A NEW METHOD FOR THE OPTIMIZATION OF FINITE BUILD STELLARATOR COILS

L. Singh[1], T. Kruger[1], C. Zhu[2], S. Hudson[2], A. Bader[1], D.T. Anderson[1]

University of Wisconsin Madison[1], Princeton Plasma Physics Laboratory[2]

Background

Stellarator Confinement

- Stellarators rely predominantly on external coils to produce rotational transform.
- Stellarators have the advantage of steady-state operation, no known disruption regime, and no Greenwald density limit.
- Necessity of rotational transform without strong plasma currents make coils fully 3-D and complex.
- Finding a coil set with desirable physics and engineering properties is a crucial step in the design and optimization of new devices

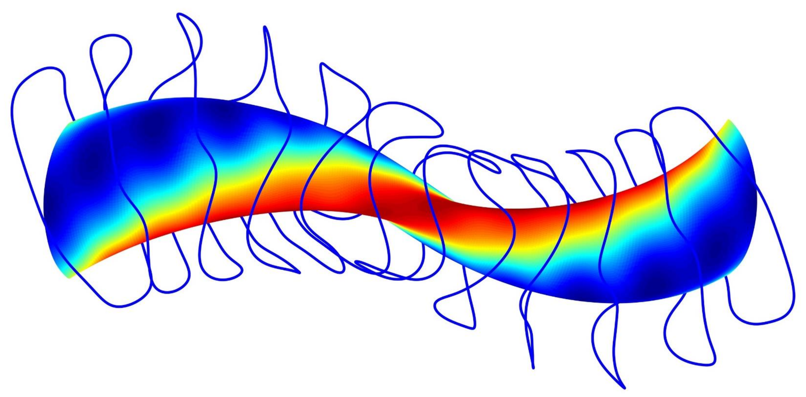

Fig 1. Plasma boundary and example single filament coil set for the HSX stellarator. Coloring indicates magnetic field strength, which varies from low (blue) to high (red). Only a single period of the device is shown.

$$R(\theta,\varphi) = \sum_{m,n} R_{mnc}\cos(m\theta - n\varphi)$$

$$Z(\theta,\varphi) = \sum_{m,n} Z_{mns}\sin(m\theta - n\varphi)$$

Fourier Boundary Parameterization

Stellarator Optimization Procedure

1. Generate magnetic equilibrium for stellarator plasma to capture physics goals
2. Generate a coil set to reproduce target magnetic field to support equilibrium

Coil Optimization Codes

- Optimization codes can target both physics and engineering goals
  - Linear codes (e.g. REGCOIL, NESCOIL) require an input "winding surface" that coils lie on.
  - Nonlinear codes (e.g. FOCUS, ONSET) can optimize coils without the winding surface using space curves, splines, and other representations.
- All existing codes ultimately produce zero-thickness single filament coils.
  - Single filaments cannot be physically realized (due to zero-thickness)
  - Close to the plasma, single filament model breaks down due to $1/R^3$ effect
  - A more accurate approximation to finite-thickness coils is the multifilament model, in which each single filament is replaced with a set of parallel single filaments.

$$\boldsymbol{r}(\theta) = \left[\sum_{n=0}^{N} \boldsymbol{X}_{c,n} cos(n\theta) + \boldsymbol{X}_{s,n} sin(n\theta)\right]$$

$$\theta \in [0, 2\pi]$$

$$\boldsymbol{X}_{c,n} = \begin{pmatrix} x_{c,n} \\ y_{c,n} \\ z_{c,n} \end{pmatrix} \quad \boldsymbol{X}_{s,n} = \begin{pmatrix} x_{s,n} \\ y_{s,n} \\ z_{s,n} \end{pmatrix}$$

C. Zhu et al., Nucl. Fusion, 58, 016008 (2018)

FOCUS single filament model. FOCUS space curves are represented by a Fourier series in each cartesian direction.

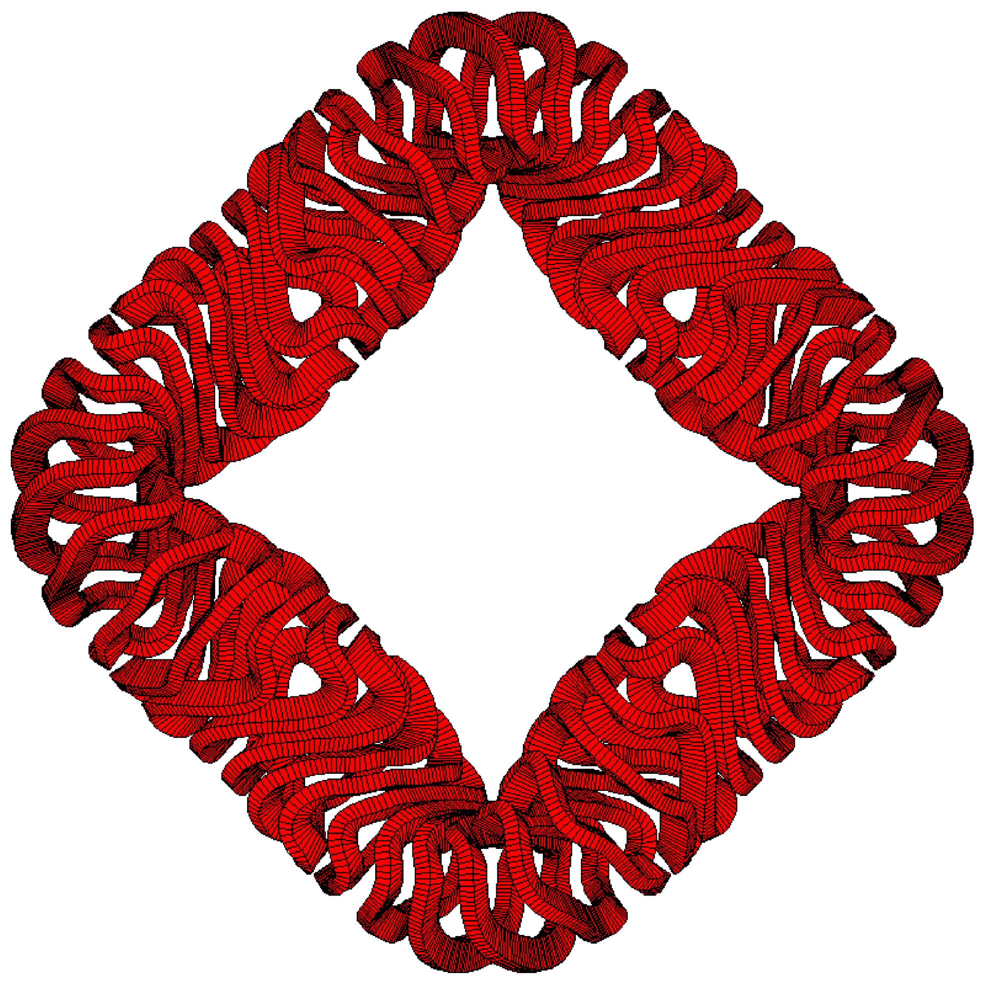

Fig 2. Example finite build coil set for the HSX stellarator at UW-Madison

The Multifilament Stellarator Coil Model

Multifilament Coil Construction

- Building on the FOCUS model, each single filament space curve is replaced with a set of evenly spaced single filaments.
- Winding pack rotation is determined by an angle function α , which is parameterized using a Fourier series. This embeds flexibility to optimize rotation of the finite build coil.

$$\boldsymbol{x}(\theta) = \boldsymbol{r}(\theta) - \boldsymbol{X}_{c,0}$$

$$\hat{\boldsymbol{t}} = \frac{\partial \boldsymbol{r}/\partial\theta}{|\partial \boldsymbol{r}/\partial\theta|}$$

$$\hat{\boldsymbol{n}} = \boldsymbol{x} - (\boldsymbol{x}\cdot\hat{\boldsymbol{t}})\hat{\boldsymbol{t}}$$

$$\hat{\boldsymbol{b}} = \hat{\boldsymbol{t}} \times \hat{\boldsymbol{n}}$$

$$\alpha(\theta) = \sum_{n=0}^{N} \alpha_{c,n}\cos(n\theta) + \alpha_{s,n}\sin(n\theta)$$

$$\hat{\boldsymbol{\rho}} = \cos(\alpha)\,\hat{\boldsymbol{n}} + \sin(\alpha)\hat{\boldsymbol{t}}\times\hat{\boldsymbol{n}}$$

$$\hat{\boldsymbol{t}}\times\hat{\boldsymbol{\rho}} = -\sin(\alpha)\,\hat{\boldsymbol{n}} + \cos(\alpha)\hat{\boldsymbol{t}}\times\hat{\boldsymbol{n}}$$

Mulitifilament Parameterization

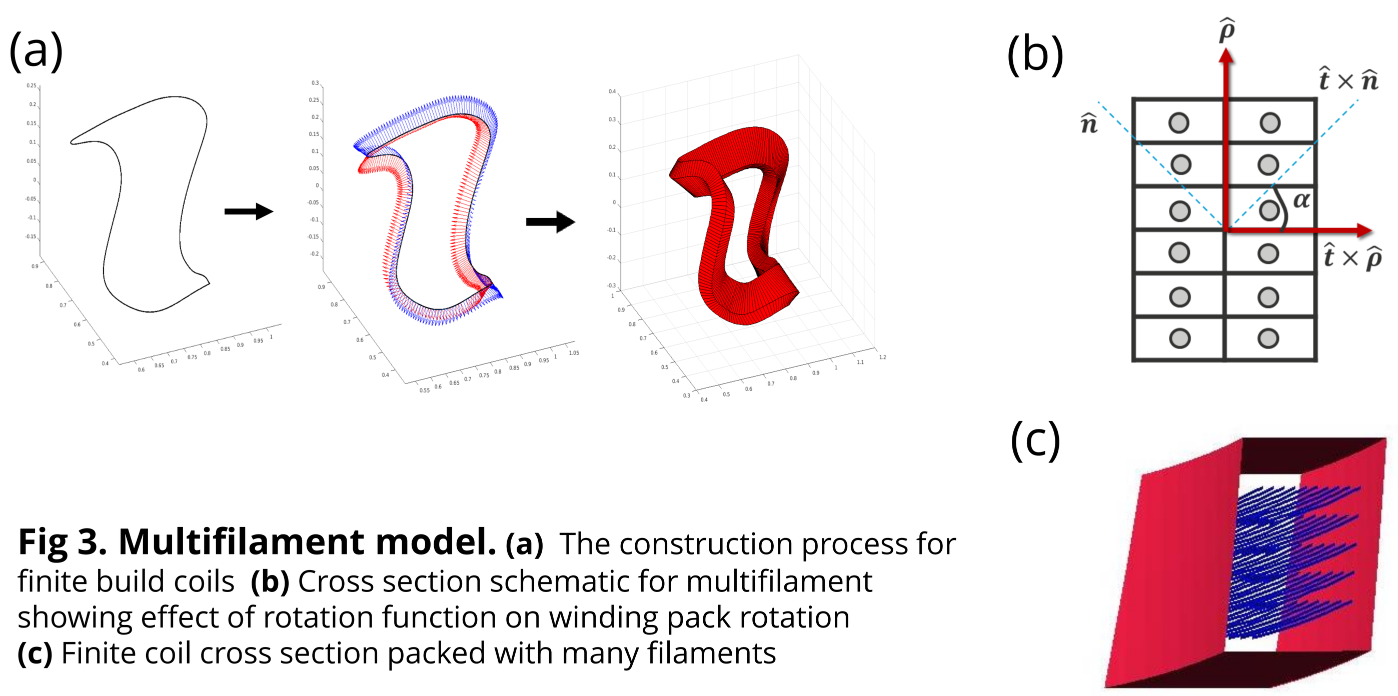

Fig 3. Multifilament model. (a) The construction process for finite build coils **(b)** Cross section schematic for multifilament showing effect of rotation function on winding pack rotation **(c)** Finite coil cross section packed with many filaments

Multifilament Optimization

- Optimization is carried out by minimizing field error objective function
  - **B** for multifilaments is determined using superposition of fields from each discretized segment.
  - Optimization of rotation function mode amplitudes is carried out using steepest descent method with central difference first derivatives.

$$\boldsymbol{B} = \frac{\mu_0}{4\pi}\hat{\boldsymbol{e}}\times\boldsymbol{R_i}\frac{2L(R_i+R_f)}{R_iR_f}\frac{1}{(R_i+R_f)^2-L^2}$$

$$f_B = \int\left(\frac{\boldsymbol{B}\cdot\hat{\boldsymbol{n}}}{B}\right)^2 dS$$

Hanson, Hirschman, Physics of Plasmas 9, 4410 (2002)

Magnetic field for a line segment and field error objective function

Application: HSX Stellarator

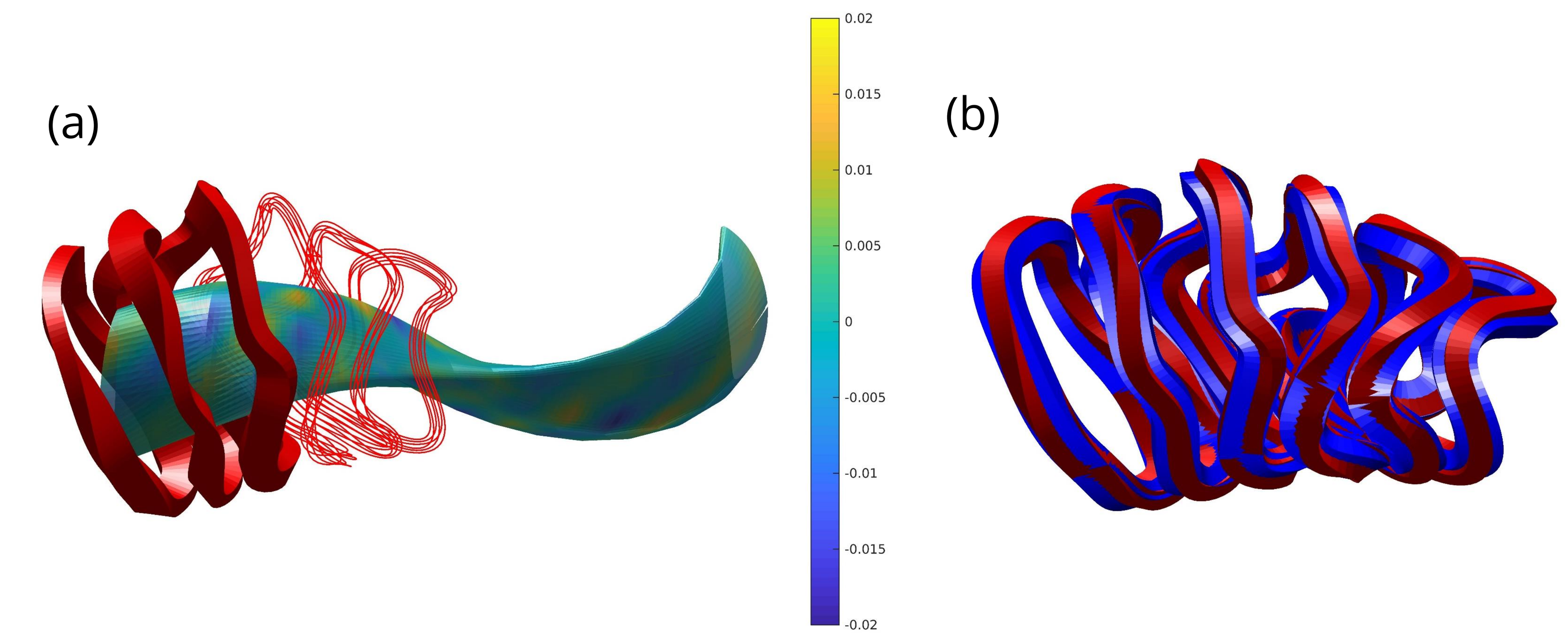

Fig. 4 Initial optimization results for HSX stellarator. (a) Optimized stellarator coils plotted over a single period of the HSX stellarator. A half period of coils is shown, the first half showing finite cross section and the second showing multifilaments. The coloring of the surface indicates $\boldsymbol{B}\cdot\hat{\boldsymbol{n}}$ error at each surface element. **(b)** Unoptimized (blue) and optimized (red) coils for one half period of the HSX stellarator. Objective function for normal error decreased by about 10% after initial optimization.

Future Work

- Add more objective functions to optimization (e.g. coil-coil and coil complexity)
- Optimize finite build centroids and build dimensions
- Implement multi-layer parallelization (OpenMP, MPI, GPU)