



# ***Plasma: Born in the Crest of a Wave & Rocked in the Cradle of the Deep....***

**PPPL SULI**

**Introduction to Fusion Energy & Plasma Physics Course**

*Plasma Waves*

**16 JUN 2021**

**CDR Royce W. James, Ph.D.**

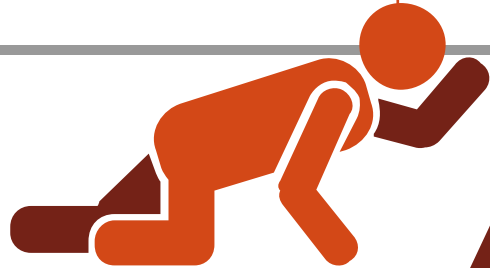
**US Coast Guard Academy**

**Air Force Institute of Technology**



# CHECK-IN ACTIVITY

**It's Not My Day!**  
SCORE: 1 - 4



**Doing Alright!**  
SCORE: 5 - 8



**Best Day Ever!**  
SCORE: 9 - 10



Dessert Poll



**COAST GUARD  
SPECTRUM**

# HERE & NOW...



AIR FORCE INSTITUTE OF TECHNOLOGY

HOME ABOUT AFIT GRADUATE EDUCATION CONTINUING EDUCATION RESEARCH CONSULTING STUDENTS LIBRARY ALUMNI CAREERS  SEARCH : QUICK LINKS



GRADUATE SCHOOL OF ENGINEERING & MANAGEMENT

GRADUATE SCHOOL HOME OUR PEOPLE ABOUT THE GRADUATE SCHOOL DEPARTMENTS PROGRAMS STUDENT RESOURCES FACULTY RESOURCES PUBLICATIONS

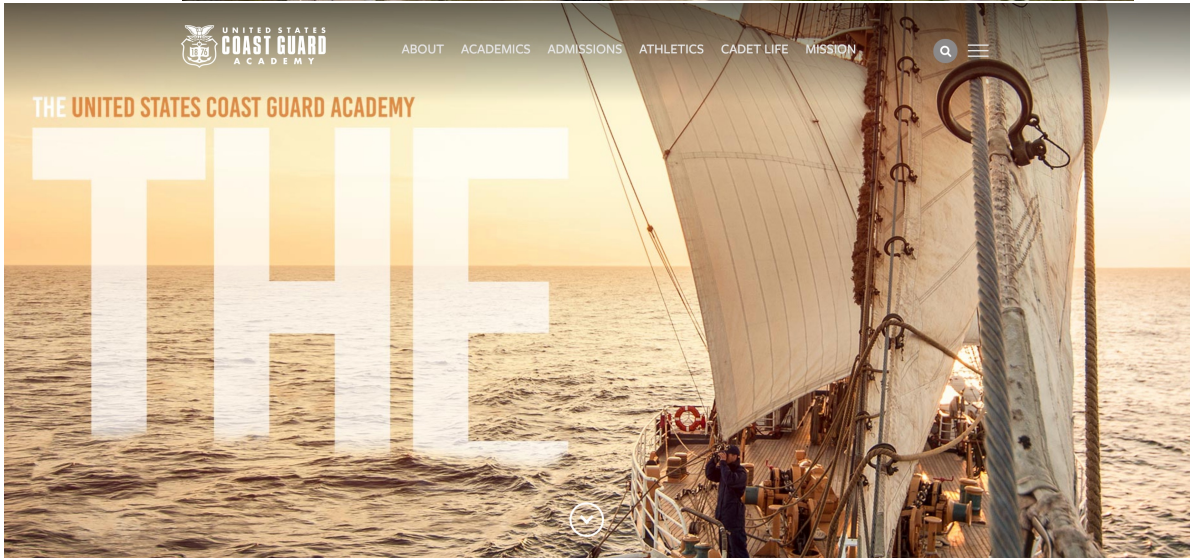
Welcome To The Graduate School Of Engineering & Management



ABOUT ACADEMICS ADMISSIONS ATHLETICS CADET LIFE MISSION

THE UNITED STATES COAST GUARD ACADEMY

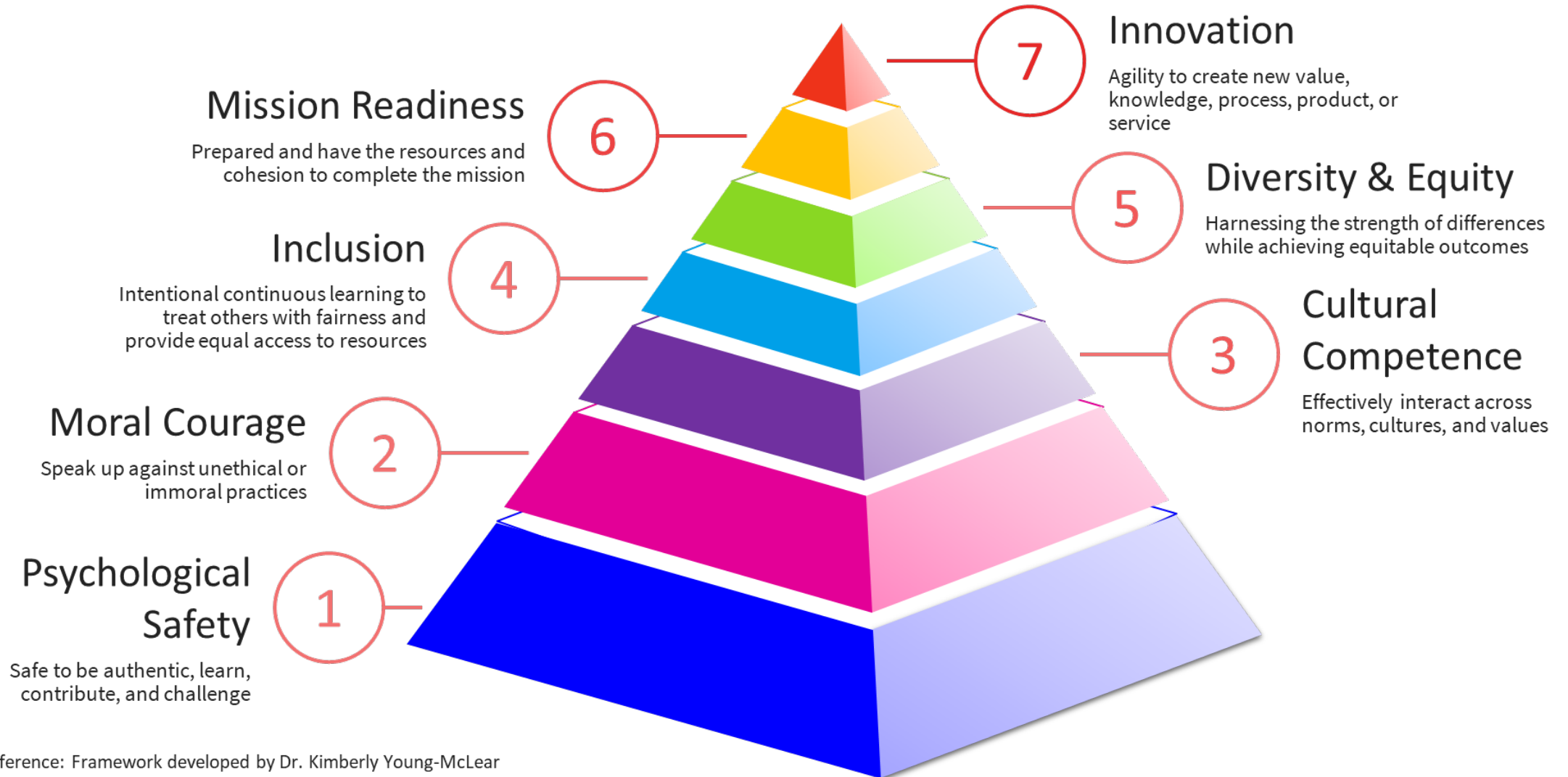
# THE



# 'HEALTHY TO INNOVATIVE' FRAMEWORK

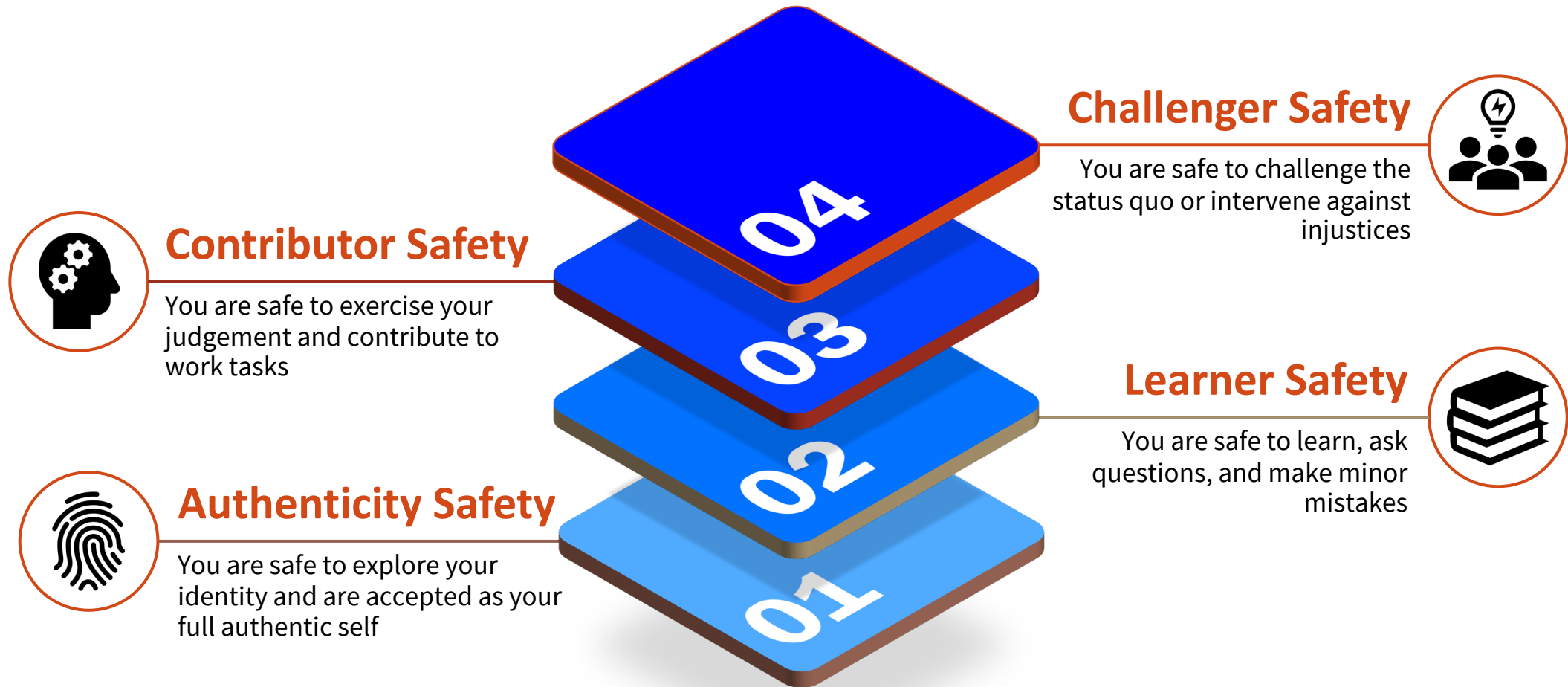


COAST GUARD  
SPECTRUM



Reference: Framework developed by Dr. Kimberly Young-McLear  
Adopted by CG Spectrum, USCGA ASEE Dean's Diversity Initiative and others

# FOUR STAGES OF PSYCHOLOGICAL SAFETY





# FLORIDA STATE UNIVERSITY



PSYCHOLOGICAL SAFETY

01  
LEVEL



01  
LEVEL

PSYCHOLOGICAL SAFETY





# Welcome to the Department of Physics

01  
LEVEL  
PSYCHOLOGICAL SAFETY



The Tokamak at Columbia University  
Located in New York City

 **COLUMBIA | ENGINEERING**  
The Fu Foundation School of Engineering and Applied Science



**STEVENS**  
INSTITUTE *of* TECHNOLOGY  
THE INNOVATION UNIVERSITY®





01  
LEVEL

PSYCHOLOGICAL SAFETY





The lasting legacy of the Greensboro Four (above from left: David Richmond, Franklin McCain, Jibreel Khazan and Joseph McNeil) was how the courageous moment grew to a revolutionary movement. ([Wikimedia Commons](#))



MORAL COURAGE

02

LEVEL

# THE MOMENT WHEN FOUR STUDENTS SAT DOWN TO TAKE A STAND

One of the great monuments to the Greensboro Sit-In is at the Smithsonian's National Museum of American History



03

LEVEL

# CULTURAL COMPETENCE

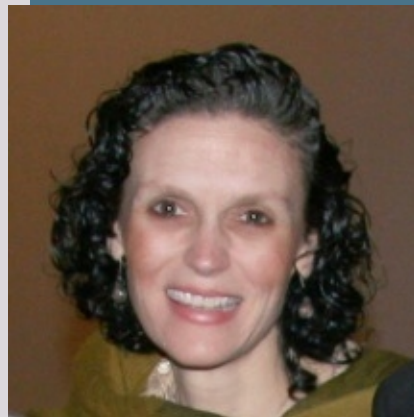


# Climate & Inclusion Town Hall, DPP 2020

This year the DPP Diversity, Equity, and Inclusion Organizing Collective Committee (OCC) is offering a workshop on evidence-based practices (EBP) that can help make the plasma science community more diverse, equitable, and inclusive. The workshop includes

- Introduction to the OCC
- Discussion of EBPs by Dr. Christine Clark
- Professionally facilitated EBP discussion sessions  
**\*\*with Registration only\*\***
- Take-home EBP toolkit

**Dr. Christine Clark**  
University of Nevada, Las Vegas  
Senior Scholar in Multicultural Education  
Founding Vice Pres. for Diversity and Inclusion



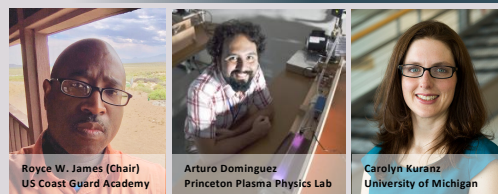
<https://www.unlv.edu/news/expert/christine-clark>

Register at:

<https://rb.gy/7matra>

Discussion slots are limited, so please register soon!

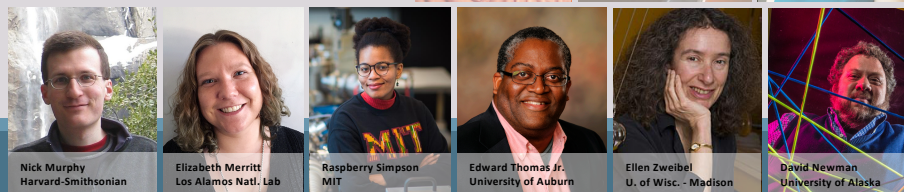
## Members of the DEI OCC



Royce W. James (Chair)  
US Coast Guard Academy

Arturo Dominguez  
Princeton Plasma Physics Lab

Carolyn Kuranz  
University of Michigan



Nick Murphy  
Harvard-Smithsonian

Elizabeth Merritt  
Los Alamos Natl. Lab

Raspberry Simpson  
MIT

Edward Thomas Jr.  
University of Auburn

Ellen Zweibel  
U. of Wisc. - Madison

David Newman  
University of Alaska

*Evidence Based Practices (EVPs)*

Section 1 of 2

## APS DPP DEI Town Hall Registration Form

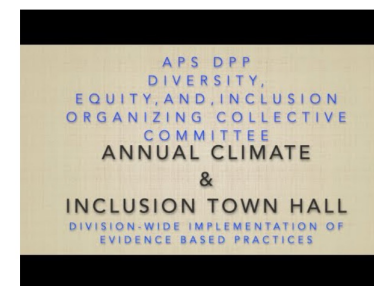
Please review the following video of our DEI Expert, Prof. Chris Clark, Ed.D. on Evidence-Based Practices (EBPs) below.

You may also want to follow Prof. Clark's presentation with this handout on EBPs:  
<https://bit.ly/2TQfRDQ>

The OCC with Prof. Clark has settled on the following 5 EBPs for the division to focus on.

- 1) addition of non-cognitive variables in graduate student admissions and in hiring industry and national laboratories personnel and faculty;
- 2) adoption of the Rooney Rule in hiring industry and national laboratories personnel and faculty;
- 3) establishment of a human relations code;
- 4) establishment of a bias incident policy; and,
- 5) establishment of a 'looping' mentorship program.

OCC Update & our DEI Expert, Prof. Chris Clark, Ed.D. on EBPs [30 min]



In the space provided, please submit a brief strategy of where and how you might approach implementing one of the above EBPs in your institution. The small group discussions at our Town Hall will be centered on this very premise to facilitate effective actions in our individual and collective work spaces. Participants will also receive a EBP Toolkit to refer to when implementing EBPs.

Long answer text



05  
LEVEL

DIVERSITY AND EQUITY

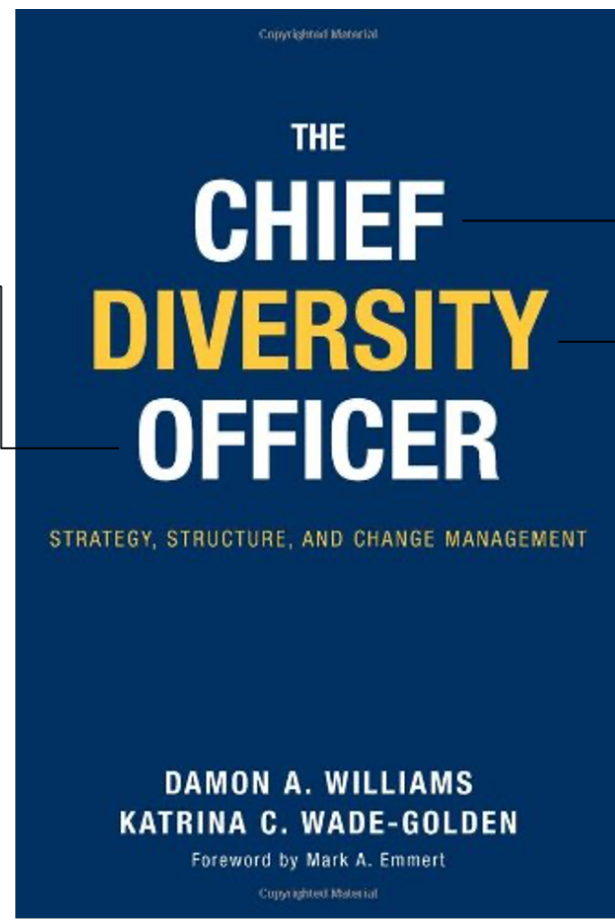


CGAPL Group - DEPS February 2017  
CGAPL Group - PPL March 2018



05  
LEVEL

DIVERSITY AND EQUITY



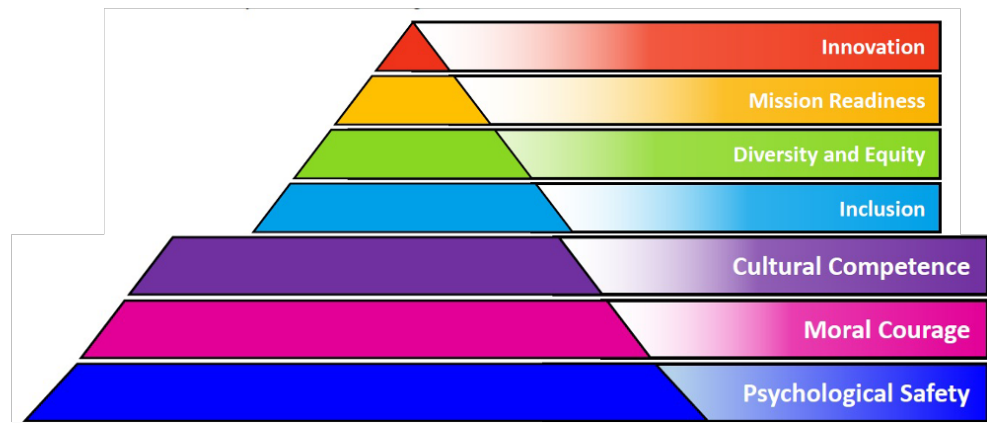
Cultural Competence

Psychologically Safety

Moral Courage

# TOTAL FORCE AIRMEN & SPACE WARFIGHTERS

Are Joint Force Social Justice  
Warriors.



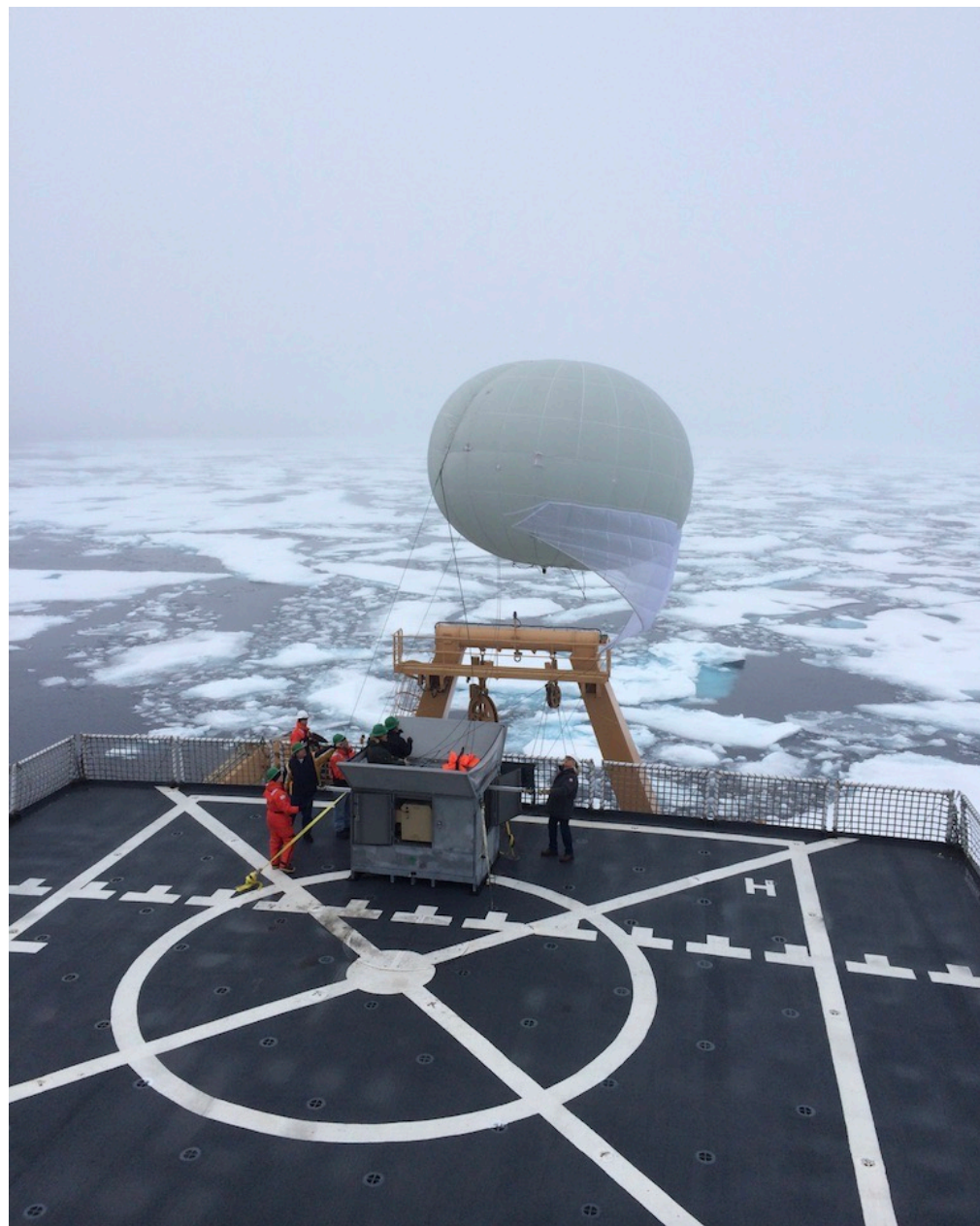
*Developed by Dr. Kimberly Young-McLear, U.S. Coast Guard Academy*

Building A Healthy and Innovative Workforce



06  
LEVEL

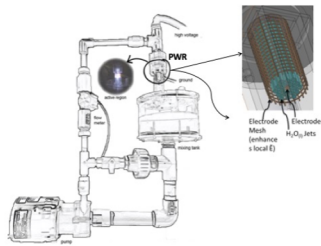
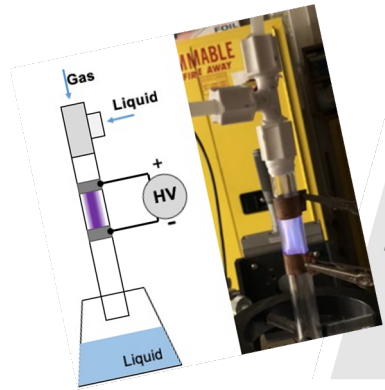
# MISSION READINESS



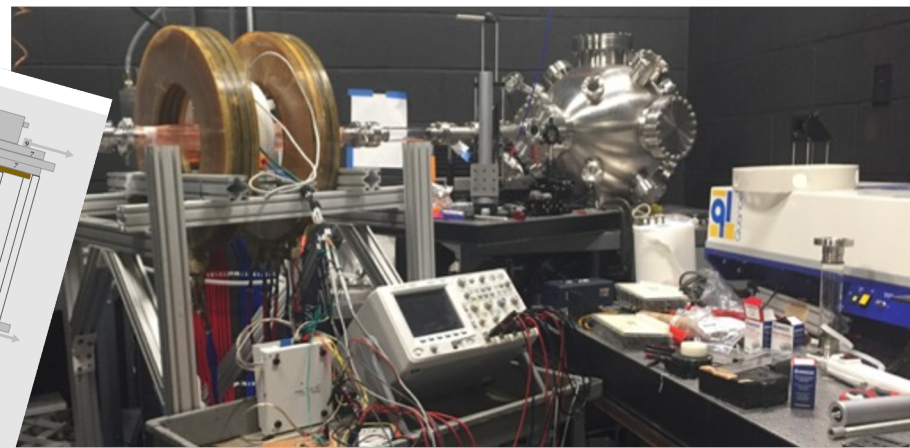
# Coast Guard Academy Plasma Lab has Multiple Experiments

LEVEL 07

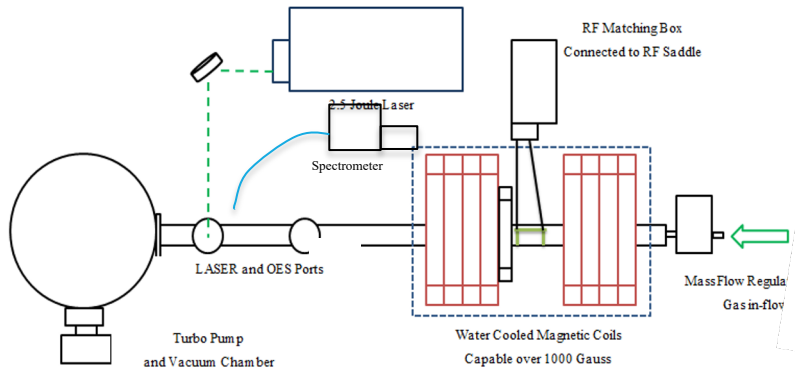
INNOVATION



Plasma Water Treatment

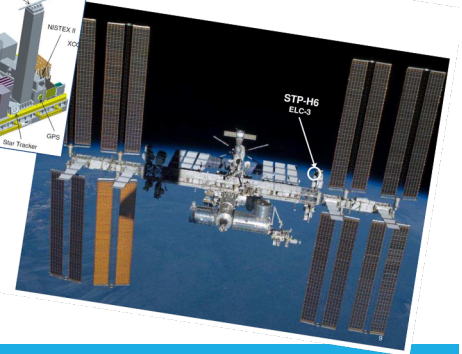
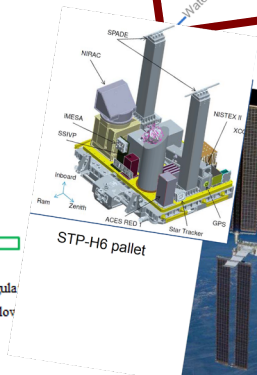
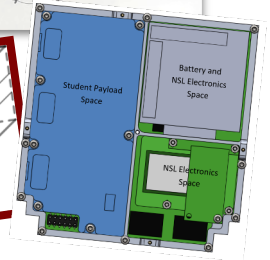
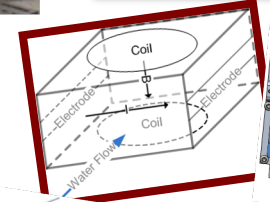


HPX



MHD Space

$$\rho \frac{d\vec{u}}{dt} = \frac{m}{V} \vec{u} = \frac{\vec{F}_A}{V} \hat{k} = -(\nabla p) + \vec{j} \times \vec{B}$$





# ON TO PLASMA WAVES: HERE ARE SOME BIG ROCKS

## Plasma Prep Tips:

- get proximate with the material
- pre-load before class/lecture (more than just notes)

### 1) Pre-Load:

- A day or two before class, read the sections for the next class and write out *in your own words* any unfamiliar definitions for symbols/vocabulary/concepts.
- Do all the examples problems in those sections (*also before class*).
- Rinse, wash, repeat....do this before every class (in all subjects) at the minimum to be ready. This base understanding is needed to get anything out of class. Don't waste your time!! Be disciplined, it will pay off!

2) Work is Class – it's your job: in class you will need to develop your physics “kata” – your profs set the level; it is for you to become proficient at that level! To do this you will need to *unabashedly* provide ideas and challenge each other's. In this you will learn the physics and become proficient in critical thinking & self-assessment.

3) Use your HW: these are your guide to tests and assessments. One of the largest stumbling blocks is overconfidence while reviewing concept/solution. Be sure to work problems without the solution in preparation.

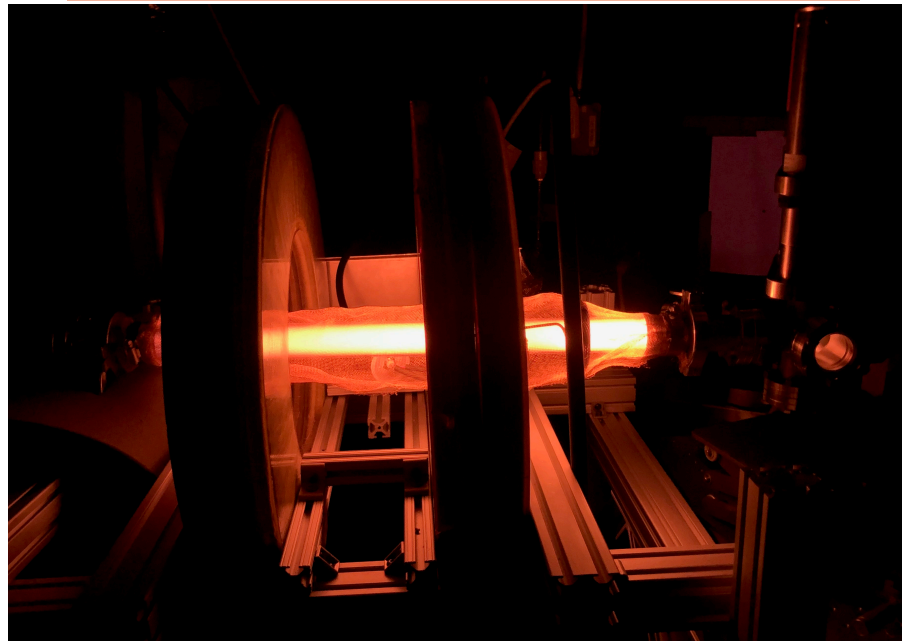


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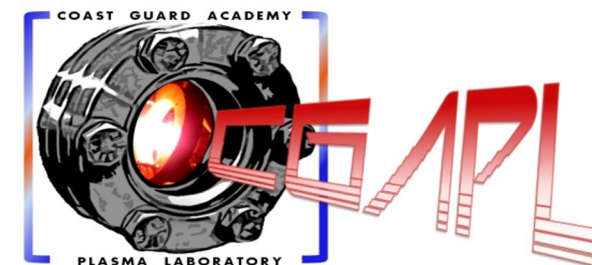
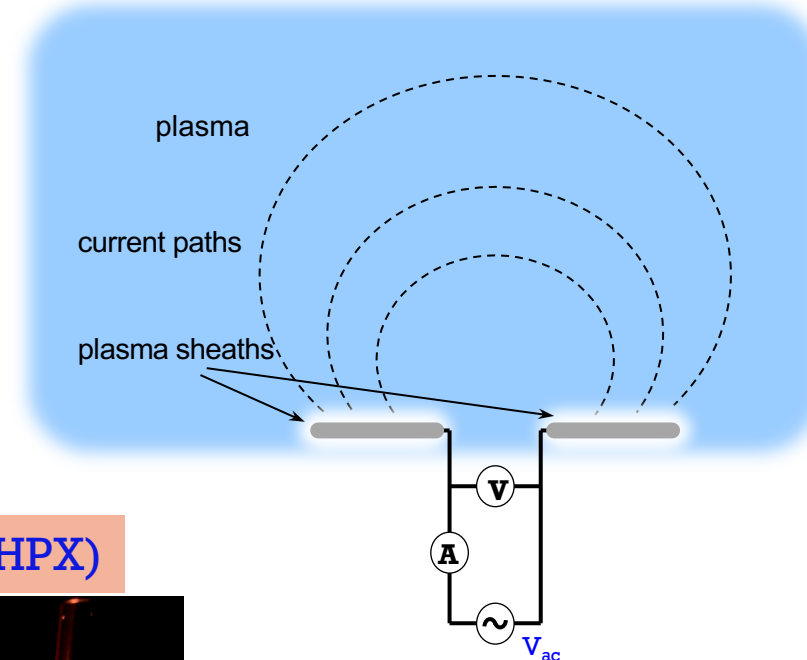
- Plasmas Perpetrating
- Plasma Oscillating
- Electron & Ion Plasma Waves
- Electrostatic Plasma Waves
- Plasma Sound Wave - Whistlers
- Ordinary & Extraordinary
- Alfvén Wave

\*FC=Chen, F.F, (2015). Introduction to Plasma Physics and Controlled Fusion (3rd ed.). Springer

Helicon Plasma Experiment (HPX)



CubeSat Impedance Probe



# PLASMAS CAN PERPETRATE AS WAVES

Periodic motion of a plasma fluid can be represented as a superposition of sinusoidal oscillations - Thanks Fourier!

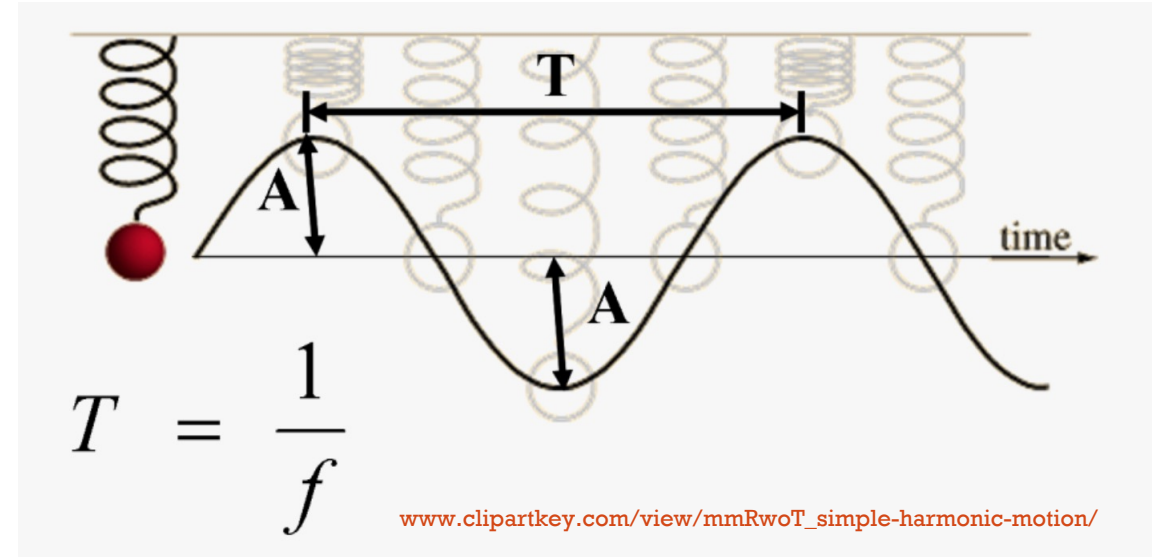
In a plasma fluid, the density ( $n$ ) can be represented as

$$n = \bar{n} \exp[i(\mathbf{k} \cdot \mathbf{r} - \omega t)]$$

where:

$\bar{n}$  - amplitude

$\mathbf{k}$  - propagation const



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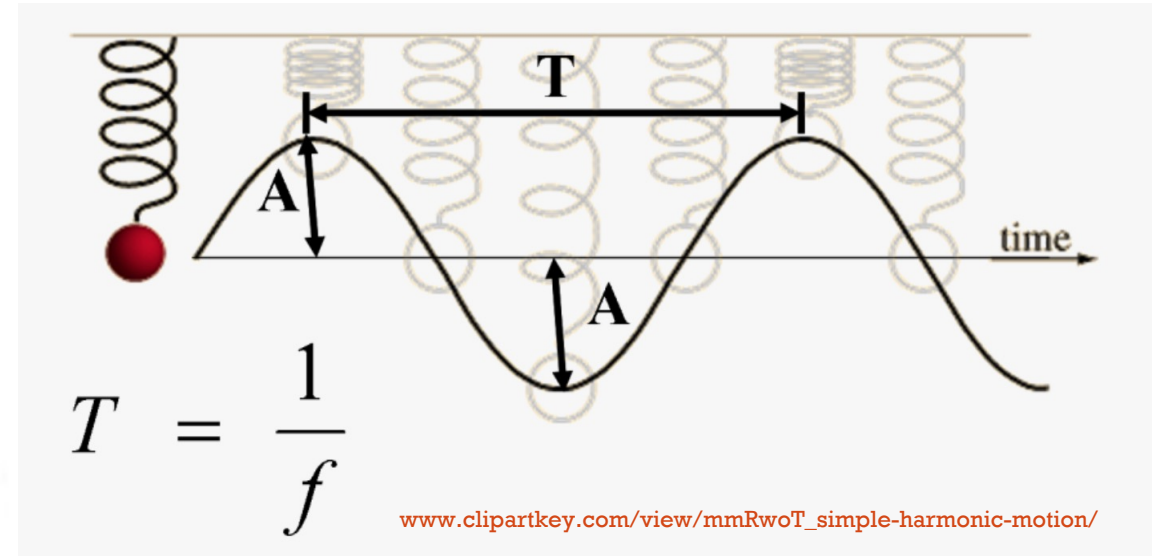
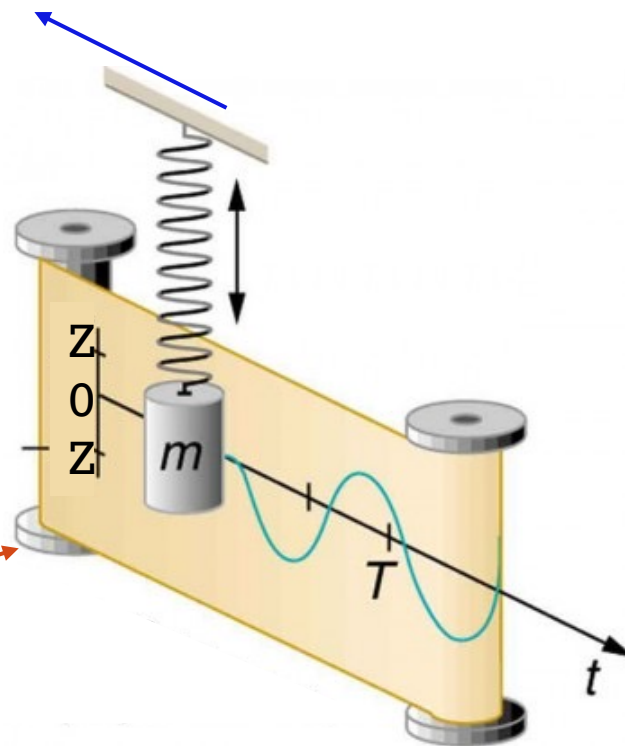
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Let's choose the x-direction for wave propagation to get

$$n = \bar{n} e^{i(kx - \omega t)}$$



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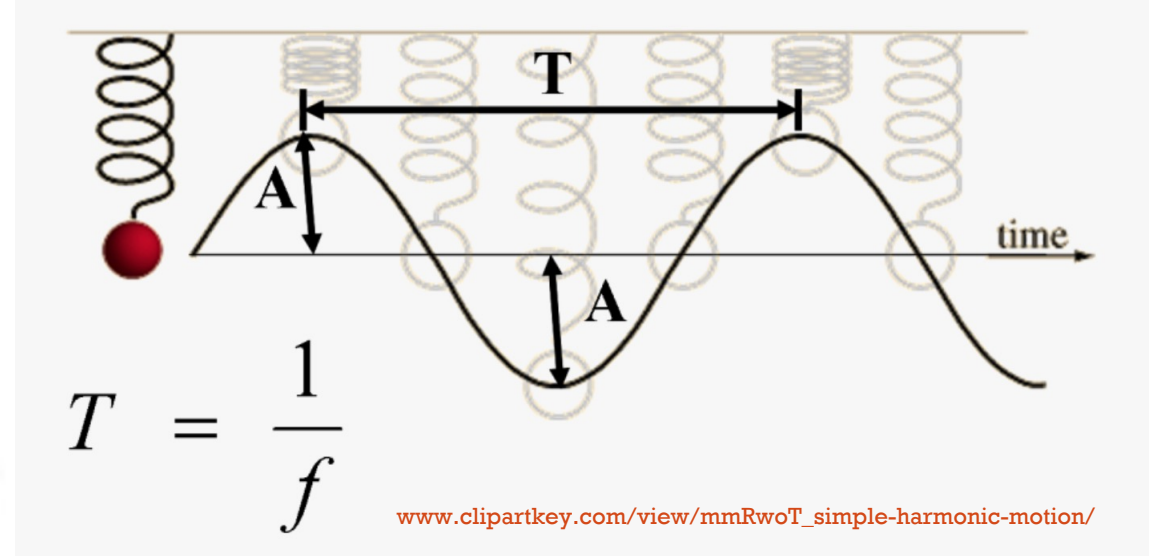
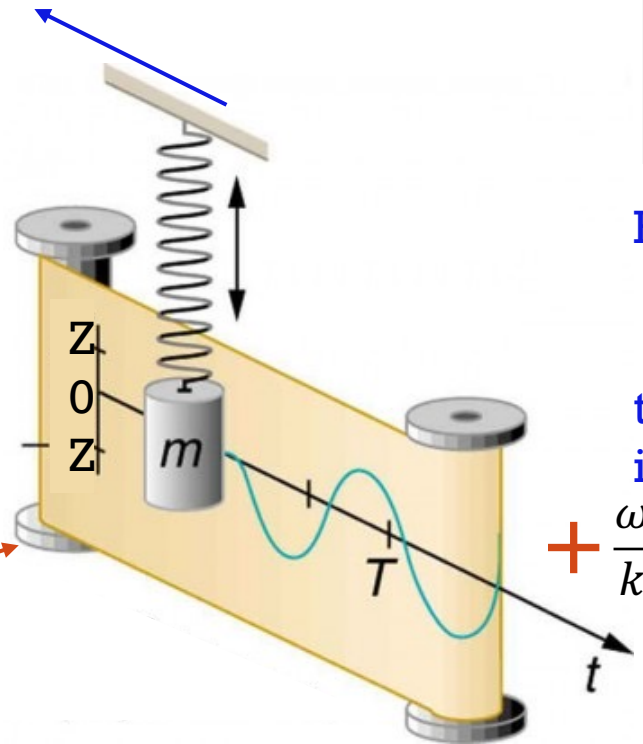
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But these are real plasmas, so

$$\text{Re}(n) = \bar{n} \cos(kx - \omega t)$$

time derivative of a point on the wave,  $(d/dt)(kx - \omega t) = 0$ , is the phase velocity

$$\frac{dx}{dt} = \frac{\omega}{k} \equiv v_{\phi}$$



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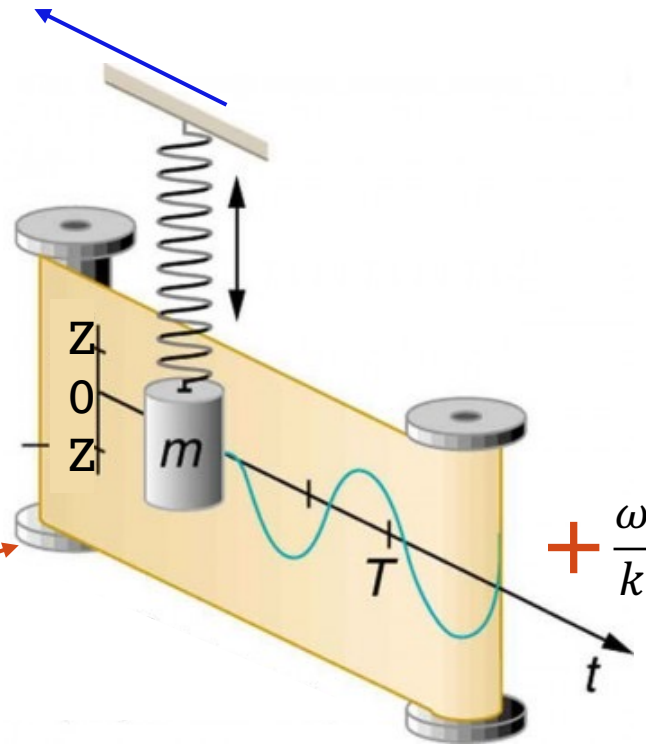
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Chose  $n$  to have no phase, but the plasma has an E-field one in the complex vector form

$$\mathbf{E} = \bar{\mathbf{E}} e^{\delta} e^{i(kx - \omega t)} \equiv \bar{\mathbf{E}}_c e^{i(kx - \omega t)}$$



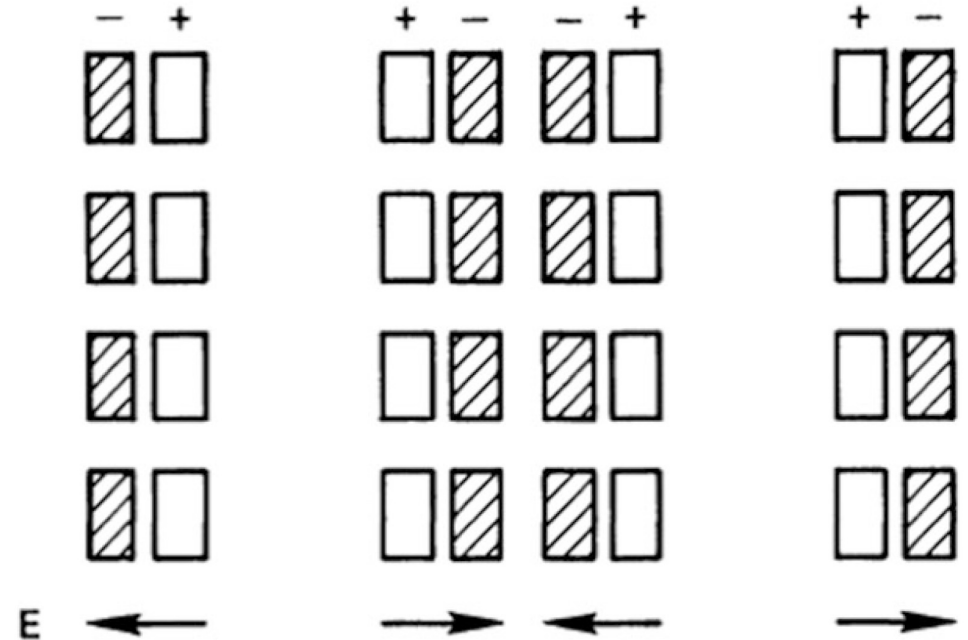
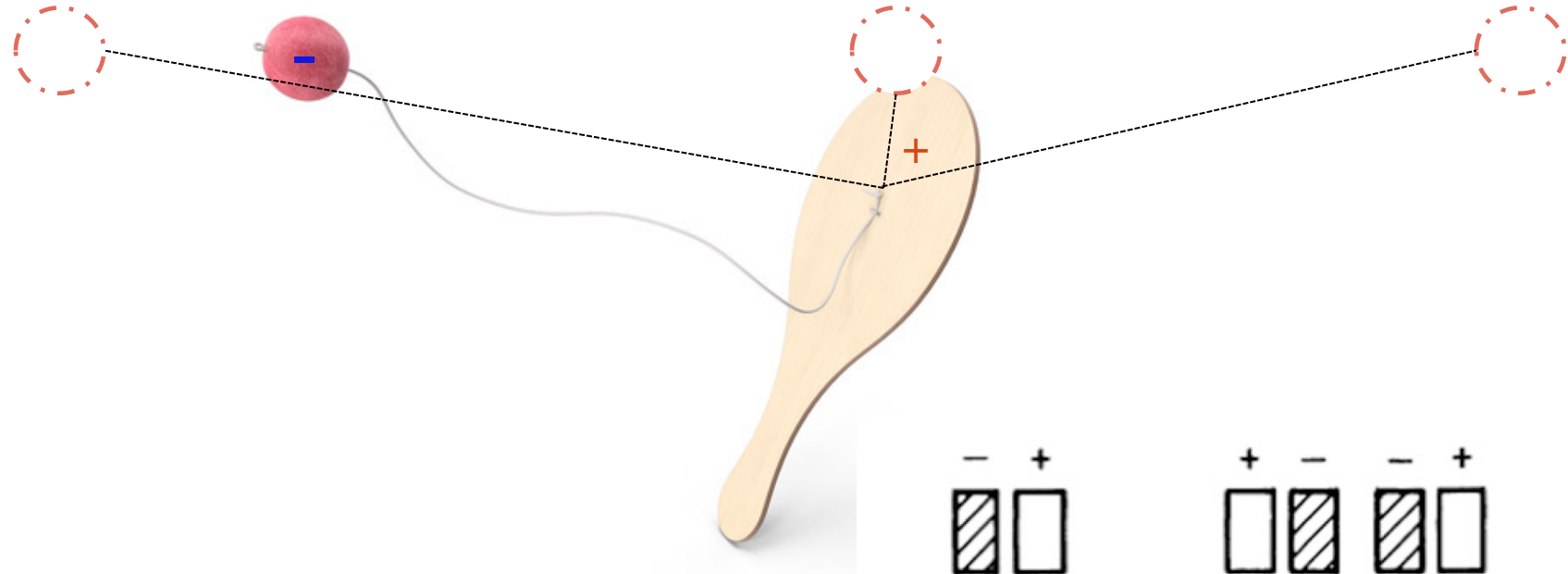
$$\frac{\Delta\omega}{\Delta k} \lim_{\Delta\omega \rightarrow 0} \rightarrow \frac{d\omega}{dk} = v_g \text{ cannot exceed } c!$$



# COULOMB FORCE DRIVES PLASMA OSCILLATION

Linearize to eliminate the high order terms

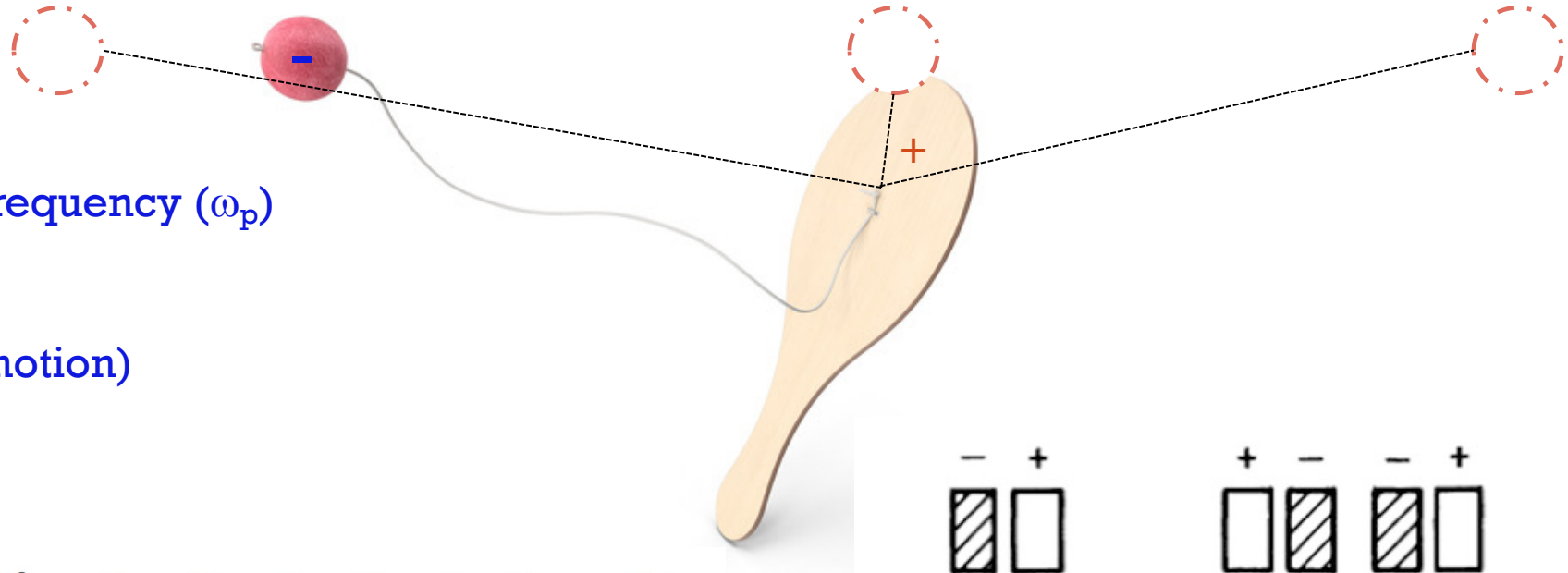
Electrons displaced and bound by coulomb force - overshoot and oscillate



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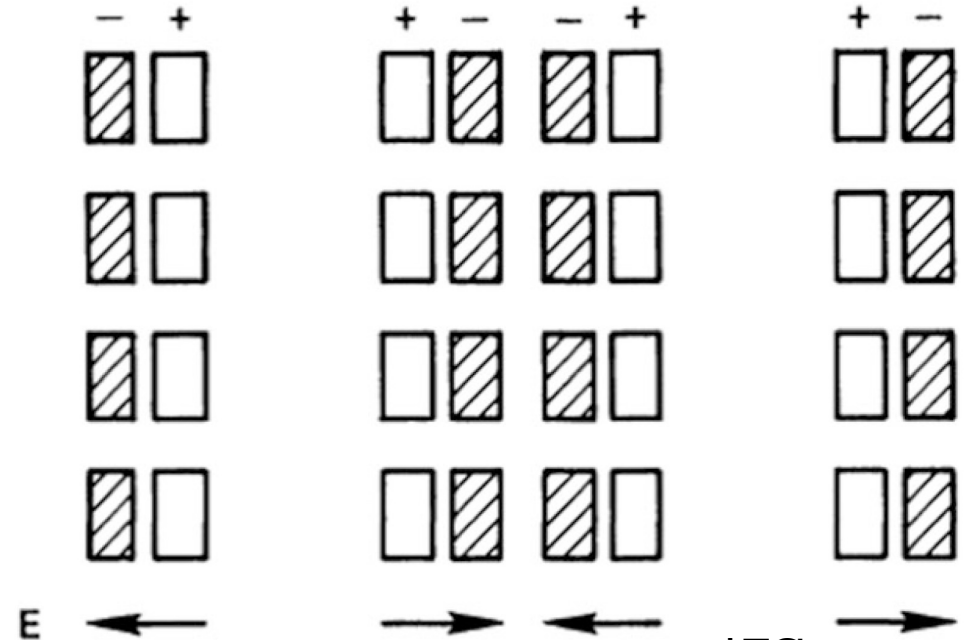
Electrons displaced and bound by coulomb force - overshoot and oscillate



## Simple Case Plasma Frequency ( $\omega_p$ )

1. No B-field
2.  $KT=0$  (no thermal motion)
3. Ions are unmoved
4. Infinite Plasma
5. 1-D, x-direction

$$\nabla = \hat{x} \partial / \partial x \quad \mathbf{E} = E \hat{x} \quad \nabla \times \mathbf{E} = 0 \quad \mathbf{E} = -\nabla \phi$$



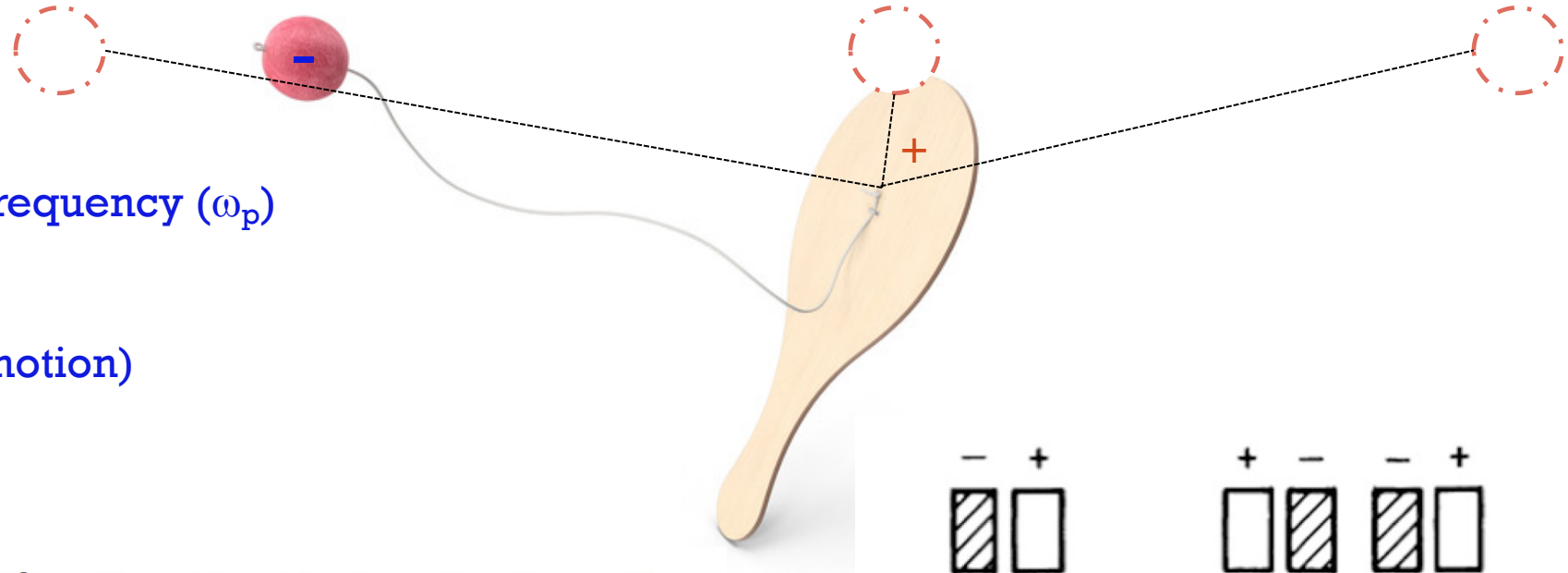
\*FC



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Equation of Motion:

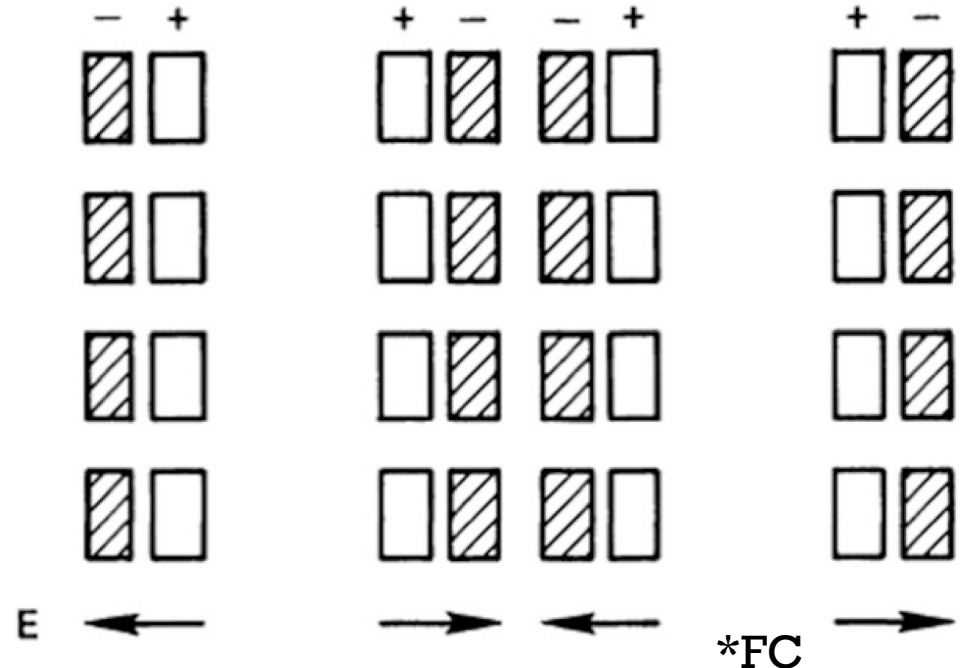
$$mn_e \left[ \frac{\partial \mathbf{v}_e}{\partial t} + (\mathbf{v}_e \cdot \nabla) \mathbf{v}_e \right] = -en_e \mathbf{E}$$

Continuity Equation:

$$\frac{\partial n_e}{\partial t} + \nabla \cdot (n_e \mathbf{v}_e) = 0$$

High frequency - use Poisson's:

$$\epsilon_0 \nabla \cdot \mathbf{E} = \epsilon_0 \partial \mathbf{E} / \partial x = e(n_i - n_e)$$



# LINEARIZE TO ELIMINATE HIGH ORDER TERMS...

Group dependent variables into equilibrium (sub<sub>0</sub>) and perturbed (sub<sub>1</sub>)

$$n_e = n_0 + n_1 \quad \mathbf{v}_e = \mathbf{v}_0 + \mathbf{v}_1 \quad \mathbf{E} = \mathbf{E}_0 + \mathbf{E}_1$$

Still in our simple case so

$$\nabla n_0 = \mathbf{v}_0 = \mathbf{E}_0 = 0$$

1. no oscillation yet
2. Electrons not displaced yet

$$\frac{\partial n_0}{\partial t} = \frac{\partial \mathbf{v}_0}{\partial t} = \frac{\partial \mathbf{E}_0}{\partial t} = 0$$

Plasma Frequency



$$\omega_p = \left( \frac{n_0 e^2}{\epsilon_0 m} \right)^{1/2} \text{ rad/sec}$$

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1. no oscillation yet
2. Electrons not displaced yet

$$\frac{\partial n_0}{\partial t} = \frac{\partial \mathbf{v}_0}{\partial t} = \frac{\partial \mathbf{E}_0}{\partial t} = 0$$

Linearized Equation of Motion:

$$m \left[ \frac{\partial \mathbf{v}_1}{\partial t} + (\mathbf{v}_1 \cdot \overset{0}{\nabla}) \mathbf{v}_1 \right] = -e \mathbf{E}_1$$

Linearized Continuity Equation:

$$\frac{\partial n_1}{\partial t} + \nabla \cdot (n_0 \mathbf{v}_1 + n_1 \overset{0}{\nabla}) = 0$$

$$\frac{\partial n_1}{\partial t} + n_0 \nabla \cdot \mathbf{v}_1 + \mathbf{v}_1 \cdot \overset{0}{\nabla} n_0 = 0$$

Linearized Poisson's:

$$\epsilon_0 \nabla \cdot \mathbf{E}_1 = -en_1$$

Plasma Frequency

→  $\omega_p = \left( \frac{n_0 e^2}{\epsilon_0 m} \right)^{1/2} \text{ rad/sec}$

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Oscillate Sinusoidally so:

$$\mathbf{v}_1 = v_1 e^{i(kx - \omega t)} \hat{\mathbf{x}}$$

$$n_1 = n_1 e^{i(kx - \omega t)}$$

$$\mathbf{E} = E_1 e^{i(kx - \omega t)} \hat{\mathbf{x}}$$

Linearized Equation of Motion:  $m \left[ \frac{\partial \mathbf{v}_1}{\partial t} + (\mathbf{v}_1 \cdot \overset{0}{\nabla}) \mathbf{v}_1 \right] = -e \mathbf{E}_1$

Linearized Continuity Equation:  $\frac{\partial n_1}{\partial t} + \nabla \cdot (n_0 \mathbf{v}_1 + n_1 \overset{0}{\nabla}) = 0$

$$\frac{\partial n_1}{\partial t} + n_0 \nabla \cdot \mathbf{v}_1 + \mathbf{v}_1 \cdot \overset{0}{\nabla} n_0 = 0$$

Linearized Poisson's:

$$\epsilon_0 \nabla \cdot \mathbf{E}_1 = -en_1$$

Substitute the time derivative and gradient:

$$-im\omega v_1 = -eE_1$$

$$\frac{\partial}{\partial t} = -i\omega$$

$$-i\omega n_1 = -n_0 ikv_1$$

$$\nabla = ik\hat{\mathbf{x}}$$

$$ik\epsilon_0 E_1 = -en_1$$

$$-im\omega v_1 = -e \frac{-e}{ik\epsilon_0} \frac{-n_0 ikv_1}{-i\omega} = -i \frac{n_0 e^2}{\epsilon_0 \omega} v_1$$

Plasma Frequency  $\longrightarrow$

$$\omega_p = \left( \frac{n_0 e^2}{\epsilon_0 m} \right)^{1/2} \text{ rad/sec}$$

# WAVES - MOVING PLASMA OSCILLATIONS

## FOR BOTH ELECTRONS & IONS

Oscillation Poll!

- Electron Waves
- Ion Waves
  - Plasma Approximation



Fig. 4.3 Synthesis of a wave from an assembly of independent oscillators

\*FC

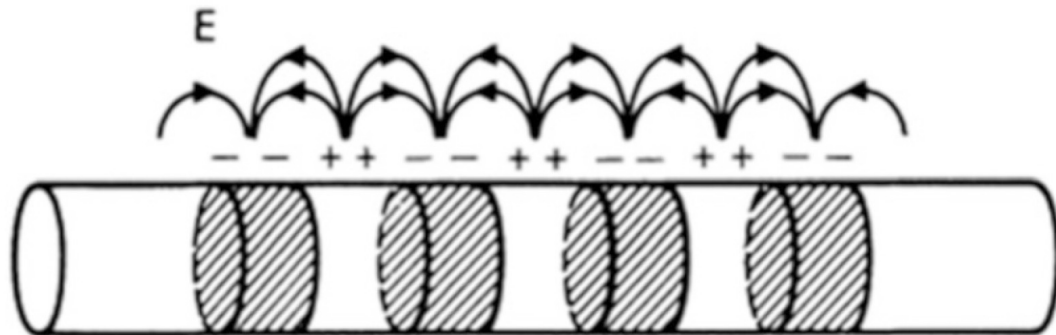


Fig. 4.4 Plasma oscillations propagate in a finite medium because of fringing fields



# WAVES - MOVING PLASMA OSCILLATIONS

## FOR BOTH ELECTRONS & IONS

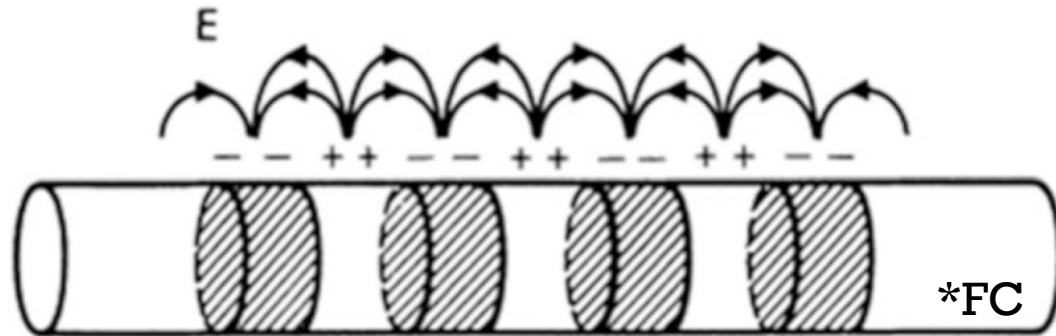


Fig. 4.4 Plasma oscillations propagate in a finite medium because of fringing fields

With thermal motion information is connected across regions - now a wave

$$\nabla p_e = 3KT_e \nabla n_e = 3KT_e \nabla(n_0 + n_1) = 3KT_e \frac{\partial n_1}{\partial x} \hat{x}$$

Linearized Equation of Motion:

$$mn_0 \frac{\partial v_1}{\partial t} = -en_0 E_1 - 3KT_e \frac{\partial n_1}{\partial x}$$

### Electron Waves

- Add  $-\nabla p_e$  to Equation of Motion with 3 degrees of freedom

Dispersion Relation (resulting freq.):

$$\omega^2 = \omega_p^2 + \frac{3}{2} k^2 v_{th}^2; \quad v_{th}^2 \equiv 2KT_e/m.$$

Group velocity is still less than c:

$$v_g = \frac{d\omega}{dk} = \frac{3}{2} \frac{k}{\omega} v_{th}^2 = \frac{3}{2} \frac{v_{th}^2}{v_\phi}$$

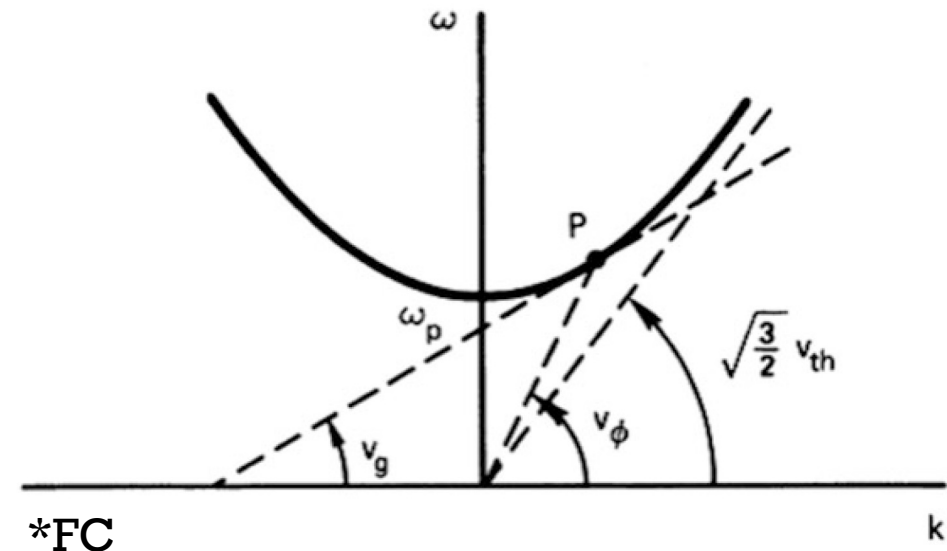


Fig. 4.5 Dispersion relation for electron plasma waves (Bohm-Gross waves)



# WAVES - MOVING PLASMA OSCILLATIONS FOR BOTH ELECTRONS & IONS

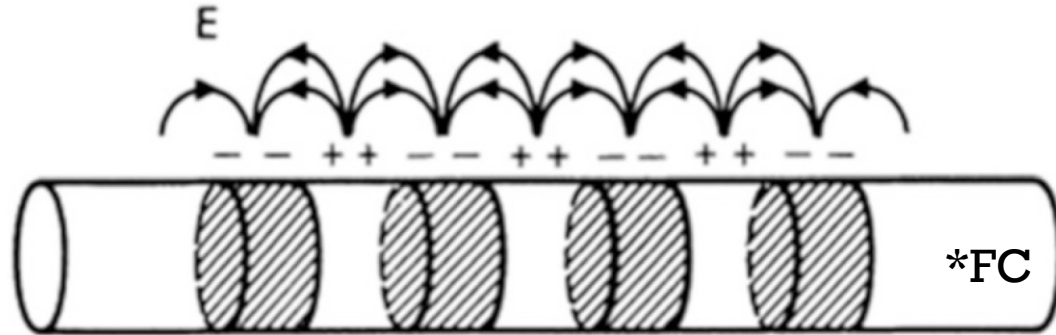


Fig. 4.4 Plasma oscillations propagate in a finite medium because of fringing fields

- Electron Waves
  - Add  $-\nabla p_e$  to Equation of Motion with 3 degrees of freedom

;

With thermal motion information is connected across regions - now a wave

$$\nabla p_e = 3KT_e \nabla n_e = 3KT_e \nabla(n_0 + n_1) = 3KT_e \frac{\partial n_1}{\partial x} \hat{x}$$

Linearized Equation of Motion:

$$mn_0 \frac{\partial v_1}{\partial t} = -en_0 E_1 - 3KT_e \frac{\partial n_1}{\partial x}$$



# WAVES - MOVING PLASMA OSCILLATIONS

## FOR BOTH ELECTRONS & IONS

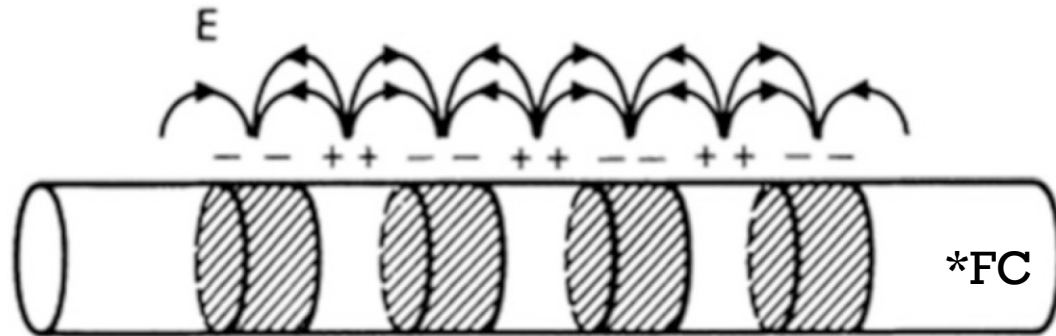


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### Electron Waves

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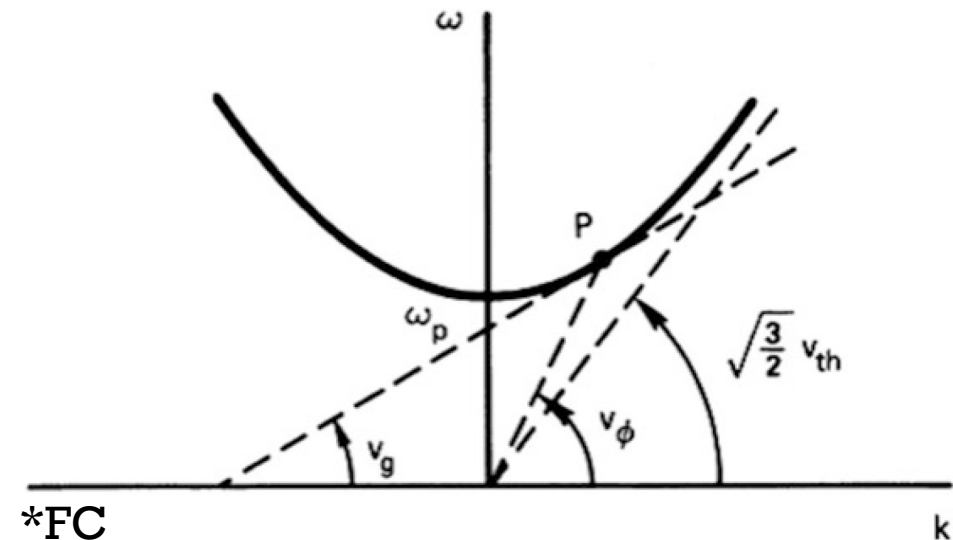


Fig. 4.5 Dispersion relation for electron plasma waves (Bohm-Gross waves)





# WAVES - MOVING PLASMA OSCILLATIONS FOR BOTH ELECTRONS & IONS

Low Frequency Oscillations &  $n_i = n_e = n$   
So, no Poisson's!

Ion Fluid Equation of Motion:

$$Mn \left[ \frac{\partial \mathbf{v}_i}{\partial t} + (\mathbf{v}_i \cdot \nabla) \mathbf{v}_i \right] = en\mathbf{E} - \nabla p = -en\nabla\phi - \gamma_i KT_i \nabla n$$

Perturbation Density for  
both Ions & Electrons:

$$n_1 = n_0 \frac{e\phi_1}{KT_e}$$

Linearized Continuity Equation:

$$i\omega n_1 = n_0 i k v_{i1}; \quad \omega^2 = k^2 \left( \frac{KT_e}{M} + \frac{\gamma_i KT_i}{M} \right); \quad \gamma_i = 3$$

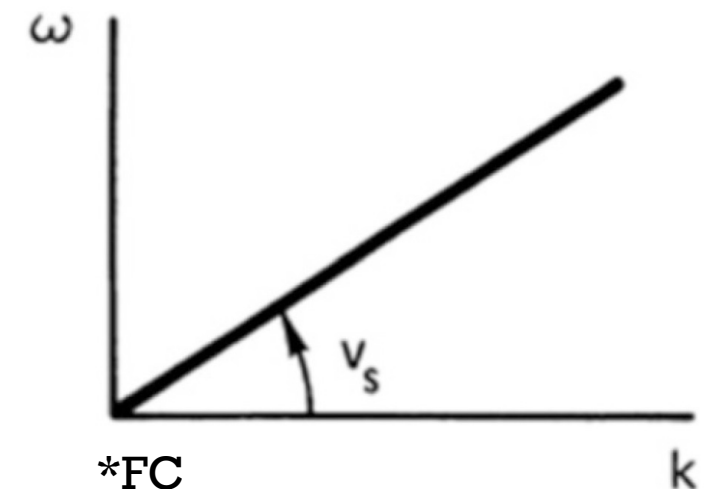
## ■ Ion Acoustic Waves

- E-Field allows for sound to travel
- Plasma Approximation

Dispersion Relation  
for ion acoustic waves:  $\frac{\omega}{k} = \left( \frac{KT_e + \gamma_i KT_i}{M} \right)^{1/2} \equiv v_s$

Acoustic velocity  
is given by:

$$v_s = (KT_e/M)^{1/2}$$



**Fig. 4.12** Dispersion relation for ion acoustic waves in the limit of small Debye length

# WAVES - MOVING PLASMA OSCILLATIONS

## FOR BOTH ELECTRONS & IONS

- Electron Waves

- Add  $-\nabla p_e$  to Equation of Motion with 3 degrees of freedom

Dispersion Relation (resulting freq.):

$$\omega^2 = \omega_p^2 + \frac{3}{2}k^2 v_{th}^2; \quad v_{th}^2 \equiv 2KT_e/m.$$

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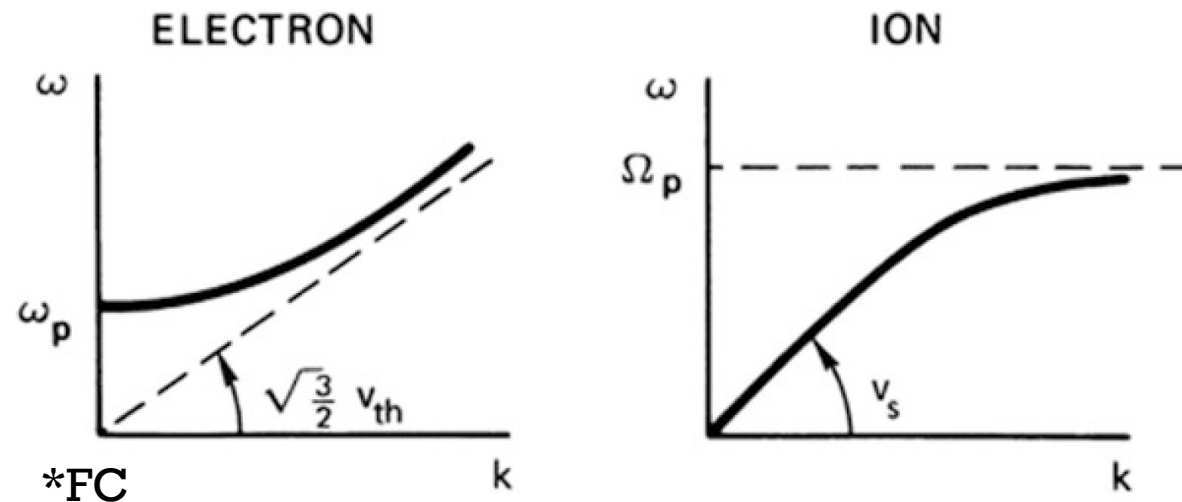
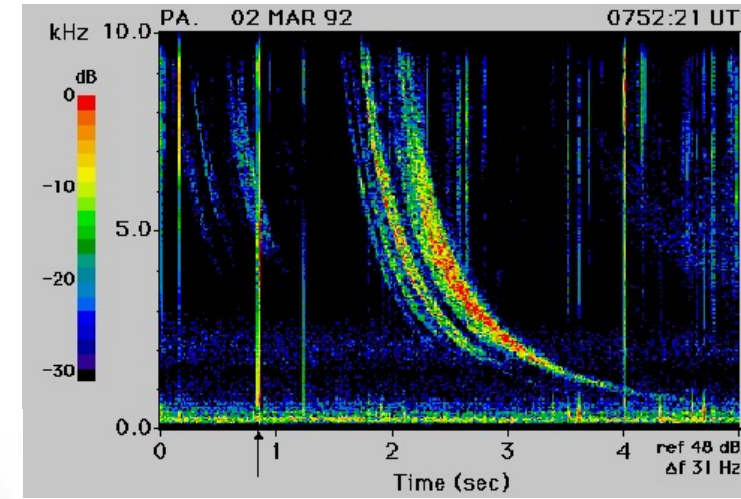


Fig. 4.13 Comparison of the dispersion curves for electron plasma waves and ion acoustic waves

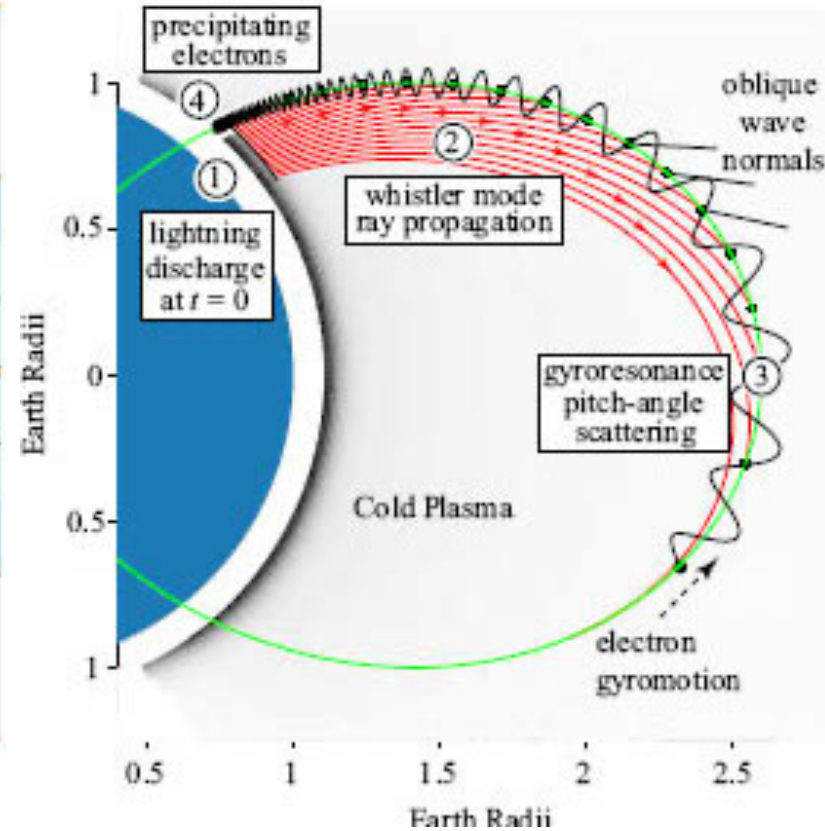
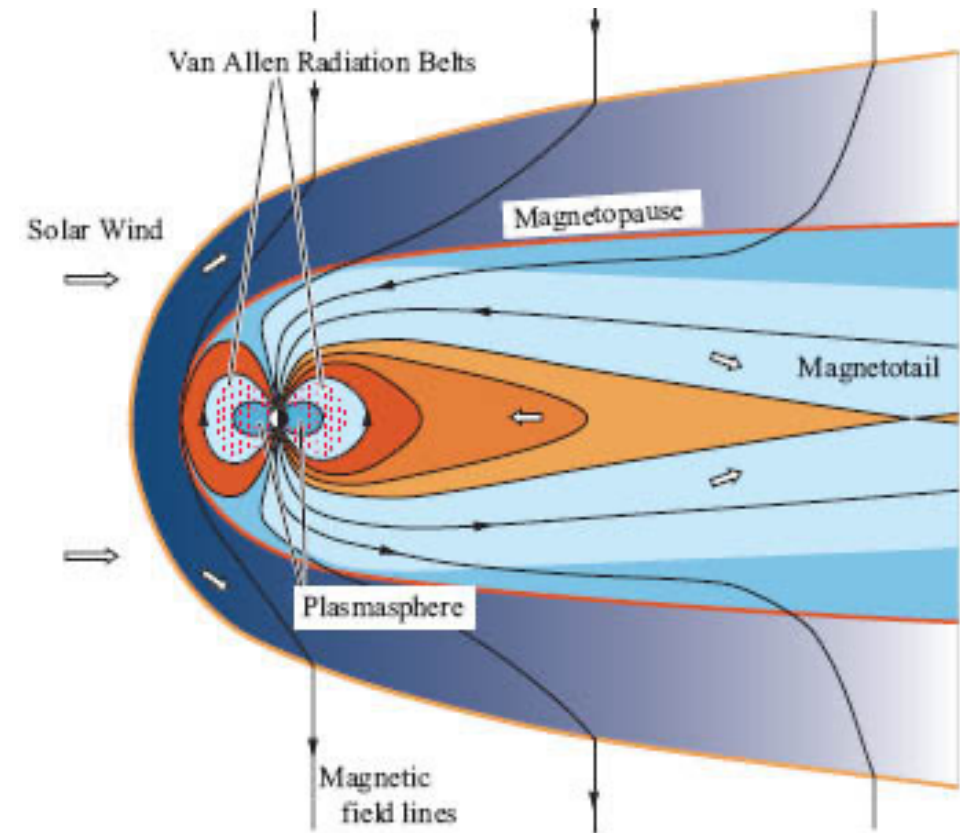


# WHISTLERS: PLASMA OSCILLATIONS IN-ZONE

[https://vlf.stanford.edu/research\\_topic\\_inlin/introduction-whistler-waves-magnetosphere/](https://vlf.stanford.edu/research_topic_inlin/introduction-whistler-waves-magnetosphere/)



Example of a whistler recorded at Palmer Station, Antarctica.



Whistlers propagate oblique to the Earth's magnetic field lines.



## 4.20 Summary of Elementary Plasma Waves

*Electron waves (electrostatic)*

$$\mathbf{B}_0 = 0 \text{ or } \mathbf{k} \parallel \mathbf{B}_0 : \quad \omega^2 = \omega_p^2 + \frac{3}{2} k^2 v_{th}^2 \quad (\text{Plasma oscillations}) \quad (4.143)$$

$$\mathbf{k} \perp \mathbf{B}_0 : \quad \omega^2 = \omega_p^2 + \omega_c^2 = \omega_h^2 \quad (\text{Upper hybrid oscillations}) \quad (4.144)$$

*Ion waves (electrostatic)*

$$\begin{aligned} \mathbf{B}_0 = 0 \text{ or } \mathbf{k} \parallel \mathbf{B}_0 : \quad \omega^2 &= k^2 v_s^2 \\ &= k^2 \frac{\gamma_e K T_e + \gamma_i K T_i}{M} \quad (\text{Acoustic waves}) \end{aligned} \quad (4.145)$$

$$\mathbf{k} \perp \mathbf{B}_0 : \quad \omega^2 = \Omega_c^2 + k^2 v_s^2 \quad (\text{Electrostatic ion cyclotron waves}) \quad (4.146)$$

or

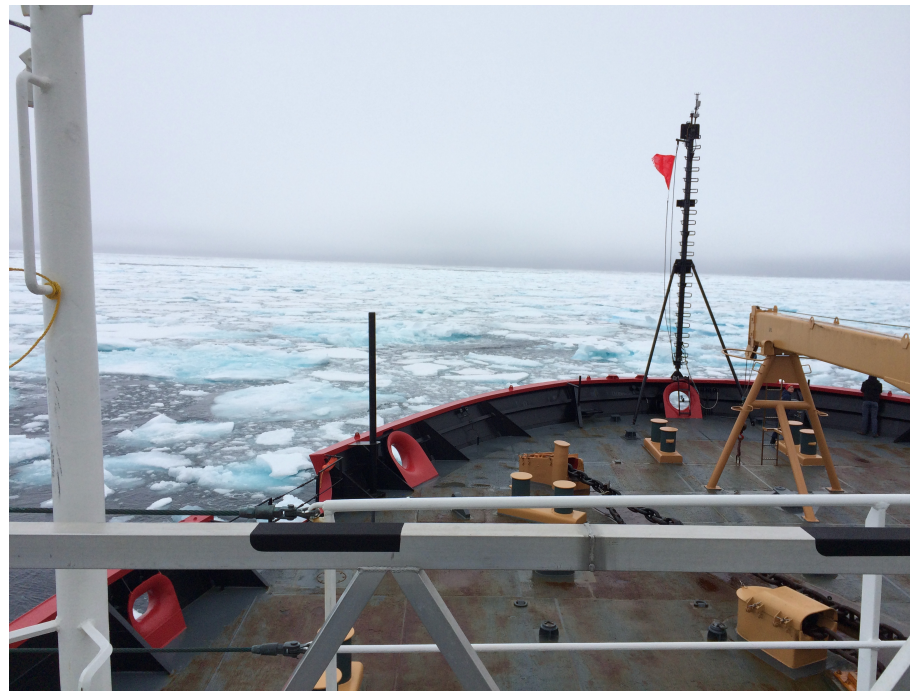
$$\omega^2 = \omega_l^2 = \Omega_c \omega_c \quad (\text{Lower hybrid oscillations}) \quad (4.147)$$

*Electron waves (electromagnetic)*

$$\mathbf{B}_0 = 0 : \quad \omega^2 = \omega_p^2 + k^2 c^2 \quad (\text{Light waves}) \quad (4.148)$$

$$\mathbf{k} \perp \mathbf{B}_0, \mathbf{E}_1 \parallel \mathbf{B}_0 : \quad \frac{c^2 k^2}{\omega^2} = 1 - \frac{\omega_p^2}{\omega^2} \quad (\text{O wave}) \quad (4.149)$$

$$\mathbf{k} \perp \mathbf{B}_0, \mathbf{E}_1 \perp \mathbf{B}_0 : \quad \frac{c^2 k^2}{\omega^2} = 1 - \frac{\omega_p^2}{\omega^2} \frac{\omega^2 - \omega_p^2}{\omega^2 - \omega_h^2} \quad (\text{X wave}) \quad (4.150)$$



# TIP OF THE ICEBERG!

$$\mathbf{k} \parallel \mathbf{B}_0 : \quad \frac{c^2 k^2}{\omega^2} = 1 - \frac{\omega_p^2 / \omega^2}{1 - (\omega_c / \omega)} \quad (\text{R wave (whistler mode)}) \quad (4.151)$$

$$\frac{c^2 k^2}{\omega^2} = 1 - \frac{\omega_p^2 / \omega^2}{1 + (\omega_c / \omega)} \quad (\text{L wave}) \quad (4.152)$$

*Ion waves (electromagnetic)*

$$\mathbf{B}_0 = 0 : \quad \text{None}$$

$$\mathbf{k} \parallel \mathbf{B}_0 : \quad \omega^2 = k^2 v_A^2 \quad (\text{Alfvén wave}) \quad (4.153)$$

$$\mathbf{k} \perp \mathbf{B}_0 : \quad \frac{\omega^2}{k^2} = c^2 \frac{v_s^2 + v_A^2}{c^2 + v_A^2} \quad (\text{Magnetosonic wave}) \quad (4.154)$$



# BIG THANKS FOR WALKING THE WAVES WITH ME!

## Progress in Development of Low Pressure High Density Plasmas on The

The small Helicon Plasma continues to progress to  $1.85 \text{ T}$  [1] of helicon, laboratory investigation frequency in the 90 to energy conservation by decreased inertial after match particle probes, system backed by a 20 MSix for HPX plasma Helicon Mode developed electromagnetic scatter

### 1 External 1.1 Uniform

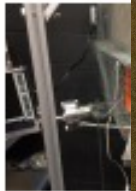
#### 1.1.1 Road to H

- E-Mode with Layer via Co
- H-Mode with Layer and re Inductive M
- Apply Extern Field to achi Mode

### 2 Faraday

#### 1.1 Tempor

- Metal mesh
- Prevents EM
- Will attach to



### 3 Particle

#### Verify M

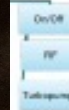
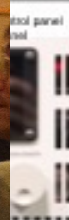
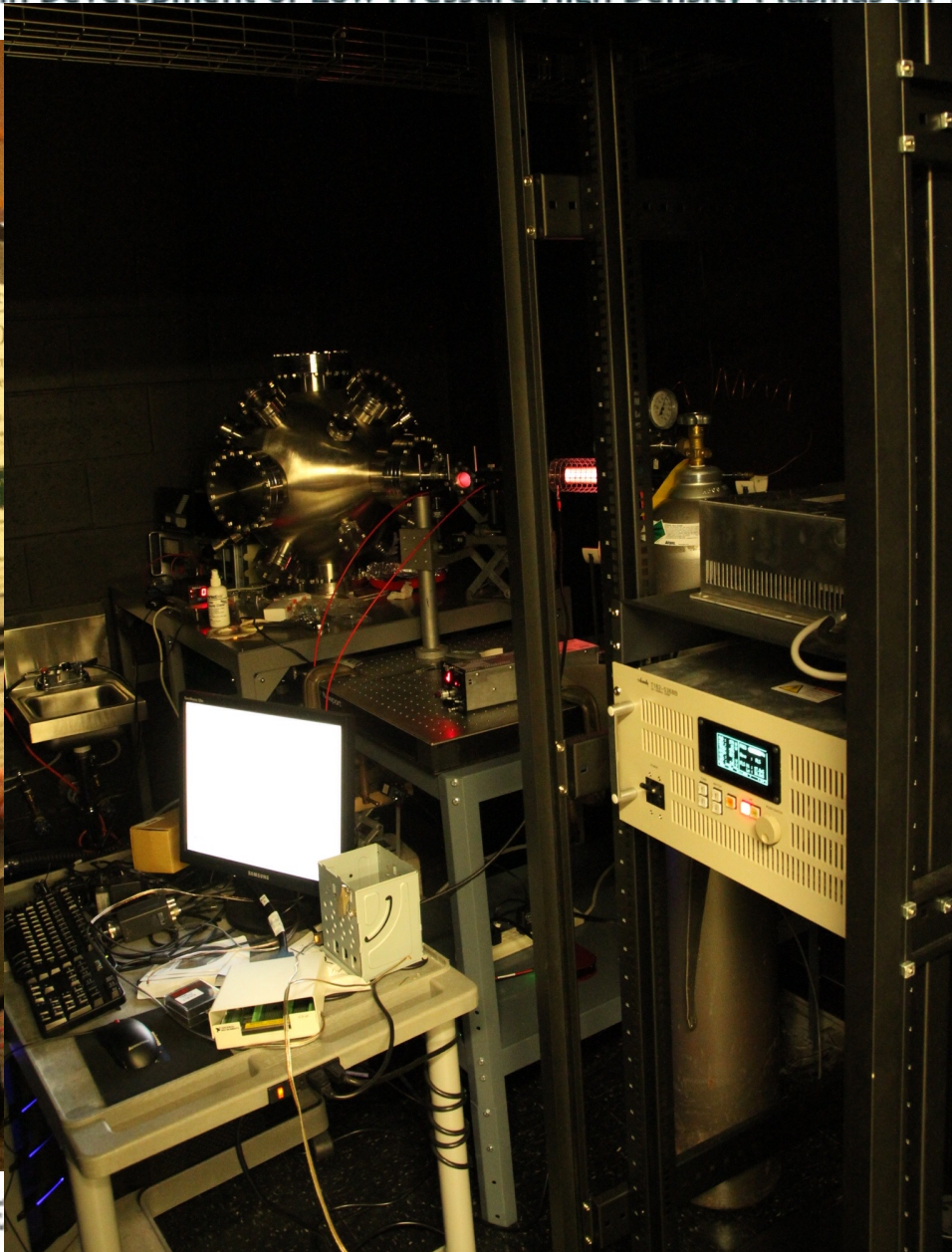
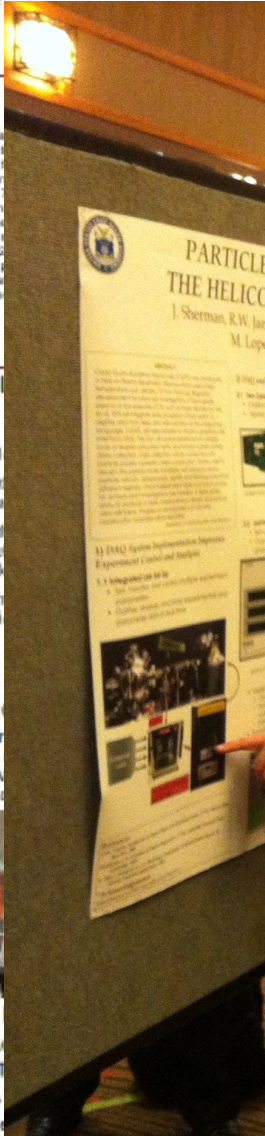
#### 3.1 OES & T

- Relatively
- HPX mod

#### References

Chen, Physics Introduction to Plasma Physics and Controlled Fusion, 2<sup>nd</sup> Ed. Plenum Press, New York, 1974.  
Hollmann, 1.1.1 Principles of Plasma Diagnostics, 2<sup>nd</sup> Ed. Cambridge University Press, Cambridge, 2004.  
K. Shi, S. Swales, et al., Preliminary Investigation of Helicon Plasma Source for Electron Beam

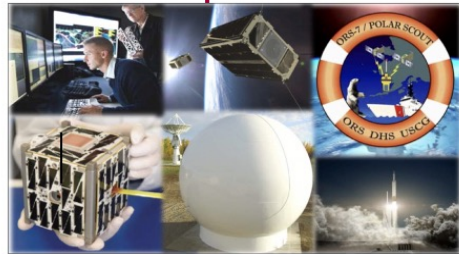
cgapl.wordpress.com



## CG Space & Energy Initiatives

### Space Initiatives

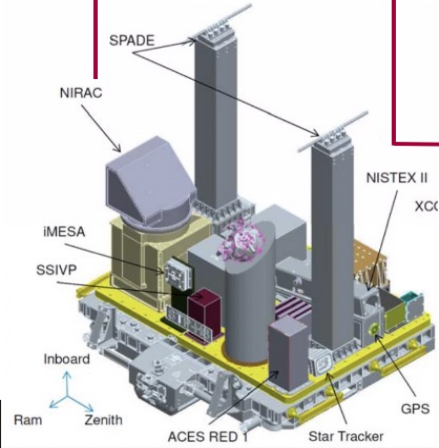
- Operation Polar Scout
- RDC/CGA Ground Station & Radome
- CubeSat [with Cyber] Ops & Research



*Polar Scout Mission & CGA Radome*

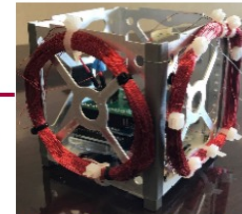
### Space & Energy Crossover

- ThinSat Research
- Remote Sensing
- Geospatial Satellite/AIS Research
- Spacecraft Plasma Propulsion Research
- Fusion/Fission
- Magnet Torquer CubeSat Attitude Control



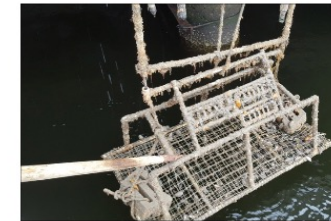
*CGA & NRL collaboration. Impedance probe to monitor plasma density on ThinSat – test for ISS diagnostic, SPADE.*

- MHD Propulsion [craft/buoys]
- Plasma Waste Water Reactor
- Corrosion Research



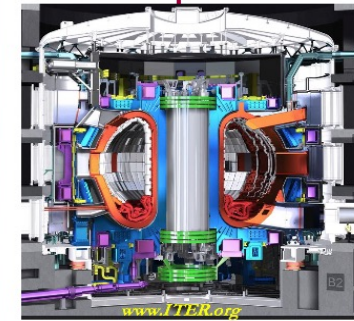
*Magnetorquer Attitude Control.*

*CGA/DOD Corrosion Research*

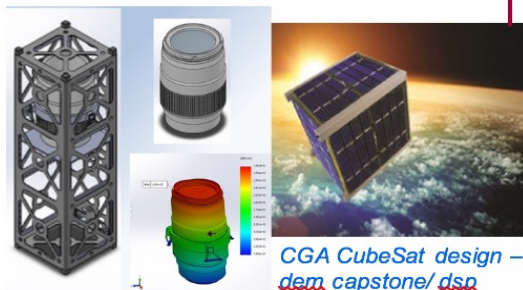


### Energy Initiatives

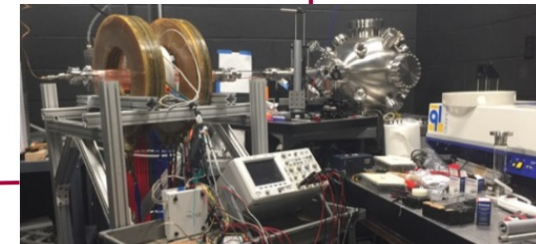
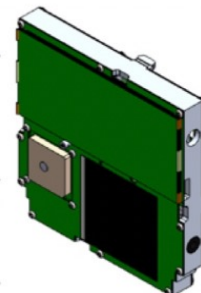
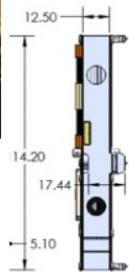
- Energy Production/Alternatives:
- Fusion Energy Research
- Wind/Wave/Solar Studies
- Oil Fingerprinting



*International Tokamak Fusion Experiment, ITER*



*CGA CubeSat design – dem capstone/dsp*



*CGA Plasma Laboratory. MHD, Plasma, Directed Energy, Fusion research.*