Plasmas as a fluid

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2023 Introduction to Fusion Energy and Plasma Physics Course, PPPL

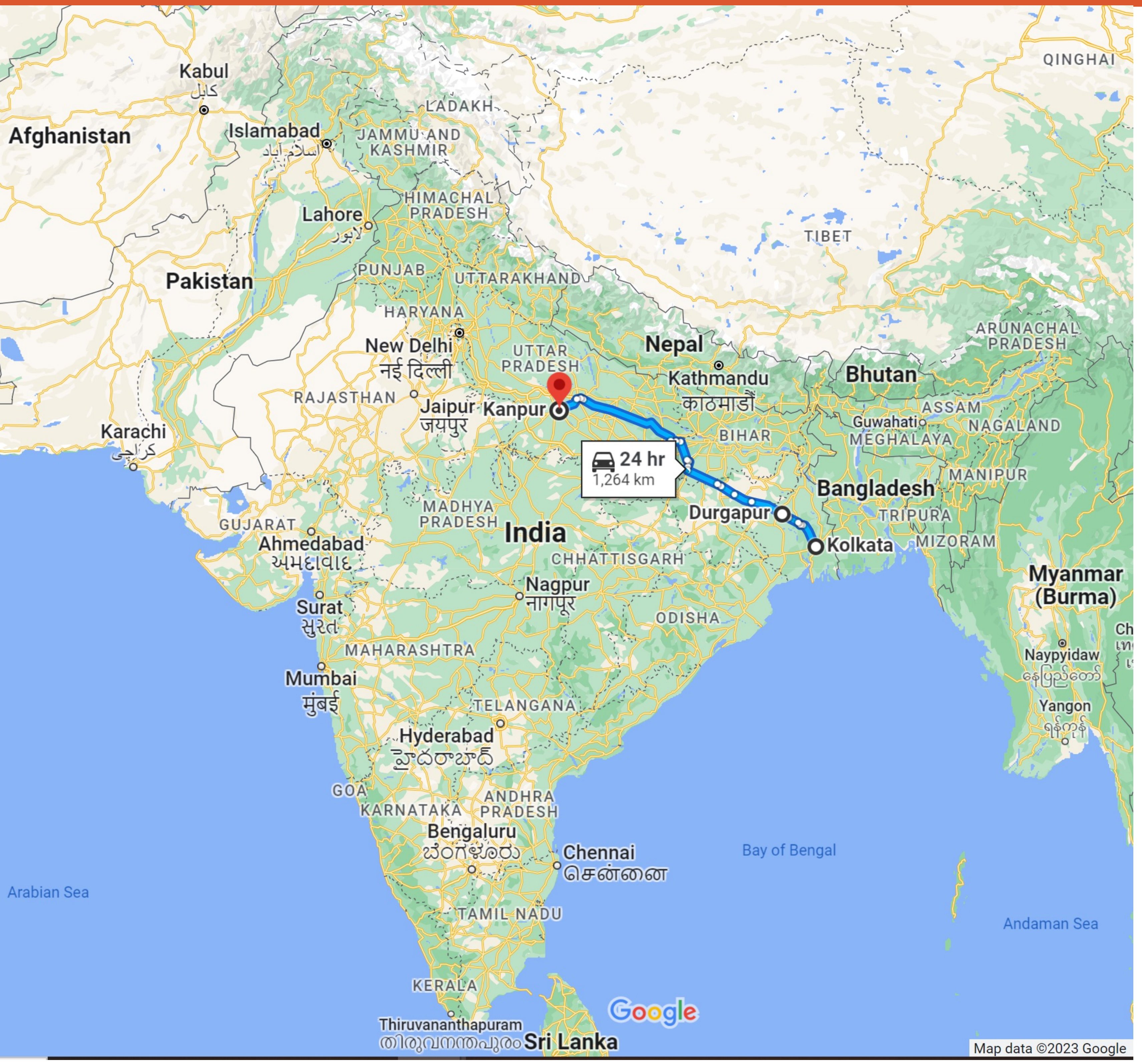
How does a scientist look like? What do they do?

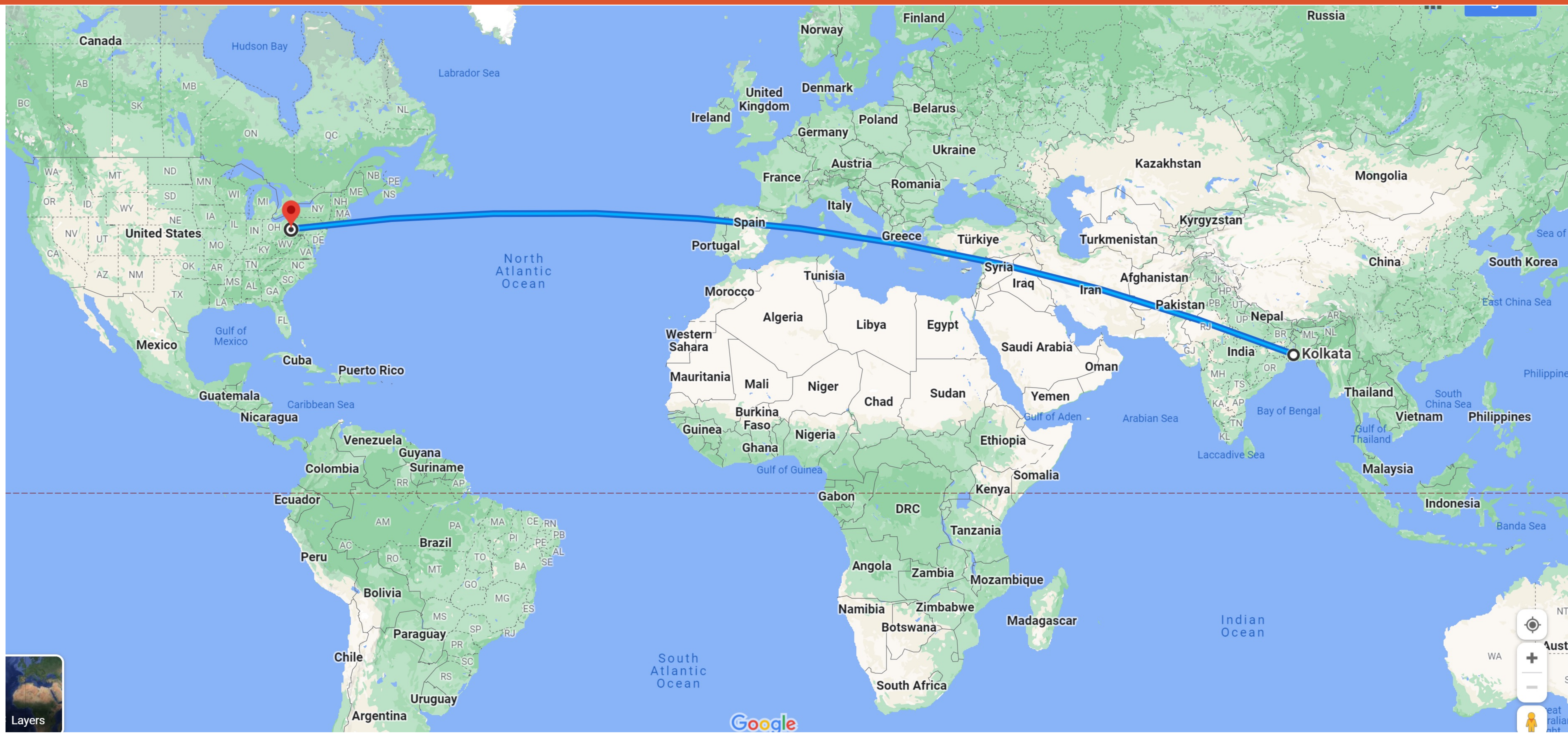
How does a scientist look like? (confused?); What do they do? (think!)

Too curious?
Too many questions?
Too confused?

Think!
Think!
Think!

My personal trajectory (transport!)





School in **Durgapur** => A mid-sized industrial town surrounded by small villages (till 10^{th} grade)

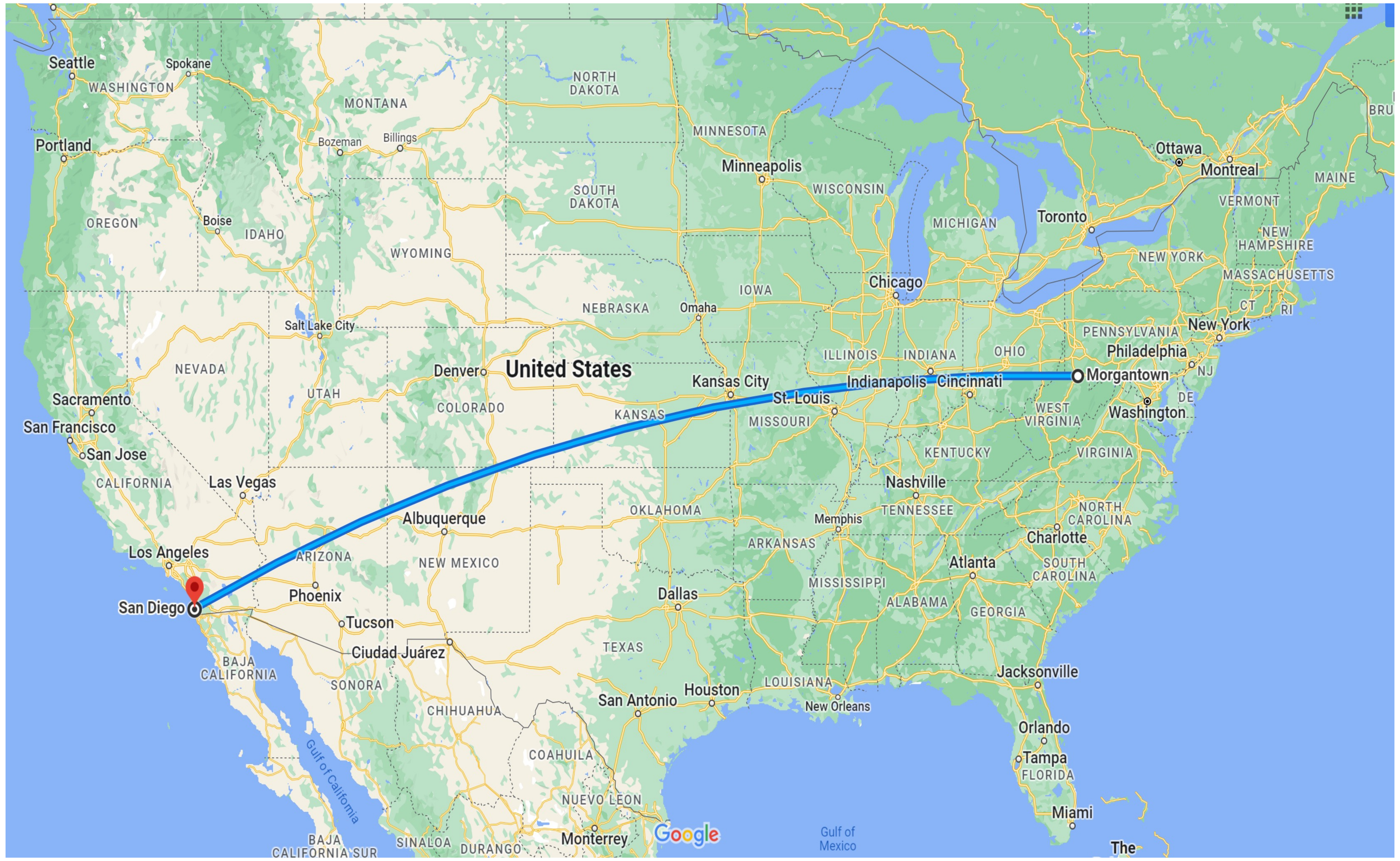
St. Xavier's College in **Kolkata** => A pretty big city (finished high school and Bachelors in Physics)

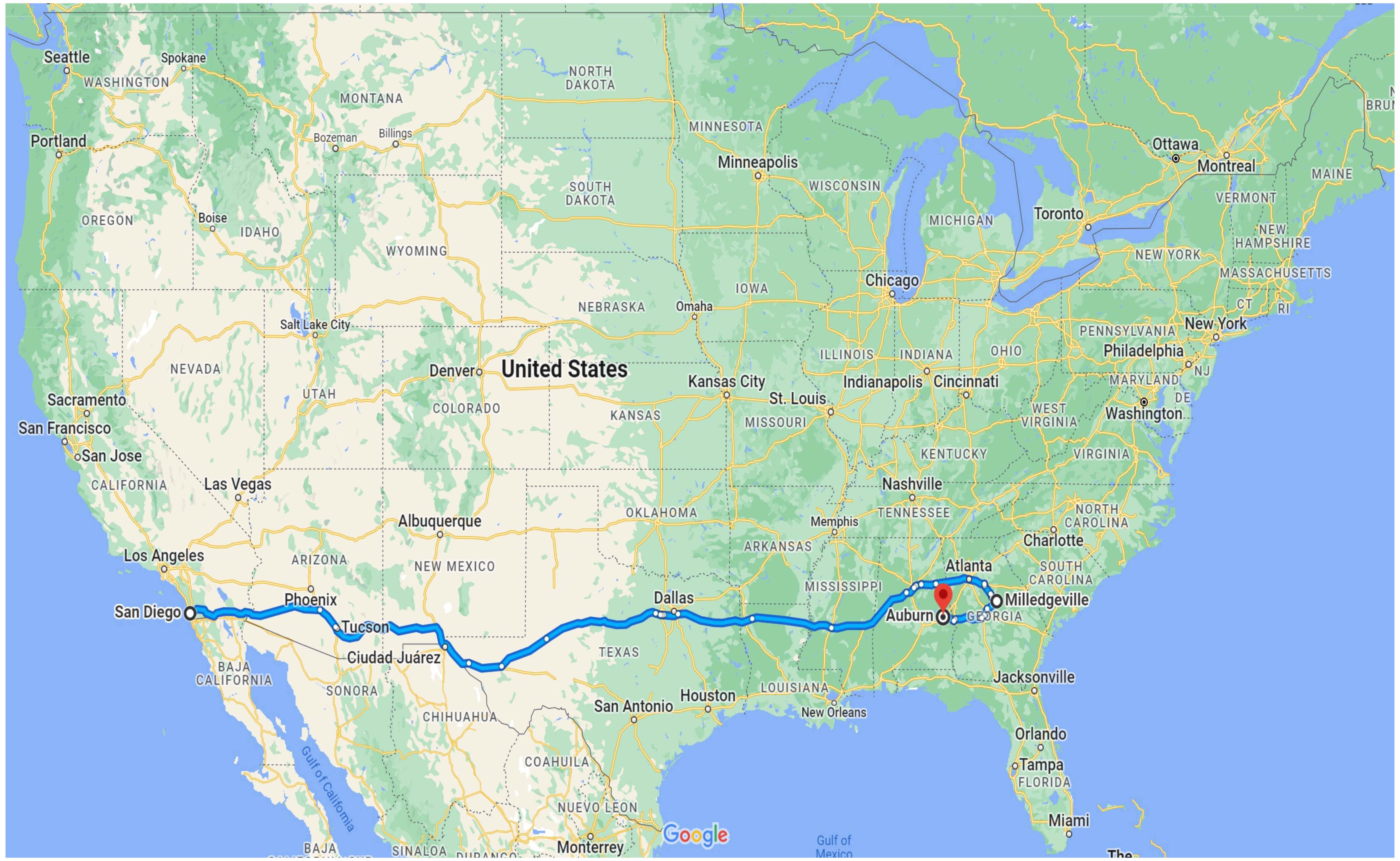
Indian Institute of Technology at **Kanpur** (IIT-Kanpur) => Masters in Physics

⇒ A pretty big jump to **Morgantown**, WV (my first ever flight! Happens to be the longest one!)

⇒ First in a large extended family to go for higher education beyond college (instead of playing soccer!)

My personal trajectory (transport!)





Ph.D. => West Virginia University at Morgantown (Go Mountaineers!) in 2010 (space plasma)

Post doctoral research scholar => Univ. of California at San Diego till 2014 (fusion edge plasma)

... continued as a Project Scientist => Univ. of California at San Diego till 2020 (divertor plasma)

Research Faculty (I get to play with cool machines and awesome students) => Auburn University (LTP)

Our trajectory of questioning and thinking for today (heuristic approach)

$\Rightarrow$ What is a “fluid”?

$\Rightarrow$ How can we describe a given plasma system?

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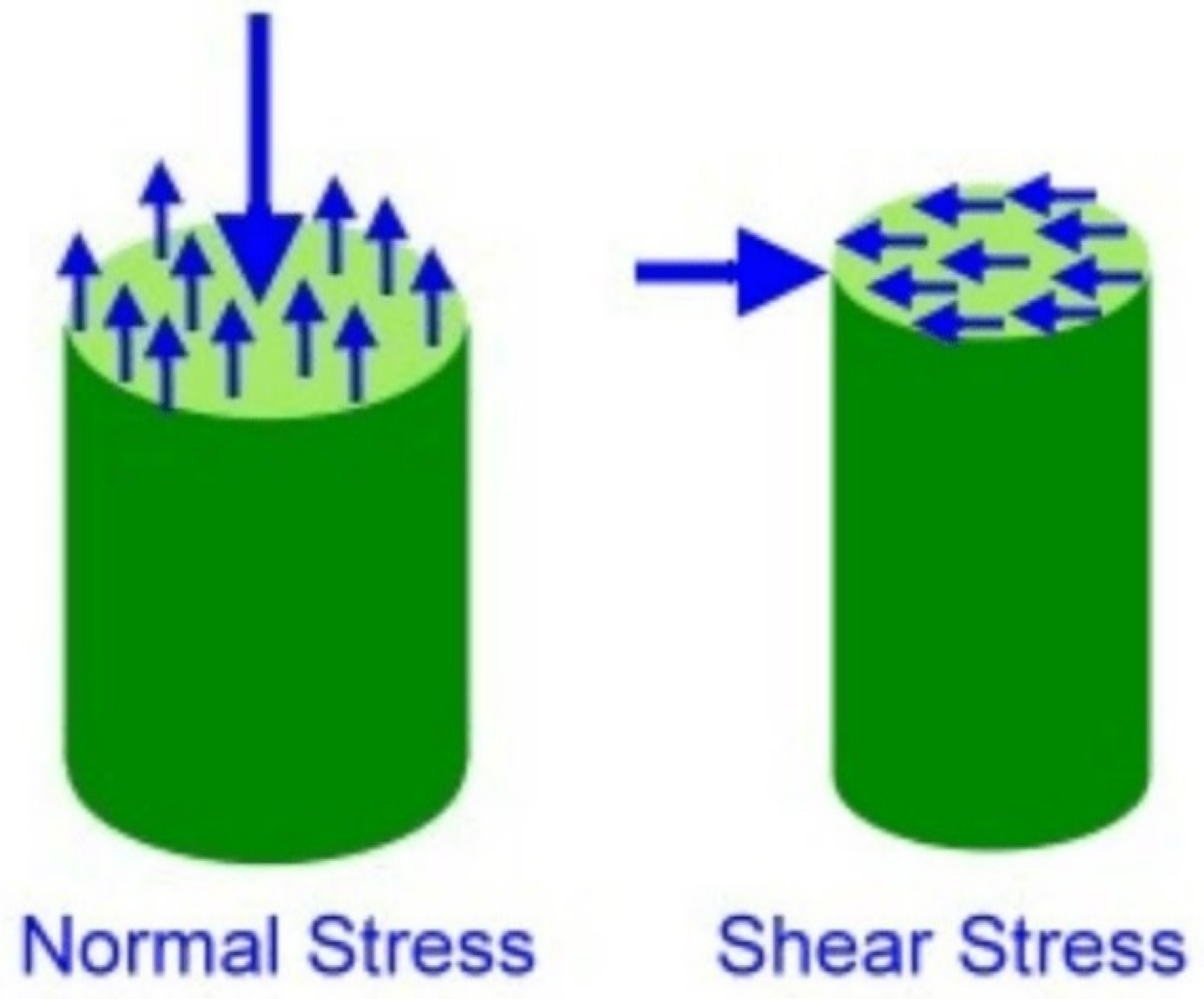
$\Rightarrow$ Some fun stuff and anecdotes and current experiments … enjoy!

Why do we give different names to different states of matter?

<u>Fluids</u> display properties such as:

$\Rightarrow$ has the ability to flow

$\Rightarrow$ lack of resistance to permanent deformation [In stark contrast to what we understand as solids]



$\Rightarrow$ A **<u>fluid</u>** is a <u>liquid</u>, <u>gas</u>, or <u>any other material</u> that continuously <u>deforms</u> (*flows*) under an applied <u>shear stress</u> (very technical definition)

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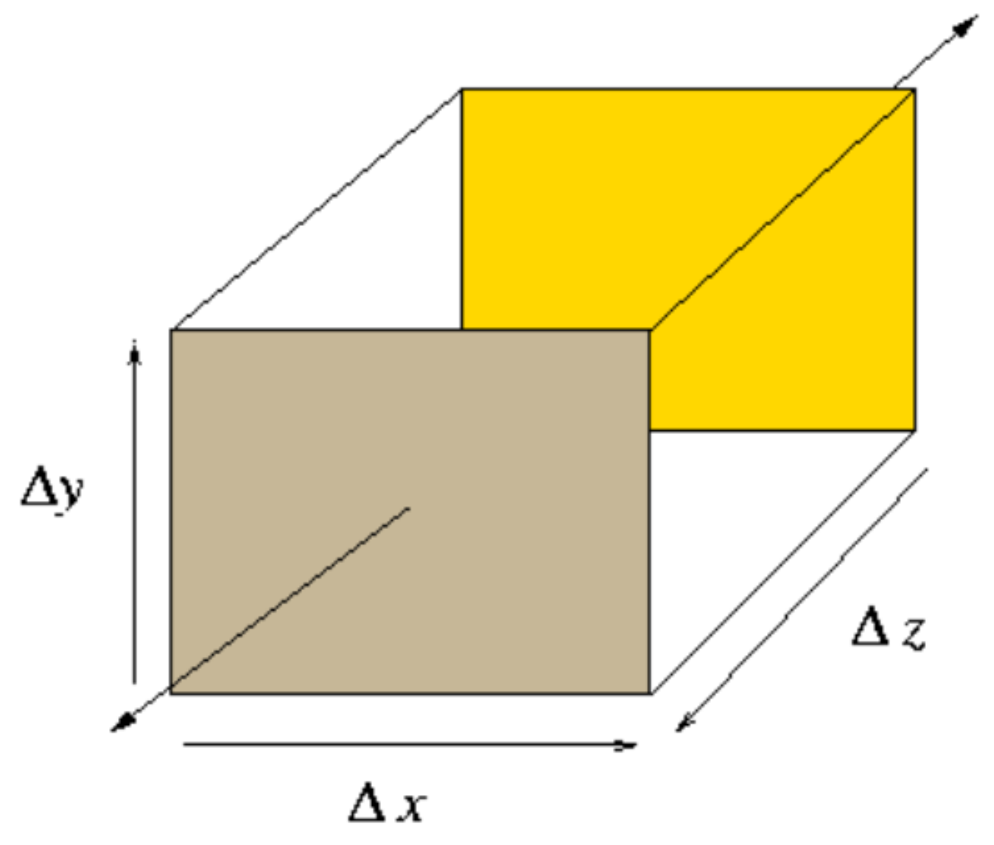
How do we study fluids (fluid dynamics)?

Some basic ideas of fluid dynamics:

$\Rightarrow$ choose a volume (much smaller than the total volume of the system being studied)

$\Rightarrow$ ensure that this "small" volume (one drop!) has a "very large" number of particles within itself

$\Rightarrow$ follow the temporal dynamics of what is happening to the "fluid" within the volume (space and time)

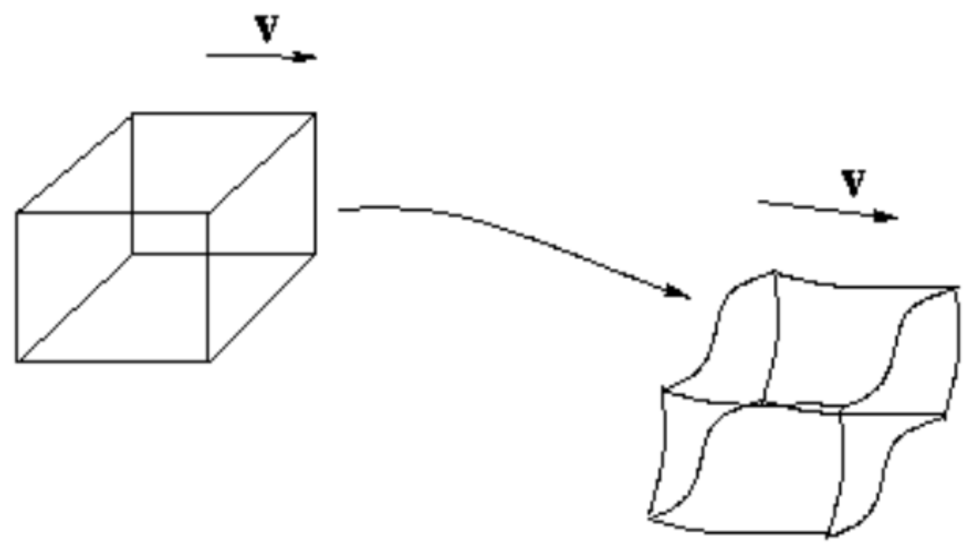


Number of particles in box $\Delta x \Delta y \Delta z$ is the volume, $\Delta V = \Delta x \Delta y \Delta z$, times the density n.

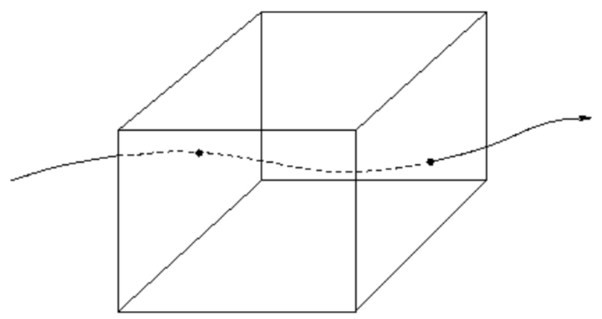
$$\boxed{\textit{Particle Conservation} \quad \frac{\partial}{\partial t} n + \nabla.(n\mathbf{v}) = 0}$$

Eulerian vs. Lagrangian formalisms (convective derivative)

1. Lagrangian. Sit on a fluid element and move with it as fluid moves.



2. Eulerian. Sit at a fixed point in space and watch fluid move through your volume element: "identity" of fluid in volume continually changing



$\frac{\partial}{\partial t}$ means rate of change at *fixed* point (Euler).

$\frac{D}{Dt} \equiv \frac{d}{dt} \equiv \frac{\partial}{\partial t} + \mathbf{v}.\nabla$ means rate of change at *moving* point (Lagrange).

$\mathbf{v}.\nabla = \frac{dx}{\partial t}\frac{\partial}{\partial x} + \frac{dy}{\partial t}\frac{\partial}{\partial y} + \frac{dz}{\partial t}\frac{\partial}{\partial z}$: change due to motion.

Remember this "convective derivative", we will come back to this concept

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⇒ Does these make sense? Some examples!

⇒ Some fun stuff and anecdotes and current experiments … enjoy!

How does all these relate to plasmas? How do we describe plasmas?

The dynamics of a plasma is determined by the self-consistent interaction between electromagnetic fields and a statistically large number of particles.

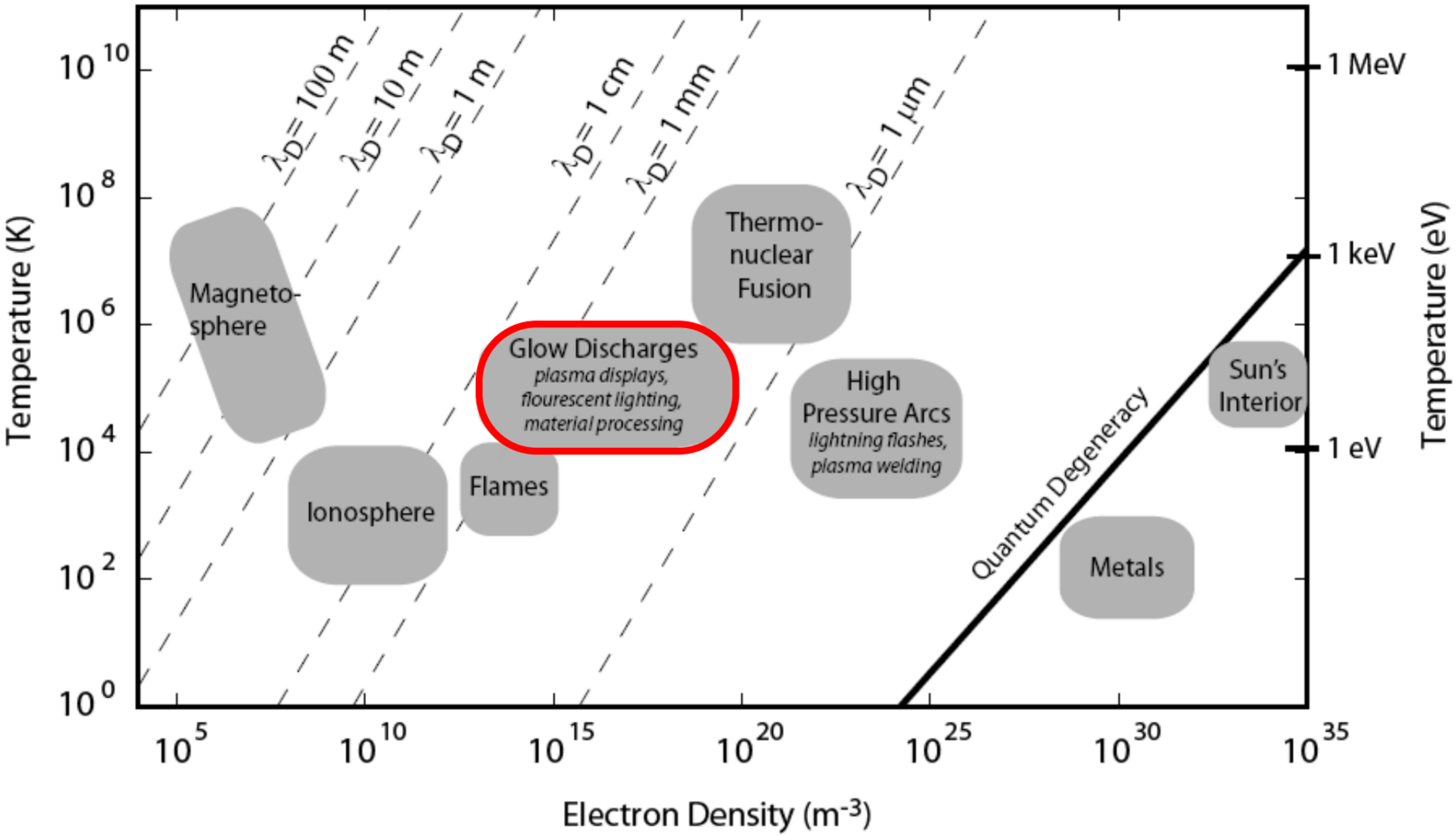
Lorentz Equation

Use knowledge of $\vec{E}\left(\vec{x},t\right)$ and $\vec{B}\left(\vec{x},t\right)$ to find $\vec{x}_j$ and $\vec{v}_j$ for every particle

Maxwell Equations

Use knowledge of $\vec{x}_j$ and $\vec{v}_j$ for every particle to find $\vec{E}\left(\vec{x},t\right)$ and $\vec{B}\left(\vec{x},t\right)$

Okay, done! But wait … is there a catch? [ALWAYS look at numbers!]



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ALWAYS look at numbers (quantification is the KEY, ALWAYS!)

$$n_p \sim 10^{15}\mathrm{m}^{-3},\ n_d \sim 10^{10}\mathrm{m}^{-3}$$

$$T_e = 3\mathrm{eV},\ T_i = \tfrac{1}{40}\mathrm{eV}$$

$$V = 3 \times 10^{-3}\mathrm{m}^3$$

Timescales

$$\tau_e \sim 10^{-9}s,\ \tau_i \sim 10^{-6}s$$

$$\tau_d \sim 10^{-2}s$$

$\Rightarrow$ Mass of an argon ion/mass of an electron ~ 73,400

$\Rightarrow$ Separation in both spatial and temporal scales

$\Rightarrow$ Impossible task for the number of particles in a realistic system!

$\Rightarrow$ Partially ionized plasmas have neutrals too (collisions)!

$$F_p = q_p \sum_{j=1}^{N} \frac{1}{4\pi\epsilon_o} \frac{q_j}{|\vec{r_i} - \vec{r_j}|^2}$$

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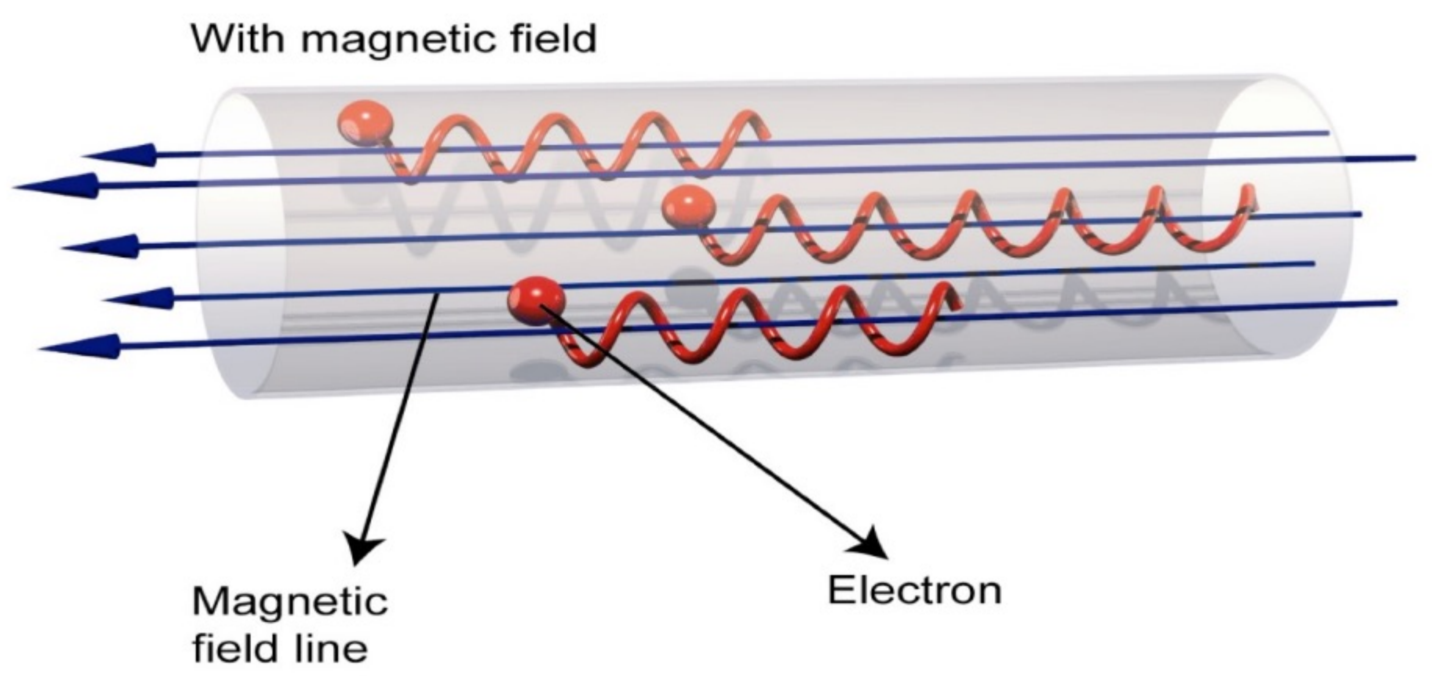
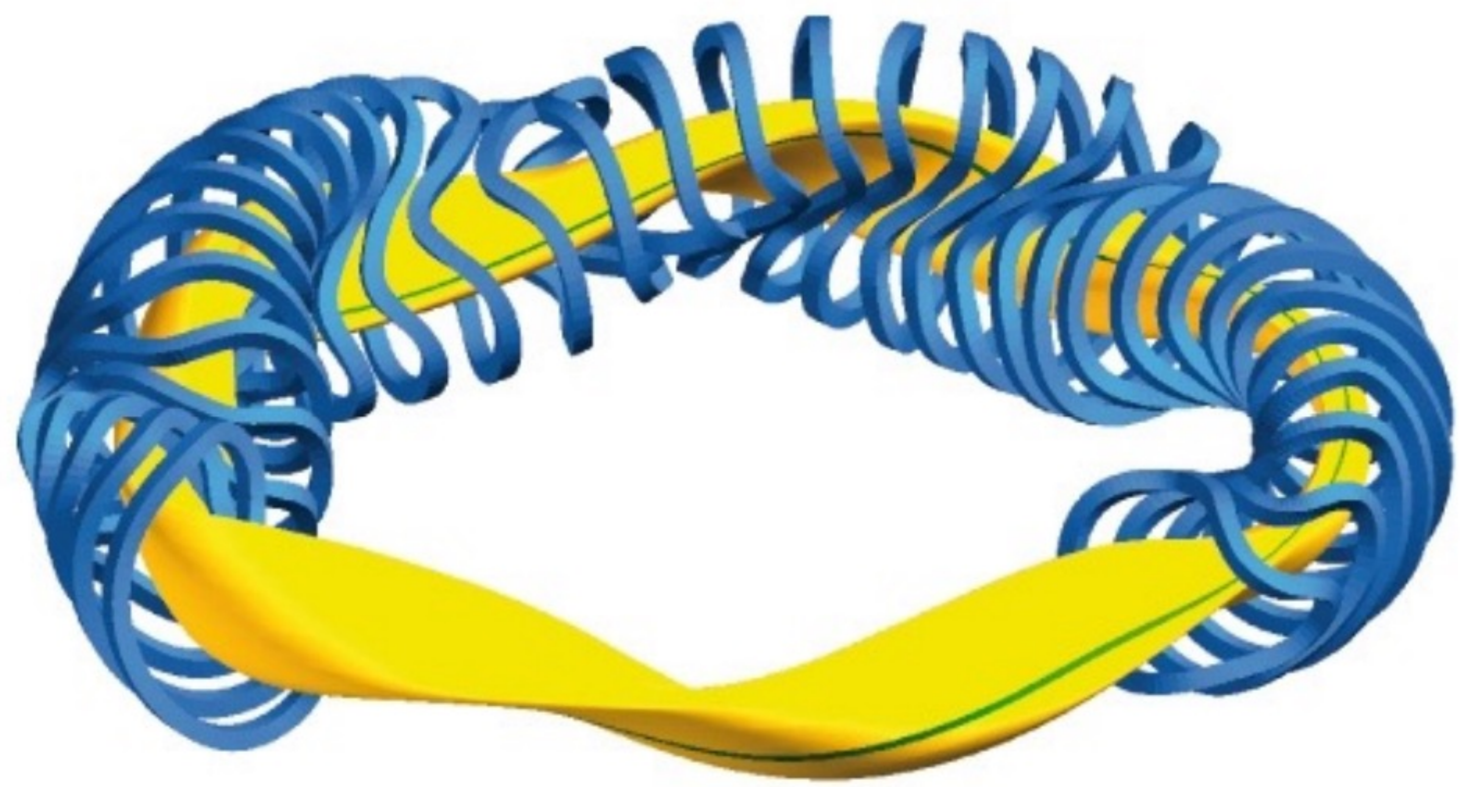
So what do we do? Any way out? Hierarchy of plasma models!



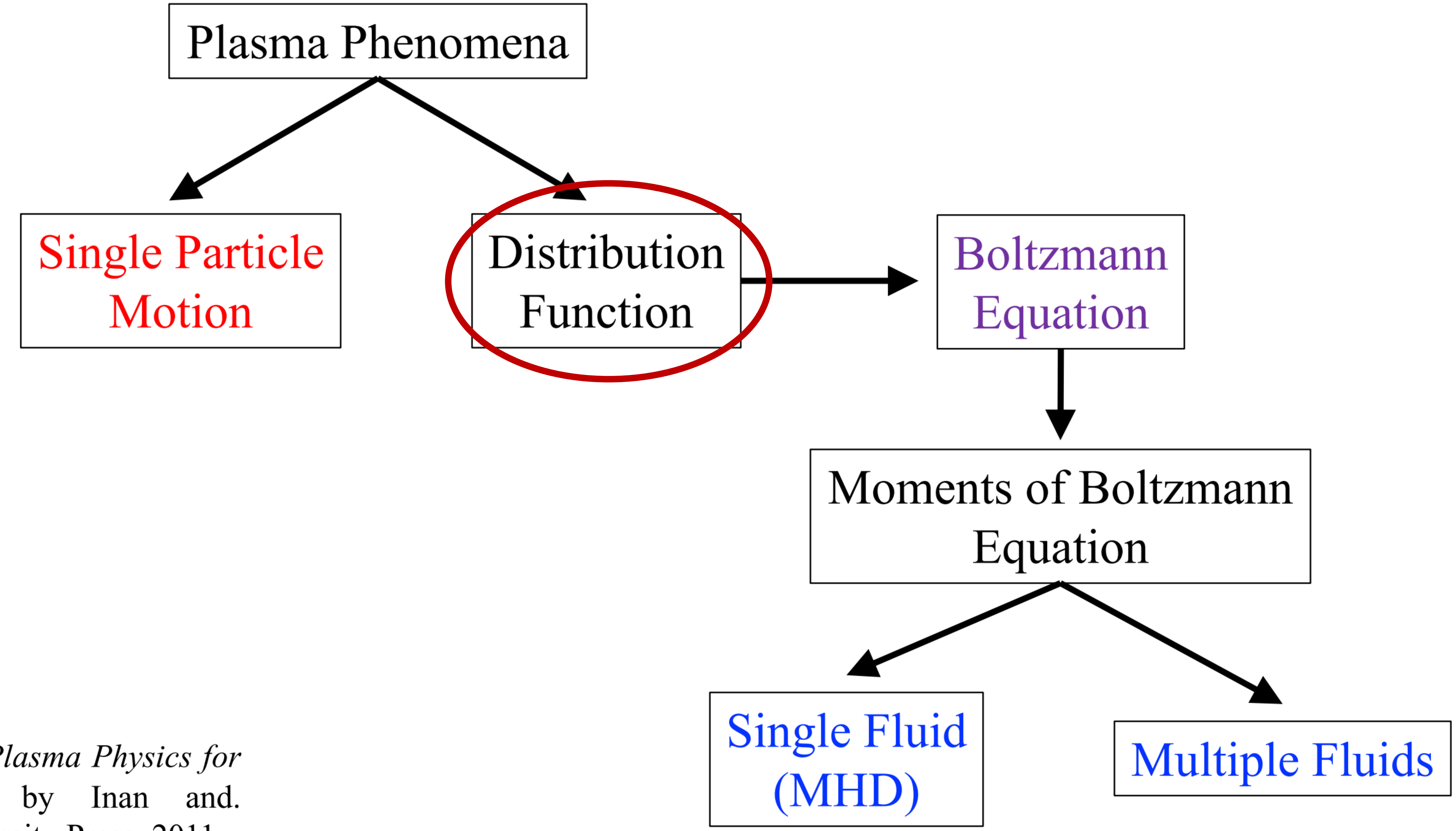


Single particle motion (particle pushing)

Kinetic Theory (distribution functions)

Fluid Model (moments, conservation eq.)

Hierarchy of plasma models continued! Distribution functions save us!



Adapted from *Principles of Plasma Physics for Engineers and Scientists* by Inan and. Gołkowski, Cambridge University Press, 2011.

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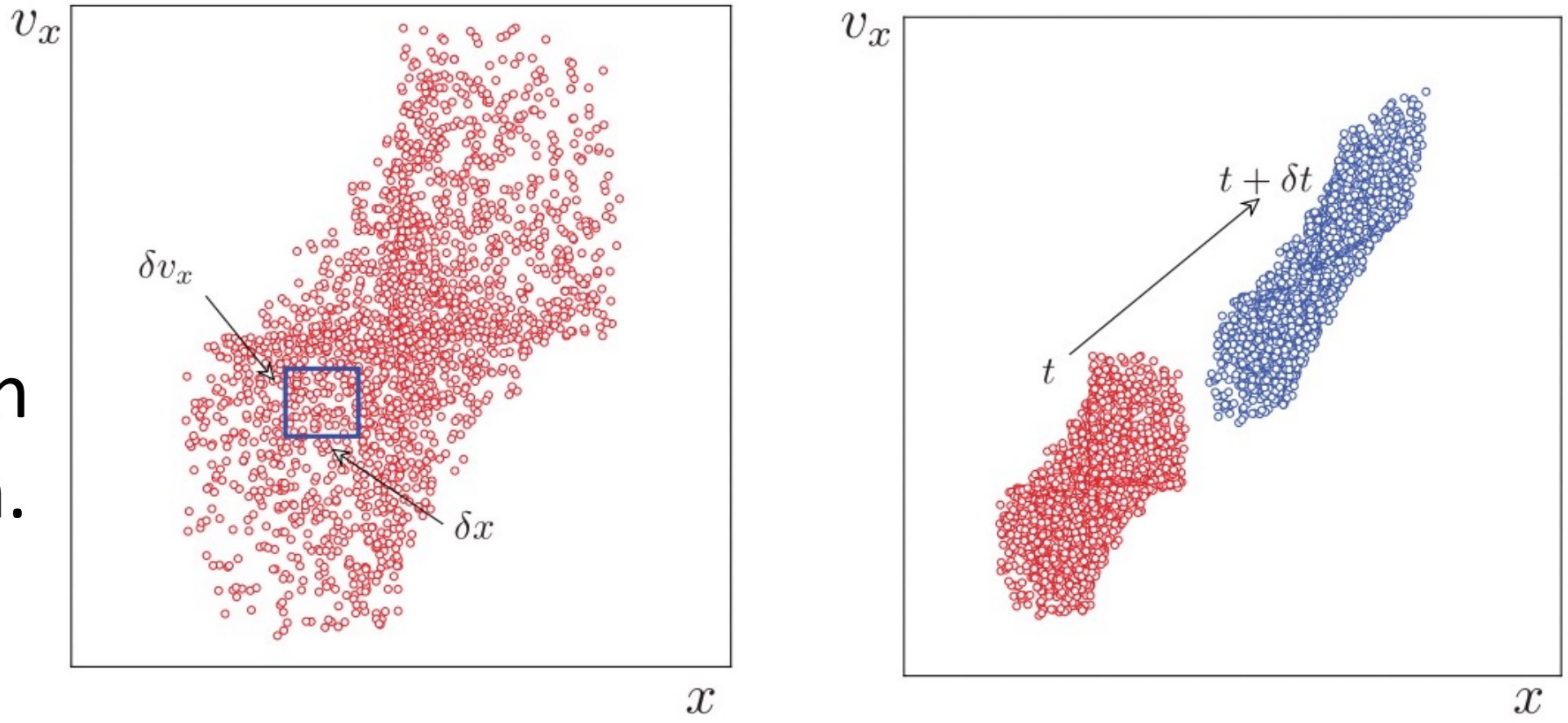
Distribution function 101 (they are everywhere … literally)

$\underline{f(\vec{x},\vec{v},t)\,d\vec{x}d\vec{v}} = \#$ of particles in a 6D phase-space volume $d\vec{x}d\vec{v}$

Interested in the evolution of the distribution function.



Cartoon figures by G. Colonna in "Plasma Modeling: Methods and Applications"

Using the full distribution function (kinetic theory of plasmas)

$f\left(\vec{x},\vec{v},t\right)d\vec{x}d\vec{v} = \#$ of particles in a 6D phase-space volume $d\vec{x}d\vec{v}$

$$0 = \frac{d}{dt}\left(f\left(\vec{x},\vec{v},t\right)\right) = \frac{\partial f}{\partial t} + \frac{d\vec{x}}{dt}\cdot\nabla f + \frac{d\vec{v}}{dt}\cdot\nabla_v f$$

Conservation of particles in phase-space

Remember the "convective derivative"? Extend it to phase-space

$$\vec{v} = \frac{d\vec{x}}{dt} \qquad \vec{a} = \frac{d\vec{v}}{dt} = \frac{q}{m}\left(\vec{E} + \vec{v}\times\vec{B}\right)$$

From Newton's law

Vlasov Equation: $\frac{\partial f}{\partial t} + \vec{v}\cdot\nabla f + \left(\vec{E} + \vec{v}\times\vec{B}\right)\cdot\nabla_v f = 0$

Boltzmann Equation: $\frac{\partial f}{\partial t} + \vec{v}\cdot\nabla f + \left(\vec{E} + \vec{v}\times\vec{B}\right)\cdot\nabla_v f = \left(\frac{\partial f}{\partial t}\right)_c$

+ Maxwell's equations

This approach involves both real space and velocity space!

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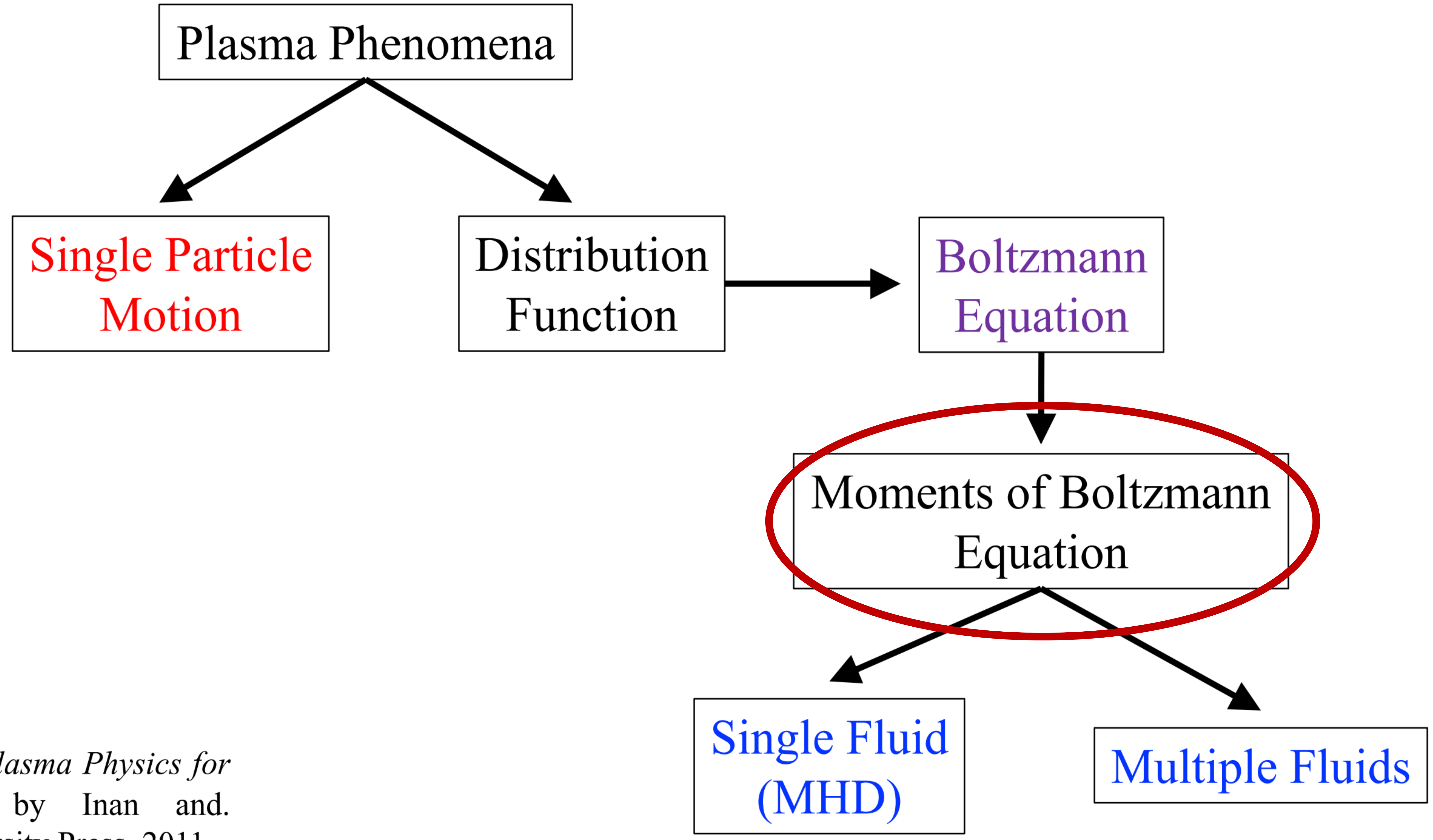
How can we go to "fluid-like" from the full kinetic theory

The full kinetic theory approach involves both real space and velocity space!
=> So, what if we average over "velocity space"?



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Taking "velocity moments" gives us the fluid quantities!

The full kinetic theory approach involves both real space and velocity space!
=> So, what if we average over "velocity space"?

Macroscopic (fluid) quantities in physical space are found by taking velocity moments of the distribution function

- Density: $n\left(\vec{x},t\right)=\int f\left(\vec{x},\vec{v},t\right)d\vec{v}$
- Mean Flow: $n\vec{V}\left(\vec{x},t\right)=\int \vec{v}f\left(\vec{x},\vec{v},t\right)d\vec{v}$
- Pressure: $\overleftrightarrow{P}\left(\vec{x},t\right)=m\int\left(\vec{v}-\vec{V}\right)\left(\vec{v}-\vec{V}\right)f\left(\vec{x},\vec{v},t\right)d\vec{v}$

Taking "velocity moments" of the Boltzmann Equations

The full kinetic theory approach involves both real space and velocity space!
=> So, what if we average over "velocity space"?

If we take moments of the Boltzmann Equation, we get the exact fluid equations.

$$\frac{\partial f}{\partial t} + \vec{v} \cdot \nabla f + \left(\vec{E} + \vec{v} \times \vec{B}\right) \cdot \nabla_v f = \left(\frac{\partial f}{\partial t}\right)_c$$

Example: 0th moment (integrate over all of the velocities, $\int v^0 \, [\;] \, d\vec{v}$)

$$\int \frac{\partial f}{\partial t} d\vec{v} + \int \vec{v} \cdot \nabla f d\vec{v} + \int \left(\vec{E} + \vec{v} \times \vec{B}\right) \cdot \nabla_v f d\vec{v} = \int \left(\frac{\partial f}{\partial t}\right)_c d\vec{v}$$

$$\frac{\partial n_s}{\partial t} + \nabla \cdot \left(n_s \vec{V}_s\right) = 0$$

Taking the higher moments of the Boltzmann equations

The full kinetic theory approach involves both real space and velocity space!
=> So, what if we average over "velocity space"?

Continuity Equation, $\int [\ ]\, d\vec{v}$

$$\frac{\partial n_s}{\partial t} + \nabla \cdot (n_s \mathbf{V}_s) = 0$$

Momentum Equation, $\int m\vec{v}\, [\ ]\, d\vec{v}$

$$mn\left(\frac{\partial \mathbf{V}_s}{\partial t} + \mathbf{V}_s \cdot \nabla \mathbf{V}_s\right) = q_s n_s \left(\mathbf{E} + \mathbf{V}_s \times \mathbf{B}\right) - \nabla \cdot \mathbf{P}_s + \mathbf{R}_s$$

Energy Equation, $\int \frac{m\vec{v}^2}{2}\, [\ ]\, d\vec{v}$

=> However, do you see a catch in this?

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The kth moment eqn. contains a term that is the (k+1)th moment

$$\begin{array}{lll} \text{Continuity,} & 0^{th} & \text{equation contains } \mathbf{v} \text{ determined by} \\ \text{Momentum,} & 1^{st} & \text{equation contains } p \text{ determined by} \\ \text{Energy,} & 2^{nd} & \text{equation contains } Q \text{ determined by } \ldots \end{array}$$

Continuity Equation, $\int [\ ]\, d\vec{v}$

$$\frac{\partial n_s}{\partial t} + \nabla \cdot (n_s \mathbf{V}_s) = 0$$

Momentum Equation, $\int m\vec{v}\, [\ ]\, d\vec{v}$

$$mn\left(\frac{\partial \mathbf{V}_s}{\partial t} + \mathbf{V}_s \cdot \nabla \mathbf{V}_s\right) = q_s n_s \left(\mathbf{E} + \mathbf{V}_s \times \mathbf{B}\right) - \nabla \cdot \mathbf{P}_s + \mathbf{R}_s$$

Energy Equation, $\int \frac{m\vec{v}^2}{2}\, [\ ]\, d\vec{v}$

We use the equation of state to close this loop!

$$p = nT$$

$$\textit{Equation of State:} \quad pn^{-\gamma} = \text{const.}$$

$$\text{Isothermal: } T = \text{const.:} \qquad \gamma = 1.$$

$$\text{Adiabatic/Isotropic: 3 degrees of freedom} \quad \gamma = \frac{5}{3}.$$

$$\text{Adiabatic/1 degree of freedom} \quad \gamma = 3.$$

$$\text{Adiabatic/2 degrees of freedom} \quad \gamma = 2.$$

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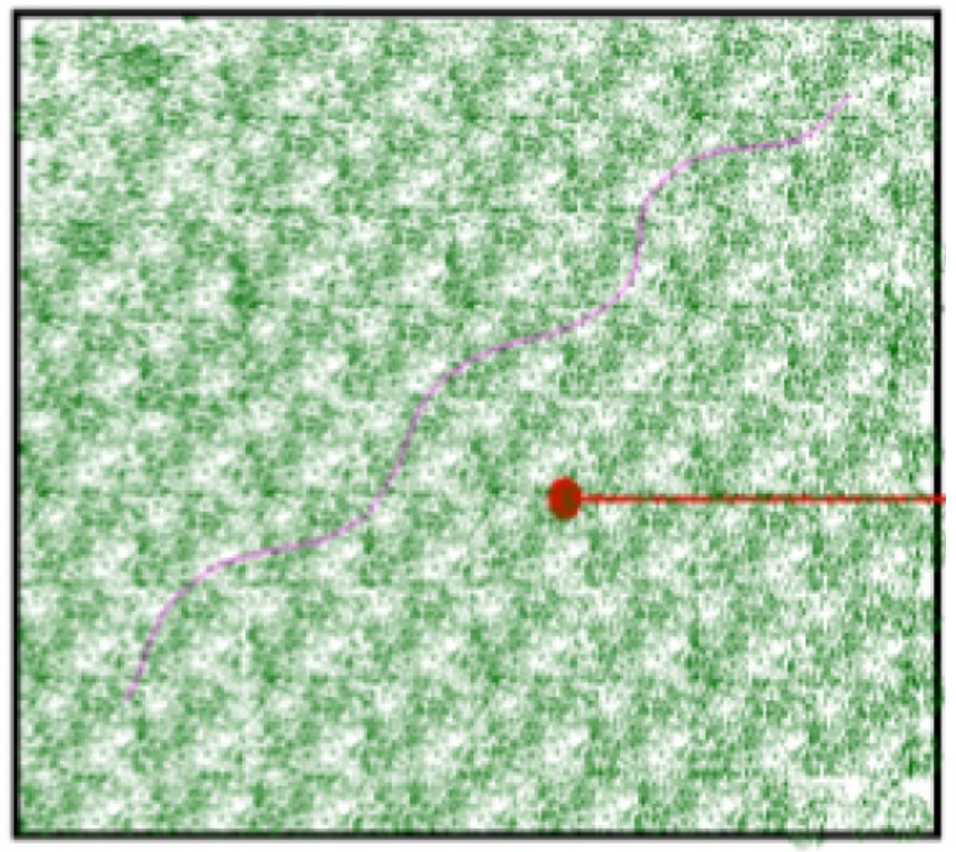
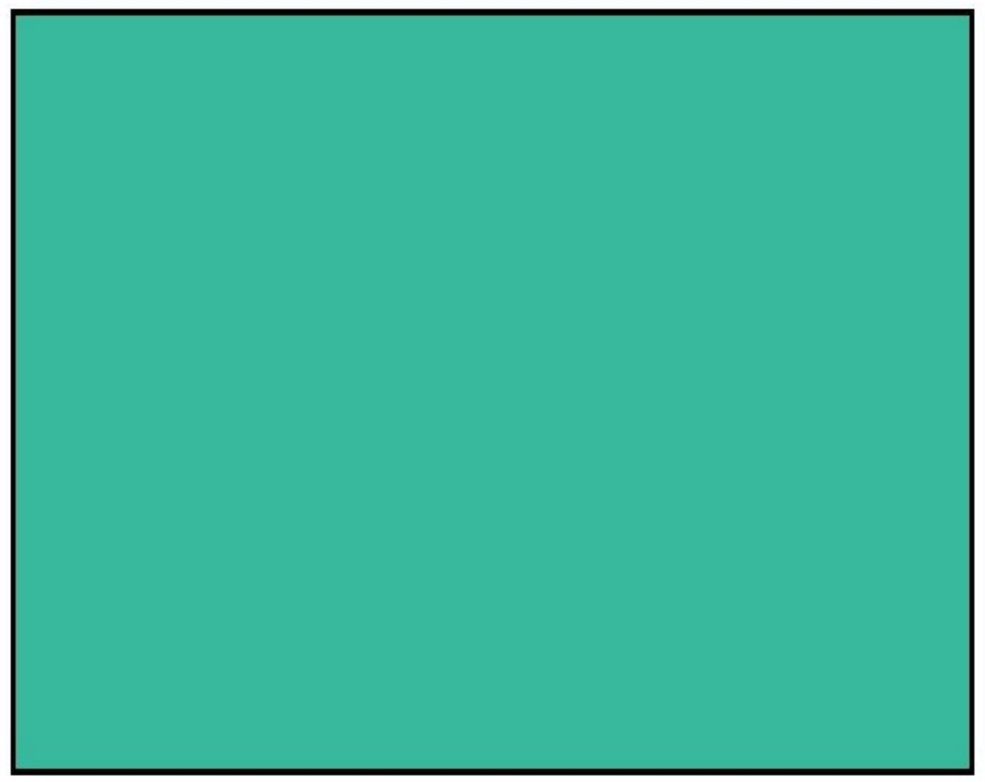
Under what conditions does this fluid model work? High collisionality!



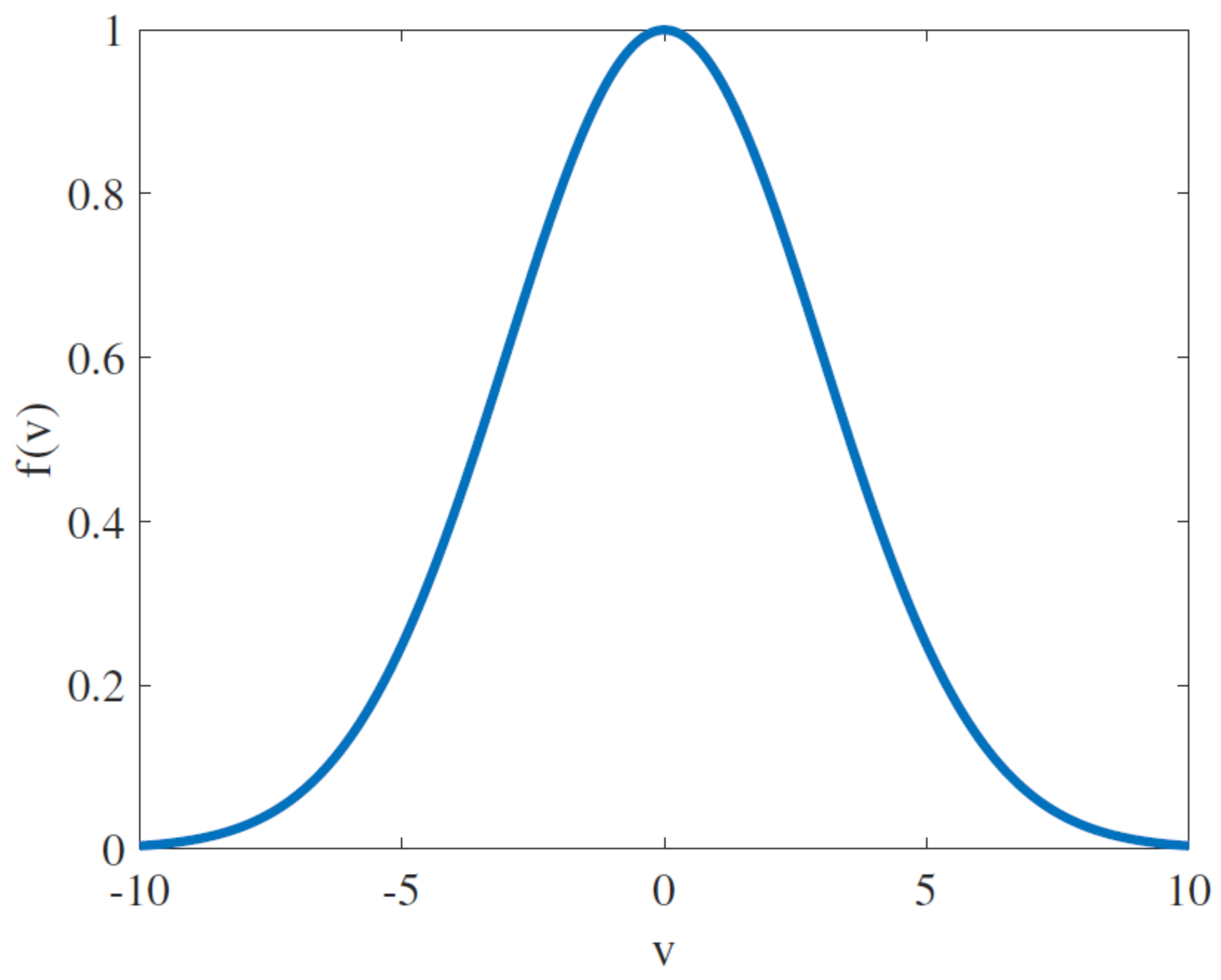
Low density – low collisionality



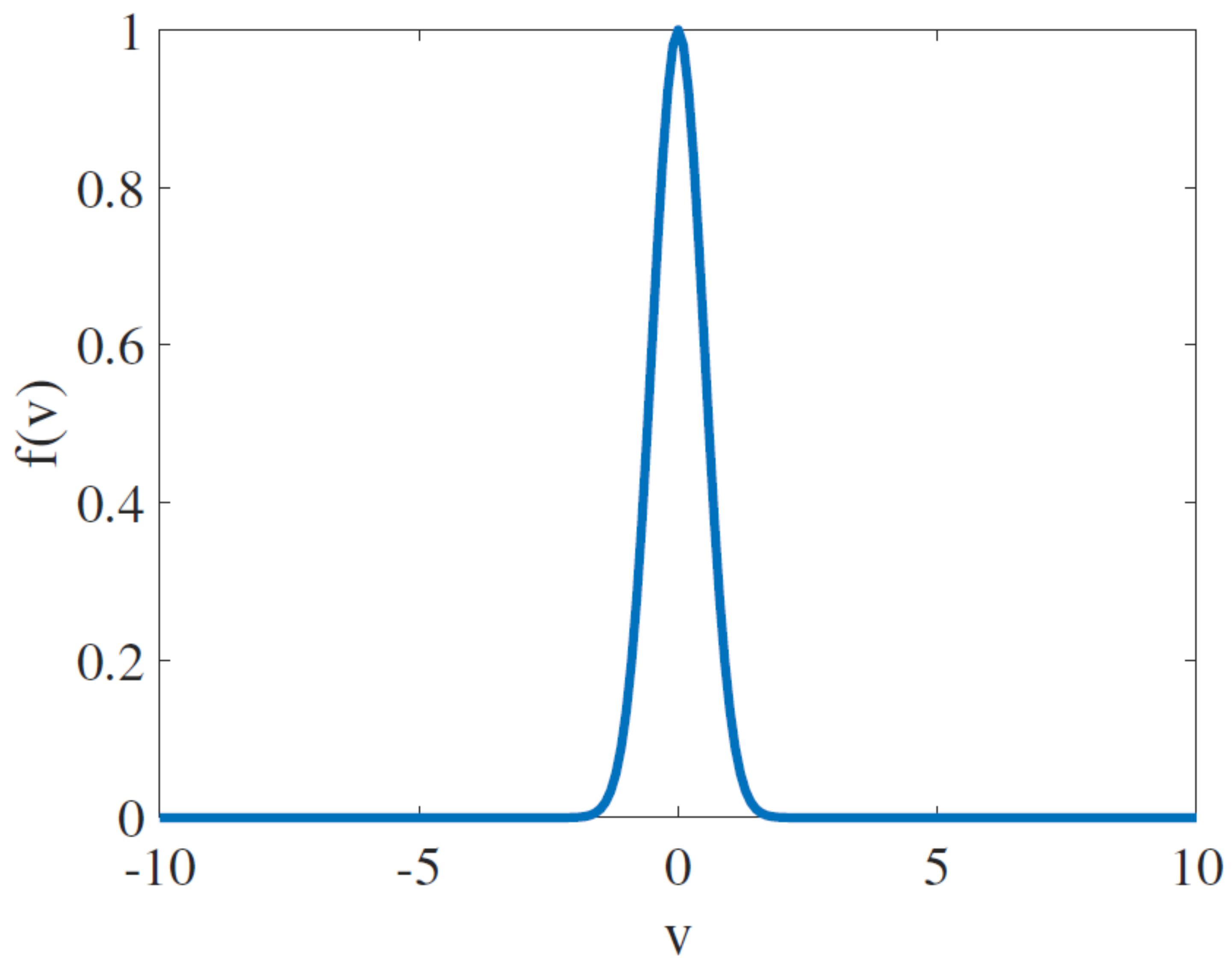
High density – high collisionality

- When there are many collisions, nearby particles remain nearby during entire evolution.
- Thermal equilibrium established locally
- **Fluid-like behavior**

Under what conditions does this fluid model work? Low Temperature!



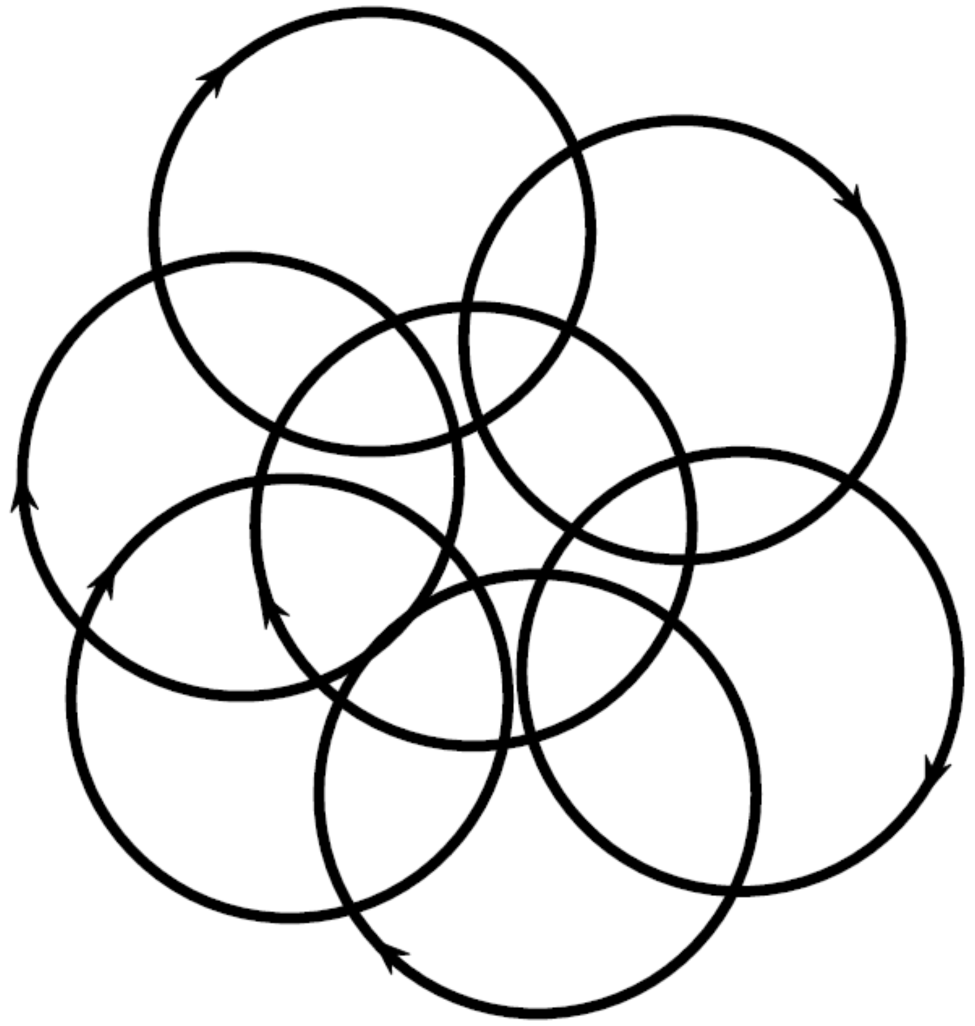
High temperature



Low temperature

- At low temperature, nearby particles have similar total velocity
- Nearby particles remain nearby during the entire evolution
- **Fluid-like behavior**

Under what conditions does this fluid model work? High magnetic field!



Weak magnetic field

Strong magnetic field

- At high magnetic field, short mean free path perpendicular to the magnetic field
- Projections of nearby particles in plane $\perp$ to magnetic field remain nearby during the entire evolution
- **Fluid-like behavior perpendicular to the magnetic field**

Two fluid equations! As if electrons and ions are two intermixing fluids

Momentum Equation

$$m_s n_s \left(\frac{\partial \vec{v}_s}{\partial t} + \vec{v}_s \cdot \nabla \vec{v}_s\right) + q_s n_s \left(\vec{E} + \vec{v}_s \times \vec{B}\right) - \nabla \cdot \overleftrightarrow{P}_s + \vec{R}_s$$

Continuity Equation,

$$\frac{\partial n_s}{\partial t} + \nabla \cdot \left(n_s \vec{V}_s\right) = 0$$

Space charge and current

$$\rho = n_i e - n_e e$$

$$\vec{j} = n_i e \vec{u}_i - n_e e \vec{u}_e$$

- Maxell's Equations

$$\nabla \cdot \vec{E} = \frac{\rho}{\epsilon_o}$$

$$\nabla \cdot \vec{B} = 0$$

$$\nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$$

$$\nabla \times \vec{B} = \mu_o \vec{J} + \frac{1}{c^2}\frac{\partial \vec{E}}{\partial t}$$

Single fluid model (MHD = magnetohydrodynamics)

One fluid model of a plasma that treats the plasma as a nonrelativistic, highly collisional, electrically conductive fluid in an electromagnetic field

- Highly collisional, so the the electron and ion temperatures have equilibrated and their particle distributions are Maxwellian
- Gyroradius is small compared to any spatial scales of interest
- Frequencies faster than the electron plasma frequency are ignored
- Non-relativistic means we can ignore the displacement currents in Ampere's Law

Single fluid model (MHD = magnetohydrodynamics)

Assumptions:

- the plasma is quasi-neutral, $n_e = n_i$
- model the electron behavior by assuming that $m_e \to 0$
- treat the plasma as a perfect conductor

- Continuity Equation

$$\frac{\partial \rho}{\partial t} + \nabla \cdot \left(\rho \vec{V}\right) = 0$$

- Momentum Equation

$$\rho \frac{\partial \vec{V}}{\partial t} = \vec{J} \times \vec{B} - \nabla p$$

- Energy Equation

$$\frac{d}{dt}\left(\frac{p}{\rho^{\frac{5}{3}}}\right) = 0$$

- Ohm's Law

$$\vec{E} + \vec{V} \times \vec{B} = 0$$

- Maxwell's Equations

$$\nabla \cdot \vec{B} = 0$$

$$\nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$$

$$\nabla \times \vec{B} = \mu_o \vec{J}$$

MHD success stories: Vertical stability of fusion plasma

MHD models and codes accurately calculate the loss of vertical stability in tokamaks

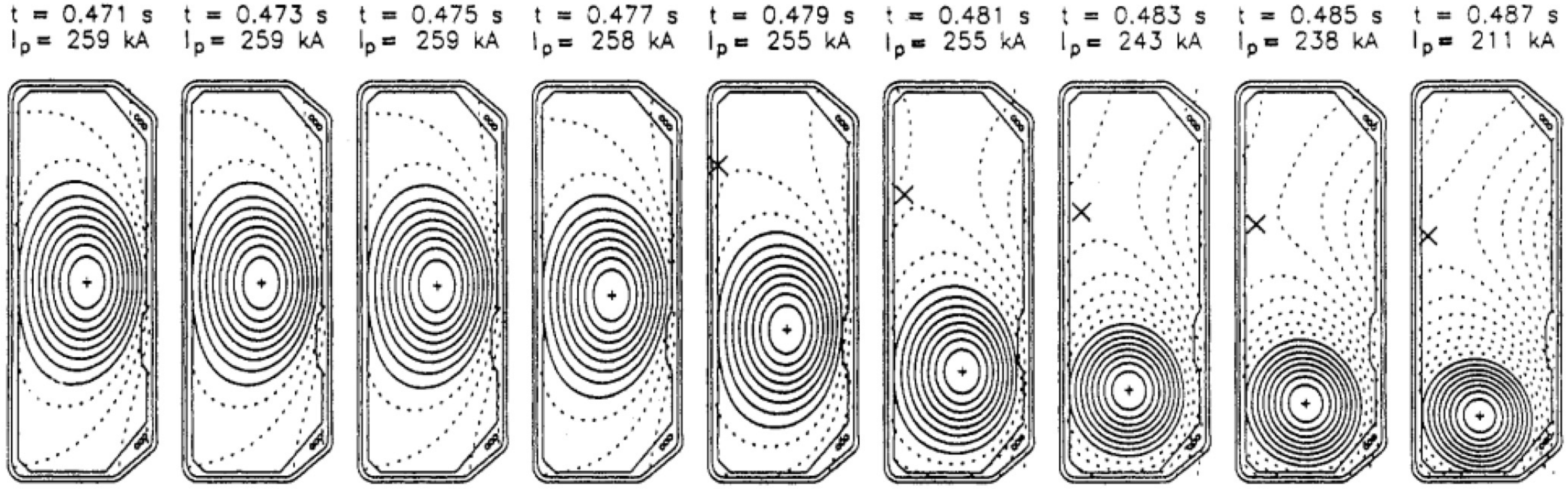
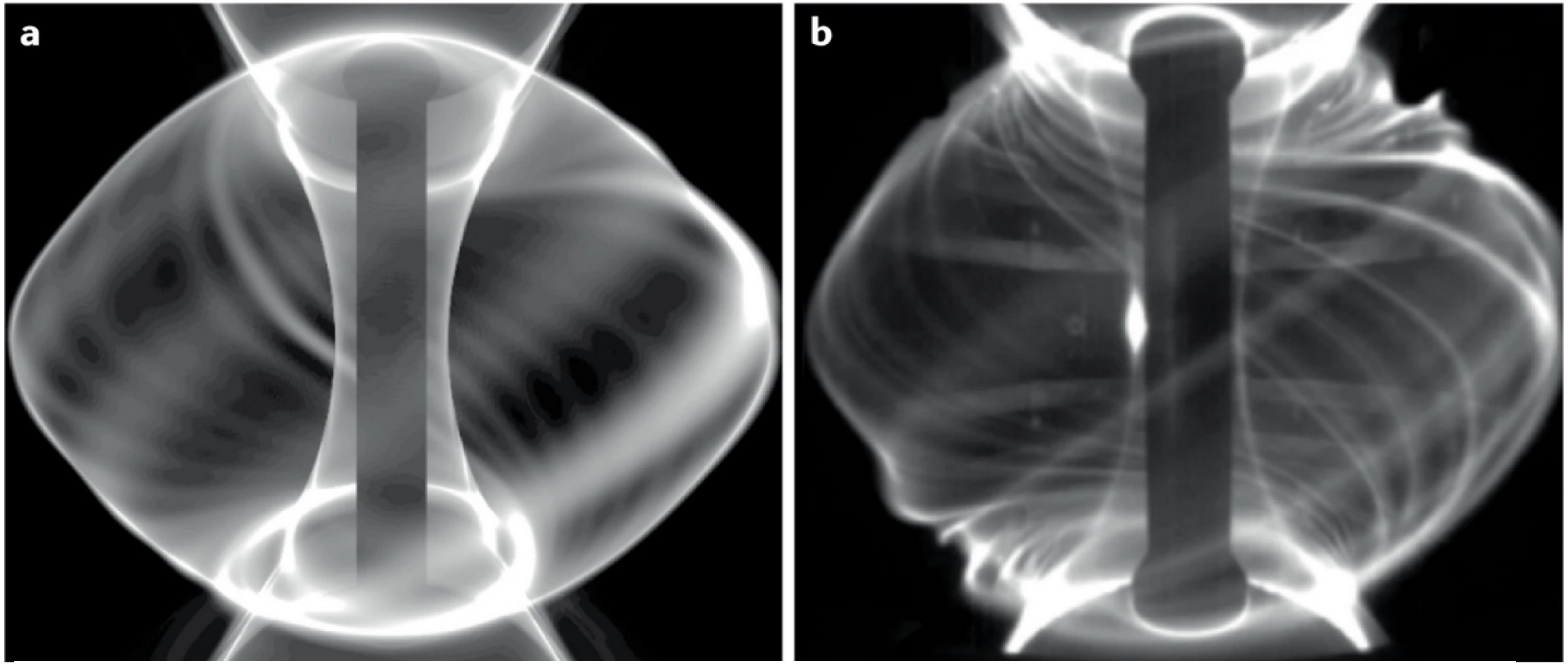


FIG. 2. Loss of vertical position control following a feedback cut at 0.470 s.

Figure from F. Hofmann *et al.*, Nuclear Fusion **37** 681 (1997)

MHD success stories: ELMs in a tokamak

MHD models and codes can capture the filamentary nature of "ballooning" modes in the edge of tokamaks



S.J.P. Pamela *et al.* 2013 *Plasma Phys. Control. Fusion* **55** 095001

Many more MHD success stories (more to come later)

$\Rightarrow$ Ideal MHD models are used to accurately predict the equilibrium magnetic configuration of fusion devices

$\Rightarrow$ MHD models are used to explain the dynamics of the solar corona

$\Rightarrow$ MHD models are used to study the structure and dynamics of the plasma around black hole accretion disks and extragalactic jets

$\Rightarrow$ MHD can explain various kinds of global instabilities (interchange, kink modes, resistive drift modes etc.)

More fluid like instabilities in plasmas: Kelvin-Helmholtz

$\Rightarrow$ KH instabilities are driven by velocity shear

$\Rightarrow$ They occur in regular fluids and also in plasmas!

https://www.americanscientist.org/article/bringing-the-stars-home

More fluid like instabilities in plasmas: Rayleigh-Taylor

$\Rightarrow$ RT instabilities occur when a heavy fluid is supported by a lighter fluid

$\Rightarrow$ They occur in regular fluids and also in plasmas!

https://www.americanscientist.org/article/bringing-the-stars-home

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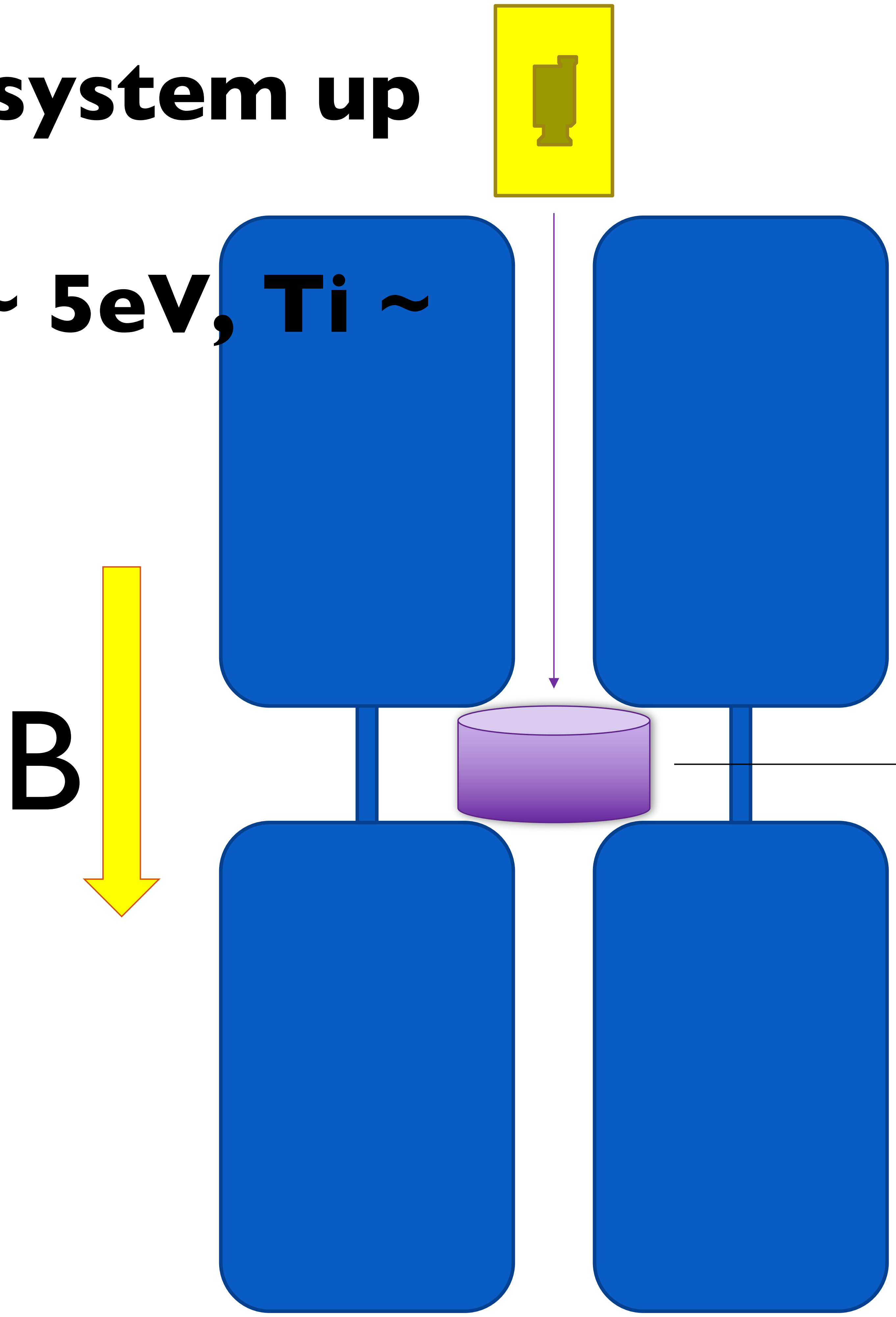
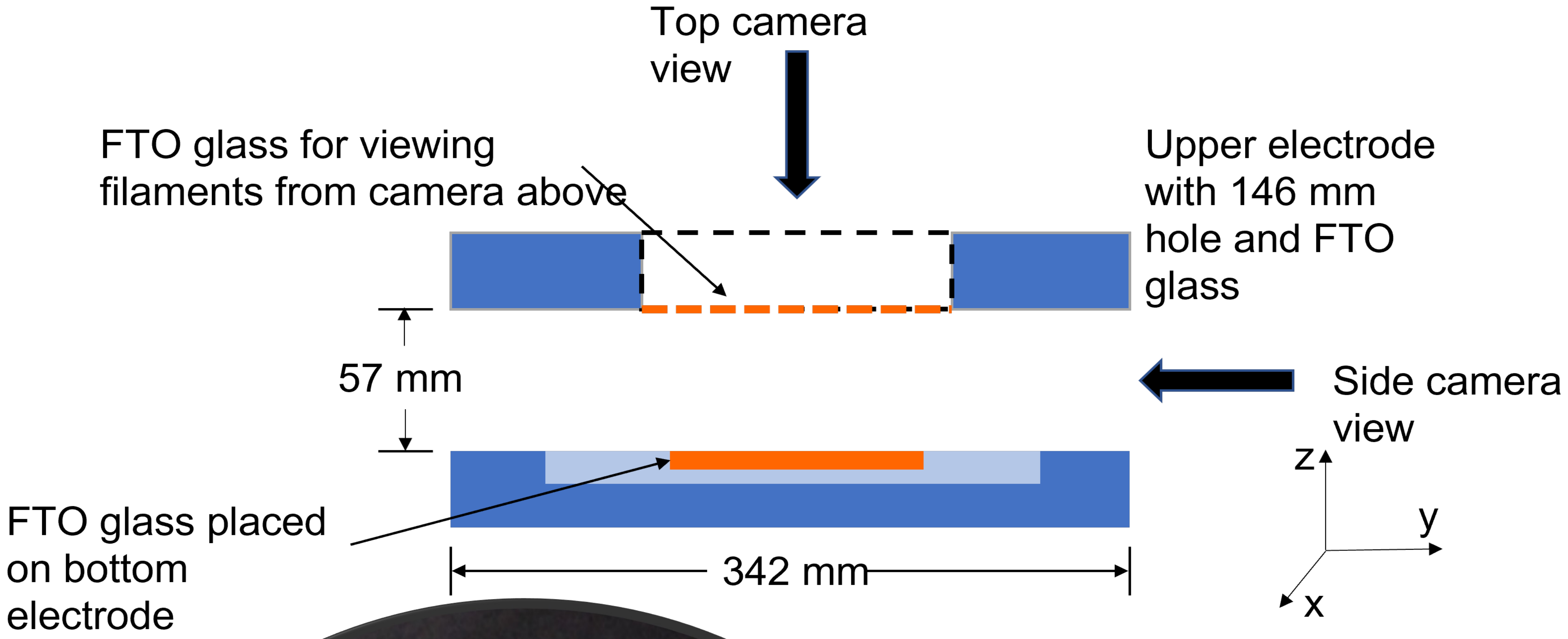
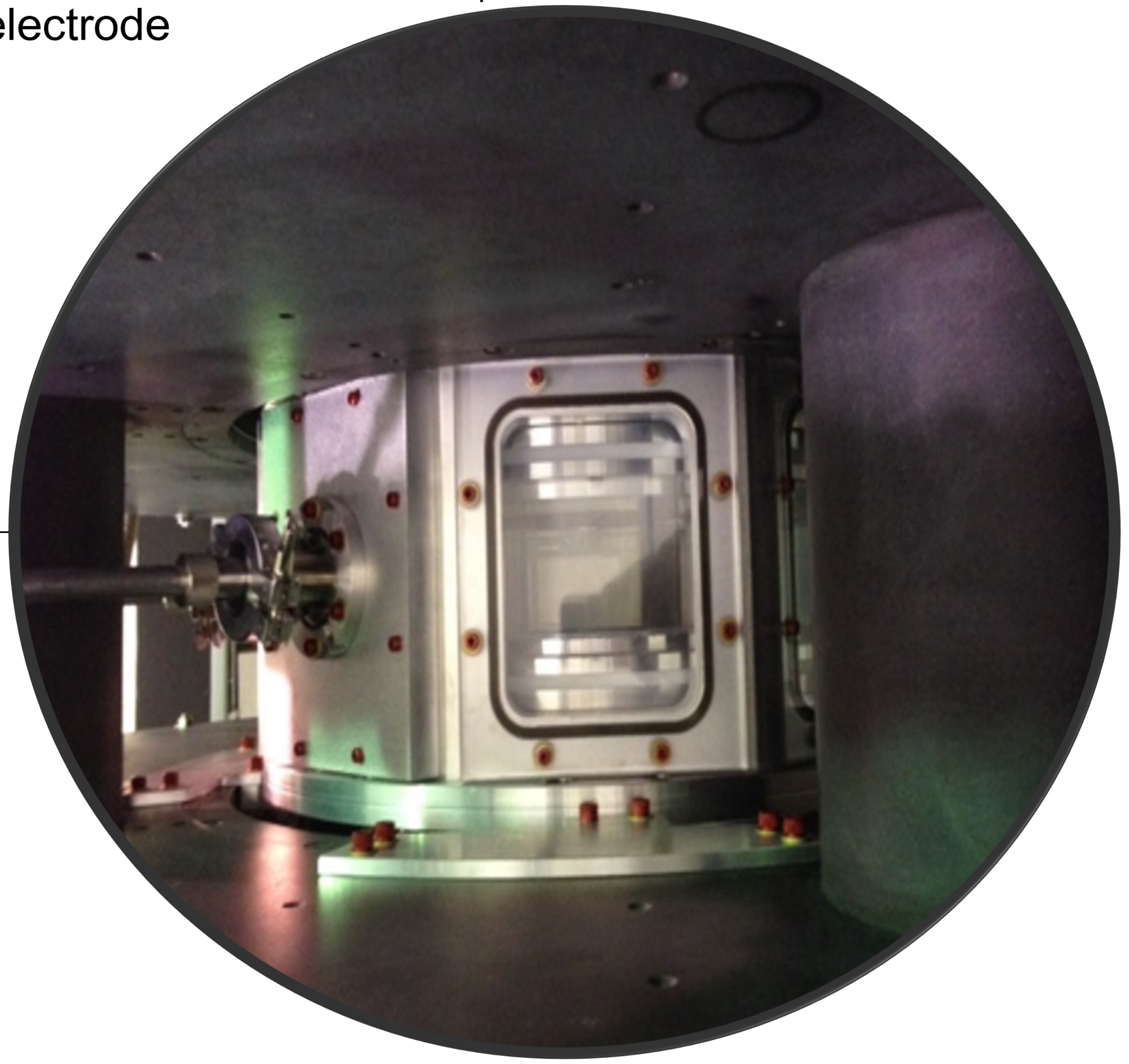
MDPX: Low temperature, low density plasmas in a high magnetic field

- **MDPX: a cryogen-free, superconducting multi-access magnet system up to 4 T**
- **CCP, n ~ 10^{15}m^{-3}, Te ~ 5eV, Ti ~ 0.025eV**







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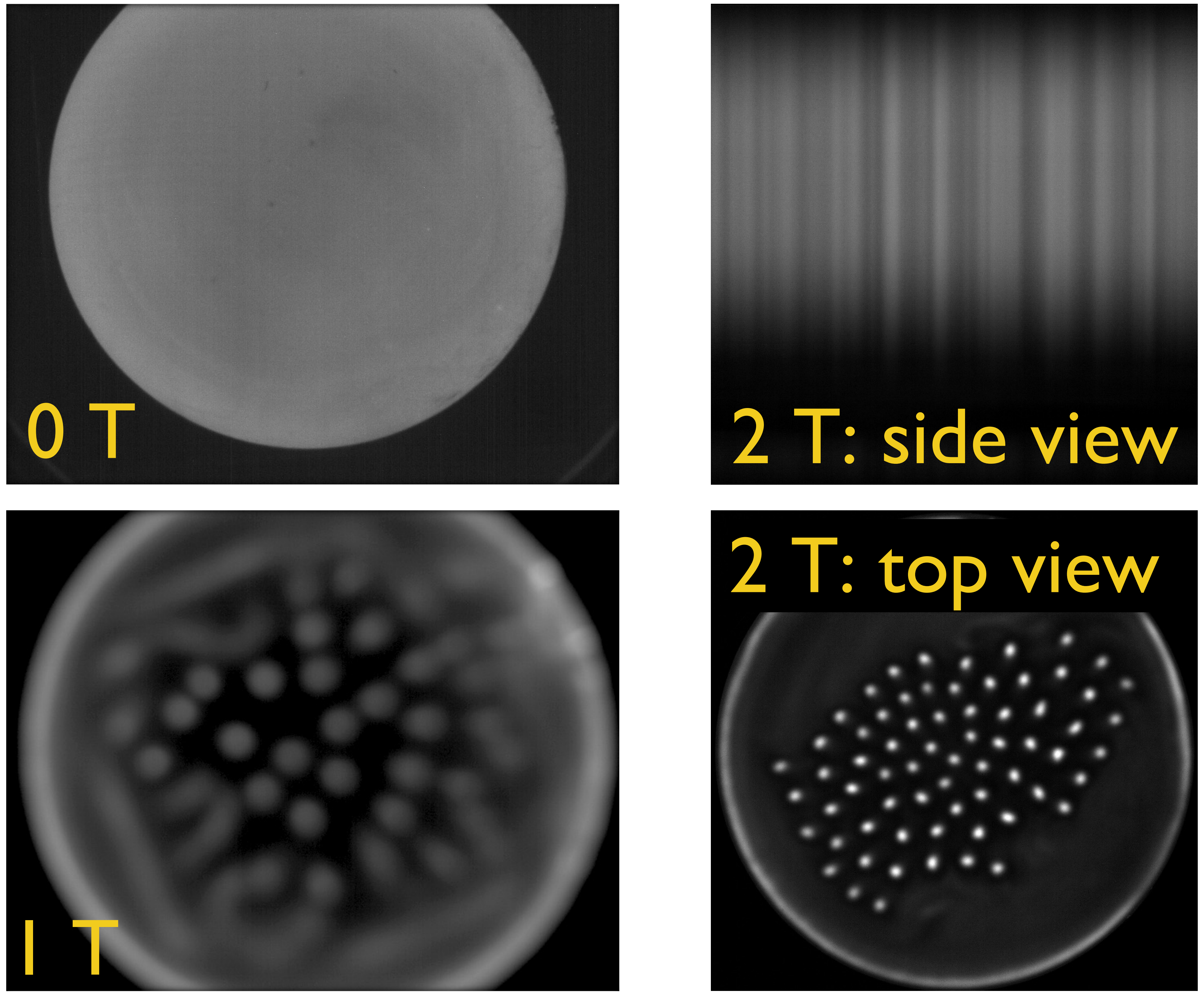
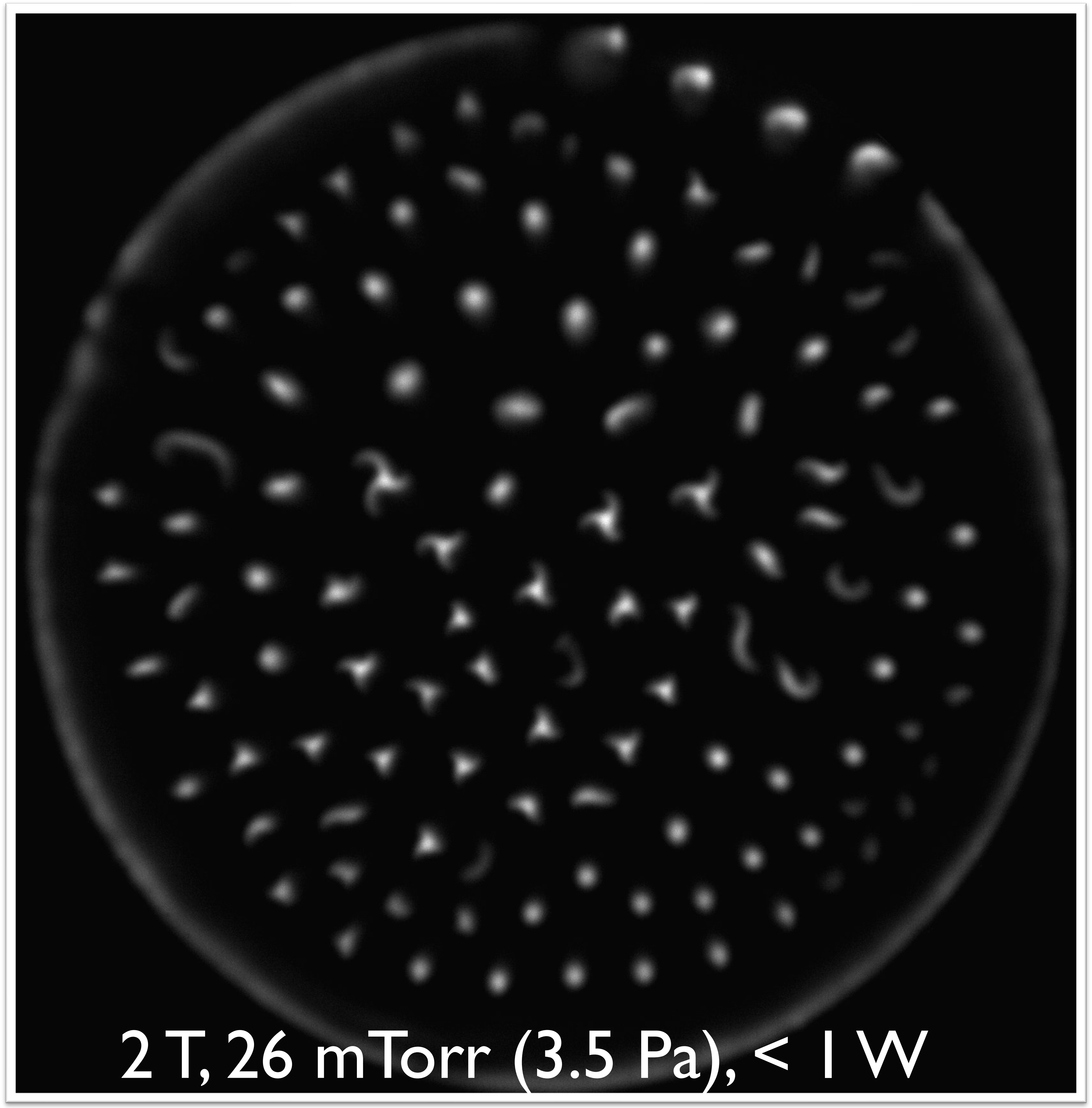
Pattern formation (filamentation) at high magnetic field

- The uniform plasma glow (neutral emission) at low B becomes non-uniform at high B

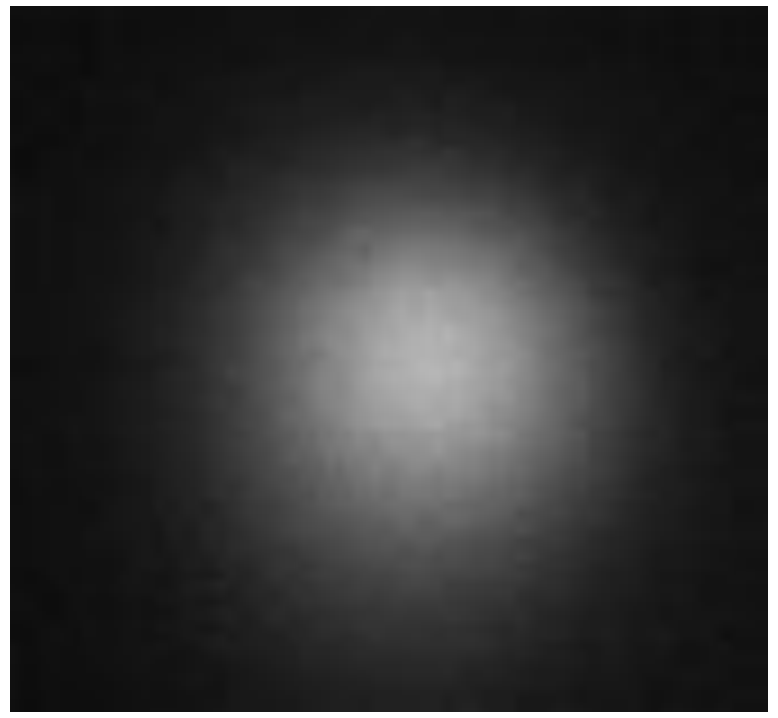


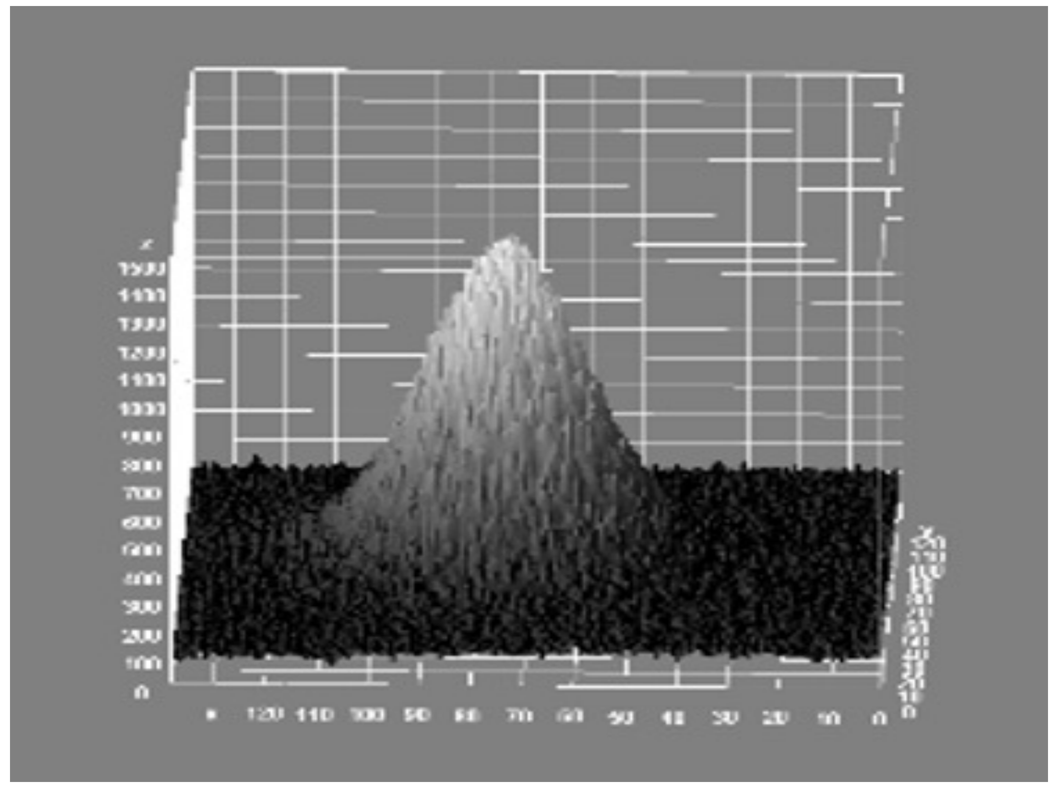
E. Thomas, et al., PPCF, **62**, 014006 (2020)
S. Williams, et al. POP, **29**, 012110 (2022)

3 most common types of stable filaments (in argon plasma)

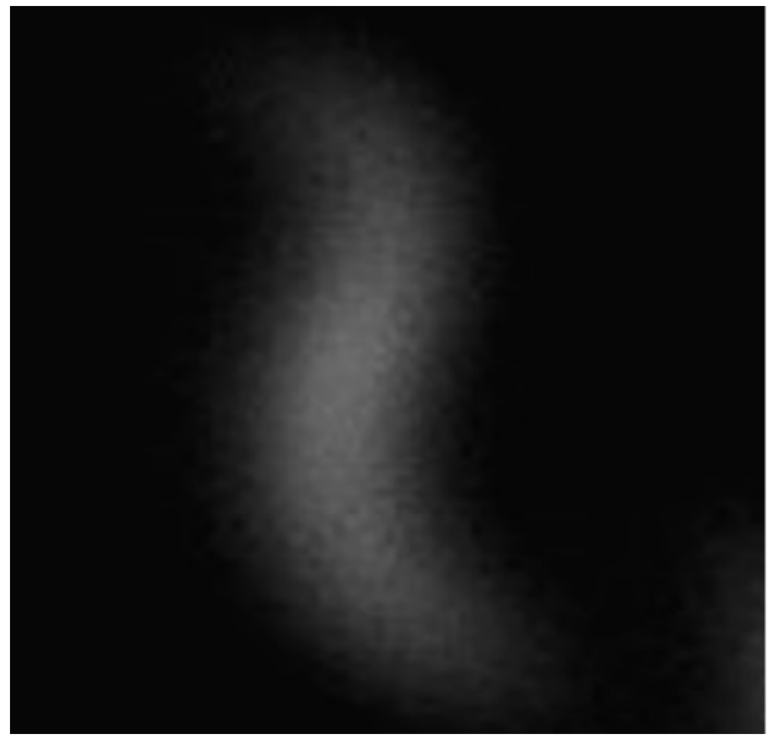


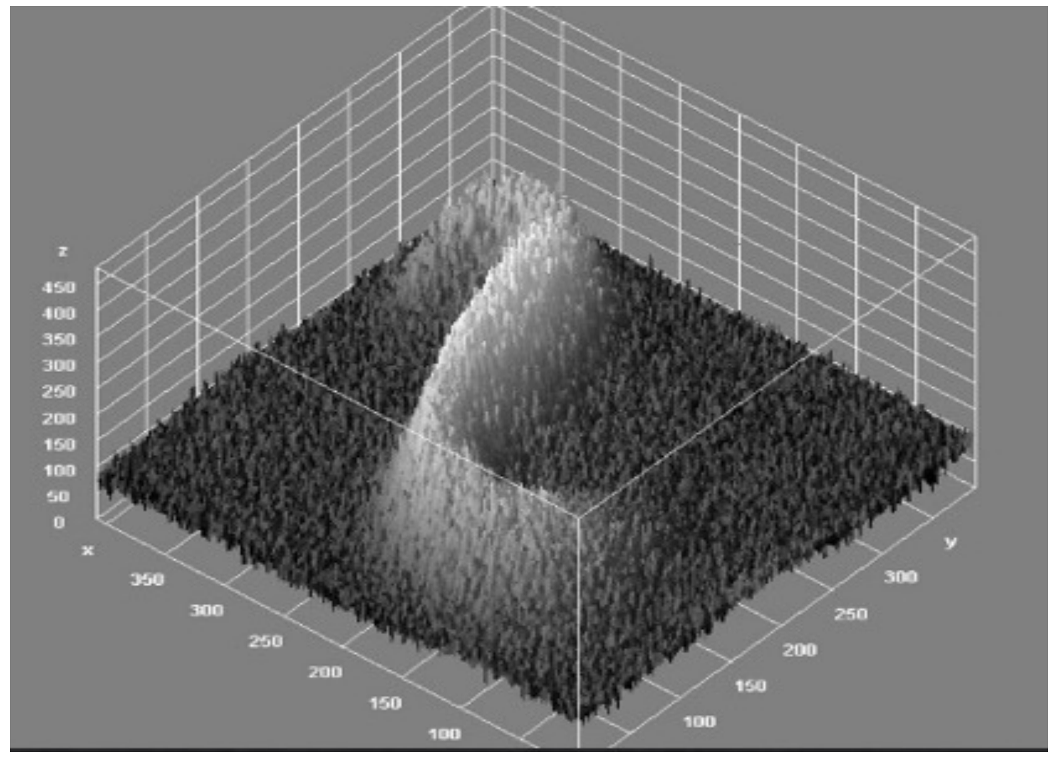
Type 1 =
Circular



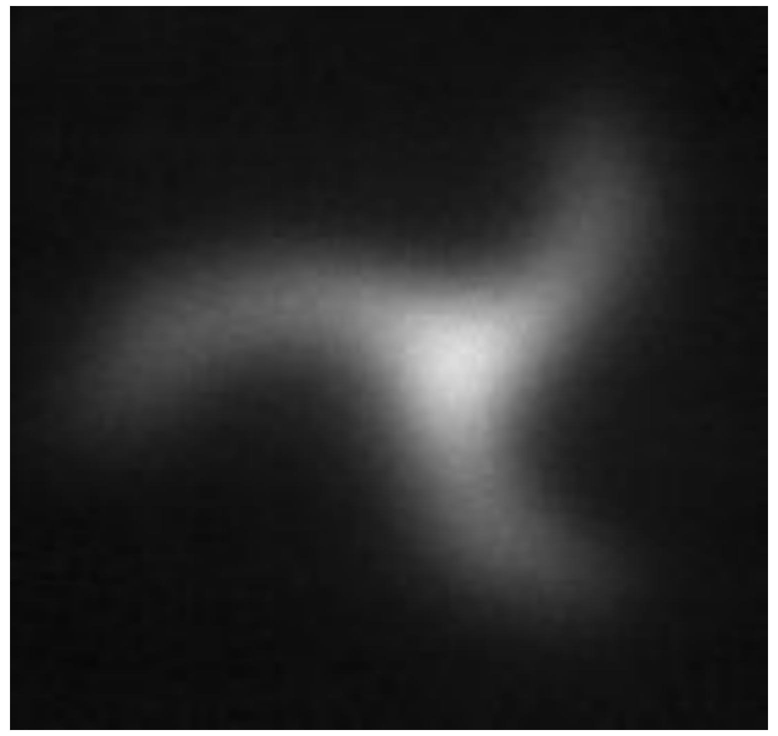


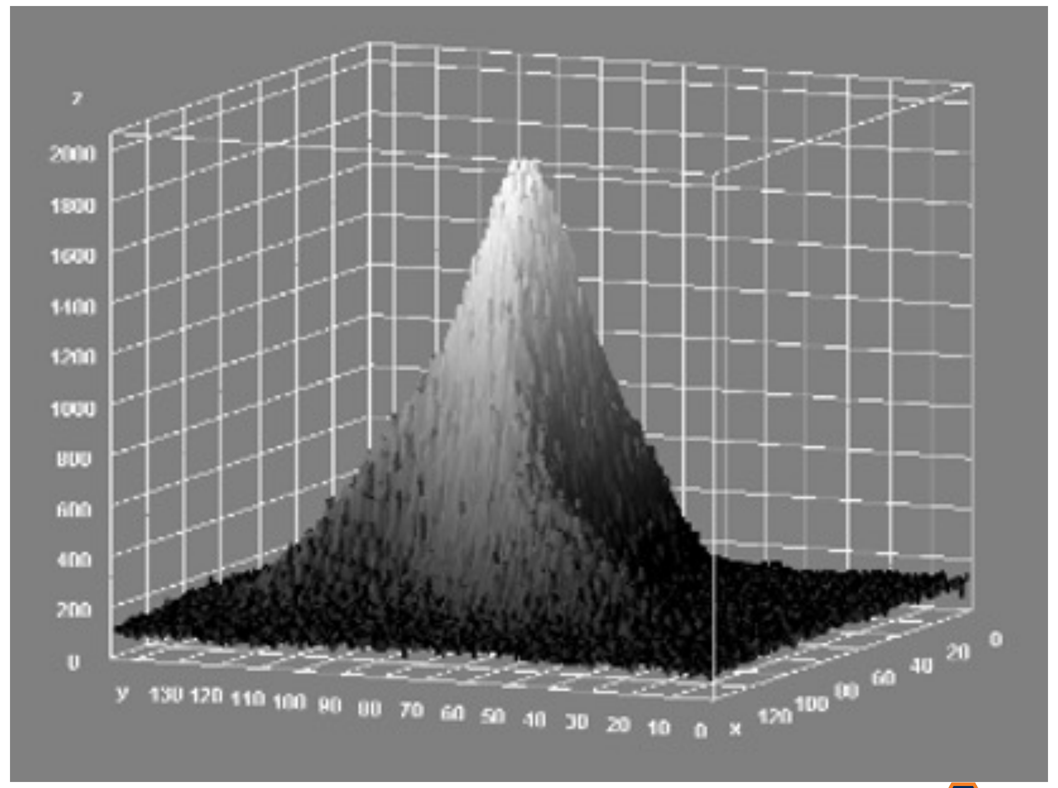
Type 2 = Elongated



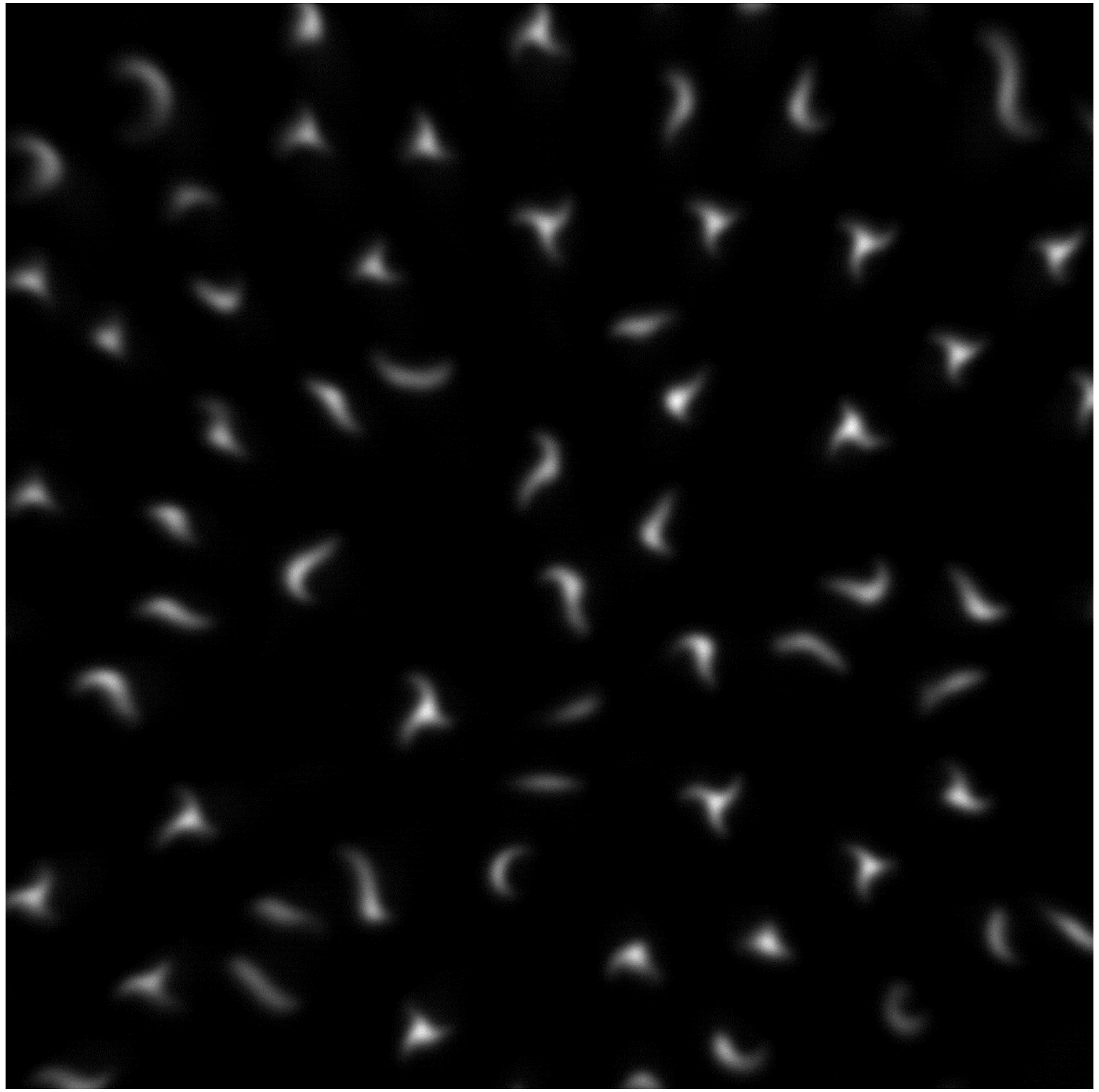


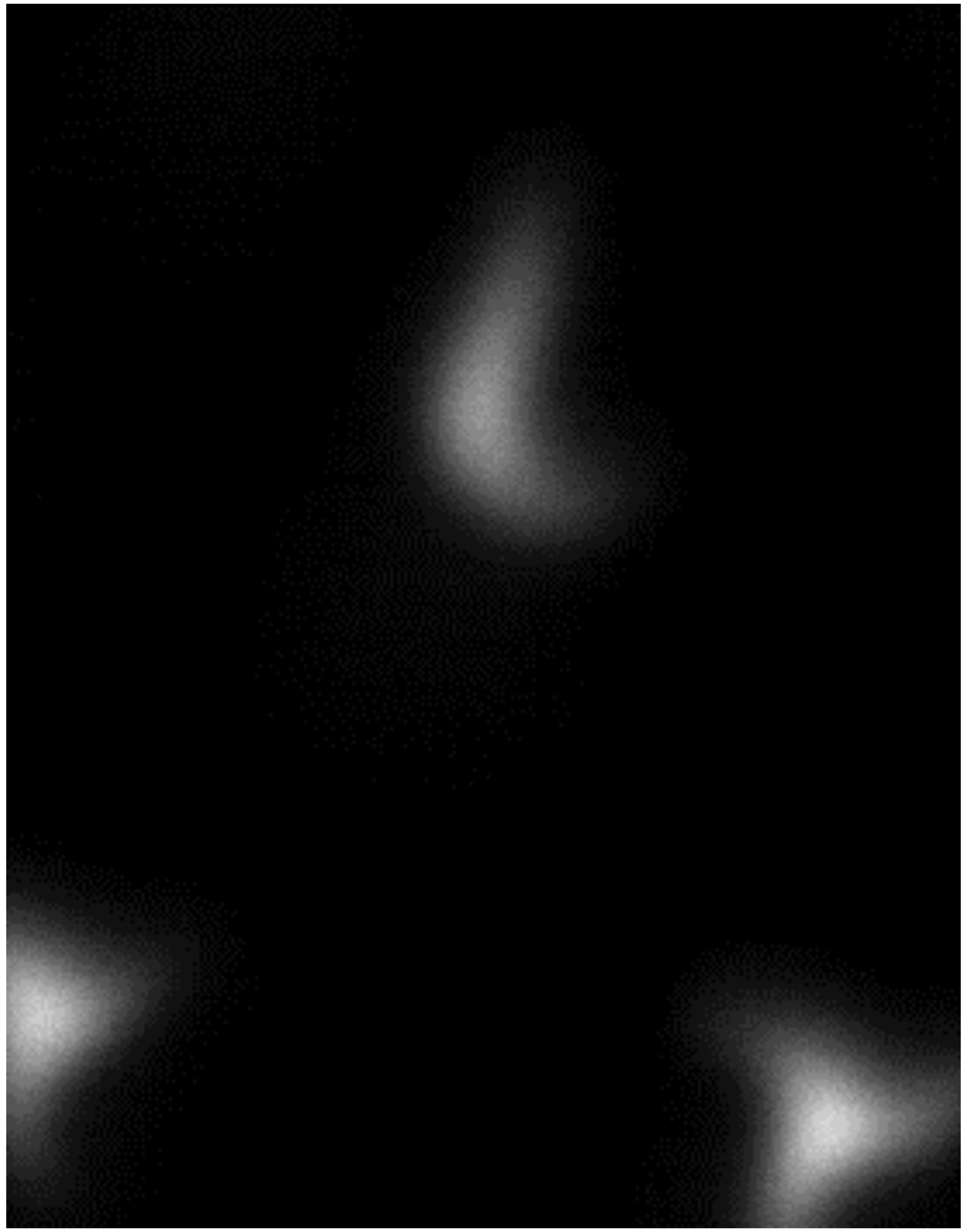
Type 3 = 3-arm spiral





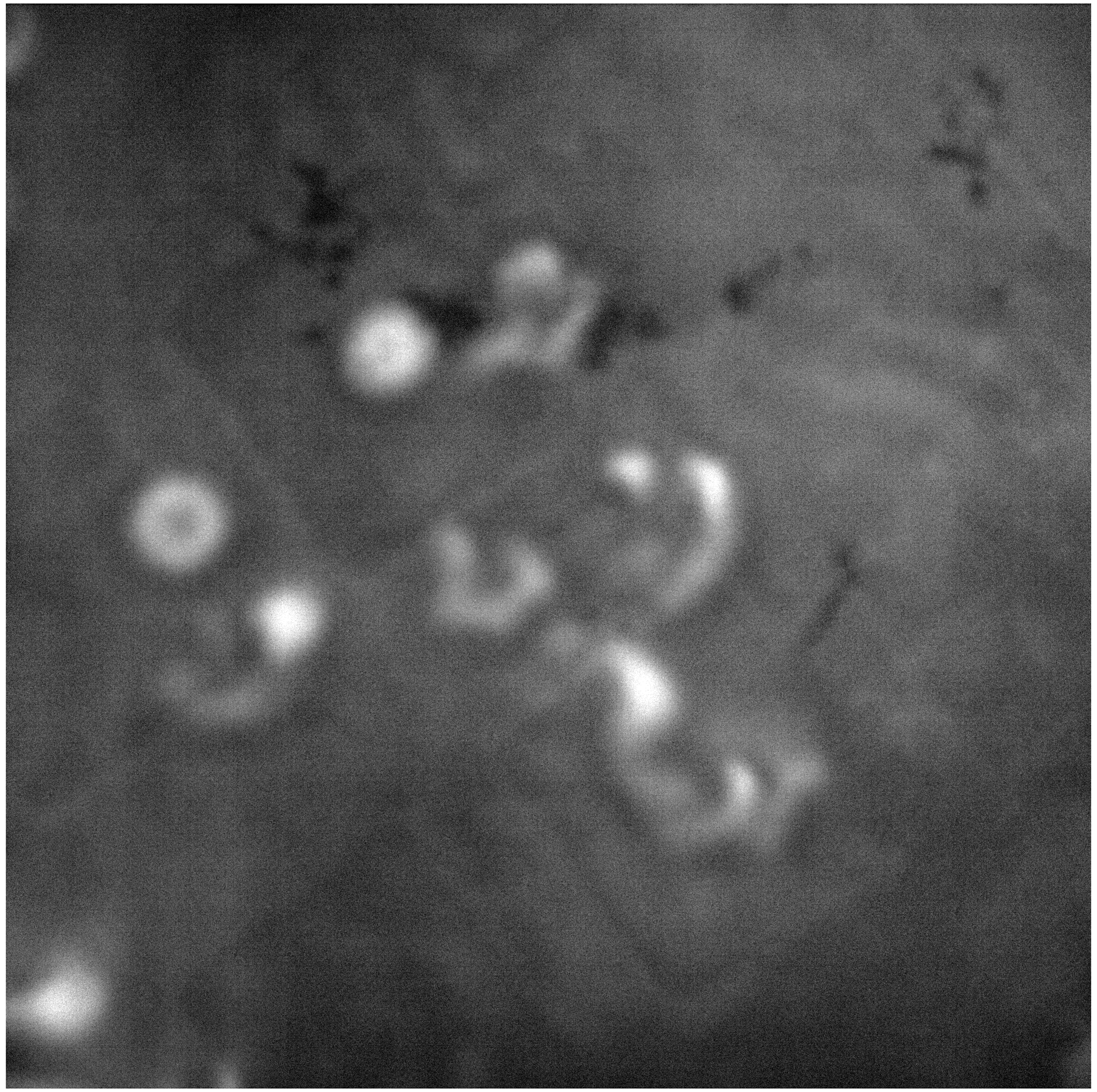
Example of temporal behavior of the filaments

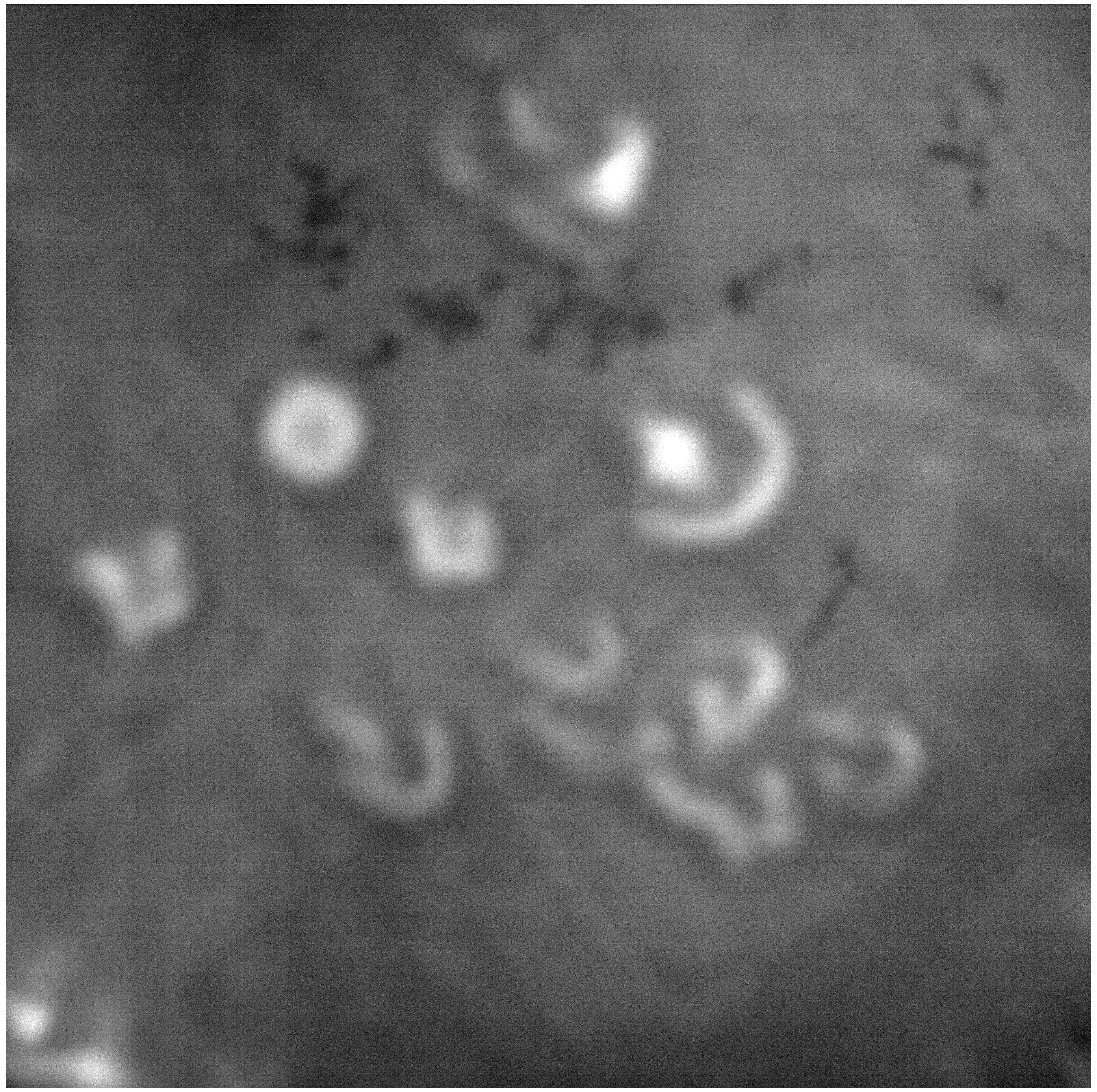




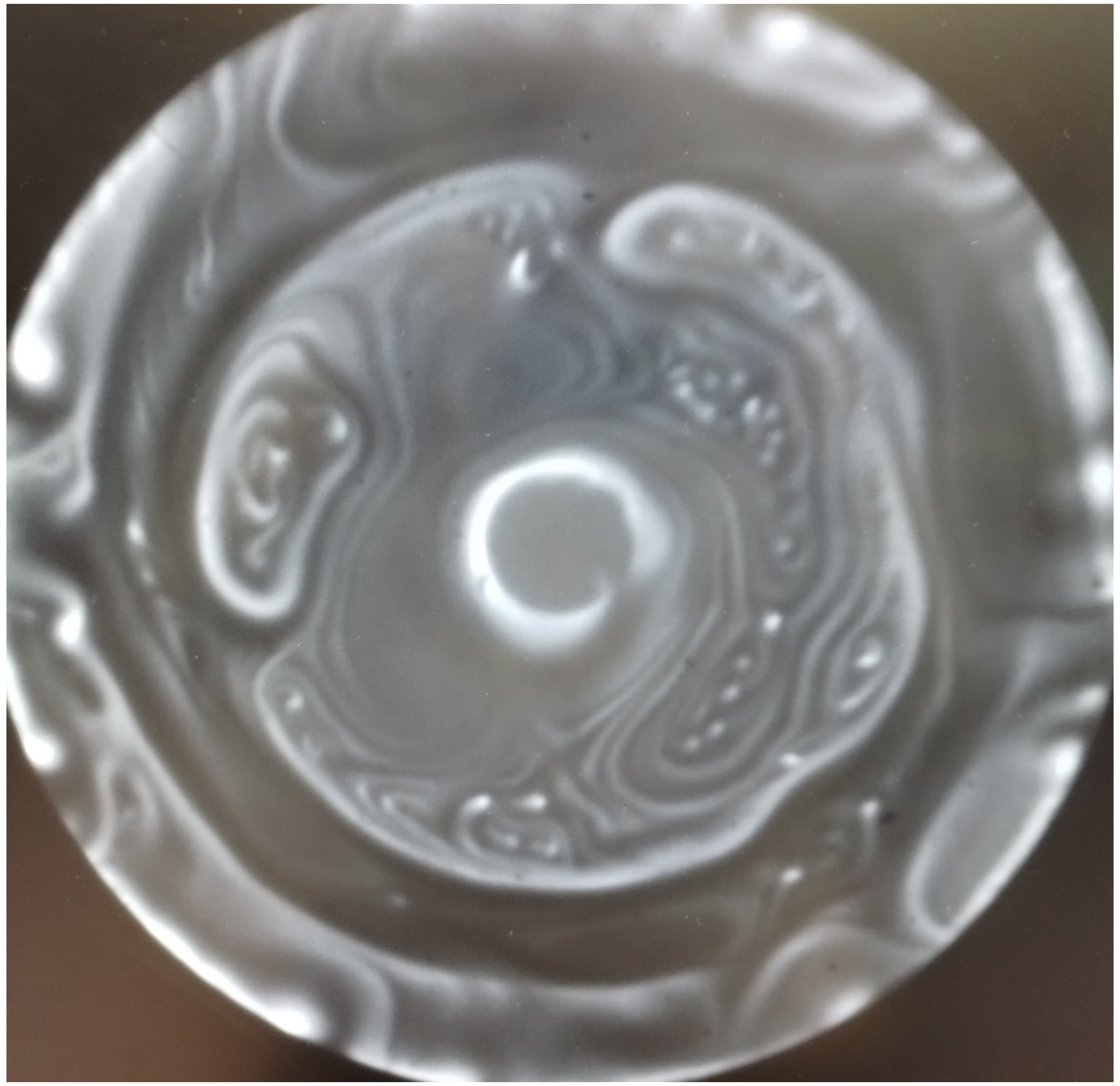
Star formation: azimuthal instability of a radially expanding filament

In Helium plasma

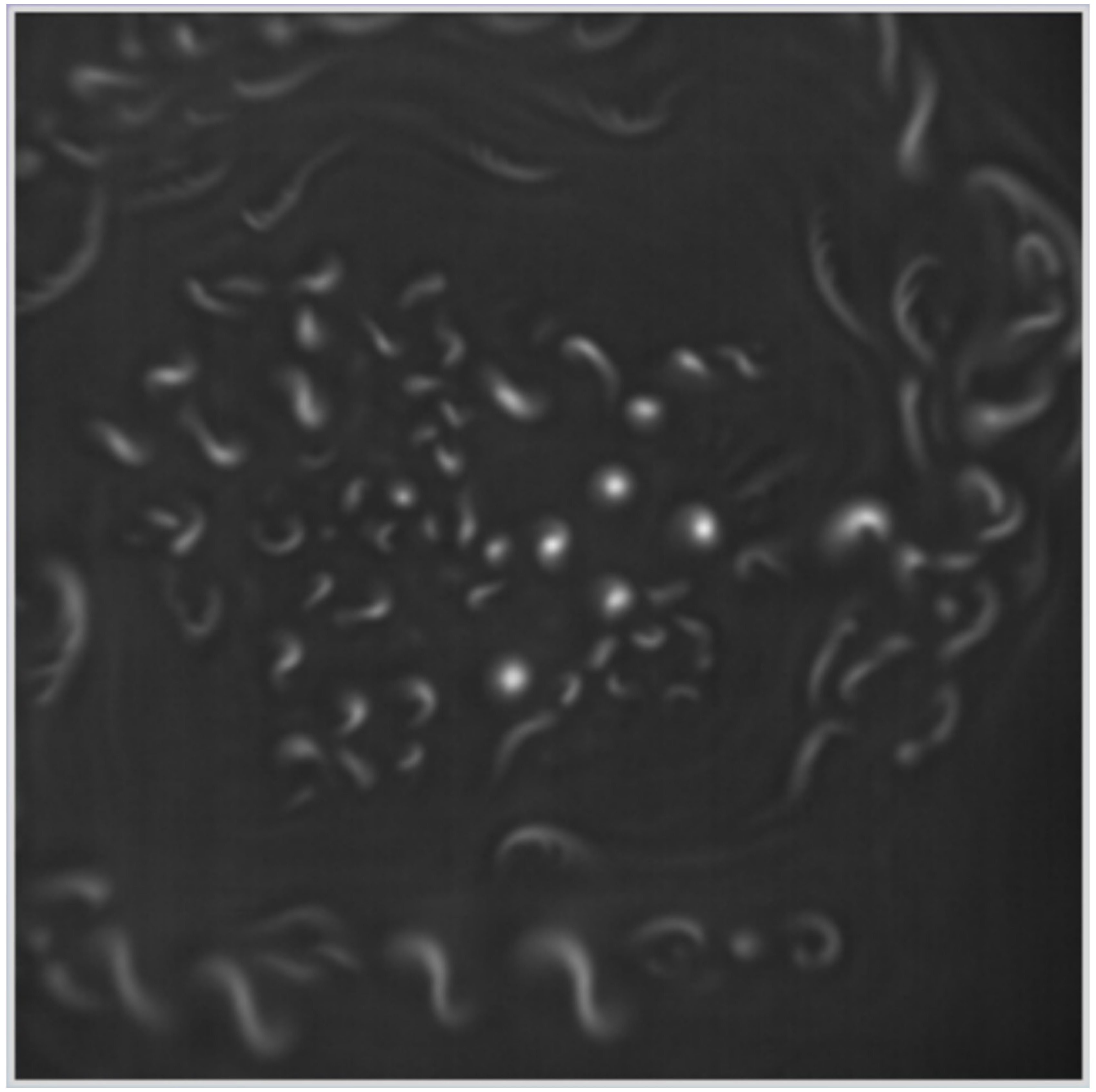




2-D fluid like behavior?



In argon plasma



In neon plasma

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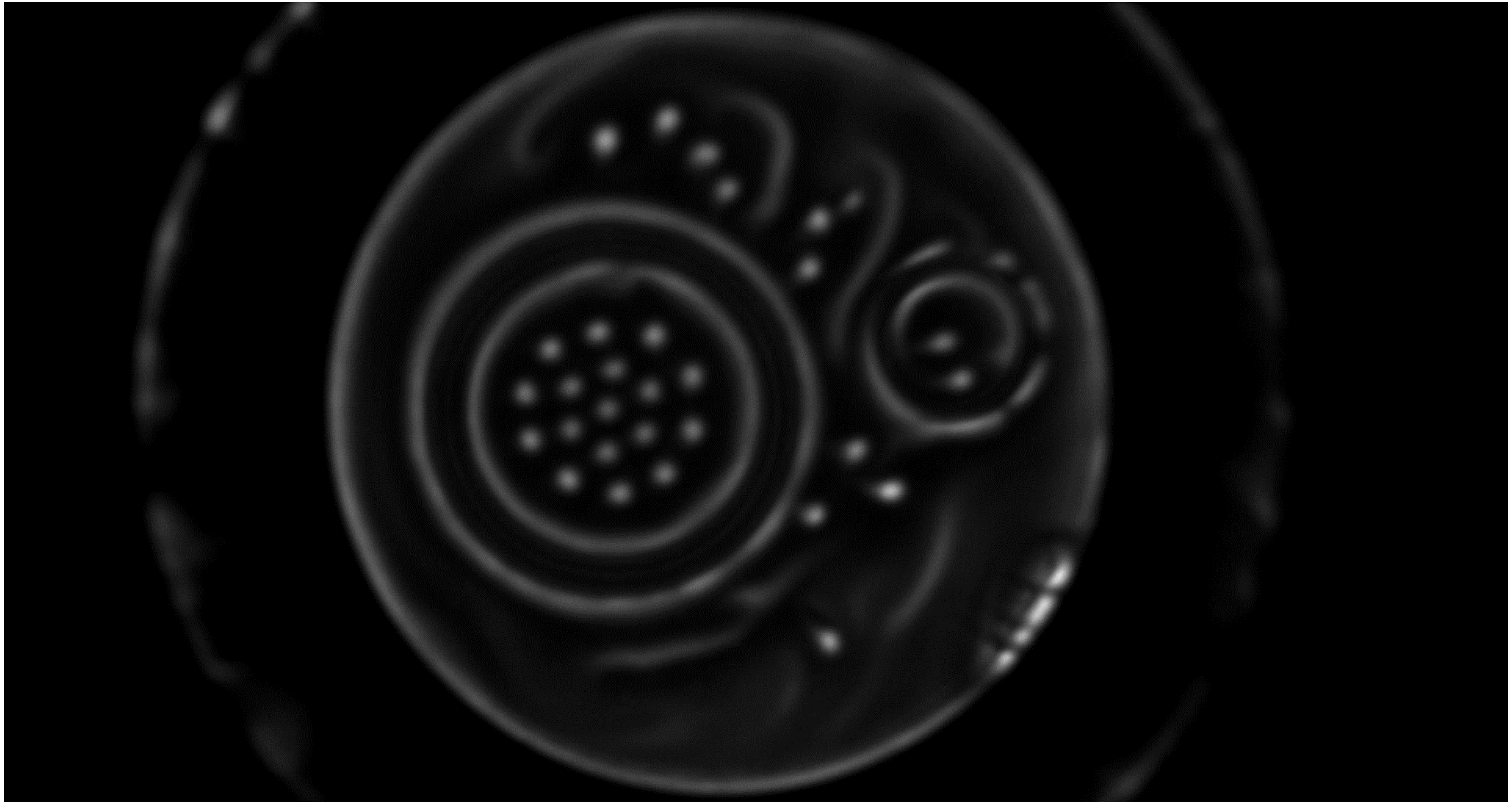
Elongated structures give rise to individual filaments (that can form lattices)



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$\Rightarrow$ All my Professors, who played an active part in teaching me ☺

Thank you very much for your time and interest

Wish you a wonderful summer of research and a great future in plasma physics ☺ …

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