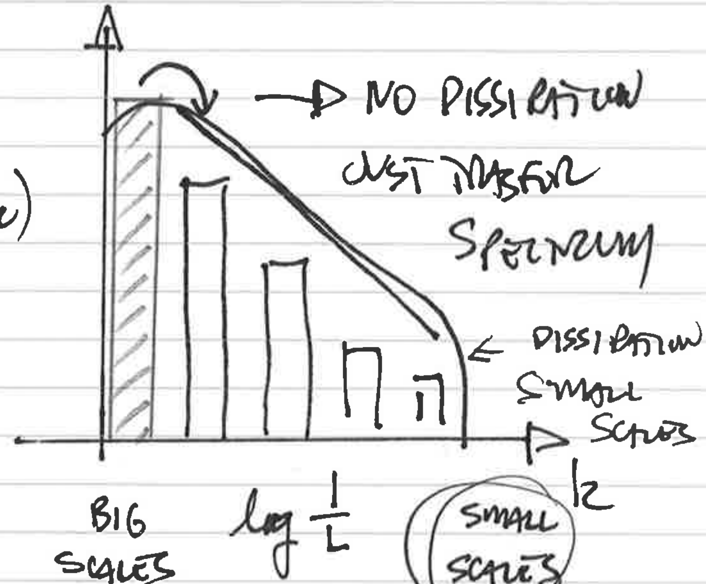
Kolmogorov 1941

SULI Lecture M. Brown (2020)

Turbulence:

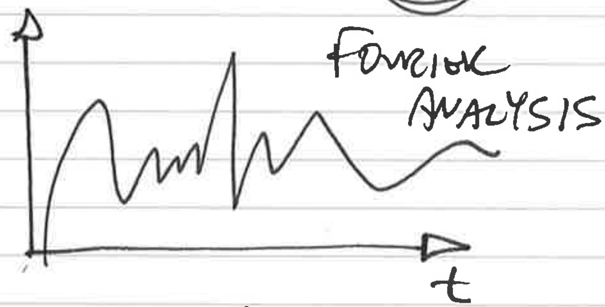
Assume: $E(k) = C \epsilon^{\alpha} k^{-\beta}$



ENERGY TRANSFER RATE

$$\epsilon = \frac{\frac{1}{2} m v^2}{\tau} \rightarrow \frac{v^2}{\tau} = \frac{v^2}{\ell / v} = \frac{v^3}{\ell}$$

ENERGY/MASS



(1) $\epsilon \sim k v^3$

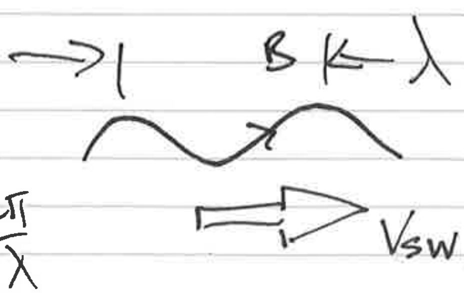
(2) $E(k) \sim v^2 / k = v^2 k^{-1}$

$$v^{2} k^{-1} = \propto k^{\alpha} v^{3\alpha} k^{-\beta} \quad \ldots \text{Find } \alpha \ \& \ \beta$$

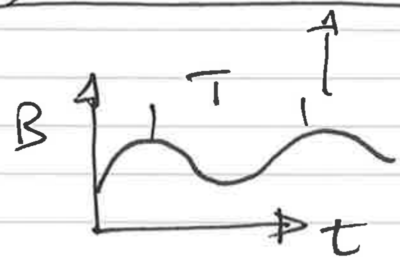
$$2 = 3\alpha \rightarrow \alpha = 2/3$$

$$-1 = \alpha - \beta \rightarrow \beta - \alpha = 1 \ \ldots \ \beta = 5/3$$

KOLMOGOROFF LAW: $\boxed{E(k) = C \epsilon^{2/3} k^{-5/3}}$



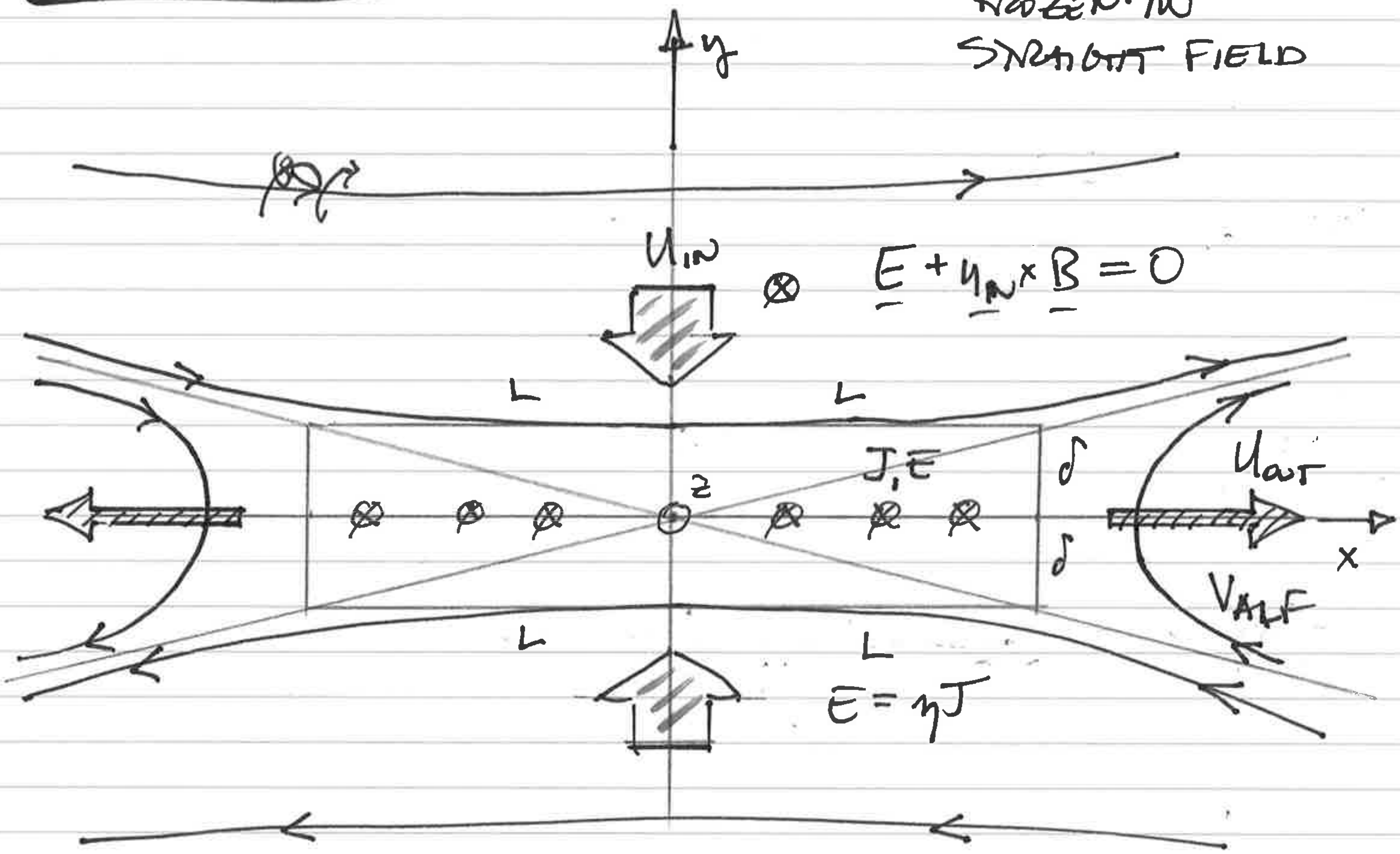
$k = \frac{2\pi}{\lambda}$



$\omega = \frac{2\pi}{T}$

RECONNECTION: PARKER·SWEET 1957

FROZEN·IN
STRAIGHT FIELD



- CONTINUITY $\quad \underline{\nabla} \cdot \underline{u} = 0 \rightarrow \oint \underline{u} \cdot \underline{da} = 0$

$$u_{IN} L = u_{OUT} \delta \qquad \boxed{\frac{u_{IN}}{V_{OUT}} = \frac{\delta}{L}}$$

- ENERGY CONSERVATION

$$\overset{IN}{\frac{B_{IN}^2}{\cancel{2}\mu_0}} = \overset{OUT}{\frac{1}{\cancel{2}}\rho u_{OUT}^2} \qquad \rho = \frac{MASS}{VOL}$$

$$u_{OUT}^2 = B_{IN}^2 / \rho\mu_0 = \boxed{V_{ALF}^2}$$

ASSUMPTIONS

- 2 DIMENSIONAL
- ✓ INCOMPRESSIBLE
- STEADY STATE $\frac{\partial}{\partial t} = 0$
- MHD NO PROTONS NO electrons
- ✓ ALL MAGNETIC ENERGY IS CONVERTED TO FLOW (KINETIC ENERGY)