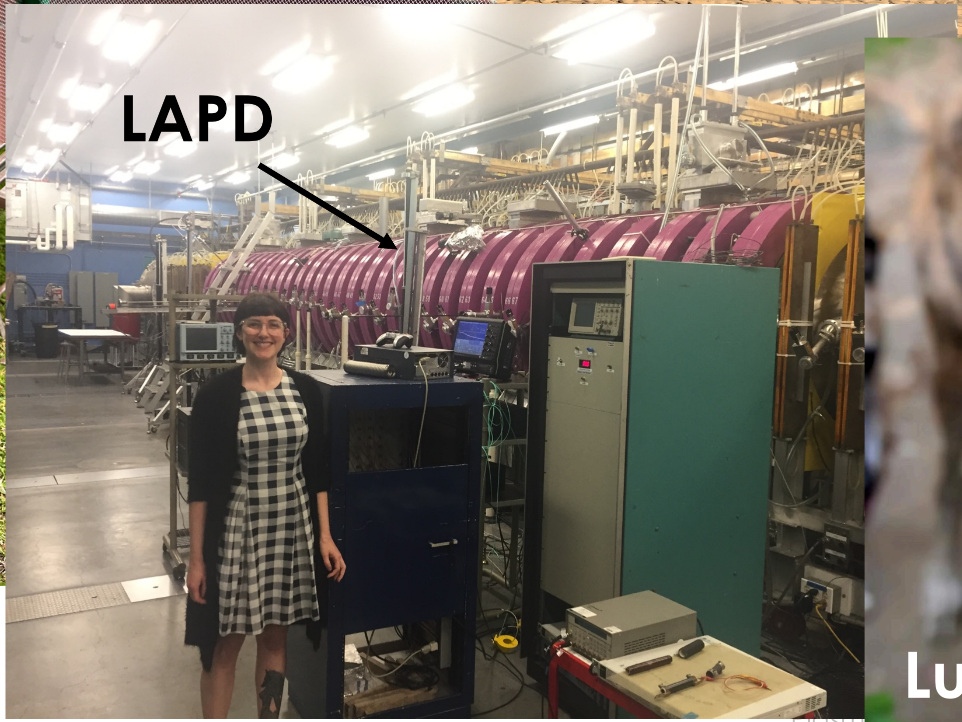
Plasma Waves

R. L. Barnett

Introduction to Fusion Energy and Plasma Physics Course,

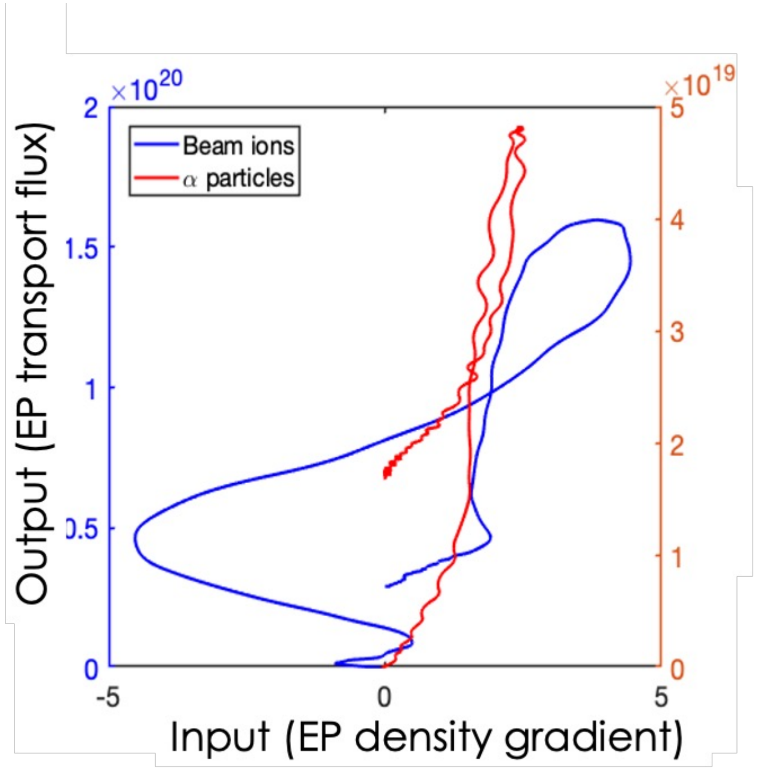
SULI, June 12th, 2024.

Who am I…



…and how did I get here?

I followed a somewhat non-traditional path:
Quite nonlinear, a bit like this energetic particle (EP) transport flux!



Courtesy of Yashika Ghai (Presenting on 06/21!)

15: Spent a year in Japan as an exchange student

16: Dropped out of high school

16 - 24: Worked a variety of jobs (aesthetician, barista, retail store manager…)

24: Enrolled in night "bridging" course, decided halfway through that I would study physics because "space is cool"

25: Enrolled in BSc, Physics major

33: Awarded PhD in Plasma Physics

On that note, a word of encouragement before we start

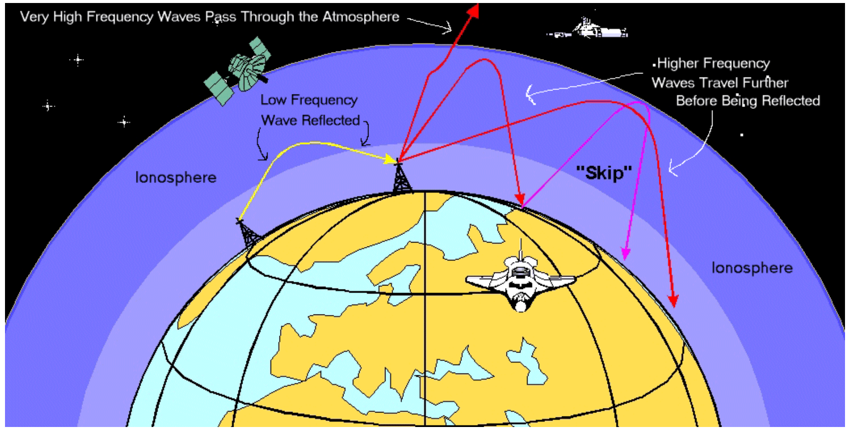
Oak Ridge National Laboratory

OAK RIDGE National Laboratory

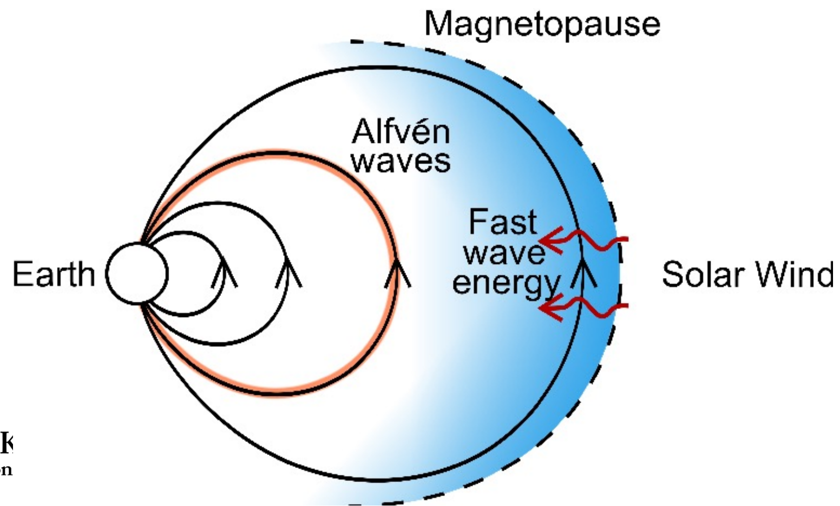
Alright enough about me, onto the waves!!

Understanding plasma waves is important for many reasons

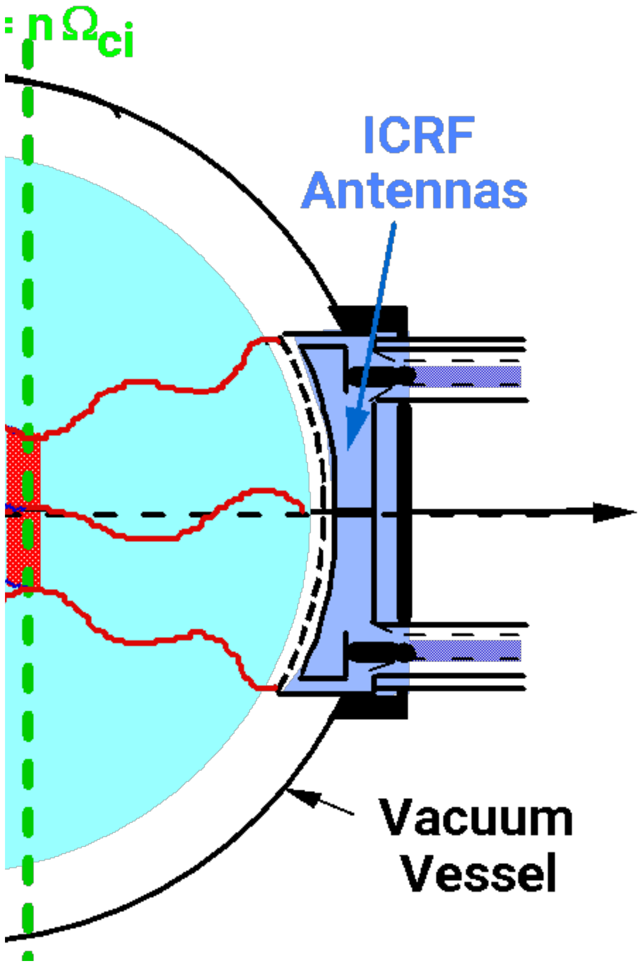
- Communication



- Geomagnetic storms



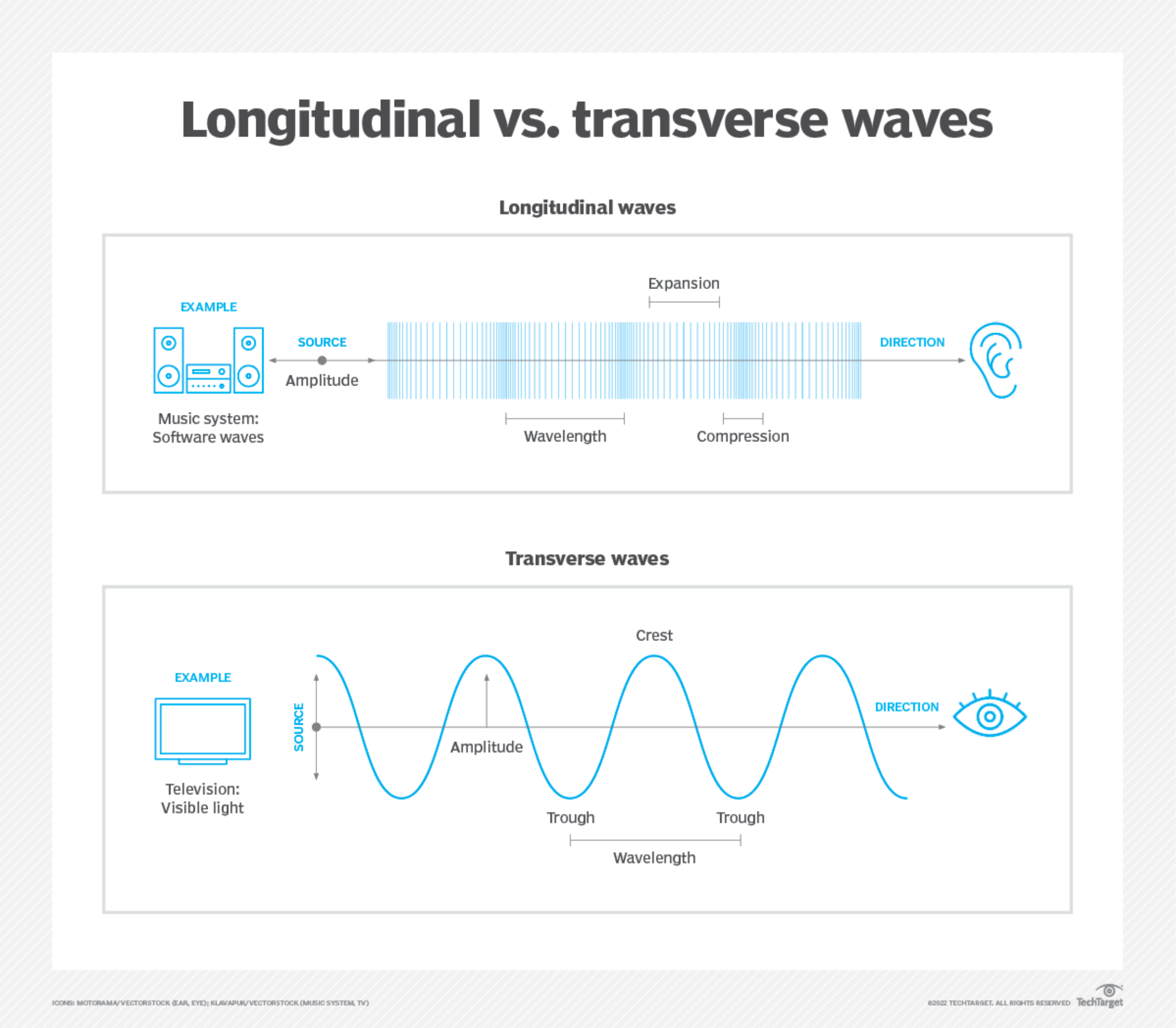
- And, of course, heating and current drive in fusion plasmas!



OAK Nation

Plasma waves: let's go back to basics…

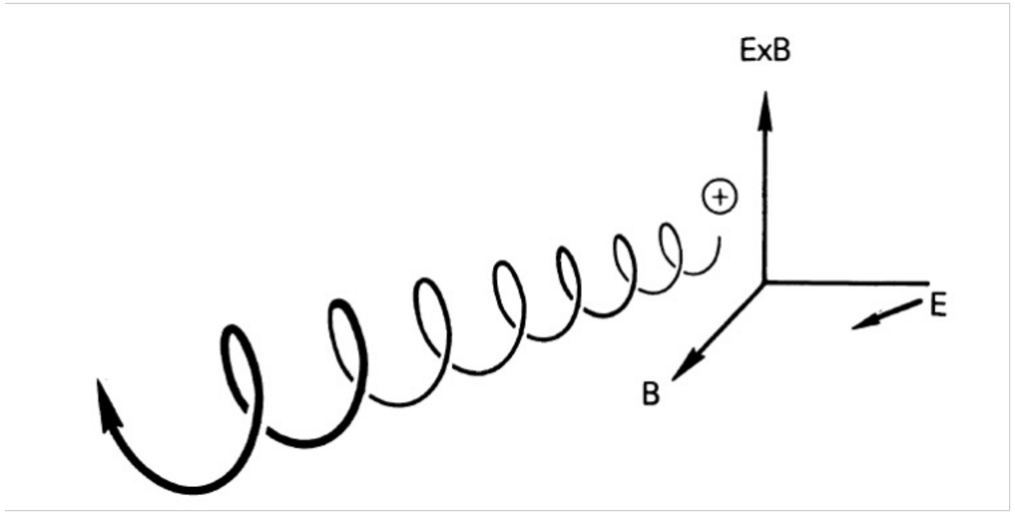
- How do we describe waves?
  - ➢ Amplitude
  - ➢ Wavelength λ
    - ➢ Wavenumber $k = 2\pi/\lambda$
  - ➢ Frequency f
    - ➢ Angular frequency $\omega = 2\pi f$
  - ➢ Velocity:
    - ➢ Phase $v_\varphi = \frac{\omega}{k}$
      Velocity of a wavefront
    - ➢ Group $v_g = d\omega/dk$
      Velocity of a wave packet



Oak Ridge National Laboratory

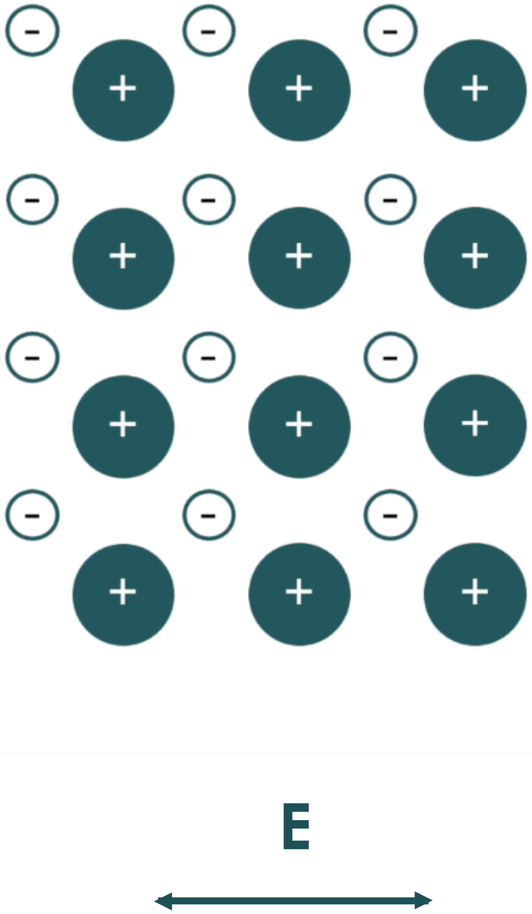
Plasma waves: a lot of the same characteristics apply, but with a bit of extra ✨spice✨

- Plasmas can act like they have "a mind of their own"… owing to two key things that make a plasma a plasma.
  - Quasi-neutrality: neutral enough to assume $Z_i n_i \approx n_e$, but not so neutral that it loses the interesting properties.
  - Collective behaviour: forces from long range electromagnetic fields dominate individual particle interactions.
- And it's not only waves, single particle (guiding centre) drifts and fluid drifts also play a huge role in plasma dynamics.



Simple motion in a plasma

Plasma oscillations



- Let's find the oscillation frequency.
- Assumptions:
  1. No thermal motion
  2. No ion motion
  3. Infinite plasma
  4. Motion only in 1D
- Need to solve the following:

$$mn_e\left[\frac{\partial \mathbf{v_e}}{\partial t} + (\mathbf{v_e}\cdot\nabla)\mathbf{v_e}\right] = -en_e\mathbf{E}$$

$$\frac{\partial n_e}{\partial t} + \nabla\cdot(n_e\mathbf{v_e}) = 0$$

$$\varepsilon_0\nabla\cdot\mathbf{E} = e(n_i - n_e)$$

Linearisation is a key concept for deriving plasma quantities

- We can assume that the fields are made up of an "equilibrium" (or slow, subscript 0) part, and a "perturbation" (fast, subscript 1) part,

$$n_e = n_0 + n_1 \quad \mathbf{v}_e = \mathbf{v}_0 + \mathbf{v}_1 \quad \mathbf{E} = \mathbf{E}_0 + \mathbf{E}_1$$

and we've assumed a neutral, uniform rest plasma, then the only remaining slow time scale quantity is n_0, so

$$mn_e\left[\frac{\partial \mathbf{v_e}}{\partial t} + (\mathbf{v_e}\cdot\nabla)\mathbf{v_e}\right] = -en_e\mathbf{E}$$
$$\frac{\partial n_e}{\partial t} + \nabla\cdot(n_e\mathbf{v_e}) = 0$$
$$\varepsilon_0\nabla\cdot\mathbf{E} = e(n_i - n_e)$$

Linearise →

$$m\frac{\partial \mathbf{v_1}}{\partial t} = -e\mathbf{E_1}$$
$$\frac{\partial n_1}{\partial t} + n_0\nabla\cdot\mathbf{v_1} = 0$$
$$\varepsilon_0\nabla\cdot\mathbf{E_1} = en_1$$

Apply Fourier transform…

- Assume sinusoidal oscillations,

$$(\mathbf{v_1}, n_1, \mathbf{E_1}) = (v_1\mathbf{x}, n_1, E_1\mathbf{x})\exp[i(kx - \omega t)]$$

and take the Fourier transform, $\nabla \rightarrow ik\mathbf{x}$, $\partial/\partial t \rightarrow -i\omega$ so our equations become,

$$mn_e\left[\frac{\partial \mathbf{v_e}}{\partial t} + (\mathbf{v_e}\cdot\nabla)\mathbf{v_e}\right] = -en_e\mathbf{E}$$

$$\frac{\partial n_e}{\partial t} + \nabla\cdot(n_e\mathbf{v_e}) = 0$$

$$\varepsilon_0\nabla\cdot\mathbf{E} = e(n_i - n_e)$$

Linearise →

$$m\frac{\partial \mathbf{v_1}}{\partial t} = -e\mathbf{E_1}$$

$$\frac{\partial n_1}{\partial t} + n_0\nabla\cdot\mathbf{v_1} = 0$$

$$\varepsilon_0\nabla\cdot\mathbf{E_1} = en_1$$

Transform →

$$-im\omega v_1 = -eE_1$$

$$-i\omega n_1 = -n_0ikv_1$$

$$ik\varepsilon_0E_1 = -en_1$$

…and finally, we arrive at the plasma frequency

$$-im\omega v_1 = -eE_1$$
$$-i\omega n_1 = -n_0 ikv_1$$
$$ik\varepsilon_0 E_1 = -en_1$$

algebra

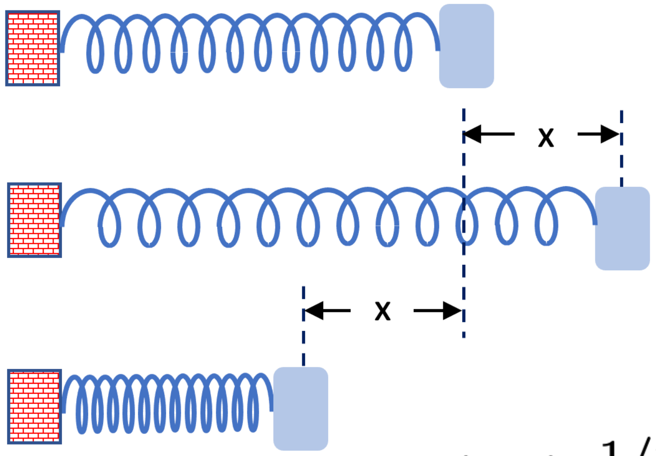
algebra

$$\omega_p = \left(\frac{nq^2}{\varepsilon_0 m}\right)^{1/2}$$

Each particle species will have its own characteristic plasma frequency given by ω_p

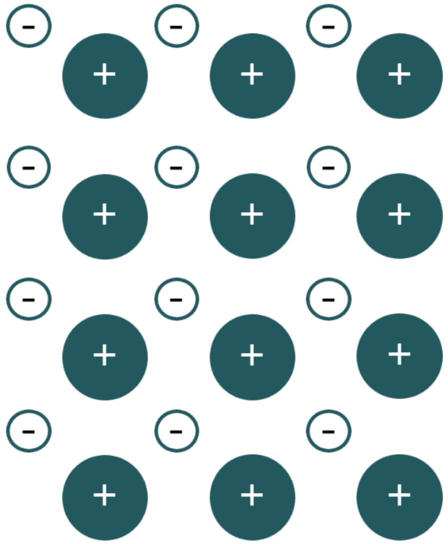
These types of oscillation show up a lot!

- Simple harmonic oscillation in a mass-spring system



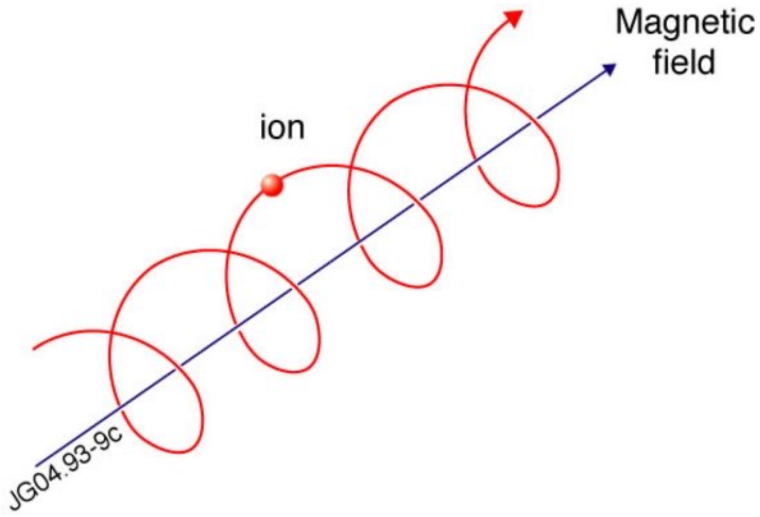
$$\omega = \left(\frac{k}{m}\right)^{1/2}$$

- Plasma oscillations



$$\omega_p = \left(\frac{nq^2}{\varepsilon_0 m}\right)^{1/2}$$

- Oscillations around a B field

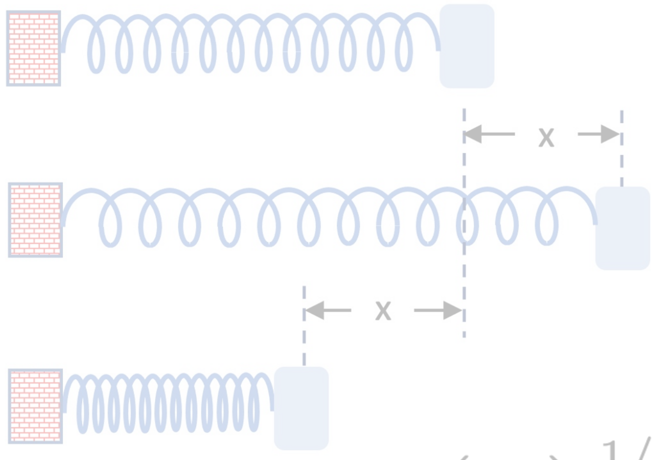


$$\Omega_c = \frac{|q|B}{m}$$

OAK RIDGE National Laboratory

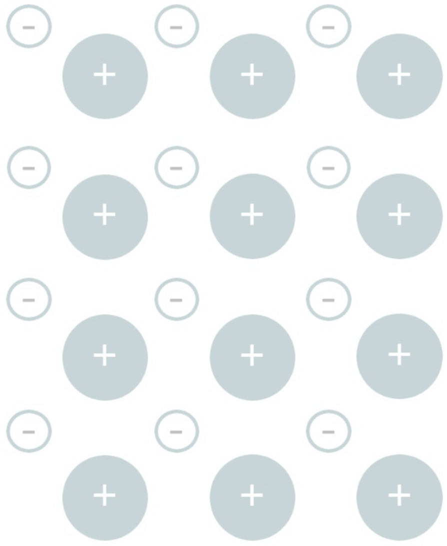
These types of oscillation show up a lot

- Simple harmonic oscillation in a mass-spring system



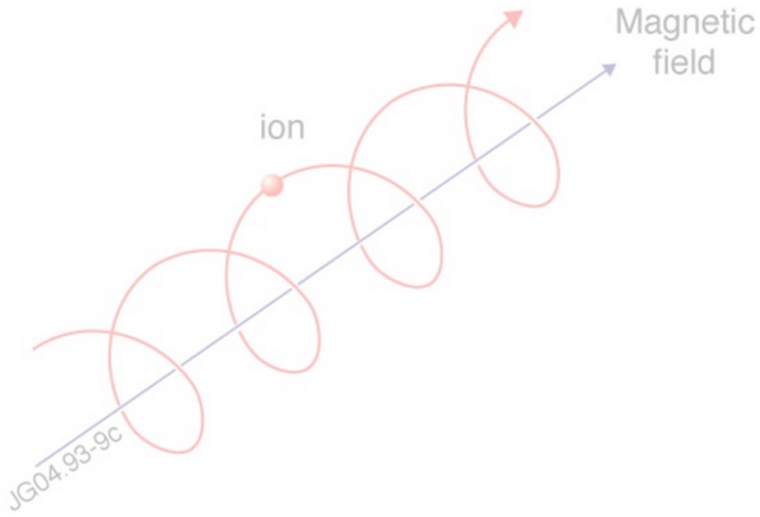
$$\omega = \left(\frac{k}{m}\right)^{1/2}$$

- Plasma oscillations



$$\omega_p = \left(\frac{nq^2}{\varepsilon_0 m}\right)^{1/2}$$

- Oscillations around a B field



$$\Omega_c = \frac{|q|B}{m}$$

You'll see these again soon!

Oak Ridge National Laboratory

The fluid description of a magnetised (cold) plasma

More general derivation will uncover many more waves!

Fluid equation of motion

$$m_j n_j \left(\frac{\partial \mathbf{v}_j}{\partial t} + \mathbf{v}_j \cdot \nabla \mathbf{v}_j \right) = q_j n_j (\mathbf{E} + \mathbf{v}_j \times \mathbf{B})$$

Maxwell's equations

$$\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t}$$

$$\nabla \times \mathbf{B} = \mu_0 \left(\mathbf{j} + \varepsilon_0 \frac{\partial \mathbf{E}}{\partial t} \right)$$

with

$$\mathbf{j} = \sum_j n_j q_j \mathbf{v}_j$$

Linearise

↓

Fourier transform

↓

Do some algebra…

The cold plasma wave equation

Linearise

Fourier transform

Do some algebra…

$$\mathbf{k} \times (\mathbf{k} \times \mathbf{E}) + k_0^2 \overline{\overline{\mathbf{K}}} \cdot \mathbf{E} = 0$$

Cold plasma dielectric tensor

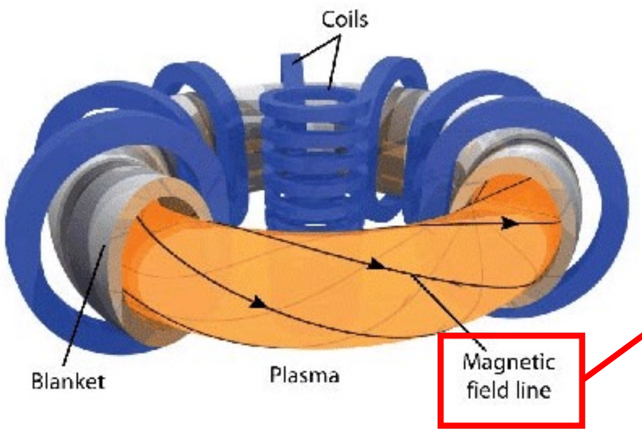
$$\overline{\overline{\mathbf{K}}} = \begin{pmatrix} S & -iD & 0 \\ iD & S & 0 \\ 0 & 0 & P \end{pmatrix}$$

- We can expand to get… (where $n = kc/\omega$)

$$\begin{pmatrix} S - n_{\parallel}^2 & -\mathrm{i}D & n_{\perp} n_{\parallel} \\ \mathrm{i}D & S - (n_{\perp}^2 + n_{\parallel}^2) & 0 \\ n_{\perp} n_{\parallel} & 0 & P - n_{\perp}^2 \end{pmatrix} \begin{pmatrix} E_x \\ E_y \\ E_z \end{pmatrix} = 0$$

An aside:

- For plasmas in a strong background magnetic field, let's define a convenient coordinate system:



∥: Parallel to B0

⊥: Perpendicular to B0

This leads to the cold plasma dispersion relation(s)

$$An_\perp^4 - Bn_\perp^2 + C = 0$$

$$A = S,$$
$$B = RL + PS - n_\parallel^2(P+S),$$
$$C = P(n_\parallel^2 - R)(n_\parallel^2 - L)$$

$$An_\parallel^4 - Bn_\parallel^2 + C = 0$$

$$A = P,$$
$$B = 2PS - n_\perp^2(P+S),$$
$$C = (n_\perp^2 - P)(Sn_\perp^2 - RL)$$

$$S \equiv \frac{1}{2}(R+L) = 1 - \sum_j \frac{\omega_{pj}^2}{\omega^2 - \Omega_{cj}^2}$$

$$D \equiv \frac{1}{2}(R-L) = \sum_j \frac{\Omega_{cj}\omega_{pj}^2}{\omega(\omega^2 - \Omega_{cj}^2)}$$

$$P = 1 - \sum_j \frac{\omega_{pj}^2}{\omega^2}$$

$$\omega_p = \left(\frac{nq^2}{\varepsilon_0 m}\right)^{1/2}$$

$$\Omega_c = \frac{|q|B}{m}$$

$$R \equiv S + D = 1 - \sum_j \frac{\omega_{pj}^2}{\omega(\omega + \Omega_{cj})}$$

$$L \equiv S - D = 1 - \sum_j \frac{\omega_{pj}^2}{\omega(\omega - \Omega_{cj})}$$

Oak Ridge National Laboratory

Why the dispersion relation is our favourite (?) tool

- The dispersion relation tells us the relationship between the spatial (**k**) and temporal (ω) properties of waves.
  - The dispersion relation contains a lot of essential information and is the same across disciplines (which often use different jargon!)
- Two key pieces of info from the dispersion relation:
  - **Cut off:** $n = 0,\ \lambda = \infty$ → the wave is evanescent
  - **Resonance:** $n = \infty, \lambda = 0$ → wave "piles up" locally

We'll see a dispersion relation showing these two phenomena in some later slides…

Brief summary of plasma waves

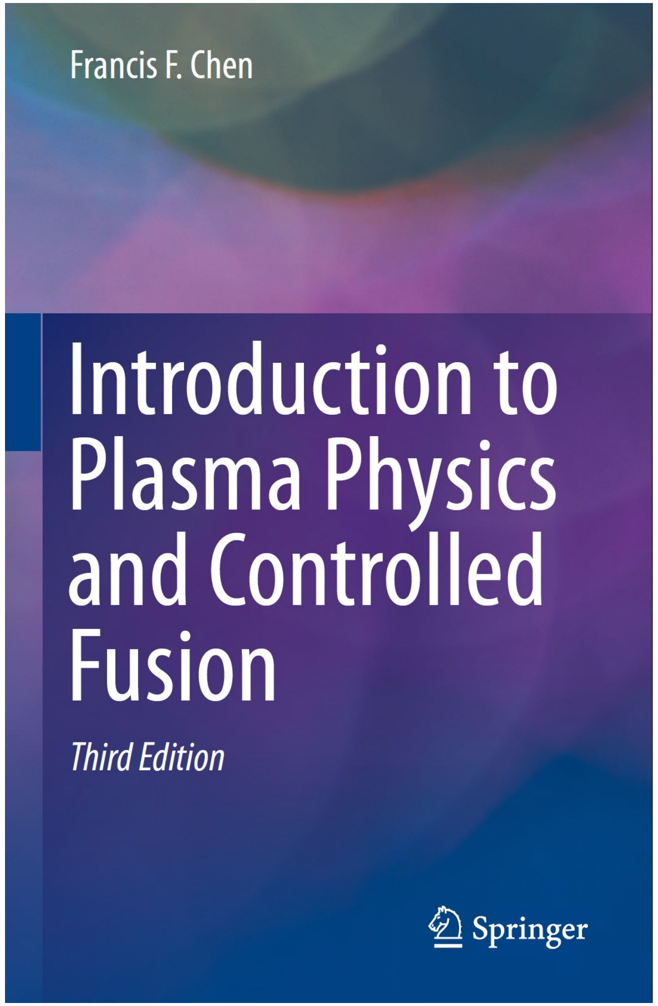
Waves with $\mathbf{k} \parallel \mathbf{B_0}$

- **Electrostatic:**
  - Electron plasma waves
  - Acoustic waves
- **Electromagnetic:**
  - R waves
  - L waves
  - Alfven waves

Waves with $\mathbf{k} \perp \mathbf{B_0}$

- **Electrostatic:**
  - Upper hybrid oscillations
  - Lower hybrid oscillations
  - Ion cyclotron waves
- **Electromagnetic:**
  - O wave
  - X wave
  - Magnetosonic wave

A helpful reference



4.20 Summary of Elementary Plasma Waves

Electron waves (electrostatic)

$\mathbf{B}_0 = 0$ or $\mathbf{k} \parallel \mathbf{B}_0$: $\quad \omega^2 = \omega_p^2 + \frac{3}{2}k^2 v_{\text{th}}^2$ (Plasma oscillations) (4.143)

$\mathbf{k} \perp \mathbf{B}_0$: $\quad \omega^2 = \omega_p^2 + \omega_c^2 = \omega_h^2$ (Upper hybrid oscillations) (4.144)

Ion waves (electrostatic)

$\mathbf{B}_0 = 0$ or $\mathbf{k} \parallel \mathbf{B}_0$: $\quad \omega^2 = k^2 v_s^2 = k^2 \frac{\gamma_e K T_e + \gamma_i K T_i}{M}$ (Acoustic waves) (4.145)

$\mathbf{k} \perp \mathbf{B}_0$: $\quad \omega^2 = \Omega_c^2 + k^2 v_s^2$ (Electrostatic ion cyclotron waves) (4.146)

or

$\omega^2 = \omega_l^2 = \Omega_c \omega_c$ (Lower hybrid oscillations) (4.147)

Electron waves (electromagnetic)

$\mathbf{B}_0 = 0$: $\quad \omega^2 = \omega_p^2 + k^2 c^2$ (Light waves)

$\mathbf{k} \perp \mathbf{B}_0, \ \mathbf{E}_1 \parallel \mathbf{B}_0$: $\quad \frac{c^2 k^2}{\omega^2} = 1 - \frac{\omega_p^2}{\omega^2}$ (**O** wave)

$\mathbf{k} \perp \mathbf{B}_0, \ \mathbf{E}_1 \perp \mathbf{B}_0$: $\quad \frac{c^2 k^2}{\omega^2} = 1 - \frac{\omega_p^2}{\omega^2} \frac{\omega^2 - \omega_p^2}{\omega^2 - \omega_h^2}$ (**X** wave)

$\mathbf{k} \parallel \mathbf{B}_0$: $\quad \frac{c^2 k^2}{\omega^2} = 1 - \frac{\omega_p^2/\omega^2}{1 - (\omega_c/\omega)}$ (*R* wave) (whistler mode) (4.151)

$\frac{c^2 k^2}{\omega^2} = 1 - \frac{\omega_p^2/\omega^2}{1 + (\omega_c/\omega)}$ (*L* wave) (4.152)

Ion waves (electromagnetic)

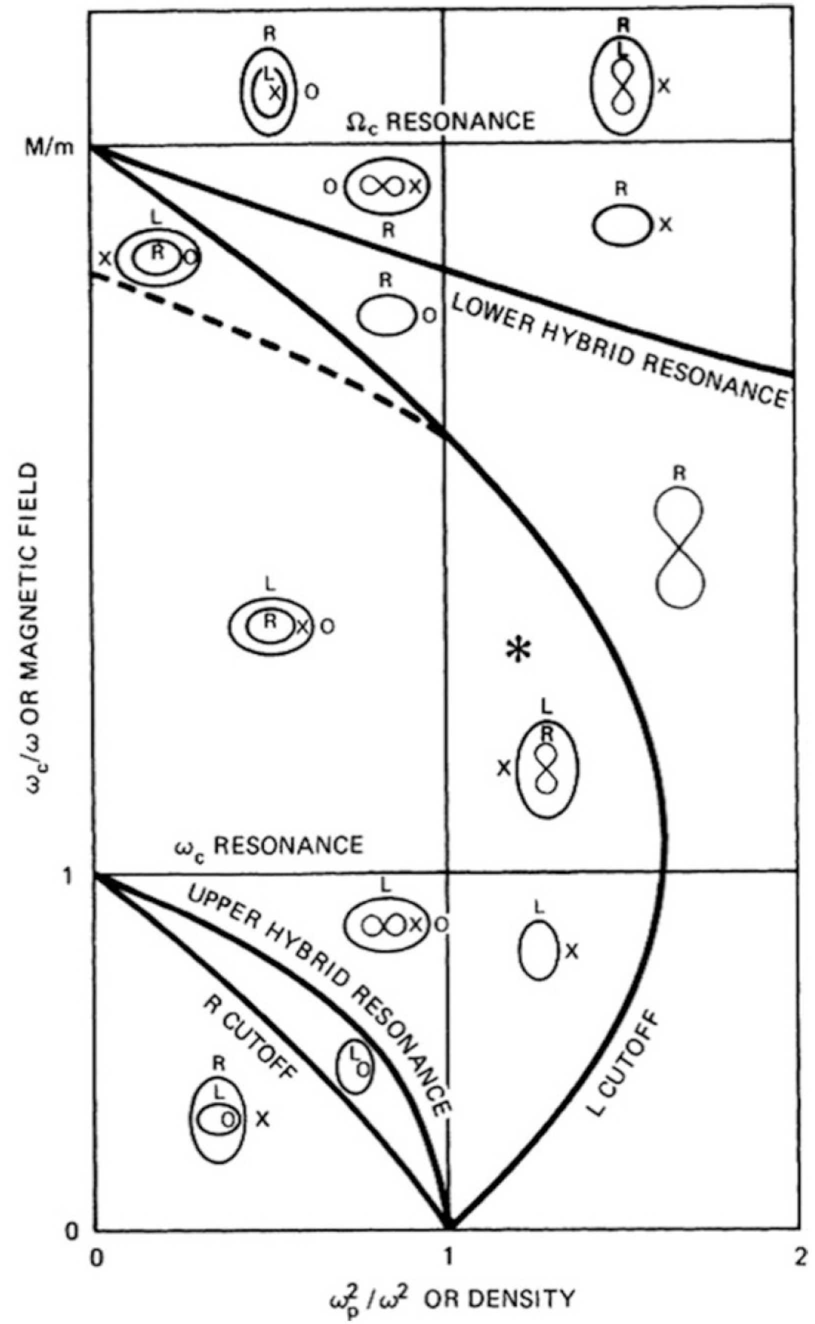
$\mathbf{B}_0 = 0$: None

$\mathbf{k} \parallel \mathbf{B}_0$: $\quad \omega^2 = k^2 v_A^2$ (Alfvén wave) (4.153)

$\mathbf{k} \perp \mathbf{B}_0$: $\quad \frac{\omega^2}{k^2} = c^2 \frac{v_s^2 + v_A^2}{c^2 + v_A^2}$ (Magnetosonic wave) (4.154)

Welcome to the zoo!

- Thinking of wave propagation as purely parallel or perpendicular is elegant but can hide some rich dynamics.
  - Modes can continuously convert into each other.
- The Clemmow–Mullaly–Allis (CMA)diagram: best friend or worst enemy?
- Don’t forget… this is only for COLD plasma waves!



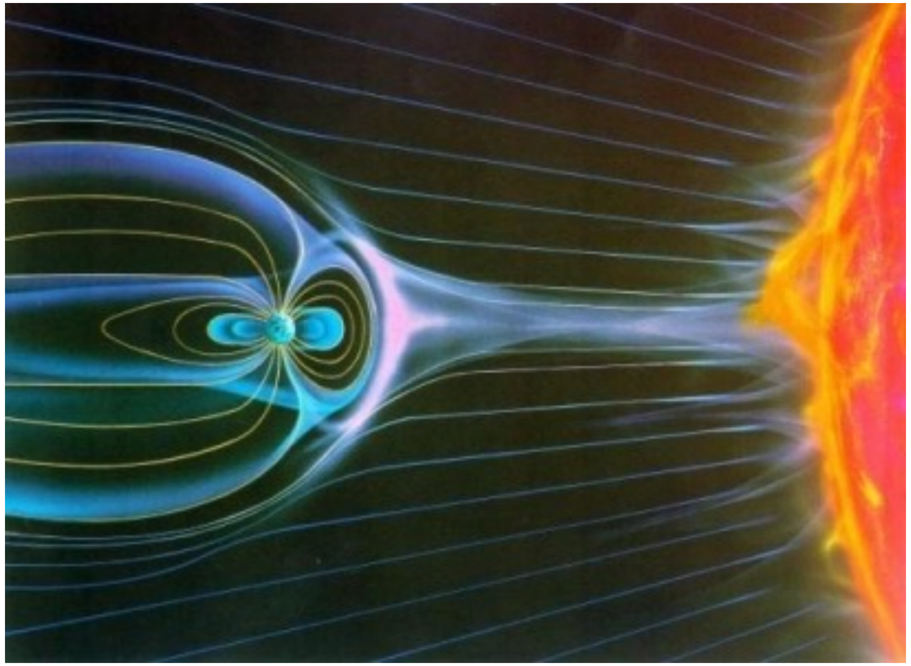
Oak Ridge National Laboratory

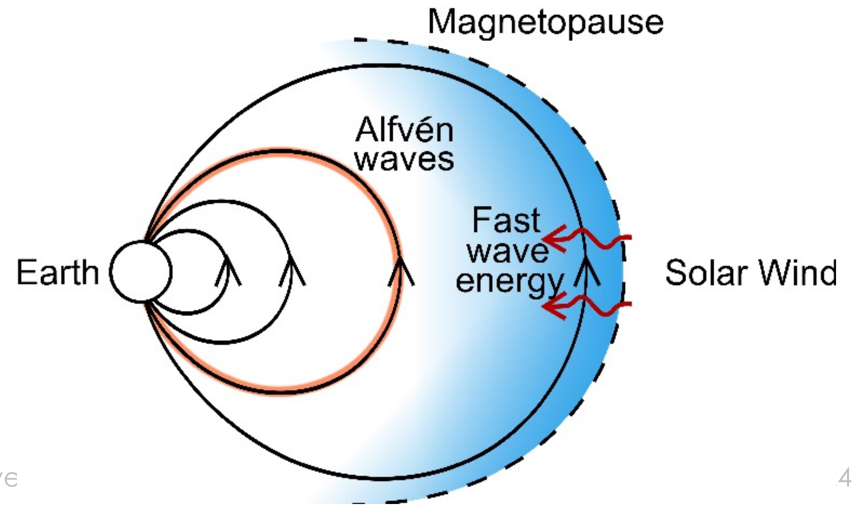
Oak Ridge National Laboratory

Quick example of waves in space plasma

A different kind of resonance!

- Solar wind compresses Earth's B field.
- A magnetosonic wave (compressional Alfven) travels towards Earth.
- This wave energy can couple to a field line resonance (FLR) in the magnetospheric cavity.
  - Imagine someone has plucked one of Earth's magnetic field lines like a guitar string!
- We can measure these fluctuations using magnetometers…



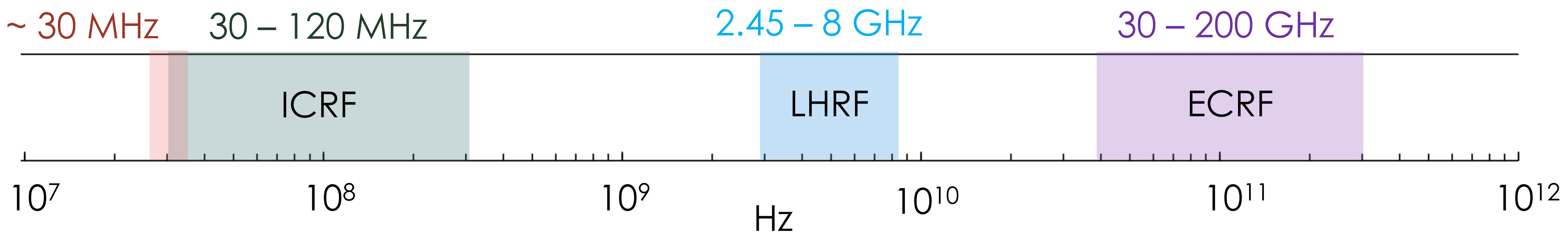


Intermission (aka please let me tell you something cool 😎)

Using waves to heat fusion plasmas

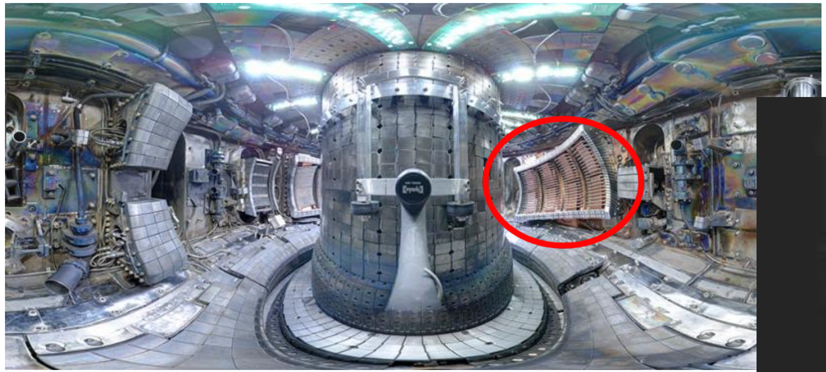
The different radio-frequency (RF) regimes used for heating

- Ion cyclotron range of frequencies (ICRF)
- High harmonic fast wave (HHFW)
- Lower hybrid range of frequencies (LHRF)
- Electron cyclotron range of frequencies (ECRF)

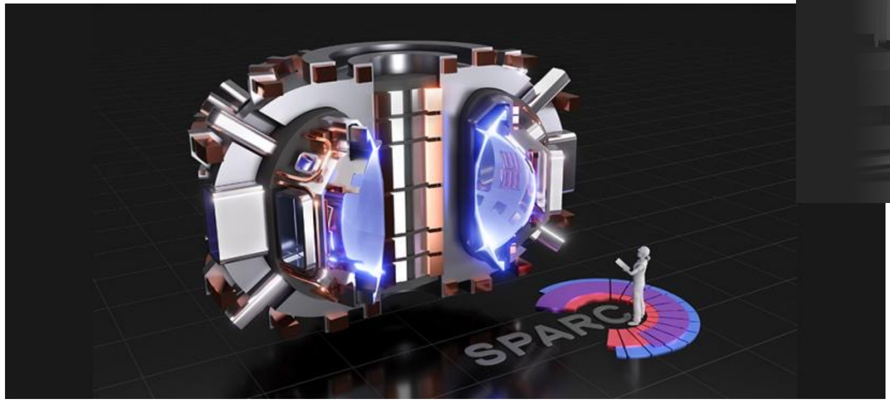


ICRF will continue to play a critical role in fusion devices

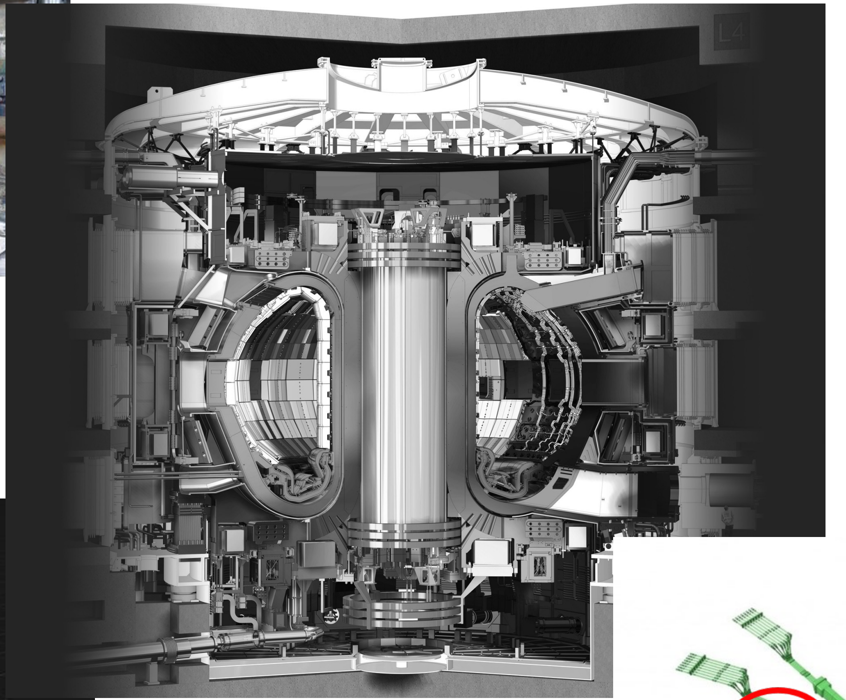
Alcator C-Mod



SPARC

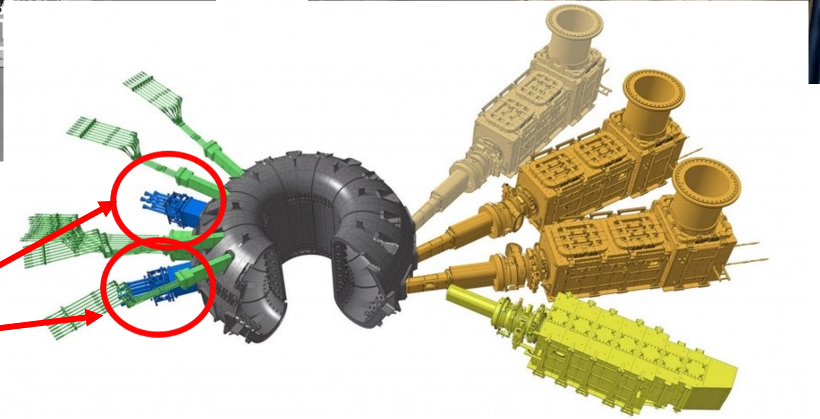


ITER



NSTX-U

ICRF systems in blue



OAK RIDGE National Laboratory

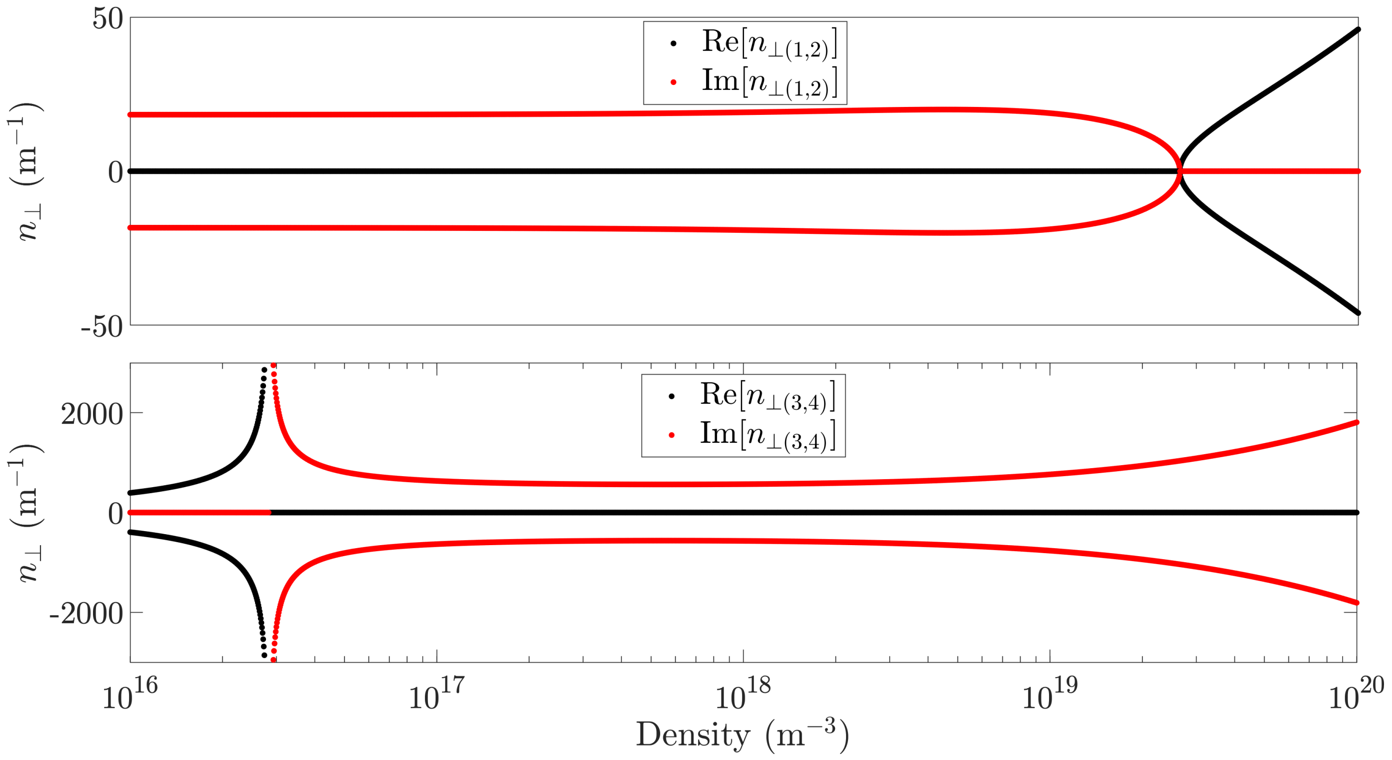
The two ICRF cold plasma wave modes

$$n^2_{\perp,F} = -\frac{(n^2_\parallel - R)(n^2_\parallel - L)}{(n^2_\parallel - S)}$$

$$n^2_{\perp,S} = -(n^2_\parallel - S)\frac{P}{S}$$

- For waves with $\omega \ll \omega_{p,e}$ (both ICRF and LH waves), the dispersion relation can be simplified.
- Now have two wave modes, termed the "fast" and the "slow" waves.
  - This is thanks to their relative phase velocities.
- For ICRF heating, we want to launch a fast wave.

Dispersion relation for the fast and slow waves



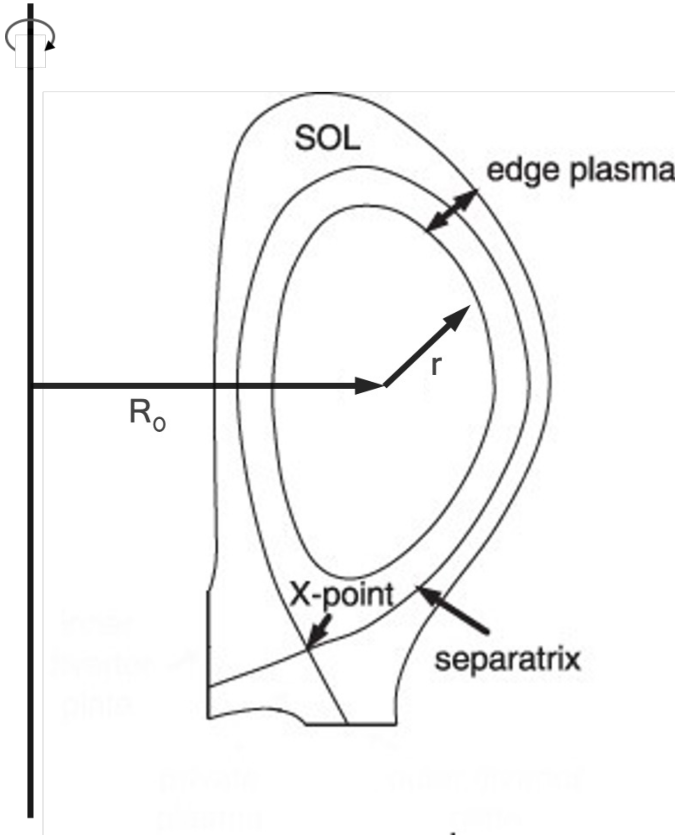
$$n_{\perp,F}^2 = -\frac{(n_\parallel^2 - R)(n_\parallel^2 - L)}{(n_\parallel^2 - S)}$$

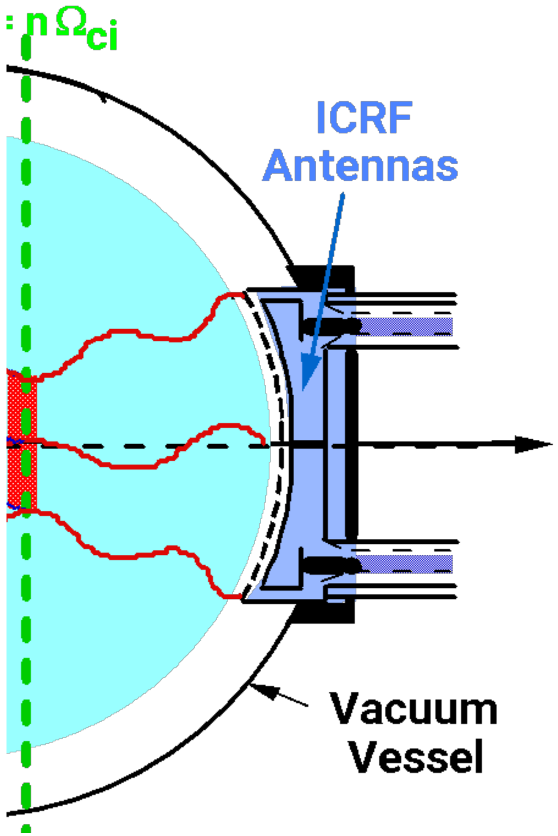
$$n_{\perp,S}^2 = -(n_\parallel^2 - S)\frac{P}{S}$$

$f = 52$ MHz, $\quad$ $B_0 = 2.5$ T, $\quad$ $m_i = 1.67\times10^{-27}$ kg

Experiments did not show expected power delivery ☹

- Coupling wave power into the core plasma is reasonably well understood…
- Getting the power to the core is the challenge!
- Nonlinear interactions become important when the stored energy is low.
  - This is usually the case close to the RF antenna.

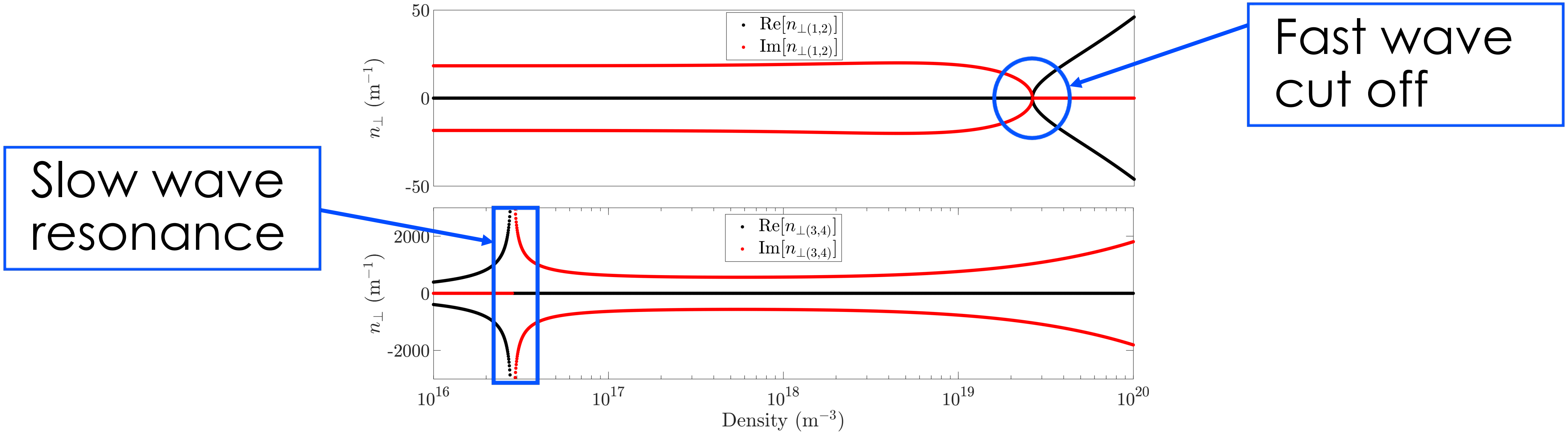




OAK RIDGE National Laboratory

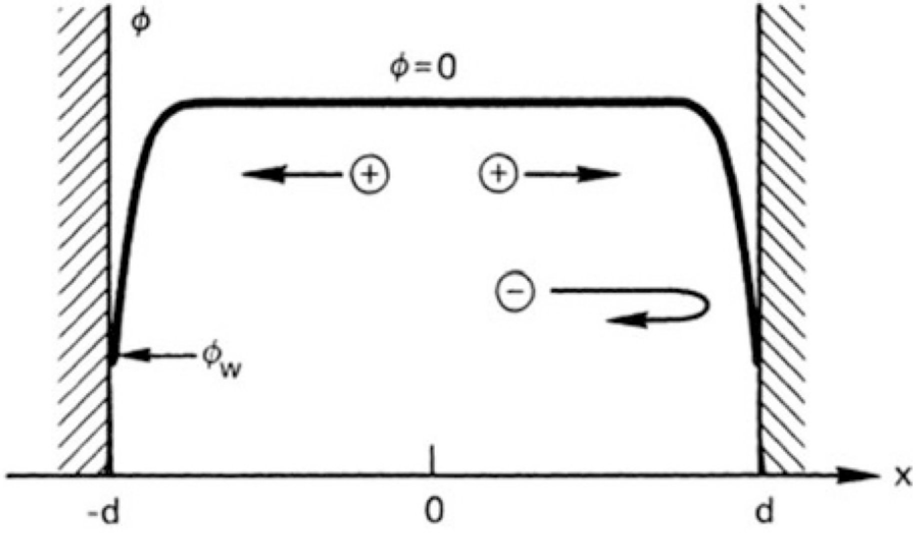
Plasma properties where heating happens are necessarily different than near the device surfaces…

- The scrape off layer (SOL) of a tokamak plasma is colder and less dense.

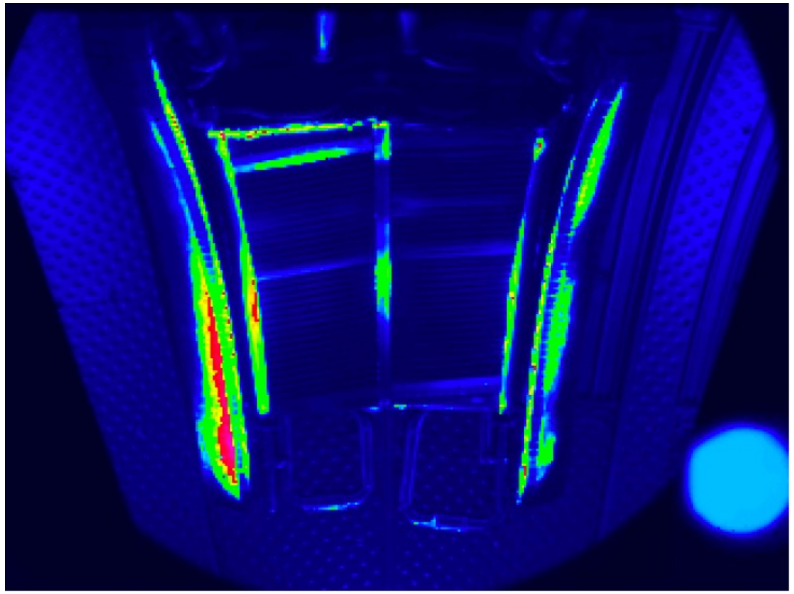


RF rectified sheaths

- A region of finite potential is formed wherever the plasma contacts any plasma facing component.
- This "sheath" acts to accelerate ions to the wall.
  - Ensures that equal number of ions and electrons hit the wall at the same time.
- This effect is enhanced during RF operation, sometimes on the order of kV!
- The increased sheath potential is attributed to a large $E_{\parallel}$.
  - Characteristic of the slow wave.



Myra et al., Nucl. Fusion, 2006



IR camera image of the antenna on Tore Supra after 15 s operation at 8MW ICRF power.

OAK RIDGE National Laboratory

Plasma density modification during ICRF operation

- Wave propagation in a plasma is governed by the density.
- Turns out that the wave energy can also *modify* the density!
  - EXB drifts, ponderomotive force…

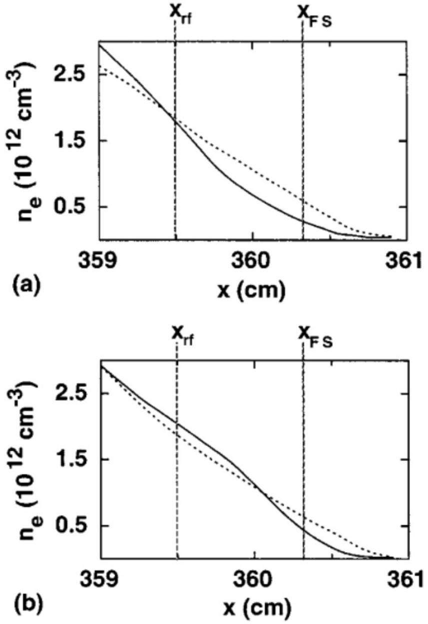
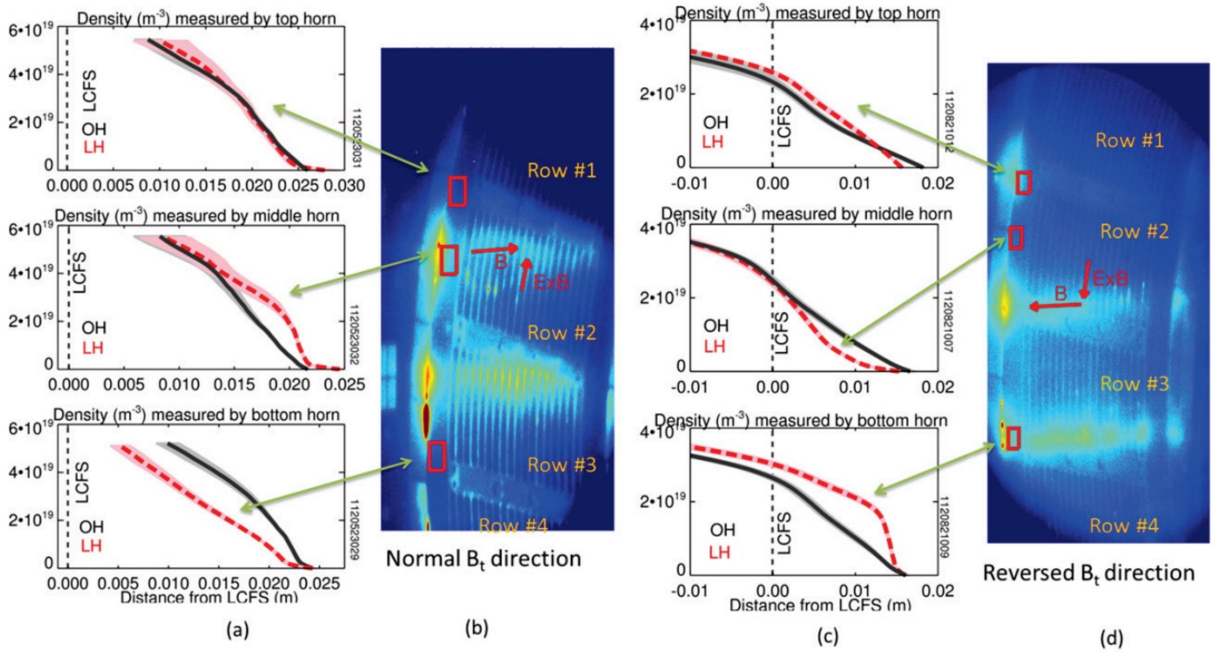


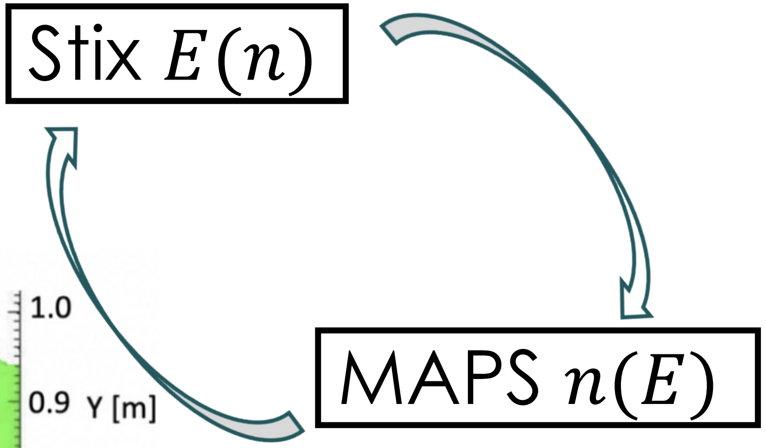
Figure (left): Measured density in front of TFTR antenna for ohmic (dotted line) and RF (solid line) operation, for different antenna phasing. From D'Ippolito D.A. et al 1998, Nucl. Fusion 38, 1543.

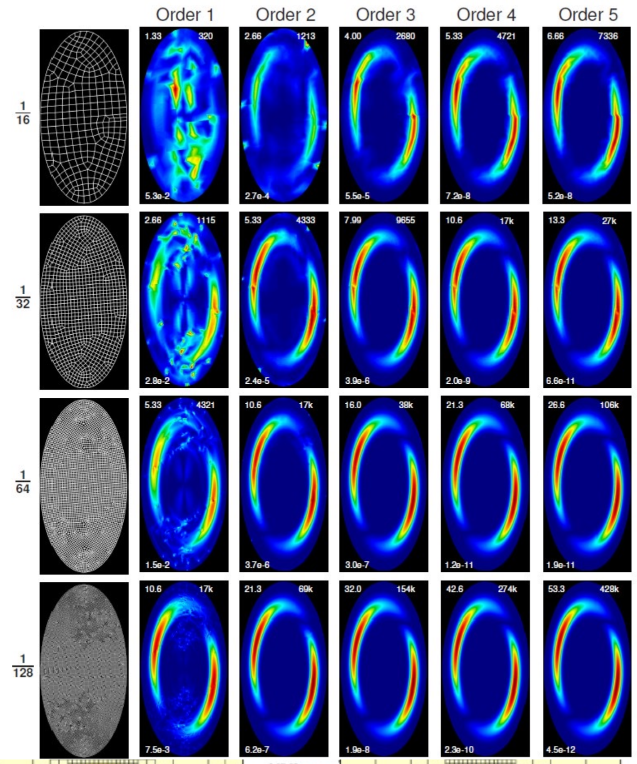
Reflectometer density profiles from three different reflectometry horns and video camera views of the LH launcher: (a) and (b) are for the normal field direction while (c) and (d) are for reversed field. From Lau et al, 2014, AIP Conf. Proc. 1580, 410–413



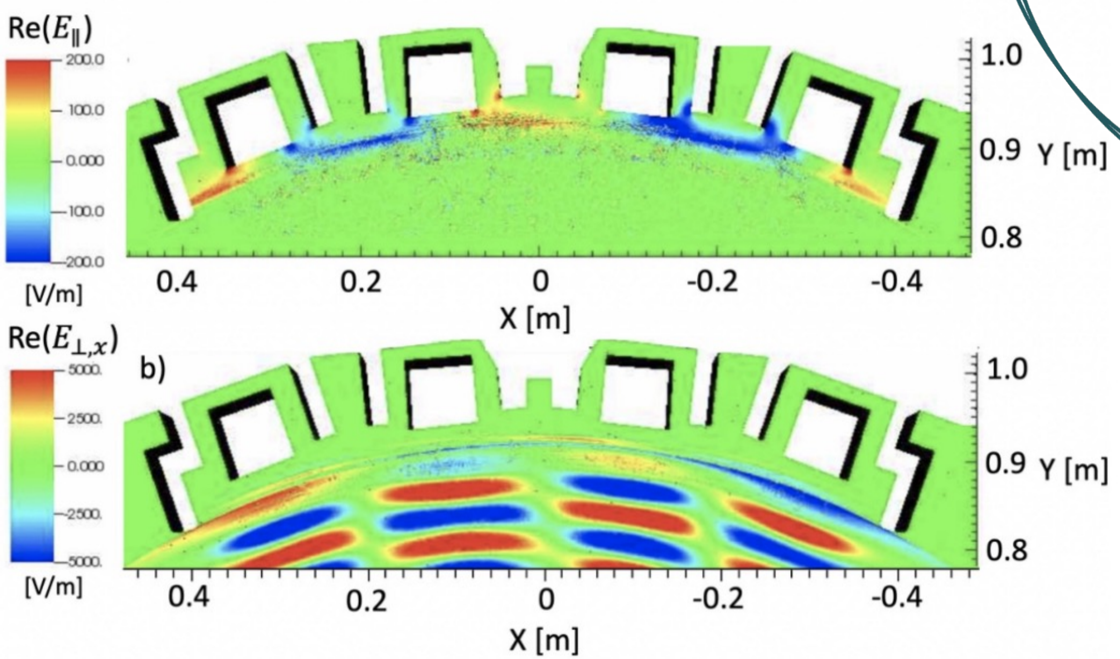
Nonlinear, coupled RF/plasma density system requires dedicated tools to solve

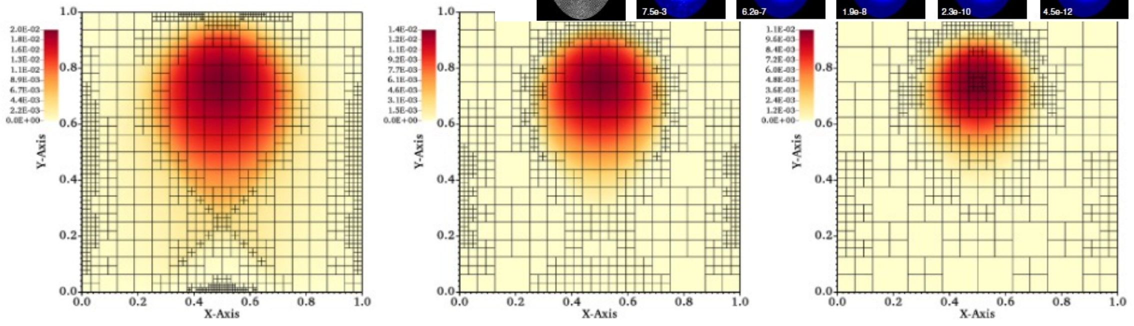
MFEM C++ library for finite element methods





C. Migliore et al 2023 Nucl. Fusion 63 106006





National Laboratory

Plasma waves in a cold (or any temp!) plasma are complex and dynamic

- But that's what makes them so fun…
- We focused mostly on cold plasma wave propagation, but there are many other types of plasma in magnetically confined fusion devices, e.g., kinetic plasmas.
- There are also many nonlinear effects, examples being RF rectified sheath potentials, ponderomotive drifts, EXB drifts…
  - We didn't get into the maths for those, it can be quite challenging.