An introduction to stellarators







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June 12, 2023





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Outline

- Magnetic confinement without symmetry
- Ingredients of stellarator confinement
- How do we "cook" a stellarator?
- Stellarators in context

Straight, uniform magnetic field provides perpendicular confinement



Straight, uniform magnetic field provides perpendicular confinement

Avoid end losses!



Toroidal magnetic confinement of charged particles *Step 1: Continuously nested toroidal surfaces*



"Magnetic surfaces" (const ψ)

Toroidal magnetic confinement of charged particles *Step 2: Rotational transform*



Toroidal magnetic confinement of charged particles *Step 2: Rotational transform*



"rotational transform"

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Magnetic confinement without symmetry The solution: rotational transform



"rotational transform"

"safety factor"











Magnetic confinement without symmetry The solution: rotational transform



"rotational transform"

"safety factor"













$\iota \approx \text{current} + \text{rotating ellipticity} + \text{axis torsion}$







 $\iota \approx \text{current} + \text{rotating ellipticity} + \text{axis torsion}$

Spitzer's figure-eight





This formed the basis for early stellarator experiments

Figure-eight (Princeton Model A) – 1953-1958



C. H. Willis, NJ Project Matterhorn (1953).



Racetrack (Princeton Model C) – 1962-1969



nature

Measurement of the Electron Temperature by Thomson Scattering in Tokamak T3

by N. J. PEACOCK D. C. ROBINSON M. J. FORREST P. D. WILCOCK UKAEA Research Group, Culham Laboratory, Abingdon, Berkshire

V. V. SANNIKOV

I. V. Kurchatov Institute, Moscow Electron temperatures of 100 eV up to I keV and densities in the range $I{-}3\times10^{13}~cm^{-3}$ have been measured by Thomson scattering on Tokamak T3. These results agree with those obtained by other techniques where direct comparison has been possible.



The tokamak enters the scene



nature

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Magnetic field integrability

NCSX Modular Coils



D. Strickler et al, IAEA (2004).

Magnetic field integrability

NCSX Modular Coils



Magnetic field integrability – Hamiltonian nature of field line flow

 $A(\rho, \theta, \phi) = \rho \nabla \theta - \chi(\rho, \theta, \phi) \nabla \phi$ $B(\rho, \theta, \phi) = \nabla \rho \times \nabla \theta - \nabla \chi \times \nabla \phi$



Magnetic field integrability – Hamiltonian nature of field line flow

 $A(\rho, \theta, \phi) = \rho \nabla \theta - \chi(\rho, \theta, \phi) \nabla \phi$ $B(\rho, \theta, \phi) = \nabla \rho \times \nabla \theta - \nabla \chi \times \nabla \phi$

$$\frac{d\theta}{d\phi} = \frac{B \cdot \nabla \theta}{B \cdot \nabla \phi} = \frac{\partial \chi(\rho, \theta, \phi)}{\partial \rho}$$
$$\frac{d\rho}{d\phi} = \frac{B \cdot \nabla \rho}{B \cdot \nabla \phi} = -\frac{\partial \chi(\rho, \theta, \phi)}{\partial \theta}$$

Hamiltonian $\rightarrow \chi(\rho, \theta, \phi)$ coordinate $\rightarrow \theta$ momentum $\rightarrow \rho$ time $\rightarrow \phi$



Magnetic field integrability – Hamiltonian nature of field line flow

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Hamiltonian $\rightarrow \chi(\rho, \theta, \phi)$ coordinate $\rightarrow \theta$ momentum $\rightarrow \rho$ time $\rightarrow \phi$

$$rac{\partial \chi(
ho, heta,\phi)}{\partial \phi} = 0
ightarrow$$
 integrability
 $m{B} \cdot
abla \chi = 0
ightarrow \chi$ is a magnetic surface labe

Magnetic field integrability – Hamiltonian nature of field line flow

$$\boldsymbol{B}(\rho,\theta,\phi) = \nabla \rho \times \nabla \theta - \nabla \chi(\rho,\theta,\phi) \times \nabla \phi$$

$$\chi(\rho, \theta, \phi) = \frac{1}{2}\rho^2 + \epsilon \cos(2\theta - \phi)$$

Integrable Non-integrable



Magnetic field integrability – Hamiltonian nature of field line flow

$$\boldsymbol{B}(\rho,\theta,\phi) = \nabla \rho \times \nabla \theta - \nabla \chi(\rho,\theta,\phi) \times \nabla \phi$$

$$\chi(\rho,\theta,\phi) = \frac{1}{2}\rho^2 + \epsilon \left[\cos(4\theta - \phi) + \cos(4\theta - 2\phi) + \cos(4\theta - 3\phi)\right]$$



Continuous toroidal symmetry yields particle confinement

Single-particle Lagrangian

$$\mathcal{L}(\boldsymbol{x}, \dot{\boldsymbol{x}}) = m \frac{|\dot{\boldsymbol{x}}|^2}{2} + q \boldsymbol{A}(\boldsymbol{x}) \cdot \dot{\boldsymbol{x}}$$

$$\frac{d}{dt}\left(\frac{\partial \mathcal{L}(R,\phi,Z,\dot{R},\dot{\phi},\dot{Z},t)}{\partial \dot{\phi}}\right) = \frac{\partial \mathcal{L}(R,\phi,Z,\dot{R},\dot{\phi},\dot{Z},t)}{\partial \phi}$$



Continuous toroidal symmetry yields particle confinement



Continuous toroidal symmetry yields particle confinement

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Confinement to ψ surfaces!

The zoology of particle orbits in 3D fields

Perturbed tokamak



ripple trapped



D.A. Spong et al, APS DPP (2014).

The zoology of particle orbits in 3D fields



D.A. Spong et al, APS DPP (2014).

Perturbed tokamak



ripple trapped



The zoology of particle orbits in 3D fields



D.A. Spong et al, APS DPP (2014).

Can we confine them all?

Perturbed

tokamak



ripple trapped



Collisional guiding center confinement



Collisional guiding center confinement



Symmetry of field strength yields particle confinement in 3D ⁸¹⁴

$$\mathcal{L}(\boldsymbol{x}, \dot{\boldsymbol{x}}) = m \frac{|\dot{\boldsymbol{x}}|^2}{2} + q\boldsymbol{A}(\boldsymbol{x}) \cdot \dot{\boldsymbol{x}}$$

$$\int \mathbf{Strongly \, magnetized}$$

$$\mathcal{L}(\psi, \theta, \phi, \dot{\psi}, \dot{\theta}, \dot{\phi}) = \mathcal{L}(\psi, \boldsymbol{B}(\psi, \theta, \phi), \dot{\psi}, \dot{\theta}, \dot{\phi})$$



Symmetry of field strength yields particle confinement in 3D ⁸¹⁵

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Quasisymmetry - a hidden symmetry of magnetic fields

Quasi-poloidal symmetry



Quasisymmetry - a hidden symmetry of magnetic fields







Quasisymmetry - a hidden symmetry of magnetic fields



...and many more

- ✓MHD stability
- ✓ Collisional "bootstrap" current
- ✓ Energetic particle confinement
- ✓ Equilibrium β limit
- ✓ Divertor configuration
- ✓ Reduction of turbulent transport
- ✓ Coil feasibility

...and many more

- ✓MHD stability
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- ✓ Energetic particle confinement ✓ Equilibrium β limit
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ASG Superconductors

- Magnetic confinement without symmetry
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Traditional two-step optimization



Boundary of equilibrium optimized for confinement

 $(\nabla \times \boldsymbol{B}) \times \boldsymbol{B} = \mu_0 \nabla p \quad \text{in } V_{\text{plasma}}$ $\boldsymbol{B} \cdot \boldsymbol{\hat{n}} \Big|_{S_{\text{plasma}}} = 0$

$$\min_{S_{\text{plasma}}} f(\boldsymbol{B}(S_{\text{plasma}}), S_{\text{plasma}})$$

Traditional two-step optimization



Traditional two-step optimization



Stellarator optimization has been experimentally verified *Helically Symmetric Experiment (HSX)*





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J. M. Canik et al, Phys. Plasmas 14, (2007).



Stellarator optimization at scale – Wendelstein 7-X



nature

Article

Demonstration of reduced neoclassical energy transport in Wendelstein 7-X

https://doi.org/10.1038/s41586-021-03687-w Received: 30 April 2020 Accepted: 2 June 2021 Published online: 11 August 2021

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Overview of stellarators & tokamaks

Tokamaks

- ✓ Automatic guiding center confinement
- ✓ Automatic integrability
- ✓ "Simpler" design
- × Requires large plasma current
- **X** Steady-state is challenging/inefficient

Stellarators

- × Confinement optimization required
- × Shaping by complicated coils/magnets
- ✓ Low recirculating power
- ✓ No plasma-terminating disruptions
- ✓ No Greenwald density limit

Plasma Phys. Control. Fusion 54 (2012) 124009 (12pp)

doi:10.1088/0741-3335/54/12/124009

Stellarator and tokamak plasmas: a comparison

P Helander, C D Beidler, T M Bird, M Drevlak, Y Feng, R Hatzky, F Jenko, R Kleiber, J H E Proll, Yu Turkin and P Xanthopoulos

Max-Planck-Institut für Plasmaphysik, Greifswald and Garching, Germany

Comparison with tokamak performance



R. C. Wolf et al, Phys. Plasmas 26, (2019).

Stellarators with "precise quasisymmetry" have been uncovered

B as a function of "Boozer" coordinates



Stellarators with "precise quasisymmetry" have been uncovered



Stellarators with "precise quasisymmetry" have been uncovered



φ

Ф

Stellarators with "precise quasisymmetry" have been uncovered



M. Landreman & E. J. Paul, Phys. Rev. Lett. 128 (2022).

Stellarators can be designed to confine energetic fusion products



Fraction of alpha-particle energy lost before thermalization

Open questions and new frontiers

Can we optimize stellarators for reduced turbulence?



J. Alucon et al, *PPCF* 62 (2020).

Can we simplify stellarator magnets?



C. Zhu et al, Nuclear Fusion, 60 (2020).

Why can stellarators routinely violate linear MHD stability limits?



A. Weller et al, Fusion Science & Tech., 50 (2006).

Can we optimize stellarators for reduced turbulence?



W7-X can operate in a "stability valley"

New theory to diagnose "available energy" for turbulence



J. Alucon et al, *PPCF* 62 (2020).

R. Mackenbach et al, PRL 128 (2022).

Why can stellarators routinely violate linear MHD stability limits?



LHD operation beyond linear ideal MHD threshold

A. Weller et al, Fusion Science & Tech., 50 (2006).

High-fidelity simulations leveraged at PPPL [A. Wright]



Can we simplify stellarator magnets?

PPPL projects use permanent magnets for confinement





C. Zhu et al, Phys. Plasmas, 29 (2022).

T. Qian et al, Nucl. Fusion, 62 (2022).

Growing private investment in stellarators







Princeton Stellarators





- Stellarators confine plasmas with non-symmetric magnetic fields
- Rotational transform (twisting field lines) can be produced through shaped magnetic fields
- Stellarators avoid headaches associated with having a large plasma current
- Although stellarators are not axisymmetric, they can be designed to have hidden symmetries
- Immense progress has been recently made, and there are exciting research opportunities

An introduction to stellarators: from magnetic fields to symmetries and optimization (arxiv:1908.05360)







- 1. Introduction
- 2. Background
- 3. Electric and magnetic fields: Maxwell's equations
- 4. Classical mechanics
- 5. Single particle motion in electromagnetic fields
- 6. Coordinate systems
- 7. Toroidal magnetic confinement
- 8. Coupling of particles and electromagnetic fields: MHD models
- 9. Magnetic coordinates
- 10. Challenges associated with 3D equilibrium fields
- 11. Models of 3D equilibrium magnetic fields
- 12. Symmetries in stellarators
- 13. Optimization for stellarator design
- 14. New frontiers of optimization

	MHD equilibrium	Force-free fields	Vacuum fields
	(surfaces assumed)		
Hyp.	$oldsymbol{J} imes oldsymbol{B} eq 0$	$oldsymbol{J} imes oldsymbol{B} = 0$	$oldsymbol{J}=0$
	abla p eq 0	$\nabla p = 0$ and $\lambda = \text{const.}$	abla p=0
	$oldsymbol{J} imes oldsymbol{B} = abla p$	$ abla imes oldsymbol{B} = \lambda oldsymbol{B}$	$\Delta \widetilde{\Phi}_B = 0$
PDE	$\nabla \cdot \boldsymbol{B} = 0$		
model	$\mu_0 oldsymbol{J} = abla imes oldsymbol{B}$	$\mu_0 oldsymbol{J} = abla imes oldsymbol{B}$	
Given	$p(\psi),\iota(\psi),\Psi_T$	λ, Ψ_T	I_P
Unkn.	В	В	$\widetilde{\Phi}_B$
With	$oldsymbol{J}$ function of $oldsymbol{B}$	$oldsymbol{J}$ function of $oldsymbol{B}$	$oldsymbol{B} = abla \Big(\widetilde{\Phi}_B + \left(\mu_0 I_P / 2 \pi ight) \phi \Big)$
BC	$oldsymbol{B}\cdot\hat{oldsymbol{n}}=0$	$oldsymbol{B}\cdot\hat{oldsymbol{n}}=0$	$\hat{m{n}}\cdot abla \widetilde{\Phi}_B + \left(\mu_0 I_P/2\pi ight)\hat{m{n}}\cdot abla \phi = m{B}\cdot\hat{m{n}}$



Join the stellarator crew at Columbia!

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Ph.D./Bridge







Faculty







