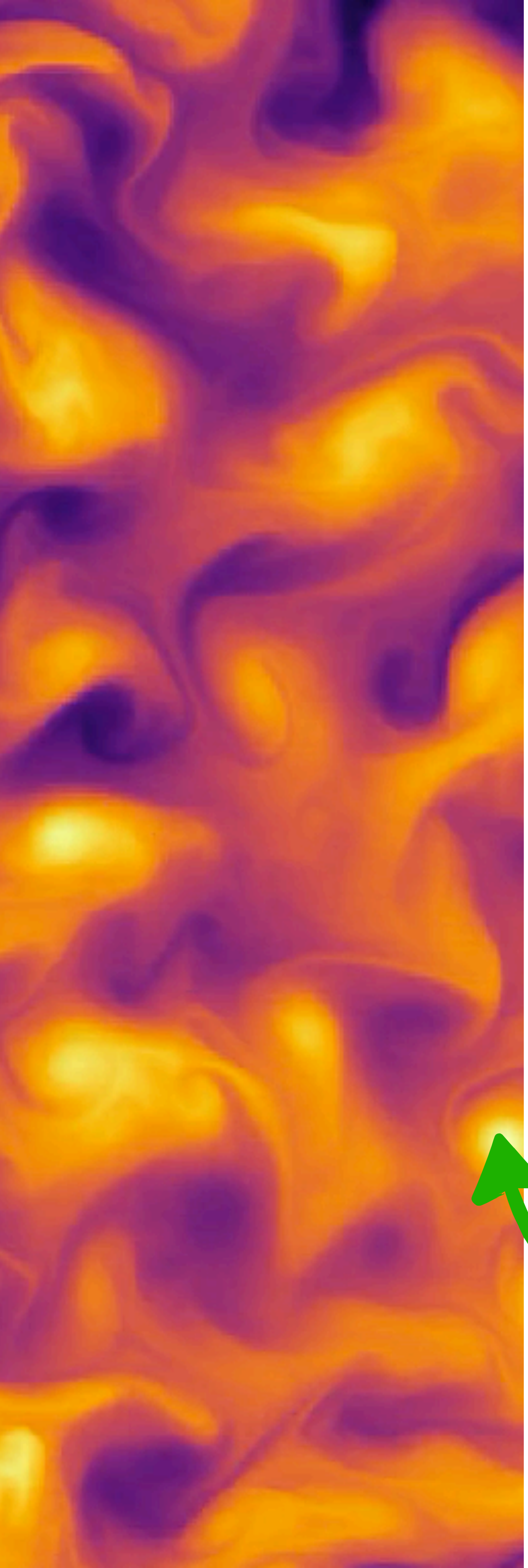

Plasma turbulence

2022 Intro to fusion energy and plasma physics course

June 15, 2022

Manaure Francisquez

mfrancis@pppl.gov

computer simulation
of magnetized plasma
turbulence

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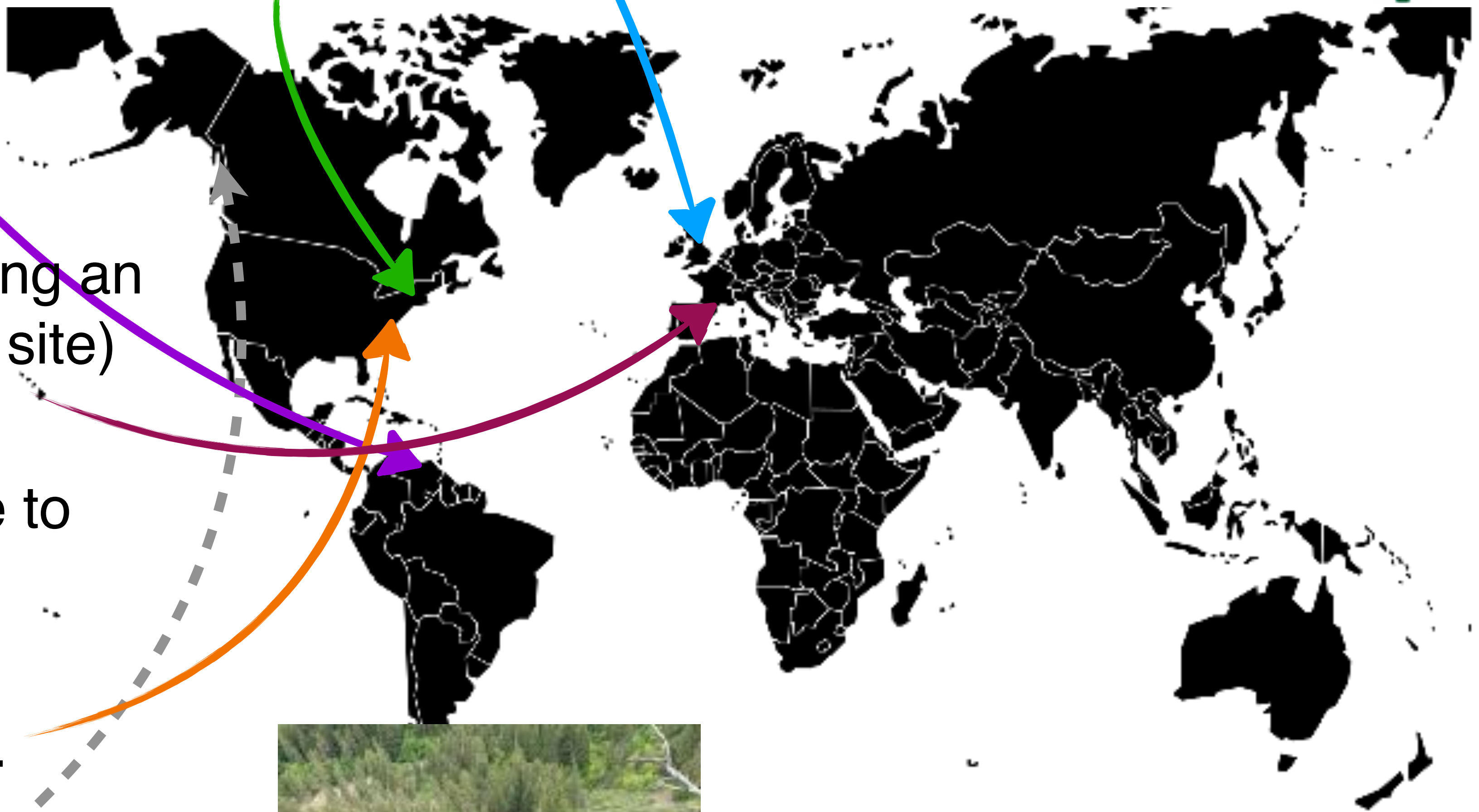
About me:

Grew up in Venezuela

love arepas

Finished school at a UWC in the UK

Did undergraduate degree at Dartmouth College



Decided to study fusion after doing an internship in Cadarache (ITER's site) modeling RF heating systems.

Went back to Dartmouth College to get a PhD.

Did a post-doc at MIT

Now I work on Gkeyll at PPPL.

Presently talking to you from here

I enjoy rock climbs in my free time

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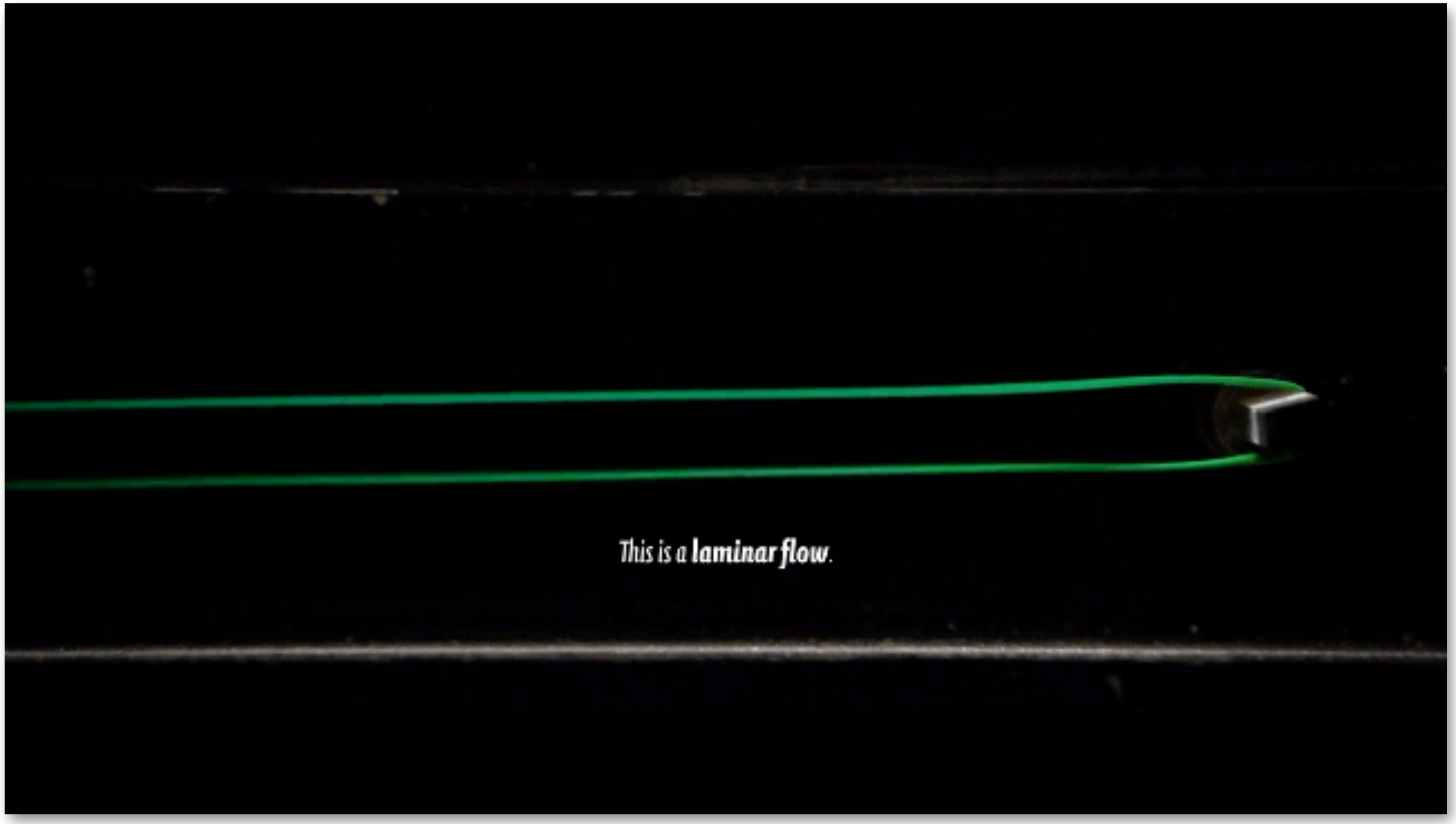
Topics in this talk:

- Turbulence: what it isn’t, what it is. & examples.
- Instability as a cause of turbulence: Rayleigh-Taylor as an example.
- Turbulent energy cascades.
- Examples of turbulence in plasmas.
- Mechanisms that can drive turbulence in plasmas.
- Ways to explore turbulence: experiment & computer simulation (including ML)

Turbulence: Everywhere in nature & in man-made fluids. We all experience it.

Let's start by seeing its onset in a channel with fluid past an obstacle oozing green dye:

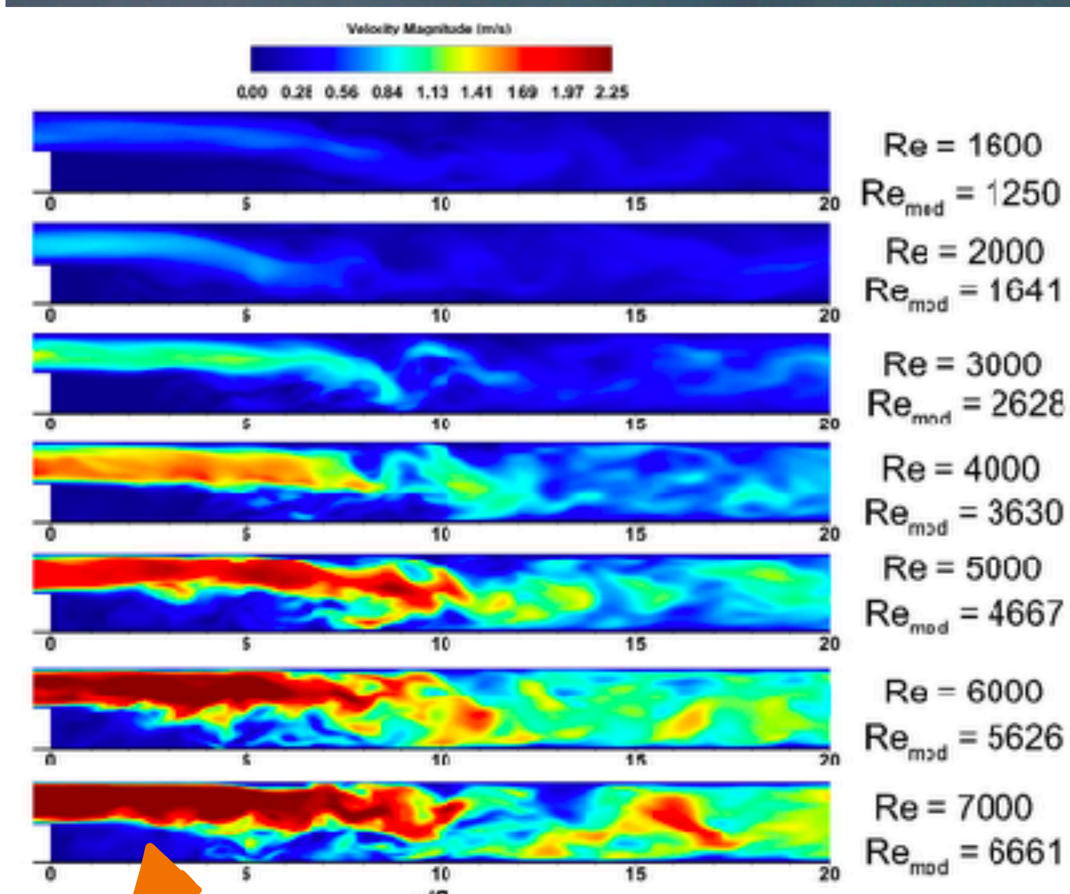


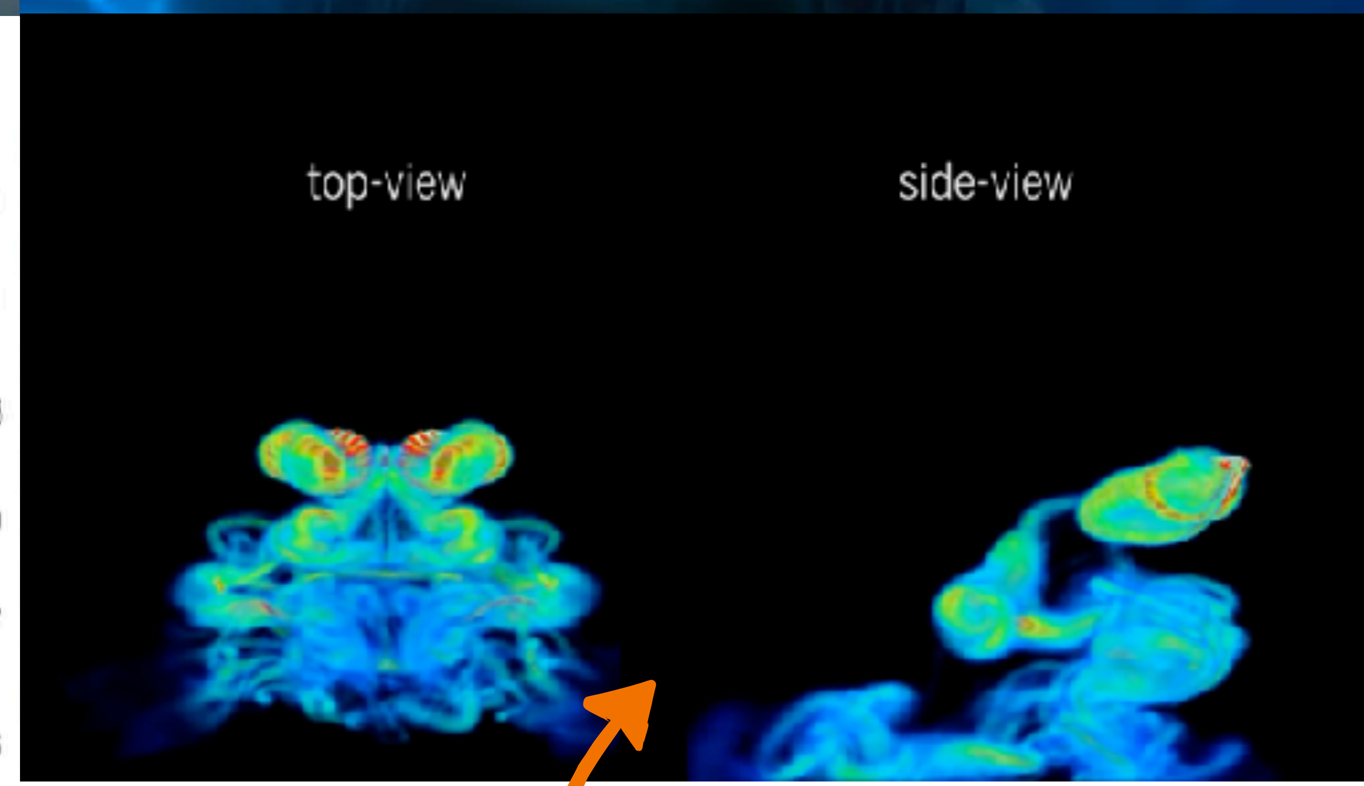
Credit:
The Lutetium Project

Visually we can say turbulence is:

- Complex
- Hard to predict
- Swirly

Turbulence: Everywhere in nature & in man-made fluids. We all experience it.





Butterfly (Fei, Yang, PRE 93, 033124 (2016)).

Velocity in blood flow (N. S. Kelly, et al. Fluids 5, 57 (2020)).

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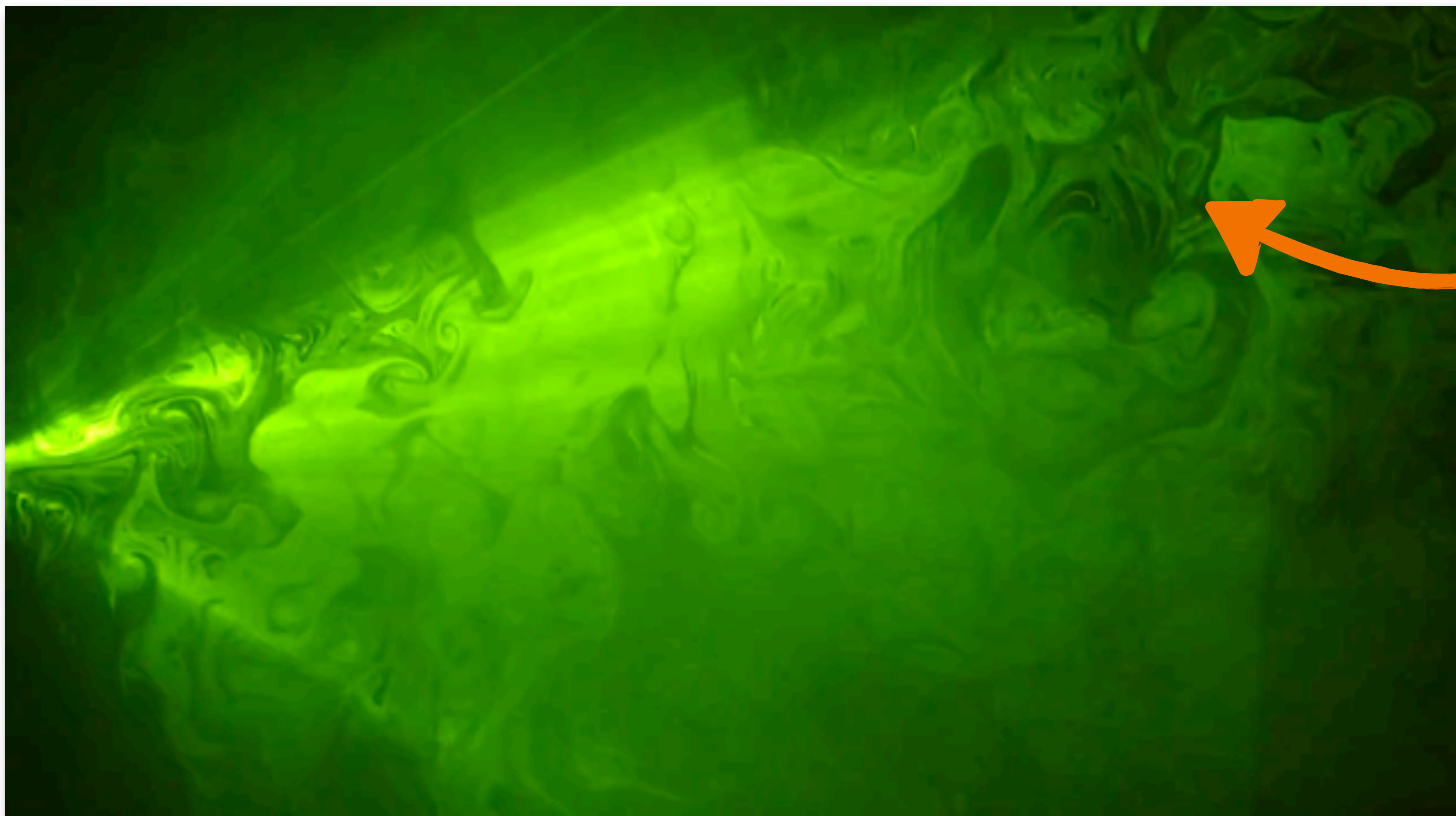
Turbulence: Everywhere in nature & in man-made fluids. We all experience it. Yet, how do we define it?

Cool at-home demo:
visualize air turbulence in your room using

- A fog machine.
- Diffracted laser light.



photons scatter off of air molecules and reach our eyes, allowing us to see the turbulent structures in the air

Credit: 3blue1brown

Other properties of turbulence more closely related to its mathematical description:

- Chaotic.
- Ergodic.
- Diffusive and mixing.
- Unstable

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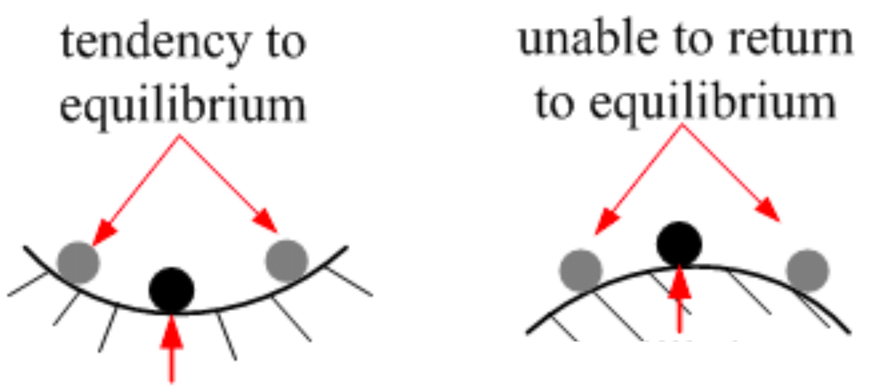
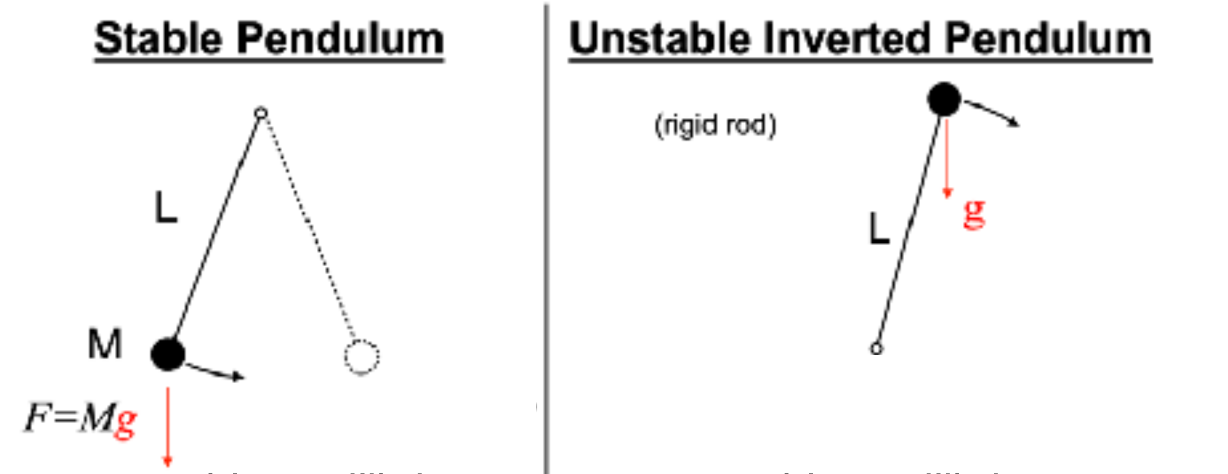
Instability is key: Turbulence arises in systems in unstable equilibrium, where perturbations can grow and lead to instability



stable equilibrium | unstable equilibrium



stable equilibrium | unstable equilibrium

- Using Newtonian mechanics ($ma = F$) one can show the frequency of these motions

$$\omega = \sqrt{\frac{g}{L}} \qquad \omega = \sqrt{-\frac{g}{L}} = i\sqrt{\frac{g}{L}} = i\gamma$$

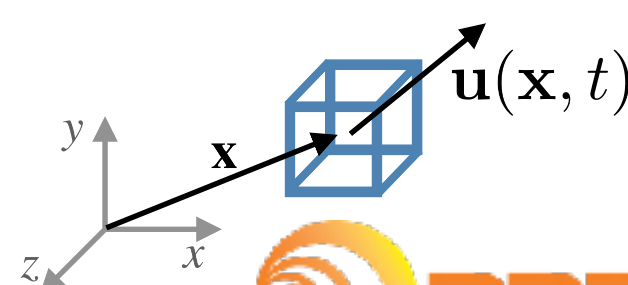
Recall the position of the pendulum: $x(t) \propto Ae^{i\omega t}$

- Fluid mechanics turbulence is often studied via the Navier-Stokes equation. For incompressible fluids:

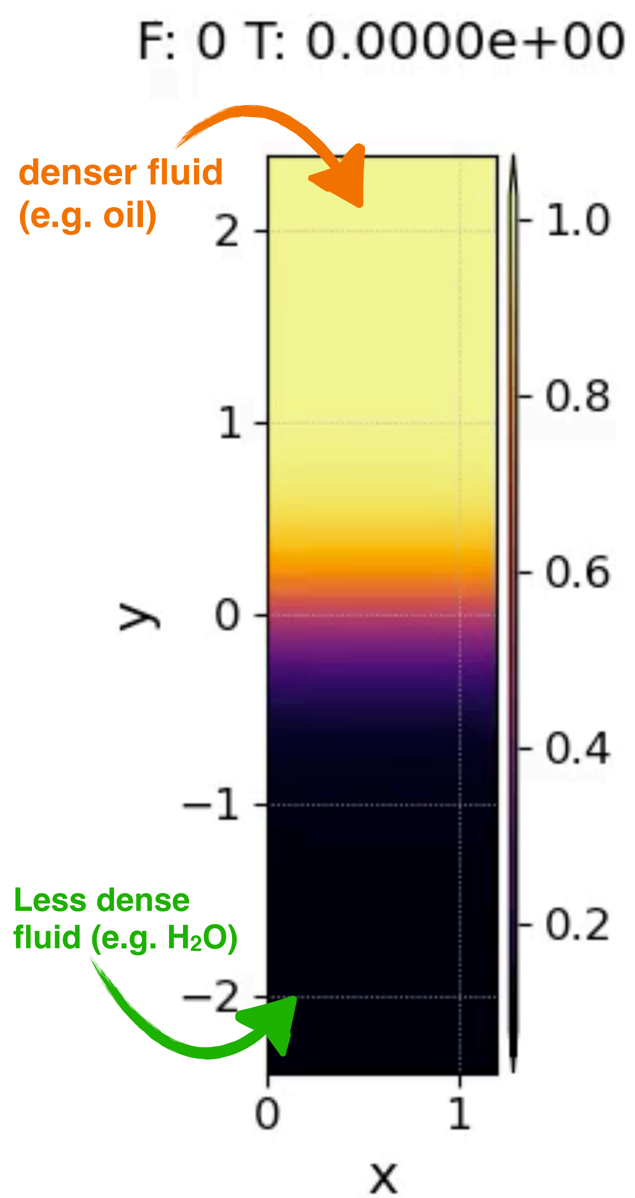
$$\underbrace{\rho\frac{D\mathbf{u}}{Dt} = \rho\left(\frac{\partial}{\partial t} + \mathbf{v}\cdot\nabla\right)\mathbf{v}}_{\text{inertia (like } ma \text{ in } F=ma)} = \underbrace{-\nabla p}_{\text{pressure force}} + \underbrace{\nu\rho\nabla^2\mathbf{u}}_{\text{viscous force}} + \underbrace{\rho\mathbf{g}}_{\text{gravitational force}}$$

p : pressure
ν : viscosity
$\mathbf{g}$: gravitational acceleration

A statement of Newton's 2nd law for an infinitesimal element at coordinate $\mathbf{x}$



Example of an unstable fluid: dense fluid on top, light fluid on the bottom



Computer simulation done with Gkeyll, a free, open-source code for modeling plasmas

How does density stratification lead to instability?

$$\rho\left(\frac{\partial}{\partial t}+\mathbf{v}\cdot\nabla\right)\mathbf{v}=-\nabla p+\nu\rho\nabla^2\mathbf{u}+\rho\mathbf{g}$$

$$\frac{\partial T}{\partial t}+\mathbf{v}\cdot\nabla T=0 \qquad\qquad \nabla\cdot\mathbf{v}=0$$

Neglect viscosity and nonlinearities, one can combine these equations to show that the frequency of sinusoidal perturbations (waves) in this scenario is given by

$$\omega=\sqrt{-\frac{g}{\rho}\frac{\partial\rho}{\partial z}}$$

and if we assume $\rho = e^{y/L}$

$$\omega=i\sqrt{\frac{g}{L}} \quad\Rightarrow\quad \gamma=\sqrt{\frac{g}{L}}$$

Compare with pendulum in previous slide.

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The perturbations grow until fully developed turbulence ensues

dense fluid

lighter fluid

Credit:
Mark Stock (Vic2d)

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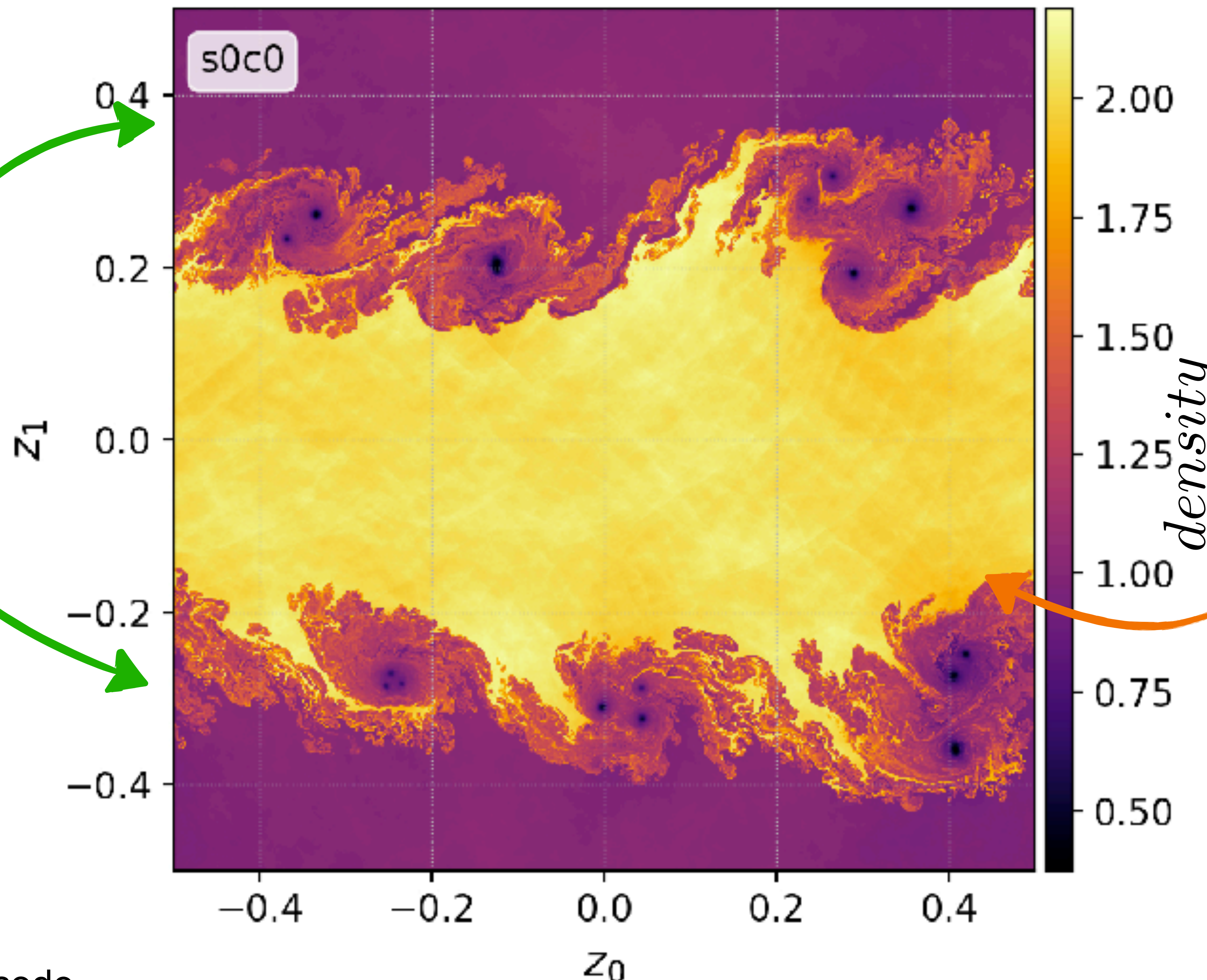
Many other sources of instability in fluids; e.g. the Kelvin-Helmholtz is driven by gradients in the fluid velocity.

Here the fluid is flowing to the right



Here the fluid is flowing to the left

Computer simulation done with Gkeyll, a free, open-source code for modeling plasmas

As turbulence evolves, we see the formation of smaller eddies, and a rich distribution of eddies of all sizes.

In the home-demo visualization of air turbulence in your room:

Credit: 3blue1brown

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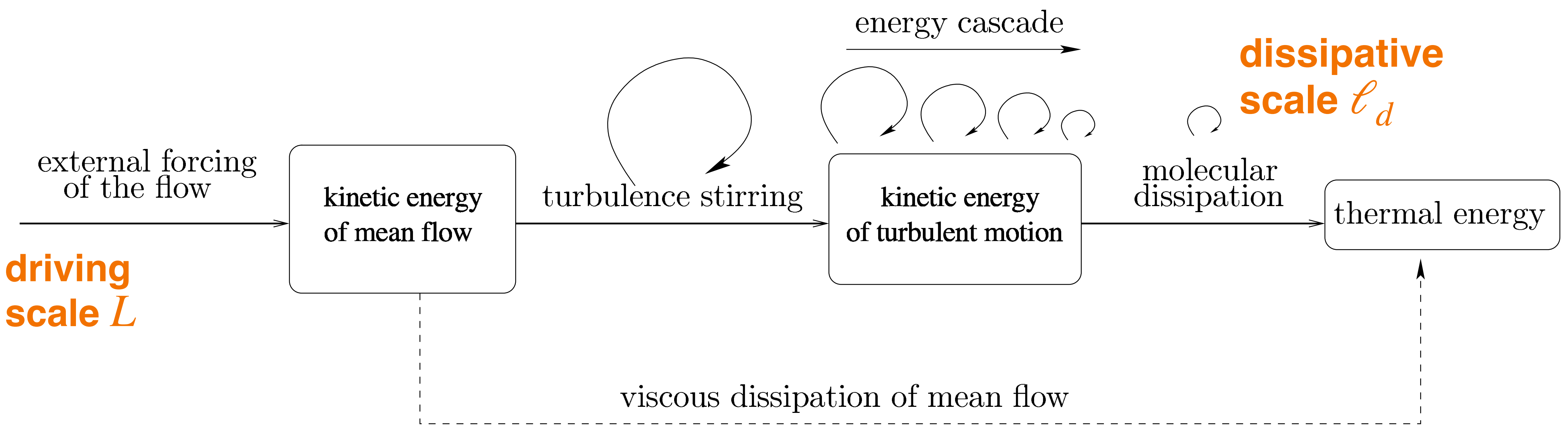
As turbulence evolves, we see the formation of smaller eddies, and a rich distribution of eddies of all sizes.

The “swirly” nature of fully developed turbulence is due to a transfer of energy from large scales structures in the fluid, to smaller and smaller swirls (eddies)

known as an “energy cascade”.

This continues to ever-smaller scales, until molecular dissipation converts kinetic energy of the eddies into thermal energy (heating up the fluid). The energy flow in a turbulence fluid is



Credit: Lévêque, E. EAS Publications Series 21, 7 (2006).

Determination of the rate of energy transfer between scales is one of the most consistent results in turbulence research

At some scales smaller than the large scale of the external forces (or driving flow inhomogeneities), but larger than the dissipative scale, this transfer of energy must be constant. Otherwise we'd see a build up of kinetic energy at one particular scale.

Russian scientist Andrey Kolmogorov determined the spectrum of the energy as a function of the size of the eddies. Similar to how special relativity stems from three basic assumptions (c=constant, c is the max speed, inertial universality of the laws of physics), Kolmogorov employed two basic assumptions[1,2].

He was concerned with differences in velocity between two points

$$\delta\mathbf{u}(\mathbf{x},\mathbf{r},t) = \mathbf{u}(\mathbf{x}+\mathbf{r},t) - \mathbf{u}(\mathbf{x},t)$$

in stationary, homogeneous and ***isotropic*** *turbulence.*

[1] Kolmogorov, A. N. Dokl. Akad. Nauk SSSR, 32, 19-21 (1941).
[2] Lévêque, E. EAS Publications Series 21, 7 (2006).

Energy transfer between scales: The Kolmogorov spectrum[1,2]

The two hypotheses were:

1. At $r \ll L$ the distributions of $\delta\mathbf{u}(\mathbf{x},\mathbf{r},t)$ are "universal", i.e. independent of the type of external force or driving mechanism, & only a function of ν (viscosity) and the mean energy dissipation rate per unit mass ($\bar{\epsilon}$).

   Units $\left.\begin{array}{r} [\nu] = m^2/s \\ [\bar{\epsilon}] = J/(kg\, s) = m^2/s^3 \end{array}\right\}$ $\left\langle|\delta\mathbf{u}(\mathbf{x},\mathbf{r},t)|^2\right\rangle = \sqrt{\nu\bar{\epsilon}}\,\Phi(r/\ell_d)$

   can also infer the dissipation length scale $\ell_d = (\nu^3/\bar{\epsilon})^{1/4}$

2. At $\ell_d \ll r \ll L$ the distributions of $\delta\mathbf{u}(\mathbf{x},\mathbf{r},t)$ are independent of ν.

$$\left\langle|\delta\mathbf{u}(\mathbf{x},\mathbf{r},t)|^2\right\rangle = b\,(\bar{\epsilon}\,r)^{2/3} = b\,\bar{\epsilon}^{2/3}\,k^{-2/3}$$

$\langle\ldots\rangle$: ensemble average

Φ : Universal dimensionless function

k : wavenumber

From this last equation deduce the energy spectrum

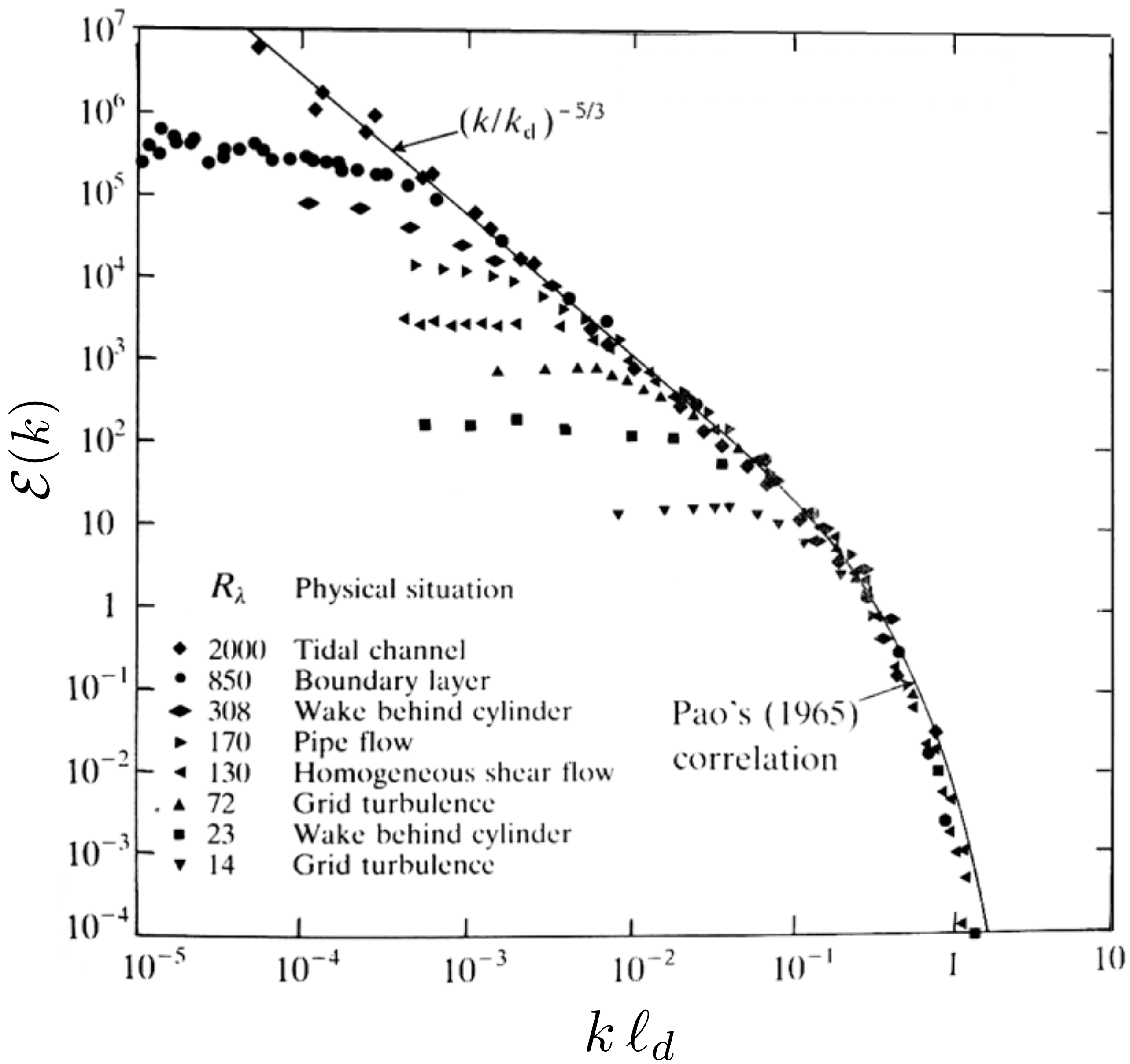
$$\boxed{\mathcal{E}(k) = C\,\bar{\epsilon}^{2/3}\,k^{-5/3}}$$ since $$\left\langle|\delta\mathbf{u}(\mathbf{x},\mathbf{r},t)|^2\right\rangle \propto E = \int_0^\infty \mathcal{E}(k)\,\mathrm{d}k$$

Kolmogorov spectrum

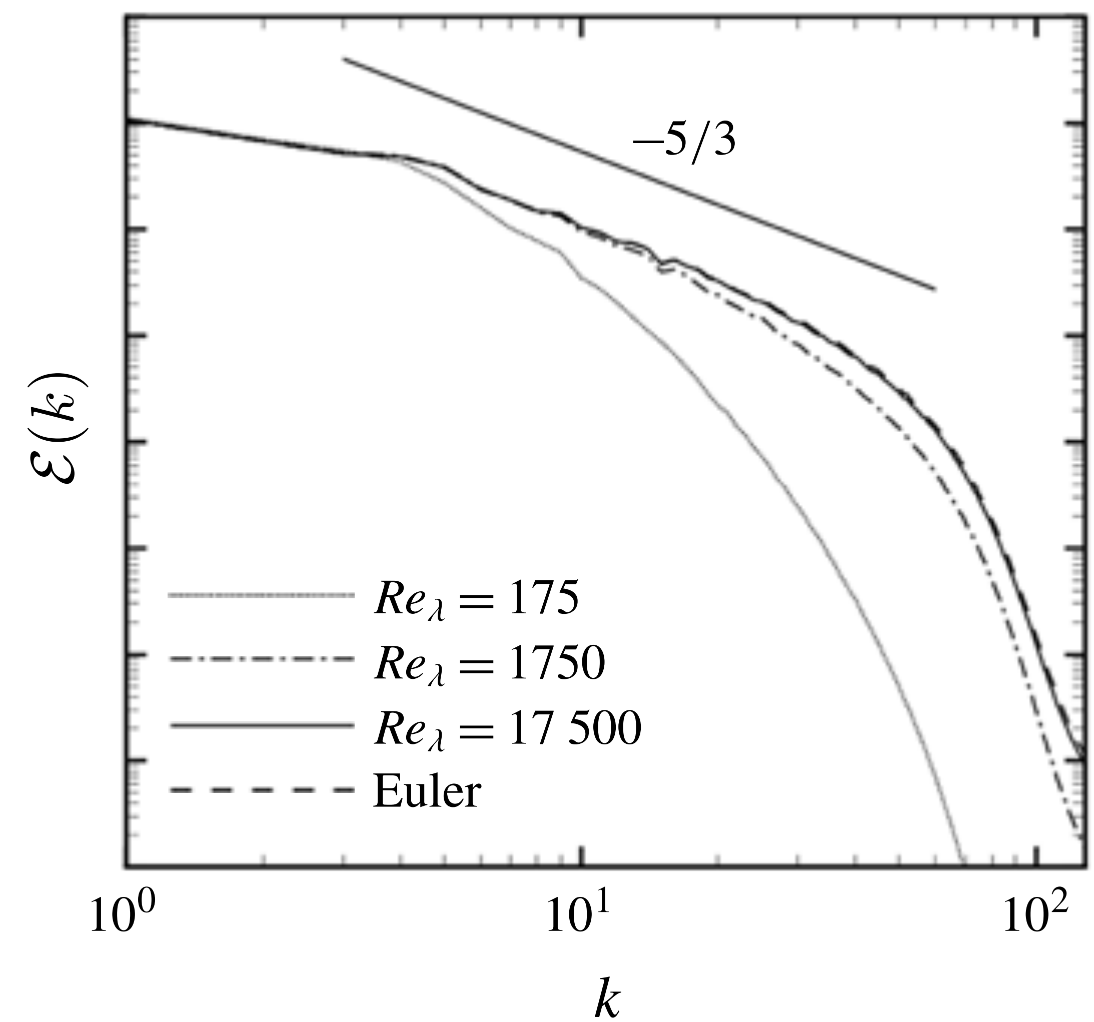
[1] Kolmogorov, A. N. Dokl. Akad. Nauk SSSR, 32, 19-21 (1941).
[2] Lévêque, E. EAS Publications Series 21, 7 (2006).

Studying, modeling & explaining turbulence is hard. But Kolmogorov's spectrum is a successful result with countless experimental & numerical confirmations

Experiments[1]:



Navier-Stokes & Euler simulations of a dense gas[2]:



[1] McComb, W. D. The Physics of Fluid Turbulence. Oxford University Press (1990).
[2] Sciacovelli, L., et al. J. Fluid Mech. 800, 140 (2016).

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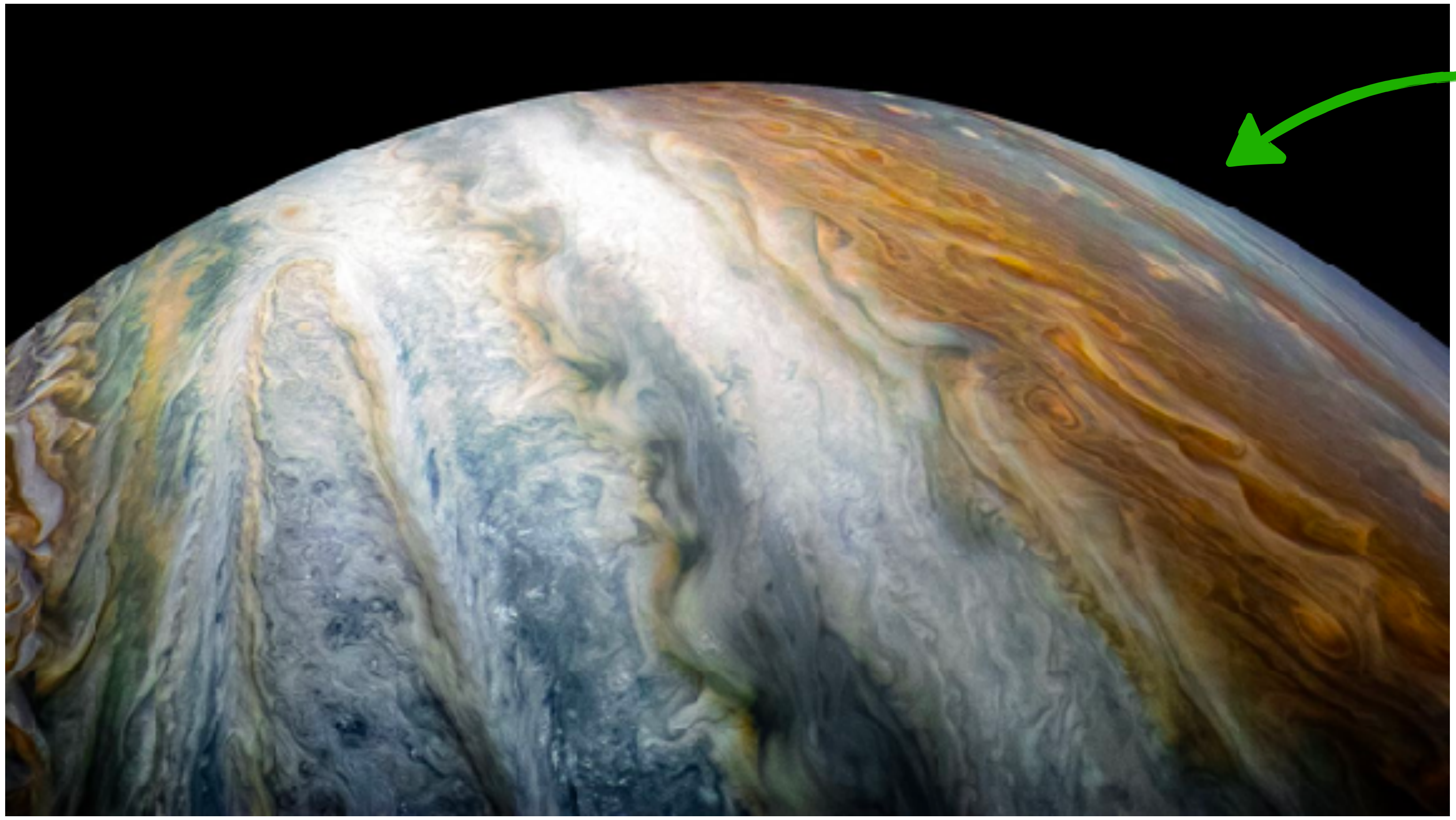
Kolmogorov's spectrum has even been accounted for in non-scientific matters…

Van Gogh's *La Nuit étoilée*

"the probability distribution function (PDF) of luminance fluctuations in some impassioned van Gogh paintings, painted at times close to periods of prolonged psychotic agitation of this artist, compares notable well with the PDF of the velocity differences in a turbulent flow as predicted by the statistical theory of Kolmogorov"[1]

[1] J. L. Aragón, et al. J. of Mathematical Imaging & Vision 30, 275 (2008).

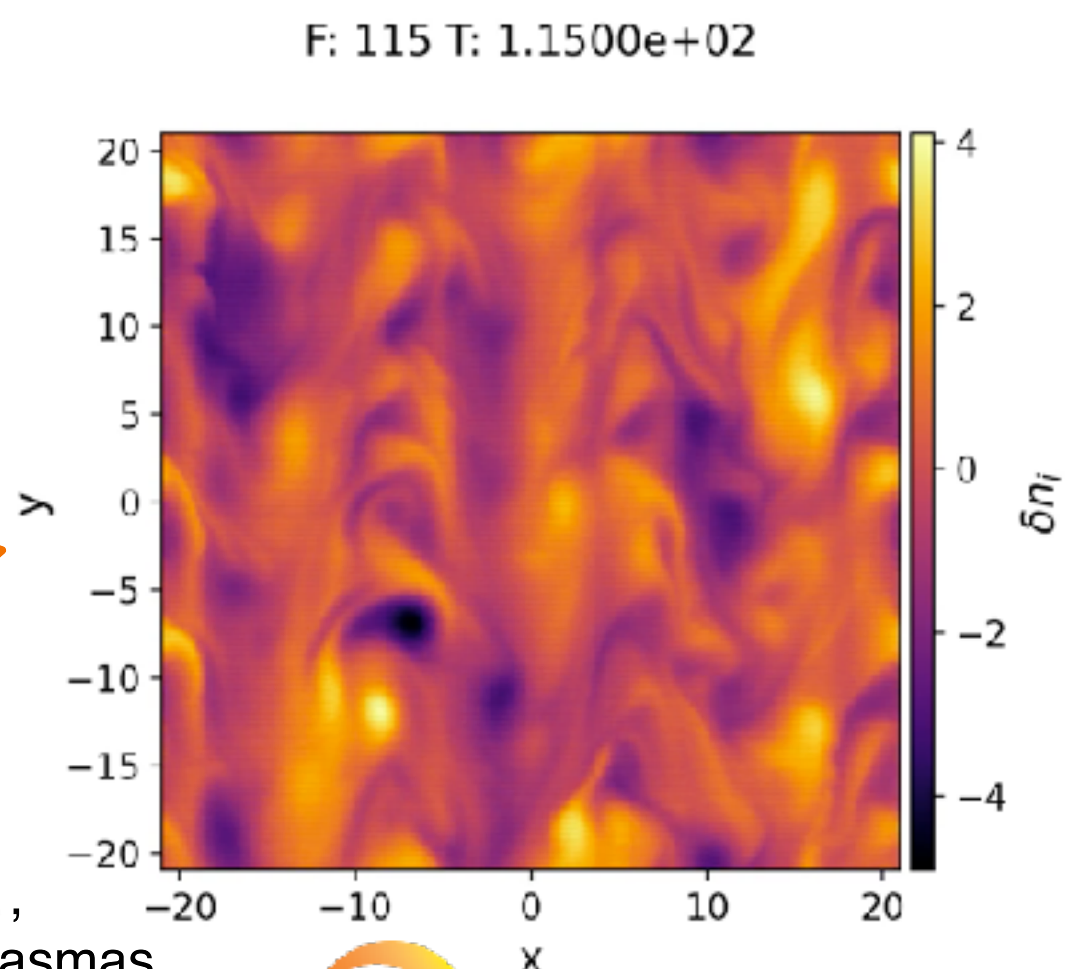
Turbulent energy transfer is usually from large to small scales, but there are instances of "inverse energy cascades"



Jupiter's turbulent atmosphere forms organized "jet streams"

Somehow energy is transferred from small scales to these large scale flows.

A similar process can occur in plasma turbulence. Here we see jet formation in a reduced-model simulation of magnetized plasma turbulence



Computer simulation done with Gkeyll, a free, open-source code for modeling plasmas

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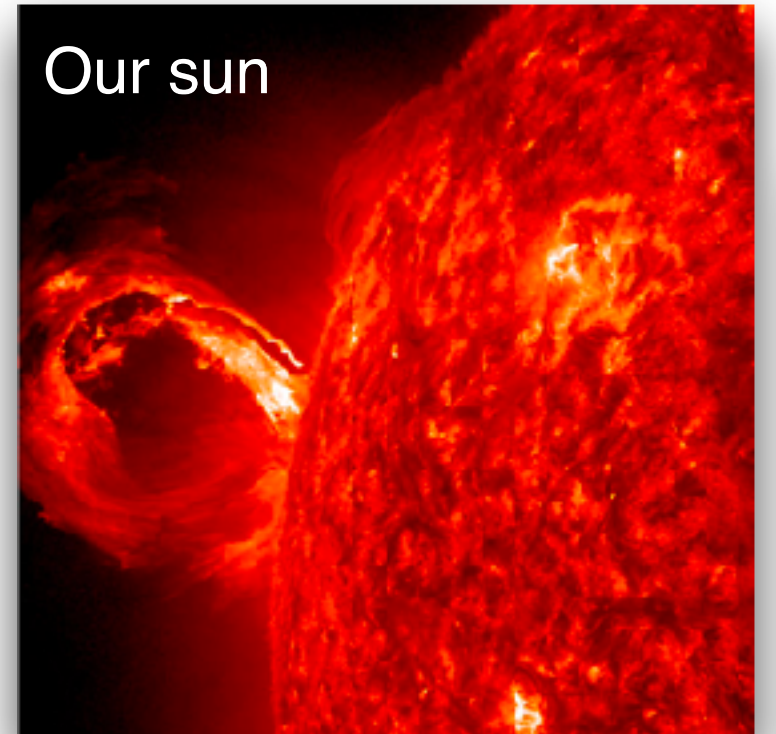
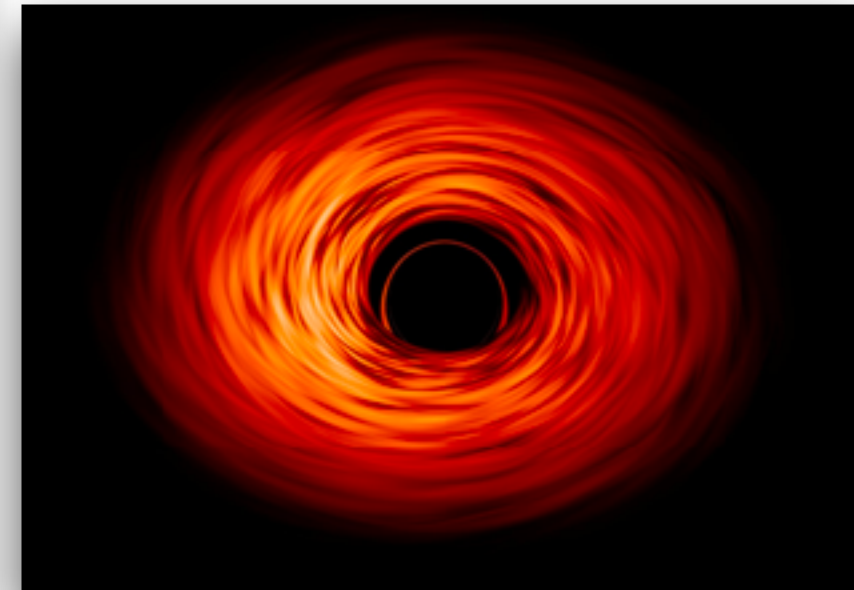
Plasma turbulence is everywhere laboratories & nature



Credit: NASA

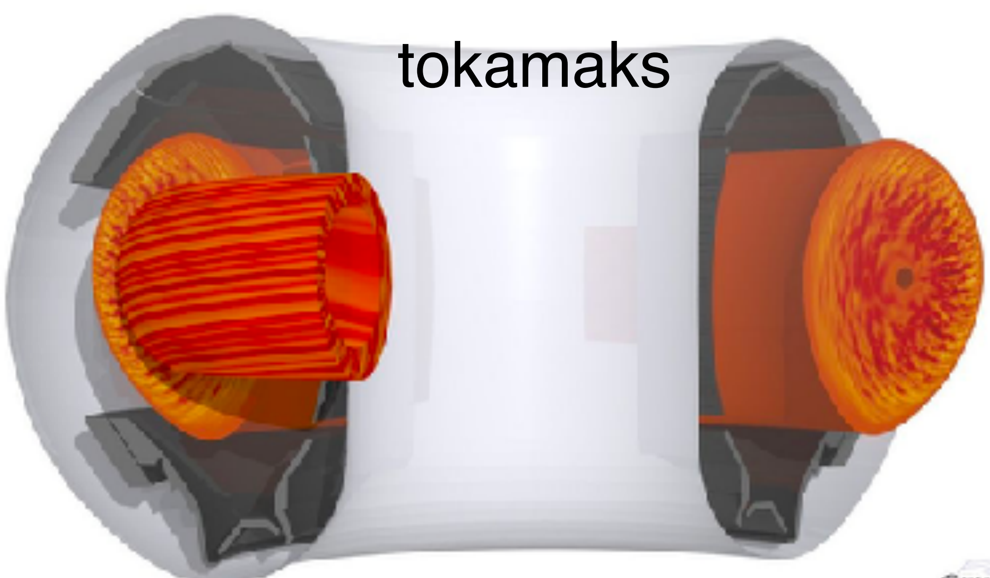
Turbulent plasma in the accretion disk of a black hole



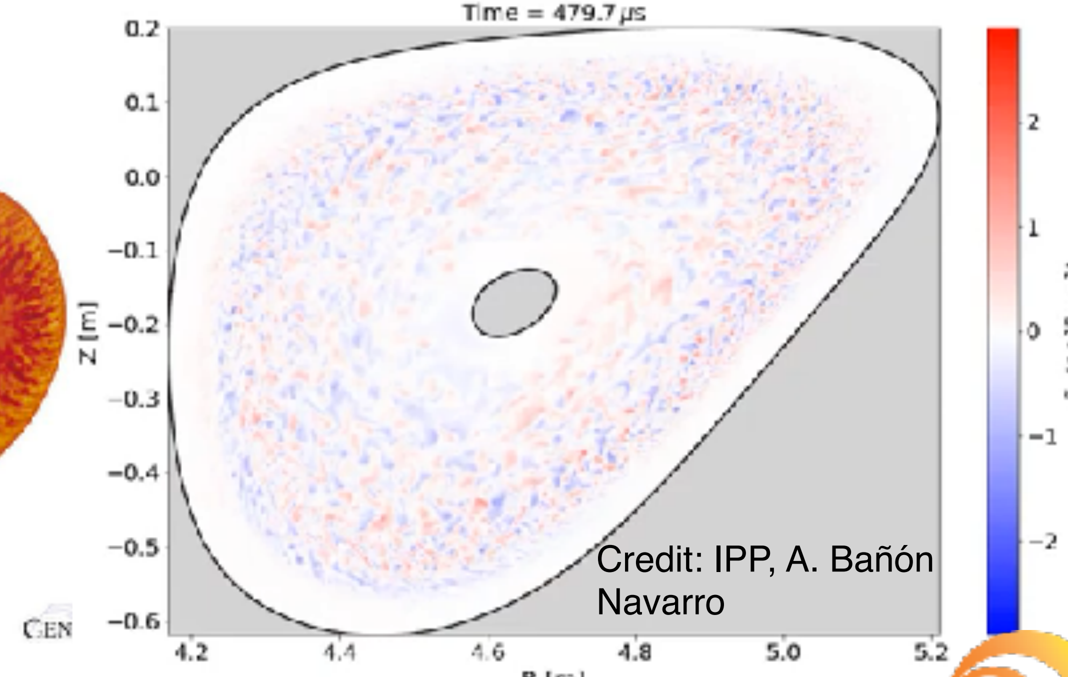
Credit: NASA

Credit: NASA

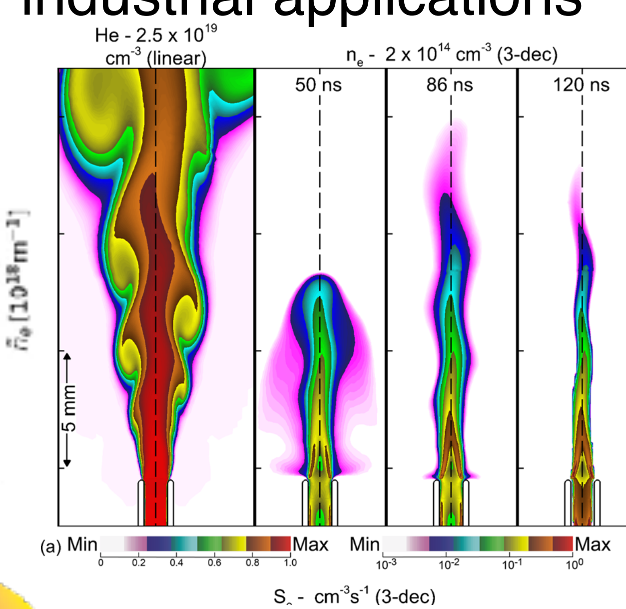
Credit: EUROfusion



stellarators



Plasma jets in medical/ industrial applications



A. M. Lietz, et al. APL 111, 114101 (2017).

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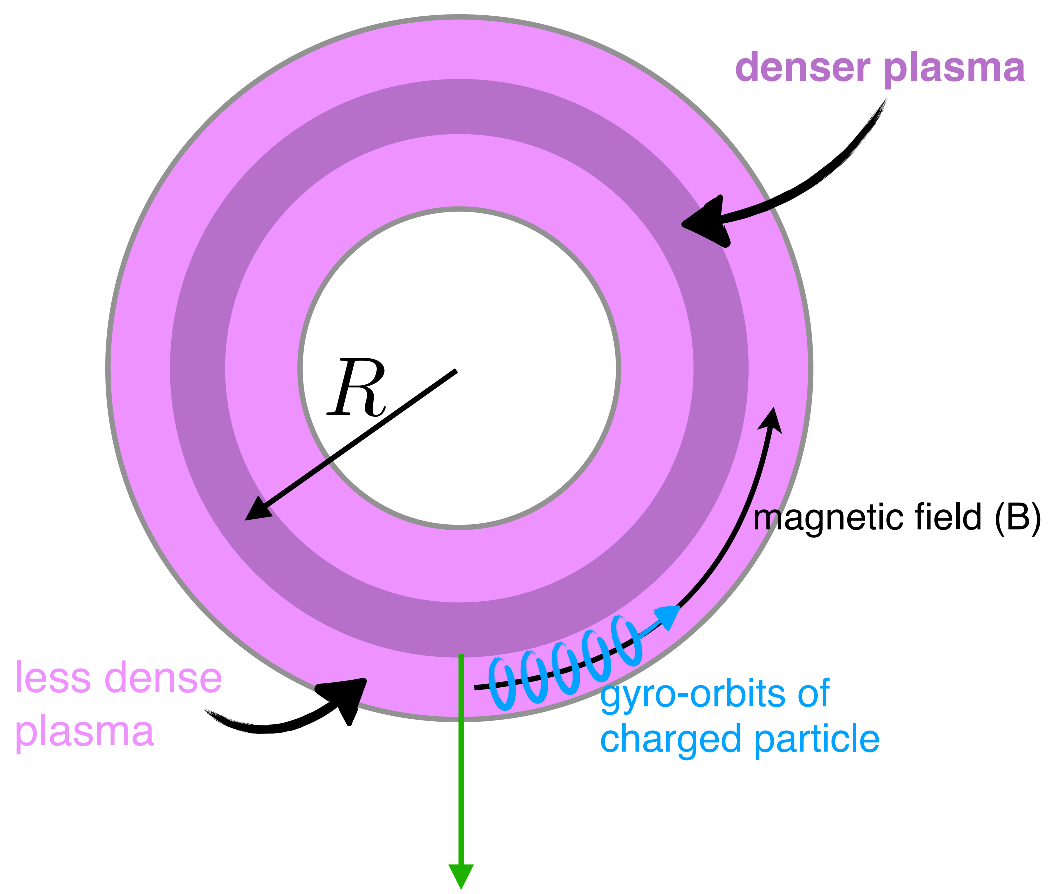
Other similarities exist between neutral fluid & plasma turbulence? Instability drives turbulence. Power law energy spectrum.

There are many drivers of instability in plasmas. Let's consider 1 from tokamaks:

Toroidal plasma (bird's-eye view)



Plasma feels an effective gravitational acceleration due to the centrifugal force:

$$g_{\text{eff}} = \frac{v^2}{R}$$

This scenario has all the ingredients of the Rayleigh-Taylor instability we studied above:

- (effective) gravity
- denser "fluid" on top of lighter "fluid"

So recycling our earlier result, we'd say that the growth rate is

$$\gamma = \sqrt{\frac{g_{\text{eff}}}{L}} = \frac{v}{\sqrt{R\,L}}$$

The length scale L here turns out to be related to the pressure gradient, e.g. $L = p/|\nabla p|$

Is the mechanism the same as in a neutral fluid?

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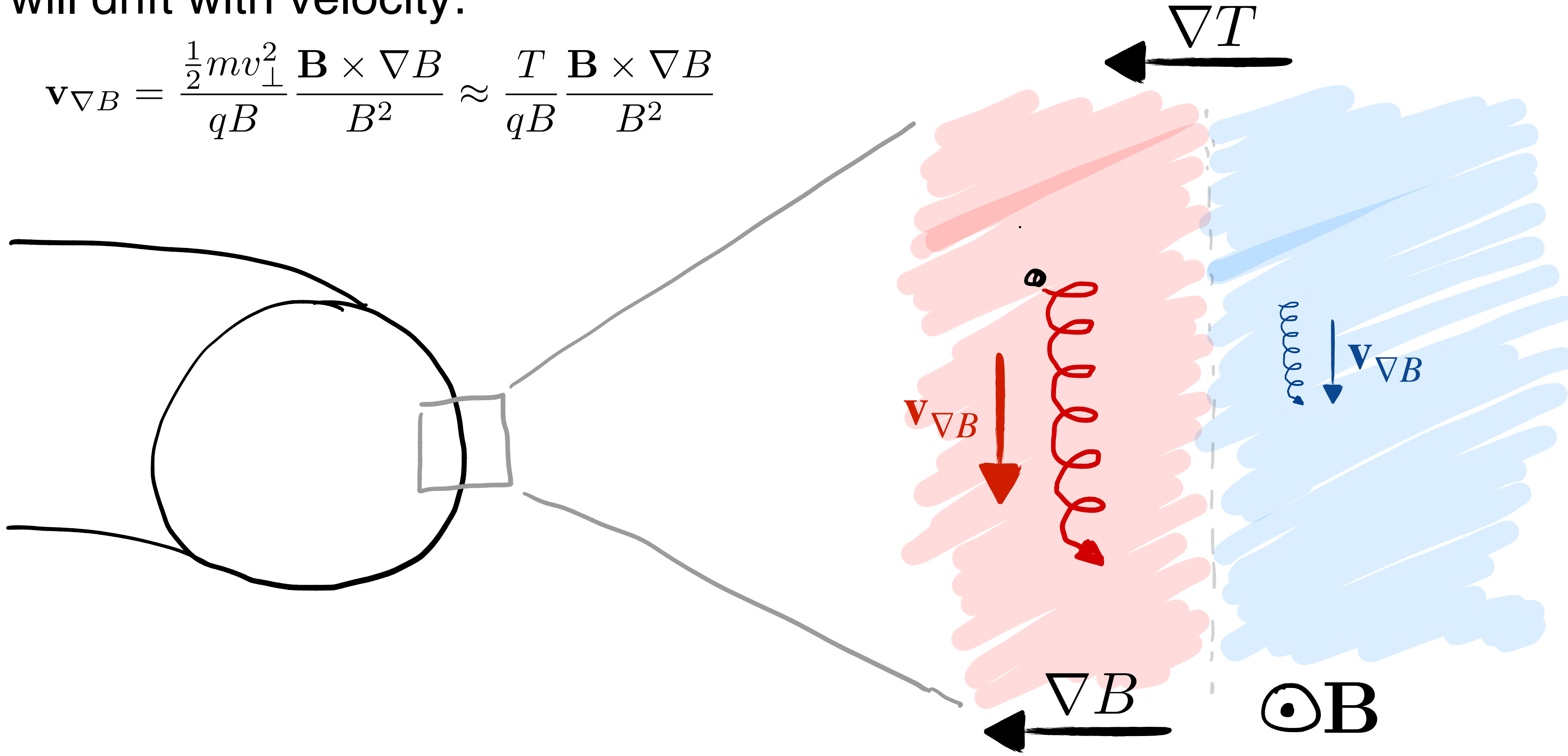
Instability in a torus can be driven by "bad curvature"

Consider a patch of plasma on the outboard region of the torus:

Particles will drift with velocity:

$$\mathbf{v}_{\nabla B} = \frac{\frac{1}{2}mv_\perp^2}{qB}\frac{\mathbf{B}\times\nabla B}{B^2} \approx \frac{T}{qB}\frac{\mathbf{B}\times\nabla B}{B^2}$$



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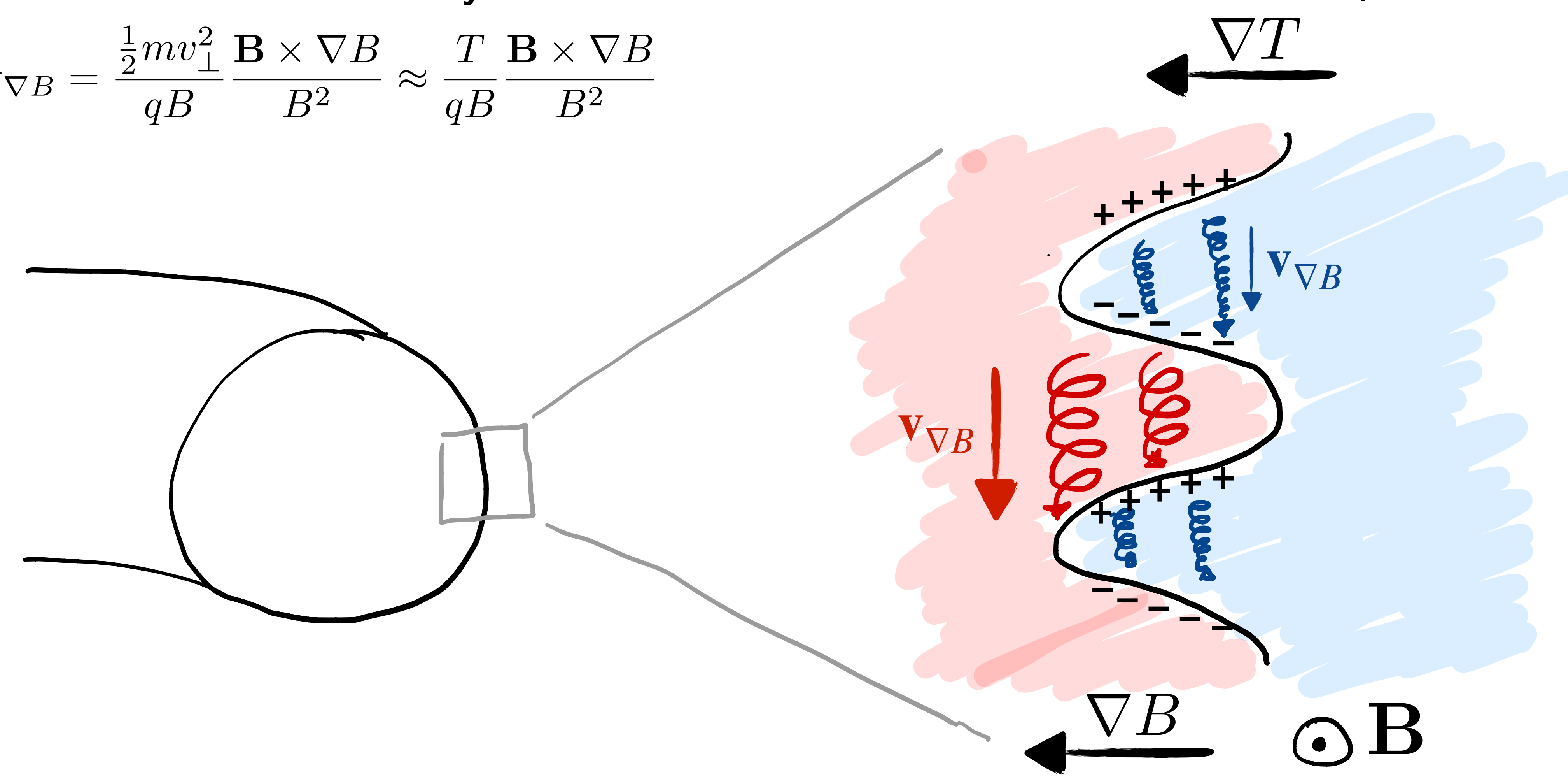
Introduce a sinusoidal perturbation…



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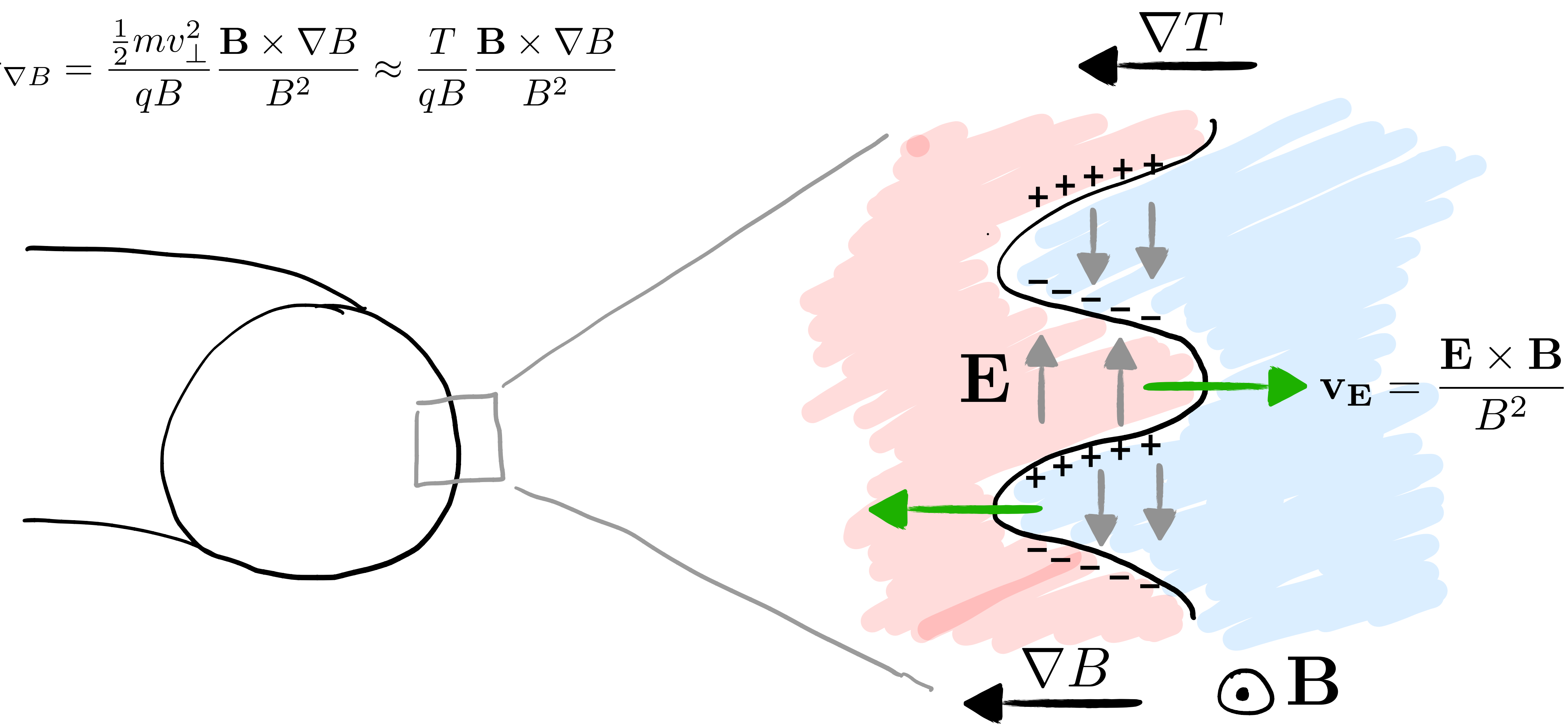
Instability in a torus can be driven by “bad curvature”

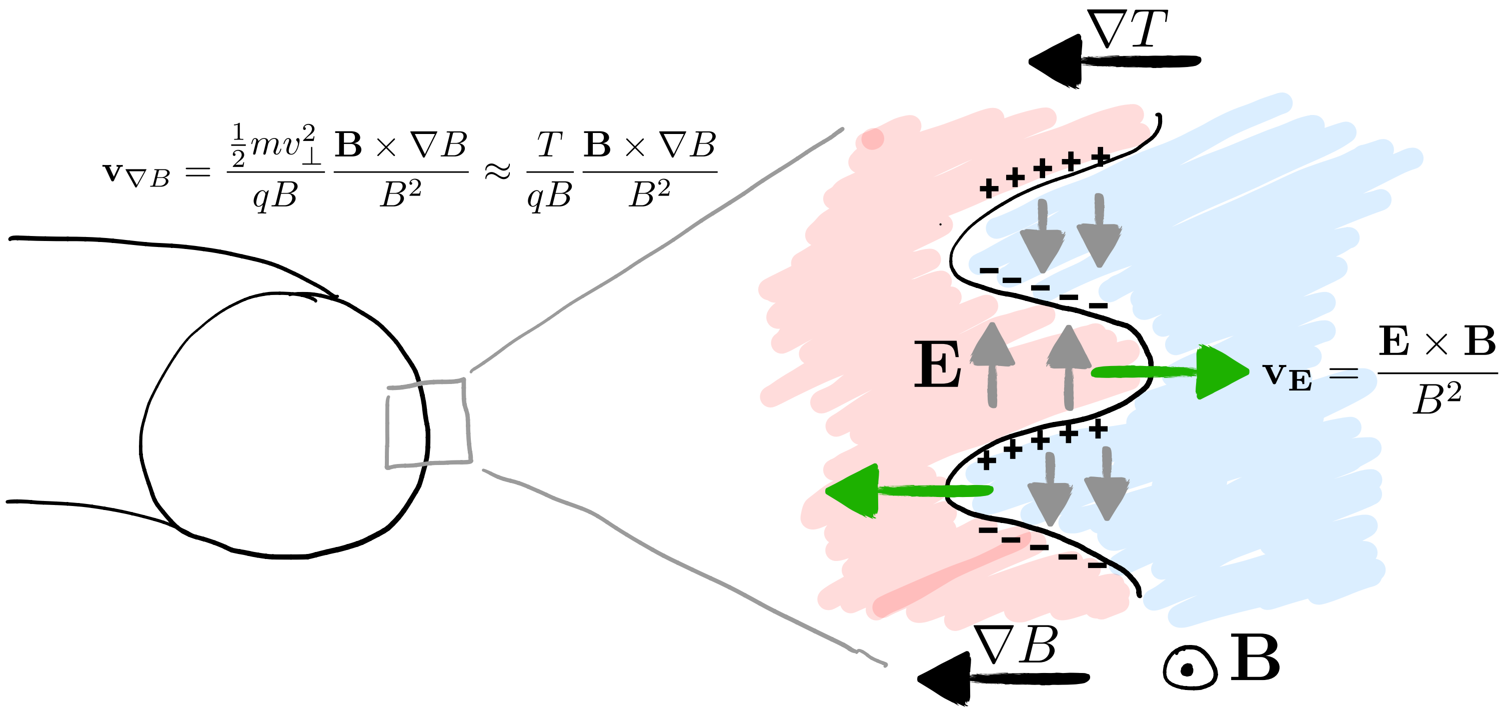
Consider a patch of plasma on the outboard region of the torus:

$$\mathbf{v}_{\nabla B} = \frac{\frac{1}{2}mv_\perp^2}{qB}\frac{\mathbf{B}\times\nabla B}{B^2} \approx \frac{T}{qB}\frac{\mathbf{B}\times\nabla B}{B^2}$$



This $E\times B$ drift reinforces the perturbation, creating an (unstable) positive feedback.

Instability in a torus can be driven by “bad curvature”

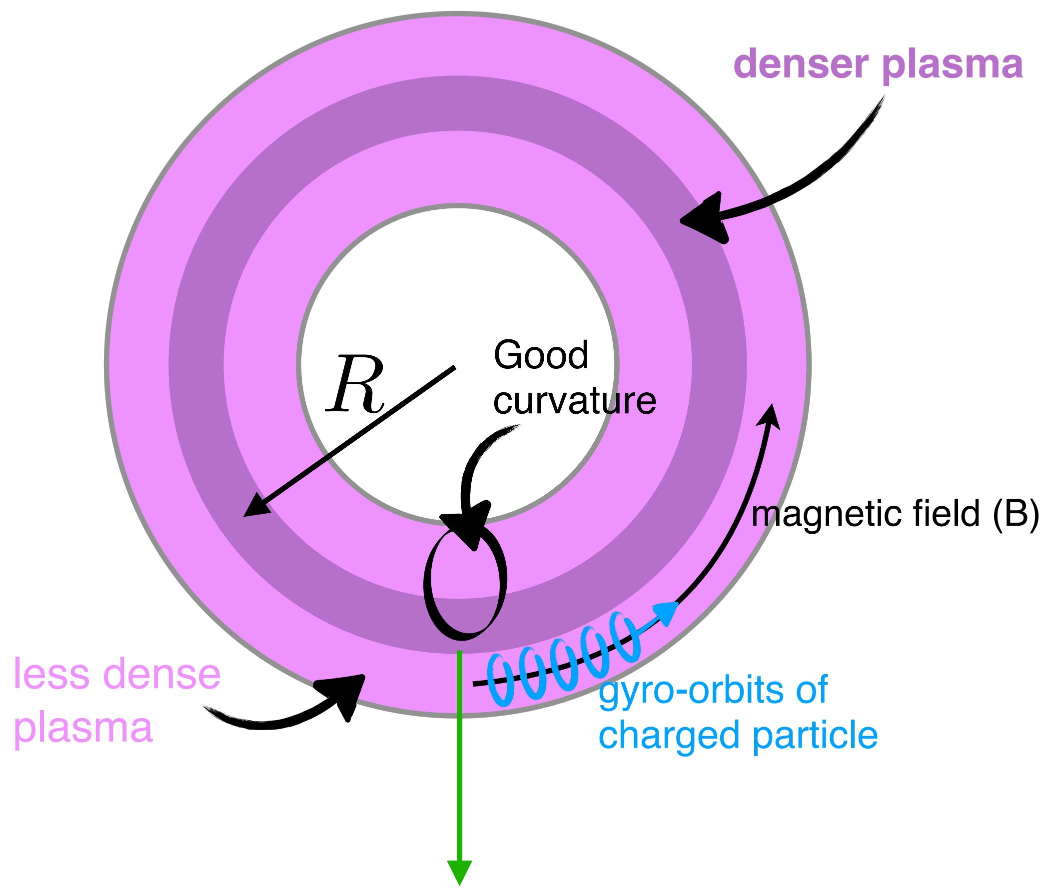


- These perturbations (eddies) have a size of order $\sim r_L = mv_\perp/(|q|B)$.
- Particles/eddies move $\sim r_L$, causing n & T to redistribute the eddy to vanish.
  $\Rightarrow$ eddy lifetime
  $$\tau_\text{eddy} \sim r_L/v_\mathbf{E}$$

- Can obtain a diffusion time estimate. Consider a diffusion equation $\frac{\partial f}{\partial t} = D\frac{d^2 f}{dx^2}$, the diffusive time scale is $\tau_d \sim L^2/D$. So we can say that energy diffusion due to the above process takes place on the timescale: $\tau_d \sim \frac{L^2}{r_L^2/\tau_\text{eddy}} \sim \frac{L^2}{r_L v_\mathbf{E}}$ $\qquad [D] = m^2/s$
- How large is $\mathbf{v}_\mathbf{E}$? $\mathbf{v}_\mathbf{E} \sim \frac{1}{BL}\phi \sim \frac{1}{|q|BL}T = \frac{v_t^2}{L\omega_c} = \frac{r_L}{L}v_t$

$\Rightarrow \tau_d \sim \frac{L^2}{r_L^2}\frac{L}{v_t}$ estimate for confinement time $\tau_E = \frac{\text{plasma energy}}{\text{rate of energy loss}}$

Energy is being lost from the fusion-producing plasma core. Is all lost? How do we stabilize RT-like & micro instabilities?

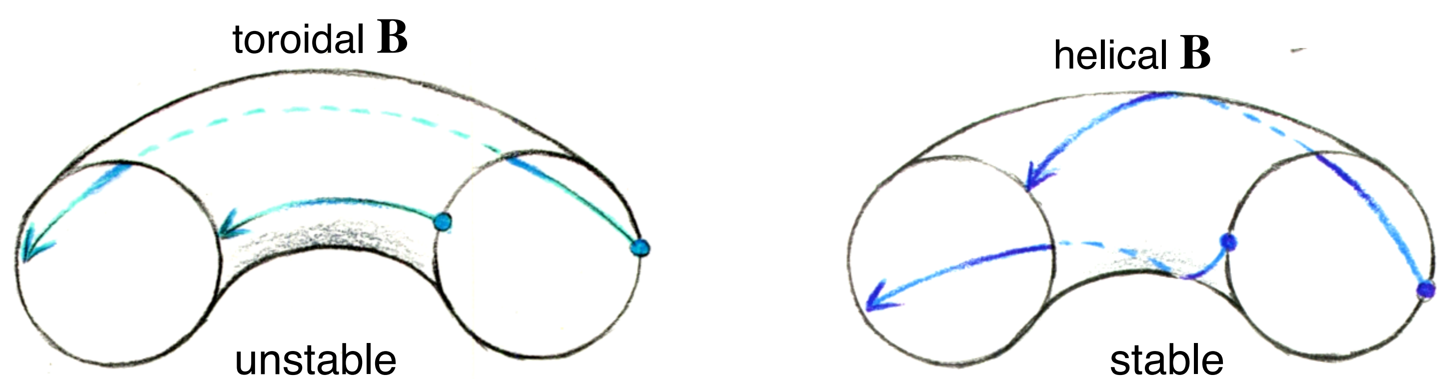


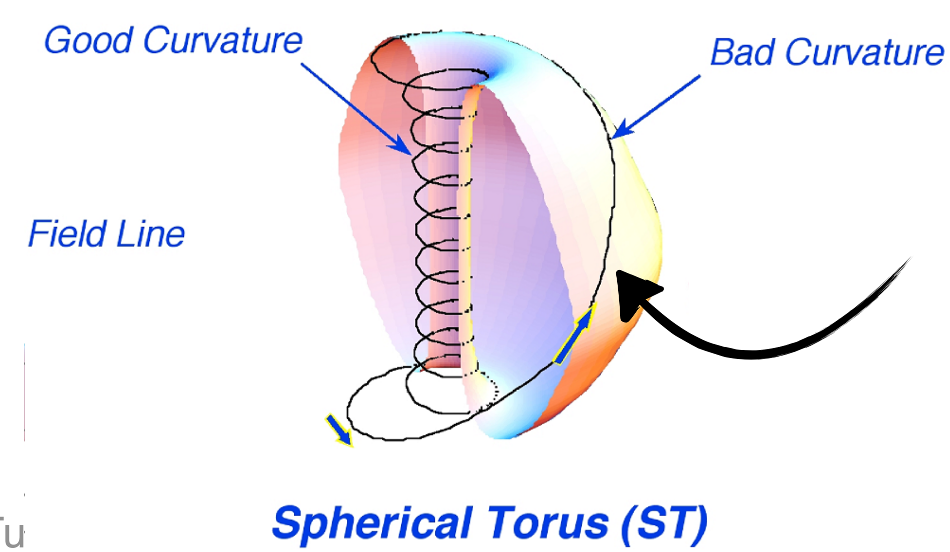
Plasma feels an effective gravitational acceleration due to the centrifugal force:

$$g_{\text{eff}} = \frac{v^2}{R}$$

Recall that this instability is driven by pressure gradient pointing opposite from the effective gravity at the outboard of the torus.

Solution:
Twist the magnetic field around so as particles move along the field line they also spend time on the "good curvature" region.





Motivation for spherical tori

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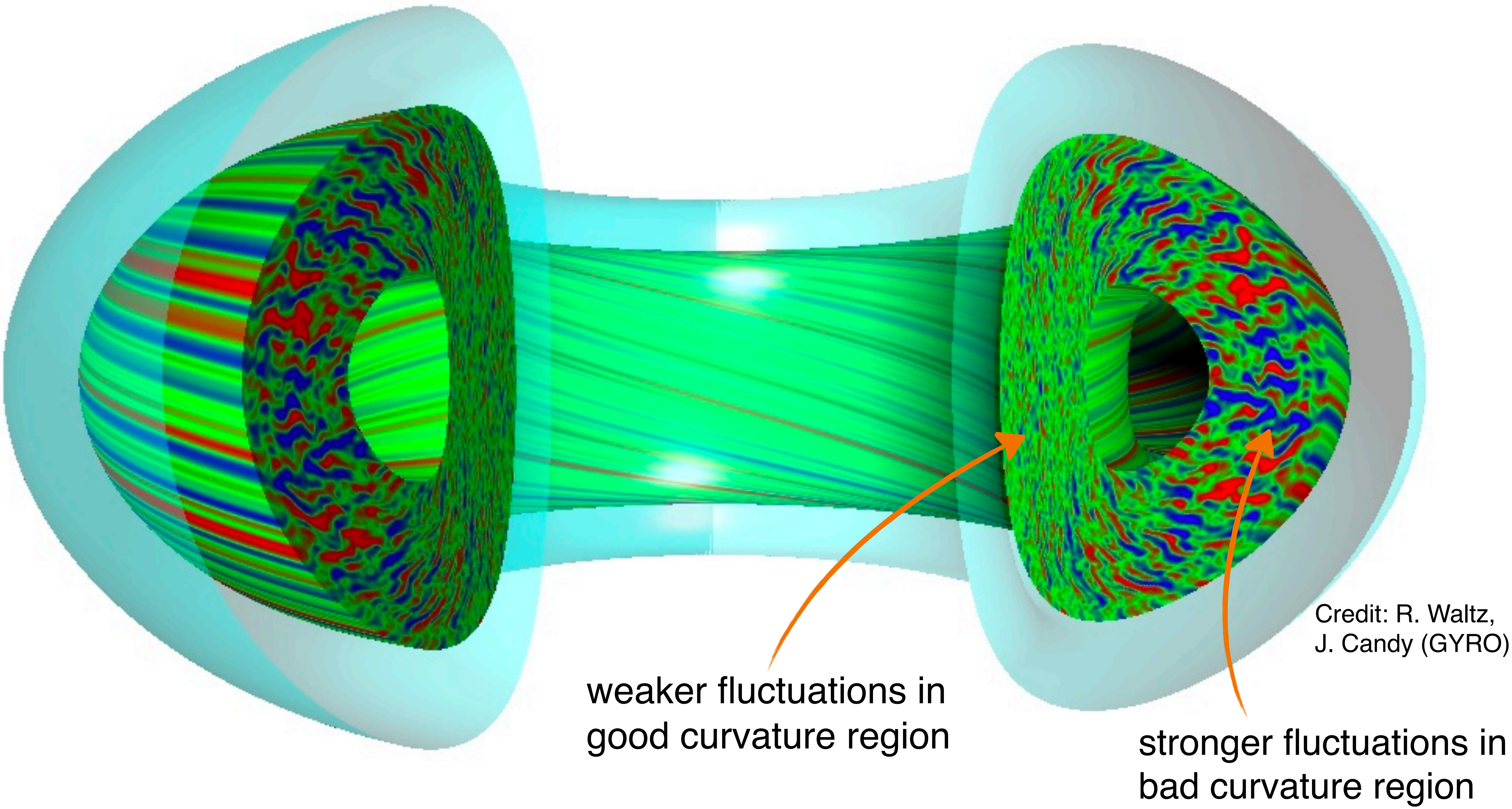
A different behavior of between good & bad curvature regions is clearly observed in computer simulations of tokamak turbulence



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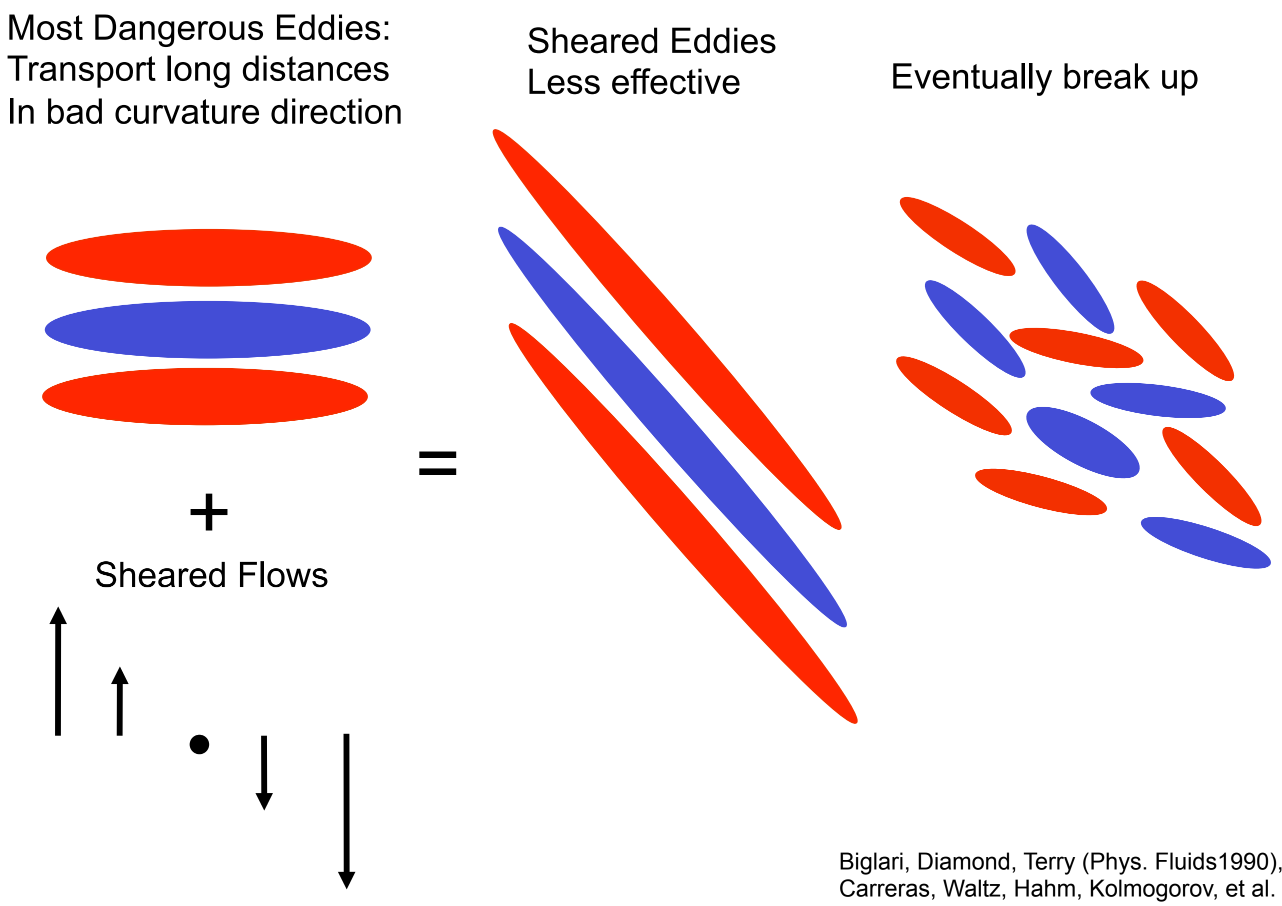
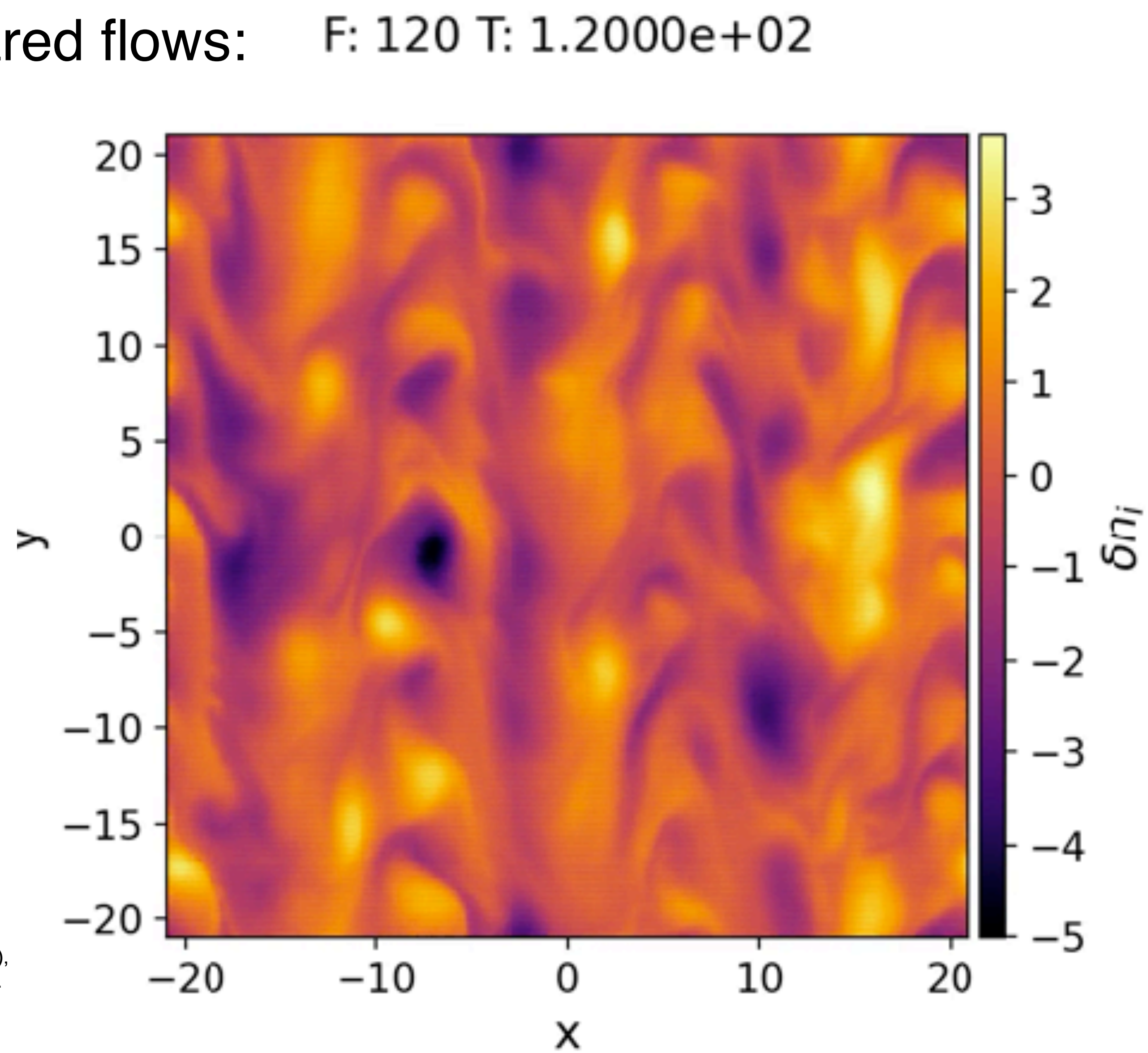
Energy is being lost from the fusion-producing plasma core. Is all lost? How do we stabilize RT-like & micro instabilities?

Turbulent transport can be mitigated by sheared flows:



Biglari, Diamond, Terry (Phys. Fluids1990), Carreras, Waltz, Hahm, Kolmogorov, et al.



Computer simulation done with Gkeyll,
a free, open-source code for modeling plasmas

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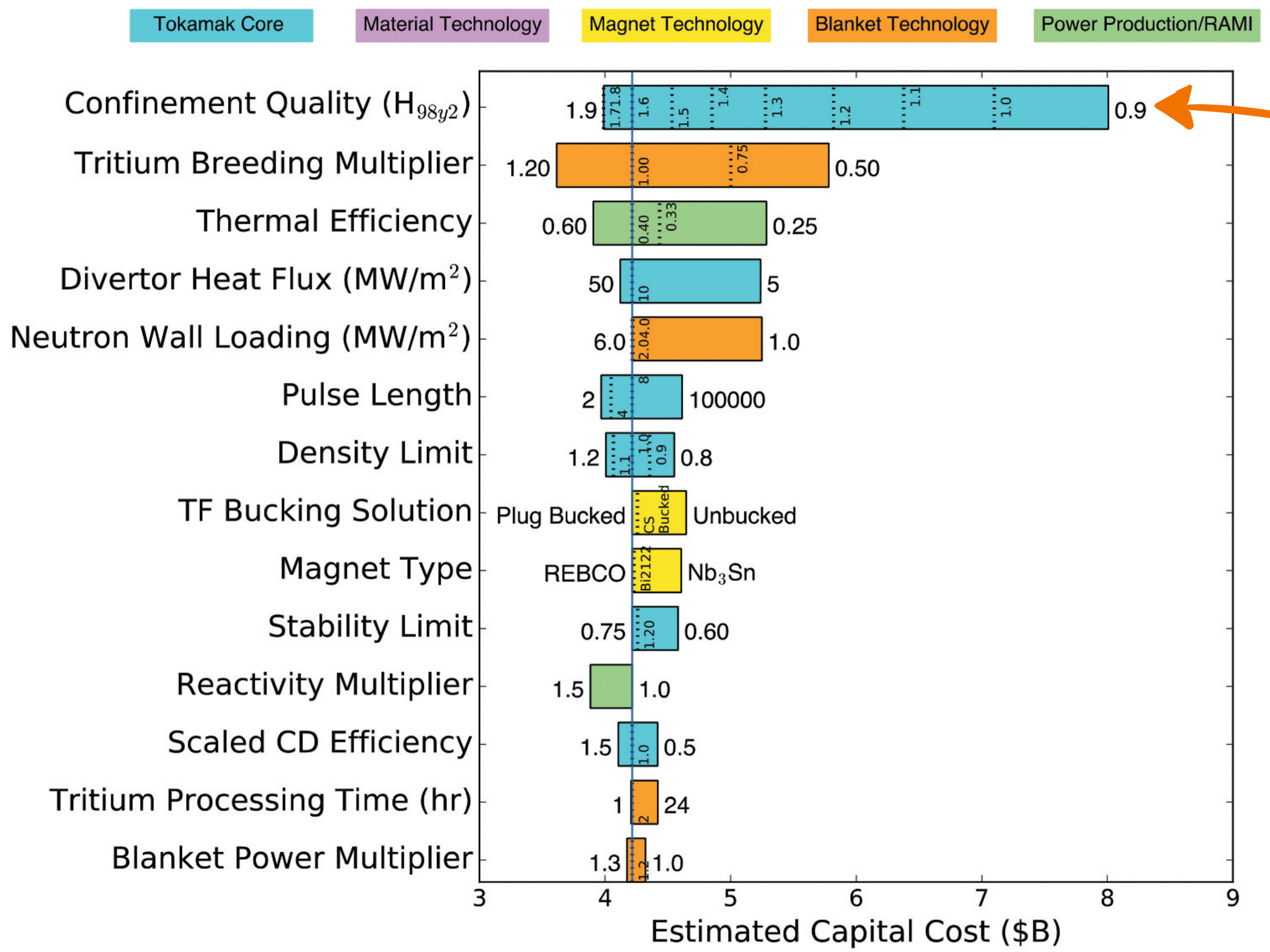
Studying, understanding and predicting turbulence is important & can have a significant impact on the cost of fusion

As an example, take a study of the sensitivity of a compact fusion power plant's capital cost:



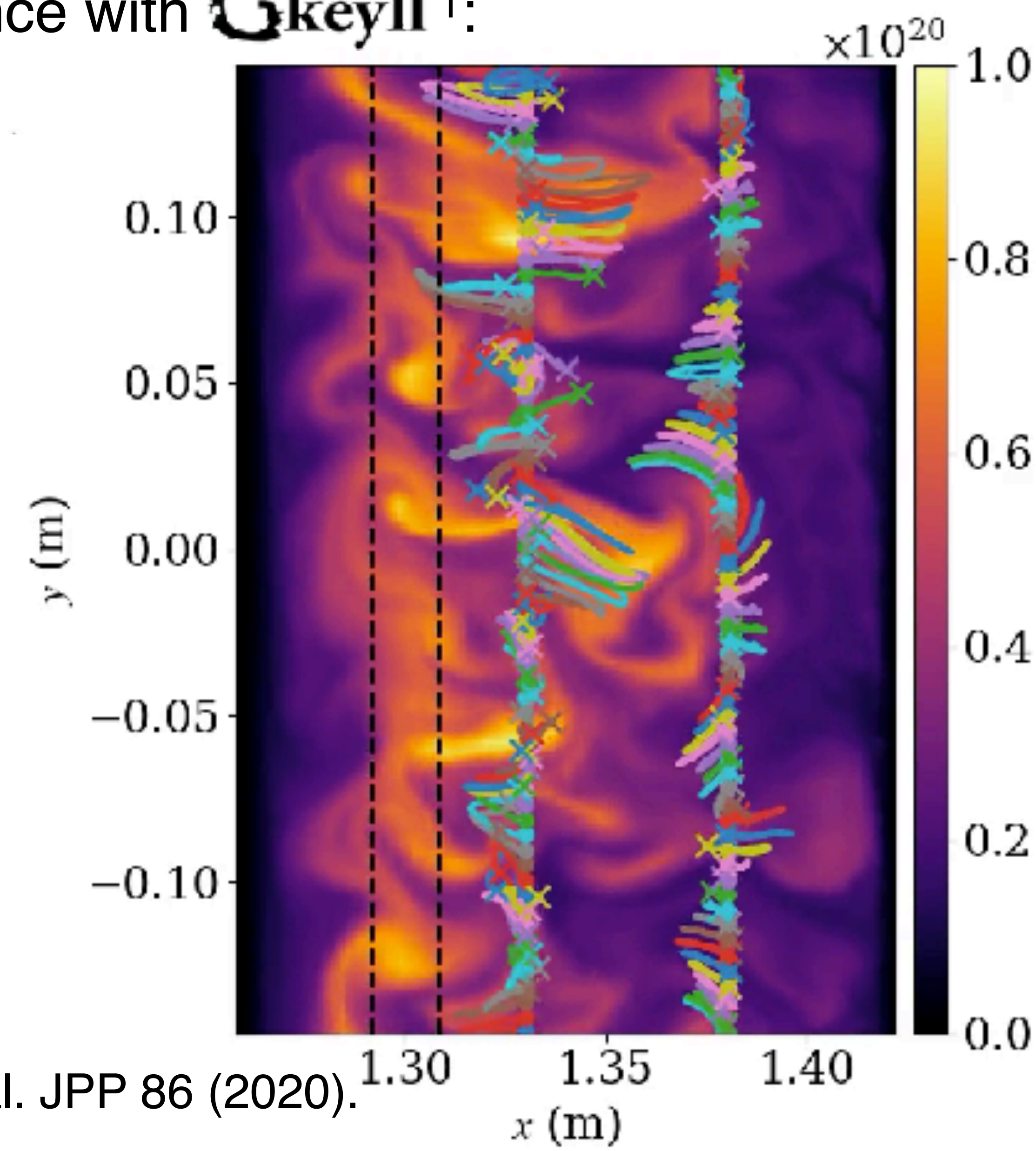
Capital cost is most sensitive to confinement quality (largely set by turbulent transport)

[1] M. R. Wade, J. A. Leuer. Fus. Sci. And Tech. 77, 119 (2021).

Turbulence is frequently studied via numerical simulation

Sophisticated computer codes that solve fluid, kinetic or gyrokinetic equations (or all of the above as is the case of the Gkeyll code I work on) have been developed around the world.

e.g. a gyrokinetic simulation of electromagnetic plasma turbulence with Gkeyll [1]:



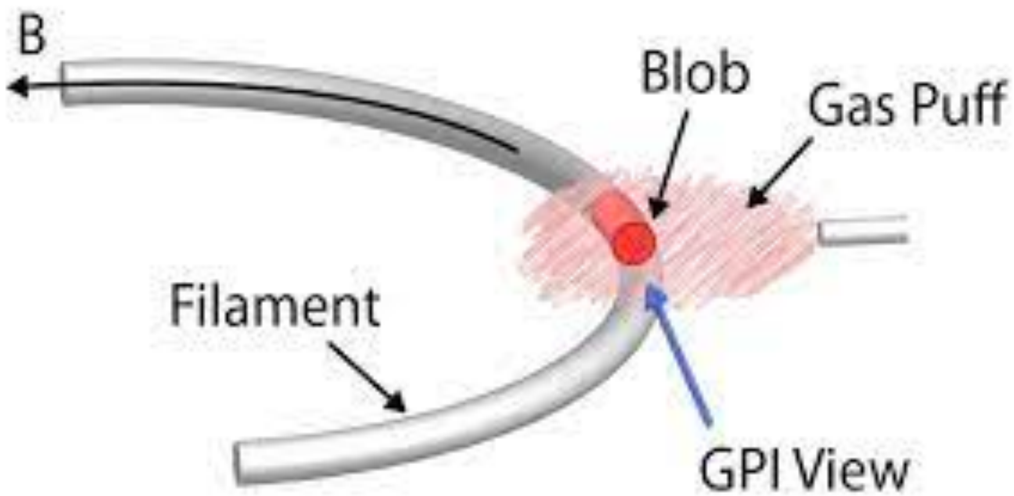
There are numerous other codes that can be used to study plasma turbulence, e.g.:

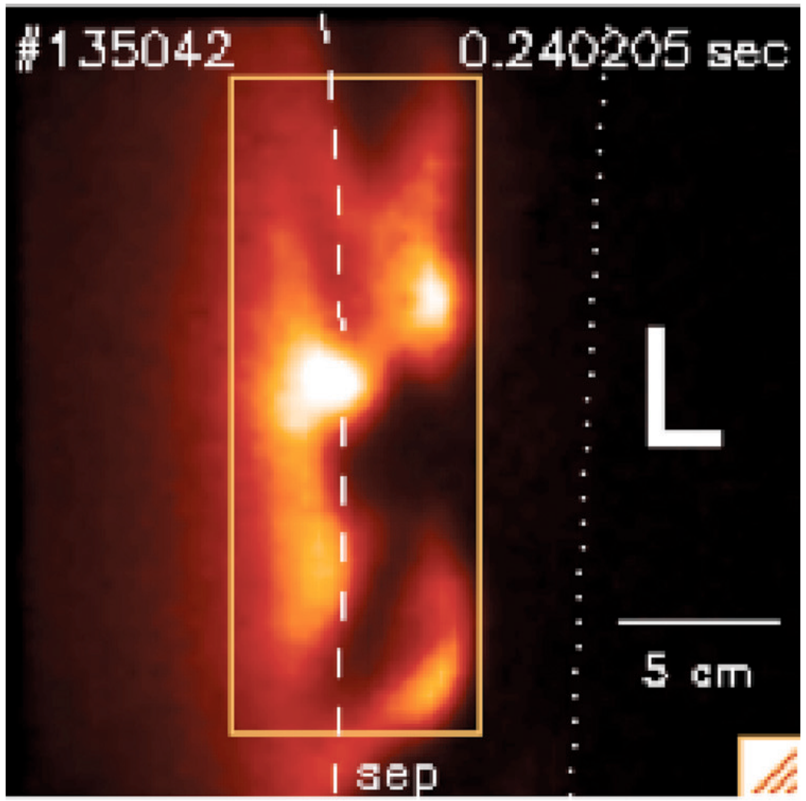
- GENE, GYRO, GS2, GX, Stella, GKV, AstroGK, XGC.
- GRILLIX, TOKAM3X, GBS.
- Athena
- Vlasiator
- Many, many others

[1]N. R. Mandell, et al. JPP 86 (2020).

Plasma turbulence is also explored experimentally. For example, via gas puff imaging

Electrons collide with puffed neutral gas, leading to ionization events and emitted light.





These collisions are more frequent where the local density is higher, allowing us to visualize plasma fluctuations

[1]S. J. Zweben, et. al. Review of Sci. Inst 88 (2017).

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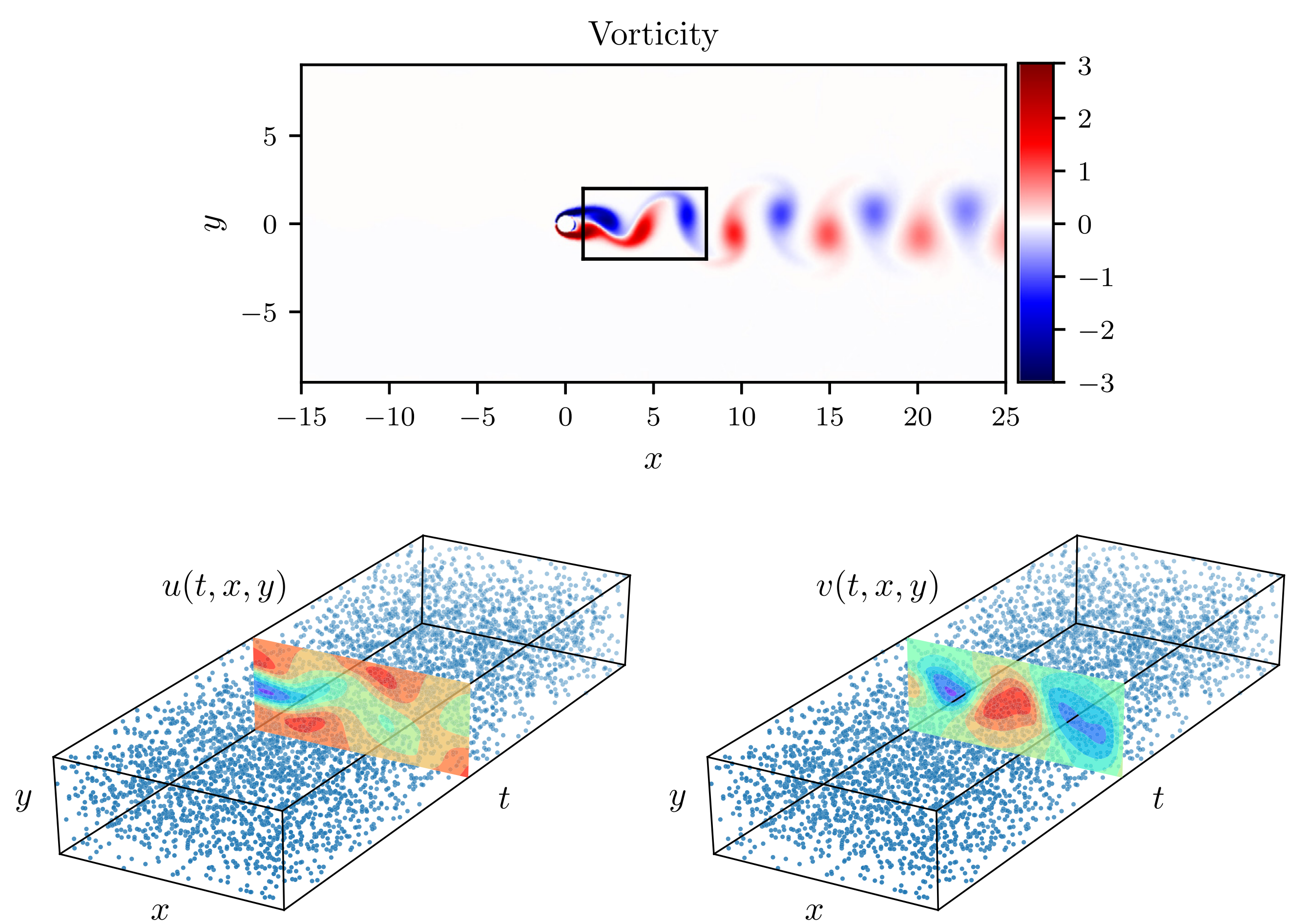
Increasingly, turbulence is also being studied w/ machine learning

One out of many examples of machine learning applied to studying turbulence is the development of physics-informed neural networks (PINN)[1]:



Authors trained a PINN using data from these locations with a loss function that incorporates the incompressible Navier-Stokes equation. Then used the PINN to advance the solution (velocity field), and obtained reasonable agreement with the Navier-Stokes-only simulations.

[1]M. Raissi et al. JCP 378, 686 (2019).

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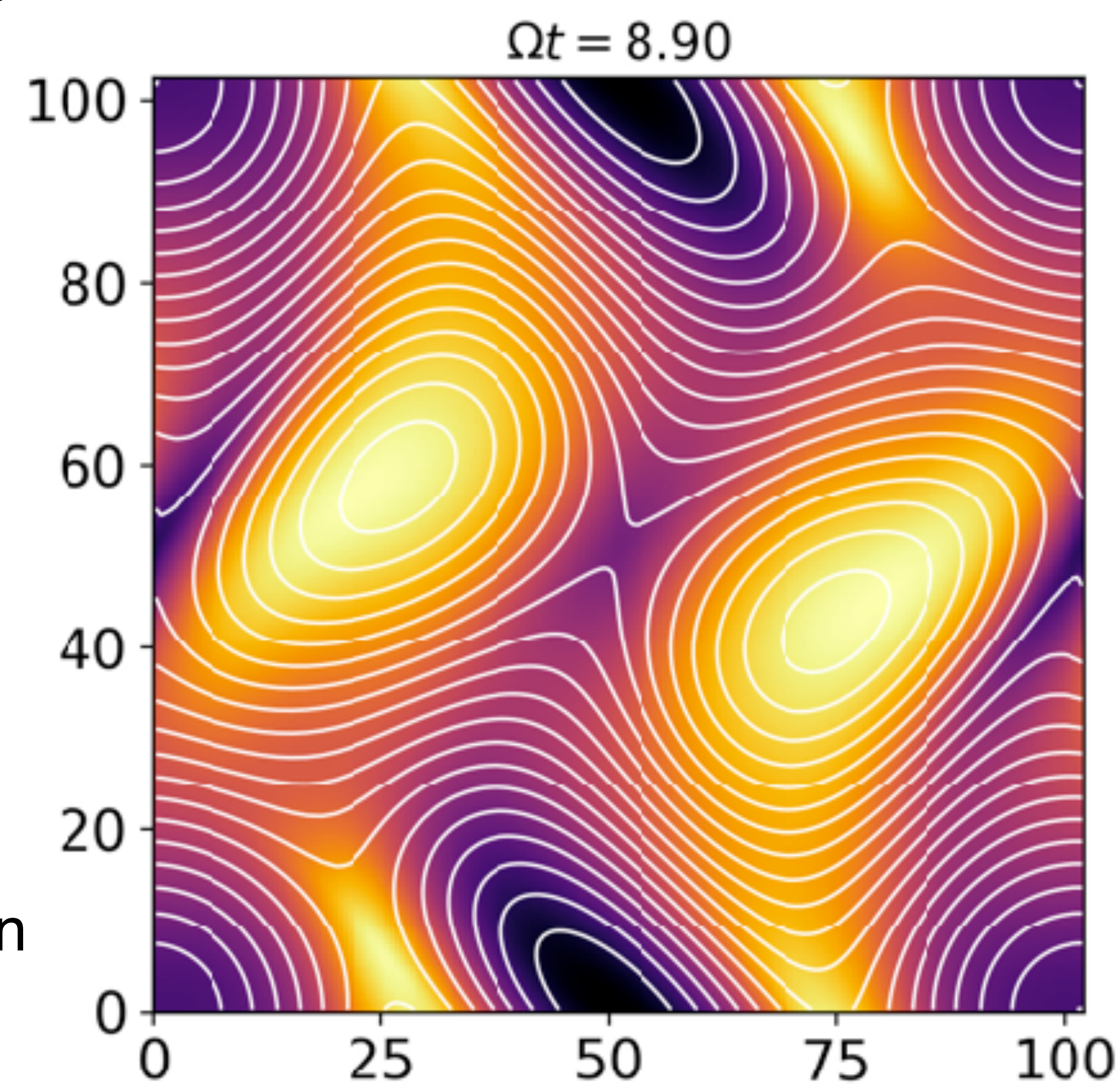
Summary

- Fluid turbulence is ubiquitous, and often driven by a system in unstable equilibrium.
- Fluid turbulence exhibits a range of scales, with an energy cascade between them that is characterized by the Kolmogorov spectrum:

$$\mathcal{E}(k) = C\,\bar{\epsilon}^{2/3}\,k^{-5/3}$$

- Some plasma instabilities have close analogs in plasmas, as is the case of the Rayleigh-Taylor instability.
- Instabilities lead to fully developed turbulence & a power law energy spectrum in plasmas too, but the turbulent transport can be mitigated by clever design of magnetic fields and sheared flows.
- Sophisticated computer codes exist for studying turbulent plasmas.
- Turbulence can strongly impact the cost of a fusion power plant.



Computer simulation done with Gkeyll,
a free, open-source code for modeling plasmas

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Additional references:

Neutral fluid turbulence:

- U. Frisch, *Turbulence: The legacy of A.N. Kolmogorov* (1995).
- S. Pope, *Turbulent flows* (2000).

Plasma turbulence:

- P. H. Diamond, *Modern plasma physics: Volume 1, Physical Kinetics of Turbulence Plasmas* (2010).
- R. J. Goldston & P. H. Rutherford, *Introduction to Plasma Physics* (1995).
- J. Krommes, *The gyrokinetic description of microturbulence in plasmas.* Annual Review of Fluid Mechanics 44, 175 (2012).

Also, if you are interested in data-mining turbulence simulations, check out the John Hopkins Turbulence Database.

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