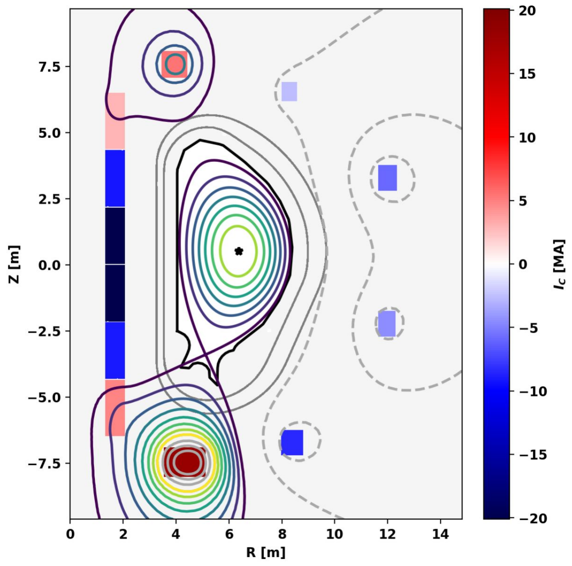
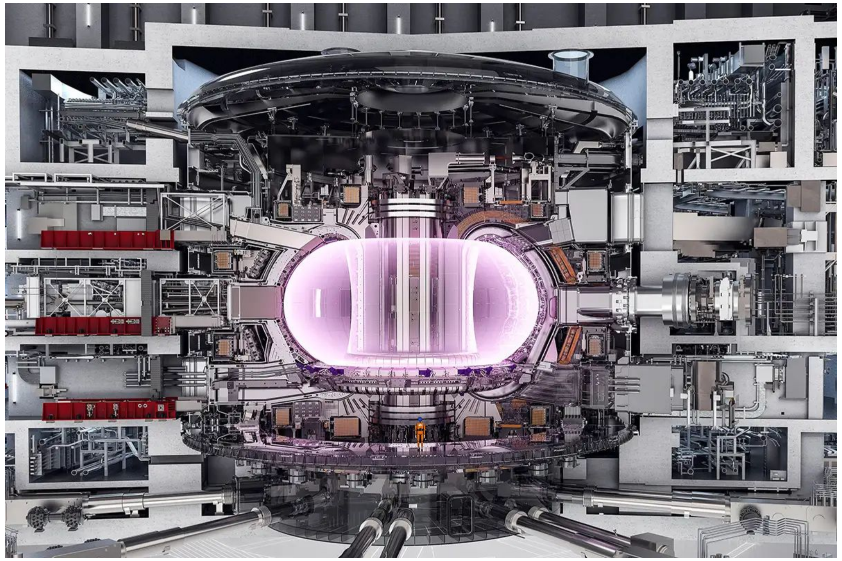
# Computational Workshop: TokaMaker

## How to design* a tokamak† using numerical tools




Chris Hansen[1]

1- *Columbia University*


* Part of the device at least
† Can be used for more than just tokamaks!



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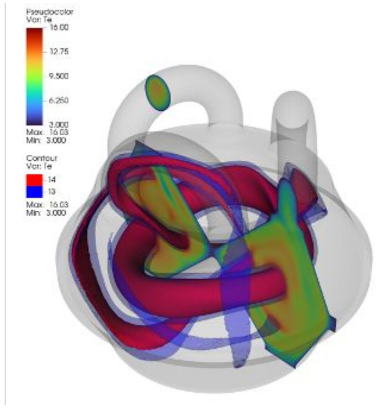
# How did I get into plasma physics and fusion?

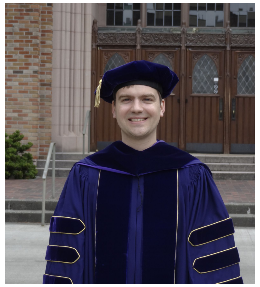
Seattle

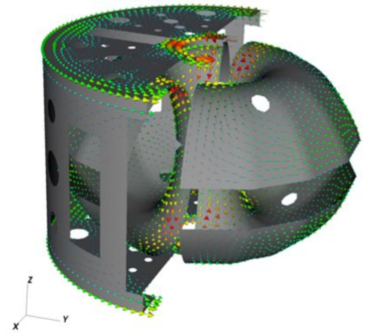
- **Grew up in Seattle, WA**
- **Undergraduate in aerospace engineering**
  - Research with U. Washington plasma group
- **PhD at University of Washington**
  - Numerical modeling of spheromaks

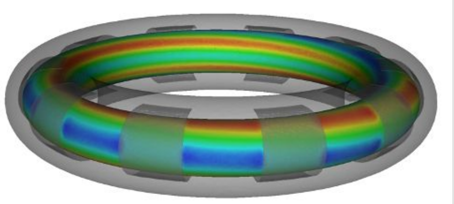
- **Post-doc working for U. Washington with Columbia U. (HBT-EP), and PPPL (LTX)**
  - Capturing 3D device features in simulations
- **Research scientist at U. Washington then Columbia U. (starting 1/2023)**
  - Work closely with students

New York









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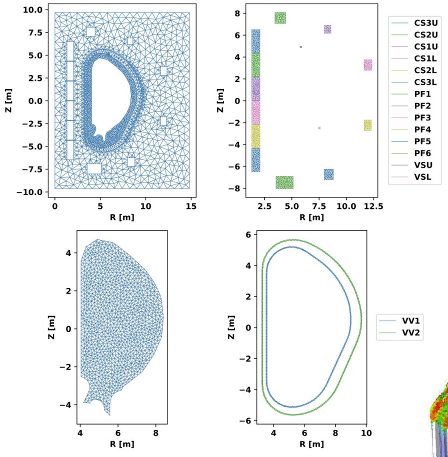
# Today we will be using the Open FUSION Toolkit an open-source suite of tools for fusion modeling and design

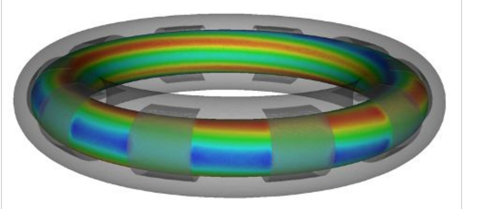
- Fully open-source on GitHub
  - All tools share the same codebase; C/C++/Fortran/Python
  - Downloadable binaries
  - Contributions welcome!

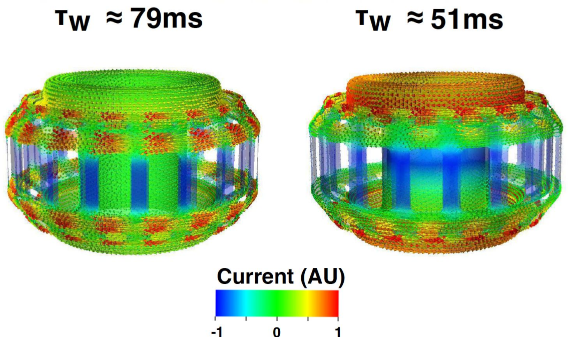
- Finite elements on unstructured grids
  - Direct meshing from CAD

- Current physics tools
  - **TokaMaker: Axisymmetric equilibrium**
  - ThinCurr: 3D thin-wall E-M modeling
  - MUG: 3D time-dependent MHD
  - Marklin: 3D Ideal MHD equilibria (const. $\lambda$)

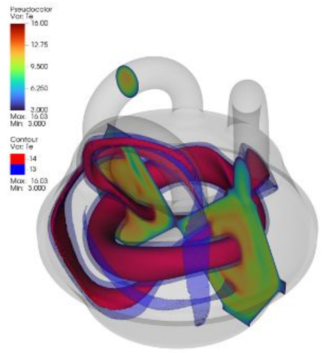



Hansen et al., *CPC* (2024)

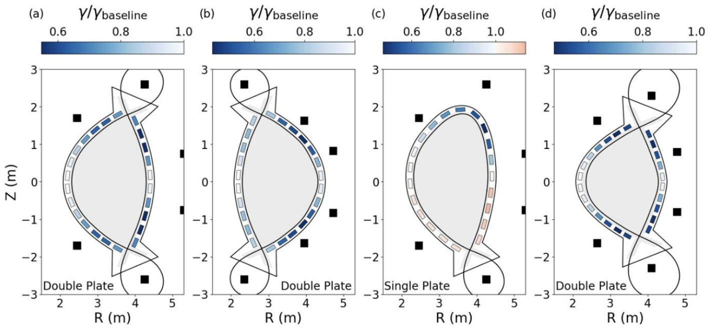





Battey et al., *NF* (2023)



Hansen et al., *PoP* (2015)



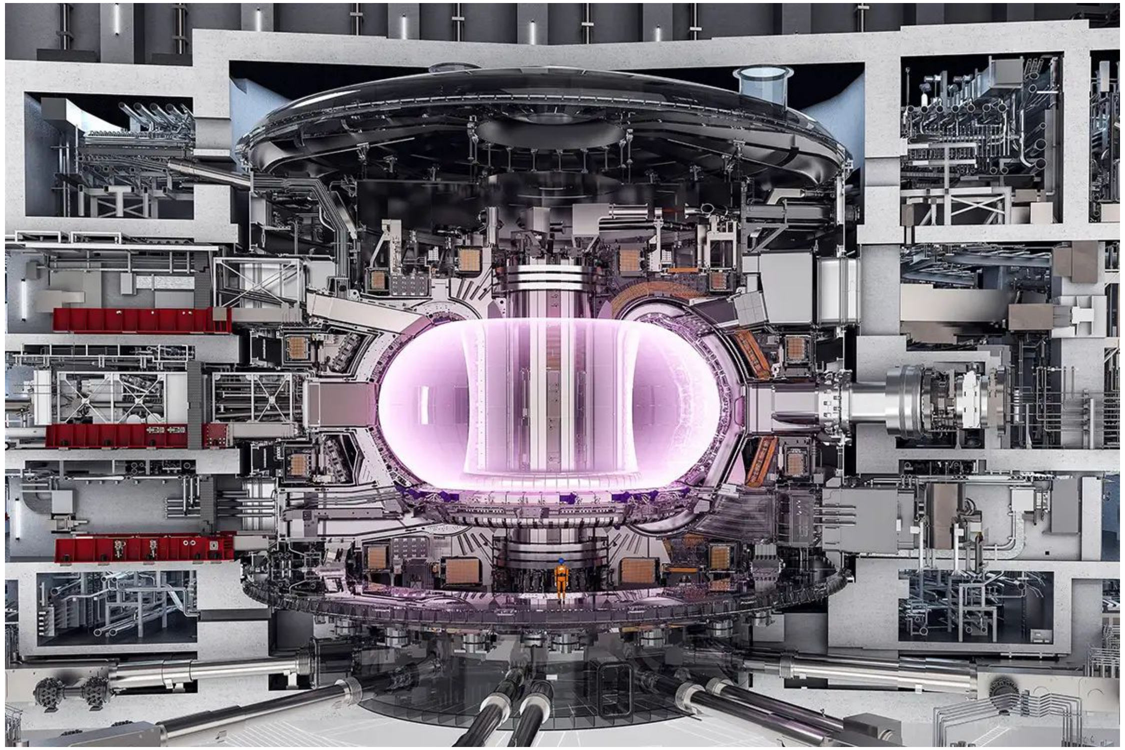

Guizzo et al., *PPCF* (2024)

https://hansec.github.io/OpenFUSIONToolkit/

# How do we design a magnetic confinement fusion power plant?

- **A fusion power plant is a very complex device with many supporting systems**
  - Core magnetized plasma
  - Systems to maintain plasma/fusion
  - Systems to convert energy to electricity
  - Systems to support all the other systems

- **Designing a new device means balancing requirements on core and surrounding systems**
  - Need models of both to evaluate tradeoffs



# How do we design a magnetic confinement fusion power plant?

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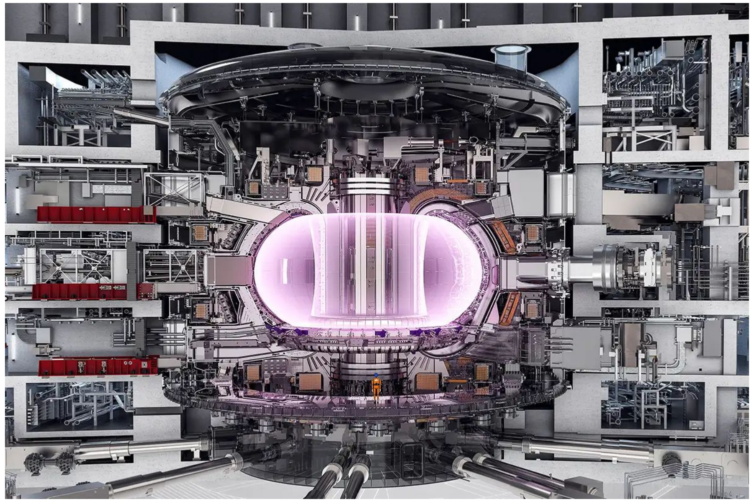

- **Designing a new device means balancing requirements on core and surrounding systems**
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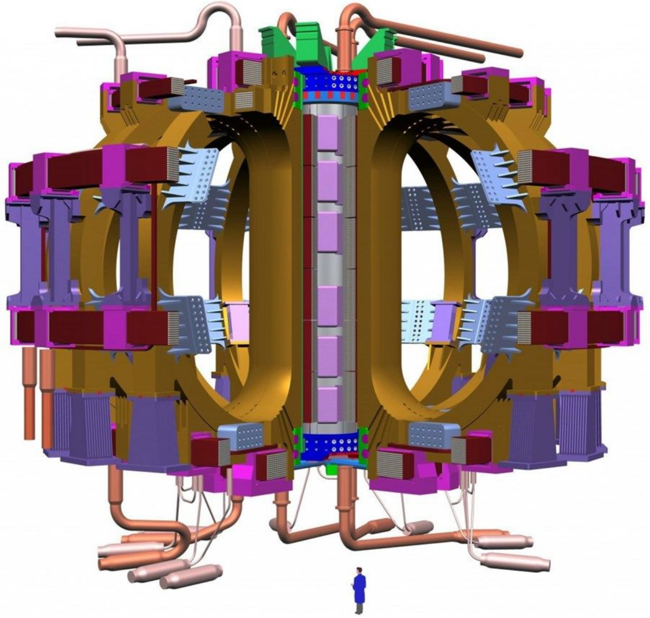
- **Core plasma is in force-balance equilibrium**
  - Force-balance timescale very fast O($\mu s$)
  - Plasma is confined by magnetic field coils; equal and opposing forces on each



# In *axisymmetric* configurations the Grad-Shafranov equation defines magnetic equilibrium

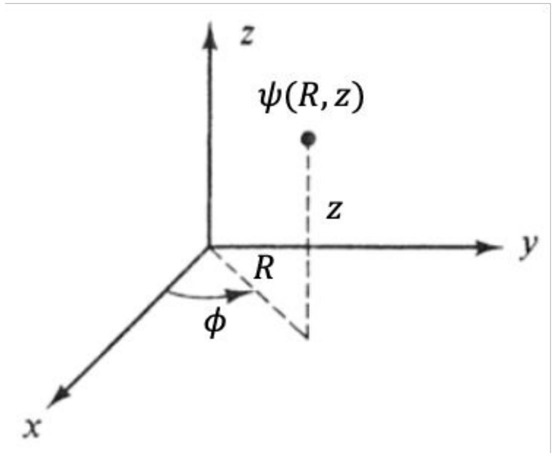
- **The Lorentz force is used to balance the pressure gradient necessary for a hot core and cold edge**

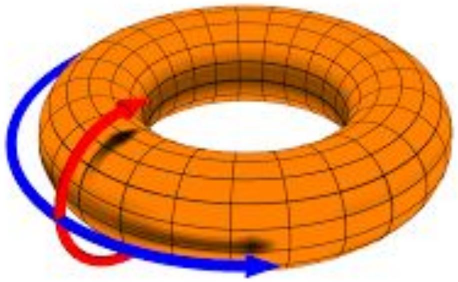
$$\boldsymbol{J}\times\boldsymbol{B} = \nabla P$$

- **If the system has a rotational symmetry $\phi$ we can express the magnetic field using a 2D scalar potential function $\psi(R,Z)$**

$$\boldsymbol{B} = \frac{1}{R}\nabla\psi\times\widehat{\boldsymbol{\phi}} + \frac{F}{R}\widehat{\boldsymbol{\phi}}$$

$$\mu_0\boldsymbol{J} = \nabla\times\boldsymbol{B} = \frac{1}{R}\frac{dF}{d\psi}\nabla\psi\times\widehat{\boldsymbol{\phi}} - \frac{\left(\frac{\partial^2\psi}{\partial R^2} - \frac{1}{R}\frac{\partial\psi}{\partial R} + \frac{\partial^2\psi}{\partial R^2}\right)}{R}\widehat{\boldsymbol{\phi}}$$



# In *axisymmetric* configurations the Grad-Shafranov equation defines magnetic equilibrium

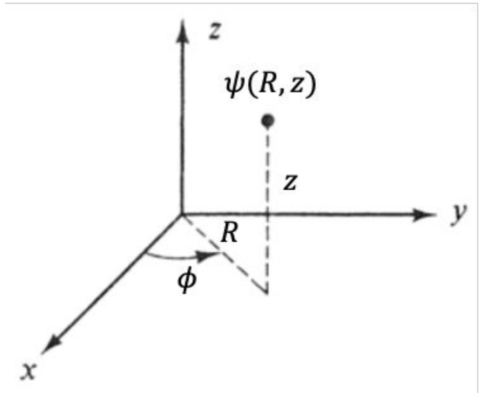
- **The Lorentz force is used to balance the pressure gradient necessary for a hot core and cold edge**

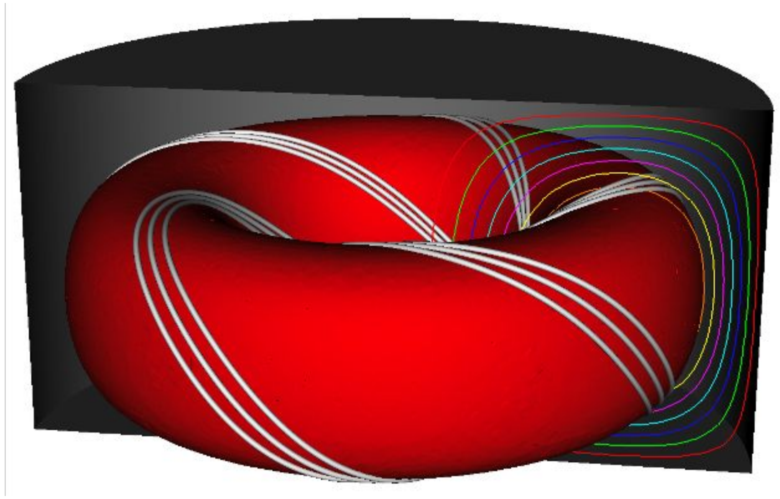
$$\boldsymbol{J} \times \boldsymbol{B} = \nabla P$$

- **If the system has a rotational symmetry $\phi$ we can express the magnetic field using a 2D scalar potential function $\psi$*(R,Z)***

$$\boldsymbol{B} = \frac{1}{R} \nabla \psi \times \widehat{\boldsymbol{\phi}} + \frac{F}{R} \widehat{\boldsymbol{\phi}}$$

- **$F$ and $P$ are each purely functions of $\psi$**
  - Pressure gradient must be perpendicular to $\boldsymbol{B}$
  - $\boldsymbol{B}$ is perpendicular to $\nabla \psi$



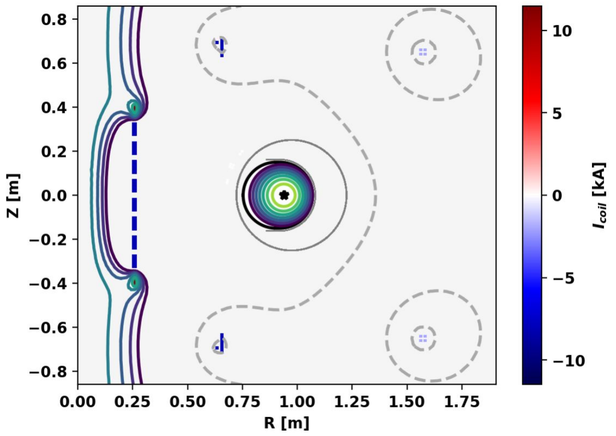



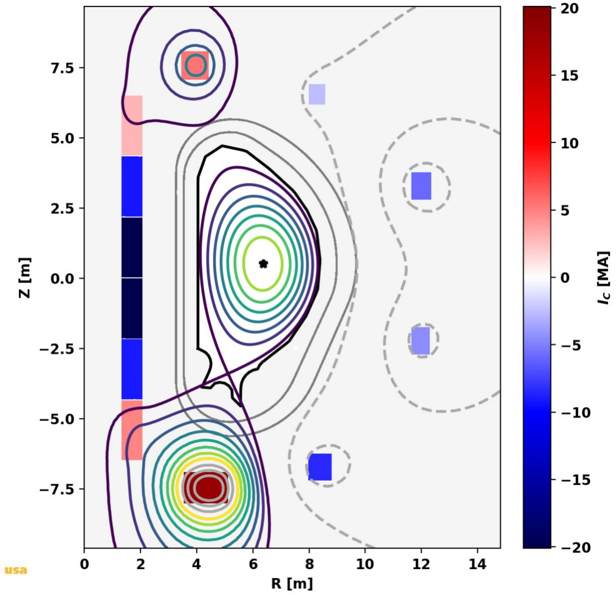
# In *axisymmetric* configurations the Grad-Shafranov equation defines magnetic equilibrium

- **With known expressions for $B$, $J$, and $P$ in terms of $\psi$ we can derive the Grad-Shafranov equation[1,2]**

$$\frac{\partial^2\psi}{\partial R^2} - \frac{1}{R}\frac{\partial\psi}{\partial R} + \frac{\partial^2\psi}{\partial R^2} = -\mu_0 R^2 \frac{dP}{d\psi} - F\frac{dF}{d\psi}$$

$$\Delta^*\psi = -\mu_0 R^2 P' - FF'$$




- **Constitutes a 2D nonlinear partial differential equation for $\psi$**
  - TokaMaker uses the finite element method
  - Toroidal current in coils act as a fixed RHS source
  - $F$ and $P$ capture limiting points


[1] - H. Grad and H. Rubin (1958)

[2] - V.D. Shafranov (1966 [1957])

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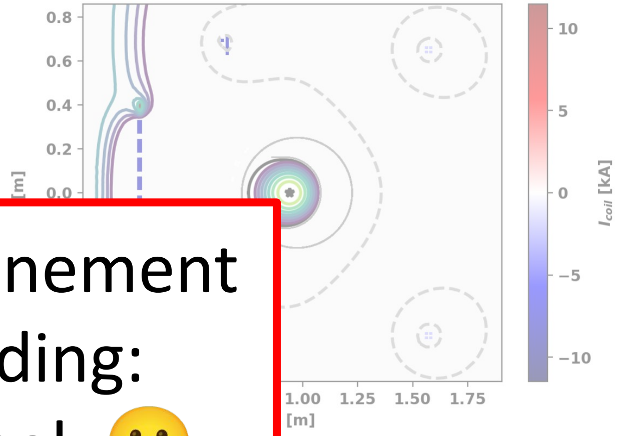
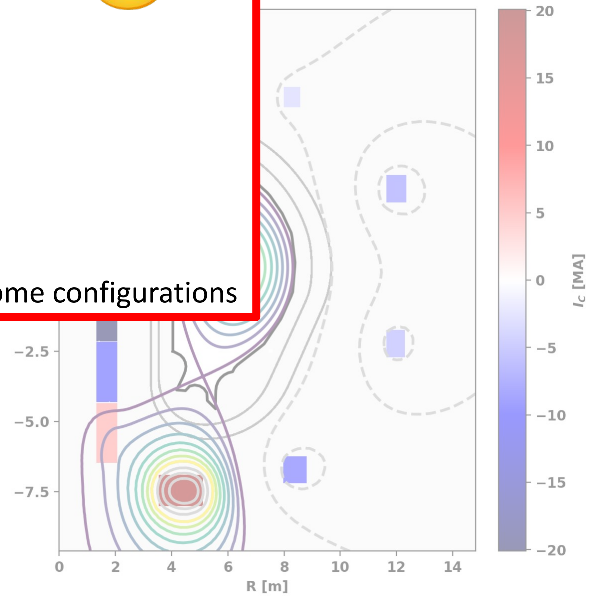
# In *axisymmetric* configurations the Grad-Shafranov equation defines magnetic equilibrium

- With known expressions for $B$, $J$, and $P$ in terms of $\psi$ we can derive the Grad-Shafranov equation[1,2]

$$\frac{\partial^2\psi}{\partial R^2} -$$

- Constitutes
  - TokaMaker solves using the finite element method
  - Toroidal current in coils act as a fixed RHS source
  - $F$ and $P$ capture limiting points




This equation works* for most magnetic confinement devices since most are axisymmetric, including:
Mirror, Z-pinch, FRC, Spheromak, RFP, Tokamak 😀

… but not stellarators 😞

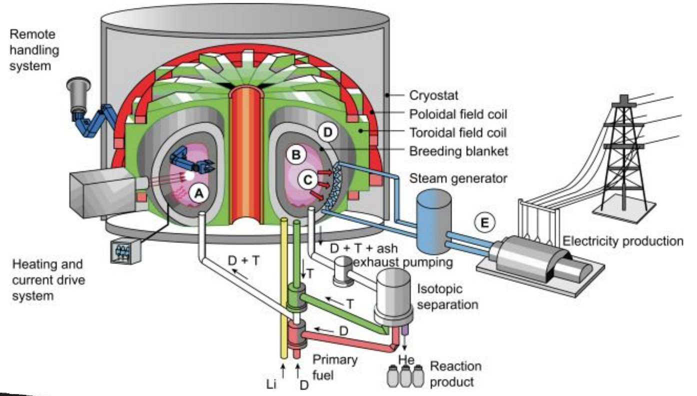
*other physics is important in some configurations


[1] - H. Grad and H. Rubin (1958)
[2] - V.D. Shafranov (1966 [1957])

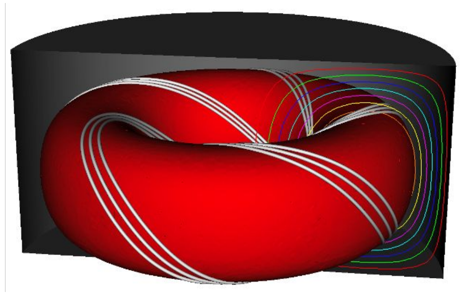
# What are some properties of plasma equilibrium that we care about for fusion?

- **Performance**
  - Pressure, core volume, etc. (plasma beta)
  - Required heating & current drive power
  - …

$$\beta = \frac{\langle P \rangle_v}{\left(\frac{|\boldsymbol{B}|^2}{2\mu_0}\right)}$$

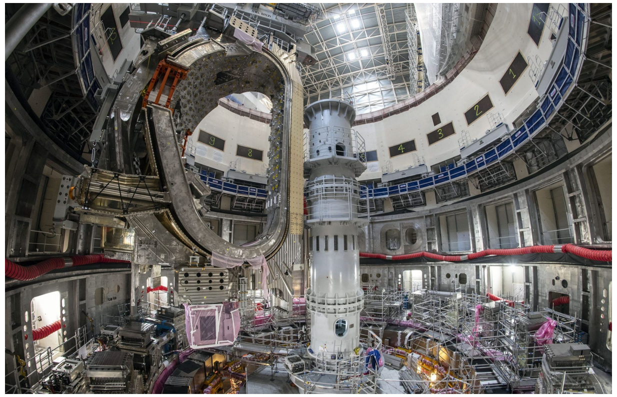



- **Stability/robustness**
  - Current and pressure profiles (safety factor)
  - Plasma shape
  - …



$$q(\psi) = \frac{N_\phi}{N_\theta}$$

- **Engineering**
  - Number, location, and capacity of coils
  - Power exhaust strategy
  - …



# Exercise 1: Computing equilibrium in fixed boundary

**Starting notebook:**
https://colab.research.google.com/github/OpenFUSIONToolkit/OpenFUSIONToolkit-Tutorials/blob/main/PPPL_SULI-2024/ex1-fixed_boundary.ipynb

**Completed notebook:**
https://colab.research.google.com/github/OpenFUSIONToolkit/OpenFUSIONToolkit-Tutorials/blob/main/PPPL_SULI-2024/ex1-fixed_boundary-Complete.ipynb

# Exercise 2: Computing equilibrium in free boundary

**Starting notebook:**

https://colab.research.google.com/github/OpenFUSIONToolkit/OpenFUSIONToolkit-Tutorials/blob/main/PPPL_SULI-2024/ex2-free_boundary.ipynb

**Completed notebook:**

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