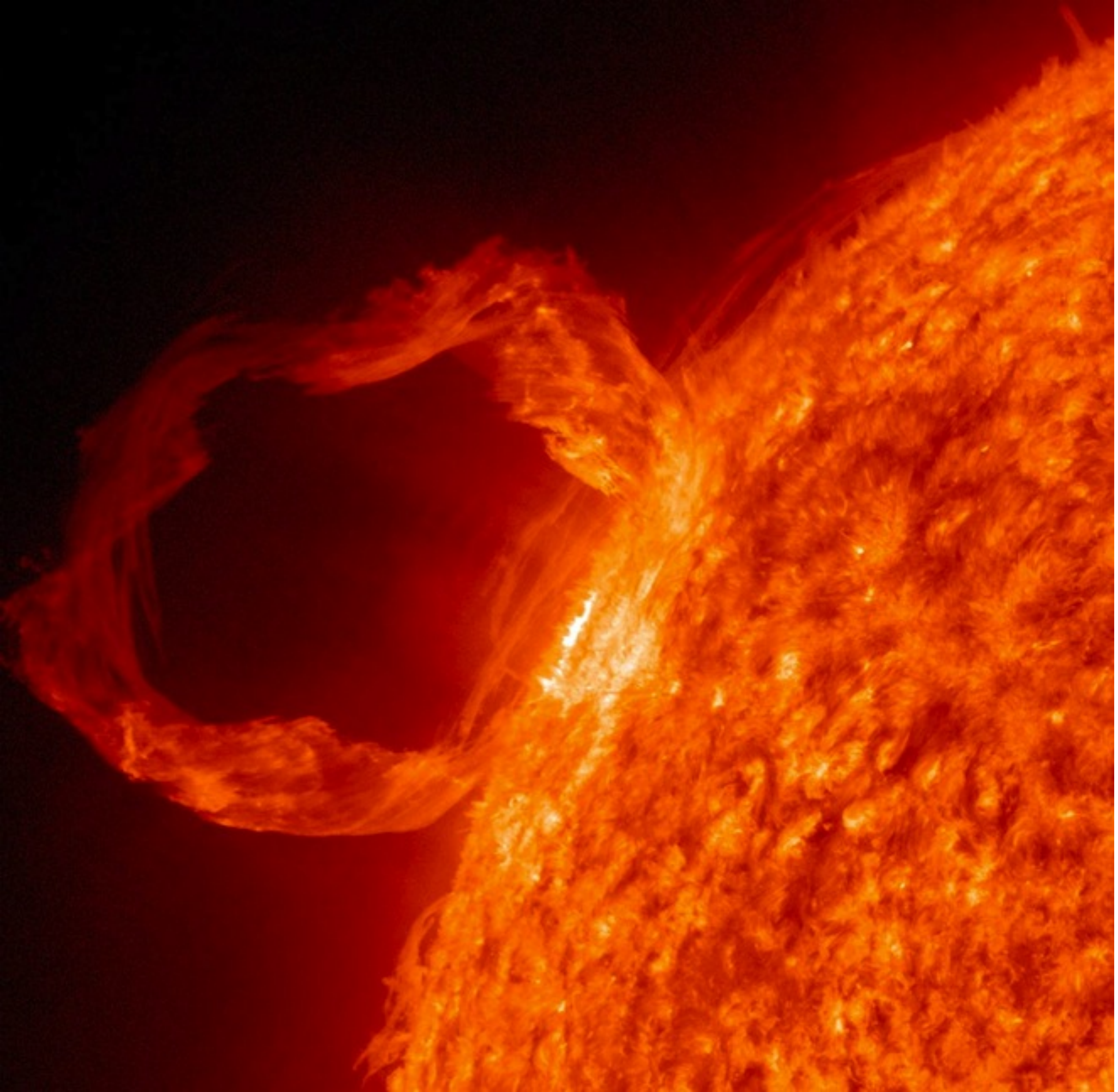

Introduction to Plasma Physics

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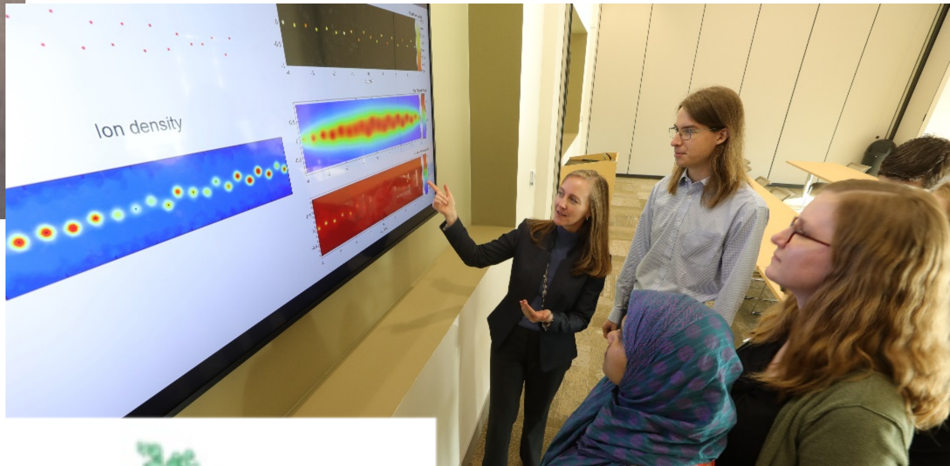

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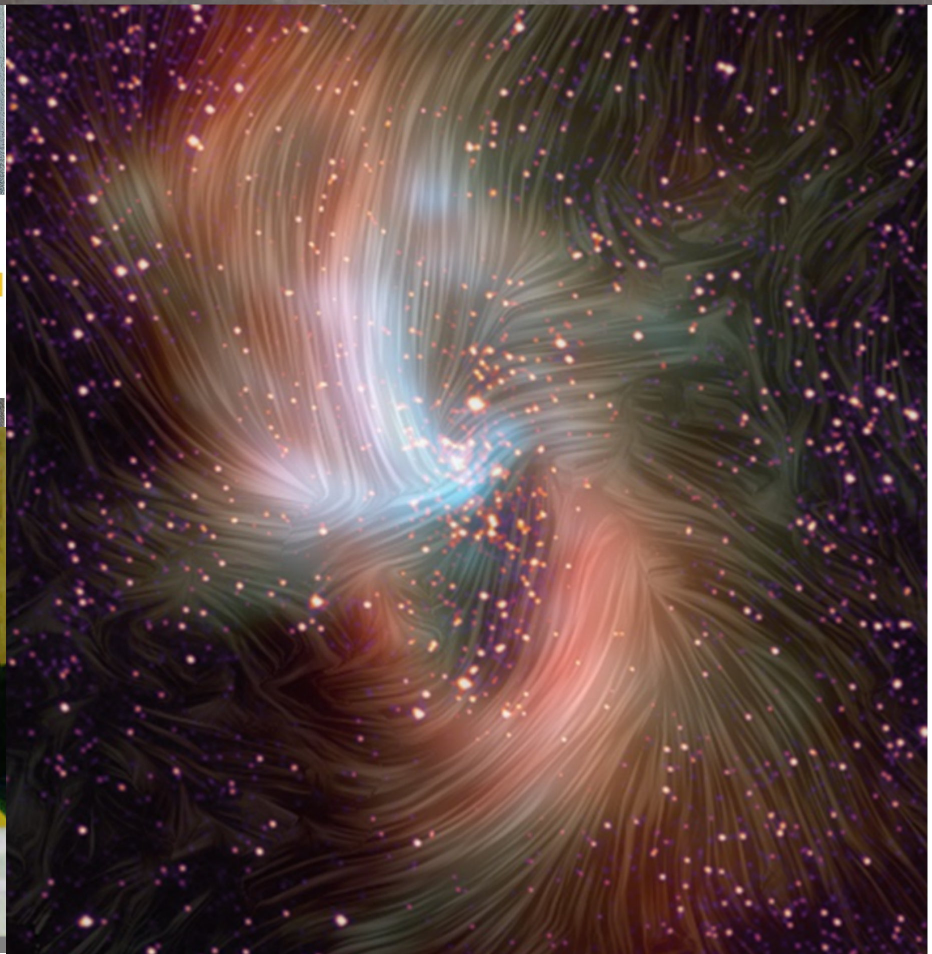

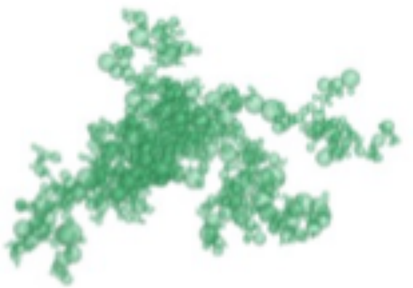

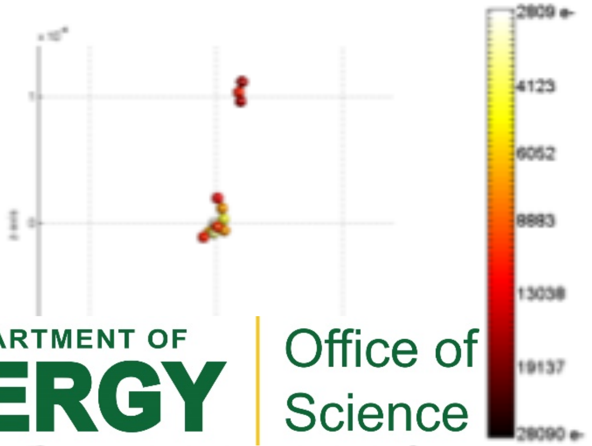

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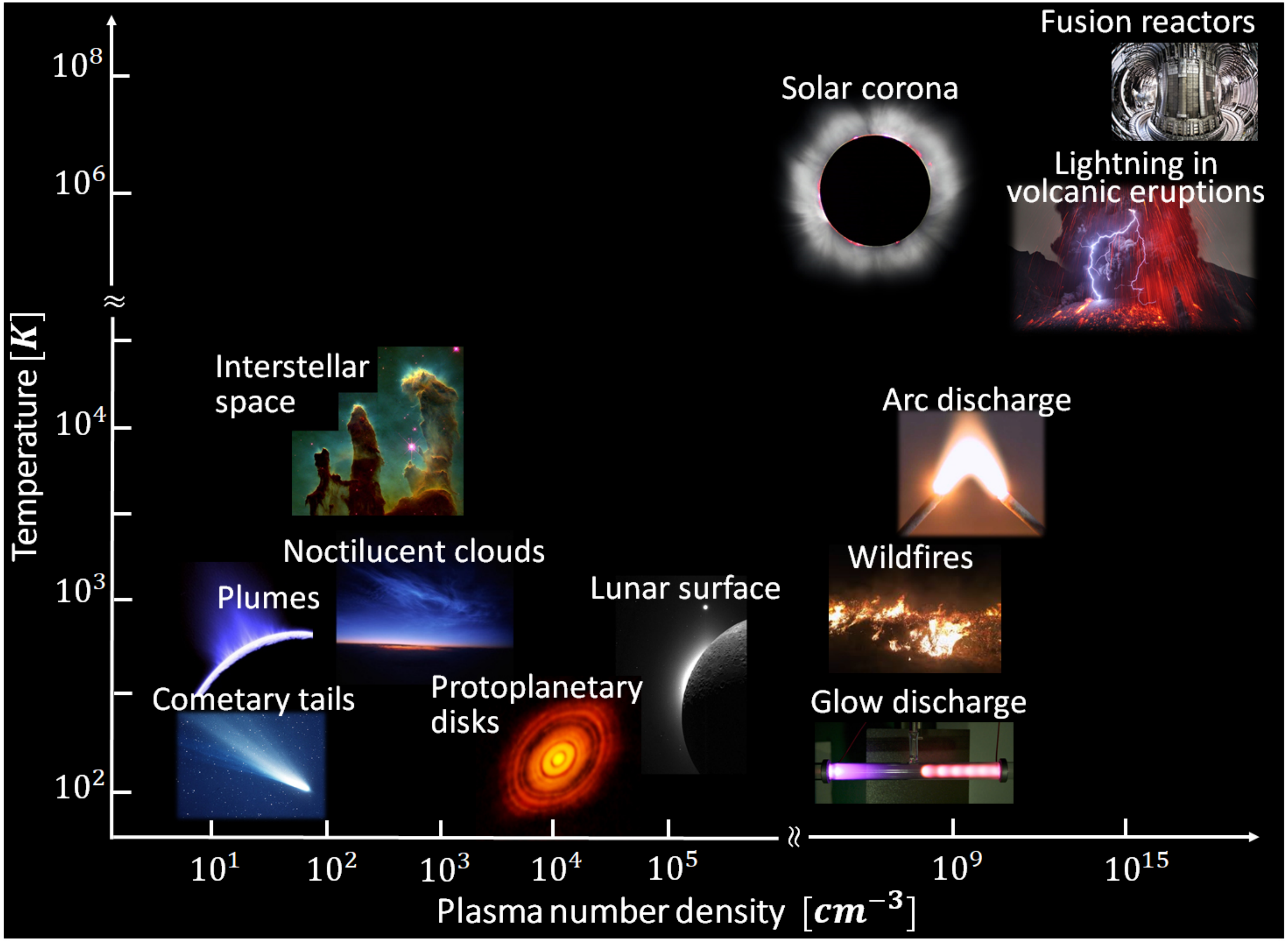
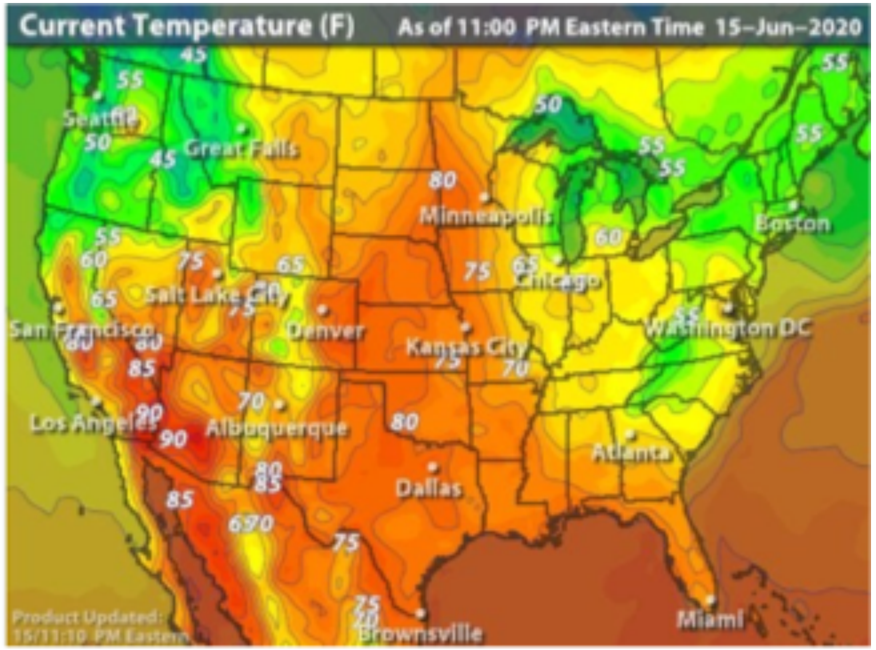
Plasma Parameter Space



Super quick math review: fields

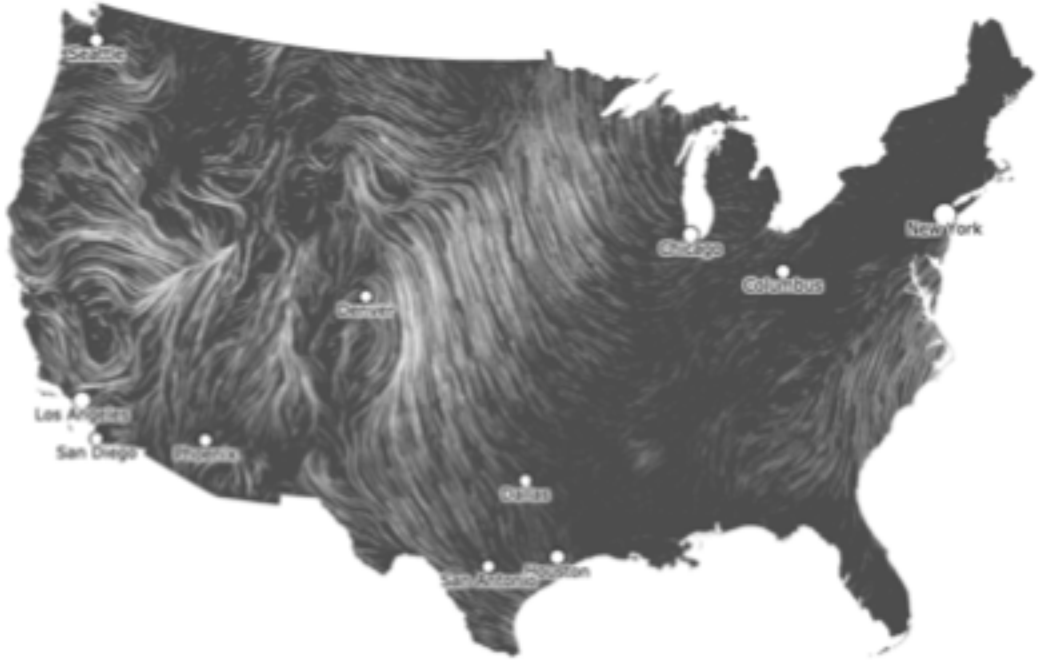
- Scalar fields: Every point in space is associated with a given value.

For example: Temperature



- Vector fields: Every point in space is associated with a given vector.

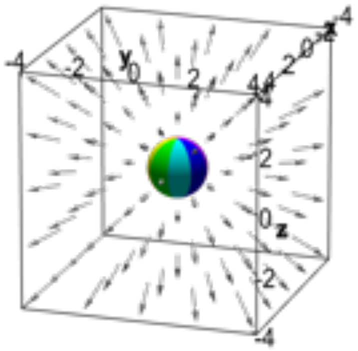
For example: Wind velocity



Super quick math review: Vector calculus

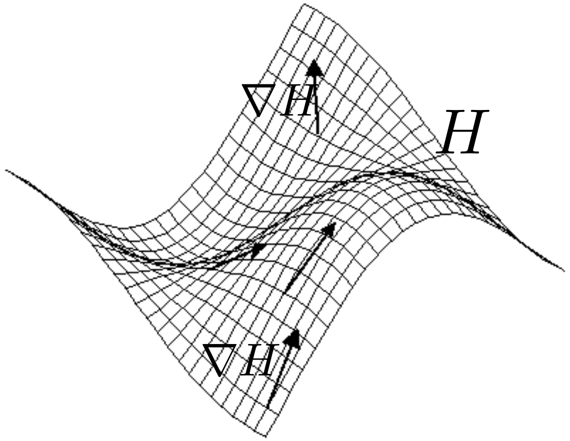
Divergence

$$\nabla \cdot \vec{V} > 0$$



$$\nabla H$$

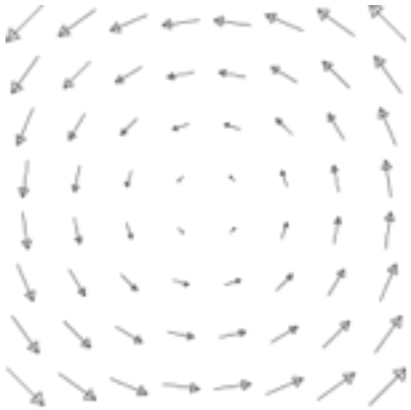
Gradient



Curl

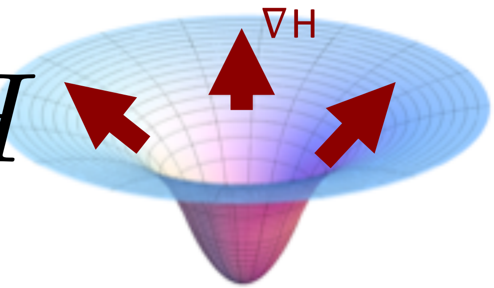
$$\nabla \times \vec{V} \neq 0$$

$\odot$



$$H$$

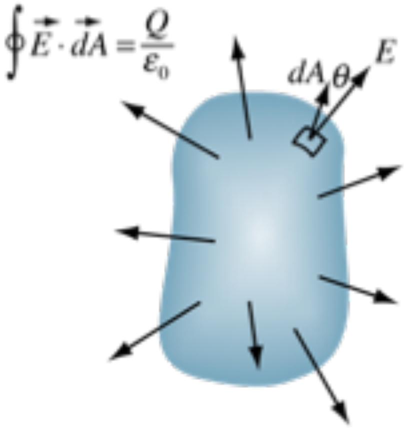
Laplacian



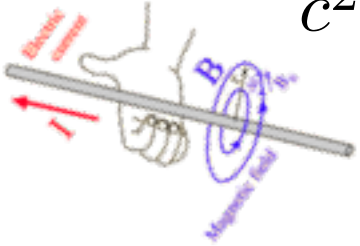
$$\nabla \cdot (\nabla H) = \nabla^2 H > 0$$

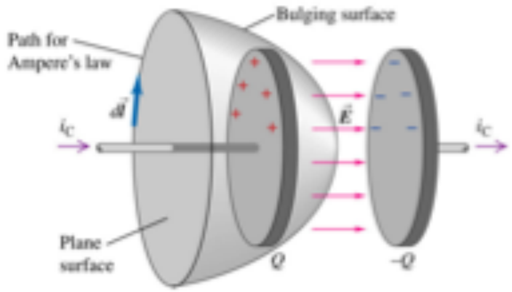
Gospel according to Maxwell

$$\nabla \cdot \vec{E} = \frac{\rho}{\epsilon_0}$$

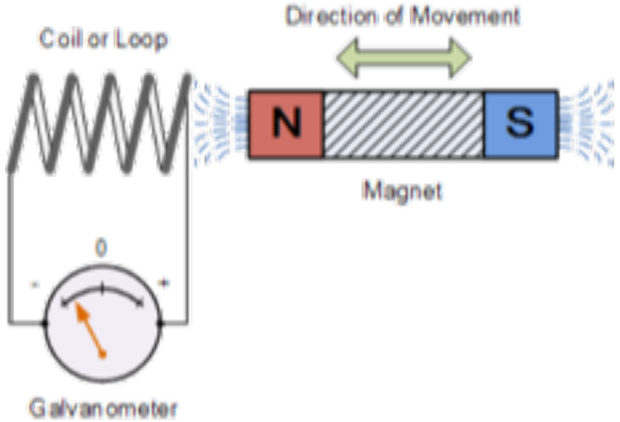


$$\nabla \times \vec{B} = \mu_0 \vec{J} + \frac{1}{c^2}\frac{d\vec{E}}{dt}$$

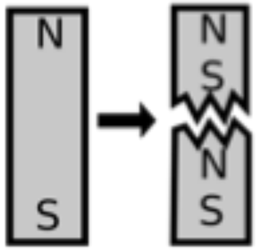


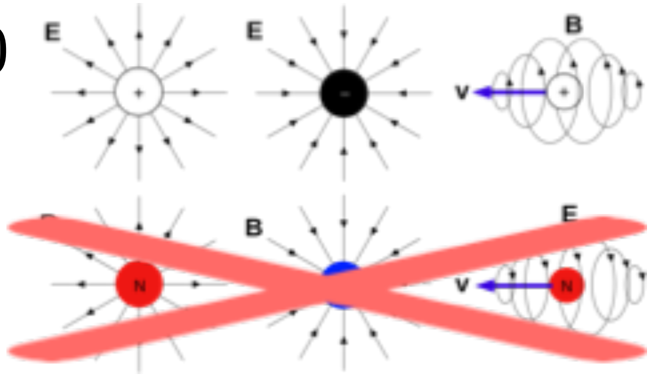


$$\nabla \times \vec{E} = -\frac{d\vec{B}}{dt}$$



$$\nabla \cdot \vec{B} = 0$$





Electric potential and Poisson’s equation

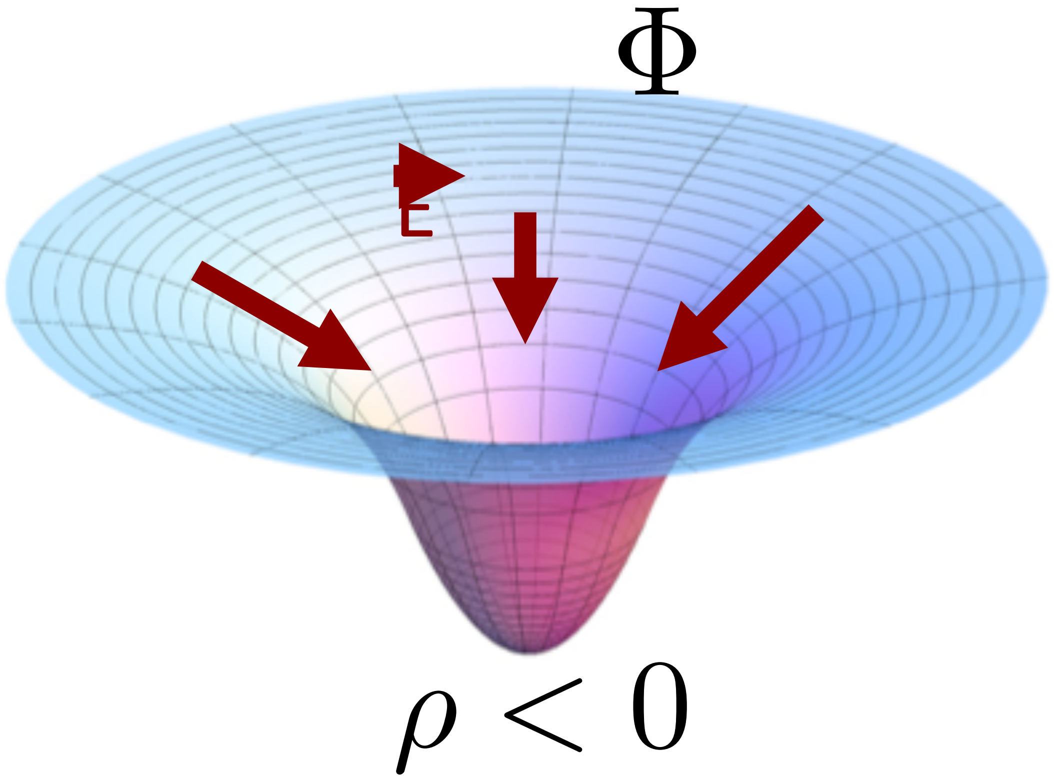
$$\vec{E} = -\nabla\Phi$$

$$\nabla \cdot \vec{E} = \frac{\rho}{\epsilon_0}$$

$$\nabla \cdot (-\nabla\Phi) = \frac{\rho}{\epsilon_0}$$

$$\nabla^2\Phi = -\frac{\rho}{\epsilon_0}$$

Potential energy of a charge in an electric potential = $q\Phi$



How much plasma is there?

- Thermal plasma – Saha Equation

$$\frac{N_{i+1}}{N_i} = \frac{2Z_{i+1}}{Z_i n_e}\left(\frac{2\pi m_e kT}{h^2}\right)^{3/2} \exp\left(-\frac{\chi_i}{kT}\right)$$

How much plasma is there?

- Thermal plasma – Saha Equation

Ionization energy of the i^{th} state

$$\frac{N_{i+1}}{N_i} = \frac{2Z_{i+1}}{Z_i n_e}\left(\frac{2\pi m_e kT}{h^2}\right)^{3/2} \exp\left(-\frac{\chi_i}{kT}\right)$$

How much plasma is there?

- Thermal plasma – Saha Equation

$$\frac{N_{i+1}}{N_i} = \frac{2Z_{i+1}}{Z_i n_e}\left(\frac{2\pi m_e kT}{h^2}\right)^{3/2} \exp\left(-\frac{\chi_i}{kT}\right)$$

Partition functions – representing degeneracy of the electron states

$$Z_j = \sum_{j=1}^{\infty} g_j e^{-(E_i - E_j)/kT}$$

How much plasma is there?

- Thermal plasma – Saha Equation

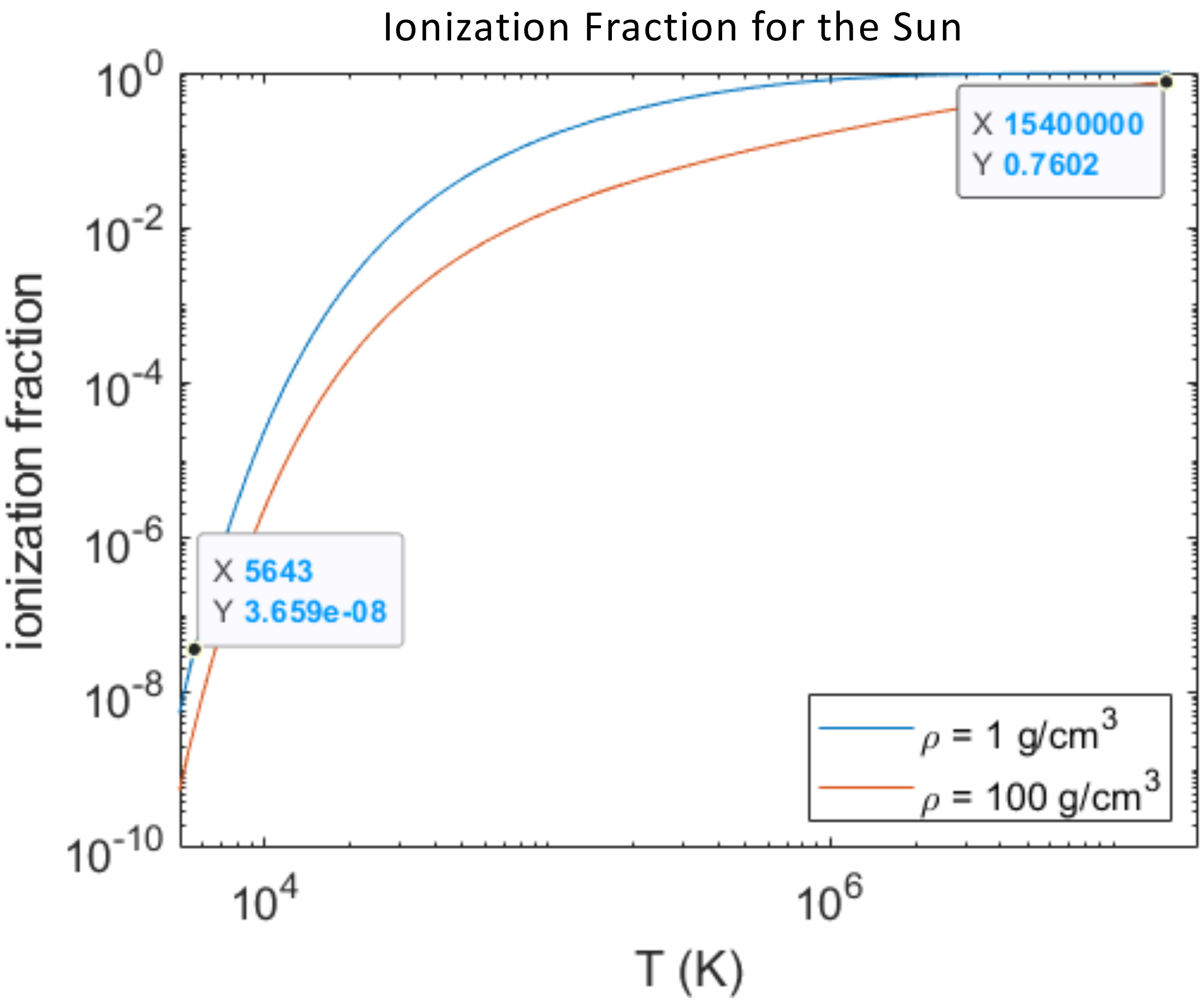
$$\frac{N_{i+1}}{N_i} = \frac{2Z_{i+1}}{Z_i n_e}\left(\frac{2\pi m_e kT}{h^2}\right)^{3/2} \exp\left(-\frac{\chi_i}{kT}\right)$$

De Broglie wavelength of electron – λ^3

How much plasma is there?

- Thermal plasma – Saha Equation

$$\frac{N_{i+1}}{N_i} = \frac{2Z_{i+1}}{n_e Z_i}\left(\frac{2\pi m}{h}\right.$$



Hydrogen plasma
N_{i+1} = $n_+ = n_e$
$N_i = n_0$
$n_H = n_0 + n_+$
Calculate n_+/n_H

How much plasma is there?

- Plasma makes up what percent of the matter in the universe?

A) 99.99%

B) 75%

C) 13.6%

D) < 0.01%

Plasma Basics

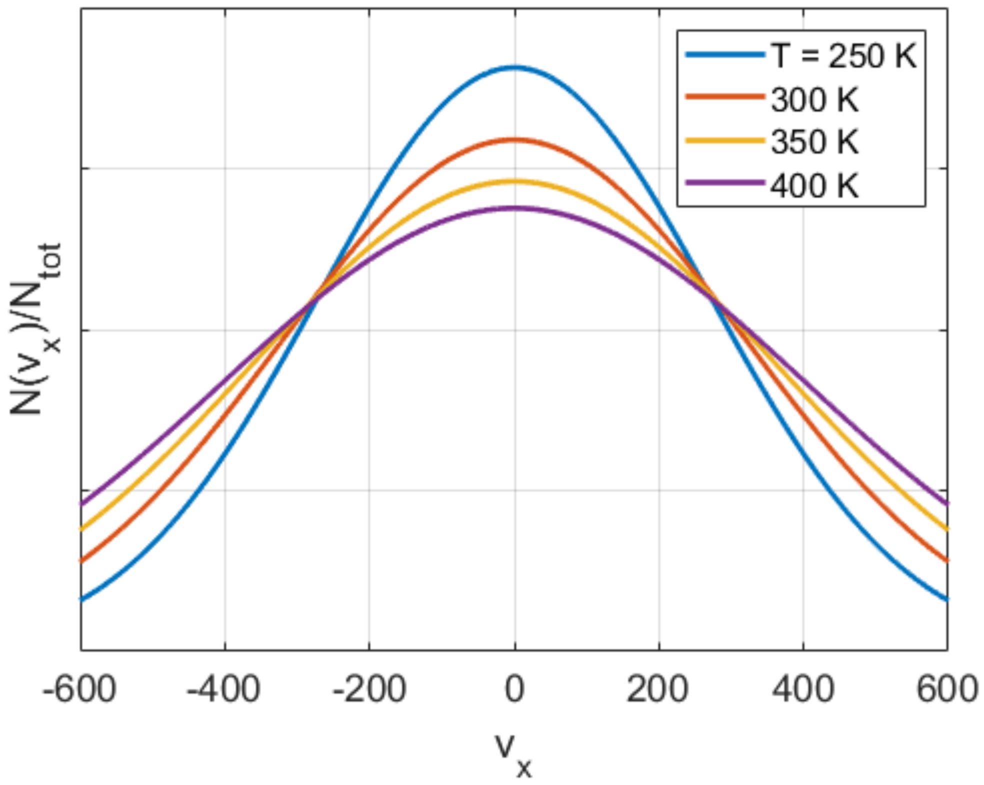
- Plasma is characterized by
  - n_s – Plasma density of species *s*
  - c_s – Thermal speed
  - T_s – Temperature
  - λ_{Ds} – Debye length
  - ω_{ps} – Plasma frequency
  - ω_{cs} – Cyclotron frequency
  - ν_{ij} – Collision frequency between species

Plasma Basics

- Thermalized plasma
- Maxwellian velocity distribution

$$f_s(v) = n_s \left(\frac{m_s}{2\pi k T_s}\right)^{3/2} e^{-\frac{m_s v^2}{2kT_s}}$$

- $v^2 = v_x^2 + v_y^2 + v_z^2$



Area under the curve = 1

Plasma Basics

- Number density

$$n_s = \int_{-\infty}^{\infty} f_s(v) d^3 v$$

$$d^3 v = v^2 \sin\theta \, dv d\theta d\phi$$

or

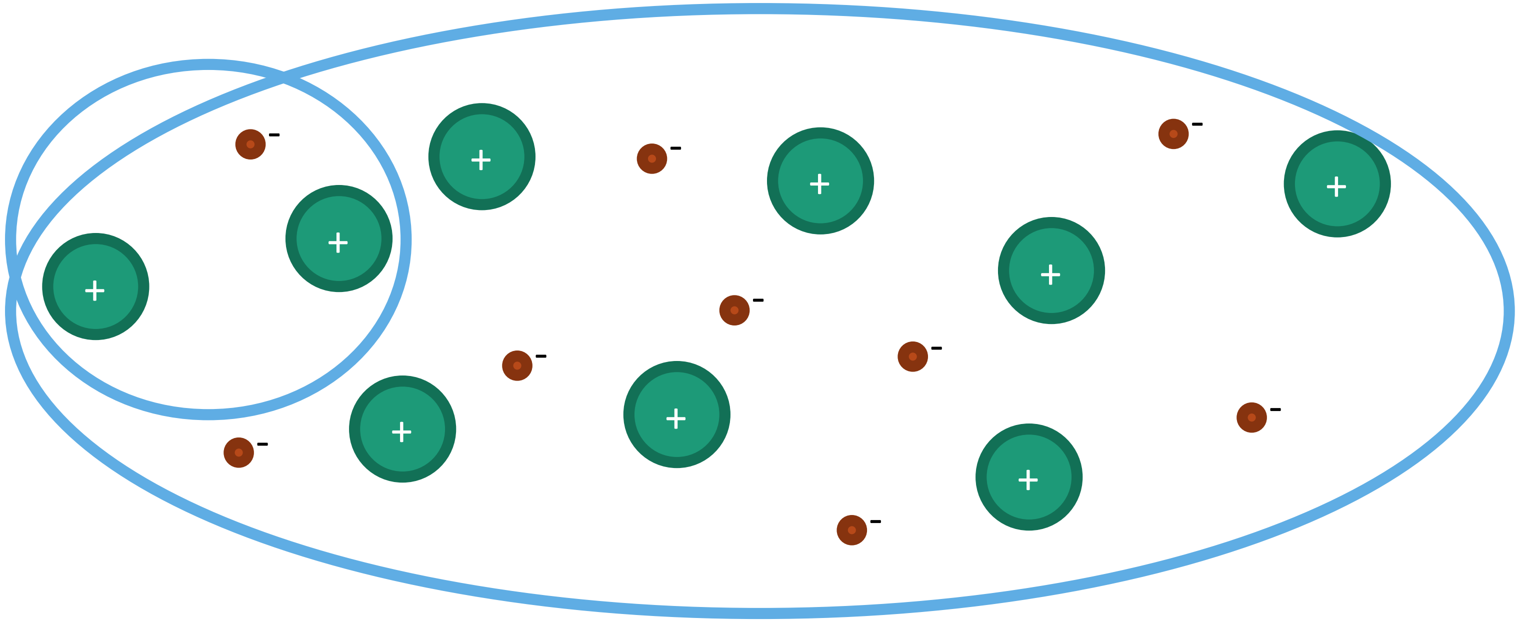
$$d^3 v = dv_x dv_y dv_z$$

This is the normalization condition for the velocity distribution

Plasma Basics

- Equilibrium: charge neutrality

$$n_i Z_i e - n_e e = 0$$



Plasma Basics

Average speed?

$$\langle v^2 \rangle = \frac{\int_{-\infty}^{\infty} v^2 f_s(v) d^3 v}{\int_{-\infty}^{\infty} f_s(v) d^3 v}$$

$$v_{rms} = \sqrt{\frac{3kT_s}{m_s}} = \sqrt{3}C_s$$

C_s = Thermal speed

$$c_s = \sqrt{\frac{kT_s}{m_s}}$$

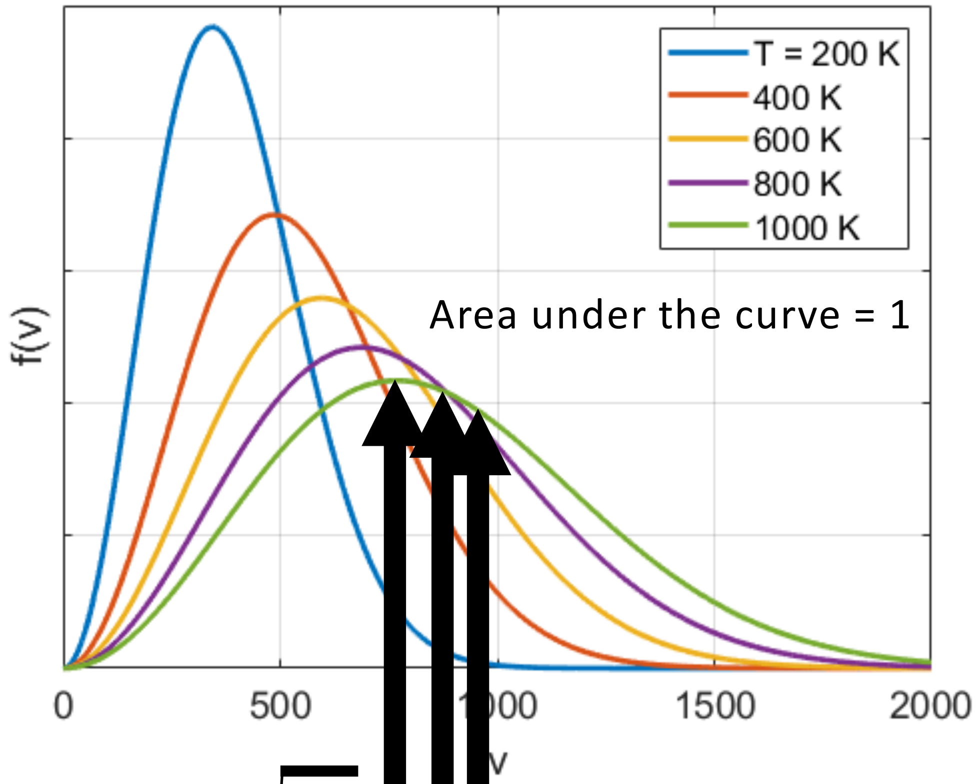
Plasma Basics

- Thermalized plasma
- Maxwell-Boltzmann probability distribution of *speeds*

(integrate over all directions for a fixed speed)

$$f_s(v) = \int_0^{2\pi}\int_0^{\pi} f_s(v) v^2 sin\theta d\theta d\phi$$

$$f_s(v) = n_s\left(\frac{m_s}{2\pi k T_s}\right)^{3/2} 4\pi v^2 e^{-\frac{m_s v^2}{2kT_s}}$$



$$v_{MP} = \sqrt{\frac{2kT}{m}}$$

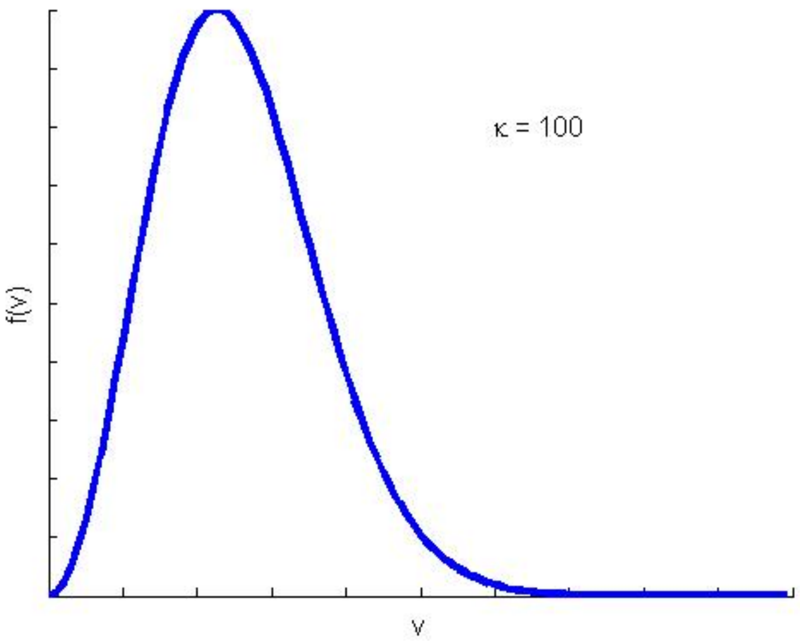
$$v_{avg} = \sqrt{\frac{8kT}{\pi m}}$$

$$v_{rms} = \sqrt{\frac{3kT}{m}}$$

Non-thermal plasma
Lorentzian (Kappa) distribution

$$f_j^\kappa(v_j) = \frac{\Gamma(\kappa+1)}{\pi^{3/2}\theta_j^3\kappa^{3/2}\Gamma(\kappa-\frac{1}{2})}\left(1+\frac{v_j^2}{\kappa\theta_j^2}\right)^{-\kappa-1},$$

$$\theta_j = \sqrt{\frac{2\kappa-3}{\kappa}}v_{Tj} \text{ and } v_{Tj} = \sqrt{\frac{k_B T_j}{m_j}}.$$



For the same plasma temperature:

- Distribution peaks at lower energy than Maxwellian distribution
- High energy tail

N. Rubab and G. Murtaza, “Dust-charge fluctuations with non-Maxwellian distribution functions”, *Phys. Scr*. **73(2)** 178-183, 2006.

Plasma Basics

Average kinetic energy

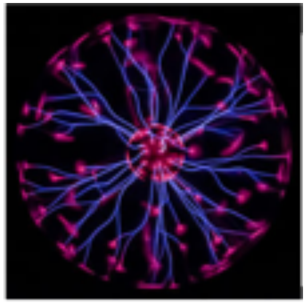
$$\left\langle \frac{1}{2} m_s v^2 \right\rangle = \frac{3}{2} kT_s$$

T_s = Temperature of the plasma

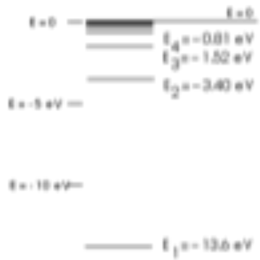
$$kT_s = \frac{1}{3} m_s v^2$$

An aside on temperatures

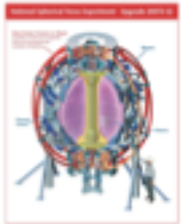
1ev = 10,000K (11,600, but whatever)

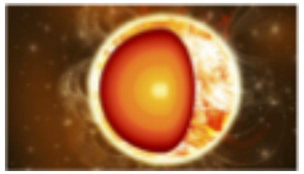


~1eV



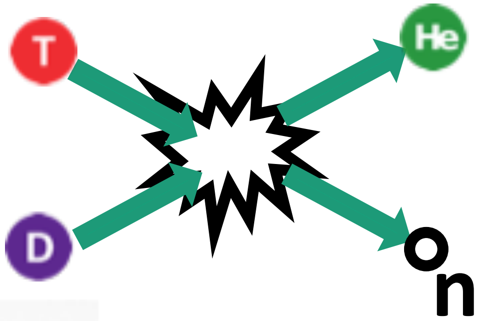
13.6eV





~1keV

~10keV

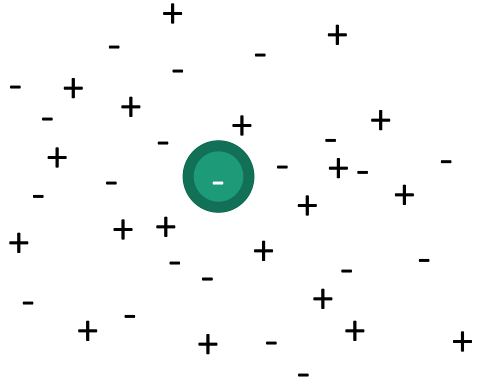


~10MeV

Plasma Basics

Debye Length

- Poisson's Equation



$$\nabla^2\Phi = -\frac{\rho_q}{\epsilon_0}$$

$$\nabla^2\Phi = -Ze\delta(\boldsymbol{x}) - \frac{e}{\epsilon_0}(n_0 - n_e)$$

$$\nabla^2\Phi = -Ze\delta(\boldsymbol{x}) - \frac{en_0}{\epsilon_0}\left(1 - \mathrm{e}^{-\frac{e\Phi}{kT_e}}\right)$$

$$n_e = n_0\mathrm{e}^{-\frac{e\Phi}{kT_e}}$$

Plasma Basics

Use $e\phi \ll kT$ to linearize equation

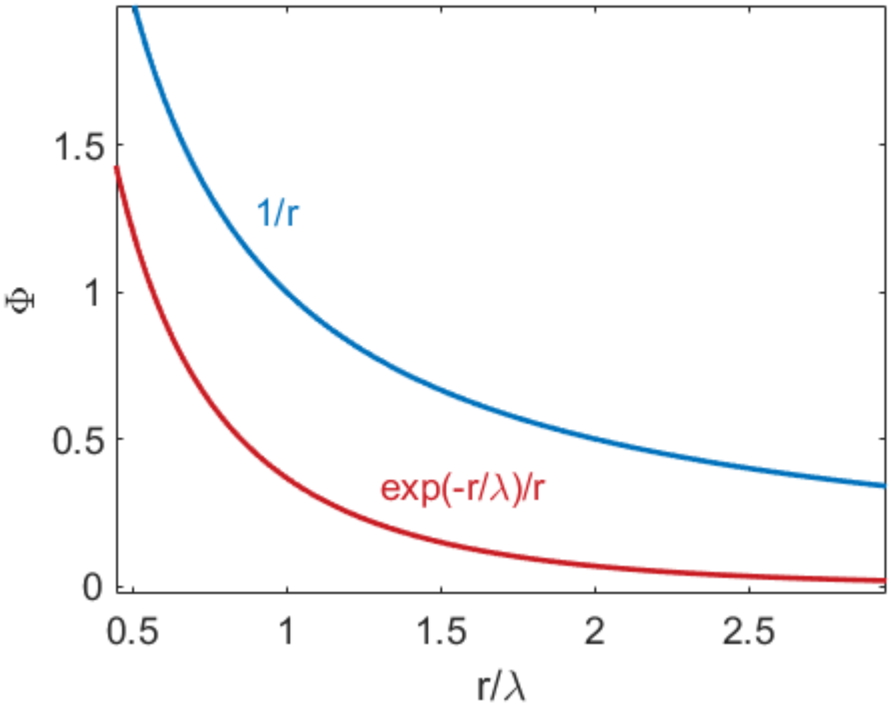
- Debye length

$$\nabla^2\Phi \approx -Ze\delta(\boldsymbol{x}) - \frac{en_0}{\epsilon_0}\left(1 - \left(1 - \frac{e\Phi}{kT_e}\right)\right)$$

$$\frac{1}{\lambda_D^2} = \frac{n_0 e^2}{\epsilon_0 k T_e}$$

$$\nabla^2\Phi - \frac{1}{\lambda_D^2}\Phi \approx -Ze\delta(\boldsymbol{x})$$

$$\Phi(r) = \frac{Ze}{4\pi\epsilon_0 r} e^{\frac{-r}{\lambda_D}}$$

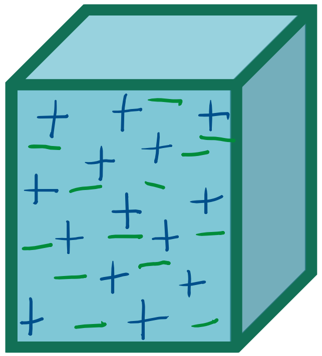


$$\lambda_D = 6.90\sqrt{\frac{T}{n_0}}\ \text{cm (cgs units)}$$

$$= 69.0\sqrt{\frac{T}{n_0}}\ \text{m (mks units)}$$

Plasma Basics

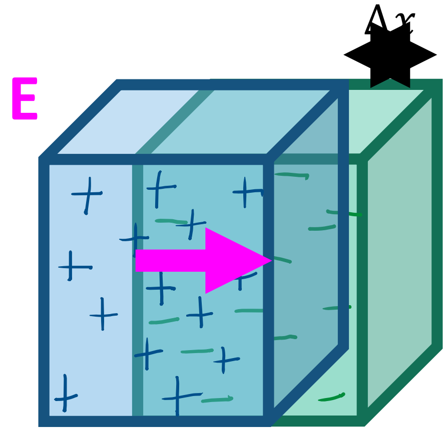
- Plasma frequency



Plasma Basics

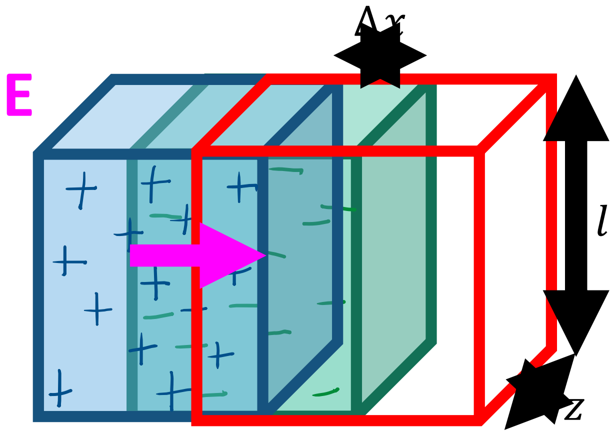
- Plasma frequency

- Use Gauss' Law to find E
- Apply Newton's 2nd Law to find equation of motion



Plasma Basics

- Plasma frequency



$$\int \vec{E} \cdot d\vec{A} = Q_{enc}/\epsilon_0$$

$$Elz = en_e(\Delta x \; l \; z)/\epsilon_0$$

$$E = en_e \Delta x/\epsilon_0$$

$$ma = F$$

$$m_e \frac{d^2 \Delta x}{dt^2} = -eE$$

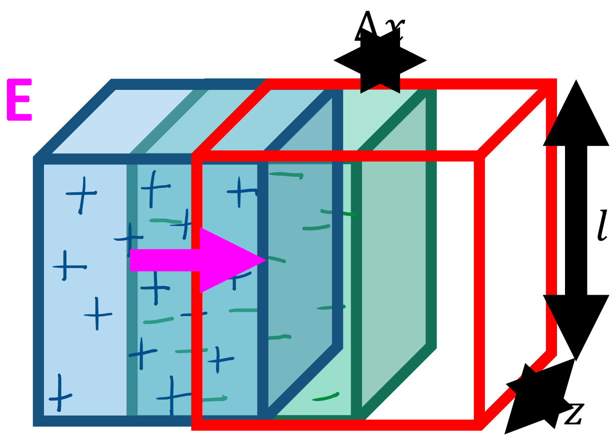
$$\ddot{x} = -\frac{n_e e^2}{m_e \epsilon_0} x$$

Compare with Hooke's Law $x = -\omega^2 x$

$$\omega_{pe}^2 = \frac{n_e e^2}{\epsilon_0 m_e}$$

Plasma Basics

- Plasma frequency



$$\omega_{ps}^2 = \frac{n_s e^2}{\epsilon_0 m_s}$$

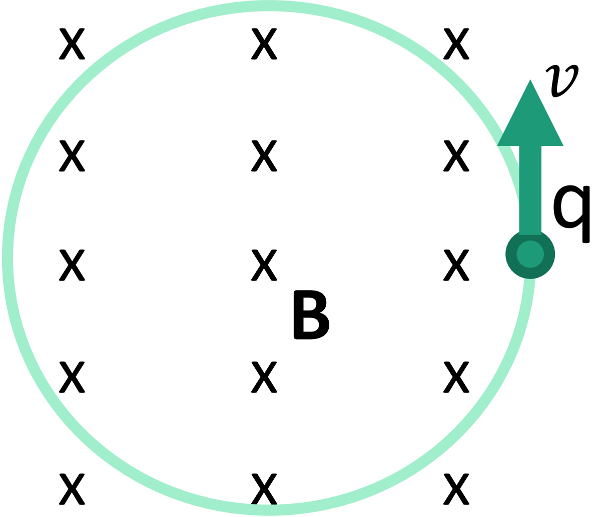
$$\omega_{ps}\lambda_{Ds} = c_s$$

If both the electrons and the ions move in the slab model, then

$$\omega_p = \sqrt{\omega_{pe}^2 + \omega_{pi}^2} \approx \omega_{pe}$$

Plasma Basics

- Cyclotron frequency



$$\omega_{cs} = \frac{q_s B}{m_s}$$

$$F = ma$$

$$q(\vec{v}\times\vec{B}) = -mv^2/r$$

$$qvB = mv^2/r$$

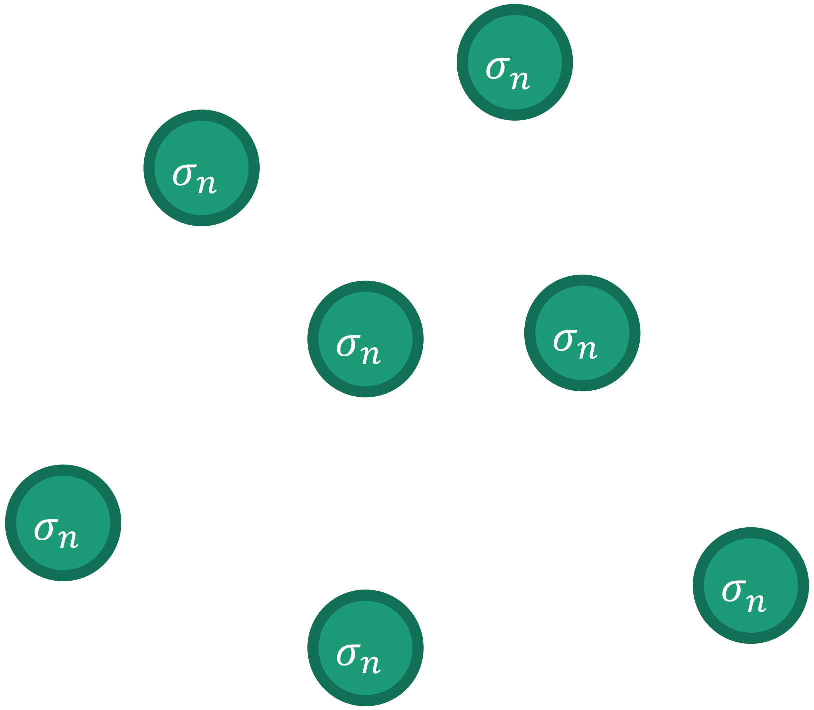
$$v = qBr/m$$

$$\omega = \frac{v}{r} = \frac{qB}{m}$$

Plasma Basics

- Collision frequency

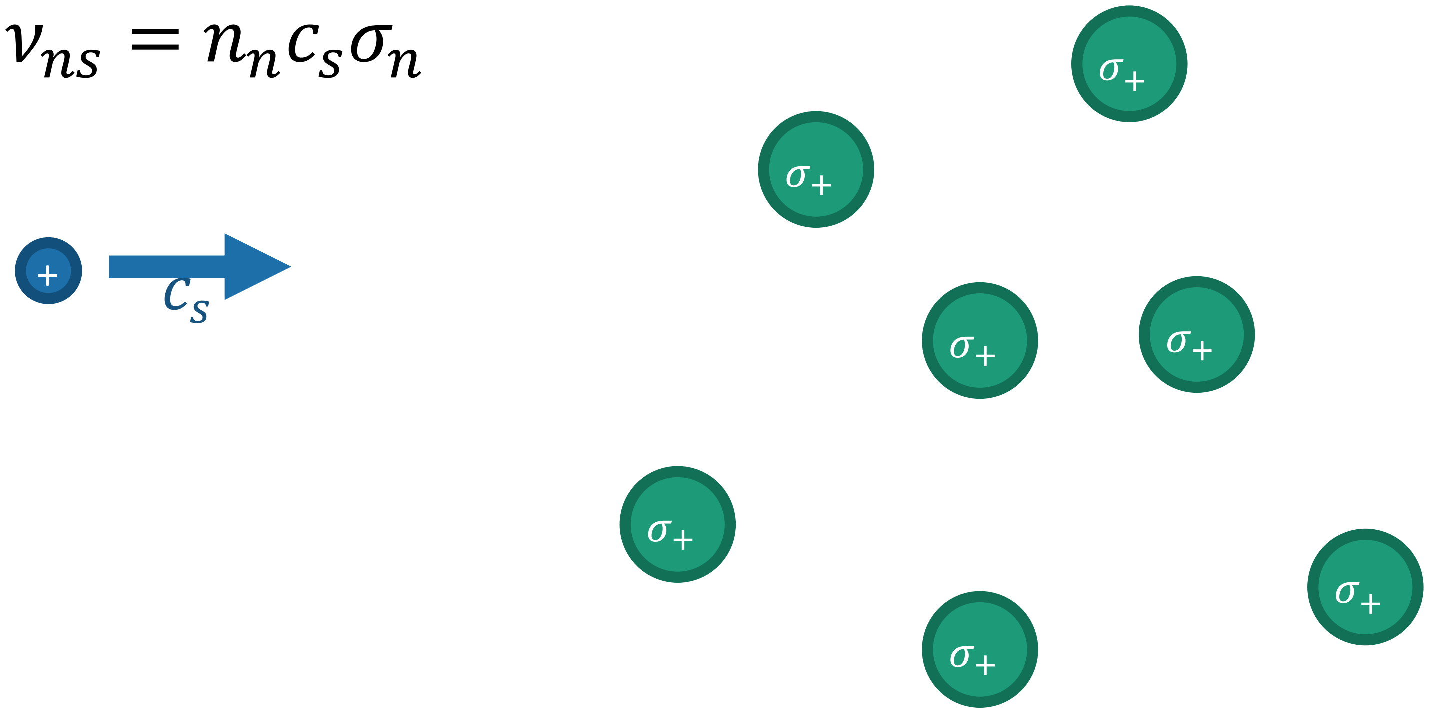
$$\nu_{ns} = n_n c_s \sigma_n$$



Plasma Basics

- Collision frequency

$$\nu_{ns} = n_n c_s \sigma_n$$



Other considerations

Number of electrons in a Debye cube

$$N_D = n_e \lambda_{De}^3$$

Macroscopic averages are meaningful:

- if $N_D \gg 1$ → treat plasma as a fluid

Debye shielding: assume $\frac{e\Phi}{kT_e} \ll 1$

Assumption only valid if $N_D \gg 1$

$$\Phi = \frac{1}{4\pi\epsilon_0}\frac{e}{r}$$

$$\Phi = \frac{1}{4\pi\epsilon_0} e n_e^{1/3}$$

$$\frac{e\Phi}{kT_e} = \frac{1}{4\pi\epsilon_0}\frac{e^2}{kT_e} n_e^{1/3}$$

$$\frac{1}{\lambda_D^2} = \frac{n_e e^2}{\epsilon_0 k T_e}$$

$$\frac{e\Phi}{kT_e} = \frac{1}{4\pi}\frac{1}{N_D^{2/3}}$$

Summary

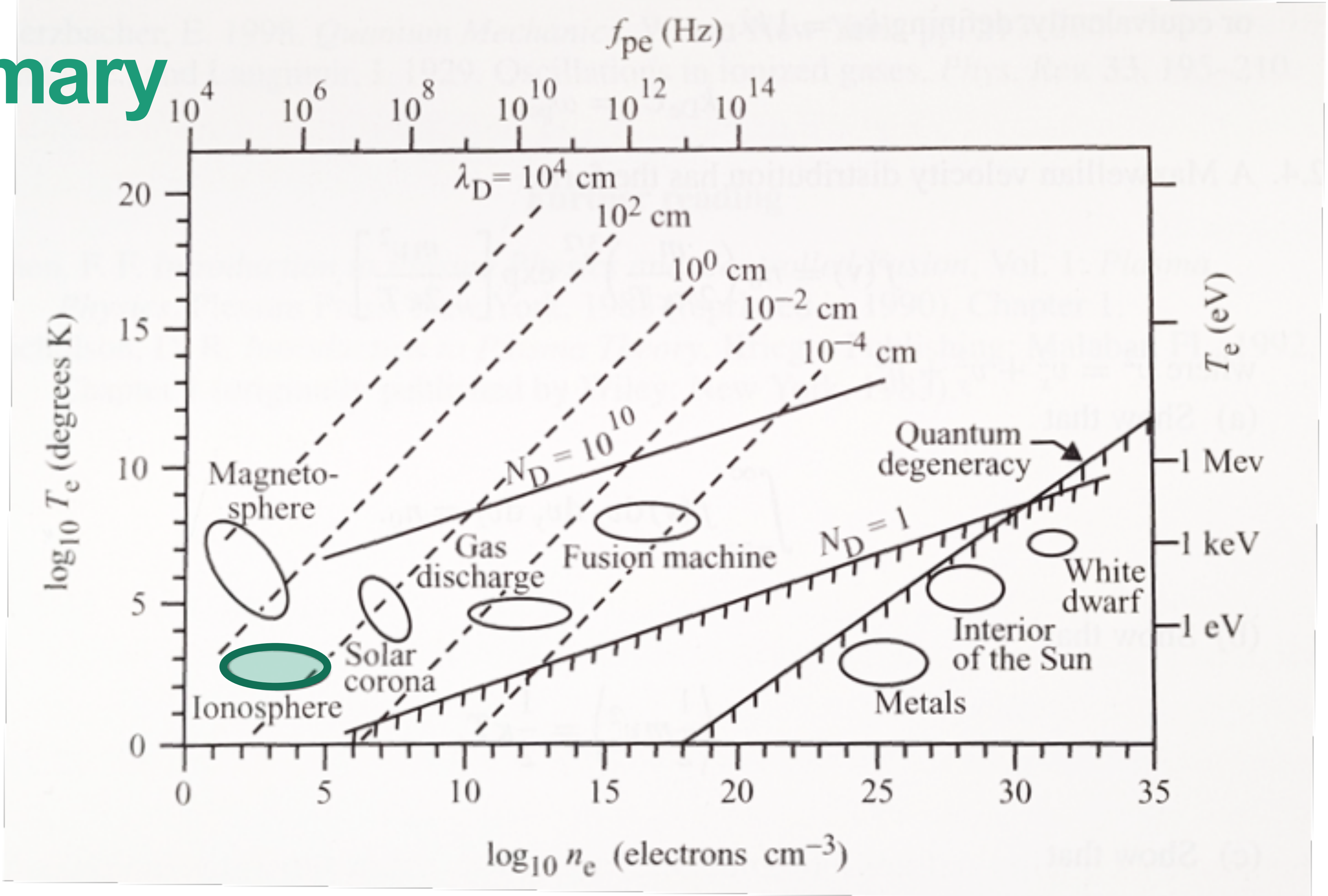
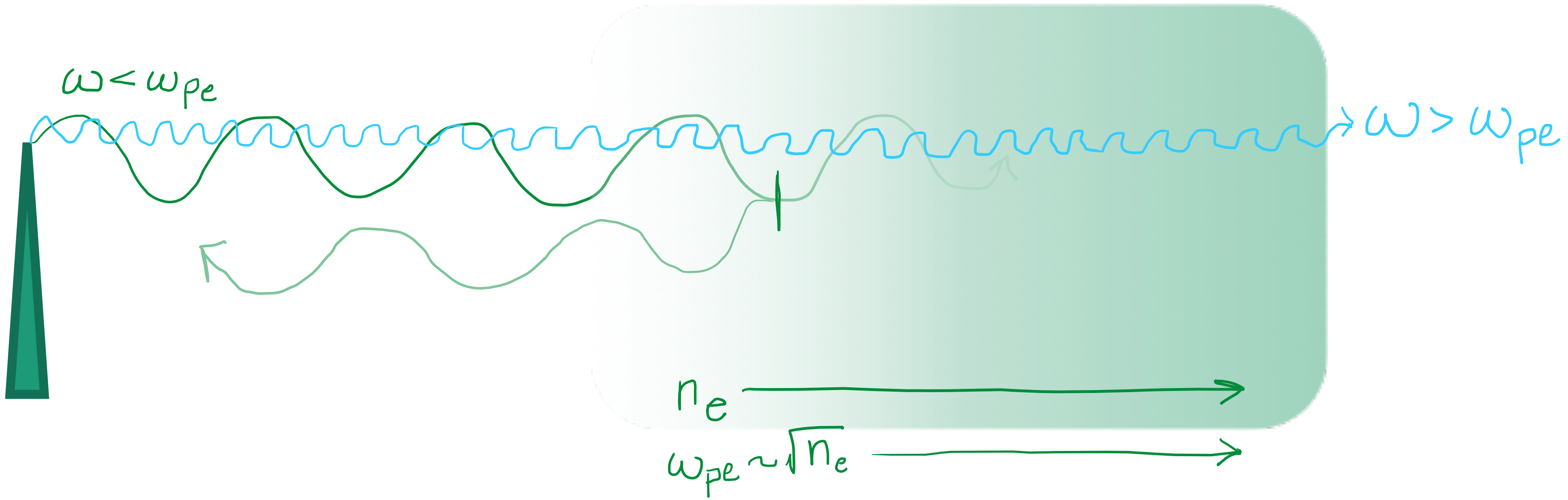


Figure 2.5. The temperature and density of various plasmas (Gurnett and Bhattacharjee, *Introduction to Plasma Physics*).

One cool application:

- Bouncing radio waves from the ionosphere



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