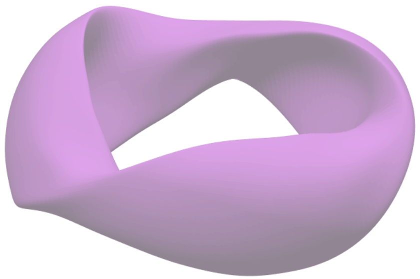
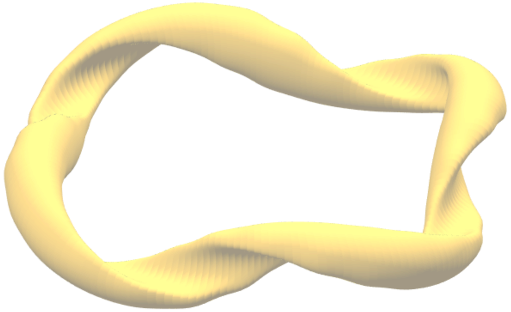
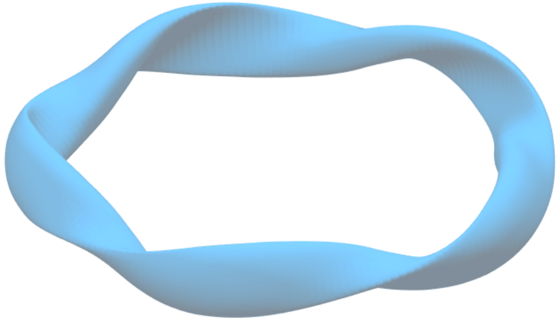
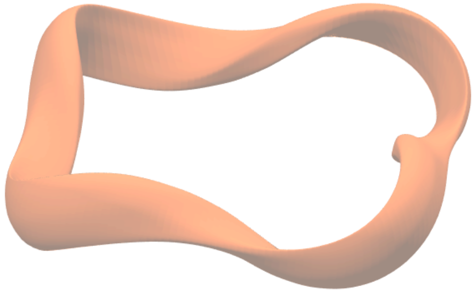
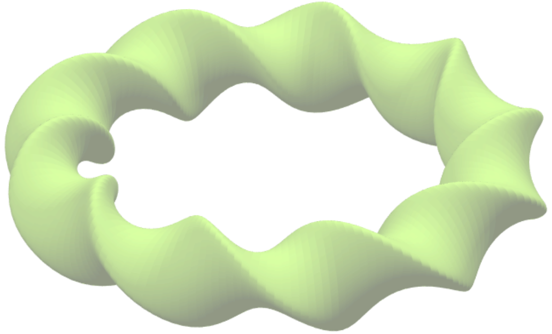
A review of the stellarator concept











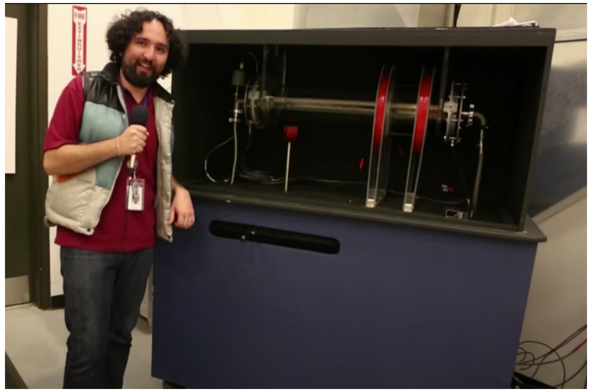
Elizabeth Paul
Princeton University/PPPL

June 21, 2021

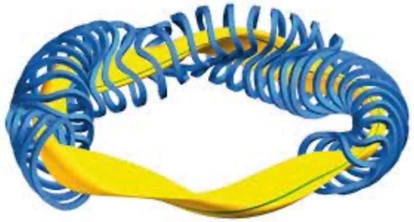
PRINCETON UNIVERSITY

PPPL

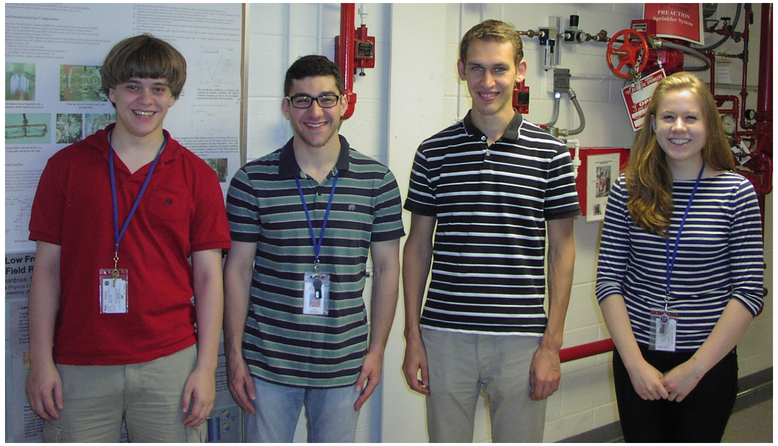
Princeton Science Action (2013)

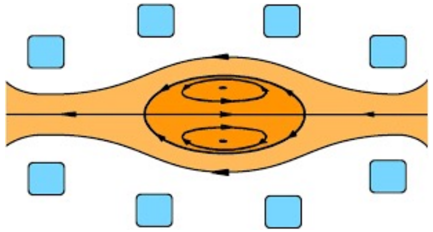


PU/PPPL Postdoc (2020-present)



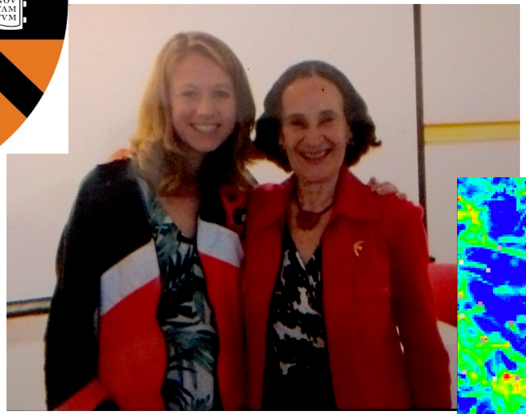
PPPL internship (2013)

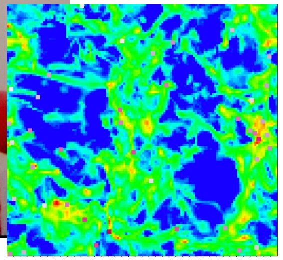




UMD Ph.D. Physics (2015-2020)

Princeton A.B. Astrophysical Sciences (2011-2015)





Outline

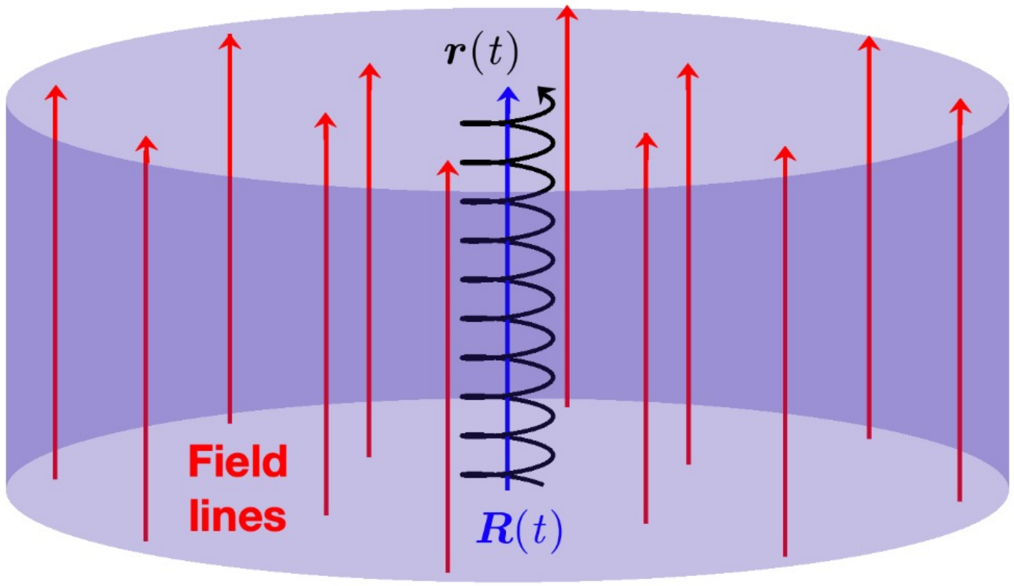
- **Magnetic confinement without symmetry**
- Ingredients of stellarator confinement
- How do we “cook” a stellarator?
- Stellarators in context

Magnetic confinement without symmetry

Single particle motion

Review: [Hussein Day 2]

Straight field provides perpendicular confinement

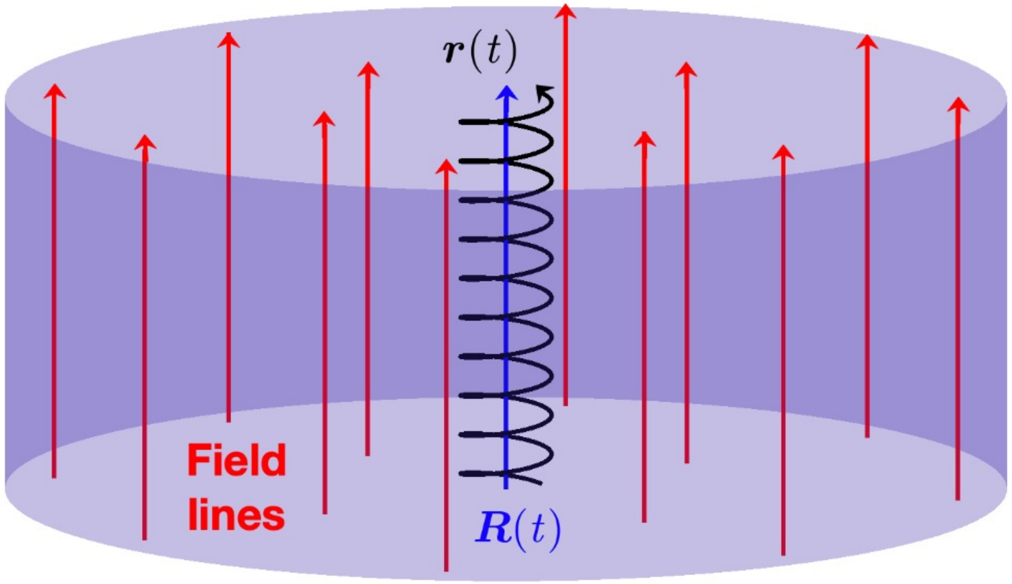


Magnetic confinement without symmetry

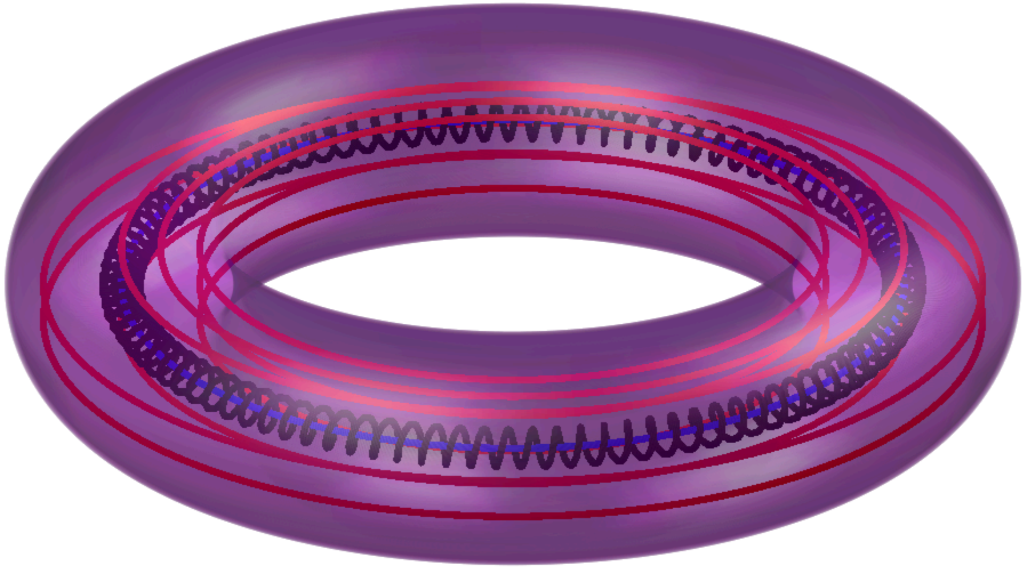
Single particle motion

Review: [Hussein Day 2]

Straight field provides perpendicular confinement

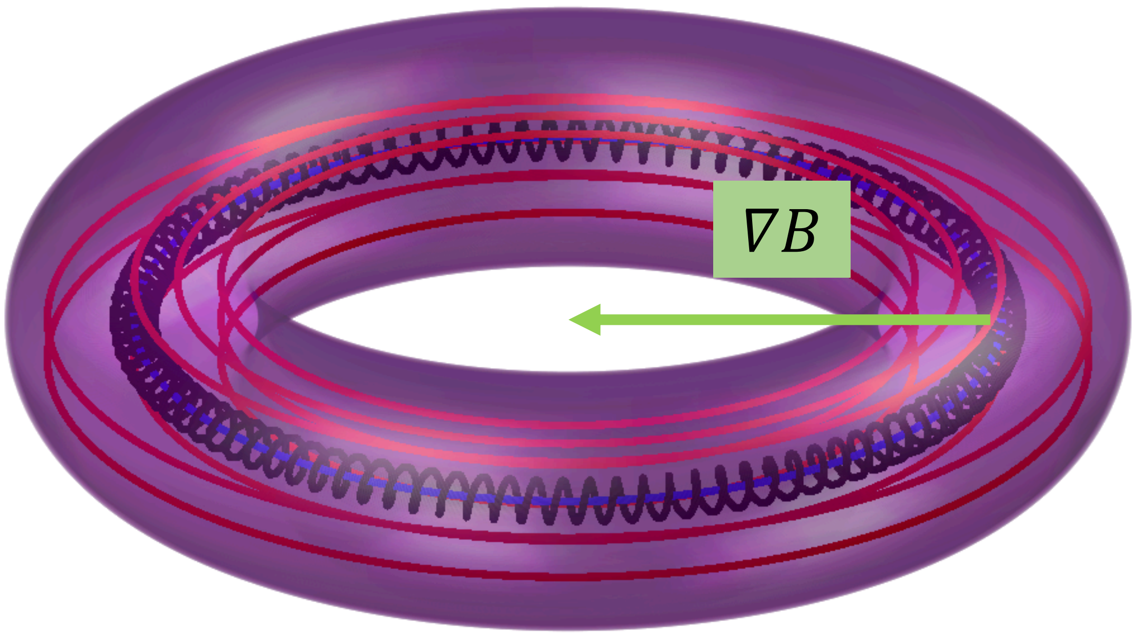


Avoid end losses!



Magnetic confinement without symmetry

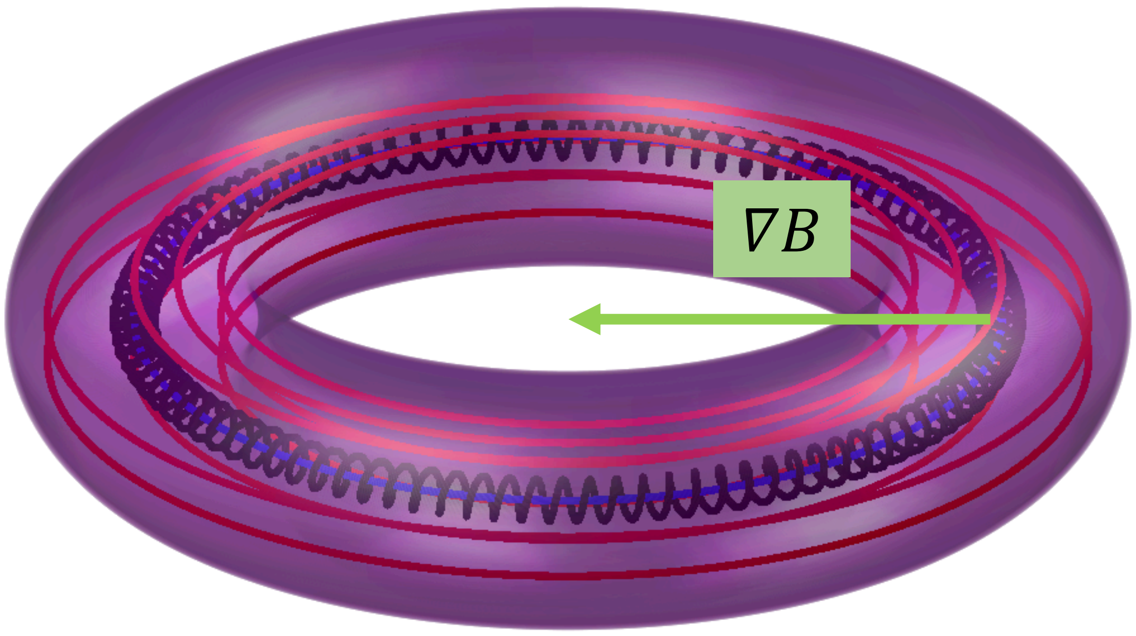
The problem: guiding center drifts



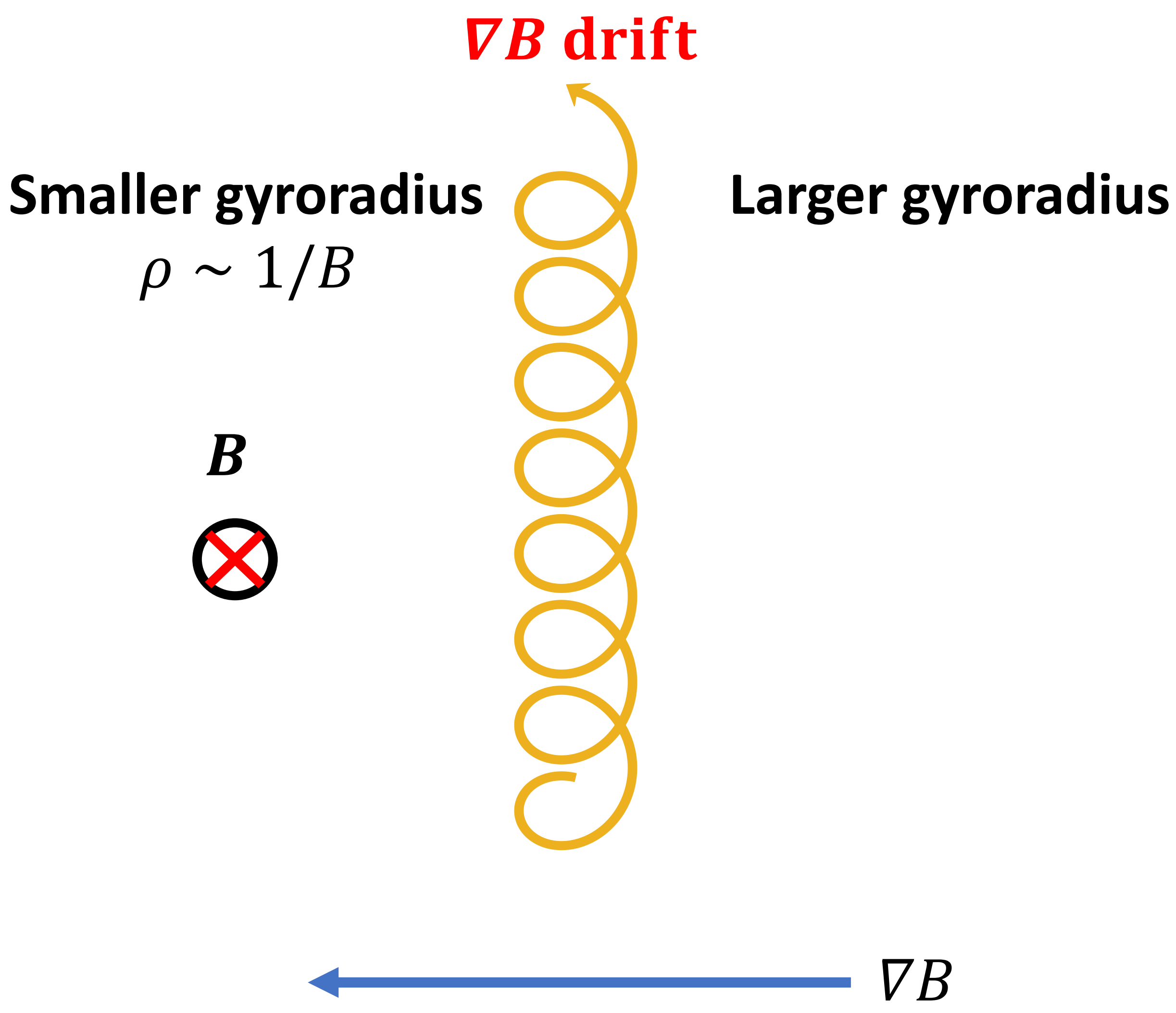
$$B \sim 1/R$$

Magnetic confinement without symmetry

The problem: guiding center drifts



$B \sim 1/R$



Magnetic confinement without symmetry

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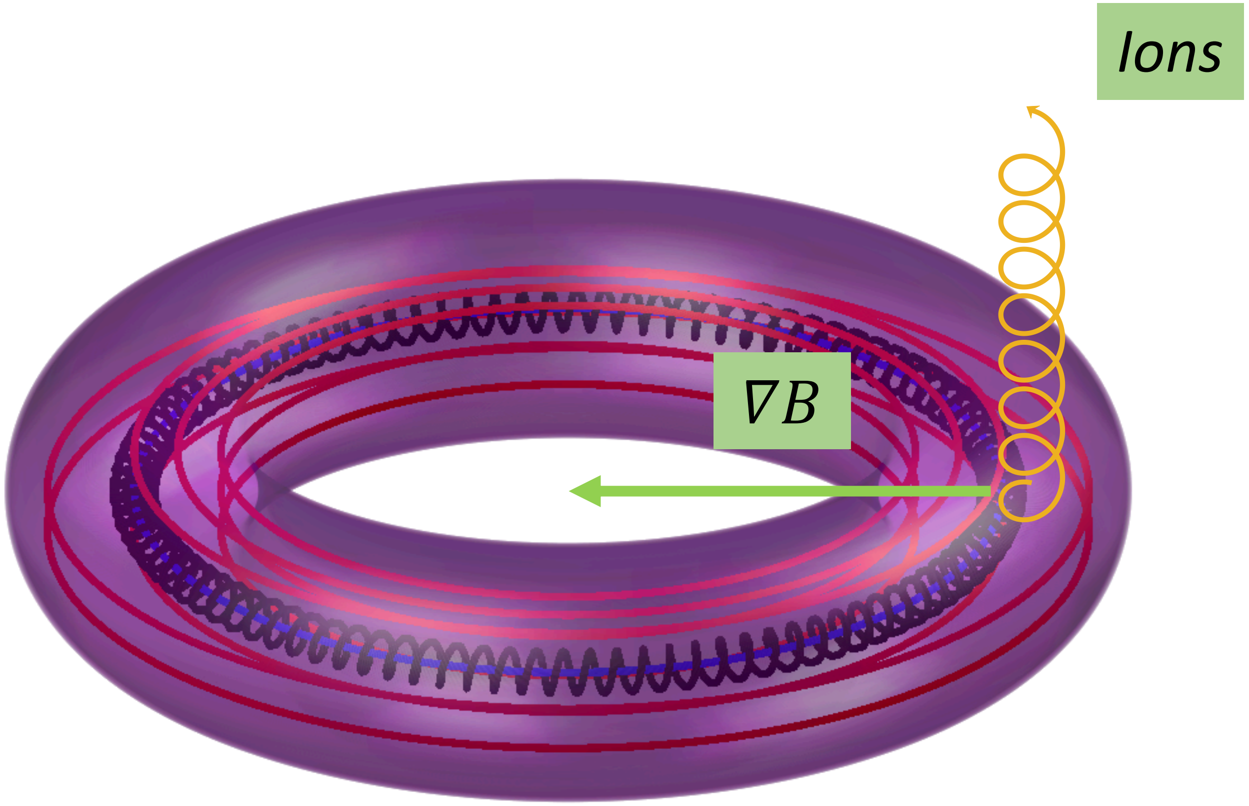

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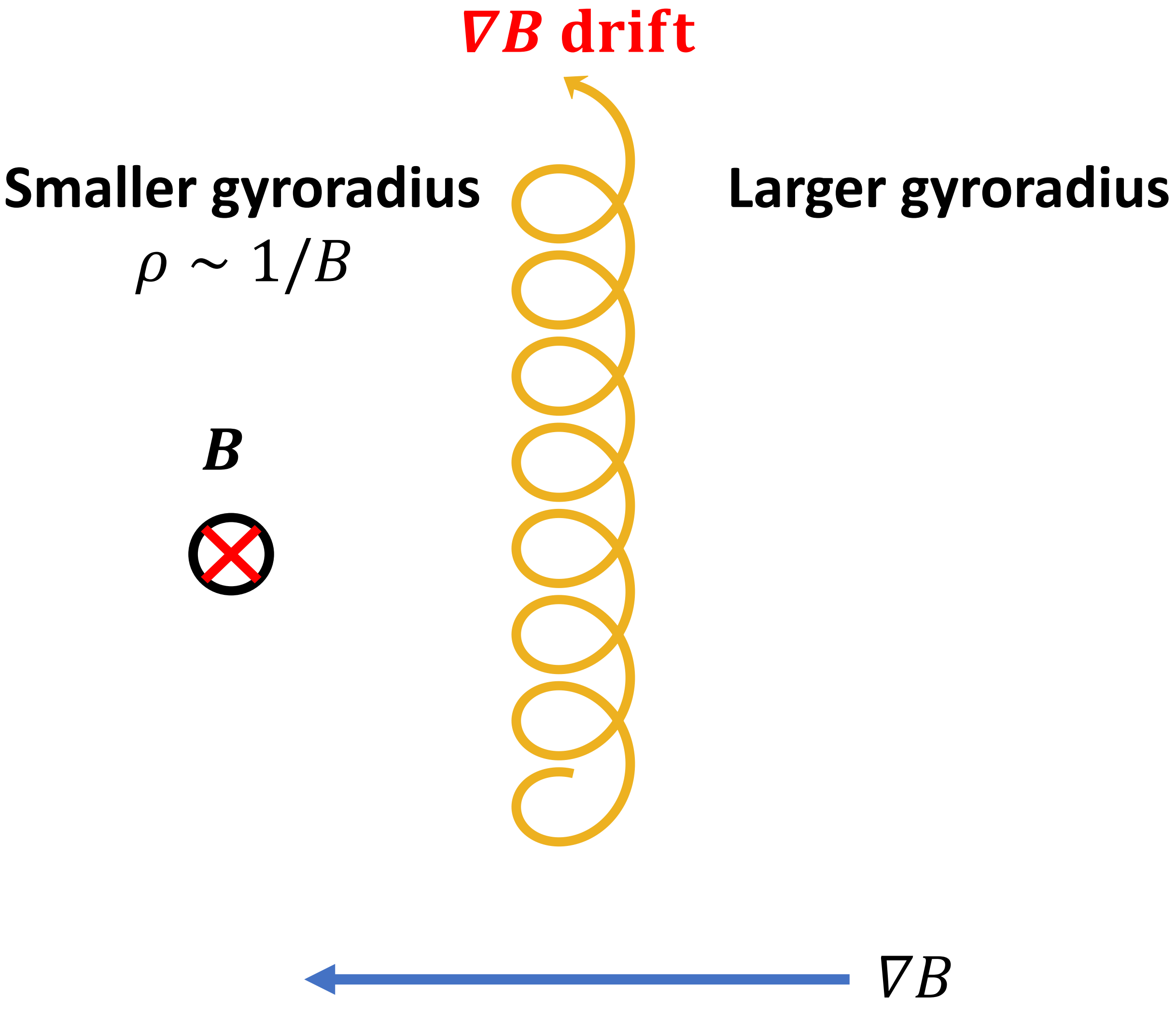
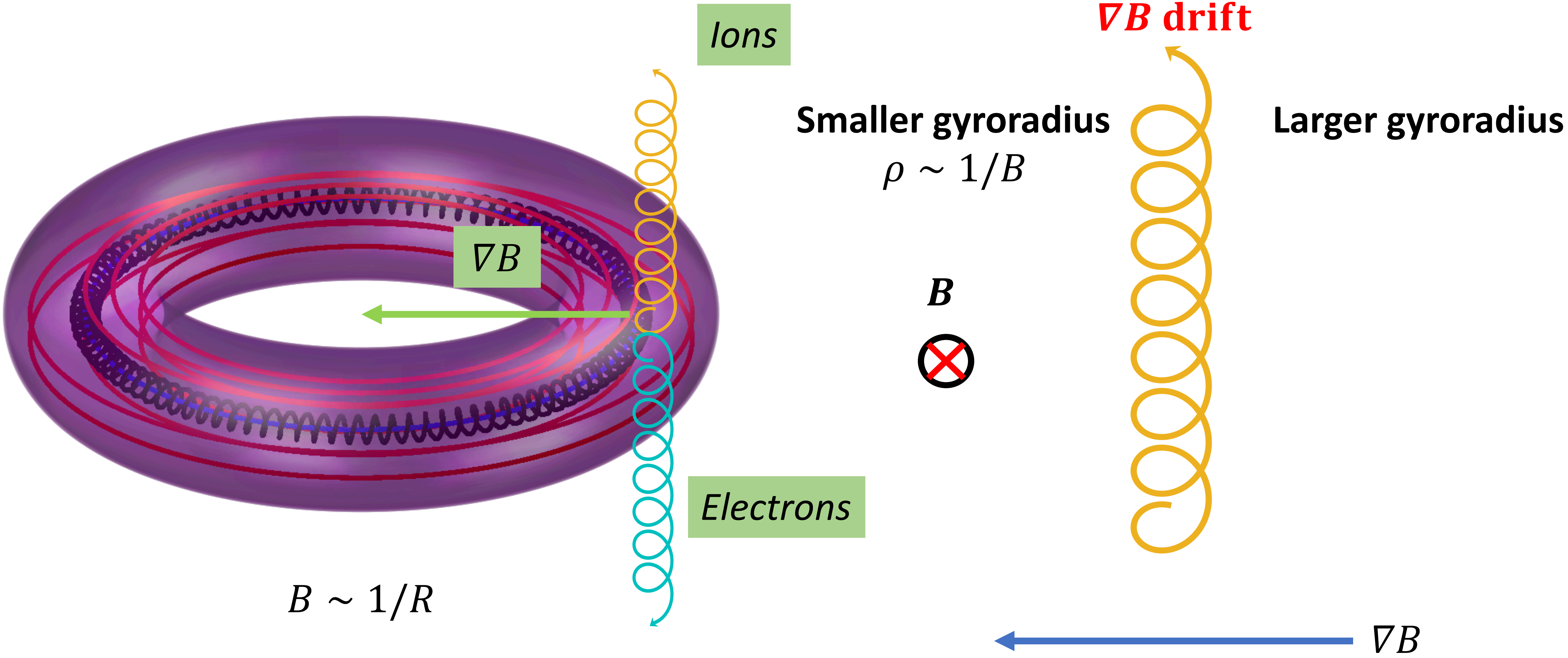


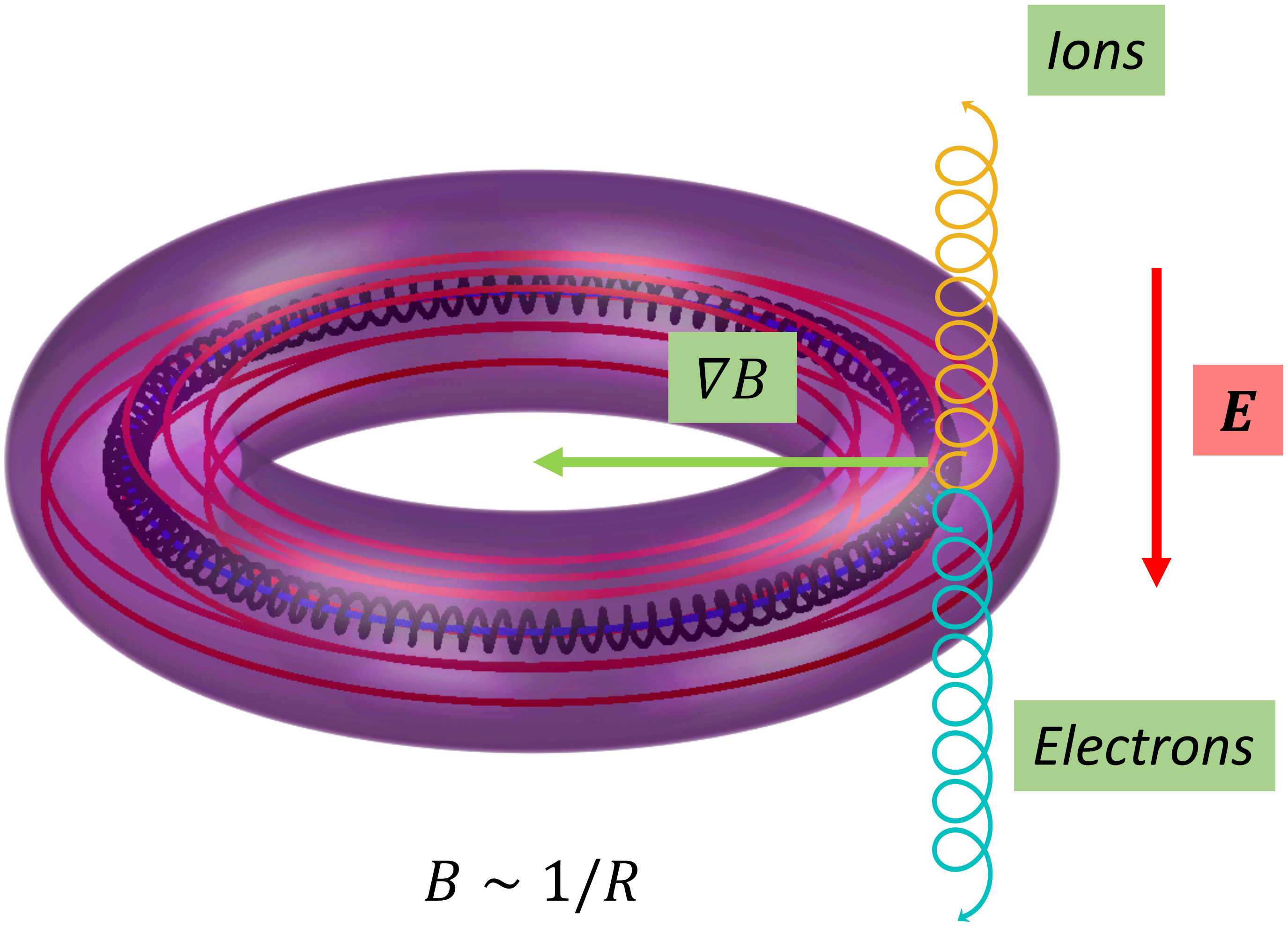
Magnetic confinement without symmetry

The problem: guiding center drifts

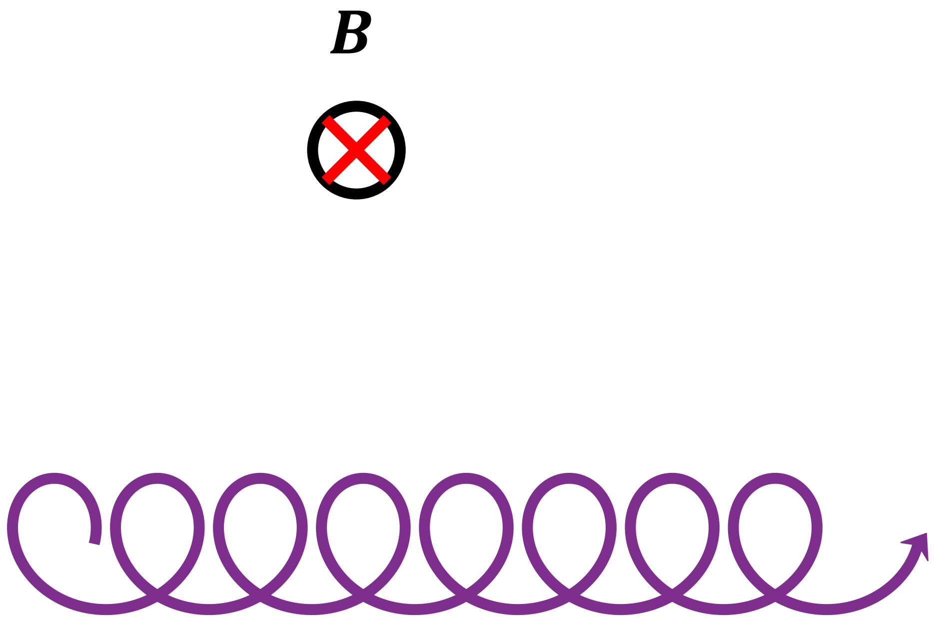


Magnetic confinement without symmetry

The problem: guiding center drifts



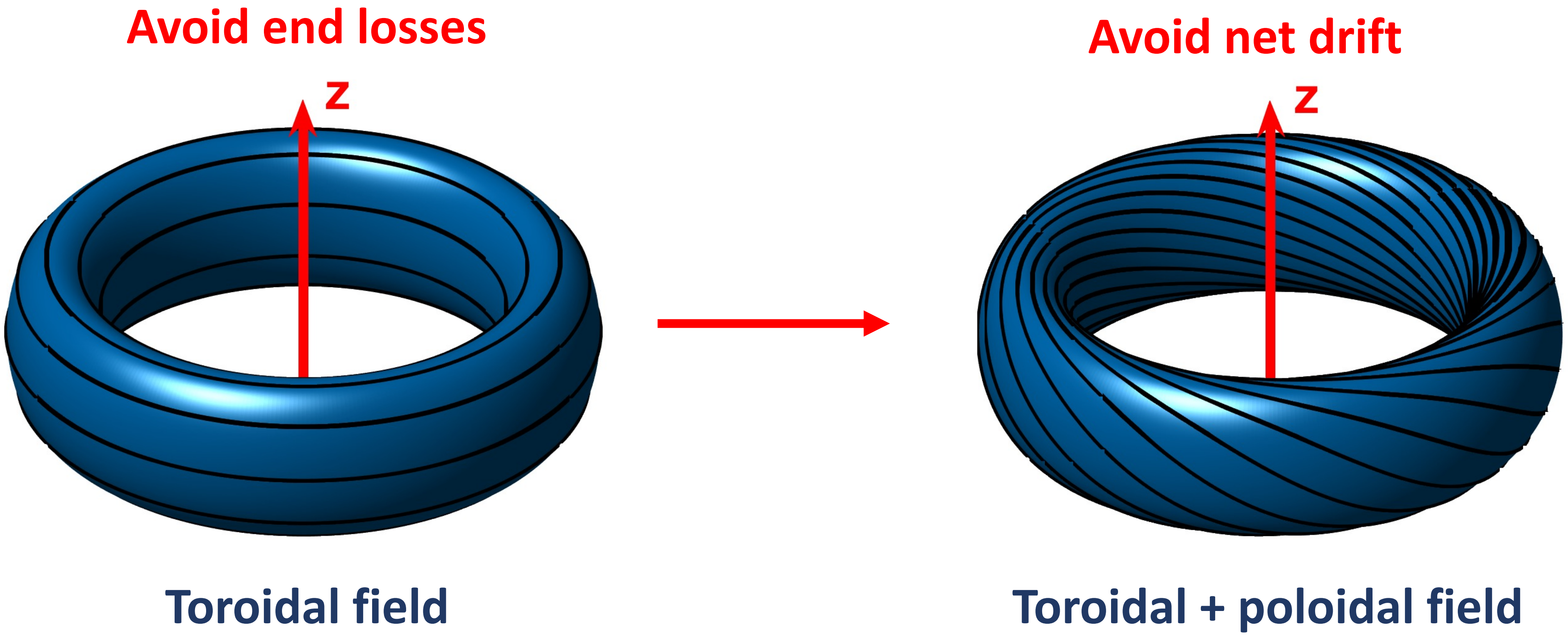
$B \sim 1/R$



$\boldsymbol{E} \times \boldsymbol{B}$ **drift**

Magnetic confinement without symmetry

The solution: rotational transform

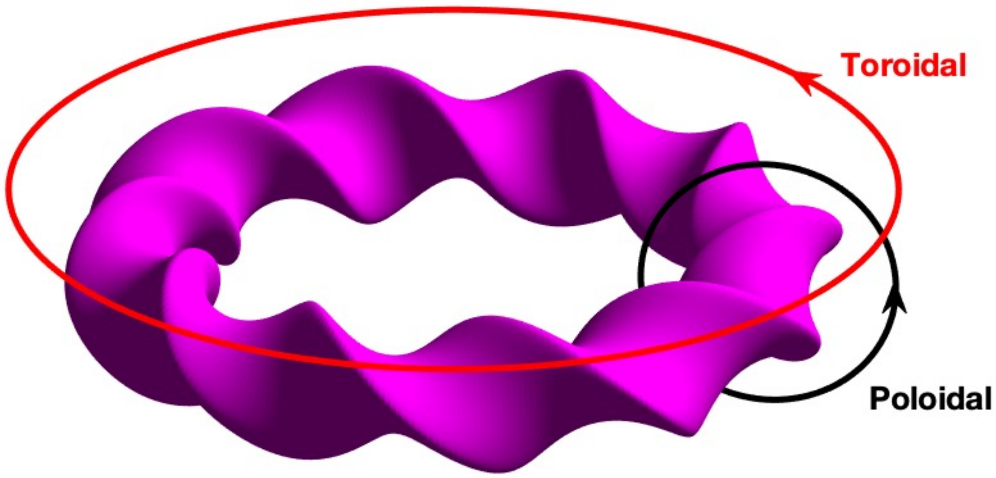


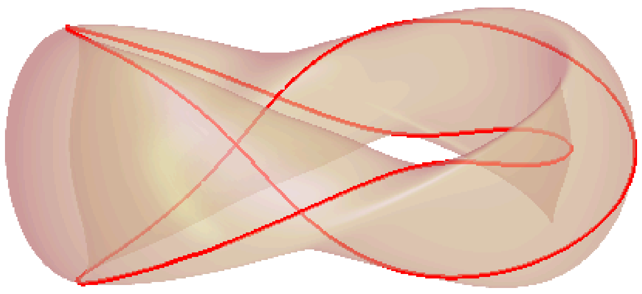
Magnetic confinement without symmetry

The solution: rotational transform

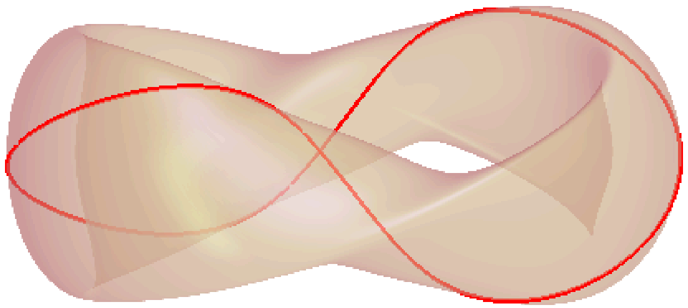
$$\iota = \frac{\#\ \text{poloidal turns}}{\#\ \text{toroidal turns}} = \frac{1}{q}$$

"rotational transform" **"safety factor"**





$\iota = 3/2$



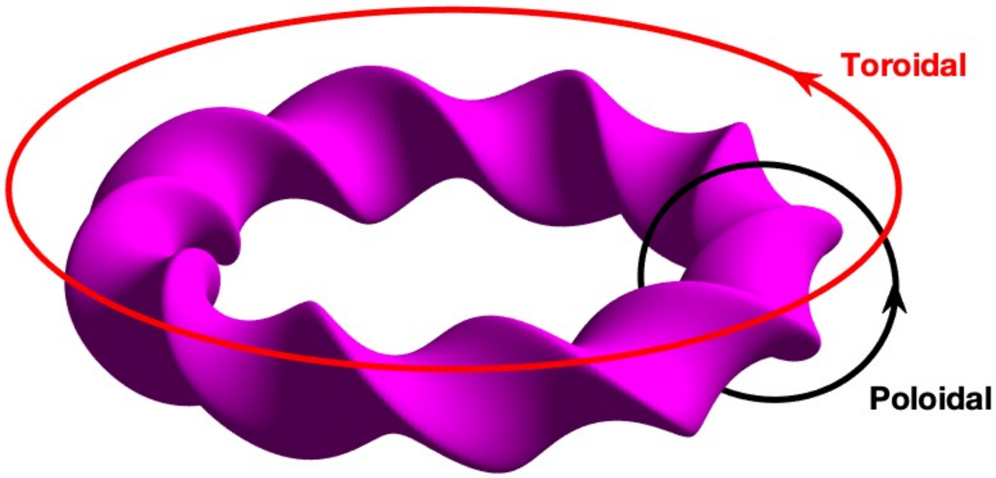
$\iota = 1$

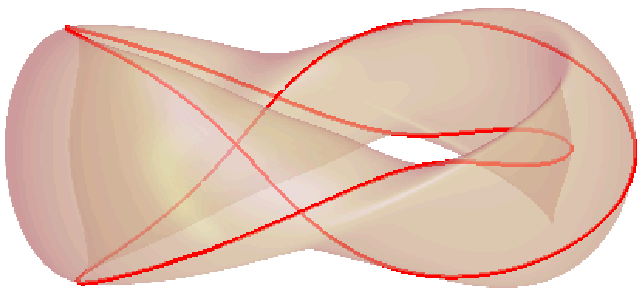
Magnetic confinement without symmetry

The solution: rotational transform

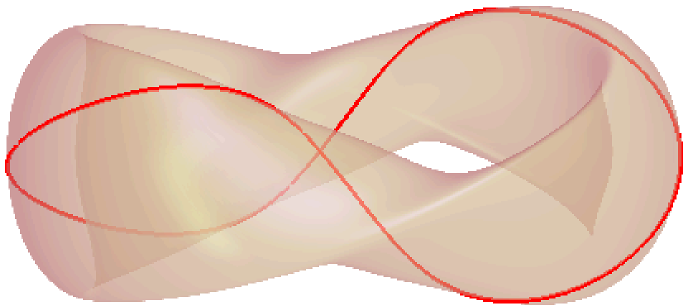
$$\iota = \frac{\#\text{ poloidal turns}}{\#\text{ toroidal turns}} = \frac{1}{q}$$

"rotational transform" **"safety factor"**





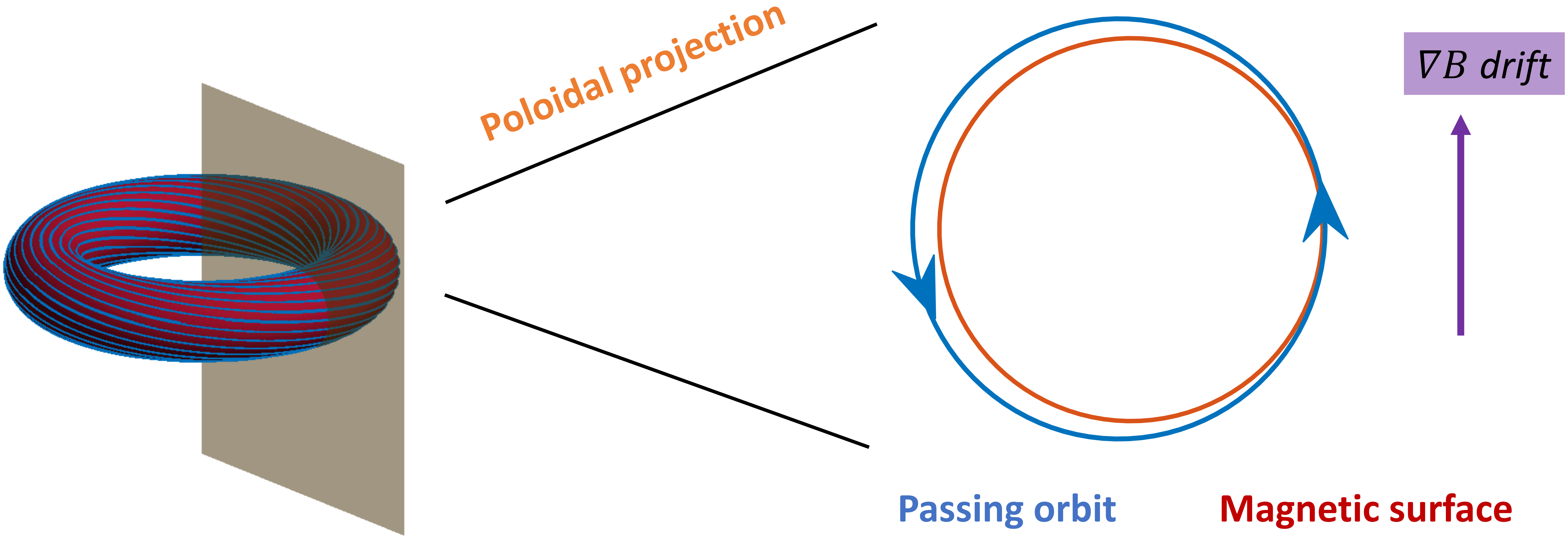
$\iota = 3/2$



$\iota = 1$

Magnetic confinement without symmetry

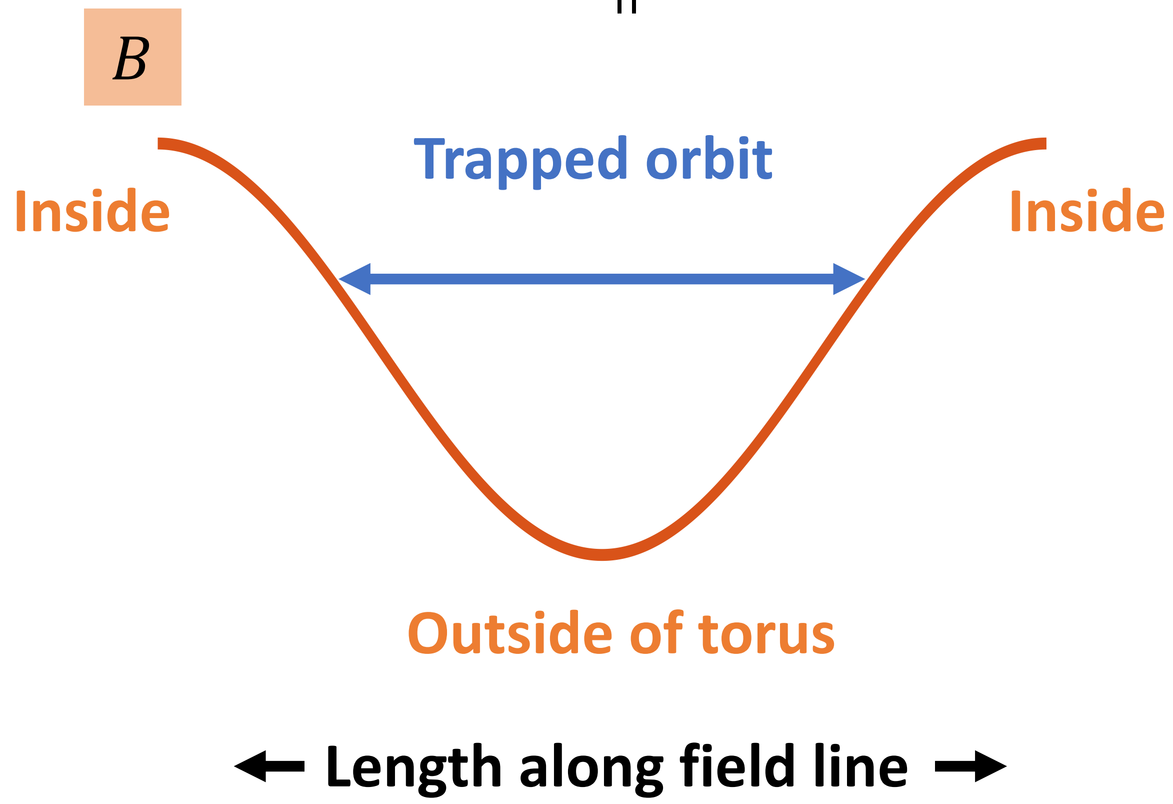
Trapped particles and drifts

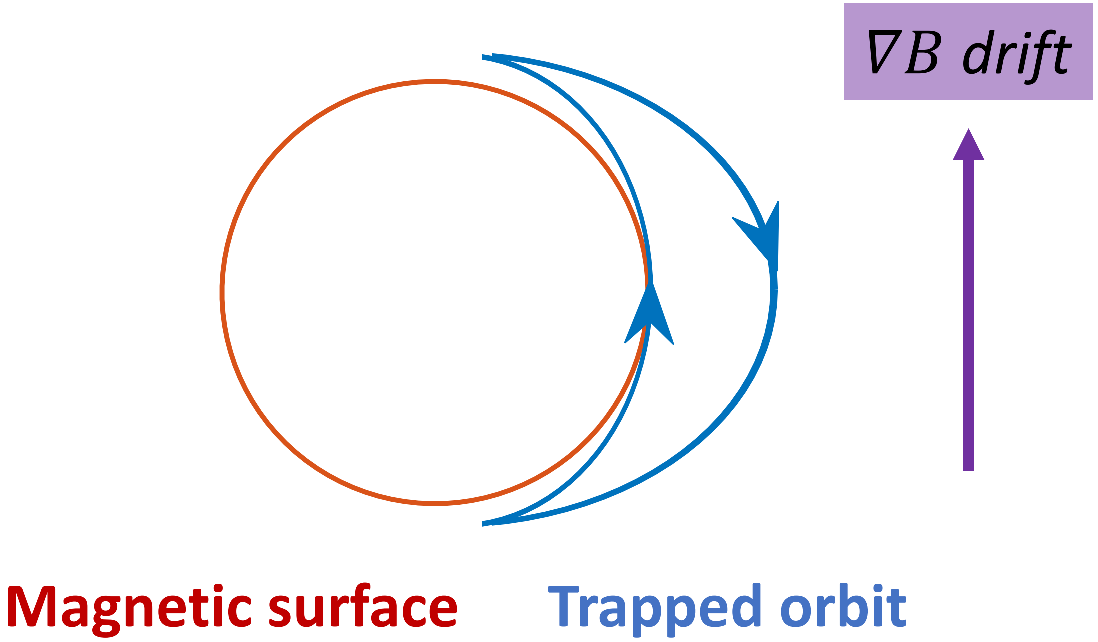


Magnetic confinement without symmetry

Trapped particles and drifts

$$E = mv_{||}^2 + \mu B$$





Magnetic confinement without symmetry

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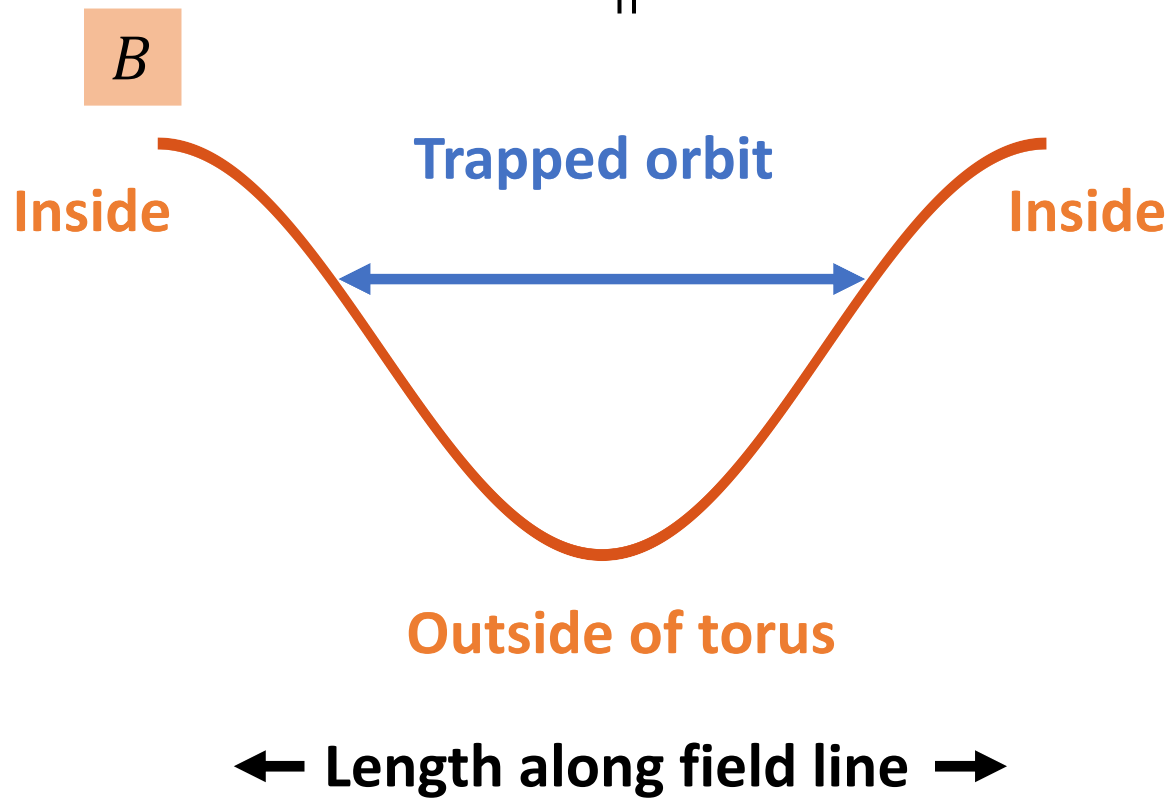
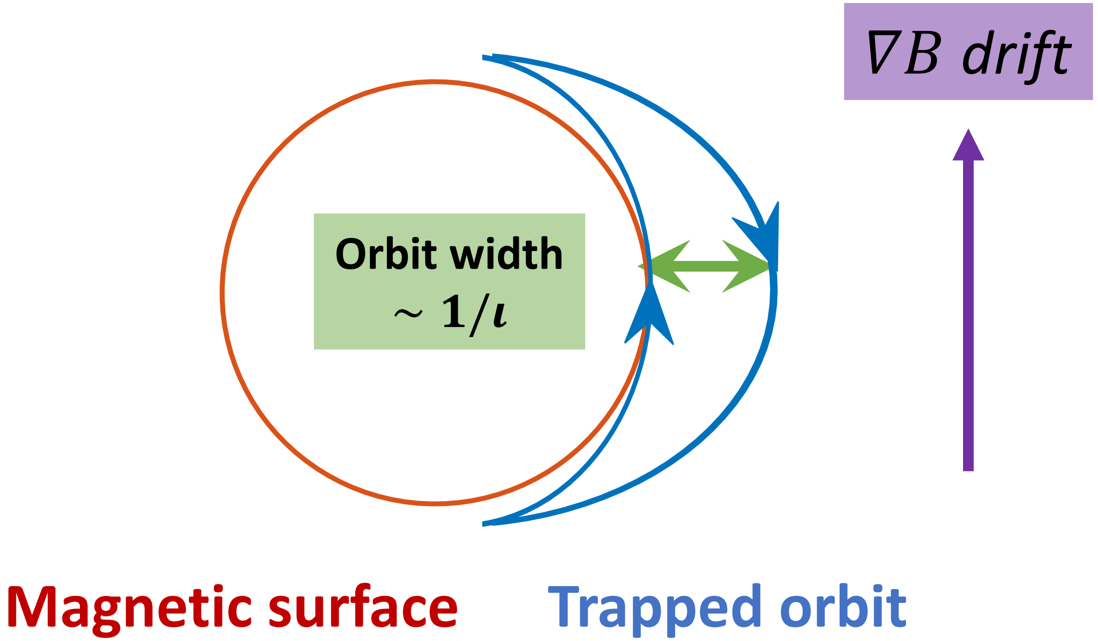
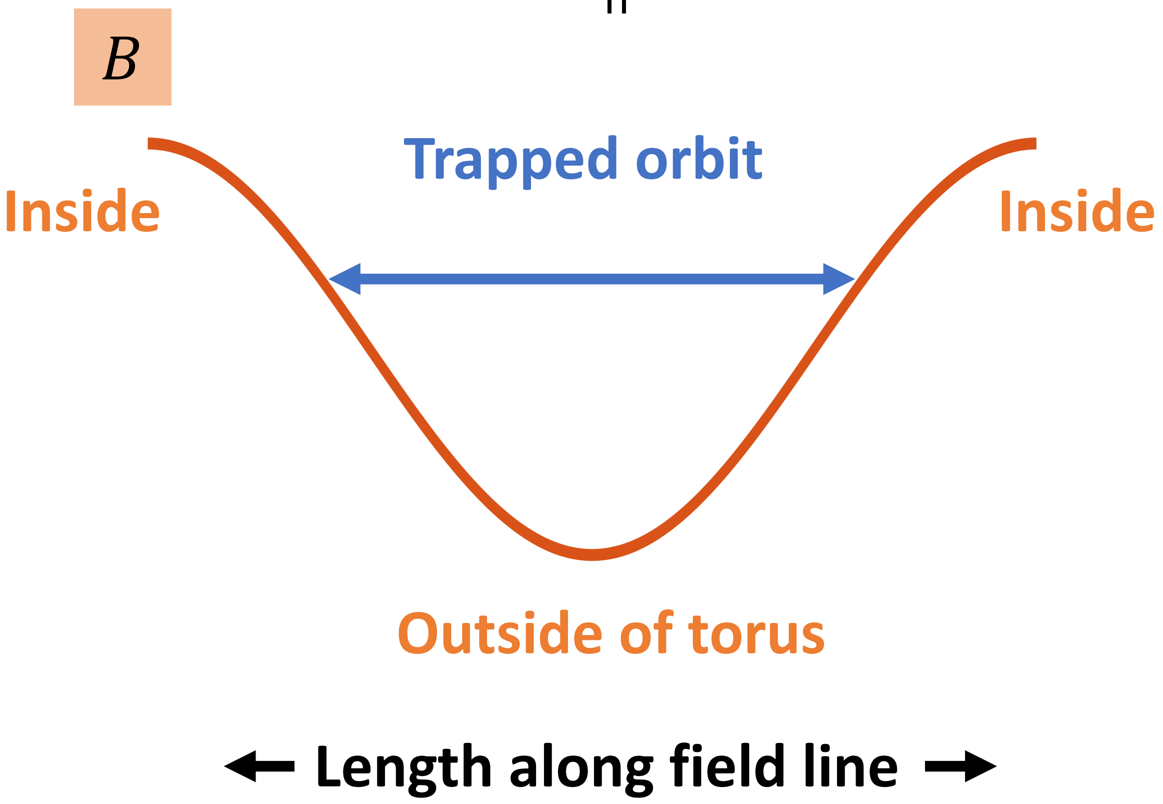


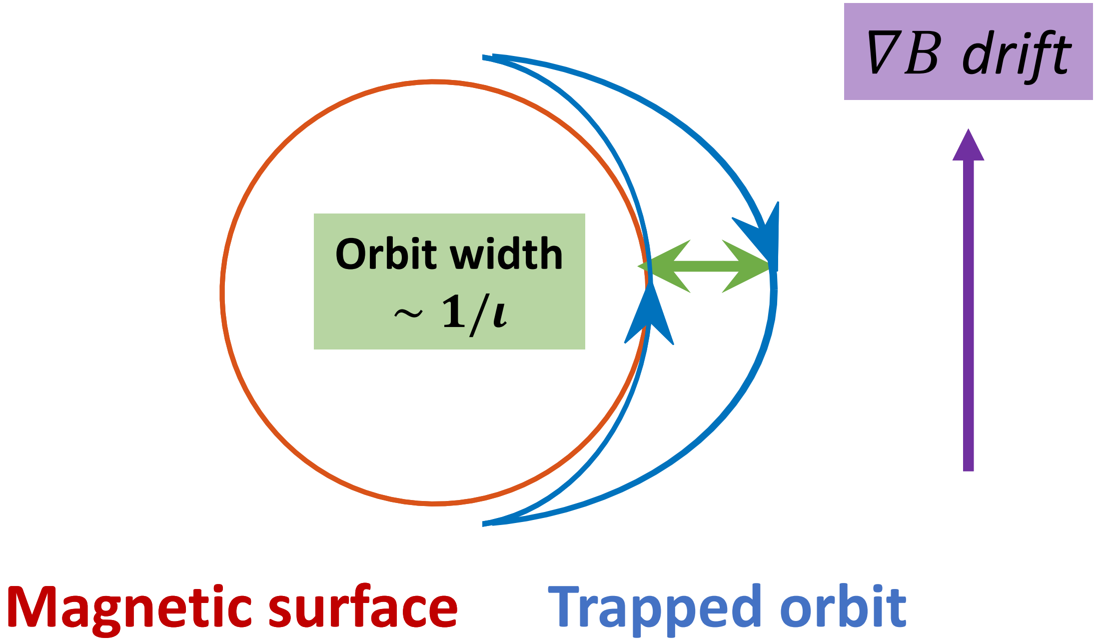


Magnetic confinement without symmetry

Trapped particles and drifts

$$\boldsymbol{E} = m v_{||}^2 + \boldsymbol{\mu} B$$

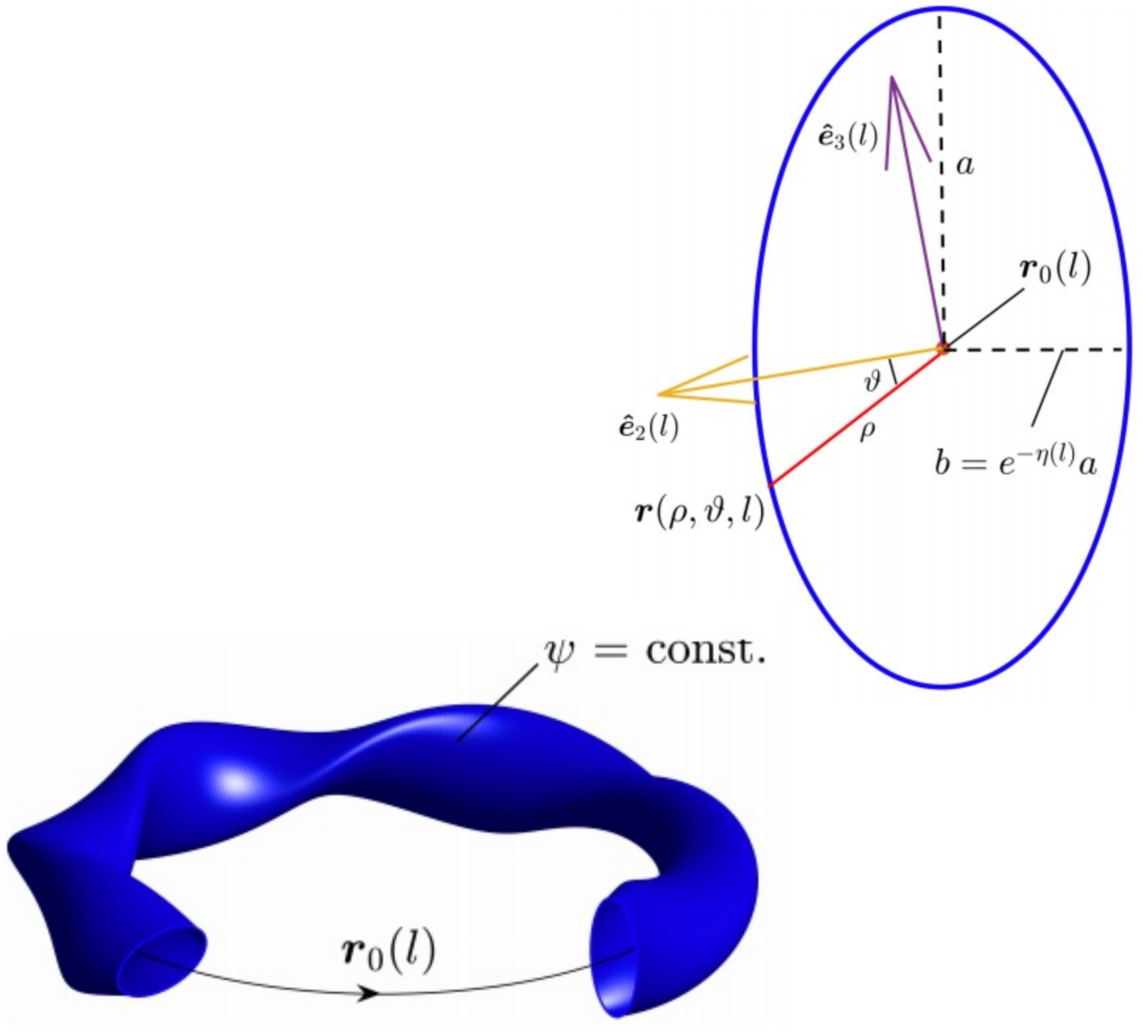




What about orbits in a stellarator?

Magnetic confinement without symmetry

Generating rotational transform



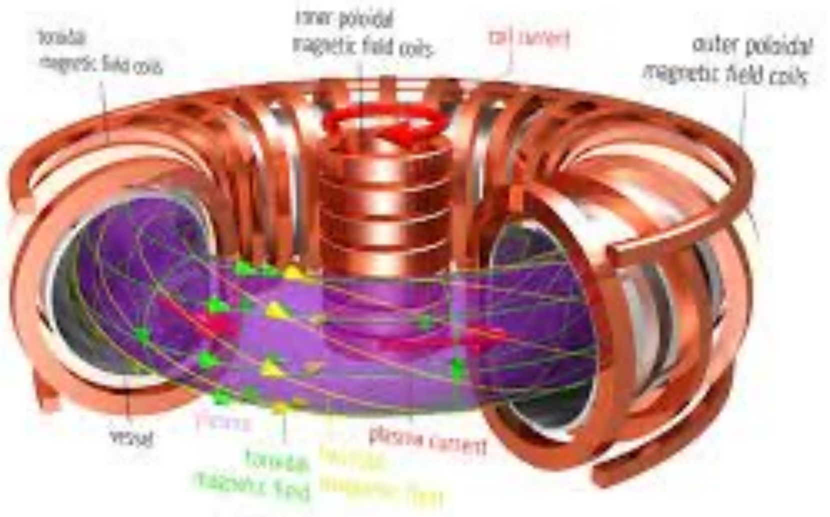
Expansion near the magnetic axis

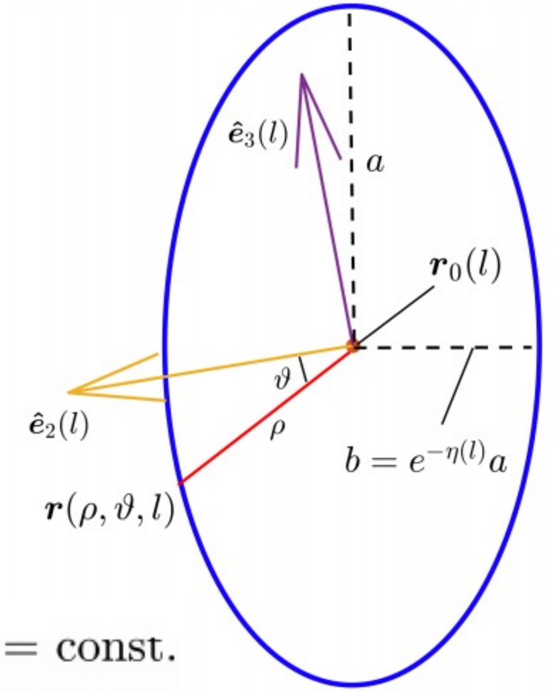
$$\iota = \frac{1}{2\pi}\int_0^L \frac{\left[\frac{\mu_0 J}{2B_0} - (\cosh\eta - 1)\delta' - \tau\right] dl}{\cosh\eta} - N$$

Magnetic confinement without symmetry

Generating rotational transform

Tokamak





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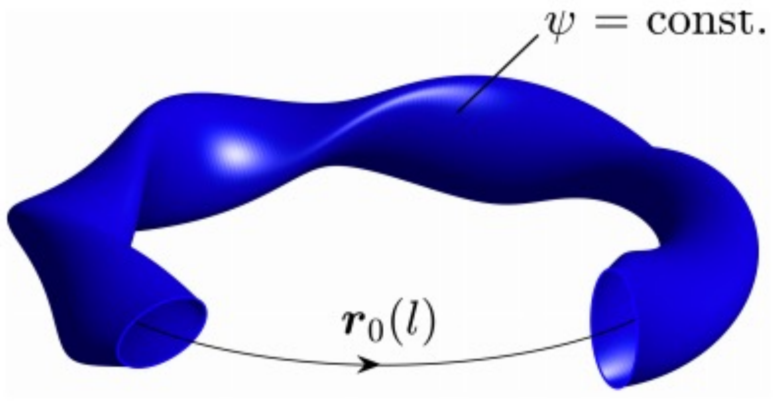
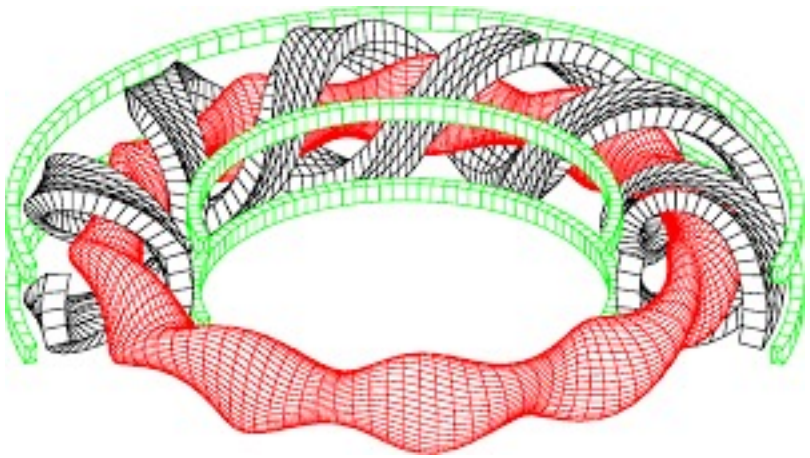
Plasma current

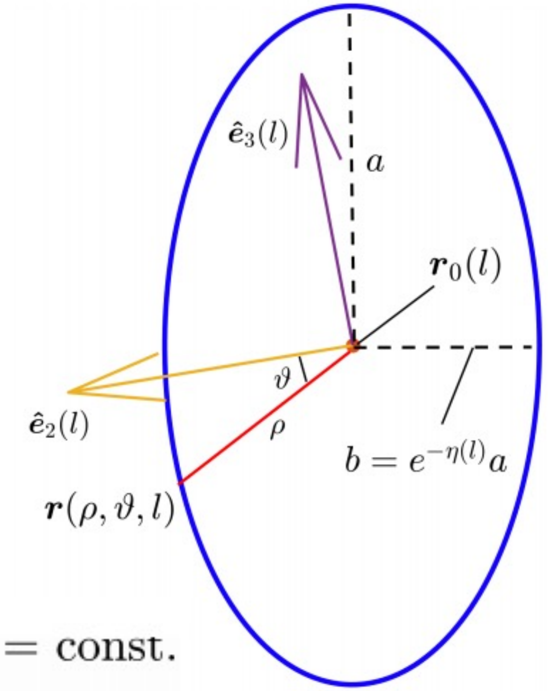


Magnetic confinement without symmetry

Generating rotational transform

Large Helical Device

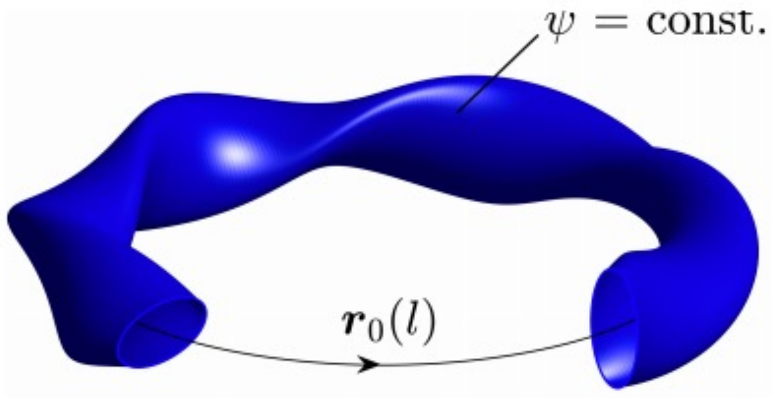




Expansion near the magnetic axis

$$\iota = \frac{1}{2\pi}\int_0^L \frac{\left[\frac{\mu_0 J}{2B_0} - (\cosh\eta - 1)\delta' - \tau\right] dl}{\cosh\eta} - N$$

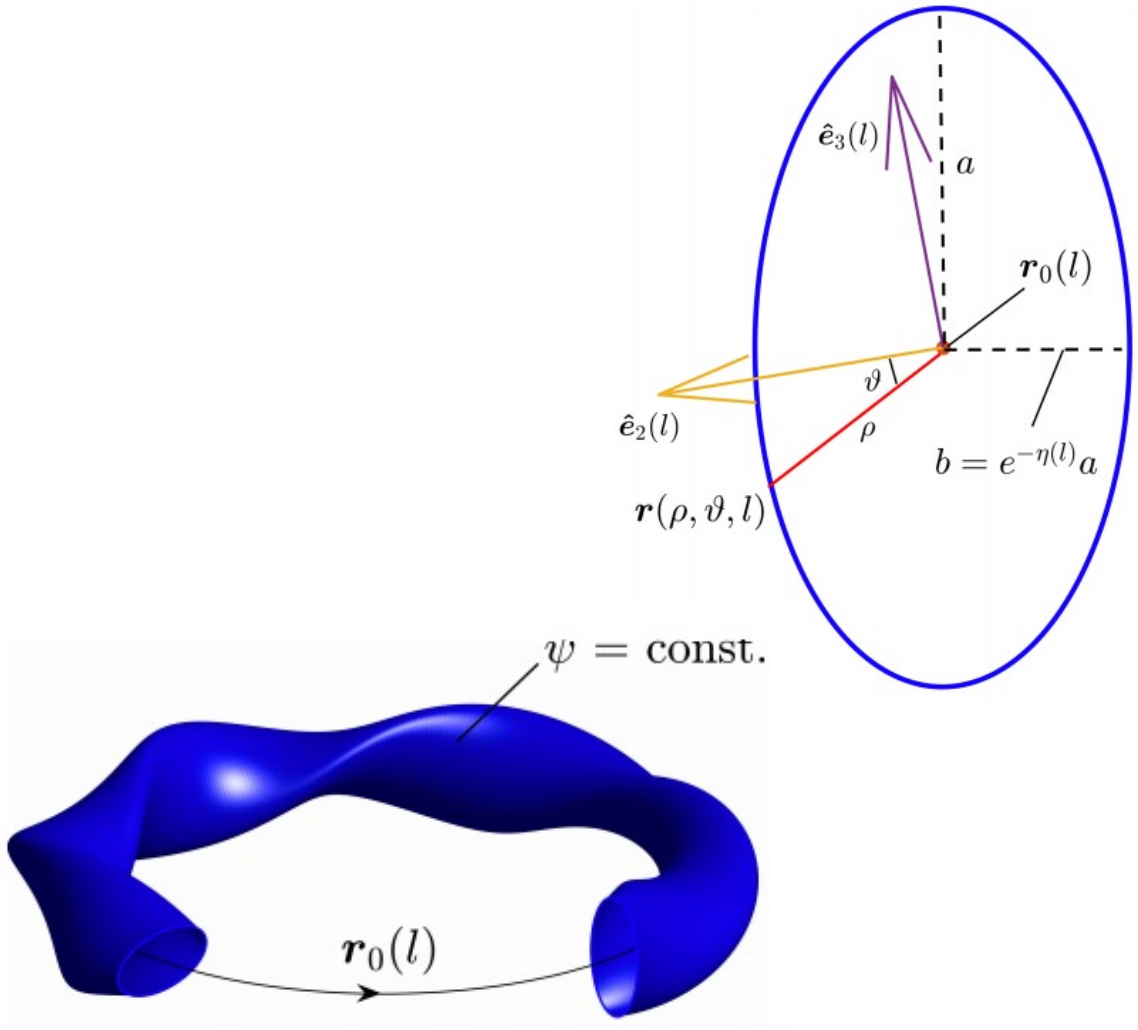
Rotating ellipticity



Magnetic confinement without symmetry

Generating rotational transform

Spitzer's figure-eight



Expansion near the magnetic axis

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Magnetic axis torsion

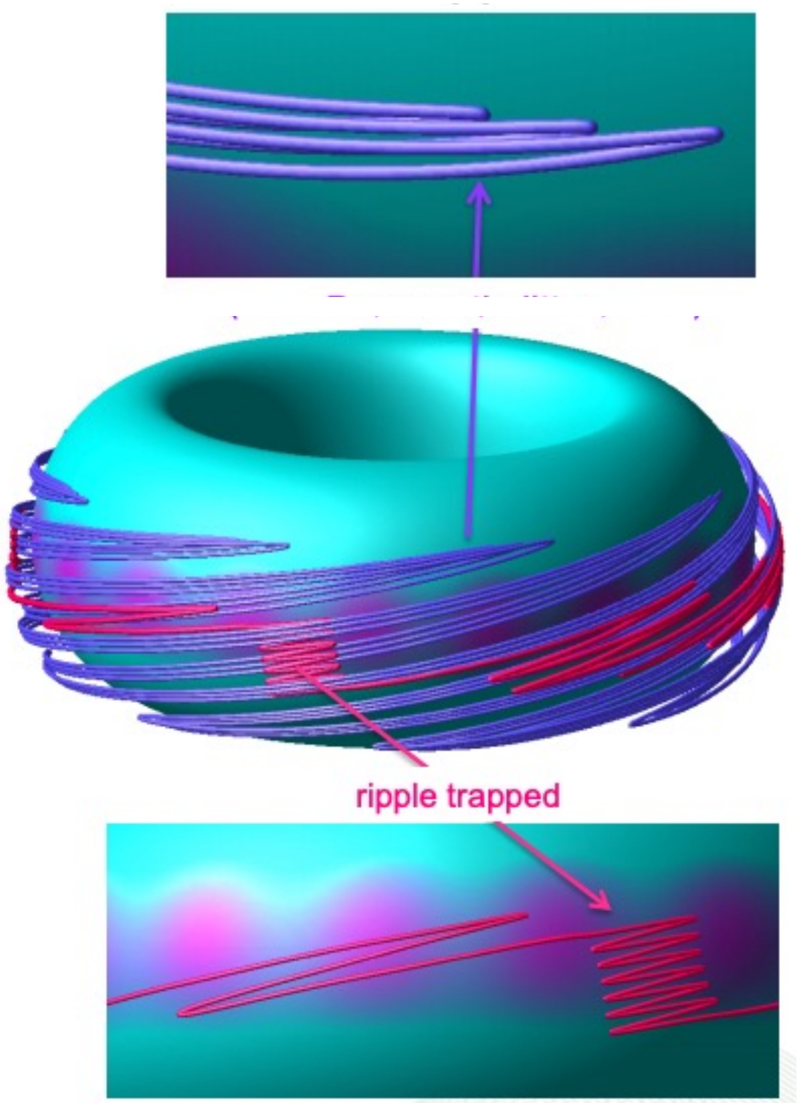
Outline

- Magnetic confinement without symmetry
- **Ingredients of stellarator confinement**
- How do we “cook” a stellarator?
- Stellarators in context

Ingredients of stellarator confinement

The zoology of particle orbits in 3D fields

Perturbed tokamak

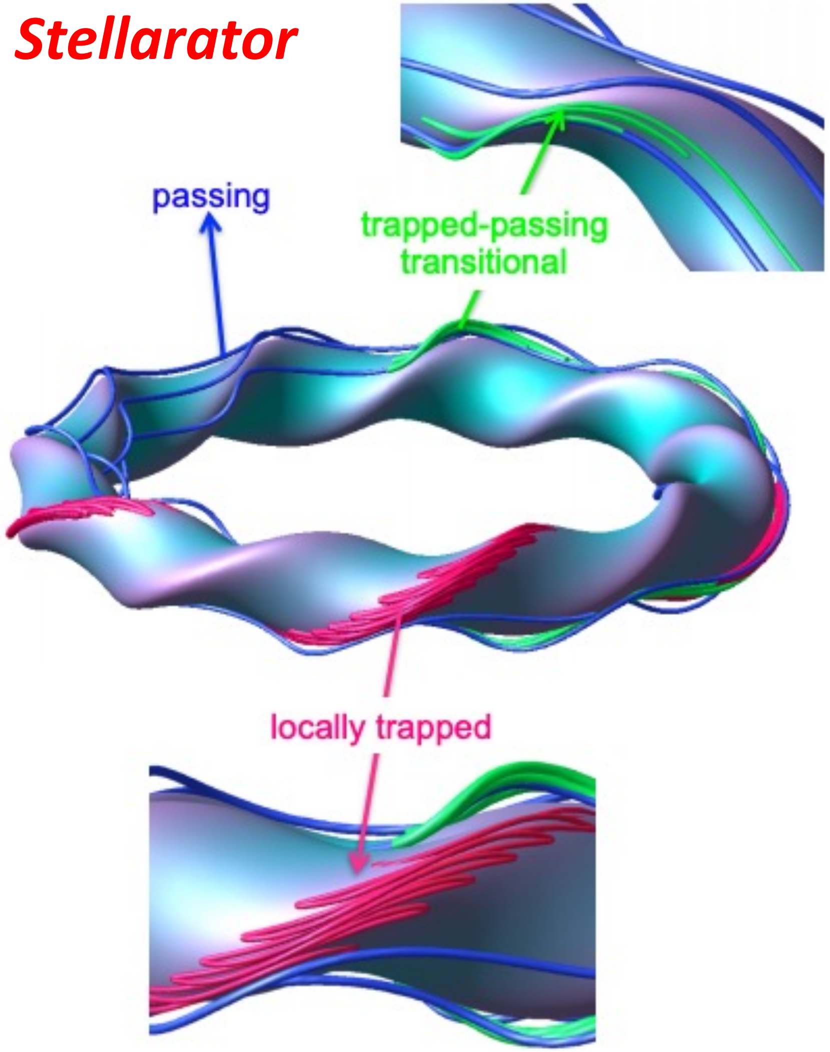


D.A. Spong et al, *APS DPP* (2014).

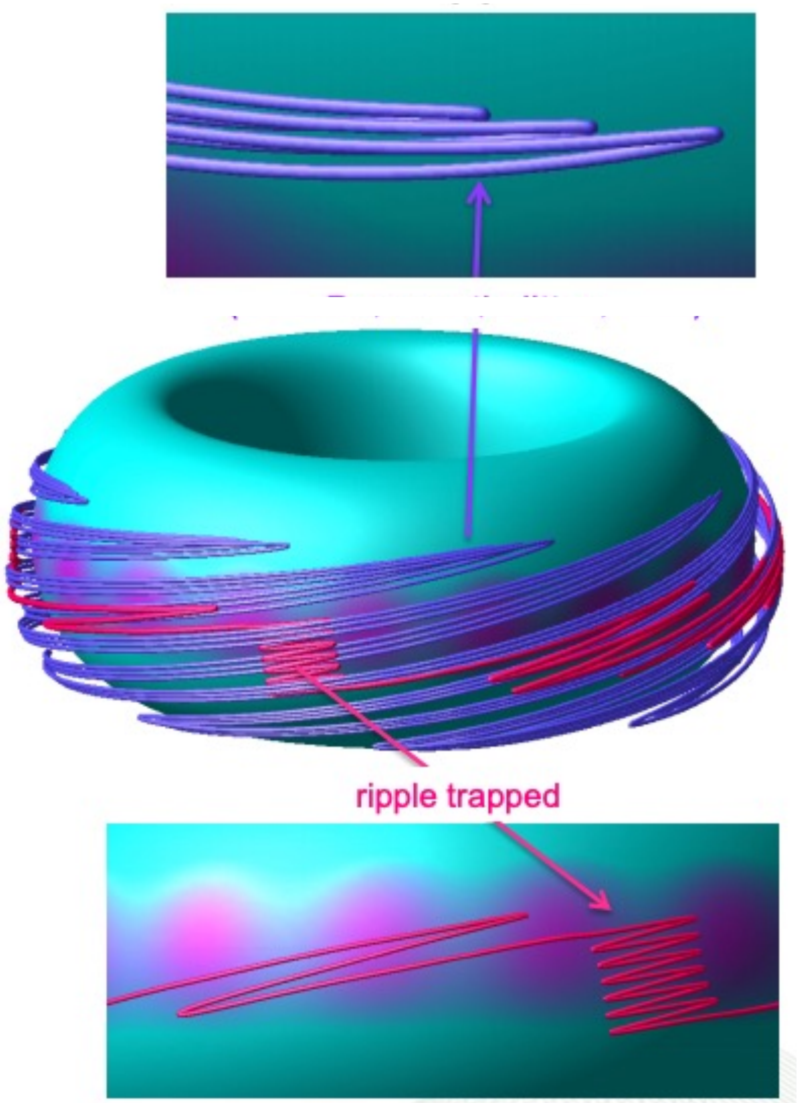
Ingredients of stellarator confinement

The zoology of particle orbits in 3D fields

Stellarator



Perturbed tokamak

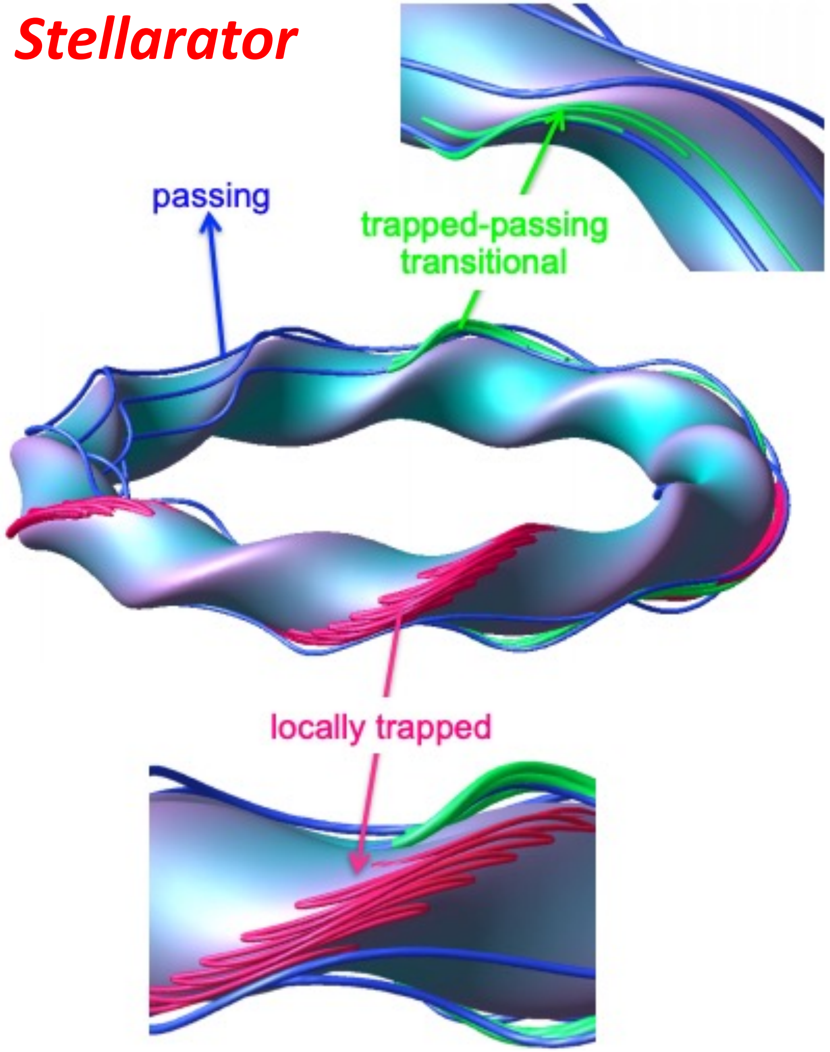


D.A. Spong et al, *APS DPP* (2014).

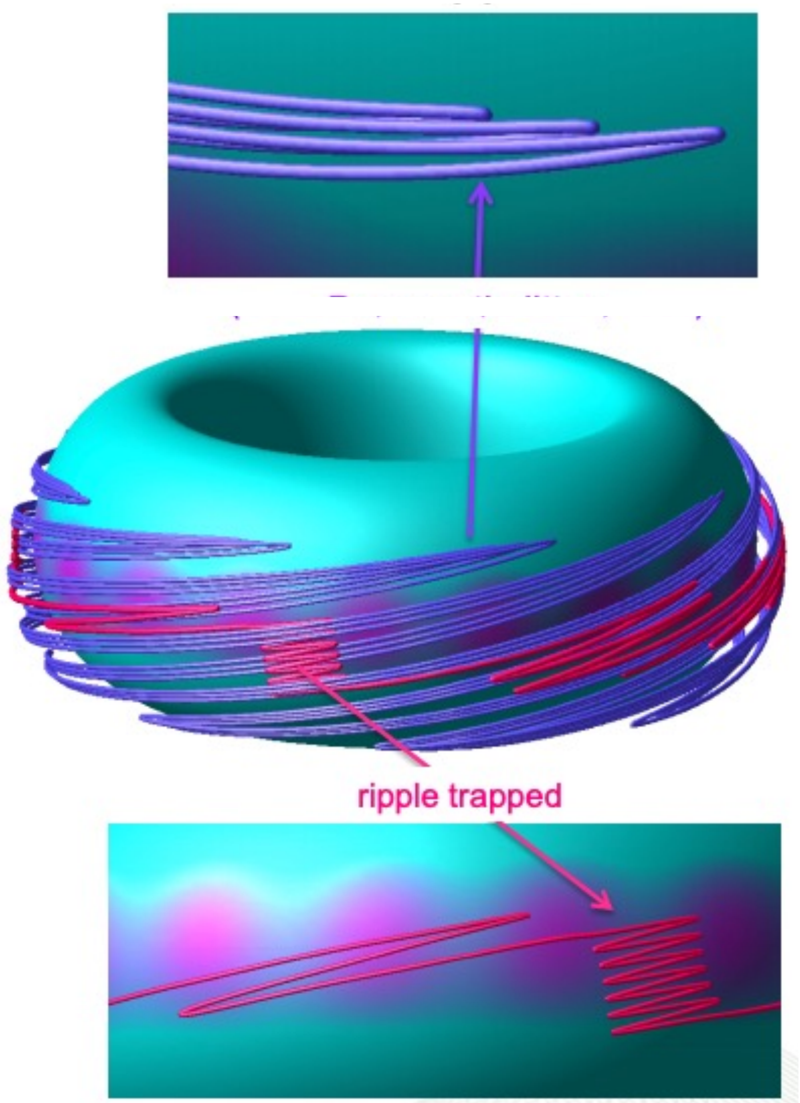
Ingredients of stellarator confinement

The zoology of particle orbits in 3D fields

Stellarator



Perturbed tokamak



Can we confine them all?

D.A. Spong et al, *APS DPP* (2014).

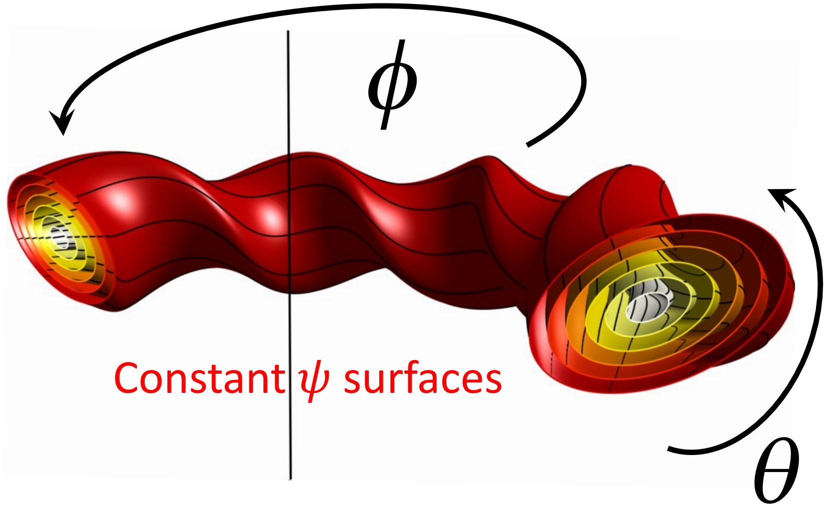
Ingredients of stellarator confinement

Collisionless guiding center confinement

$$\mathcal{L}(\boldsymbol{x}, \dot{\boldsymbol{x}}) = m\frac{|\dot{\boldsymbol{x}}|^2}{2} + q\,\boldsymbol{A}(\boldsymbol{x}) \cdot \dot{\boldsymbol{x}}$$

↓ Average over fast gyration

$$\mathcal{L}(\psi, \theta, \phi, \dot{\psi}, \dot{\theta}, \dot{\phi}) = \mathcal{L}(\psi, B(\psi, \theta, \phi), \dot{\psi}, \dot{\theta}, \dot{\phi})$$



Ingredients of stellarator confinement

Collisionless guiding center confinement - Axisymmetry

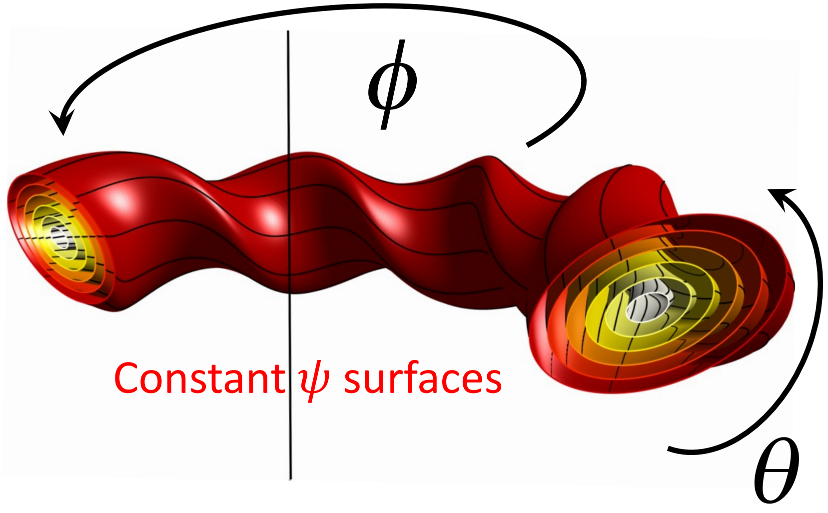
$$\mathcal{L}(\boldsymbol{x}, \dot{\boldsymbol{x}}) = m\frac{|\dot{\boldsymbol{x}}|^2}{2} + q\,\boldsymbol{A}(\boldsymbol{x}) \cdot \dot{\boldsymbol{x}}$$

Average over fast gyration

$$\mathcal{L}(\psi, \theta, \phi, \dot{\psi}, \dot{\theta}, \dot{\phi}) = \mathcal{L}(\psi, B(\psi, \theta, \phi), \dot{\psi}, \dot{\theta}, \dot{\phi})$$

$$\frac{\partial B}{\partial \phi} = 0 \rightarrow \frac{dp_\phi}{dt} = 0$$

$$p_\phi = mRv_\phi + qRA_\phi$$



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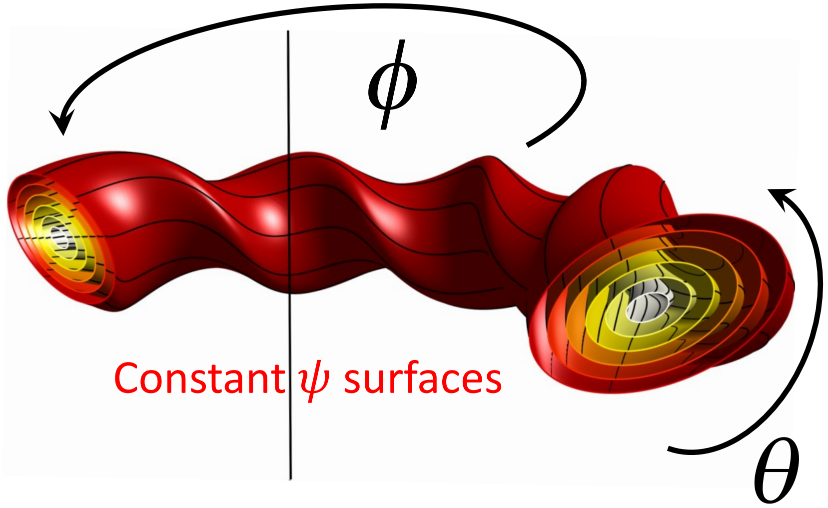
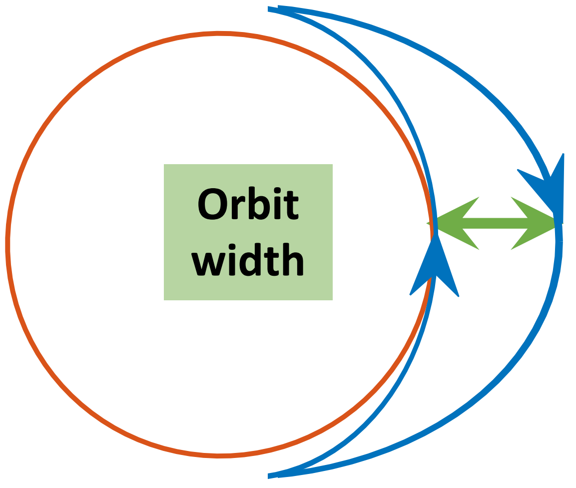
Const. on ψ surfaces



$$\frac{mRv_\phi}{qRA_\phi} \sim \frac{\text{orbit width}}{\text{system size}} \ll 1$$



Ingredients of stellarator confinement

Collisionless guiding center confinement - Quasisymmetry

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Average over fast gyration

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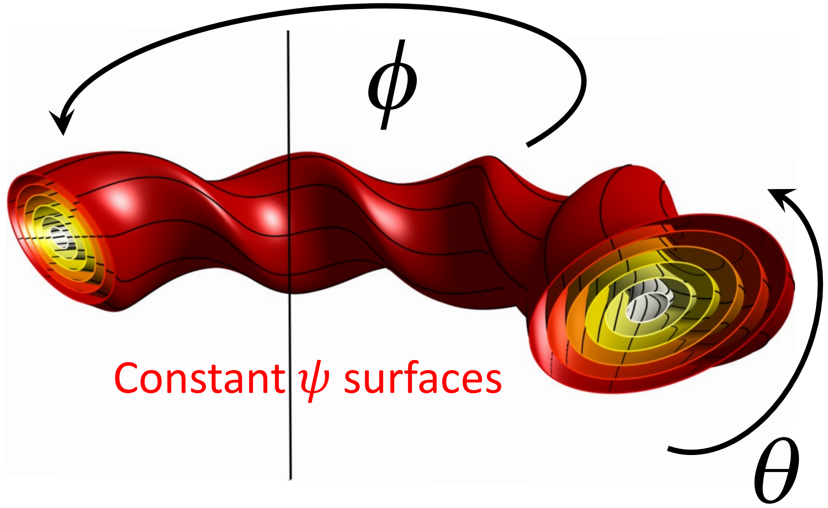


$$\eta = M\theta - N\phi$$
$$\chi = M'\theta - N'\phi$$

$$\frac{\partial B}{\partial \eta} = 0 \rightarrow \frac{dp_\eta}{dt} = 0$$

$$p_\eta = mv_\eta + qF(\psi)$$

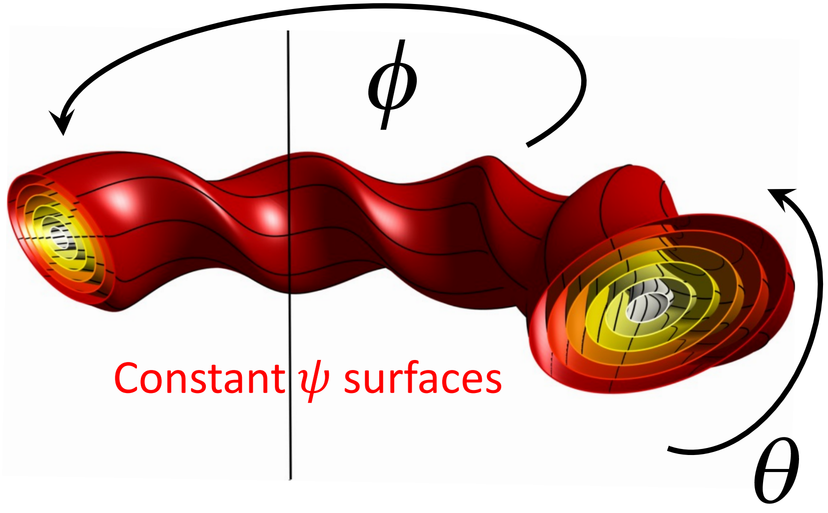
Ingredients of stellarator confinement

Collisionless guiding center confinement - Quasisymmetry

$$\mathcal{L}(\boldsymbol{x},\dot{\boldsymbol{x}}) = m\frac{|\dot{\boldsymbol{x}}|^2}{2} + q\,\boldsymbol{A}(\boldsymbol{x})\cdot\dot{\boldsymbol{x}}$$

Average over fast gyration

$$\mathcal{L}(\psi,\theta,\phi,\dot{\psi},\dot{\theta},\dot{\phi}) = \mathcal{L}(\psi, B(\psi,\theta,\phi),\dot{\psi},\dot{\theta},\dot{\phi})$$



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Const. on ψ surfaces

Ingredients of stellarator confinement

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↓ Average over fast gyration

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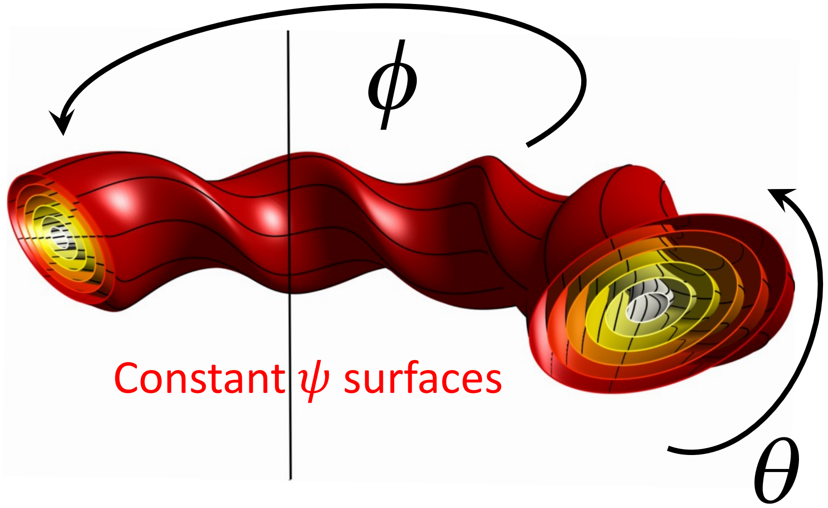


$$\eta = M\theta - N\phi$$
$$\chi = M'\theta - N'\phi$$

$$\frac{\partial B}{\partial \eta} = 0 \rightarrow \frac{dp_\eta}{dt} = 0$$

$$p_\eta = mv_\eta + qF(\psi)$$

Const. on ψ surfaces

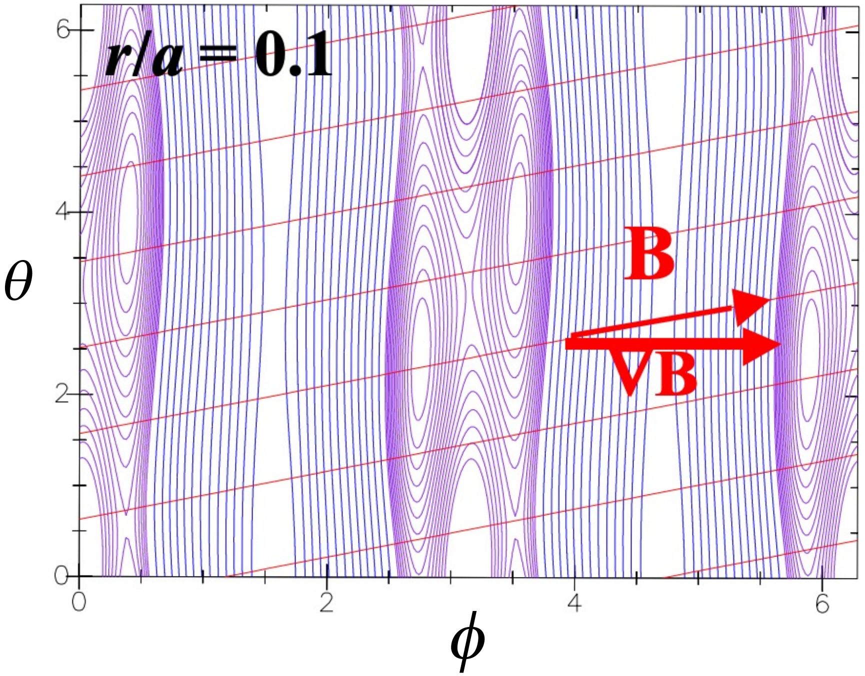
What does this look like?

Ingredients of stellarator confinement

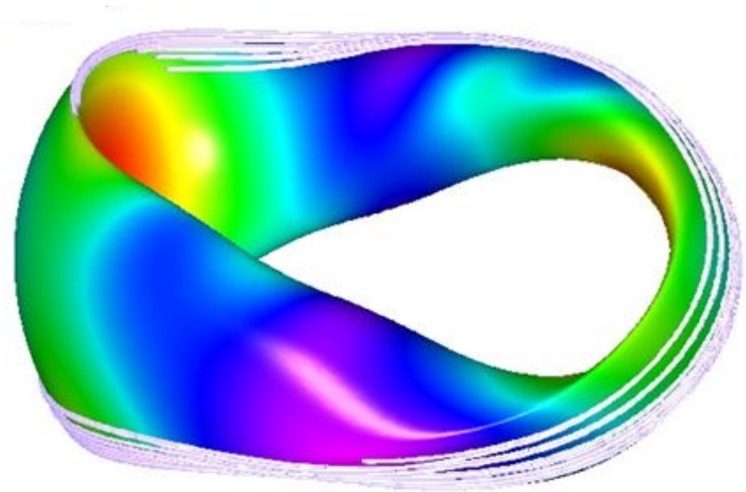
Quasi-poloidal symmetry – Quasi-Poloidal Stellarator (QPS)

$$\eta = M\theta - N\phi$$

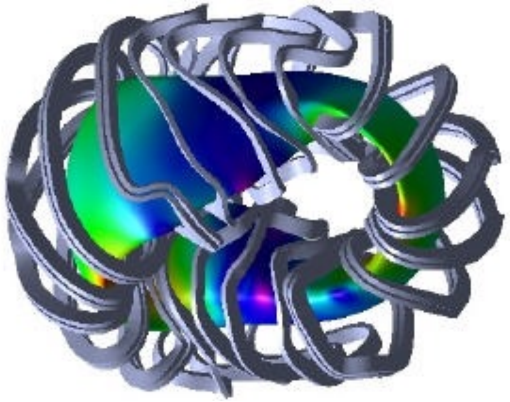
B on ψ surface



Toroidal "bananas"



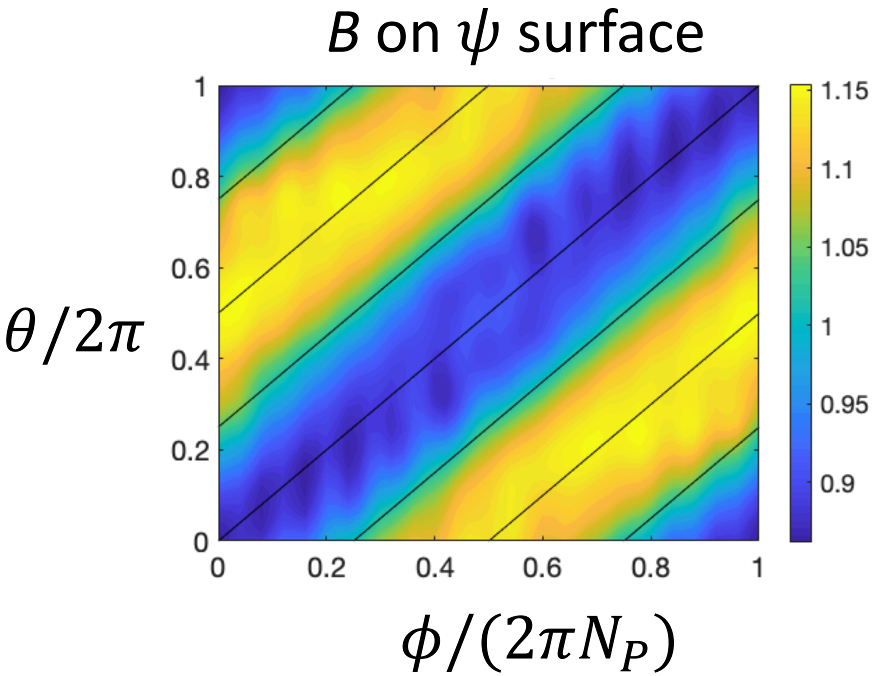
$$\frac{\partial B}{\partial \theta} \approx 0 \rightarrow \frac{dp_\theta}{dt} \approx 0$$



D.A. Spong et al, *IAEA* (2003).

Ingredients of stellarator confinement

Quasi-helical symmetry – Helically Symmetric eXperiment (HSX)

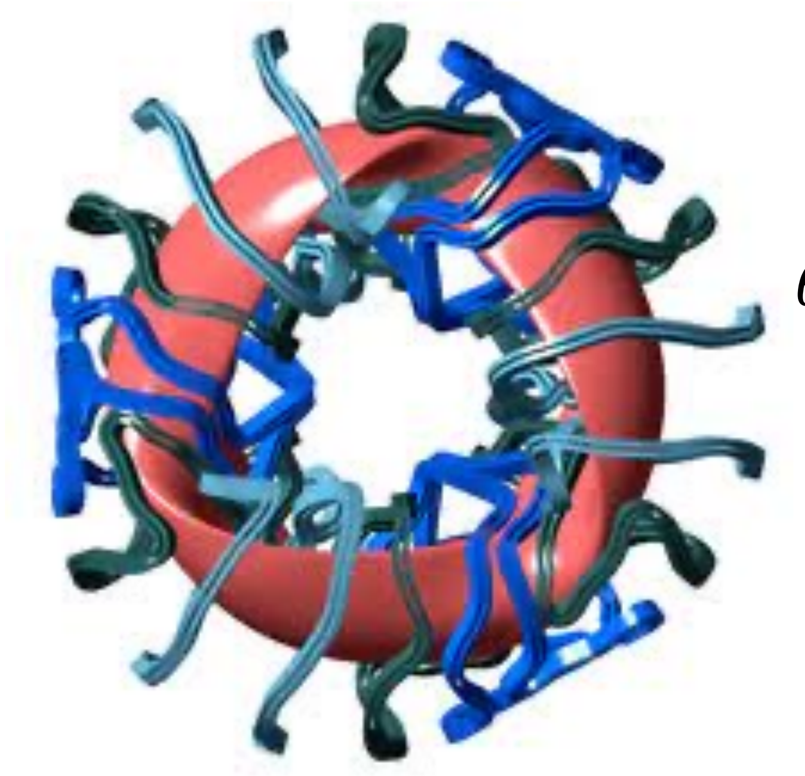


$$\frac{\partial B}{\partial \eta} \approx 0 \to \frac{dp_\eta}{dt} \approx 0 \qquad \eta = \theta - \phi$$

Ingredients of stellarator confinement

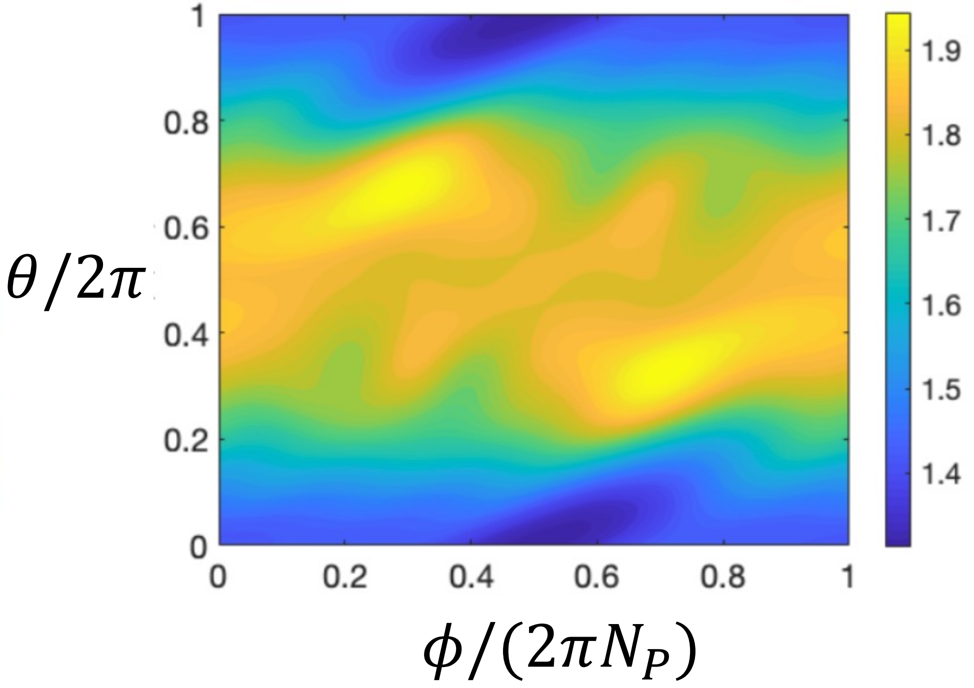
Quasi-axisymmetry – National Compact Stellarator eXperiment (NCSX)

$$\eta = M\theta - N\phi$$



$$\frac{\partial B}{\partial \phi} \approx 0 \rightarrow \frac{dp_{\phi}}{dt} \approx 0$$

B on ψ surface



Ingredients of stellarator confinement

Collisionless guiding center confinement - Omnigeneity

$H(p, q, \lambda) \rightarrow$ **Hamiltonian** depending on slowly varying **parameter**

$\frac{\dot{\lambda}}{\lambda} \ll \omega \rightarrow$ **parameters** varying slowly in comparison with **frequency**

$J(H, \lambda) = \oint p(H, q, \lambda)\, dq$ = **adiabatic invariant** is conserved

Ingredients of stellarator confinement

Collisionless guiding center confinement - Omnigeneity

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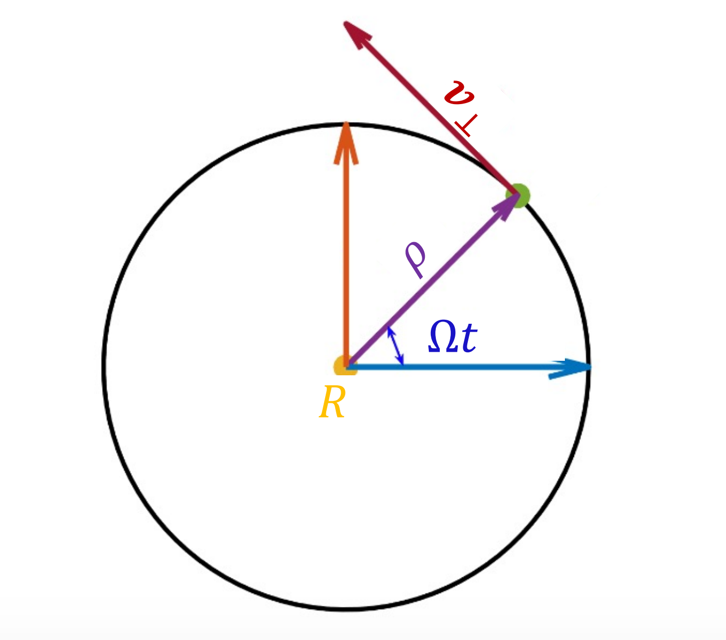


Recall - Magnetic moment conservation

Assuming $\dot{B}/B \ll \Omega$,

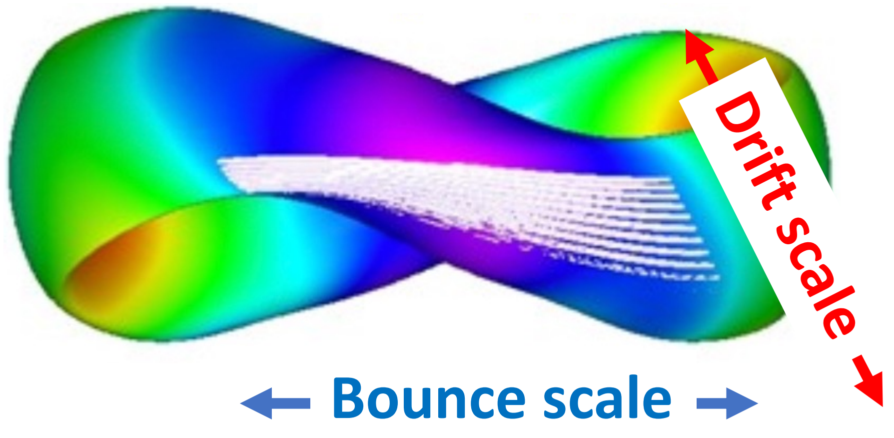
$$J = \oint p_\theta d\theta = \oint m v_\perp \rho\, d\theta = \left(\frac{4\pi\, m}{q}\right) \frac{m v_\perp^2}{2B} = \text{const.}$$

$\mu =$ "**magnetic moment**"

Ingredients of stellarator confinement

Collisionless guiding center confinement - Omnigeneity

Trapped orbit

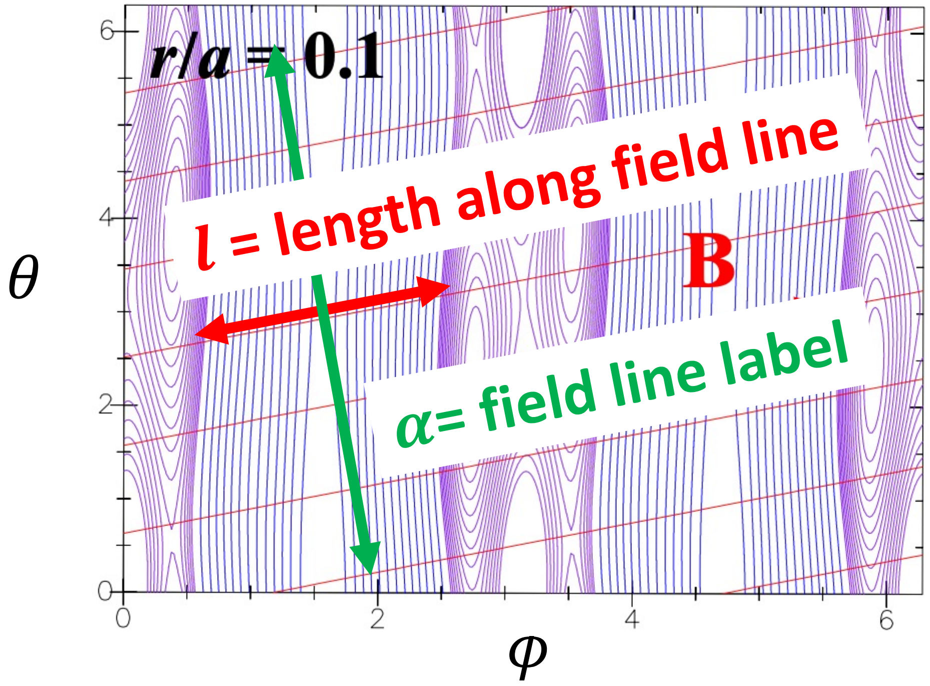


D.A. Spong et al, *IAEA* (2003).

Assuming drift frequency ≪ bounce frequency,

$$J_{||}(\psi, \alpha, E, \mu) = \oint dl\ v_{||}(\psi, \alpha, E, \mu) = \text{const.}$$

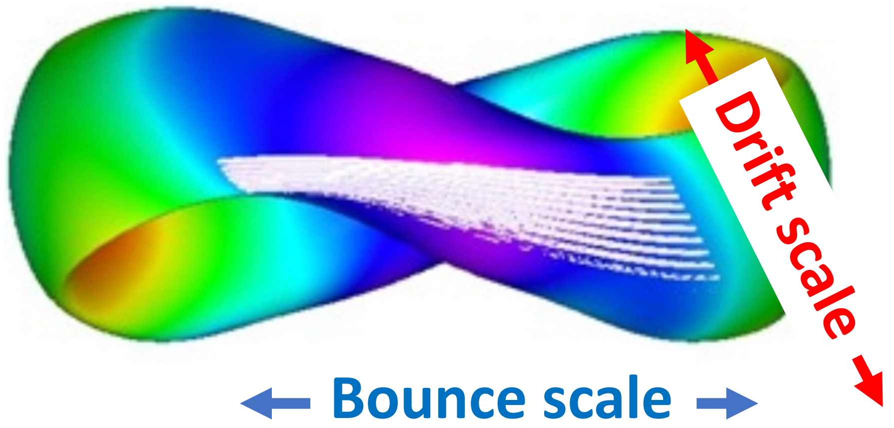
"parallel adiabatic invariant"



Ingredients of stellarator confinement

Collisionless guiding center confinement - Omnigeneity

Trapped orbit

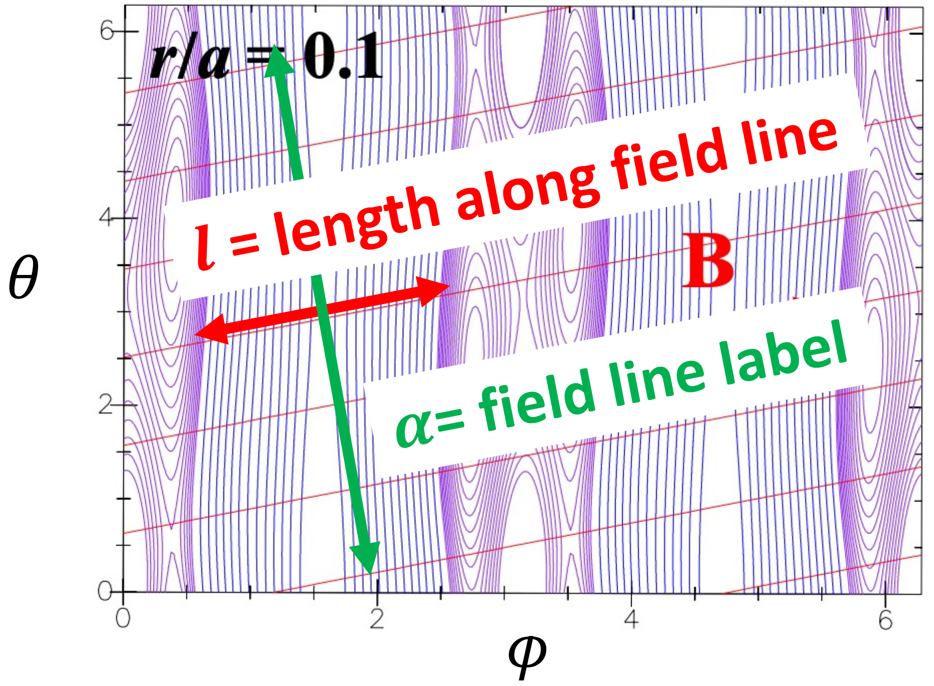


D.A. Spong et al, *IAEA* (2003).

Assuming drift frequency ≪ bounce frequency,

$$J_{||}(\psi,\alpha,E,\mu) = \oint dl\ v_{||}(\psi,\alpha,E,\mu) = \text{const.}$$

"parallel adiabatic invariant"

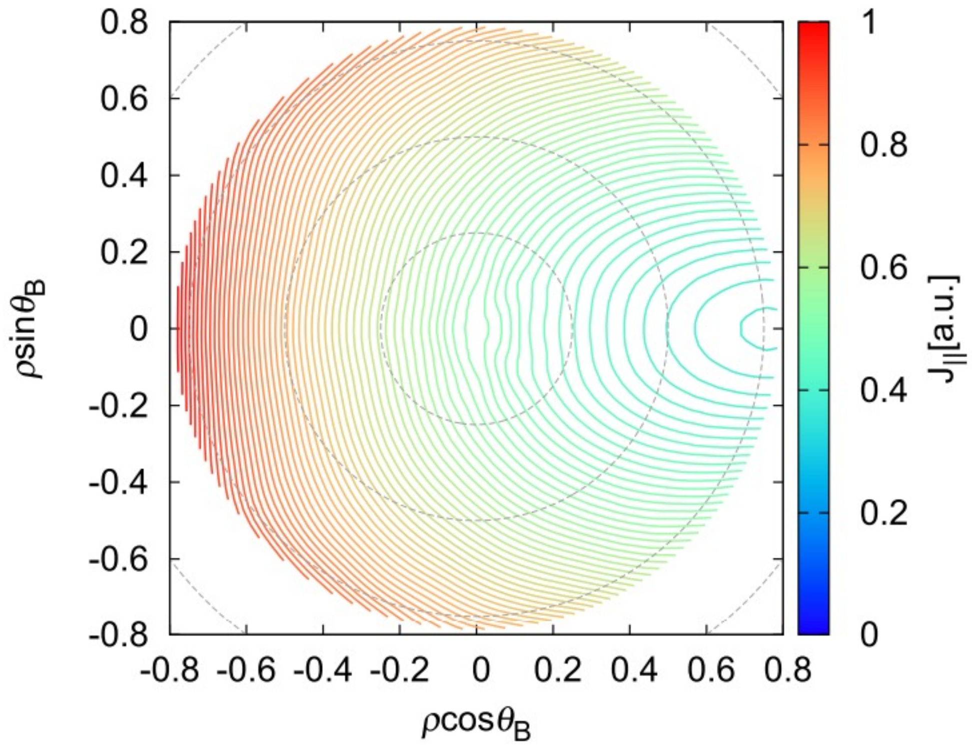


$$\frac{\partial J_{||}}{\partial \alpha} = 0 \rightarrow \text{"omnigeneity"}$$

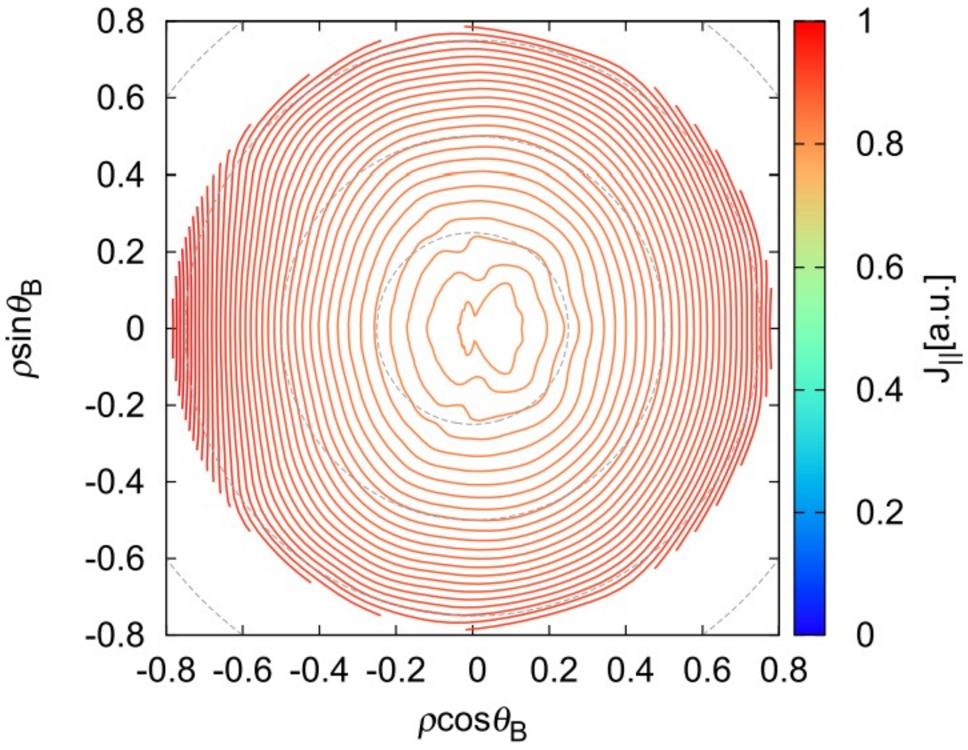
Ingredients of stellarator confinement

Collisionless guiding center confinement - Omnigeneity

Far from omnigeneous



Close to omnigeneous

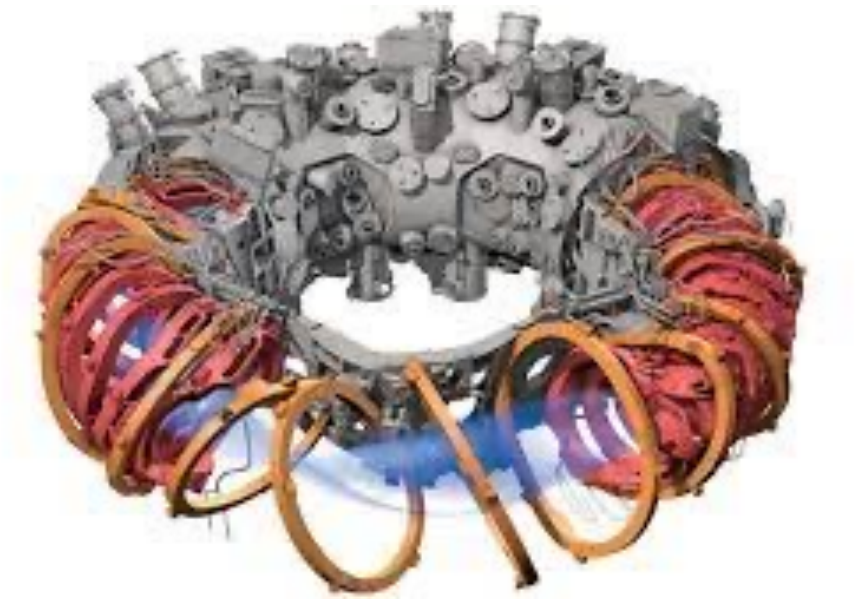


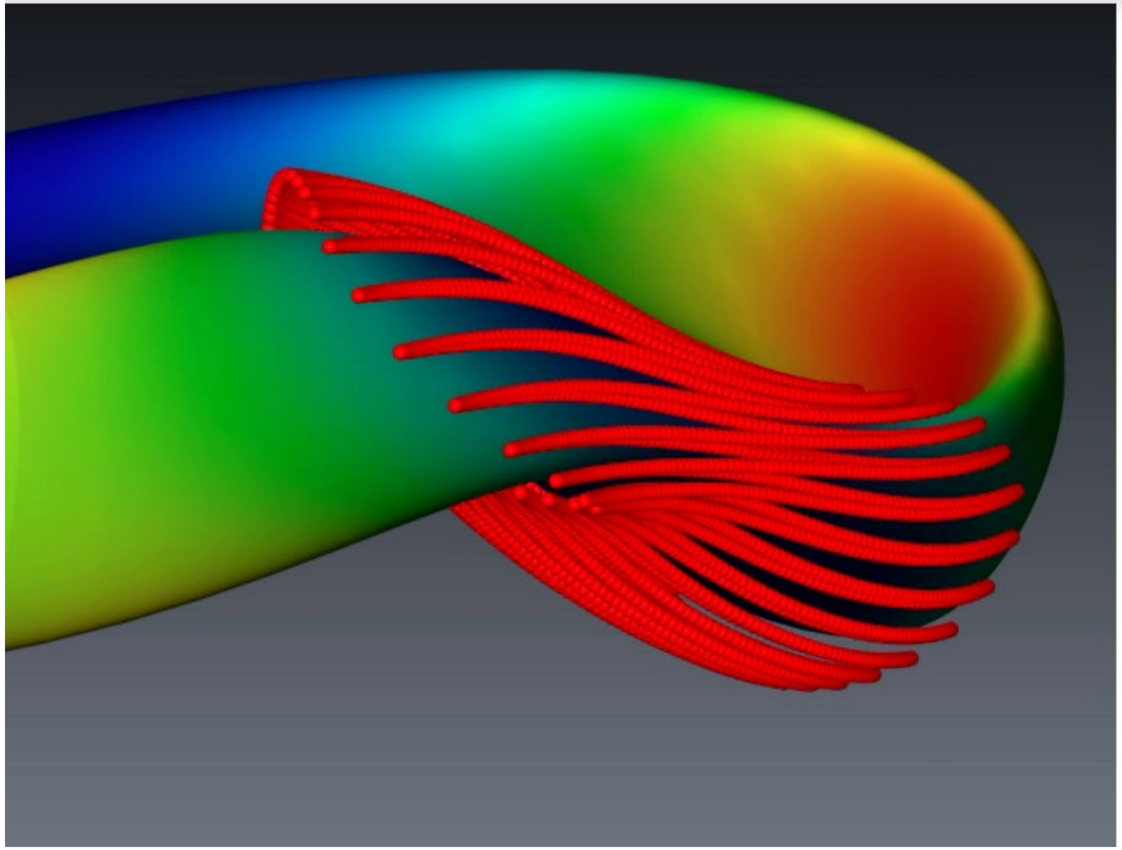
H. Yamaguchi, *Nuclear Fusion 59* (2019).

Ingredients of stellarator confinement

Collisionless guiding center confinement - Omnigeneity

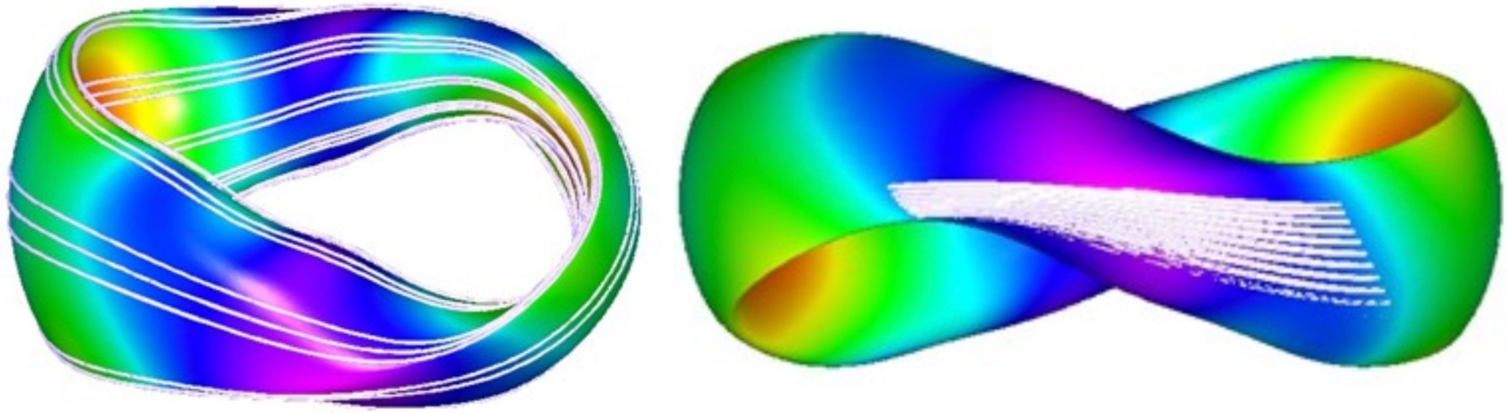
Wendelstein 7-X





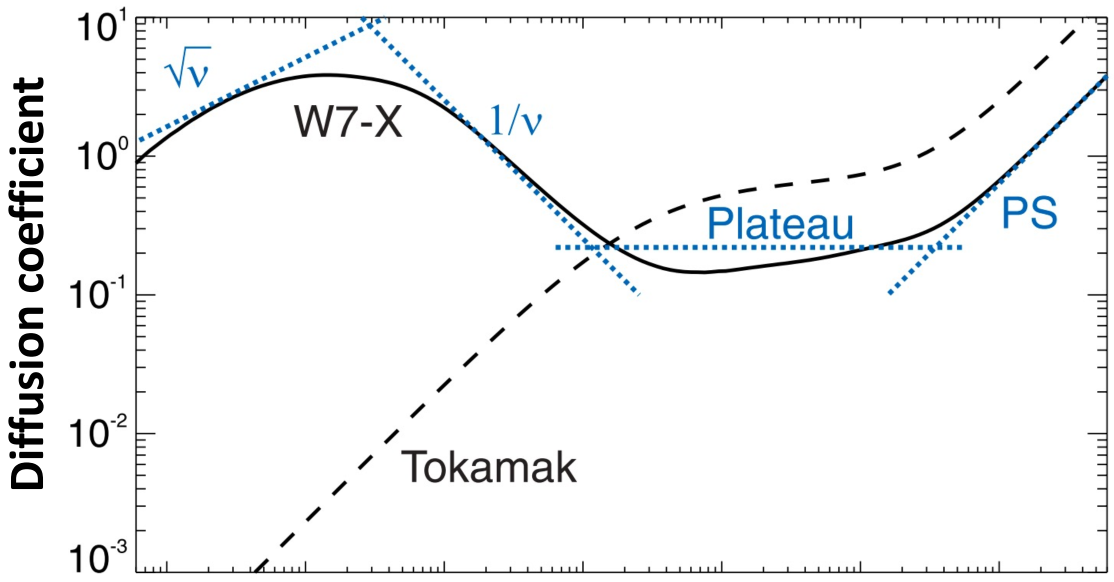
Ingredients of stellarator confinement

Collisional guiding center confinement



D.A. Spong et al, *IAEA* (2003).

P. Helander, *Rep. Prog. Phys.*, 77 (2014).



$\nu_* = \nu R/(\iota v)$

"drift kinetic equation"

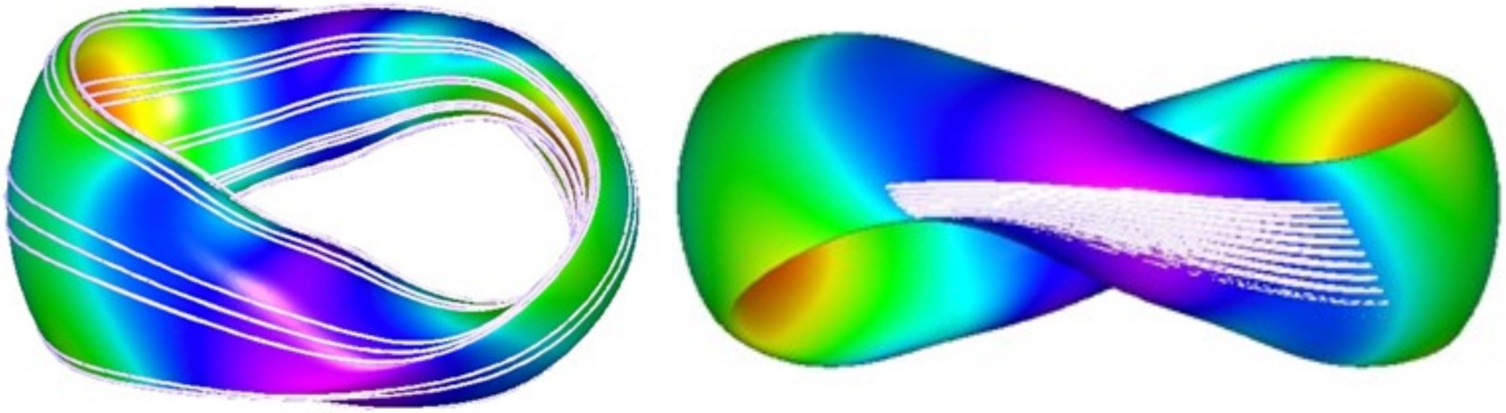
$$(v_{||}\hat{\boldsymbol{b}} + \boldsymbol{v}_E + \boldsymbol{v}_m) \cdot \nabla f = C(f)$$

Guiding center motion **Collisions**

Ingredients of stellarator confinement

Collisional guiding center confinement

Review: [Parra Day 3]



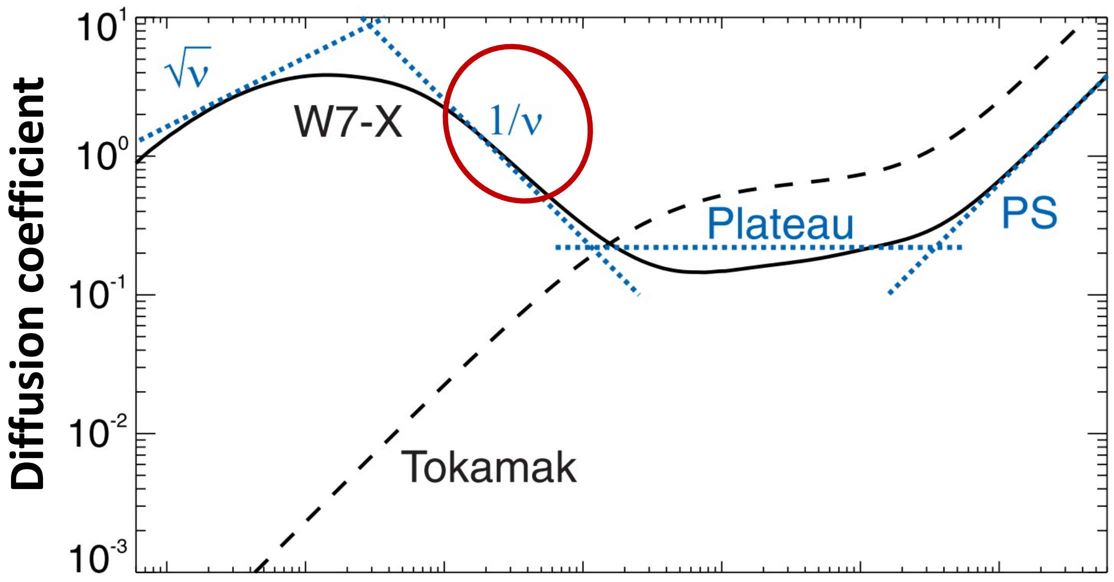
D.A. Spong et al, *IAEA* (2003).

$$D \sim \frac{\Delta r^2}{\Delta t} \sim \frac{(v_d \cdot \nabla r\ /v)^2}{1/\nu}$$

$$(v_{||}\hat{\boldsymbol{b}} + \boldsymbol{v}_E + \boldsymbol{v}_m) \cdot \nabla f = C(f)$$

$$v_d \cdot \nabla\alpha \ll \nu_{eff} \ll v_d \cdot \nabla r/L$$

P. Helander, *Rep. Prog. Phys.*, 77 (2014).

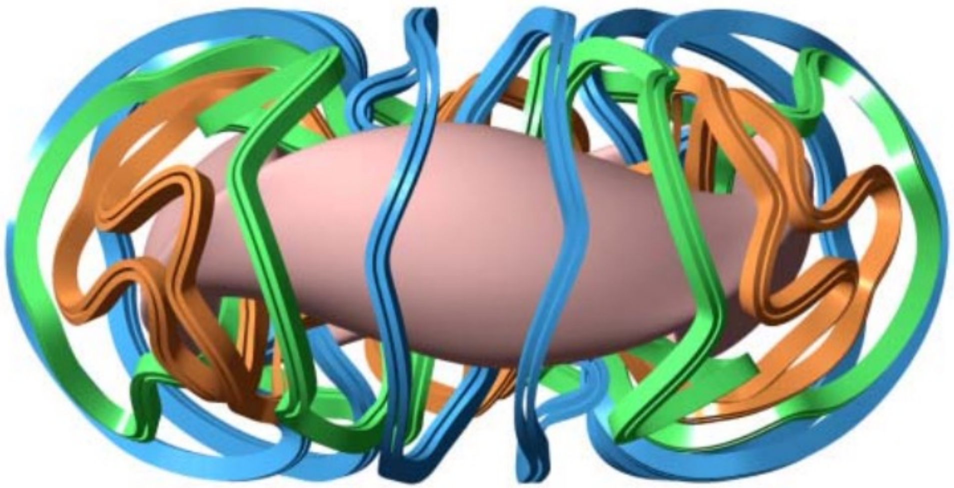


$\nu_* = \nu R/(\iota v)$

Ingredients of stellarator confinement

Magnetic field integrability

NCSX Modular Coils

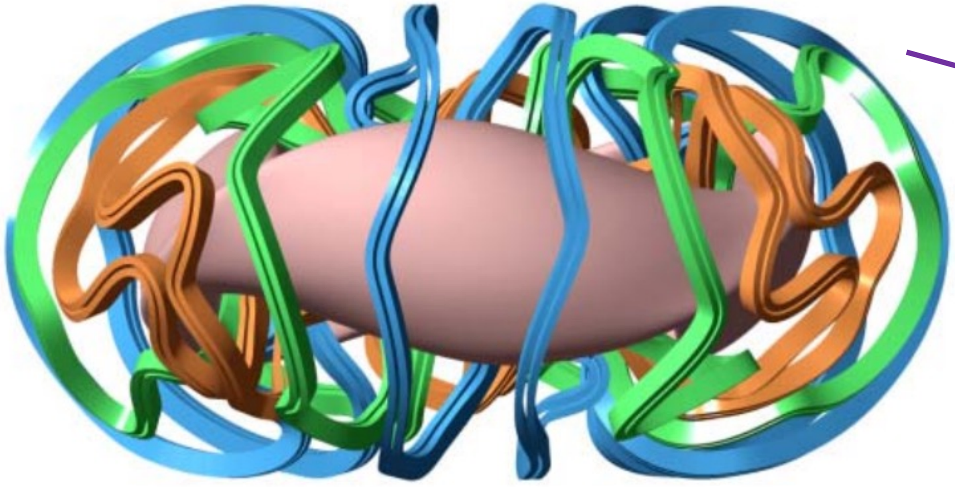


D. Strickler et al, *IAEA* (2004).

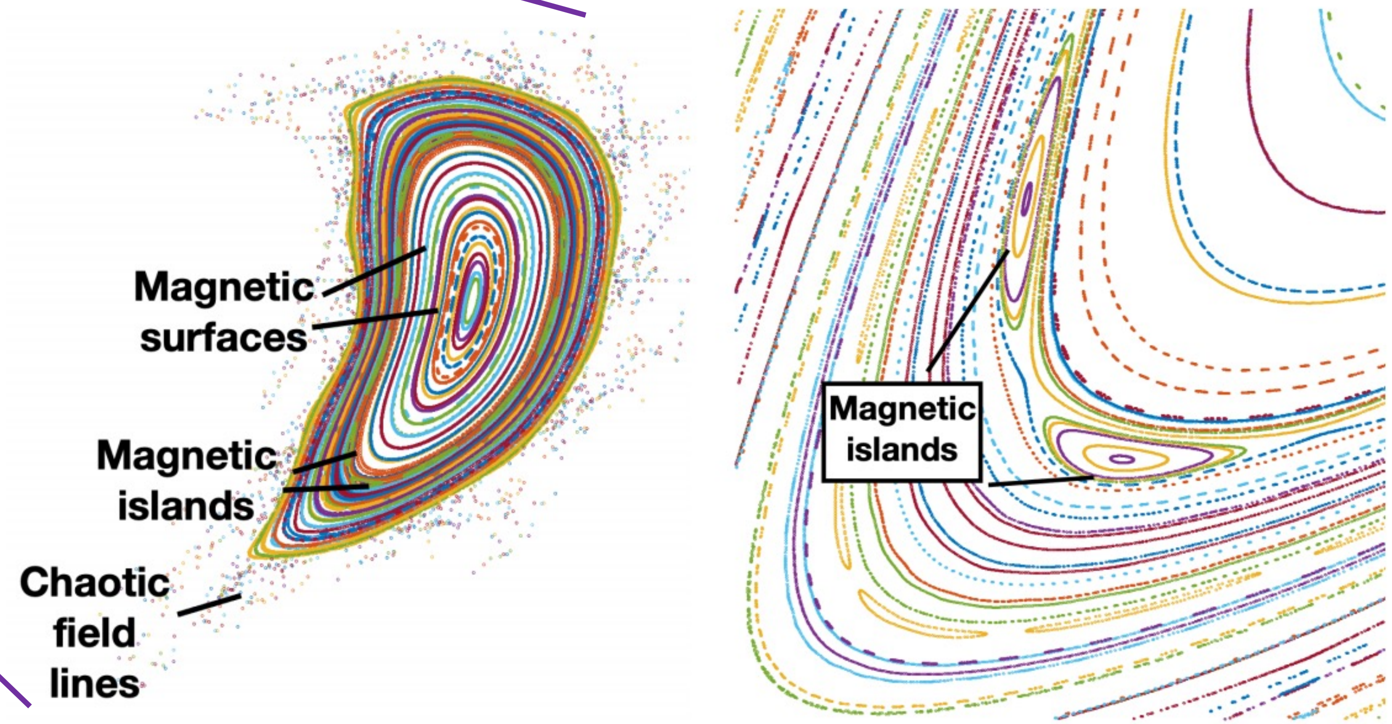
Ingredients of stellarator confinement

Magnetic field integrability

NCSX Modular Coils



D. Strickler et al, *IAEA* (2004).

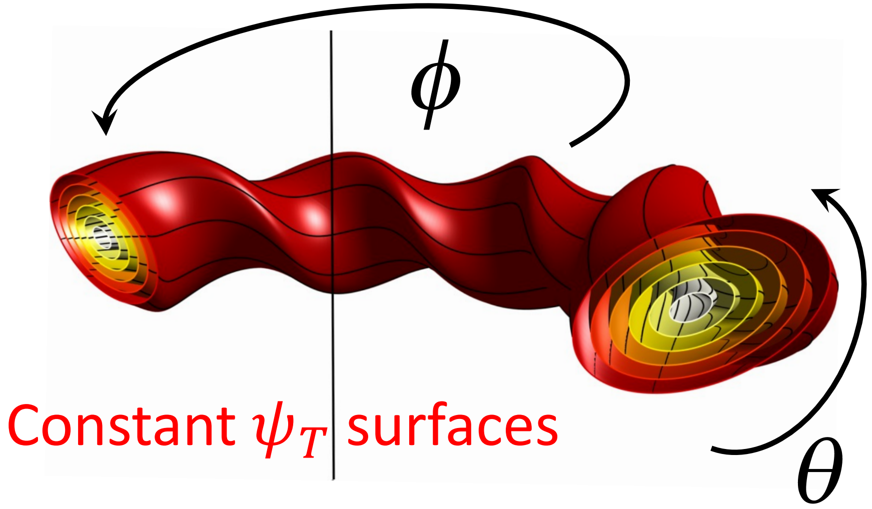


Ingredients of stellarator confinement

Magnetic field integrability – Hamiltonian nature of field line flow

$$\boldsymbol{A}(\psi_T, \theta, \phi) = \psi_T \nabla\theta - \psi_P(\psi_T, \theta, \phi)\nabla\phi$$

$$\boldsymbol{B}(\psi_T, \theta, \phi) = \nabla\, \psi_T \times \nabla\theta - \nabla\psi_P \times \nabla\phi$$



Ingredients of stellarator confinement

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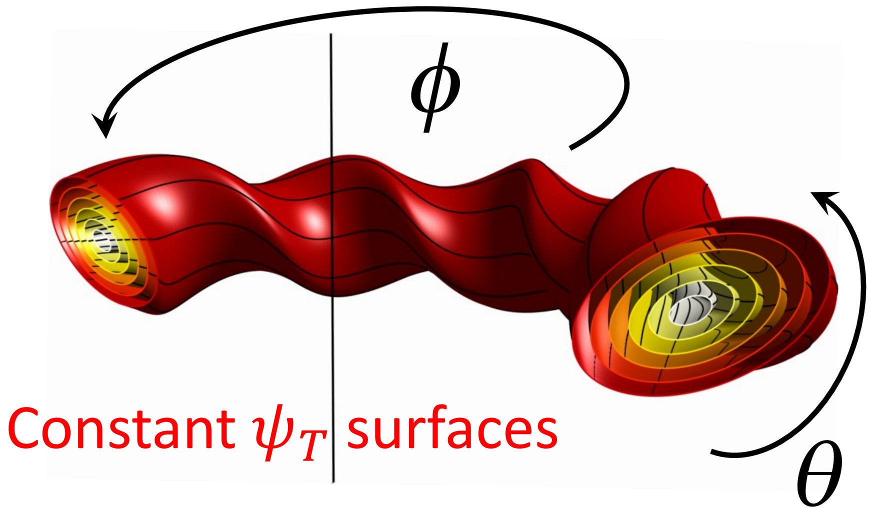

$$\frac{d\theta}{d\phi} = \frac{\boldsymbol{B}\cdot\nabla\theta}{\boldsymbol{B}\cdot\nabla\phi} = \frac{\partial\psi_P}{\partial\psi_T}$$

$$\frac{d\psi_T}{d\phi} = \frac{\boldsymbol{B}\cdot\nabla\psi_T}{\boldsymbol{B}\cdot\nabla\phi} = -\frac{\partial\psi_P}{\partial\theta}$$

Hamiltonian → ψ_P
coordinate → θ
momentum → ψ_T
time → ϕ

Ingredients of stellarator confinement

Magnetic field integrability – Hamiltonian nature of field line flow

$$\boldsymbol{A}(\psi_T,\theta,\phi) = \psi_T\nabla\theta - \psi_P(\psi_T,\theta,\phi)\nabla\phi$$
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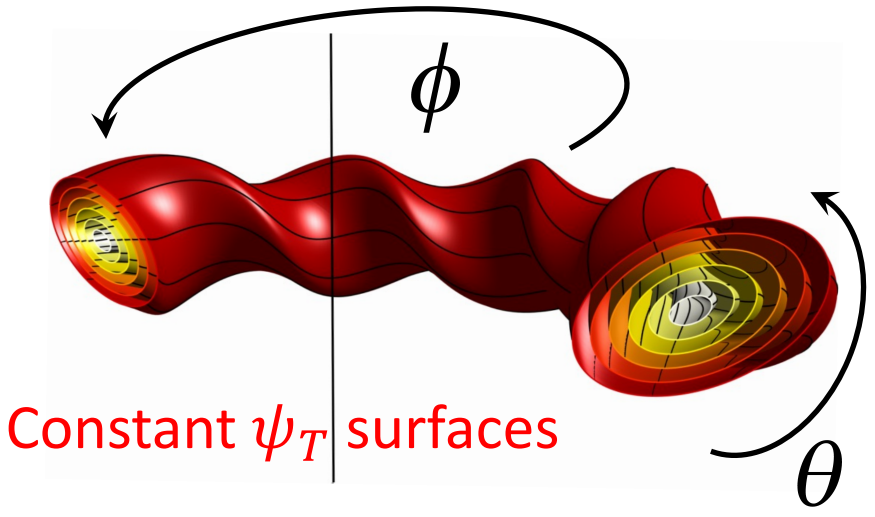


Hamiltonian → ψ_P
coordinate → θ
momentum → ψ_T
time → ϕ

$$\frac{\partial\psi_P(\theta,\psi_T,\phi)}{\partial\phi} = 0$$ → integrability

$\boldsymbol{B}\cdot\nabla\psi_p$= 0 [i.e., ψ_p is flux label]

Ingredients of stellarator confinement

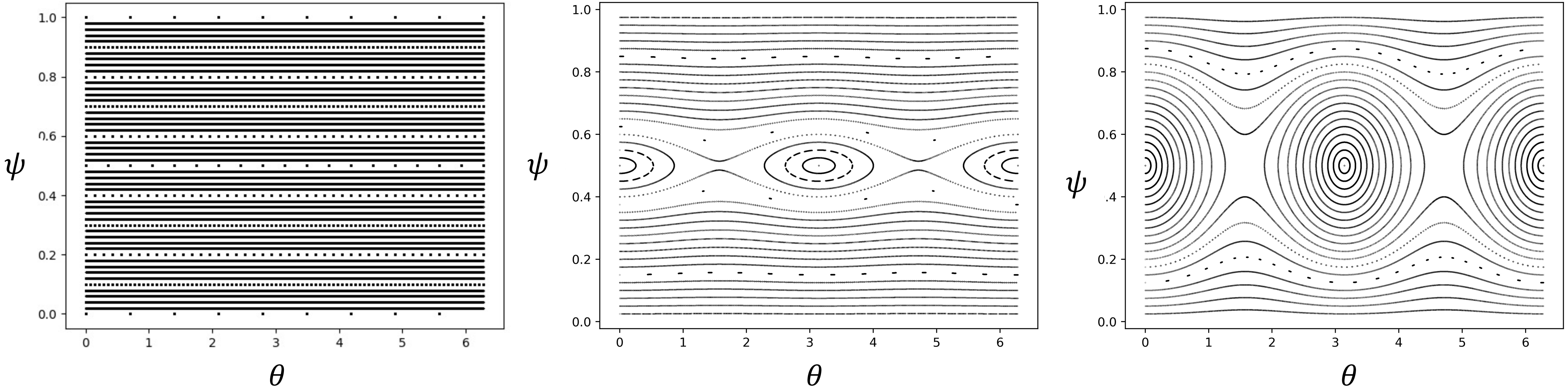
Magnetic field integrability – Hamiltonian nature of field line flow

$$\boldsymbol{B}(\psi_T, \theta, \phi) = \nabla\, \psi_T \times \nabla\theta - \nabla\psi_P \times \nabla\phi$$

$$\psi_P = \frac{1}{2}\psi_T^2 + \epsilon \cos(\theta - \phi)$$

Integrable **Non-integrable**

$\epsilon = 0$ $\epsilon = 0.01$ $\epsilon = 0.1$



Ingredients of stellarator confinement

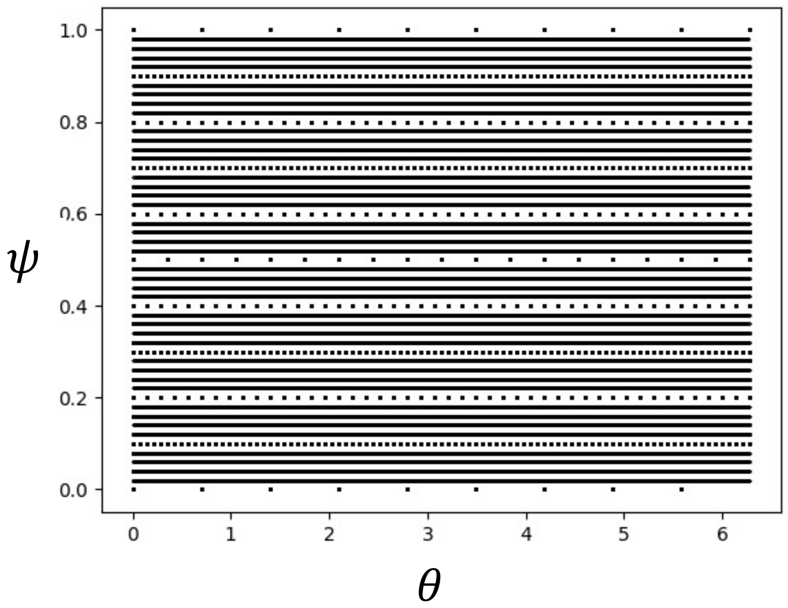
Magnetic field integrability – Hamiltonian nature of field line flow

$$\boldsymbol{B}(\psi_T, \theta, \phi) = \nabla\,\psi_T \times \nabla\theta - \nabla\psi_P \times \nabla\phi$$

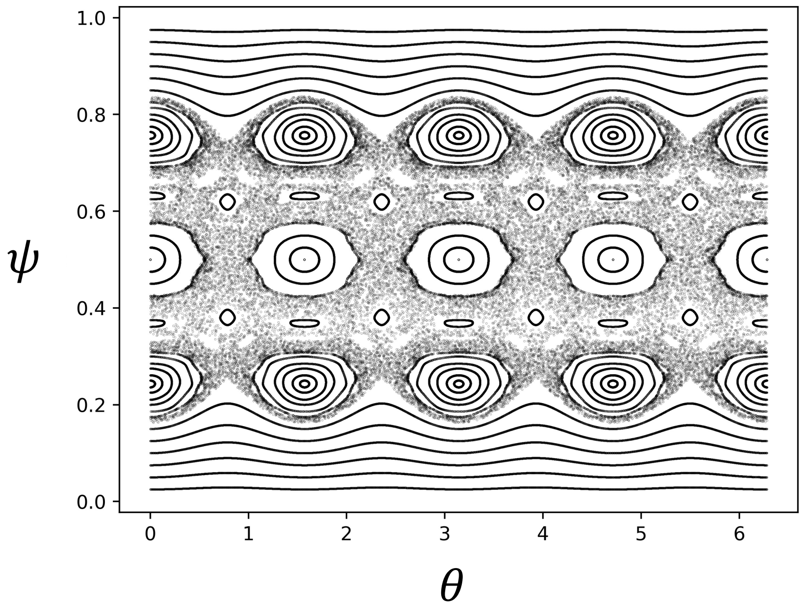
$$\psi_P = \frac{1}{2}\psi_T^2 + \epsilon\left[\cos(4\theta - \phi) + \cos(4\theta - 2\phi) + \cos(4\theta - 3\phi)\right]$$

Integrable **Non-integrable**

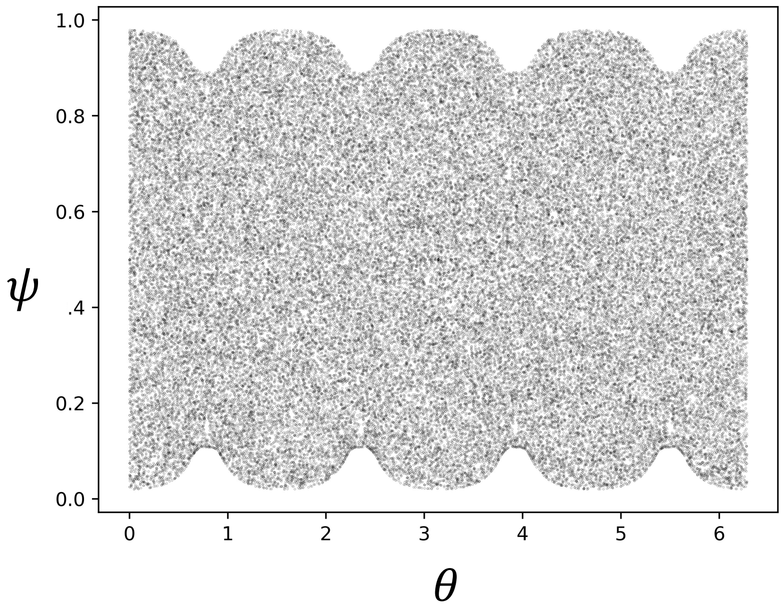
$\epsilon = 0$



$\epsilon = 0.01$



$\epsilon = 0.1$



Ingredients of stellarator confinement

…and many more

- ✓MHD stability
- ✓Collisional "bootstrap" current
- ✓Energetic particle confinement
- ✓Equilibrium β limit
- ✓Divertor configuration
- ✓Reduction of turbulent transport
- ✓Coil feasibility

Ingredients of stellarator confinement

…and many more

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- ✓Reduction of turbulent transport
- ✓**Coil feasibility**

ASG Superconductors

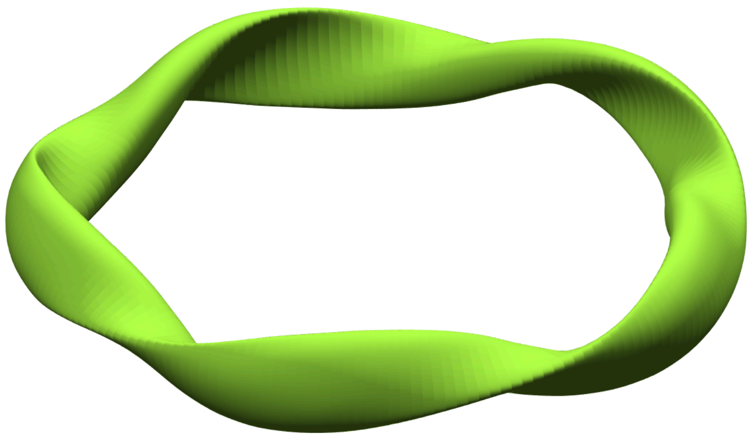
Outline

- Magnetic confinement without symmetry
- Ingredients of stellarator confinement
- **How do we “cook” a stellarator?**
- Stellarators in context

How do we "cook" a stellarator?

MHD equilibrium optimization

Review: [Wright Day 3]



$$(\nabla\times\boldsymbol{B})\times\boldsymbol{B} = \mu_0\nabla p \quad \text{in } V_{\text{plasma}}$$

$$\boldsymbol{\nabla}\cdot\boldsymbol{B} = 0 \quad \text{in } V_{\text{plasma}}$$

$$\boldsymbol{B}\cdot\hat{\boldsymbol{n}} = 0 \quad \text{on } S_{\text{plasma}}$$

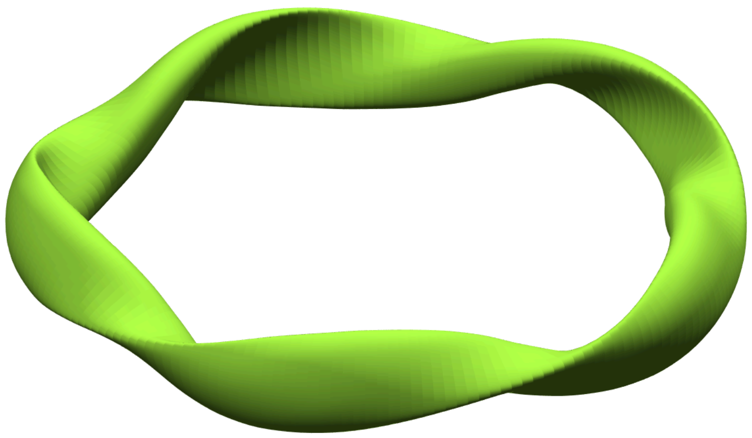
$$\text{Given } p(\psi),\, I_T(\psi)$$

$$\min_{S_{\text{plasma}}} f\left(\boldsymbol{B}\left(S_{\text{plasma}}\right), S_{\text{plasma}}\right)$$

How do we "cook" a stellarator?

MHD equilibrium optimization

Review: [Wright Day 3]



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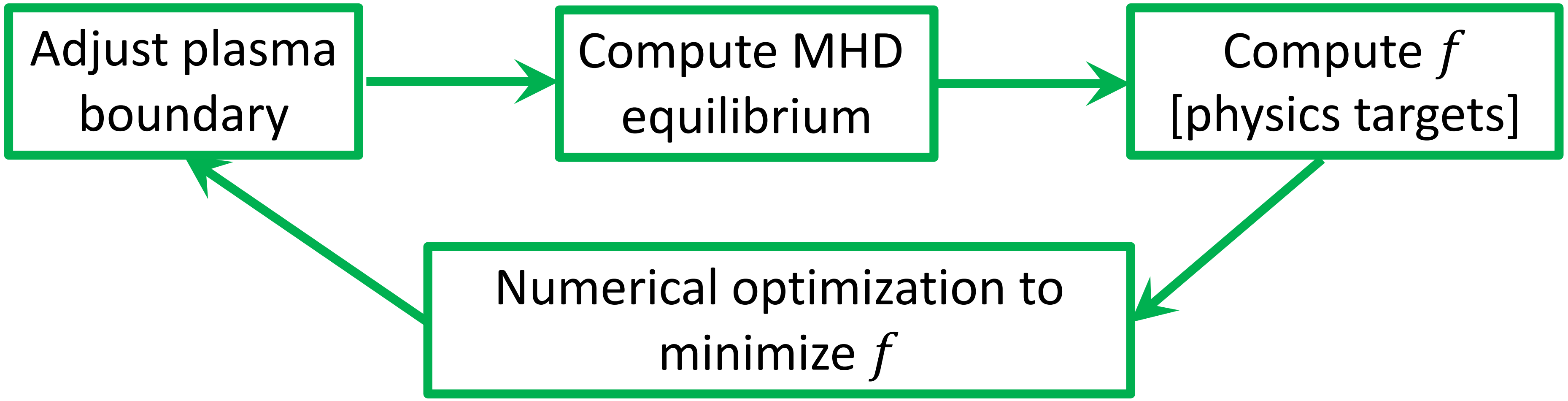
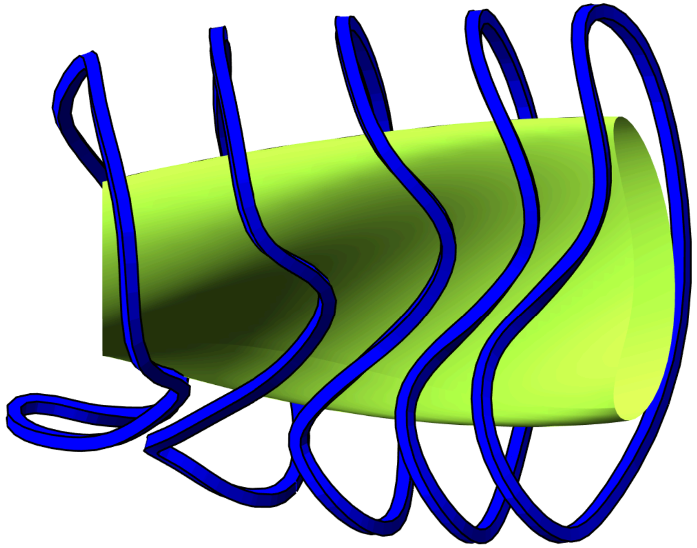


How do we "cook" a stellarator?

Coil optimization

Total normal field to plasma boundary

$$\boldsymbol{B}(\boldsymbol{x})\cdot\widehat{\boldsymbol{n}}(\boldsymbol{x}) = \boldsymbol{B}_P(\boldsymbol{x})\cdot\widehat{\boldsymbol{n}}(\boldsymbol{x}) + \frac{\mu_0}{4\pi}\int_{\mathbb{R}^3\backslash V_{\text{plasma}}} d^3x' \frac{\boldsymbol{J}_C(\boldsymbol{x}')\times(\boldsymbol{x}-\boldsymbol{x}')\cdot\widehat{\boldsymbol{n}}(\boldsymbol{x})}{|\boldsymbol{x}-\boldsymbol{x}'|^3}$$



$$\min_{J_C}\left(\int_{S_{\text{plasma}}} d^2x\,\left(\boldsymbol{B}\cdot\widehat{\boldsymbol{n}}\right)^2 + (\text{coil complexity})\right)$$

Outline

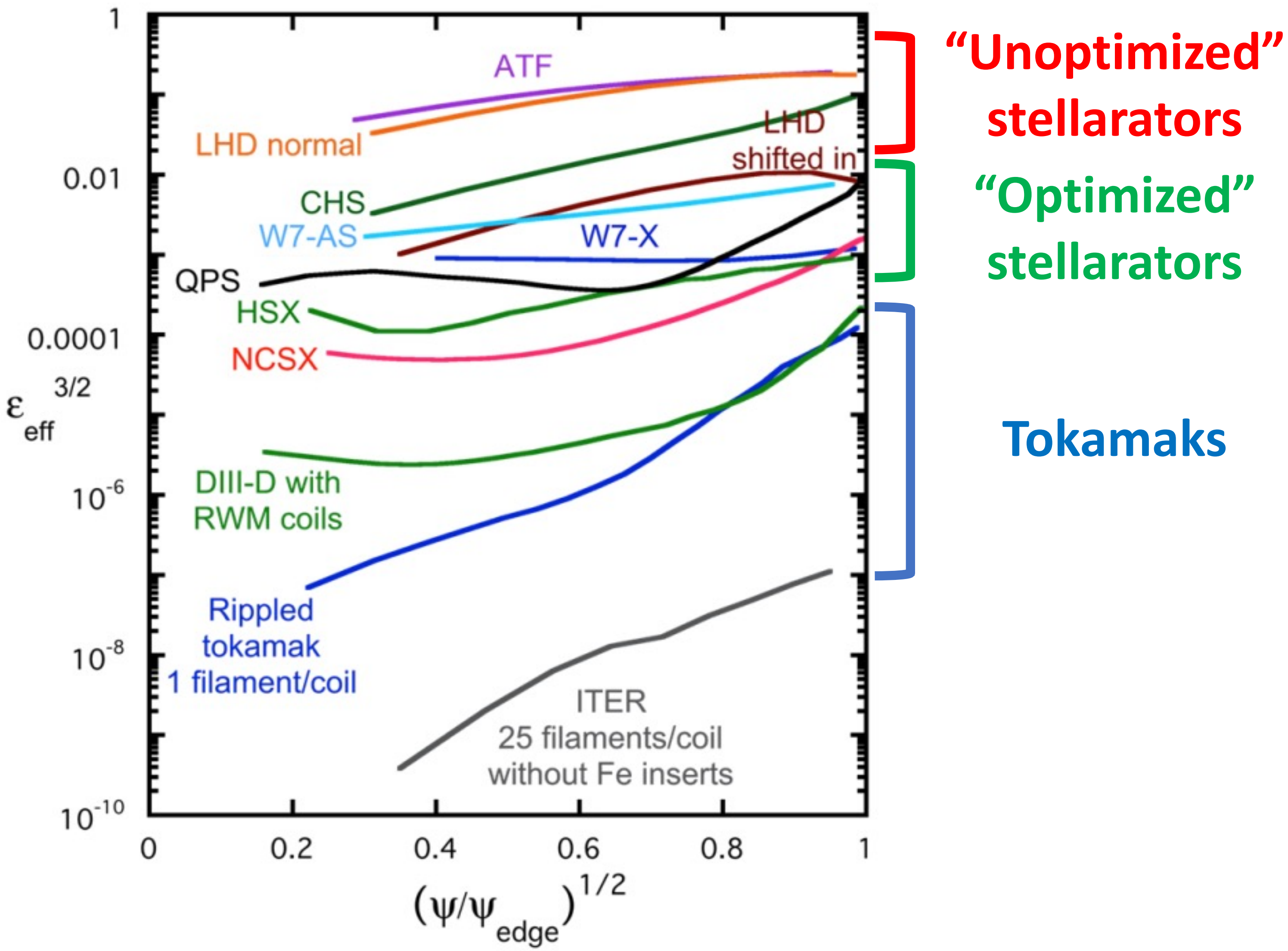
- Magnetic confinement without symmetry
- Ingredients of stellarator confinement
- How do we “cook” a stellarator?
- **Stellarators in context**

Stellarators in context

The result of optimization

ϵ_{eff} = measure of $1/\nu$ collisional transport

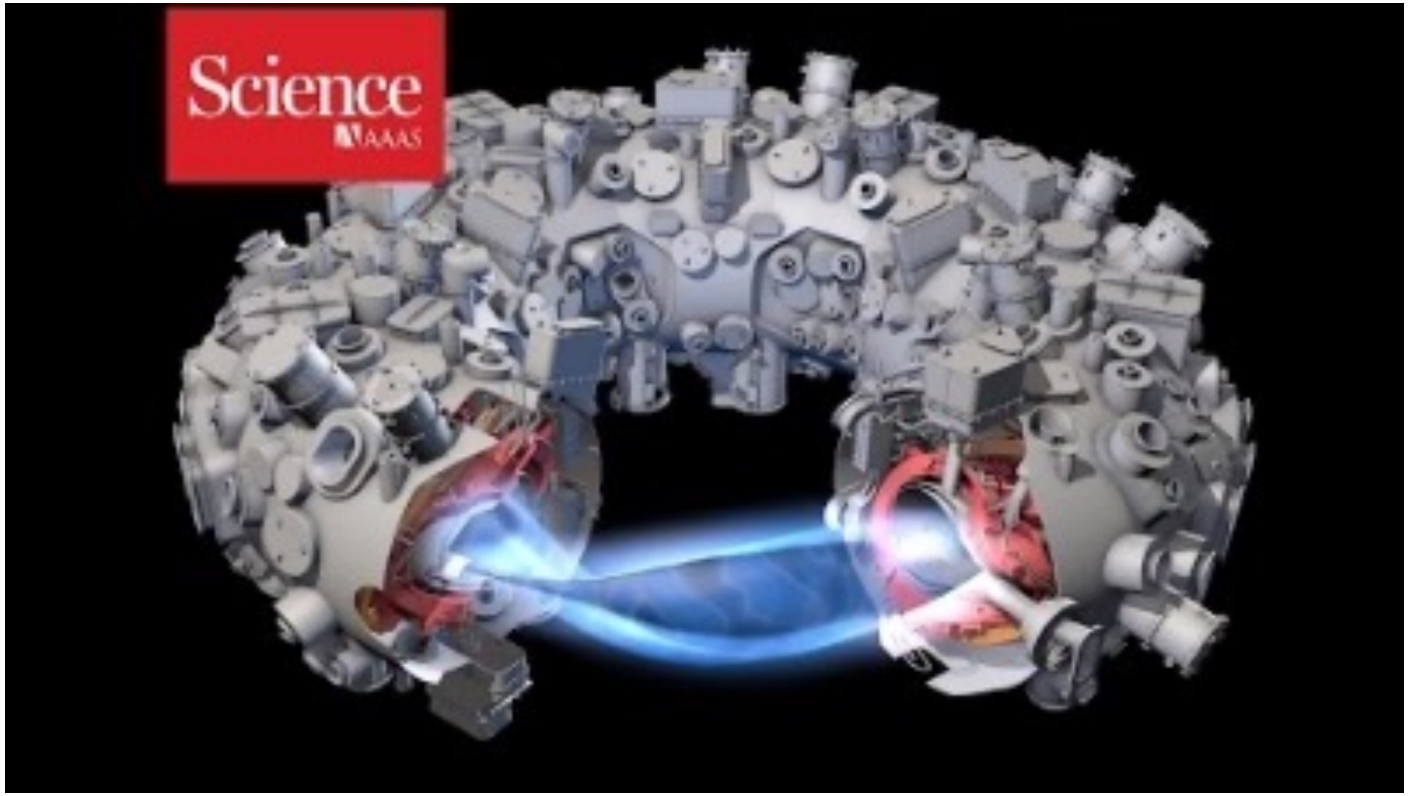
(= 0 in perfect axisymmetry)



D.A. Spong et al, *APS DPP* (2014).

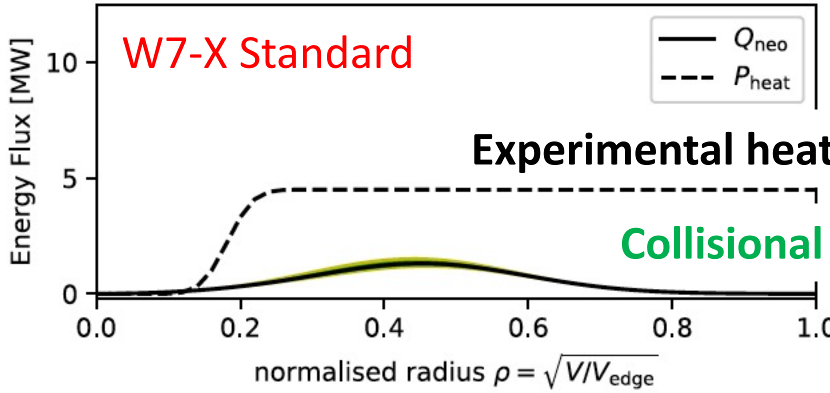
Stellarators in context

The result of optimization – Wendelstein 7-X



Stellarators in context

The result of optimization – Wendelstein 7-X

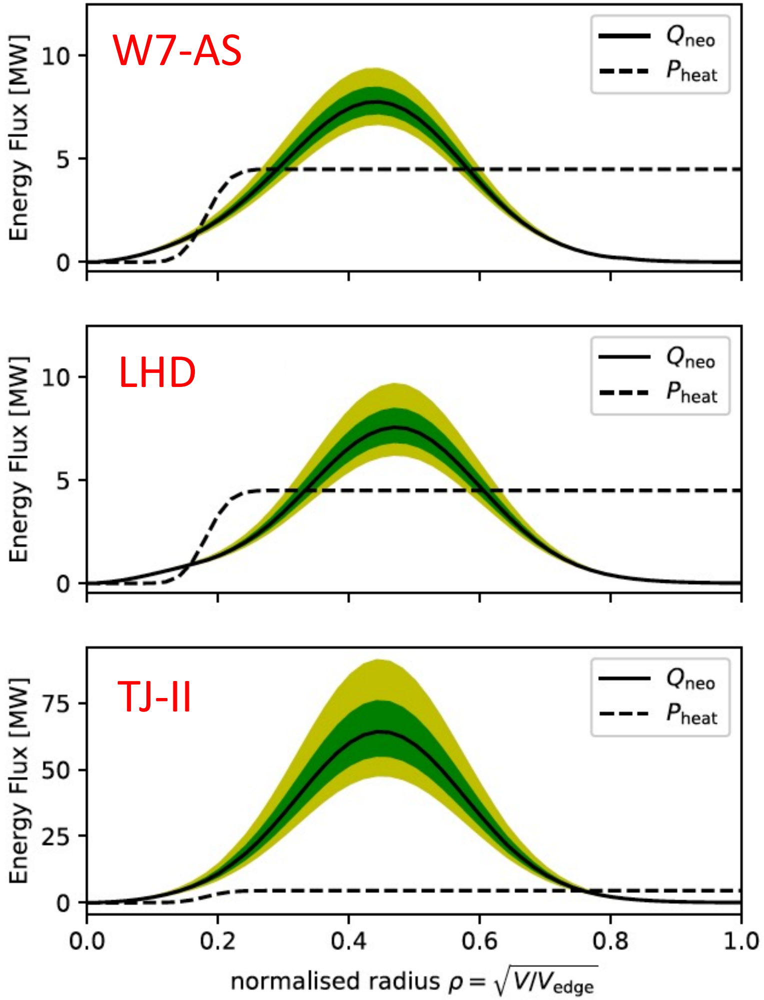


Experimental heat flux

Collisional transport

Without optimization, record-breaking W7-X shot would be impossible

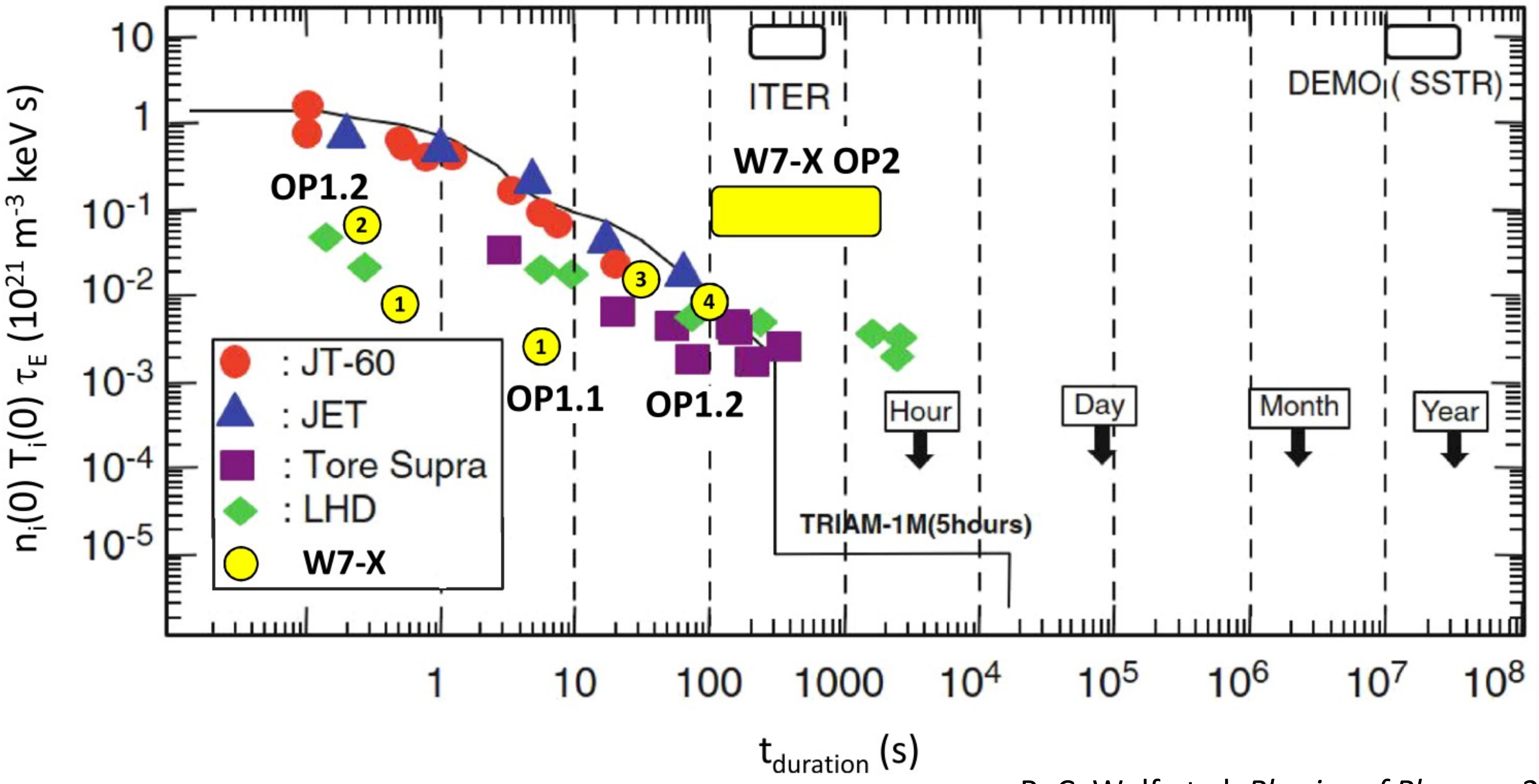
"Not optimized" (scaled to W7-X)



T. Sunn Pedersen et al, *IAEA* (2021).

Stellarators in context

Comparison with tokamak performance



R. C. Wolf et al, *Physics of Plasmas* 26, (2019).

Stellarators in context

Overview of stellarators & tokamaks

Tokamaks

- ✓ Automatic guiding center confinement
- ✓ Automatic integrability
- ✓ Simpler design
- ✗ Requires large plasma current
- ✗ Steady-state is challenging/inefficient

Stellarators

- ✗ Confinement optimization required
- ✗ Shaping by complicated coils/magnets
- ✓ Low recirculating power
- ✓ No plasma-terminating disruptions
- ✓ No Greenwald density limit
- ✓ “softer” pressure/β limit

Plasma Phys. Control. Fusion **54** (2012) 124009 (12pp) doi:10.1088/0741-3335/54/12/124009

Stellarator and tokamak plasmas: a comparison

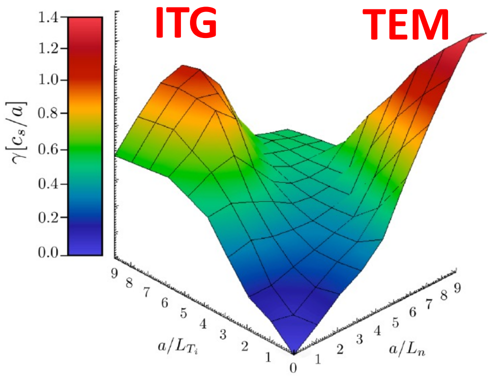
P Helander, C D Beidler, T M Bird, M Drevlak, Y Feng, R Hatzky, F Jenko, R Kleiber, J H E Proll, Yu Turkin and P Xanthopoulos

Max-Planck-Institut für Plasmaphysik, Greifswald and Garching, Germany

Stellarators in context

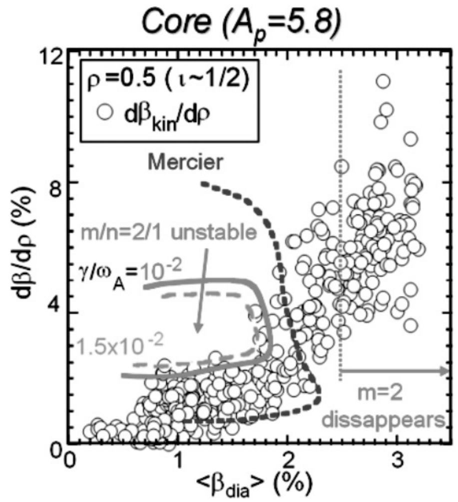
Open questions and new frontiers

Can we optimize stellarators for reduced turbulence?



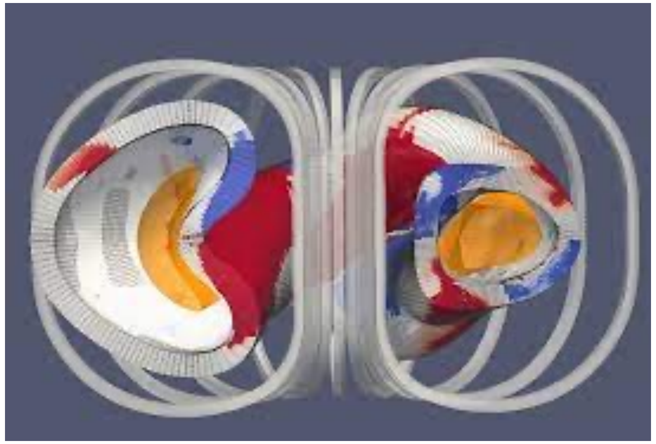
J. Alucon et al, *PPCF* 62 (2020).

Why can stellarators routinely violate linear MHD stability limits?



A. Weller et al, *Fusion Science & Tech.*, 50 (2006).

Can we confine stellarators with permanent magnets?



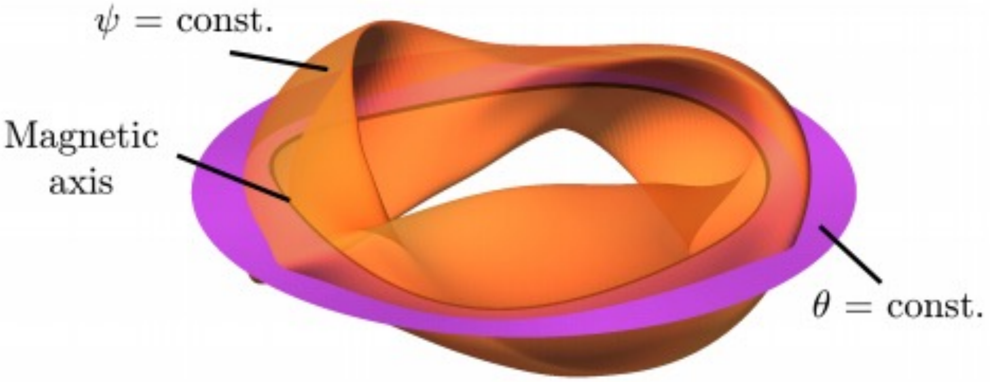
C. Zhu et al, *Nuclear Fusion*, 60 (2020).

An introduction to stellarators: from magnetic fields to symmetries and optimization (arxiv:1908.05360)



1. Introduction
2. Background
3. Electric and magnetic fields: Maxwell's equations
4. Classical mechanics
5. Single particle motion in electromagnetic fields
6. Coordinate systems
7. Toroidal magnetic confinement
8. Coupling of particles and electromagnetic fields: MHD models
9. Magnetic coordinates
10. Challenges associated with 3D equilibrium fields
11. Models of 3D equilibrium magnetic fields
12. Symmetries in stellarators
13. Optimization for stellarator design
14. New frontiers of optimization

	MHD equilibrium (surfaces assumed)	Force-free fields	Vacuum fields
Hyp.	$\boldsymbol{J}\times\boldsymbol{B}\neq 0$ $\nabla p\neq 0$	$\boldsymbol{J}\times\boldsymbol{B}=0$ $\nabla p=0$ and $\lambda=$ const.	$\boldsymbol{J}=0$ $\nabla p=0$
PDE model	$\boldsymbol{J}\times\boldsymbol{B}=\nabla p$ $\nabla\cdot\boldsymbol{B}=0$ $\mu_0\boldsymbol{J}=\nabla\times\boldsymbol{B}$	$\nabla\times\boldsymbol{B}=\lambda\boldsymbol{B}$ $\mu_0\boldsymbol{J}=\nabla\times\boldsymbol{B}$	$\Delta\widetilde{\Phi}_B=0$
Given	$p(\psi)$, $\iota(\psi)$, Ψ_T	λ, Ψ_T	I_P
Unkn.	$\boldsymbol{B}$	$\boldsymbol{B}$	$\widetilde{\Phi}_B$
With	$\boldsymbol{J}$ function of $\boldsymbol{B}$	$\boldsymbol{J}$ function of $\boldsymbol{B}$	$\boldsymbol{B}=\nabla\left(\widetilde{\Phi}_B+(\mu_0 I_P/2\pi)\,\phi\right)$
BC	$\boldsymbol{B}\cdot\hat{\boldsymbol{n}}=0$	$\boldsymbol{B}\cdot\hat{\boldsymbol{n}}=0$	$\hat{\boldsymbol{n}}\cdot\nabla\widetilde{\Phi}_B+(\mu_0 I_P/2\pi)\,\hat{\boldsymbol{n}}\cdot\nabla\phi=\boldsymbol{B}\cdot\hat{\boldsymbol{n}}$



Thank you, SULI!