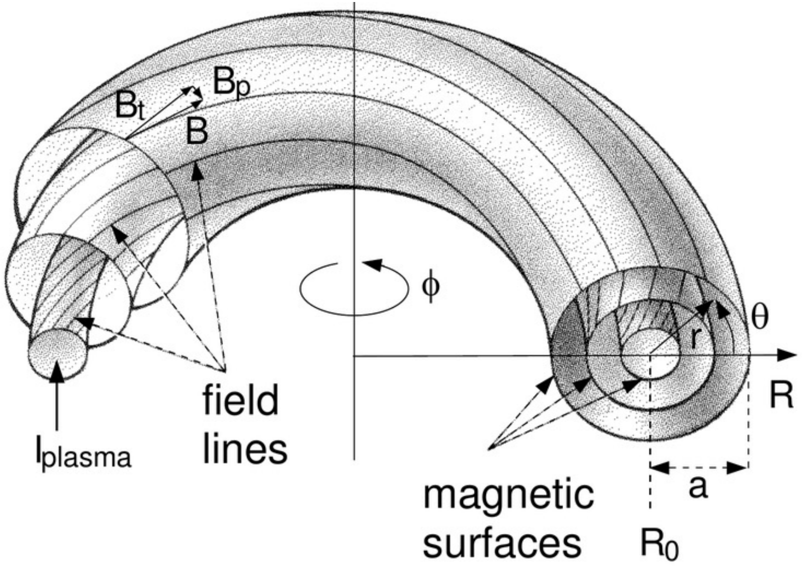








This results in embedded flux surfaces



Two very important concepts:

(1) Magnetic field lines lie on nested surfaces of constant magnetic flux, Ψ

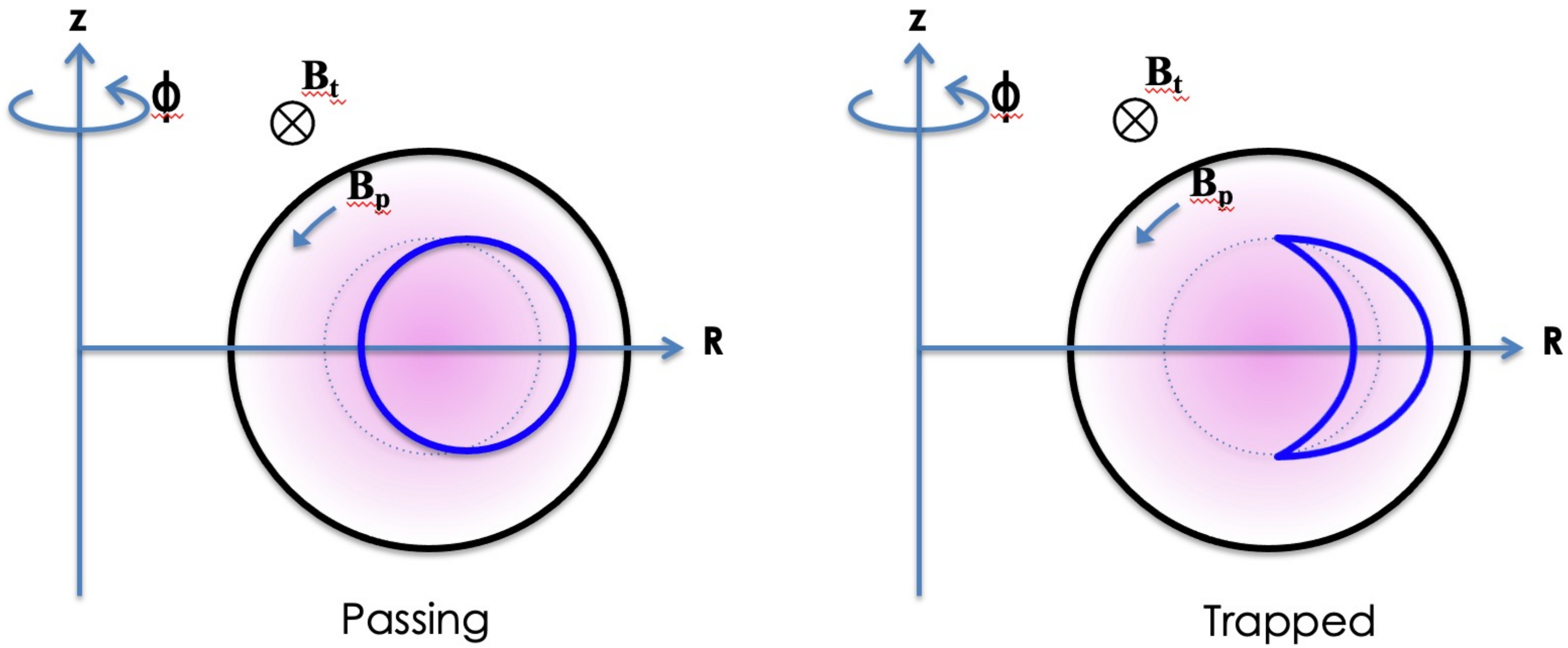
(2) Safety factor:

$$q = \frac{toroidal\ transits}{poloidal\ transits}$$

$$= \frac{d\phi}{d\theta} \text{ or "pitch"}$$

Iota: $\iota = \frac{\#\ poloidal\ turns}{\#\ toroidal\ turns}$ ***= 1/q***

Particles drift off the flux surfaces but return (or bounce)

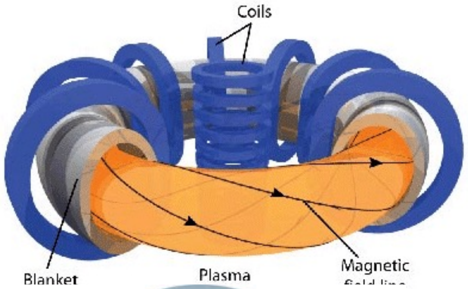


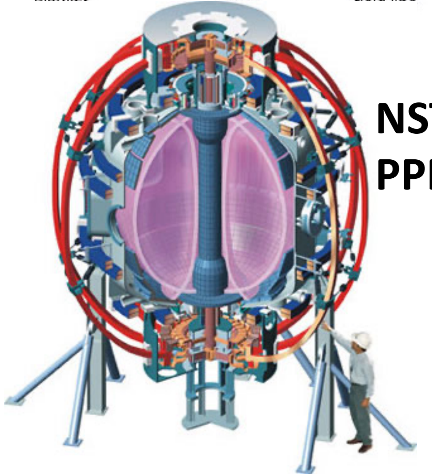
Roadmap

- Plasma in a Magnetic Field
- Magnetic Mirrors
- Plasma in a Magnetic + Electric field
- Generalized drift
- Drifts in a curved geometry
- Toroidal confinement
- Tokamaks and Stellarators
- …and Beyond

The two most-studied configurations are

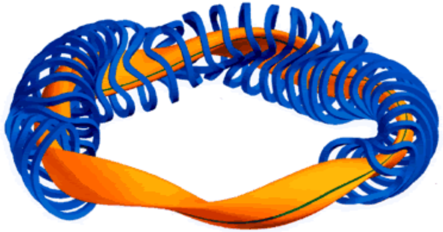
Tokamak



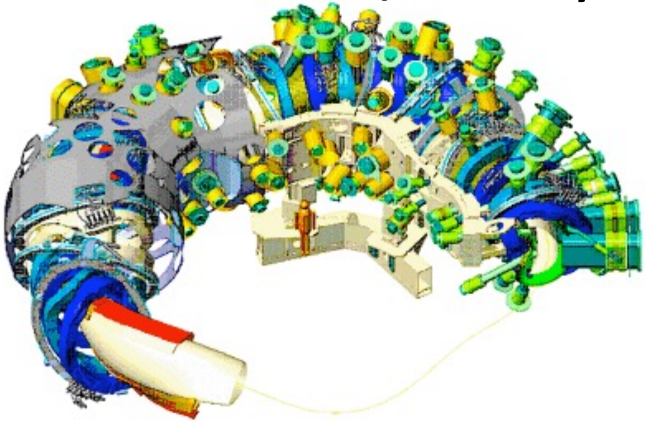


NSTXU @ PPPL

Stellarator

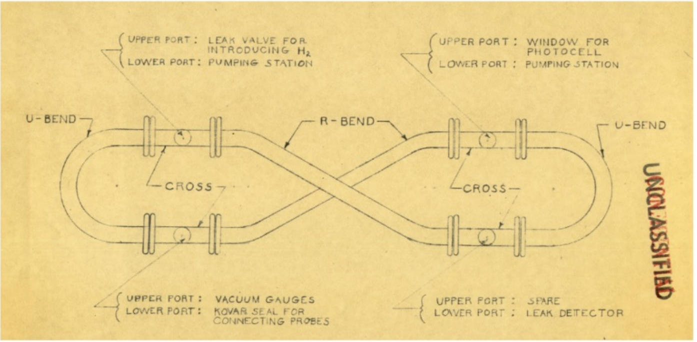


W7X @ Max-Planck / Germany

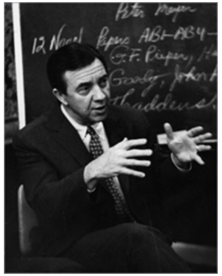


Early stellarator experiments

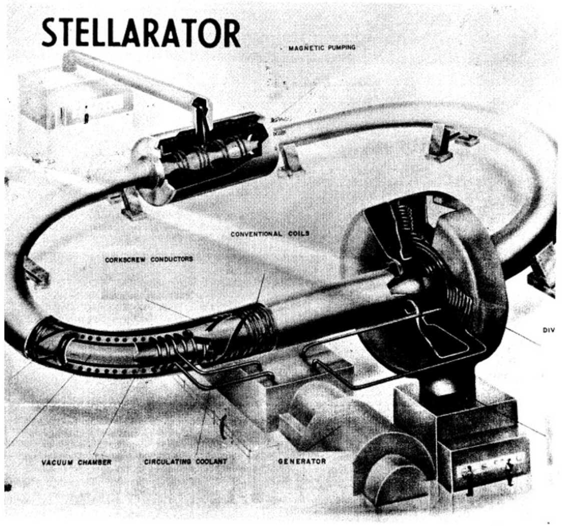
Figure-eight (Princeton Model A) – 1953-1958



C. H. Willis, *NJ Project Matterhorn* (1953).



Racetrack (Princeton Model C) – 1962-1969



Early stellarator experiments

Spiraling coils give it the necessary rotation at the turns

Racetrack stellarator was taken to the 1958 Atoms for Peace conference.

Tokamaks enter the game

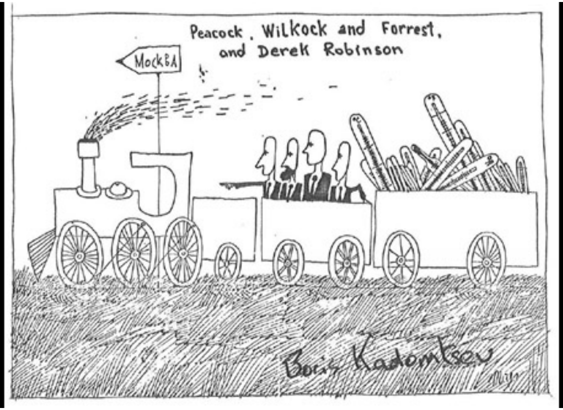
nature

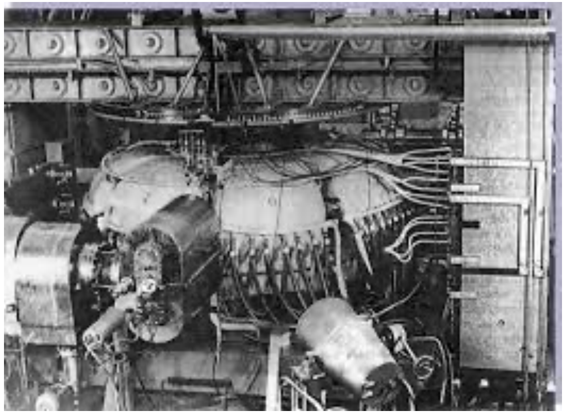
Measurement of the Electron Temperature by Thomson Scattering in Tokamak T3

by
N. J. PEACOCK
D. C. ROBINSON
M. J. FORREST
P. D. WILCOCK
UKAEA Research Group,
Culham Laboratory,
Abingdon, Berkshire

V. V. SANNIKOV
I. V. Kurchatov Institute,
Moscow

Electron temperatures of 100 eV up to 1 keV and densities in the range $1\text{–}3 \times 10^{13}$ cm^{-3} have been measured by Thomson scattering on Tokamak T3. These results agree with those obtained by other techniques where direct comparison has been possible.





Tokamaks are the most studied

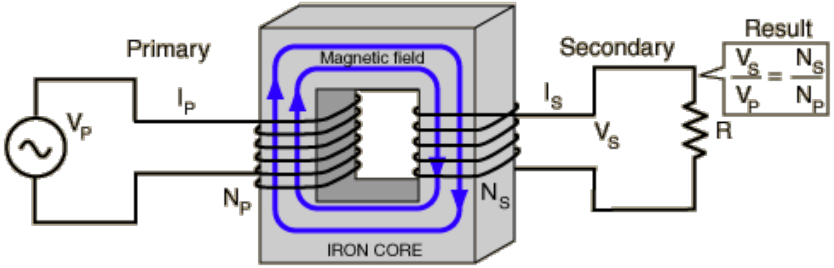
- Tokamaks confine with an *externally* produced toroidal field and a *plasma current* produced poloidal field

- TOKAMAK is a Russian acronym:
  TOroidal'naya KAmera s MAgnitnymi KAtushkami
  "Toroidal Chamber with Magnetic Coils"

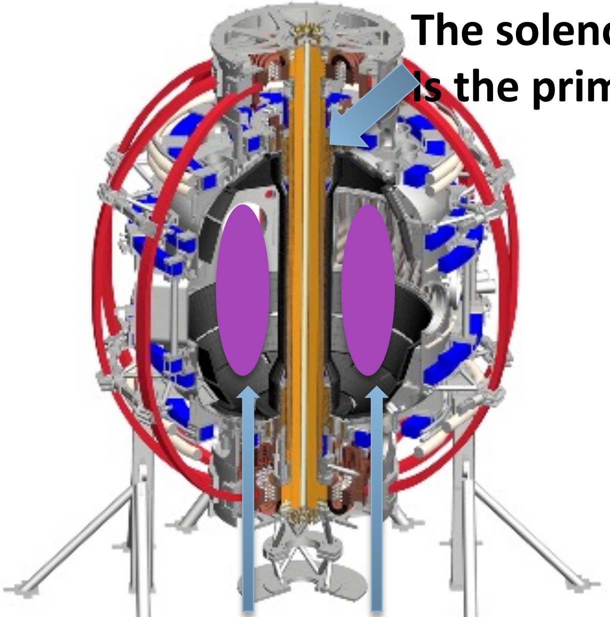
- Leading magnetic confinement concept in terms of number of facilities and funding

A tokamak relies on a central solenoid

The poloidal field is induced using a central solenoid



This makes it inherently non-steady state.

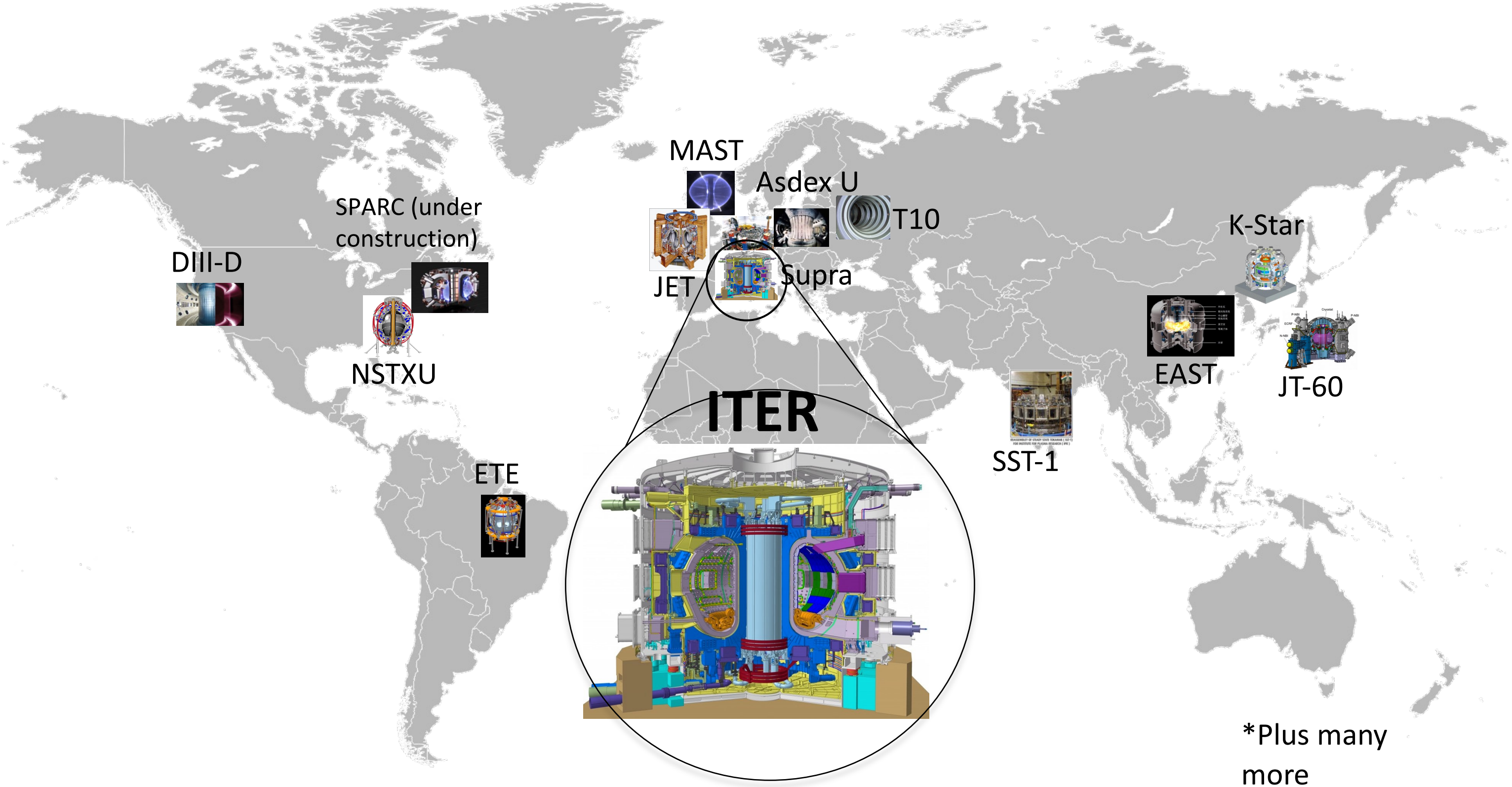


The solenoid is the primary

The plasma is the secondary

Jumping ring demo is a good analogy

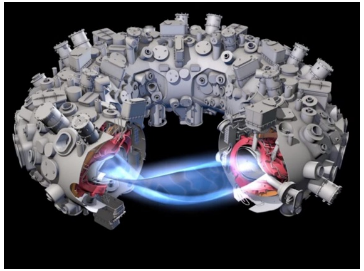
There are many tokamaks around the world studying different plasma parameters and shapes



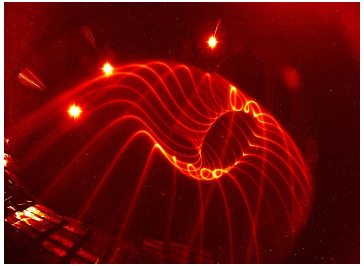
*Plus many more

Quasi-symmerties have brought back the Stellarator

In recent years, Stellarators have made a comeback with the discovery of quasi-symmetries. These configurations have shown fusion-relevant confinement properties



Wendelstein 7-X

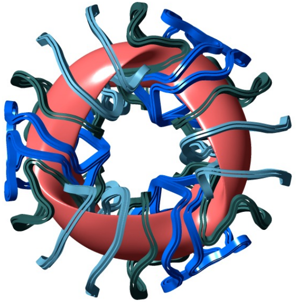


POPULAR MECHANICS HOME LATEST STORIES SCIENCE MILITARY POP MECH

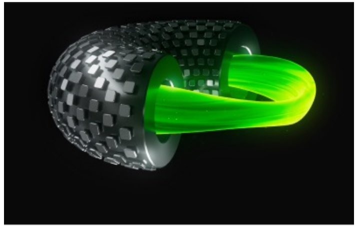
German Nuclear Fusion Experiment Sets Records

The stellarator was largely replaced by the tokamak in the 1960s, but Germany's Wendelstein 7-X could be bringing the reactor back from the dead.

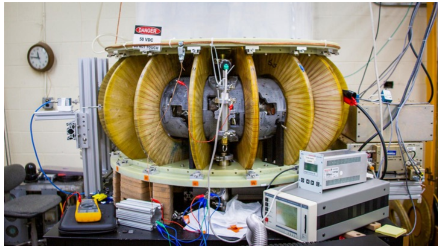
BY JAY BENNETT PUBLISHED: JUN 26, 2018 12:31 PM EDT



NCSX NATIONAL COMPACT STELLARATOR EXPERIMENT



THEA ENERGY



MUSE

Tokamaks vs Stellarators

Tokamaks

✓ Automatic guiding center confinement
✓ Much more studied
✓ "Simpler" design
✗ Requires large plasma current
✗ Steady-state is challenging/inefficient

Stellarators

✗ Confinement optimization required
✗ Shaping by complicated coils/magnets
✓ Low recirculating power
✓ No plasma-terminating disruptions
✓ No Greenwald density limit
✓ "softer" pressure/β limit

Plasma Phys. Control. Fusion **54** (2012) 124009 (12pp) doi:10.1088/0741-3335/54/12/124009

Stellarator and tokamak plasmas: a comparison

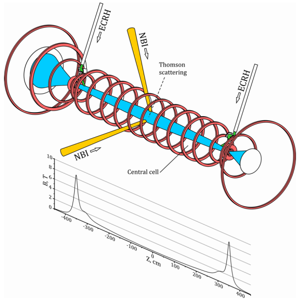
P Helander, C D Beidler, T M Bird, M Drevlak, Y Feng, R Hatzky, F Jenko, R Kleiber, J H E Proll, Yu Turkin and P Xanthopoulos

Max-Planck-Institut für Plasmaphysik, Greifswald and Garching, Germany

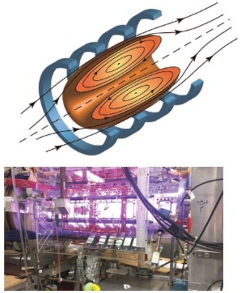
Roadmap

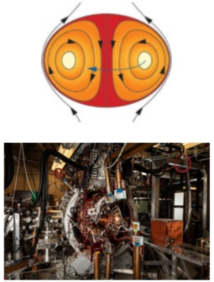
- Plasma in a Magnetic Field
- Magnetic Mirrors
- Plasma in a Magnetic + Electric field
- Generalized drift
- Drifts in a curved geometry
- Toroidal confinement
- Tokamaks and Stellarators
- …and Beyond

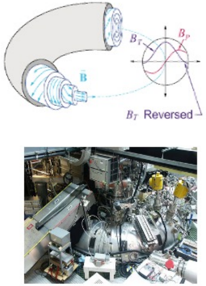
There are a variety of magnetic approaches, from simple topologies to more complex ones

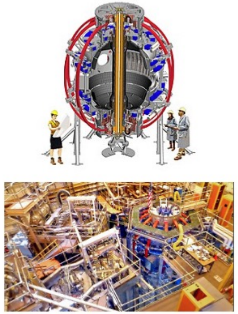


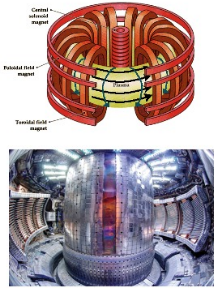
Mirror

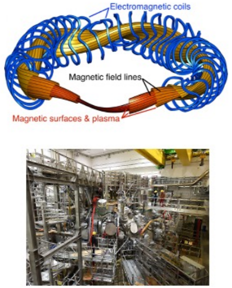












Field-reversed configuration (FRC)

Spheromak

Reversed-Field Pinch (RFP)

Spherical Tokamak

Tokamak

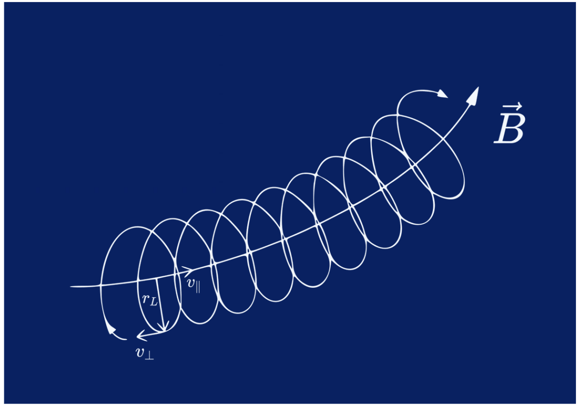
Stellarator

More topological simplicity

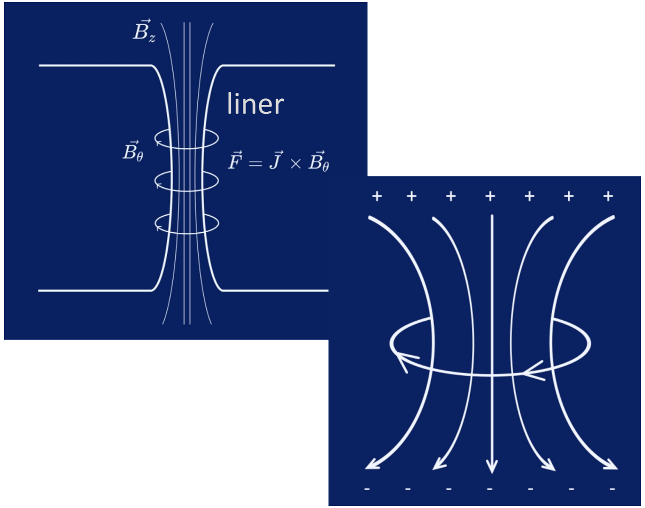
More B-field cost/complexity

There are even concepts between Inertial and Magnetic

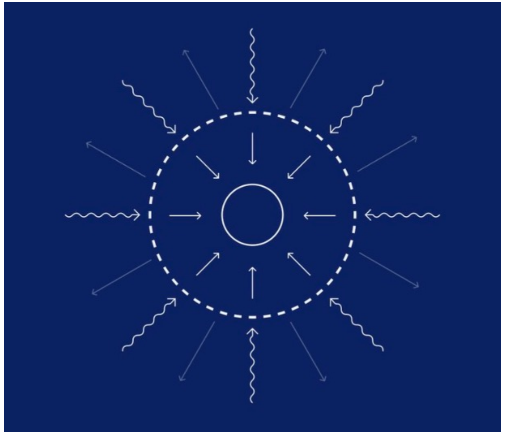
Magnetic confinement fusion (MCF)



Magneto-inertial fusion (MIF) and Z pinch

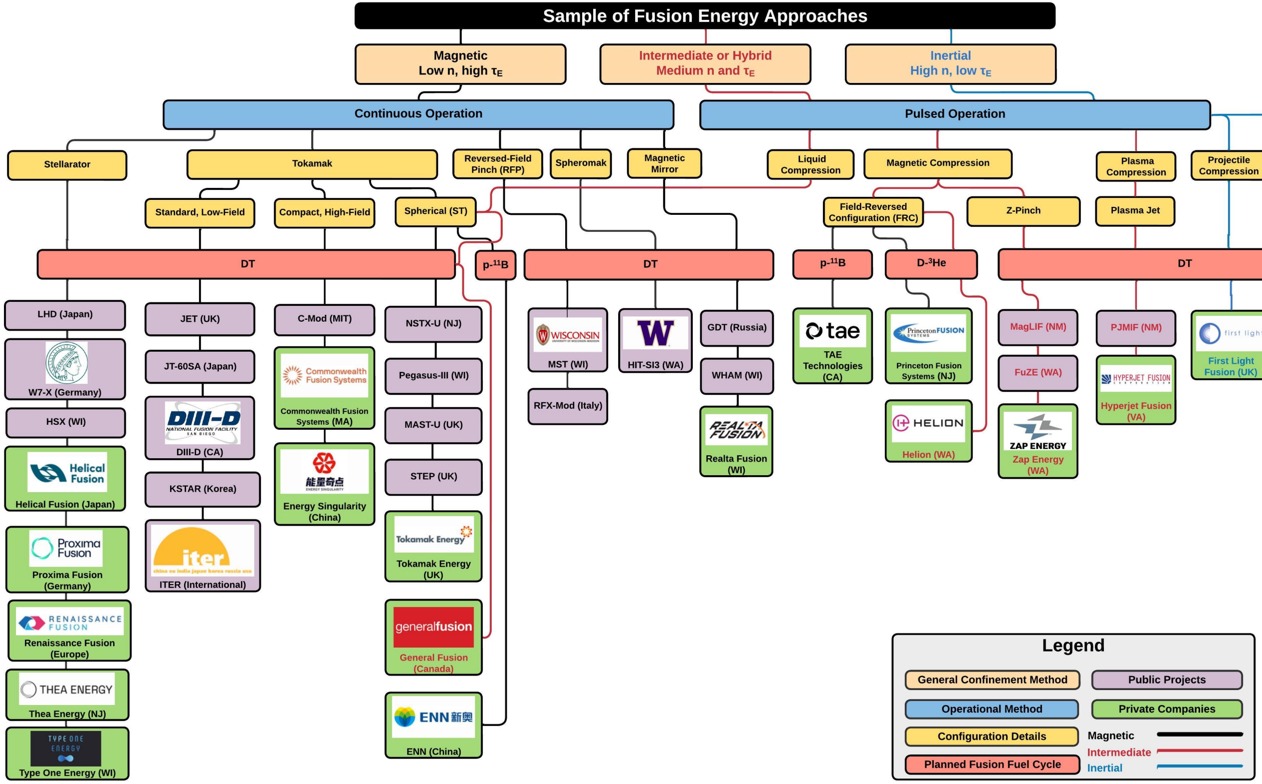


Inertial confinement fusion (ICF)



Drawings by Yousef Lawrence (Twitter @FusionHub0

There are many fusion approaches and companies



Created By: Derek Sutherland & Andy Freeberg, Zap Energy, Inc.

Last Modified: September 2023