Summer 2019 Homework Sessions June 13th-14th

Session I 3:00-5:00PM

LOCATION:	**Thursday: B318 Friday: Auditorium**	**LSB Commons**	**Display Wall**	**A118-IT Mtg Rm.**	**Theory Lounge**	**B153**	**Theory Conference Room**
INSTRUCTOR:	**Brian Kraus**	**Alexandra LeViness**	**Andy Alt**	**Sierra Jubin**	**Himawan Winarto**	**Joe Abbate**	**Valentin Skoutnev**
	Barbara Garcia	Ryan Golant	Michaela Hennebury	Hibiki Yamazaki	Andrew Christopher He	Courtney L Johnson	Kellin Murphy
	Gabriel Antonio Gonzalez	Shun Kamiya	Adeline Hennebury	Kota Yanagihar	Trace Johnson	Ryan Arbon	Le Viet Nguyen
	Andrew Herschberg	Keisuke Kanda	Esha Rao	*Rob Goldston PEI student*	Kai Torrens	Promise Oluwagbope Adeb	Wyatt Pauley
	Natalie Cannon	Shinichiro Kojima	Luquant Singh	Andrew Brown	Joshua James Luoma	Landon David Bevier	James Robinson
	Carlos Andre Catalano	Matthew Barber	Marion Elizabeth Smedberg	Joshua Latham	Stephen Yan	Andy Brown	Jamal Johnson
	Marco Andres Miller	Loukas Carayannopoul	Jace Christian Waybright	Oleksandr Redin Yardas	Cole Alexander Love-Bal	Alexander Liu	Justin Cohen
	Colin Myrick	James LeCompte	Eric Wolf				Henry Fetsch
	Samantha Ann Pereira	Cristian Arens	Laura Natalia Zaidenberg				Daniel Thomas
	Paul Simmerling	Ben Alessio	Ish Kaul				
	Katrina Teo						
	Mikayla Washington						
	Anna Martha Wolz						

Session II 5:45-6:50PM

Anyone not on the list can stay in Auditorium and participate

Summer Program in Plasma Physics and Fusion Engineering

Problem Set #1

June 11, 2018

The following homework is intended to introduce you to the course material and to set you thinking about some of the things you will be learning. If you cannot answer all or any of these questions, do not worry about it. Keep the questions in mind, and, by the end of the week, you should be able to answer them.

Problem 1 *Fusion Reactor and its parameters*

A typical first-phase-ITER plasma is composed of ionized deuterium at 10keV (assume the ion and electron temperatures are equal) with a density of $10^{20}\,\mathrm{m}^{-3}$ (or $10^{14}\,\mathrm{cm}^{-3}$) in a magnetic field of 5T (or 50kG). Calculate the following lengths for the plasma and compare them to the size of the plasma, a torus with major radius 6.2m and minor radius 2m.

(a) Debye length

(b) electron and ion gyroradii

(c) electron and ion mean free paths

(d) gyroradius of a 3.5 MeV α particle (the product of a D-T fusion reaction)

In order to estimate the mean free paths, take the ratio of the thermal velocity to the collision rate. The $\ln \Lambda = 20$ in the formula for collision rates.

(e) For a $1000MW$ electric fusion power plant the size of ITER, how much tritium in grams is used up per hour?

(f) For the same power plant, how long would it take to burn up a 50:50 DT mix?

(g) For the same power plant, if each neutron stopped displaces one atom in the blanket, how long would it take to displace most of the atoms in a $0.5m$ thick blanket?

(h) Given that about one atom in 6000 of hydrogen in ocean water is deuterium, and that the average electricity use per capita in the US is 1700 W, approximately what volume of seawater provides enough D (then fused with a proportional amount of Tritium) to power your life for a year?

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Problem 2 *Pressure and Energy Density*

Using the parameters above, for ITER find:

(a) The plasma pressure in Torr.

(b) The kinetic energy density.

(c) The magnetic energy density.

(d) How many joules are stored in the magnetic fields.

(e) How much is this compared to the energy in TNT.

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Problem 3 *Probes and Sheaths*

This question is concerned with the plasma sheath – the region that forms between a plasma and any surface in contact with it.

(a) Say that an isolated object suddenly appears in our uniform proton-electron plasma, describe in words what happens. Why does the sheath form?

(b) It can be shown that ions are accelerated towards the surface by the sheath potential so that they reach the surface with the Bohm velocity, $c_s = \sqrt{k_B T_e / m_i}$. Here k_B is Boltzmann's constant, T_e is the electron temperature and m_i is the ion mass. Assuming the electrons are thermal (i.e., have a Maxwellian velocity distribution), what is the voltage difference between the plasma and the object.

(c) Imagine that we now connect a wire to our object, meaning the net current to the object is no longer constrained to be zero. Describe in words how we can use this to measure plasma properties. Which properties can we measure? What happens if the electrons do not have a Maxwellian distribution function.

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Problem 4 *Plasma Oscillations*

Consider a homogeneous cold electron plasma, where the ions may be assumed to be an infinitely massive neutralizing background. Suppose that a small fraction of the electrons in the slab $x_1 < x < x_2$ are removed and then placed at $x_1 + L < x < x_2 + L$.

(a) Linearize the fluid equations for the electrons and derive the equation for plasma oscillations.

(b) When might the nonlinear terms that you neglected be important?

(c) Sketch the subsequent evolution of the electron density, the electric field, and the electron fluid velocity. Sketch the trajectory of a single electron.

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Summer Program in Plasma Physics and Fusion Engineering

Problem Set #2

June 11, 2018

Problem 1 *Z-Pinch*

Consider a long cylinder of plasma with a current passing through the axis of the cylinder in the $\hat{z}$ direction. Ignore variation along z, and assume for simplicity that the current density can be written as $\vec{J} = J(r)\hat{z}$, i.e., as cylindrically symmetric. The current heats the plasma through the plasma resistivity, creating a pressure $p = p(r)$.

(a) Show:

$$\vec{B} = b_\theta(r)\hat{\theta} = \frac{\mu}{r}\int_0^r rJ(r)dr. \tag{1}$$

(b) The time to reach force balance is short, at which point $\nabla p = \vec{J} \times \vec{B}$. Show that:

$$\frac{dp}{dr} = -\frac{\mu}{r}J(r)\int_0^r rJ(r)dr. \tag{2}$$

(c) Where is the pressure the highest? Can you see why this might be called a *Z-pinch*? These devices, however, succumb to a number of MHD instabilities, although recently there has been reported stabilization during the current rise.

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Problem 2 *Distribution Function*

Suppose N electrons are distributed with distribution function $f(x, v, t)$ in 1D. That is, $f(x, v, t)dxdv$ is the probable number of electrons located at position x with velocity v at time t within a volume element $dxdv$.

(a) In terms of $f(x, v, t)$, what is the electron density $n(x, t)$? And the electron current density $j(x, t)$?

(b) Suppose at $t = 0$,

$$f(x, v, t = 0) = \begin{cases} 1, & \text{for } |x| < x_0,\ |v| < v_0; \\ 0, & \text{otherwise.} \end{cases} \tag{3}$$

Sketch $f(x, v, t)$ for non-colliding particles in the absence of external forces.

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Problem 3 *Non-Maxwellian Distribution Function*

Most of the schemes for extracting energy through nuclear fusion are *thermonuclear*, in other words, the plasma ions fusing have a thermal distribution of energies. Why might one wish for a non thermal distribution? Why might one be stuck with a thermal distribution?

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Problem 4 *Wave Kickin' (Quasilinear Diffusion)*

Consider an ion immersed in a uniform homogeneous magnetic field pointing in the $\hat{z}$ direction. The ion also interacts with a a wave, traveling in the $\hat{x}$ direction with phase velocity v_0. As the ion gyrates, it is struck by the wave every time its $\hat{x}$ directed velocity v_x is equal to the wave phase velocity, i.e., $v_x = v_0$. Assume that the interaction with the wave is instantaneous, so it results in no change to the ion position, but $v_x \to v_x + \Delta v$.

(a) Sketch the ion motion prior to interaction with the wave. Sketch the ion motion subsequent to an interaction with the wave.

(b) As a function of Δv, calculate the change in the gyro center of the ion as a result of interacting with the wave once.

(c) As a function of Δv, calculate the change in the energy on the ion as a result of interacting with the wave once.

(d) If the ion experiences many interactions with the wave, with the sign Δv random, sketch the trajectory of the ion in energy/gyrocenter space.

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