Shape Gradients and Magnetic Tolerance in MUSE

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MUSE Stellarator: Motivation

- Since plasma is sensitive to small magnetic field errors, need super high-precision field, complicated coil design
- Tabletop stellarator
- Permanent magnets in addition to 16 circular, planar coils
- Goal: science education, test permanent magnet design before large-scale implementation
- Searching for field achievable with inboard dipoles, feasible tolerances

Shape Gradients and Tolerance

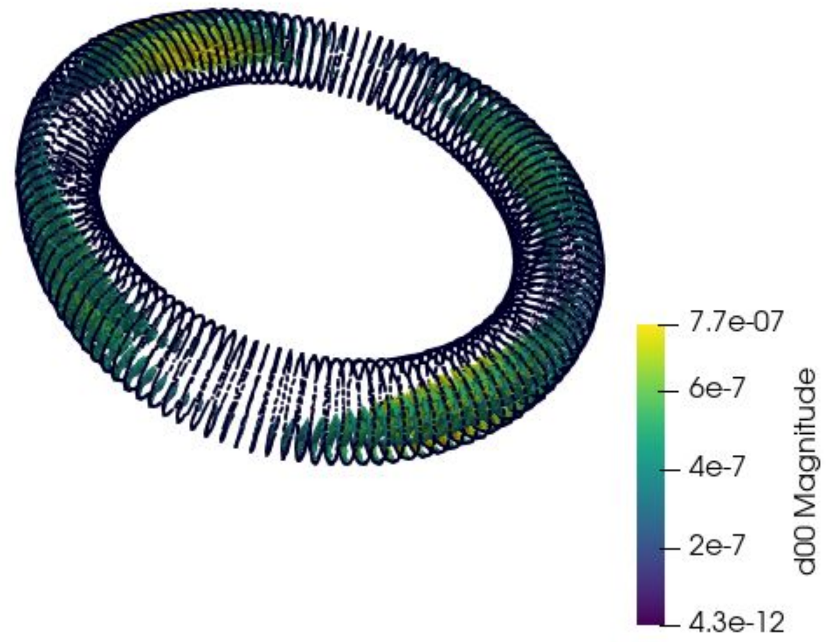
- Shape gradient S:
  - Shows sensitivity of quantity f (e.g., Bn, quasi-axisymmetry) to changes in parameters p_j (e.g., Fourier modes, dipole position)
  - Gives spatially local information
- Tolerance: 1/S
  - Engineering tolerance

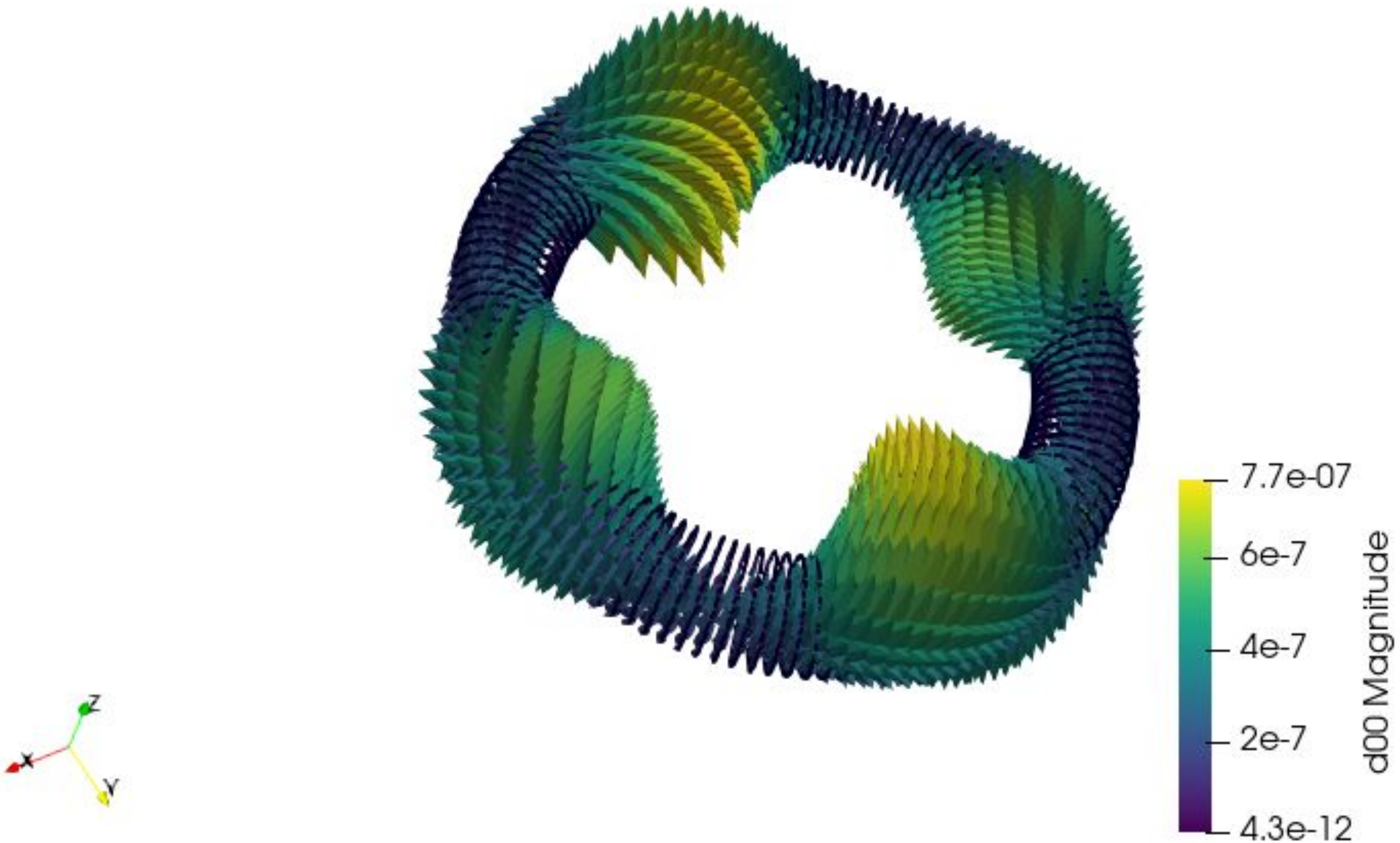
FAMUS

- FORTRAN optimization code
- Input:
  - Dipole location, orientation
  - Target B-field, plasma boundary
- Penalty system accounts for diagnostic port locations $\rho = p^q, p \in [0, 1]$
- Normal field error $F_B(\rho, \theta, \phi) = \iint_S (\mathbf{B}_M \cdot \mathbf{n} - B_n^{tgt})^2 \mathrm{d}a$
- Uses $\frac{\mathrm{d}F_B}{\mathrm{d}x}$, $\frac{\mathrm{d}F_B}{\mathrm{d}y}$, and $\frac{\mathrm{d}F_B}{\mathrm{d}z}$ to optimize system

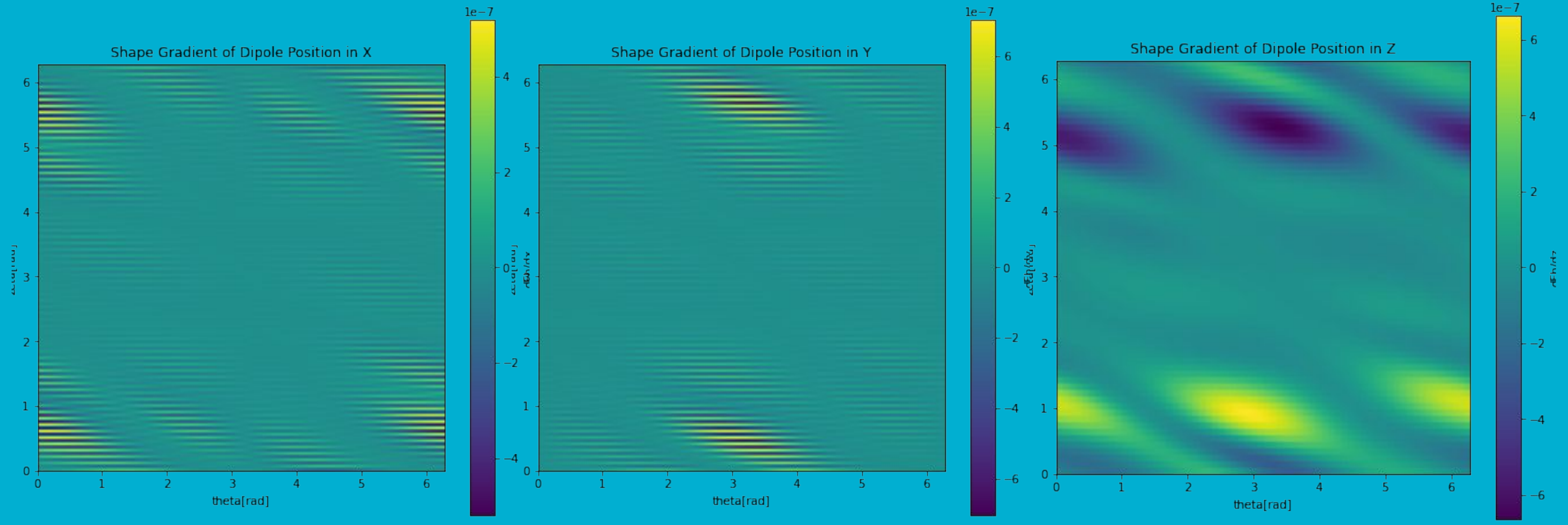
Elliptical Plasma Shape

- Calculated shape gradient of normal field error
- Rotating elliptical stellarator with two plasma periods
- Dipoles in grid-like configuration over toroidal surface
- Vector of derivative values associated with each dipole



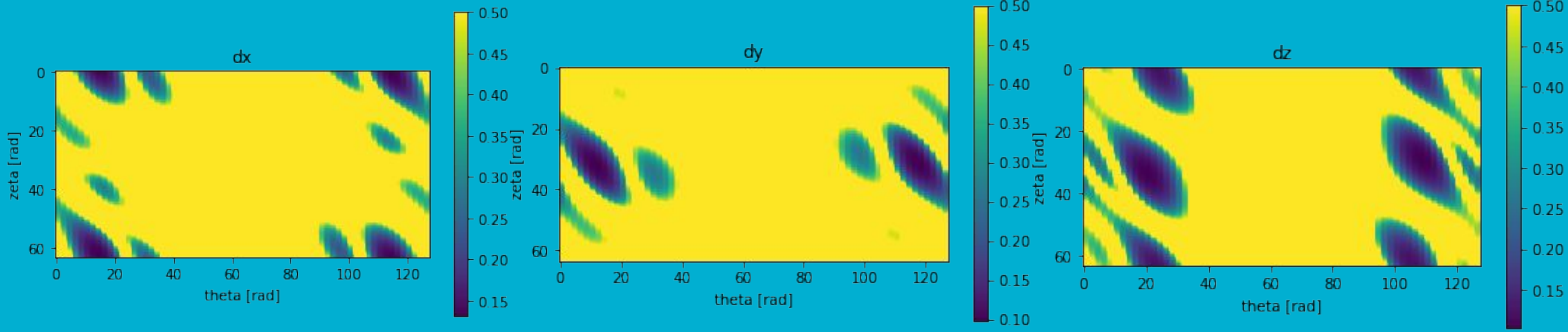

7.7e-07
6e-7
4e-7
2e-7
4.3e-12
d00 Magnitude
Z
X
Y

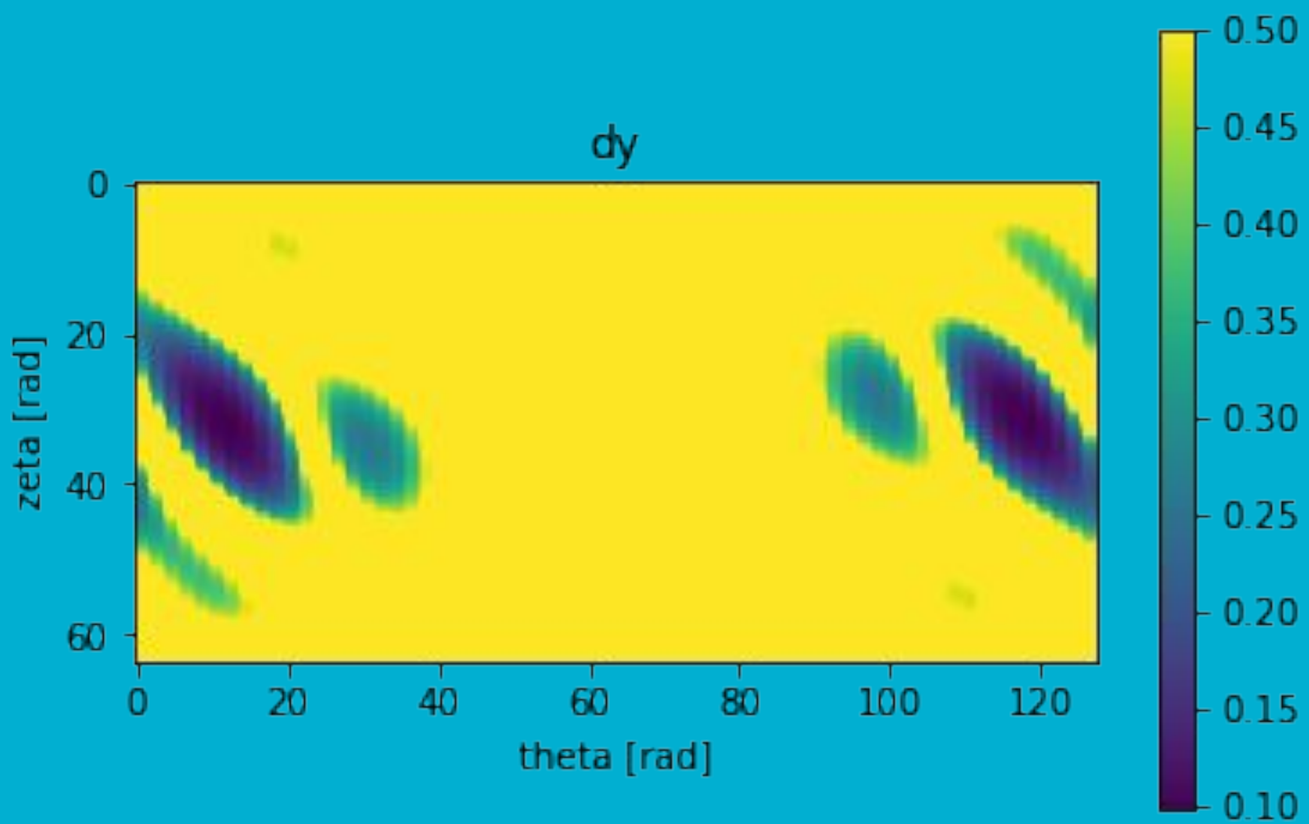
Visualizing Shape Gradients: Ellipse



Problems With the Shape Gradient Approach

- How far would dipoles have to move to result in a 5% change in $\mathbf{F}_B$?



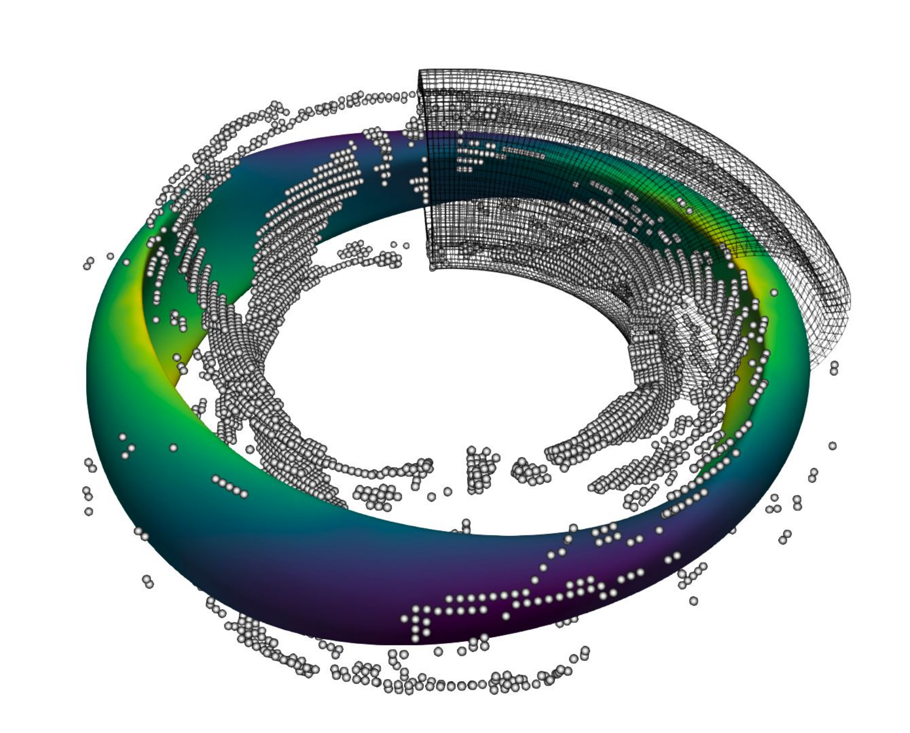

dy
zeta [rad]
theta [rad]
0
20
40
60
0
20
40
60
80
100
120
0.50
0.45
0.40
0.35
0.30
0.25
0.20
0.15
0.10

Solution: Hessian Matrix Method

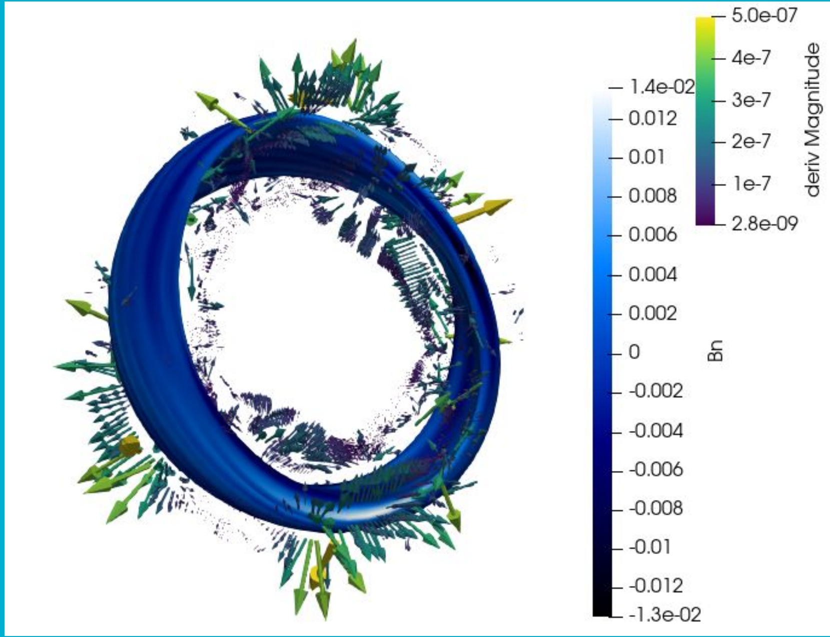
- Calculate the Hessian matrix of $\mathbf{F}_B$
- Taylor expansion of $\mathbf{F}_B$: $F(\mathbf{X}) = F(\mathbf{X}_0) + (\mathbf{X} - \mathbf{X}_0)^T \cdot \mathbf{g}_0$
$$+ \frac{1}{2}(\mathbf{X} - \mathbf{X}_0)^T \cdot \mathbf{H}_0 \cdot (\mathbf{X} - \mathbf{X}_0) + \ldots$$
- Assume $|\Delta\mathbf{X}| = |\mathbf{X} - \mathbf{X}_0|$ small
- $\Delta F(\mathbf{X}) = F(\mathbf{X}) - F(\mathbf{X}_0) \approx \frac{1}{2}(\Delta\mathbf{X})^T \cdot \mathbf{H}_0 \cdot \Delta\mathbf{X}$
- Eigenvalues of Hessian quantify sensitivity: $\mathbf{v}_i^T \cdot \mathbf{H}_0 \cdot \mathbf{v}_i = \lambda_i$

MUSE Candidate: NCSX

- Plotted all dipoles with $\rho > 0.9$
- Mostly meets "inboard" goal
- Major radius: 32 cm
- Minor radius: 6cm
- Plasma volume: 33L
- Mean $\mathbf{B}$ field: 0.2 T
- Two-period quasi-axisymmetric stellarator




5.0e-07
4e-7
3e-7
2e-7
1e-7
1.6e-09
deriv Magnitude
X
Y
Z


5.0e-07
4e-7
3e-7
2e-7
1e-7
2.8e-09
deriv Magnitude
1.4e-02
0.012
0.01
0.008
0.006
0.004
0.002
0
-0.002
-0.004
-0.006
-0.008
-0.01
-0.012
-1.3e-02
Bn

Conclusion

- We can use FAMUS to calculate derivatives that illustrate system sensitivity (i.e., the shape gradient) to dipole displacement
- Inverse of shape gradient gives engineering tolerances, but these values are not useful for small S
- Use Hessian matrix method in case of small S
- Can use this information to evaluate candidate fields for MUSE

Future Work

- Calculate shape gradient for quasisymmetry, rotational transform
- Assess the impact of dipole position on these quantities
- Find optimum dipole configuration that preserves desired values of these quantities
- Use the Hessian matrix method to assess system sensitivity to $\frac{\mathrm{d}F_B}{\mathrm{d}x}$
- Update FAMUS

Acknowledgements

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