Intro to Magneto-Hydrodynamics (MHD)

by
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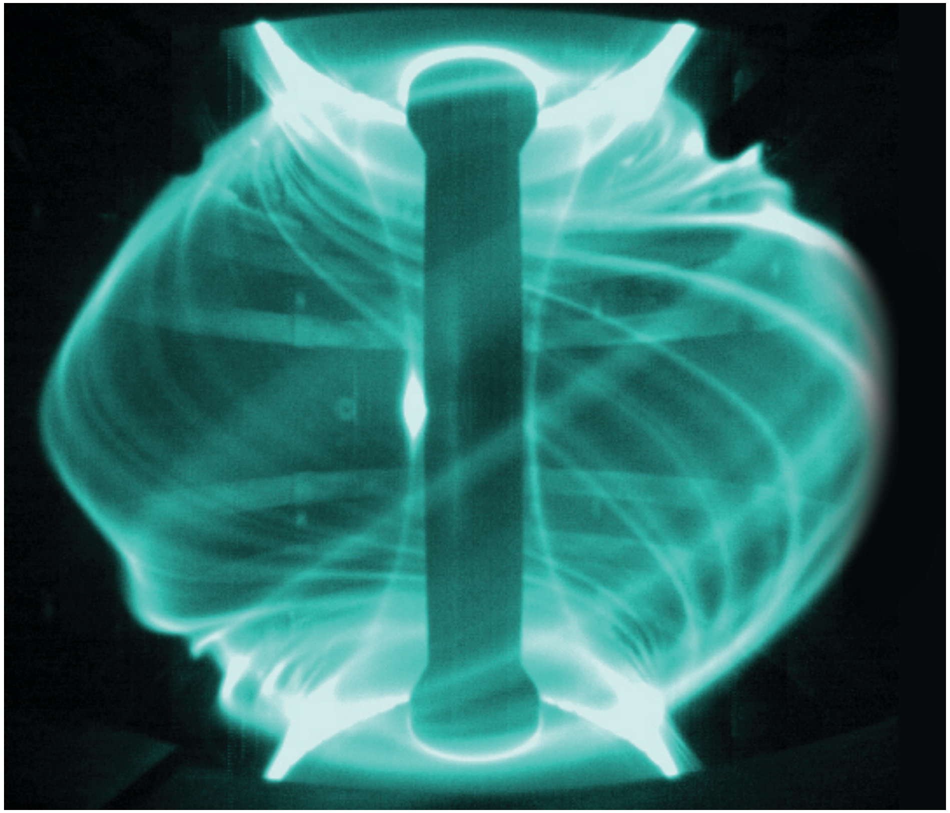
with cited contributions

Presented at the
SULI One-Week Course
PPPL

June 10^{th} 2019



Courtesy: MAST / CCFE

Outline of Presentation

- Pre-amble: Why the MHD model?
- Development of the MHD Equations
- MHD Equilibrium: 1-D, 2-D, 3-D Configurations
- MHD and its Relation to Global Operational Limits
- Brief Tour of Common MHD Instabilities and Their Control

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- **Brief Tour of Common MHD Instabilities and Their Control**

Pre-amble: Why is MHD important?

- Magneto-Hydro-Dynamics "MHD" (or the *fluid model*) provides a relatively simple way to compute the equilibrium and stability of a fusion plasma

MHD *cannot* tell you:

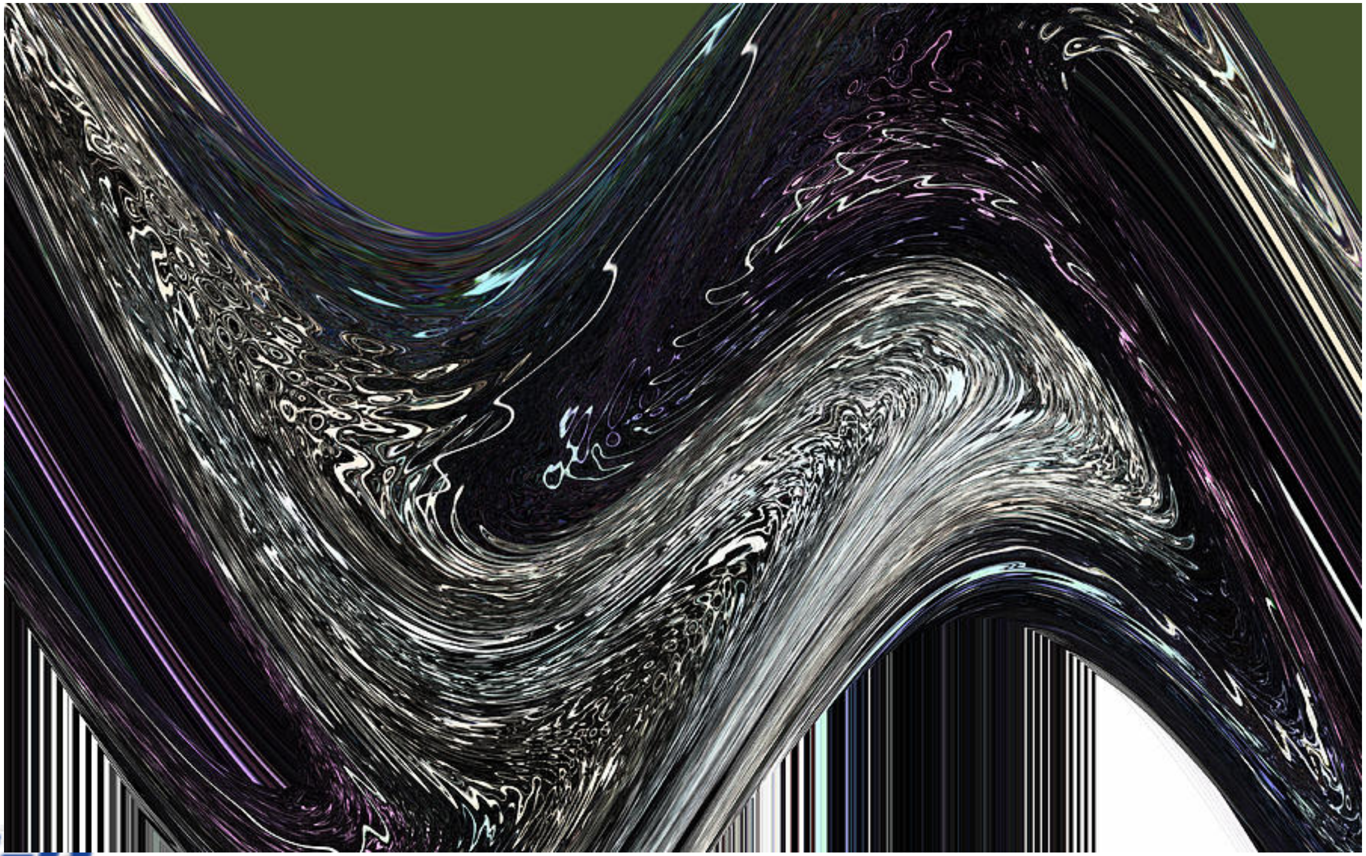
- How much fusion *will* I get in my plasma for a given input power?

MHD *can* tell you:

- What is the maximum pressure (fusion) my plasma can sustain?
- Where should I place my magnets to control the plasma?
- What are those wiggles and crashes in my plasma?

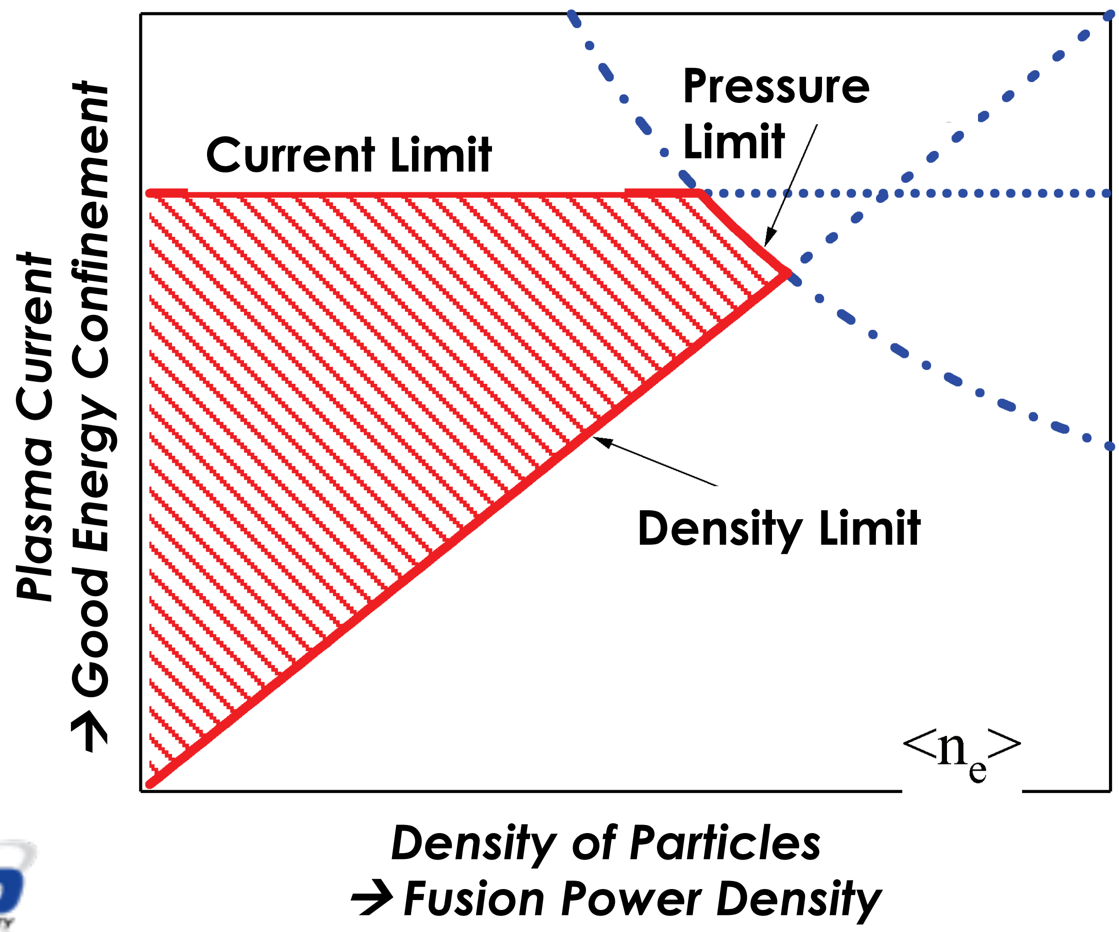
Example 1: MHD Describes Conducting Fluids … Liquid Metal

Key Variables:
Magnetic Field, Flow, Current



Liquid Metals Flowing is a photograph by Gregory Lafferty

Example 2: Most Tokamak Operational Limits are Governed by MHD



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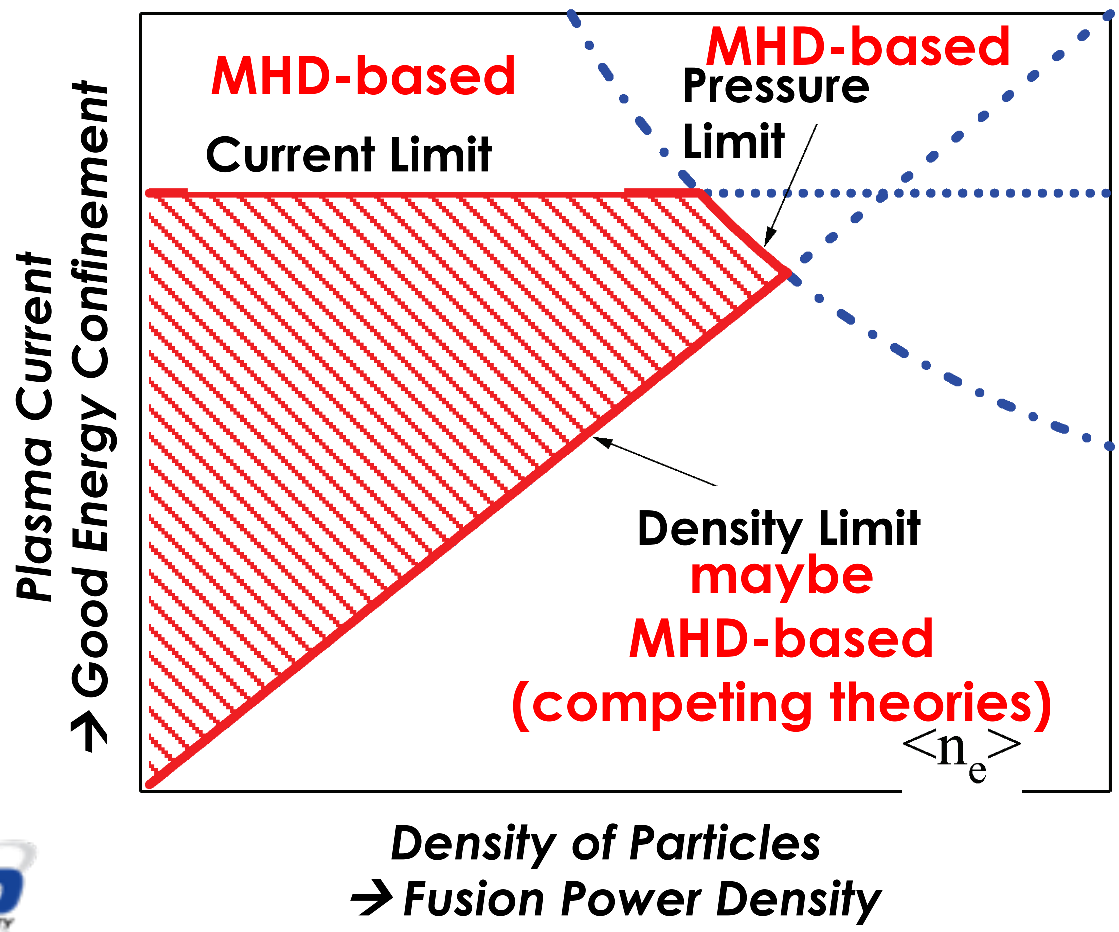
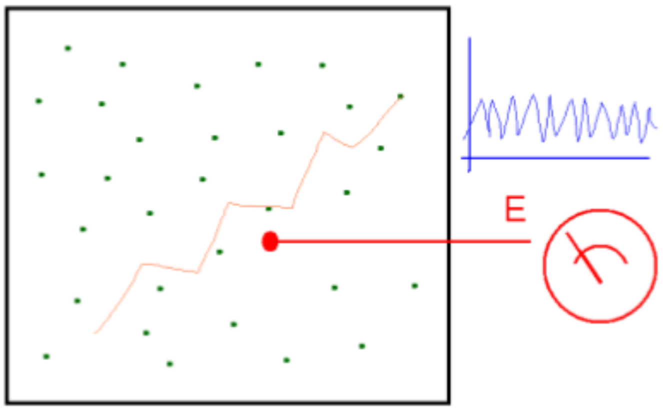
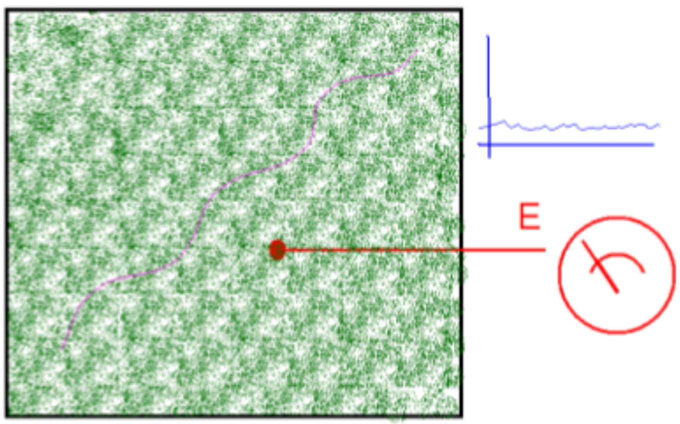


Guiding Principle: Zoom Out ! Single Particles Become Well Described by Aggregate Properties

Too few particles



Good # particles

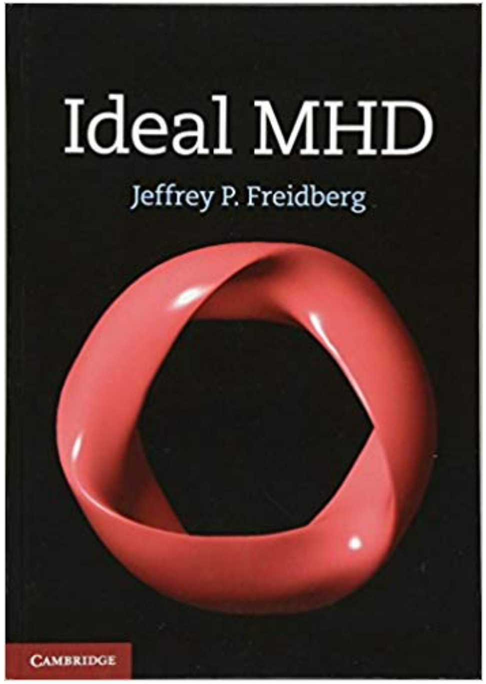


- **Aggregate Quantities**
  - Number of particles
  - Mean velocity
  - Kinetic Energy

The spatial scale considered is rather big: "macro-scale"

Functionally this means no smaller than ~~ few % of device radius

Excellent Textbooks on MHD are Available



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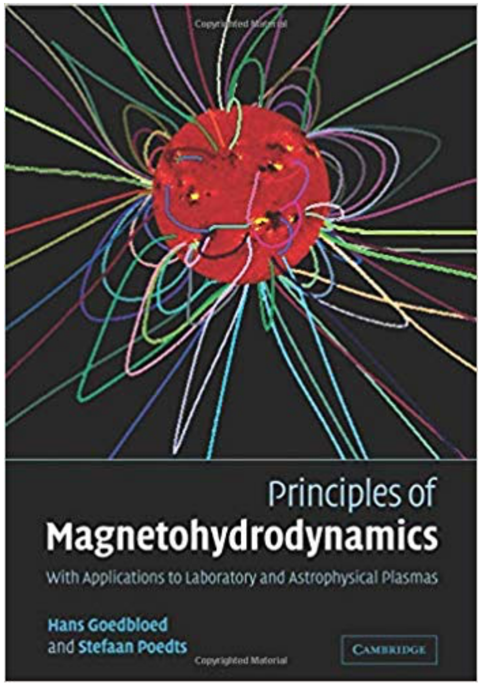

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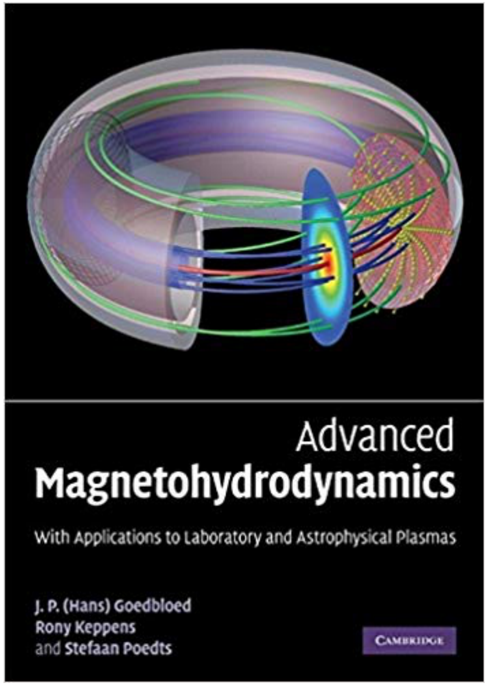

ISBN-10: 052170524X

We have only ~ one hour for a graduate-level topic
…my treatment will try to be conceptual
☺ … ! you should read the books ! … ☺

About Me

- **Born: Toronto, Ontario, Canada**
- **B.Sc.E: Kingston, Ontario (2007)**
- **Ph.D: Madison, Wisconsin (2012)**
- **@ DIII-D/General Atomics since 2012**

DIII-D Control Room c. 2019

<u>My Roles (@ DIII-D):</u>

- **Tokamak physics operator**
  - (pressing buttons / turning knobs)
- **Research on MHD instabilities**
  - "ELMs", "Error Fields", "Runaways"
- **NOT a theorist: I dont derive equations**
- **AM an "Experimentalist"**
  - Design scans, collect & analyze data
- **NOT a professor / lecturer !**

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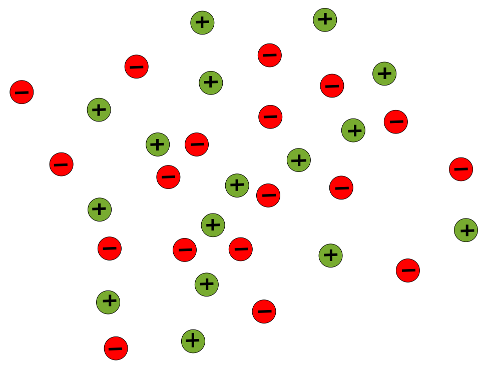
MHD Starting point: Equations for Single-Particle Motion Are Simple, Right?



Force **F** on single particle:

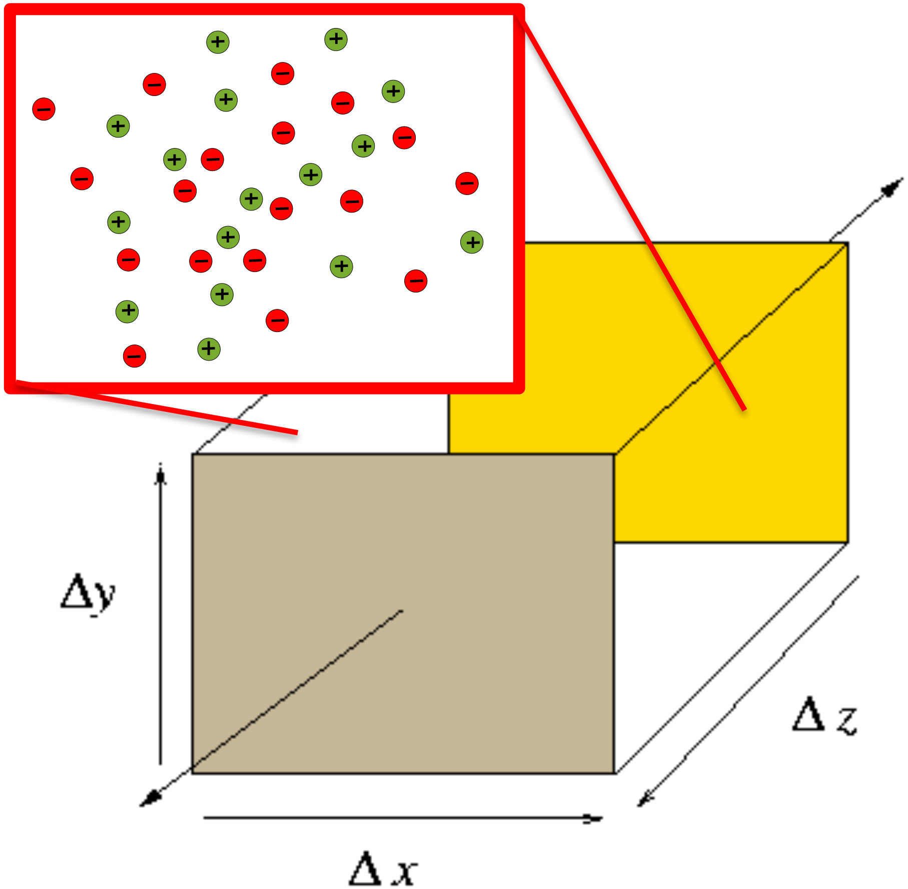
$$\mathbf{F} = q(\mathbf{E} + \mathbf{v} \times \mathbf{B})$$

E set up by all other particles
B set up by all other particles

- **Natural idea: Move each particle according to F = *ma***
- **Difficulty 1: Many particles,**
  - $N \sim 10^{20}$–10^{22} in magnetic fusion grade plasmas
- **Difficulty 2: Force depends on position of all other particles**

Impossible to directly compute (and wasteful to try)

Basic Idea of a Fluid Model: Look at what happens in a box that encompasses large # of particles



- **Replace discrete particles with smooth *distribution function***

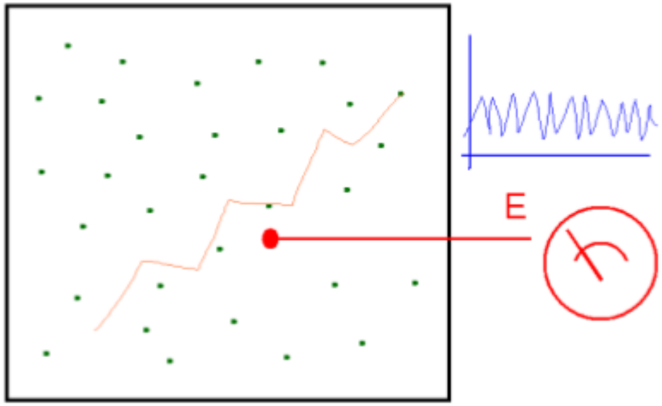
$$f(\mathbf{x}, \mathbf{v}, t)$$

defined such that:

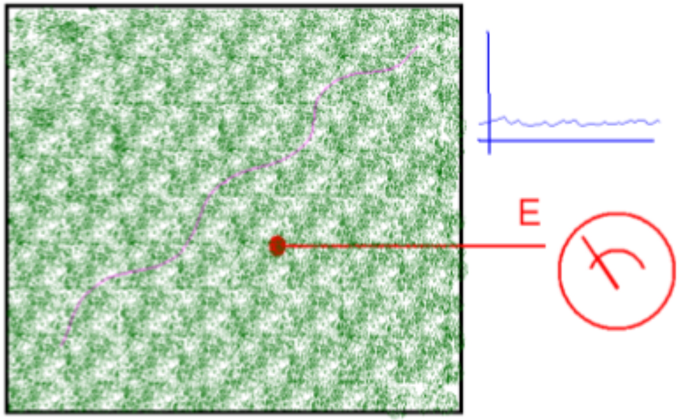
- $\int f(\mathbf{x}, \mathbf{v}, t)\, d\mathbf{x}\, d\mathbf{v}$ **= # of particles in 6D phase-space volume *dx dv***
- **7 dimensions:**
  - 3 spatial (x, y, z)
  - 3 velocity (v_x, v_y, v_z)
  - 1 temporal (t)

Guiding Principle: If enough particles in the box it will have well defined aggregate properties

Too few particles



Good # particles



- **Expected quantities to be regular in the box (derive some later)**
  - Number of particles
  - Mean velocity
  - Kinetic Energy

- **This sets a key aspect of MHD:**

The spatial scale considered is rather big: "macro-scale"

Functionally this means no smaller than ~~ few % of device radius

Fluid Moments are how we mathematically formalize aggregate quantities of the distribution function

$$0^{th} \text{ moment} \quad \int \quad d^3\mathbf{v}$$

$$1st \text{ moment} \quad \int \quad \mathbf{v}d^3v$$

$$2nd \text{ moment} \quad \int \quad \mathbf{v}\mathbf{v}d^3v$$

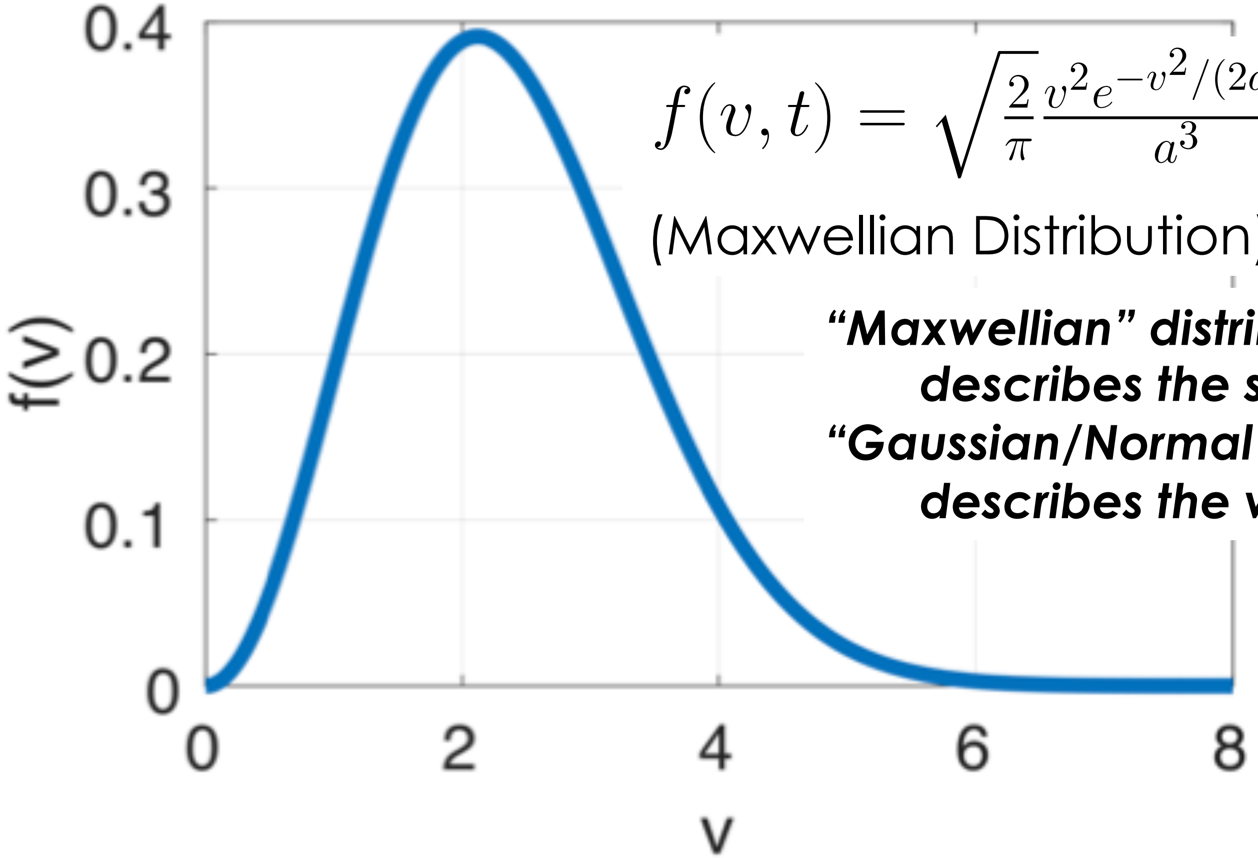
$$n(\mathbf{x},t) = \iiint f(\mathbf{x},\mathbf{v},t)d\mathbf{v} \qquad \text{Density}$$

$$n\mathbf{V}(\mathbf{x},t) = \iiint \mathbf{v}f(\mathbf{x},\mathbf{v},t)d\mathbf{v} \qquad \text{Mean flow}$$

$$\mathbf{P}(\mathbf{x},t) = m\iiint (\mathbf{v}-\mathbf{V})(\mathbf{v}-\mathbf{V})fd\mathbf{v} \qquad \text{Pressure tensor}$$

Let's look at an example (*Maxwellian* Distribution)

Fluid Moments and their Relation to the Distribution



$$f(v,t) = \sqrt{\frac{2}{\pi}}\frac{v^2 e^{-v^2/(2a^2)}}{a^3}$$

(Maxwellian Distribution)

"Maxwellian" distribution describes the speed (1-D)

"Gaussian/Normal" distribution describes the velocity (3-D)

Distribution

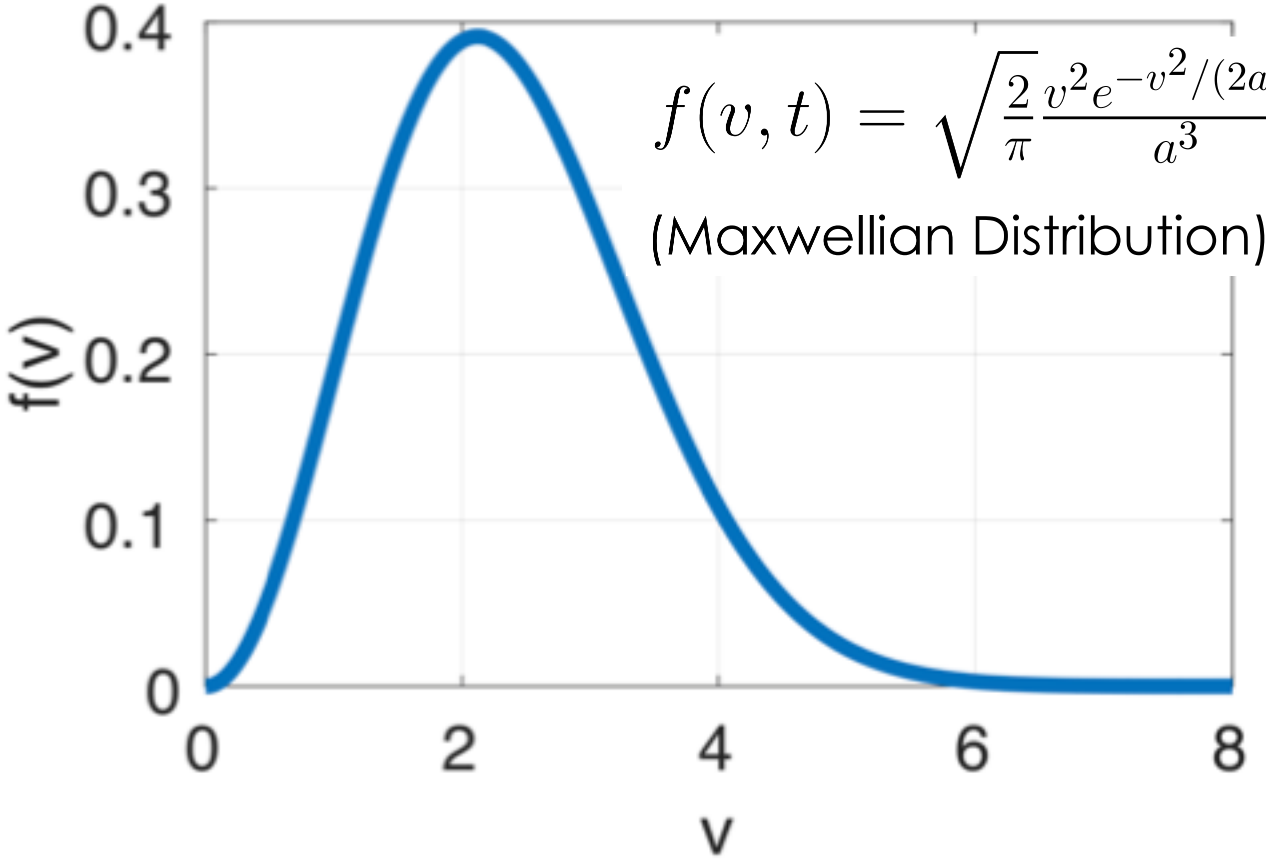
Speed (m/s)

Fluid Moments and their Relation to the Distribution
Fluid Density: Number of Particles (area under curve)

$$n(t) = \int f(v,t)dv \equiv 1$$

$$f(v,t) = \sqrt{\frac{2}{\pi}}\frac{v^2 e^{-v^2/(2a^2)}}{a^3}$$

(Maxwellian Distribution)



Distribution

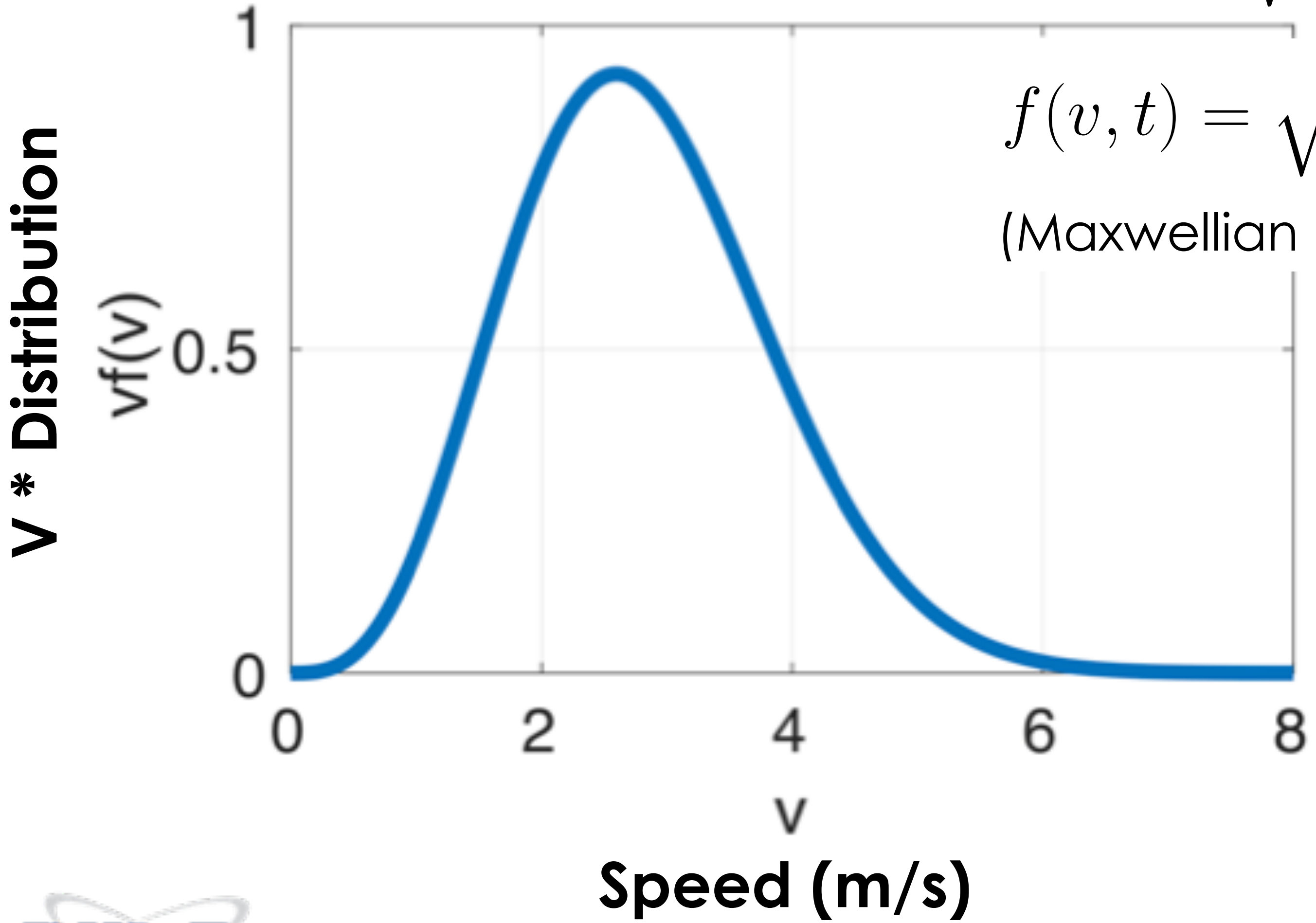
Speed (m/s)

Fluid Moments and their Relation to the Distribution
Fluid Velocity: Mean Value of Distribution

$$V(t) = \frac{1}{n}\int v f(v,t)dv \equiv 2a\sqrt{\frac{2}{\pi}}$$

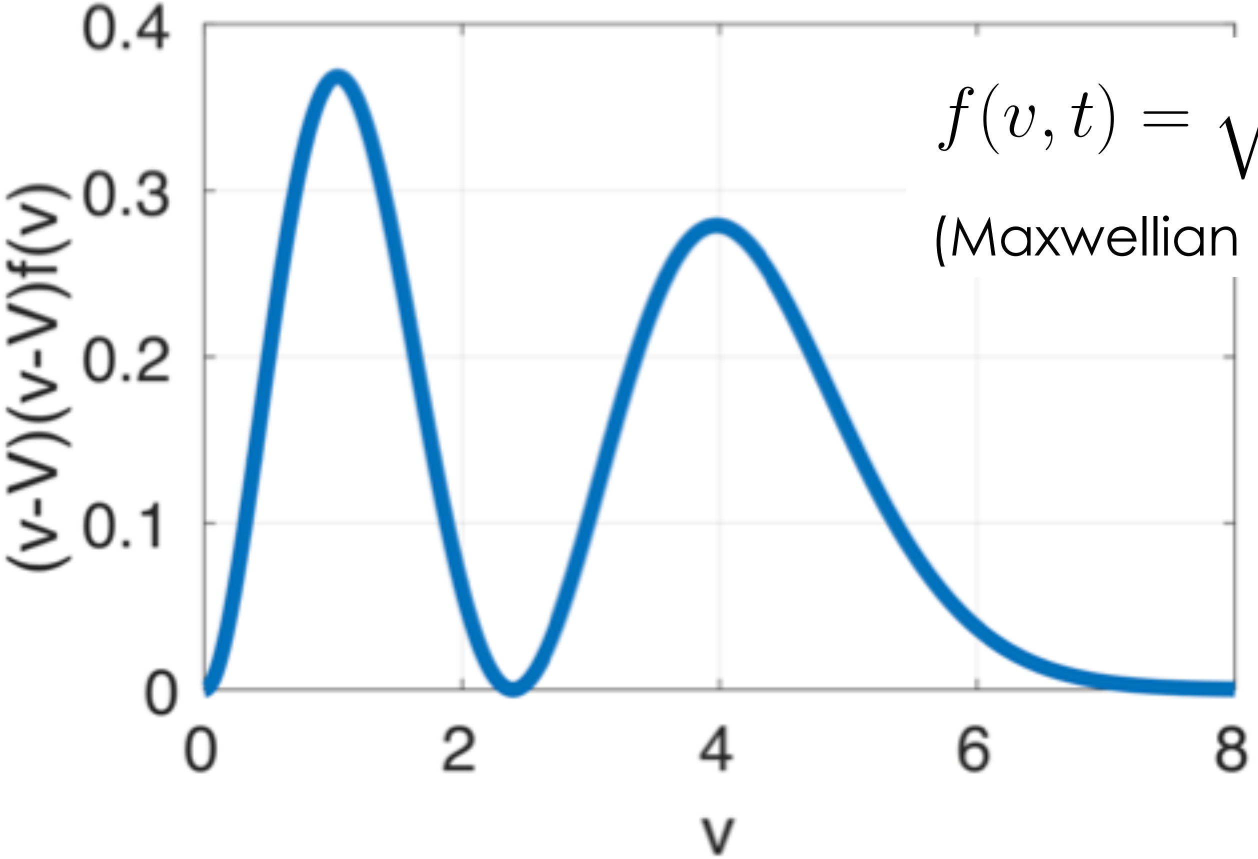
$$f(v,t) = \sqrt{\frac{2}{\pi}}\frac{v^2 e^{-v^2/(2a^2)}}{a^3}$$

(Maxwellian Distribution)



Fluid Moments and their Relation to the Distribution
Fluid Pressure: Variance of Distribution

$$P(t) = \frac{1}{m}\int (v-V)(v-V)f(v,t)dv \equiv \frac{a^2}{\pi}(3\pi - 8)$$



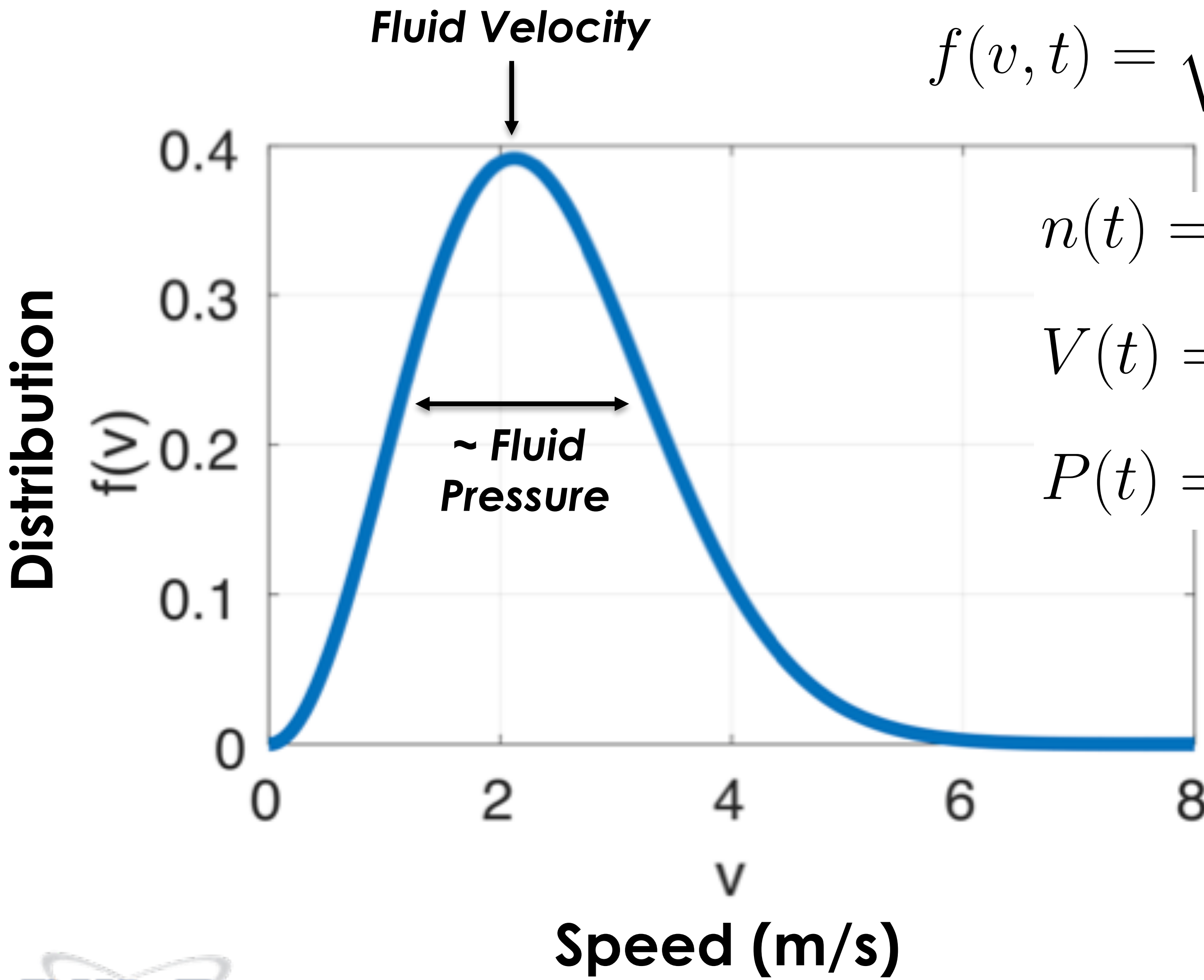
$$f(v,t) = \sqrt{\frac{2}{\pi}}\frac{v^2 e^{-v^2/(2a^2)}}{a^3}$$

(Maxwellian Distribution)

Speed (m/s)

Power of Fluid Moments:
Convert Complicated Distributions to Single Numbers!

$$f(v,t) = \sqrt{\frac{2}{\pi}}\frac{v^2 e^{-v^2/(2a^2)}}{a^3}$$



$$n(t) = 1$$

$$V(t) = 2a\sqrt{\frac{2}{\pi}}$$

$$P(t) = \frac{a^2}{\pi}(3\pi - 8)$$

How can we write F = m a for the distribution function?

- **Enforce that particles can not be created or destroyed**

$$\frac{df}{dt} = 0$$

- **Apply it to the 7-D *f* (x, v, *t*) (Wikipedia "*convective derivative*")**

$$\frac{df}{dt} = \frac{\partial f}{\partial t} + \frac{d\mathbf{x}}{dt} \cdot \nabla f + \frac{d\mathbf{v}}{dt} \cdot \nabla_{\mathbf{v}} f = 0$$

- **We already know some of these terms!**

$$\frac{d\mathbf{x}}{dt} = \mathbf{v} \qquad \frac{d\mathbf{v}}{dt} = \frac{q}{m}\left(\mathbf{E} + \mathbf{v} \times \mathbf{B}\right)$$

- **Plug into the convective derivative: *Vlasov Equation***

$$\frac{\partial f}{\partial t} + \mathbf{v} \cdot \nabla f + \frac{q}{m}\left(\mathbf{E} + \mathbf{v} \times \mathbf{B}\right) \cdot \nabla_{\mathbf{v}} f = 0$$

Taking Moments of the Boltzmann Equation Gives Rise to the Fluid Equations

- *Vlasov Equation* **does not include collisions**

$$\frac{\partial f}{\partial t} + \mathbf{v} \cdot \nabla f + \frac{q}{m}(\mathbf{E} + \mathbf{v} \times \mathbf{B}) \cdot \nabla_{\mathbf{v}} f = 0$$

- ***Boltzmann Equation*** **includes collisions**

$$\frac{\partial f}{\partial t} + \mathbf{v} \cdot \nabla f + \frac{q}{m}(\mathbf{E} + \mathbf{v} \times \mathbf{B}) \cdot \nabla_{\mathbf{v}} f = \left(\frac{\partial f}{\partial t}\right)_c$$

- ***MHD Equations are fluid moments of the Boltzmann equation***

$$\int \left[\frac{df}{dt} - \left(\frac{\partial f}{\partial t}\right)_C\right] d\mathbf{v} \longleftarrow \text{Continuity Equation}$$

$$\int m\mathbf{v} \left[\frac{df}{dt} - \left(\frac{\partial f}{\partial t}\right)_C\right] d\mathbf{v} \longleftarrow \text{Momentum Equation}$$

$$\int \frac{mv^2}{2} \left[\frac{df}{dt} - \left(\frac{\partial f}{\partial t}\right)_C\right] d\mathbf{v} \longleftarrow \text{Energy Equation}$$

Example: Derivation of the Continuity Equation (For a Single Species)

- **We have to take a fluid moment of the Boltzmann equation:**

$$\int \left[\underbrace{\frac{\partial f}{\partial t}}_{1} + \underbrace{\mathbf{v} \cdot \nabla f}_{2} + \underbrace{\frac{q}{m}(E + \mathbf{v} \times B) \cdot \nabla_v f}_{3} \right] d\mathbf{v}$$

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1 2 3

$$1 \int \left[\frac{\partial f}{\partial t}\right] d\mathbf{v} = \frac{\partial\left[\int f\, d\mathbf{v}\right]}{\partial t} = \frac{\partial}{\partial t} n(x,t)$$

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$$3 \int \left[\frac{q}{m}(E+\mathbf{v}\times B)\cdot\nabla_v f\right] d\mathbf{v} = \int [\nabla_v\cdot[\ldots]] d\mathbf{v} = 0$$

Example: Derivation of the Continuity Equation (For a Single Species)

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$$1\ \int \left[\frac{\partial f}{\partial t}\right]d\mathbf{v} = \frac{\partial\left[\int f\,d\mathbf{v}\right]}{\partial t} = \frac{\partial}{\partial t}n(x,t)$$

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$$3\ \int \left[\frac{q}{m}(E+\mathbf{v}\times B)\cdot\nabla_v f\right]d\mathbf{v} = \int [\nabla_v\cdot[\ldots]]d\mathbf{v} = 0$$

$$\frac{\partial}{\partial t}n(x,t) + \nabla\cdot n(x,t)\mathbf{v} = 0$$

This Process Continues to Derive the Higher Order Moments

$$\frac{\partial n_s}{\partial t} + \nabla \cdot (n_s \mathbf{V}_s) = 0 \qquad \text{Continuity}$$

$$mn\left(\frac{\partial \mathbf{V}_s}{\partial t} + \mathbf{V}_s \cdot \nabla \mathbf{V}_s\right) = q_s n_s\left(\mathbf{E} + \mathbf{V}_s \times \mathbf{B}\right) - \nabla \cdot \mathbf{P}_s + \mathbf{R}_s \quad \text{Momentum}$$

$$\frac{d}{dt}\left(\frac{3}{2}p_s\right) + \frac{5}{2}p_s \nabla \cdot \mathbf{V}_s + \boldsymbol{\pi}_s : \nabla \mathbf{V}_s + \nabla \cdot \mathbf{q}_s = 0 \qquad \text{(Energy)}$$

- **Notice a *hierarchy* is present:**
  - Quantity needed to solve (N)th equation is given by (N+1)th equation
- **This continues forever, and is called the “Closure Problem”**
  - Approximations are ***essential*** to the fluid model

The Final Step to Derive MHD Equations is to Combine Electron and Ion Species

- **Single-Species Momentum Equation:**

$$mn\left(\frac{\partial \mathbf{V}_s}{\partial t} + \mathbf{V}_s \cdot \nabla \mathbf{V}_s\right) = q_s n_s \left(\mathbf{E} + \mathbf{V}_s \times \mathbf{B}\right) - \nabla \cdot \mathbf{P}_s + \mathbf{R}_s$$

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- **Combined Momentum Equation:**

$$m_e n_e \left(\frac{\partial \mathbf{V}_e}{\partial t} + \mathbf{V}_e \cdot \nabla \mathbf{V}_s\right) + m_i n_i \left(\frac{\partial \mathbf{V}_i}{\partial t} + \mathbf{V}_i \cdot \nabla \mathbf{V}_i\right) =$$

$$e(n_e - n_i)\mathbf{E} + e(n_e \mathbf{V}_e - n_i \mathbf{V}_i) \times \mathbf{B} - \nabla \cdot (\mathbf{P}_e + \mathbf{P}_i) + \mathbf{R}_e + \mathbf{R}_i$$

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Quasi-neutrality (both) $n_e = n_i = n$

Mass Density (ion) $\rho = n(m_i + m_e) \approx nm_i$

Current (both) $J = en(\mathbf{V}_i - \mathbf{V}_e)$

Mass Flow (ion) $\mathbf{V} \approx \mathbf{V}_i$

Pressure (both) $p = \mathbf{P}_i + \mathbf{P}_e$

The Final Step to Derive MHD Equations is to Combine Electron and Ion Species

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$$\cancel{e(n_e - n_i)\mathbf{E}} + e(n_e \mathbf{V}_e - n_i \mathbf{V}_i) \times \mathbf{B} - \nabla \cdot (\mathbf{P}_e + \mathbf{P}_i) \cancel{+ \mathbf{R}_e + \mathbf{R}_i}$$

Quasi-neutrality (both)	$n_e = n_i = n$
Mass Density (ion)	$\rho = n(m_i + m_e) \approx n m_i$
Current (both)	$J = en(\mathbf{V}_i - \mathbf{V}_e)$
Mass Flow (ion)	$\mathbf{V} \approx \mathbf{V}_i$
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The Final Step to Derive MHD Equations is to Combine Electron and Ion Species

- **Single-Species Momentum Equation:**

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- **Combined Momentum Equation:**

$$\cancel{m_e n_e \left(\frac{\partial \mathbf{V}_e}{\partial t} + \mathbf{V}_e \cdot \nabla \mathbf{V}_s\right)} + \boxed{m_i n_i}\left(\frac{\partial \mathbf{V}_i}{\partial t} + \mathbf{V}_i \cdot \nabla \mathbf{V}_i\right) =$$

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Current (both) $J = \boxed{en(\mathbf{V}_i - \mathbf{V}_e)}$

Mass Flow (ion) $\mathbf{V} \approx \mathbf{V}_i$

Pressure (both) $p = \boxed{\mathbf{P}_i + \mathbf{P}_e}$

The Final Step to Derive MHD Equations is to Combine Electron and Ion Species

- **Single-Species Momentum Equation:**

$$mn\left(\frac{\partial \mathbf{V}_s}{\partial t} + \mathbf{V}_s \cdot \nabla \mathbf{V}_s\right) = q_s n_s \left(\mathbf{E} + \mathbf{V}_s \times \mathbf{B}\right) - \nabla \cdot \mathbf{P}_s + \mathbf{R}_s$$

- **Combined Momentum Equation:**

$$\rho\left(\frac{\partial \mathbf{V}}{\partial t} + \mathbf{V} \cdot \nabla \mathbf{V}\right) = \mathbf{J} \times \mathbf{B} - \nabla p$$

Quasi-neutrality (both)	$n_e = n_i = n$
Mass Density (ion)	$\rho = n(m_i + m_e) \approx \boxed{nm_i}$
Current (both)	$J = \boxed{en(\mathbf{V}_i - \mathbf{V}_e)}$
Mass Flow (ion)	$\mathbf{V} \approx \mathbf{V}_i$
Pressure (both)	$p = \boxed{\mathbf{P}_i + \mathbf{P}_e}$

Final Ideal MHD Equations: (Including Maxwell's Laws)

$$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \mathbf{V}) = 0$$ **Continuity**

$$\rho \frac{d\mathbf{V}}{dt} = \mathbf{J} \times \mathbf{B} - \nabla p$$ **Momentum** (~ ion)

$$\frac{d}{dt}\left(\frac{p}{\rho^{5/3}}\right) = 0$$ **Energy**

$$\mathbf{E} + \mathbf{V} \times \mathbf{B} = \mathbf{0}$$ **Current** (~ electron)

$$\left.\begin{aligned} \nabla \times \mathbf{E} &= -\frac{\partial \mathbf{B}}{\partial t} \\ \nabla \times \mathbf{B} &= \mu_0 \mathbf{J} \\ \nabla \cdot \mathbf{B} &= 0 \end{aligned}\right\}$$ **Maxwell's Laws**

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Final Ideal MHD Equations: (Including Maxwell's Laws)

$$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \mathbf{V}) = 0$$ **Continuity**

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$$\frac{d}{dt}\left(\frac{p}{\rho^{5/3}}\right) = 0$$ **Energy**

$$\mathbf{E} + \mathbf{V} \times \mathbf{B} = \mathbf{0}$$ **Current** (~ electron)

$\partial \mathbf{B}$

Key Variables:
B-field, E-field, Pressure (p), Flow (V), Current (J)

Some Words on the Philosophy of the MHD Approach

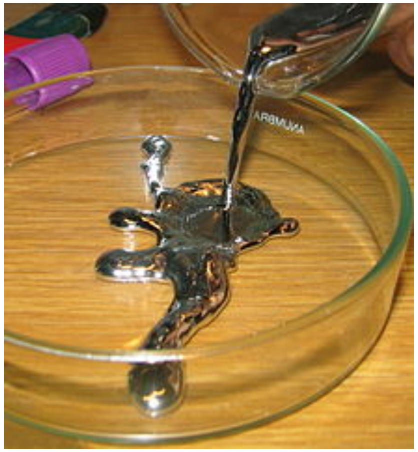
- **The purpose of ideal MHD is to study the macroscopic behavior of the plasma**

- **MHD can be used to design machines that avoid large scale instabilities (we'll discuss some later)**

- **Regime of interest**
  - Typical length scale: the radius of the device (~ 1 meter)
  - Typical velocities: Ion thermal velocity (~ 500 km/s)
  - Typical time scale: Radius / velocity ~ 2 microseconds (< ~100s kHz)

MHD is the Perfect Model for a Liquid Metal … but Not Actually for a Fusion Plasma

- **Fluid models work best when density is high and collisions are frequent**
  - Allows equilibrated (Maxwellian) distributions
  - Allows moments to capture distribution well
  - Solves "Closure" problem

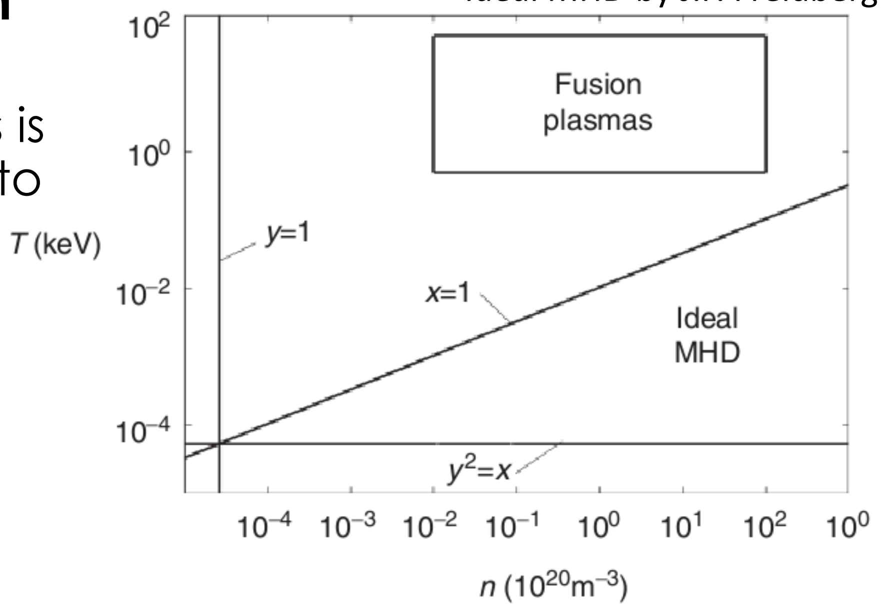


Mercury at room temperature [Wikipedia].

- **The regime of validity of ideal MHD does NOT coincide with the fusion plasma regime**
  - The collisionality of fusion plasmas is too low for the ideal MHD model to be valid !

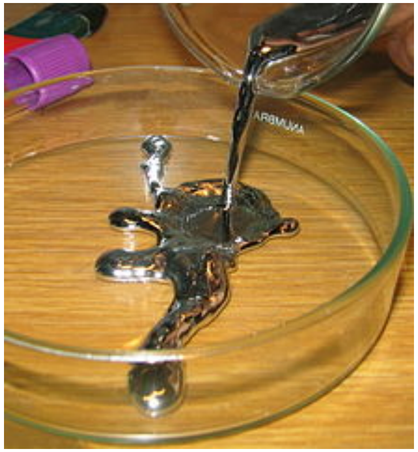
Why do we still use it? Because it works !

Ideal MHD by J.P. Freidberg



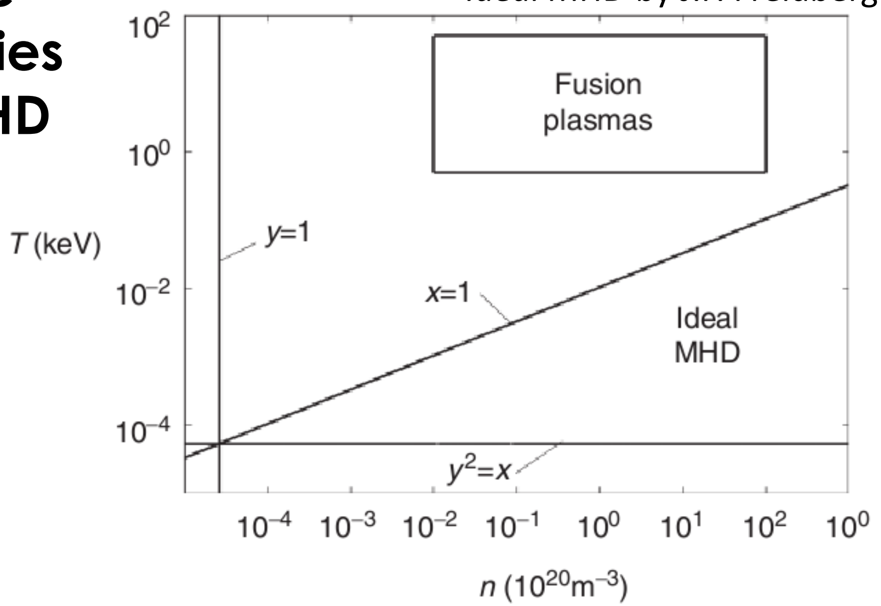
MHD is the Perfect Model for a Liquid Metal … but Not Actually for a Fusion Plasma

- **Success of MHD is not due to luck but to subtle physical reasons**
- **This is because ideal MHD is accurate for dynamics perpendicular to the fields lines**
- **Can show that collisionless kinetic models for macroscopic instabilities are more optimistic than ideal MHD**
- **Designs based on ideal MHD calculations are conservative designs**



Mercury at room temperature [Wikipedia].

Ideal MHD by J.P. Freidberg



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Review of Concepts – MHD Equations

- The fluid approach is all about describing a distribution of particles in terms of their aggregate properties
  - Density, Flow, Energy

- "Fluid Moments" are taken of the underlying Boltzmann equation to derive the "Fluid Equations" for ions and electrons

- Electron and ion equations are combined and simplified to give the "MHD Equations"

- The main goal of the MHD approach is to describe the macroscopic phenomena / instabilities of the plasma

- MHD is not technically valid for fusion plasmas but it works !

Outline of Presentation

- Pre-amble: Why the MHD model?
- Development of the MHD Equations
- MHD Equilibrium: 1-D, 2-D, 3-D Configurations
- MHD and its Relation to Global Operational Limits
- Brief Tour of Common MHD Instabilities and Their Control

Static MHD Equilibrium Equations are a Dramatic Reduction of the Ideal MHD Equations

Full Ideal MHD

$$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \mathbf{V}) = 0$$

$$\rho \frac{d\mathbf{V}}{dt} = \mathbf{J} \times \mathbf{B} - \nabla p$$

$$\frac{d}{dt}\left(\frac{p}{\rho^{5/3}}\right) = 0$$

$$\mathbf{E} + \mathbf{V} \times \mathbf{B} = \mathbf{0}$$

$$\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t}$$

$$\nabla \times \mathbf{B} = \mu_0 \mathbf{J}$$

$$\nabla \cdot \mathbf{B} = 0$$

+

Constraints

$$\partial / \partial t = 0$$

$$\mathbf{V} = \mathbf{0}$$

=

Equilibrium MHD

$$\nabla \cdot \mathbf{B} = 0$$

$$\nabla \times \mathbf{B} = \mu_0 \mathbf{J}$$

$$\mathbf{J} \times \mathbf{B} = \nabla p$$

Static MHD Equilibrium Equations are a Dramatic Reduction of the Ideal MHD Equations

Full Ideal MHD

$$\cancel{\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \mathbf{V}) = 0}$$

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$$\cancel{\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t}}$$

$$\nabla \times \mathbf{B} = \mu_0 \mathbf{J}$$

$$\nabla \cdot \mathbf{B} = 0$$

+

Constraints

$$\partial / \partial t = 0$$

$$\mathbf{V} = \mathbf{0}$$

=

Equilibrium MHD

$$\nabla \cdot \mathbf{B} = 0$$

$$\nabla \times \mathbf{B} = \mu_0 \mathbf{J}$$

$$\mathbf{J} \times \mathbf{B} = \nabla p$$

only 3 quantities to solve
Pressure (P), Current (J), Mag Field (B)

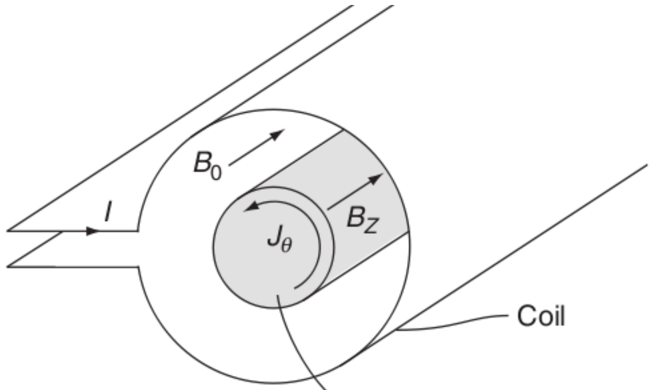
No separation of density vs temperature
No electric fields allowed

Two Very Simple MHD Equilibrium Configurations Can be Easily Obtained in Cylindrical Geometry

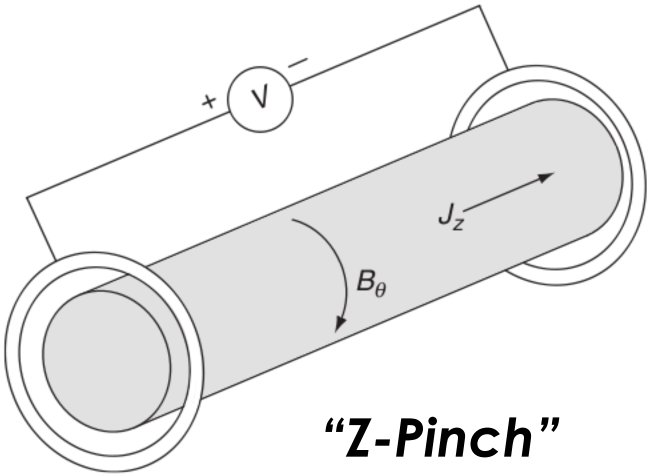
$$\mathbf{J} \times \mathbf{B} = \nabla p$$

$$J_\theta B_z = \frac{dp}{dr}$$

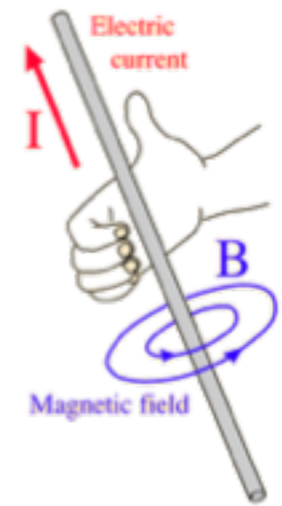
$$J_z B_\theta = \frac{dp}{dr}$$



"Theta-Pinch"

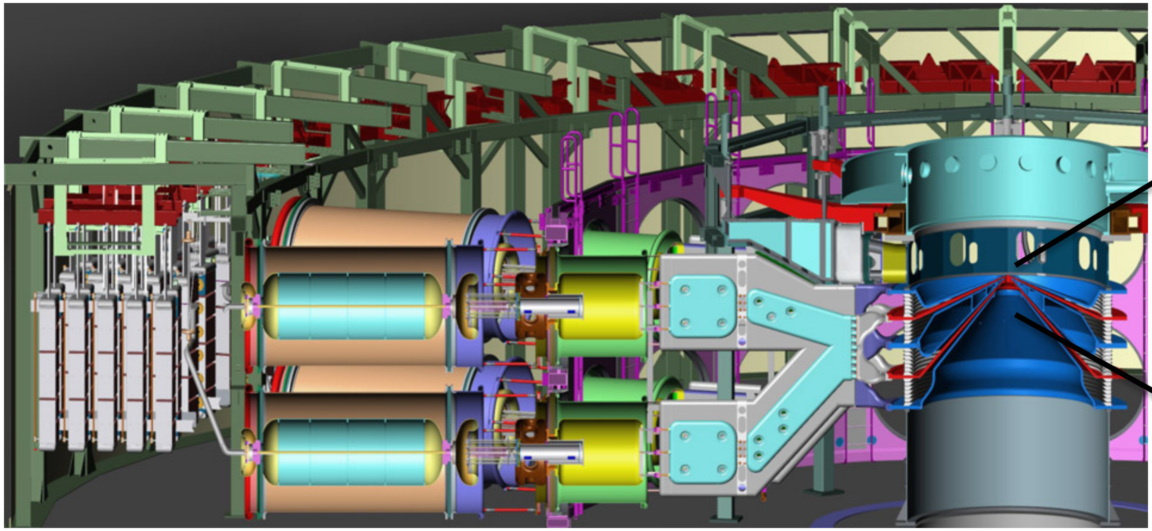


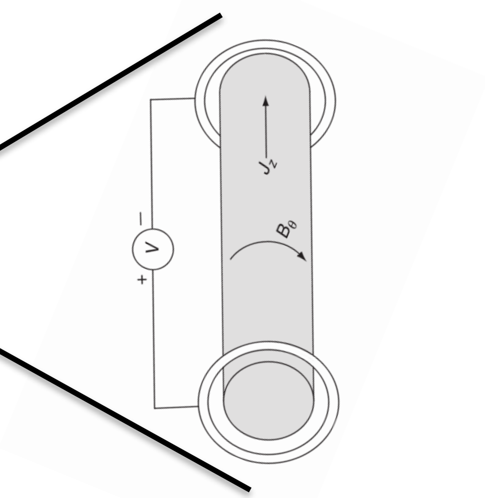
"Z-Pinch"

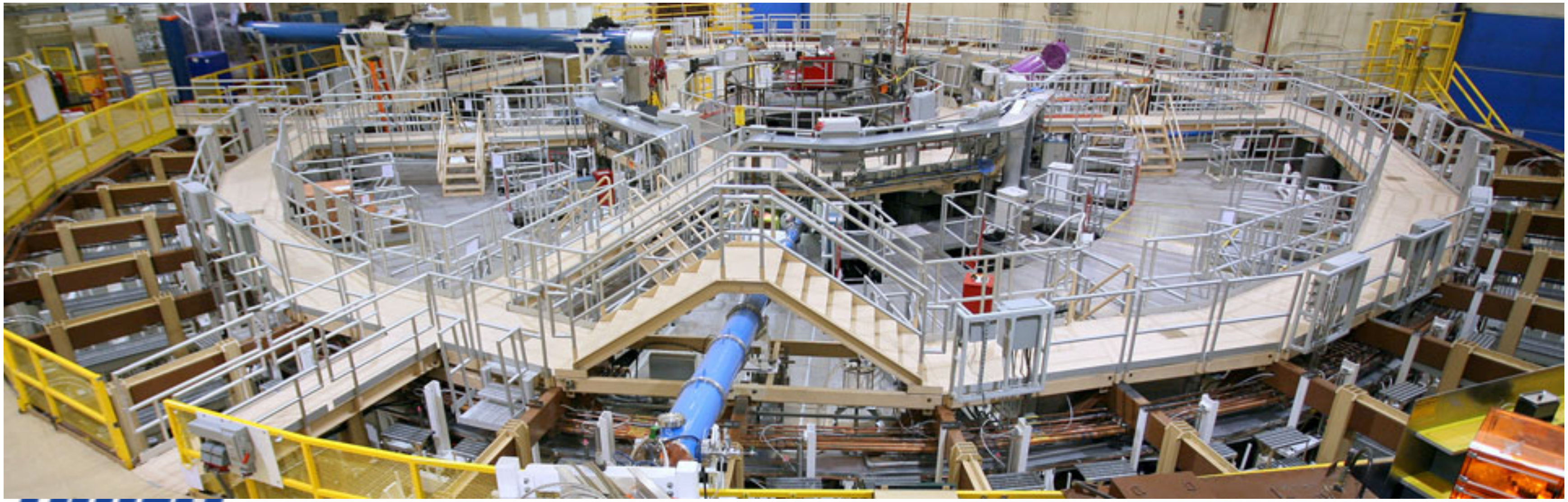


- Radial Pressure gradient = hot core separated from a cold wall

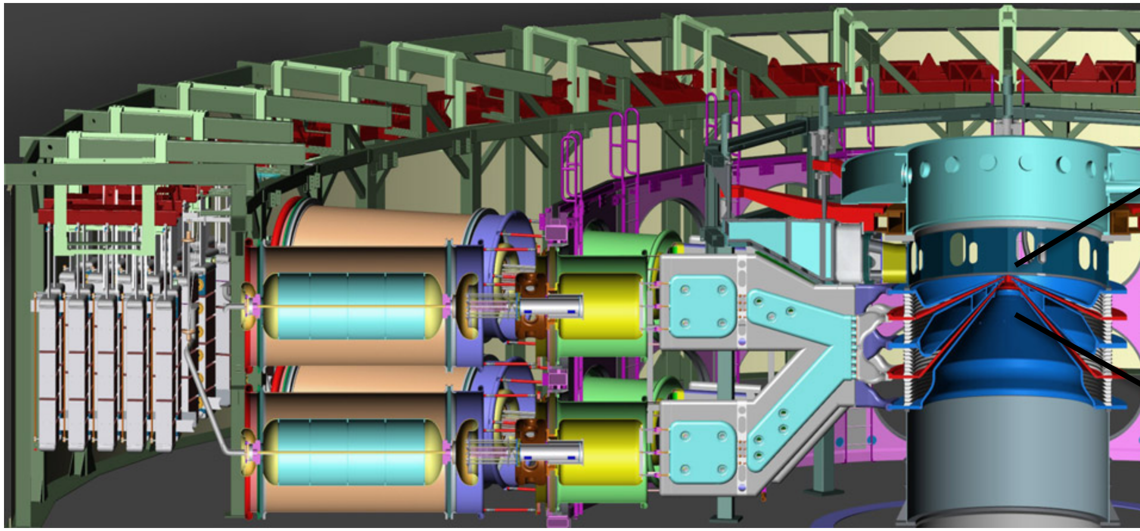
Sandia National Lab Operates a Very Large Z-Pinch

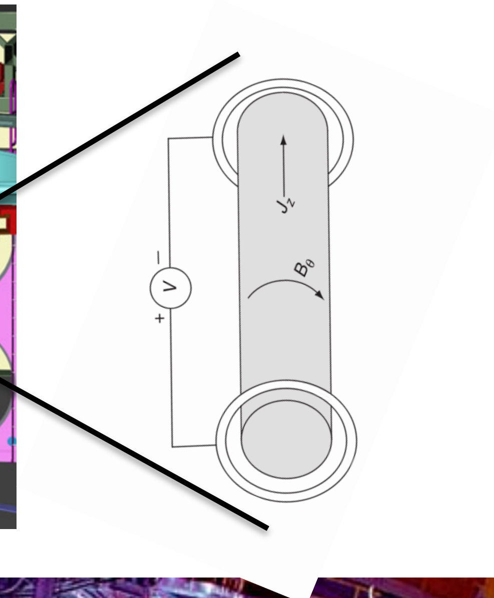


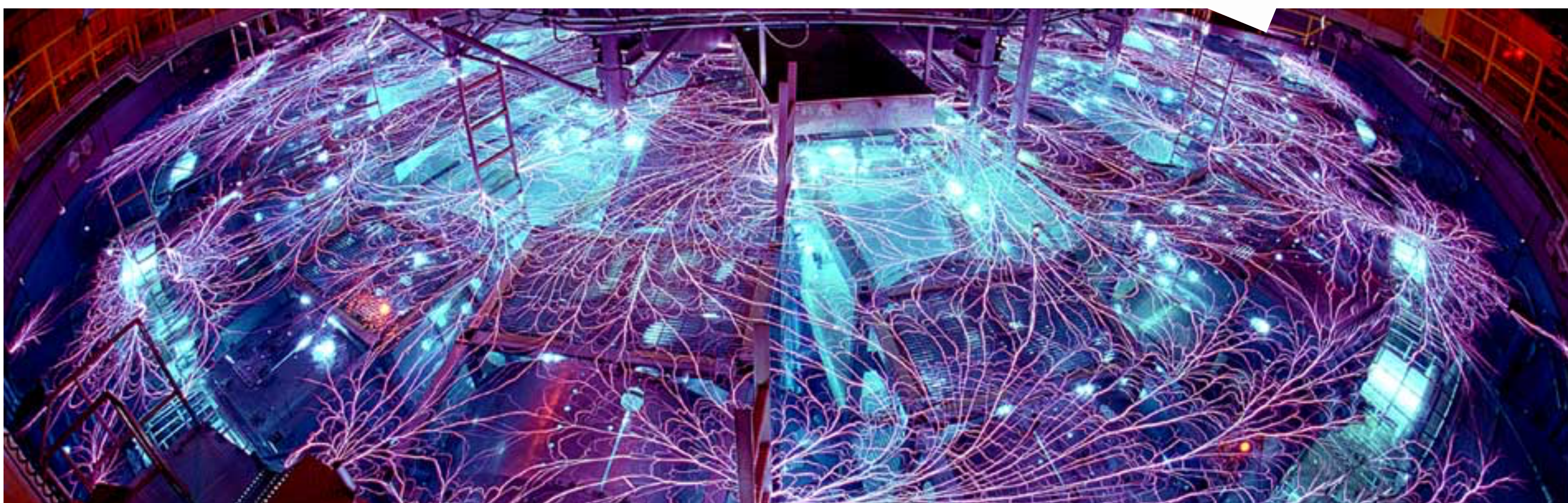




Sandia National Lab Operates a Very Large Z-Pinch



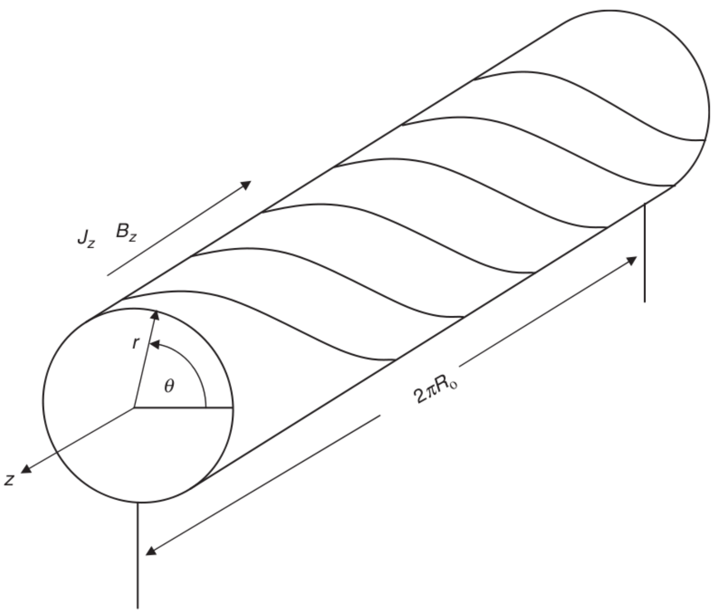




Screw Pinch Combines Z and Theta Pinch

$$\mathbf{J} \times \mathbf{B} = \nabla p$$

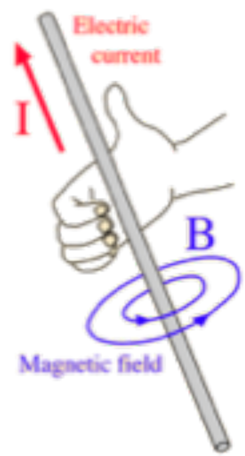
$$J_z B_\theta + J_\theta B_z = \frac{dp}{dr}$$



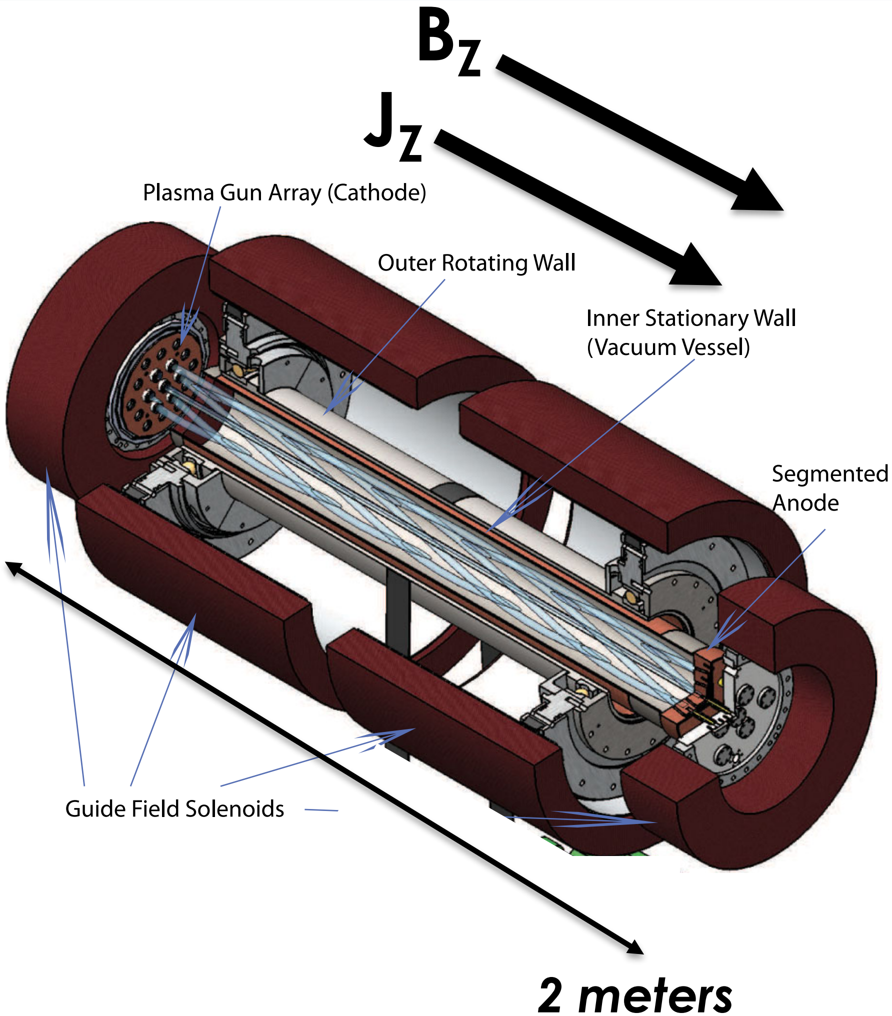
- **Pressure gradient supported by axial field (B_Z) and axial currents (J_Z)**

- **B_θ comes from J_Z**
  - (Ampere's Law)

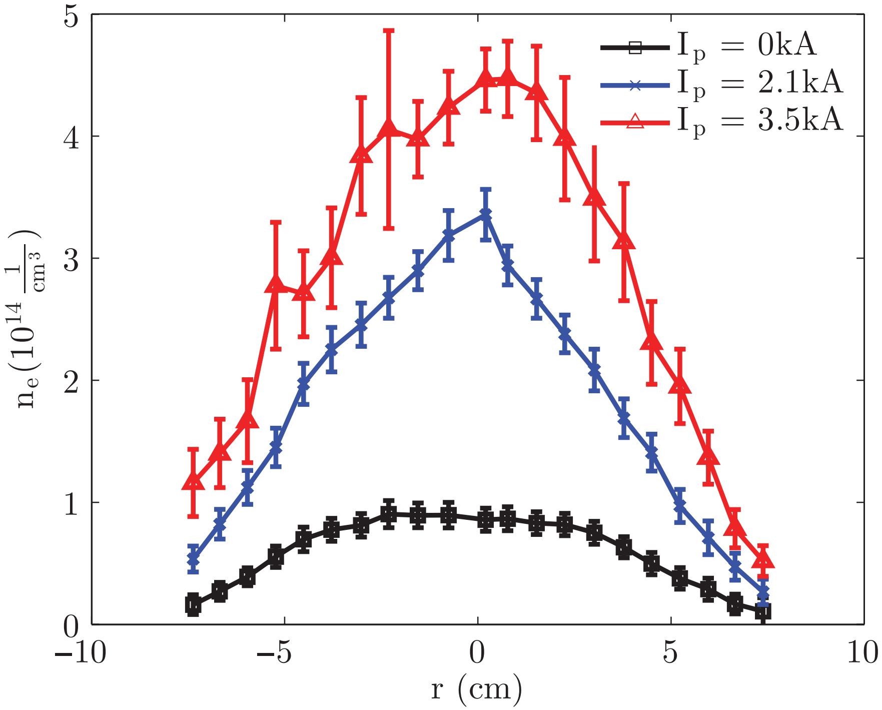
- **J_θ comes from diamagnetic drift**



I conducted my PhD Research on a Screw Pinch

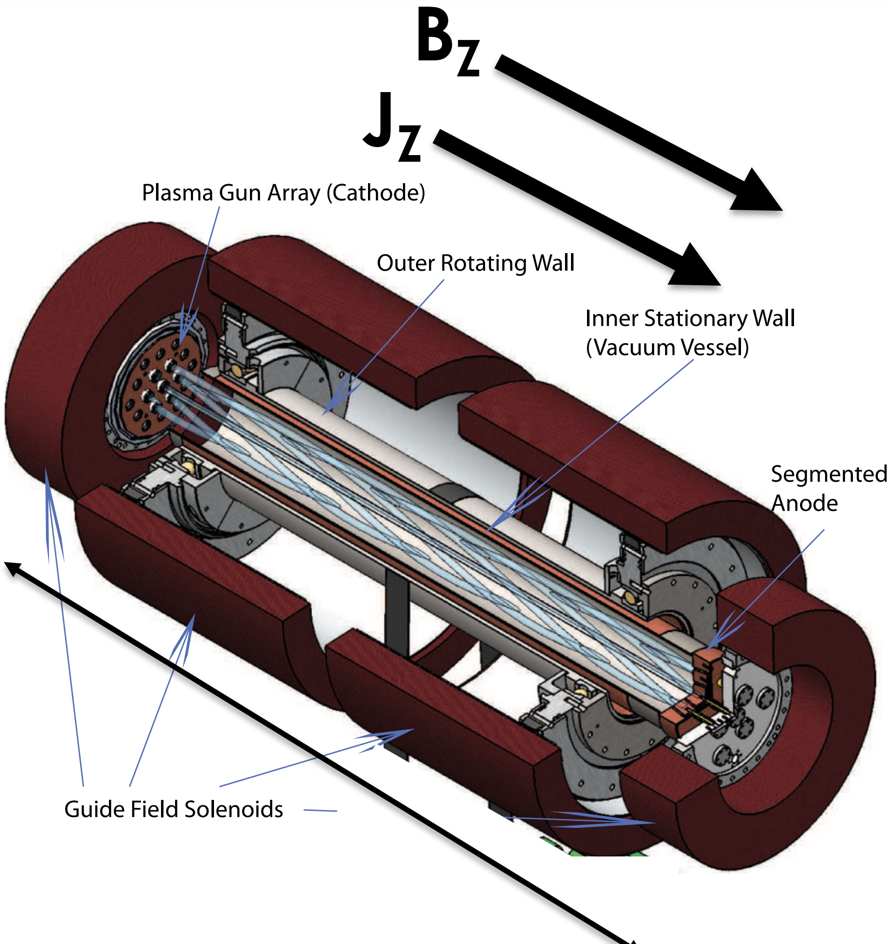


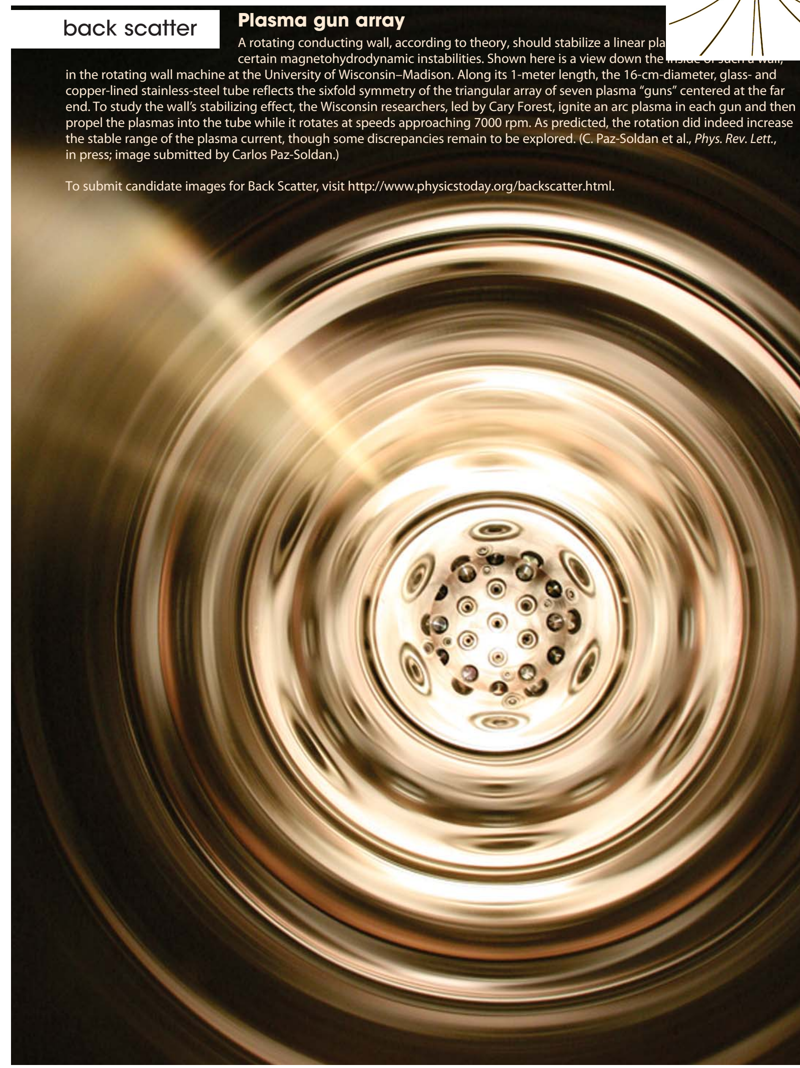
Pressure gradients! $\frac{dp}{dr}$



C. Paz-Soldan et al, Rev Sci Instrum. 2011

I conducted my PhD Research on a Screw Pinch




back scatter

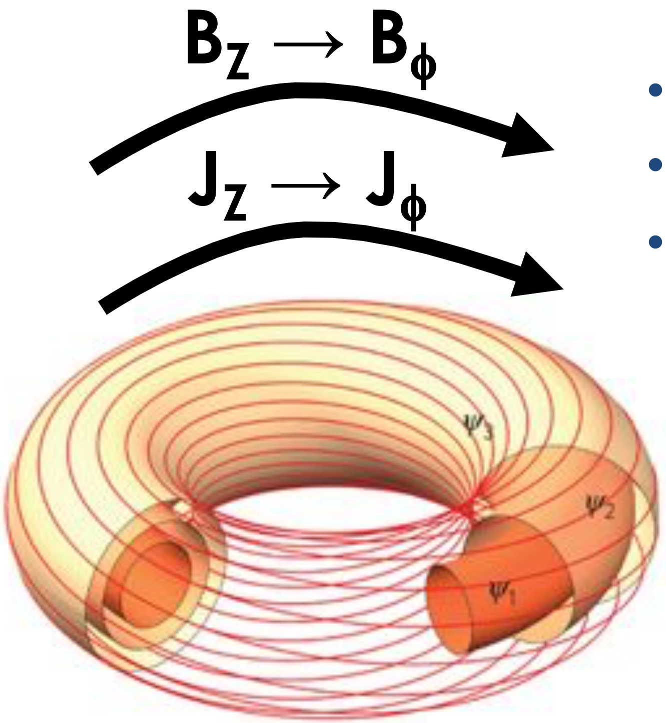
Plasma gun array

A rotating conducting wall, according to theory, should stabilize a linear pla
certain magnetohydrodynamic instabilities. Shown here is a view down the
in the rotating wall machine at the University of Wisconsin–Madison. Along its 1-meter length, the 16-cm-diameter, glass- and copper-lined stainless-steel tube reflects the sixfold symmetry of the triangular array of seven plasma “guns” centered at the far end. To study the wall’s stabilizing effect, the Wisconsin researchers, led by Cary Forest, ignite an arc plasma in each gun and then propel the plasmas into the tube while it rotates at speeds approaching 7000 rpm. As predicted, the rotation did indeed increase the stable range of the plasma current, though some discrepancies remain to be explored. (C. Paz-Soldan et al., *Phys. Rev. Lett.*, in press; image submitted by Carlos Paz-Soldan.)

To submit candidate images for Back Scatter, visit http://www.physicstoday.org/backscatter.html.

C. Paz-Soldan et al, Phys Today 2011

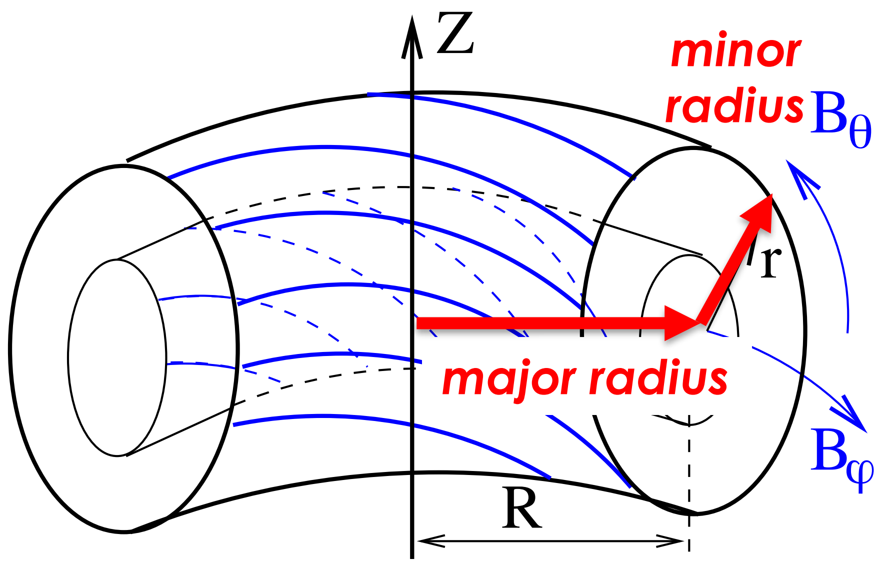
A Tokamak is Basically a Screw Pinch Whose Ends Wrap Around and Connect … Forming a *Torus*



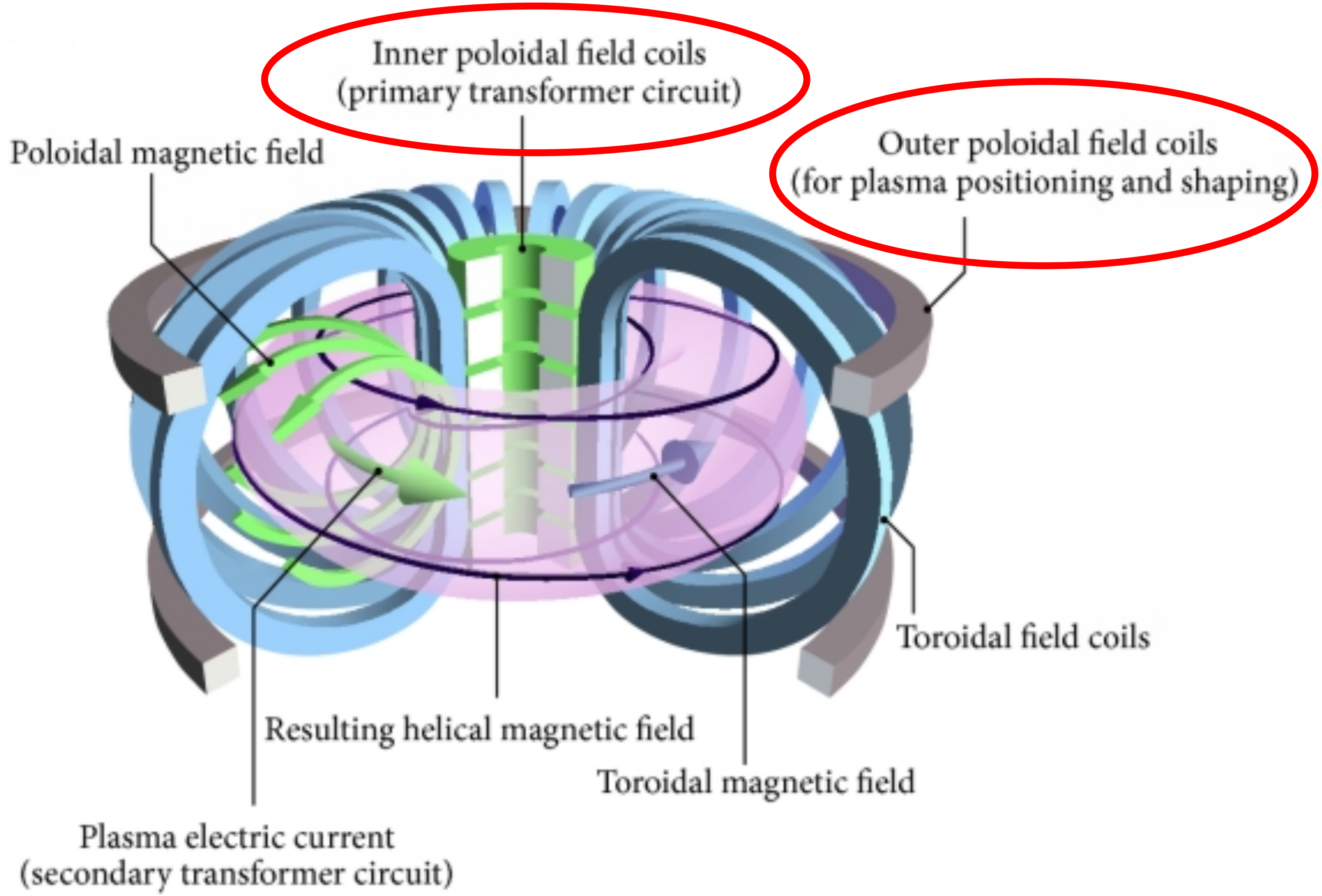
- B_ϕ is called the "Toroidal Field"
- J_ϕ is called the "Plasma Current"
- B_θ (from J_ϕ) is called the "Poloidal Field"

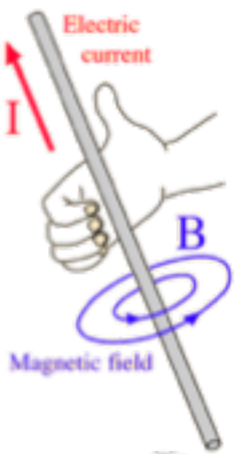
  Geometry: Major radius, minor radius

  Geometry: Toroidal, Poloidal direction



Tokamaks Require Additional Coils (Beyond Toroidal Field) to Provide *Toroidal* MHD Equilibrium

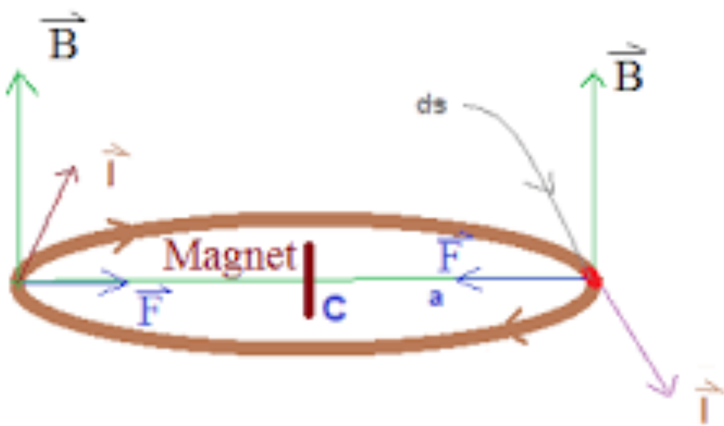


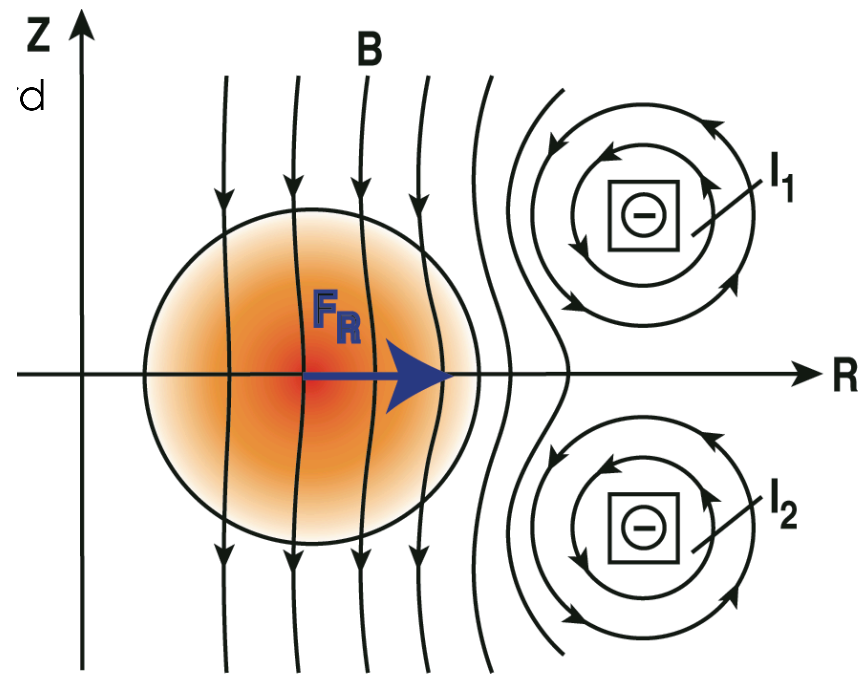


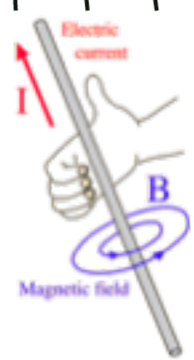
Courtesy Wikipedia

"Poloidal Field Coils" Are Needed to: Counter the "Hoop Force" and Control Radial Position

- **Plasma naturally wants to expand radially outward from hoop force**
- **Coils are needed to stop this**



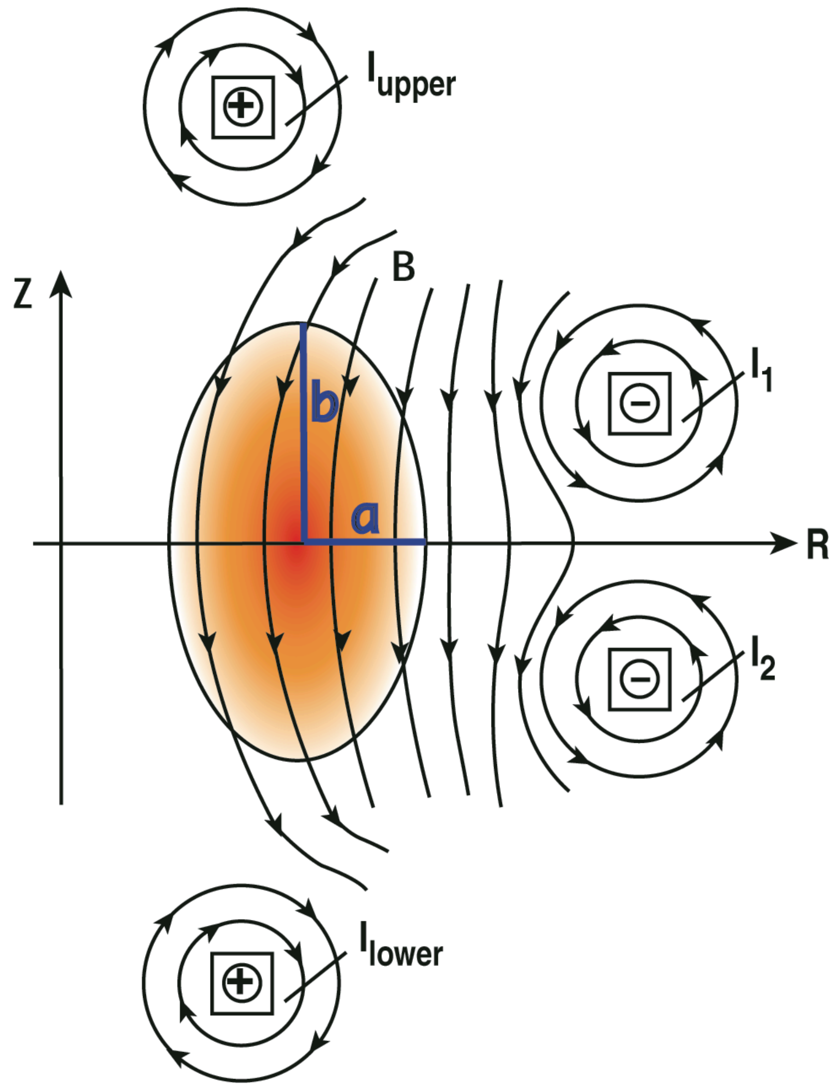




Courtesy M. Walker, GA

"Poloidal Field Coils" Are Needed to: Allow Plasma to "Elongate" and Improve Performance

- **Elongation increases the cross section of the plasma**
  - Area ~ pi*a*b
- **More room for fusion at the same major radius**
- **Elongation is *unstable* and requires *active control***
  - We'll revisit this later



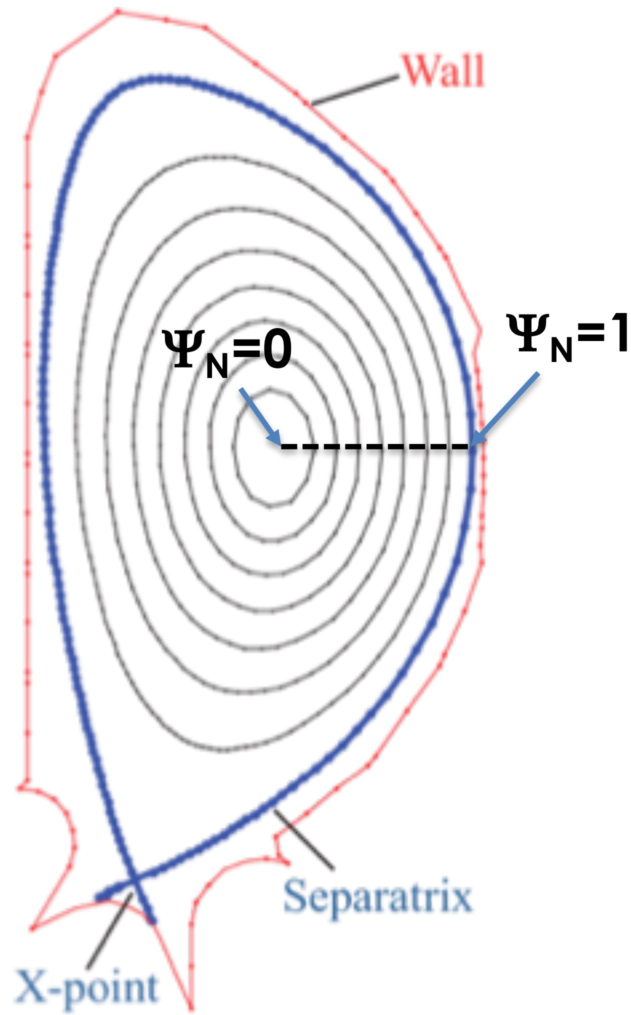
Courtesy M. Walker, GA

Axisymmetric Toroidal Equilibria are Described by the "Grad Shafranov" Equation[1,2]

Grad Shafranov:

$$R\frac{\partial}{\partial R}\left(\frac{1}{R}\frac{\partial \Psi}{\partial R}\right)+\frac{\partial^2\Psi}{\partial Z^2}=-\mu_0 R^2\frac{dp}{d\Psi}-F\frac{dF}{d\Psi}$$

- **Provides a solution for the Flux (Ψ) as a function of space (R, Z) and Pressure (p) and current (F)**
  - Flux as a function of space: **Ψ(R,Z)** is the basic coordinate
- **Contours of equal flux are called "*Flux Surfaces*"**
  - Pressure is constant on a flux surface
- **Outermost flux surface is called the "*Separatrix*"**
- **We label radius by "normalized flux"**
  - Core = 0, Separatrix = 1

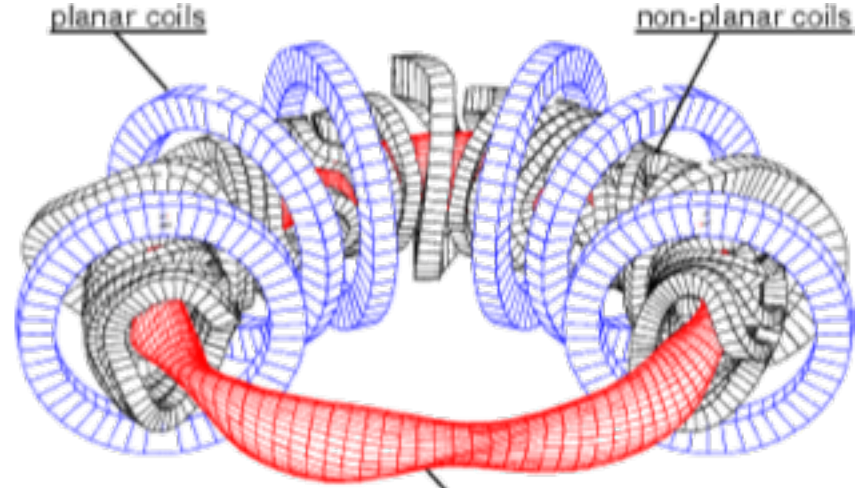


ITER Equilibrium

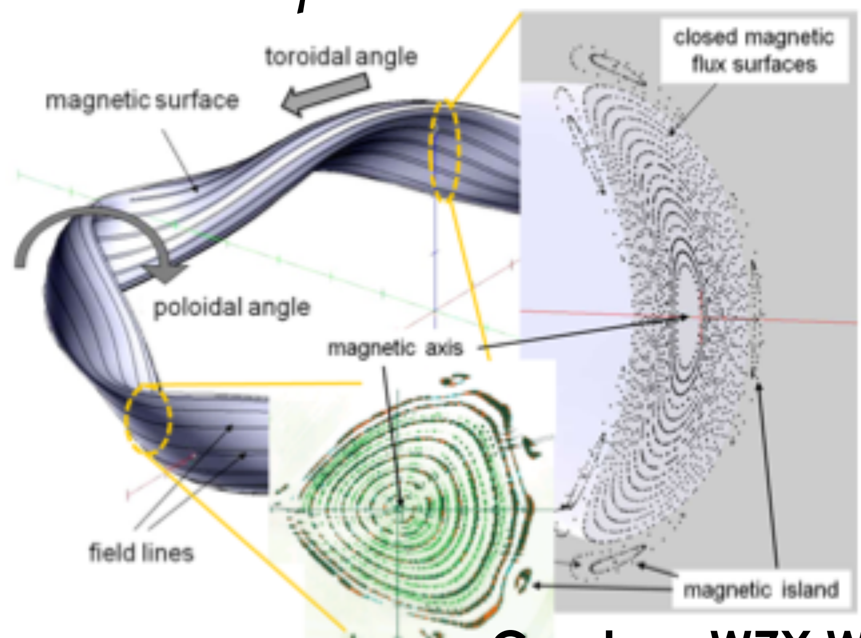
1. *Proceedings of the Second United Nations Conference on the Peaceful Uses of Atomic Energy*, Vol. 31, p.190
2. *Sov. Phys. JETP* **6**, 545 (1958)

3D Configurations: Also described by Ideal MHD and Also Defined by Flux Surfaces

- **Require powerful computers to design 3D configurations**
- **Equations are the same; flux surfaces play the same role**
  - They just morph toroidally
- **Advantage: *no current necessary within the plasma***
  - *Removes a free energy source for MHD instability*
- ***Disadvantage: engineering complexity increases***



$$\mathbf{J} \times \mathbf{B} = \nabla p$$



Courtesy W7X Web

3D Configurations: Also described by Ideal MHD and Also Defined by Flux Surfaces

Flux surfaces can be imaged by an electron beams lighting fluorescent rods

Courtesy W7X Web, Matthias Otte

Review of Concepts - Equilibrium

- The main equilibrium equation: $\mathbf{J} \times \mathbf{B} = \nabla p$
- Simplest configurations: Theta and Z-pinch
- Combine Theta & Z to form Screw Pinch $J_z B_\theta + J_\theta B_z = \frac{dp}{dr}$
- Wrap screw pinch into a torus = tokamak
- Poloidal field coils control and elongate tokamak
- 2-D Toroidal equilibria governed by Grad-Shafranov Equation
- 3-D Configurations (Stellarators) obey same MHD equations

$$\mathbf{J} \times \mathbf{B} = \nabla p$$

Outline of Presentation

- Pre-amble: Why the MHD model?
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- MHD Equilibrium: 1-D, 2-D, 3-D Configurations
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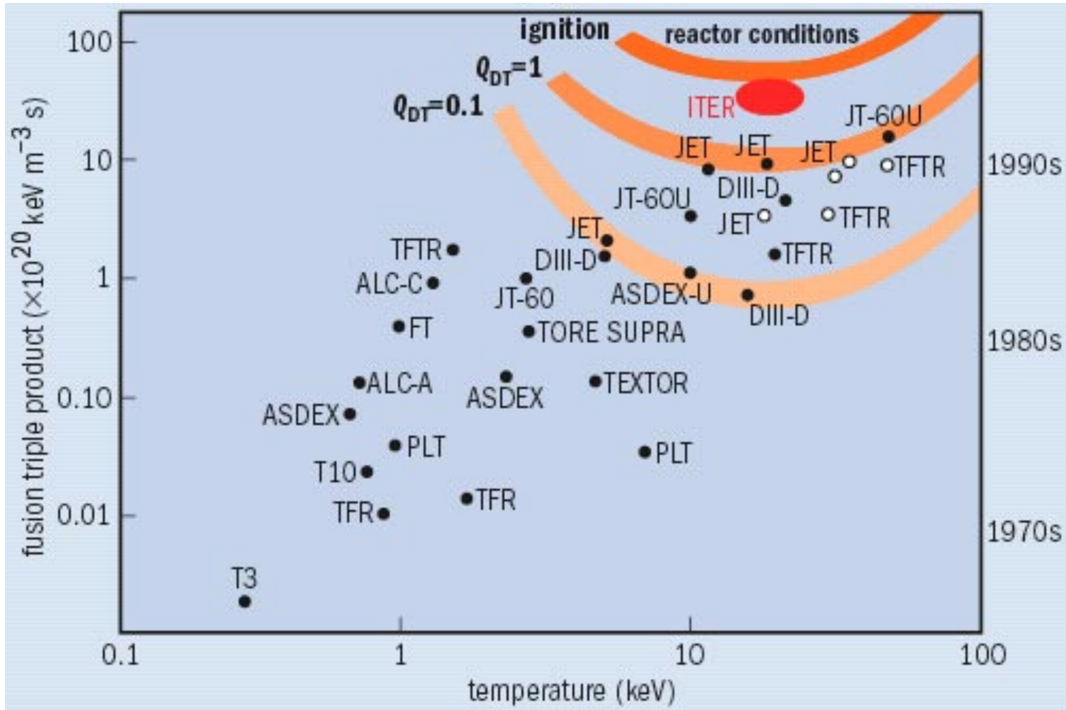
MHD Stability Sets the Most Fundamental Limits to Achieving Controlled Magnetic Fusion

τ_E = $\frac{\text{Energy}}{\text{Power}}$

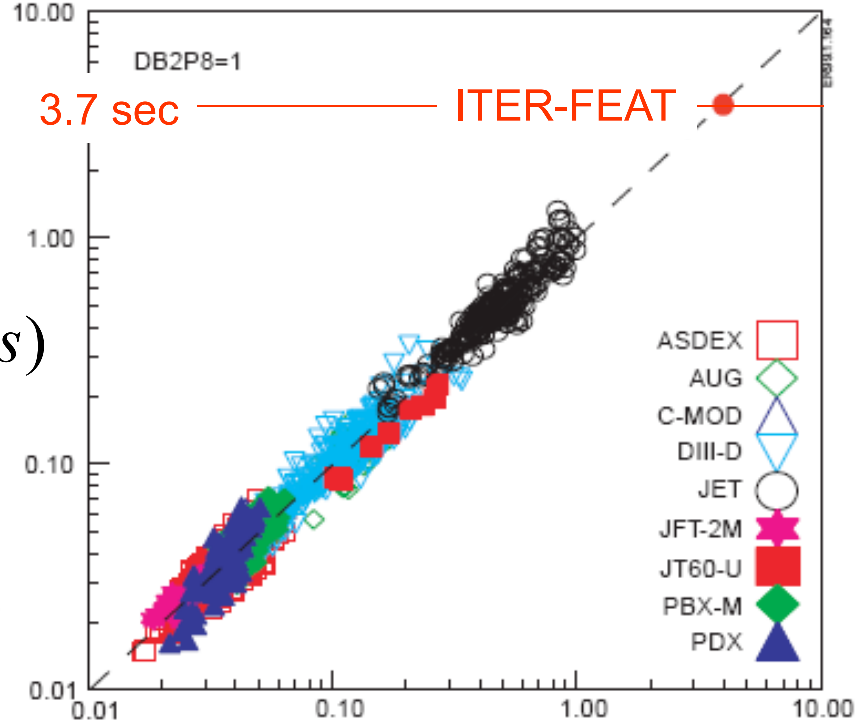


Condition for ignition: $\boxed{p\tau_E \geq 8\ bar.s}$ $\quad T_{min} \sim 15\text{keV}$

- **Pressure (p) comes directly from MHD limits**
- **Energy confinement time (τ_E) depends on quantities limited by MHD**

Energy Confinement Time (τ_E) Increases with Quantities Limited by MHD Considerations

$$\tau_{\mathrm{E,th}}^{\mathrm{exp}}\ (s)$$



$$\tau_{\mathrm{E,th}}^{98(\mathrm{y},2)}\ (s)$$

τ_E = **Energy / Power**

Prediction for ITER

τ_E = 3.7 s

5.3 T; 15 MA;
n = 1 10^{20} m^{-3} = 0.85 n_{GW}
P = 87 MW

Size (strongest!) — Plasma current — *Power (bad)*

$$\tau_{\mathrm{E,th}}^{98(\mathrm{y},2)} = 0.0562 I^{0.93} B^{0.15} P^{-0.69} n^{0.41} M^{0.19} R^{1.97} \varepsilon^{0.58} \kappa^{0.78} \propto R^2 I_p P^{-2/3}$$

6

Current (strong) — *B-field (weak)* — inverse aspect ratio — elongation

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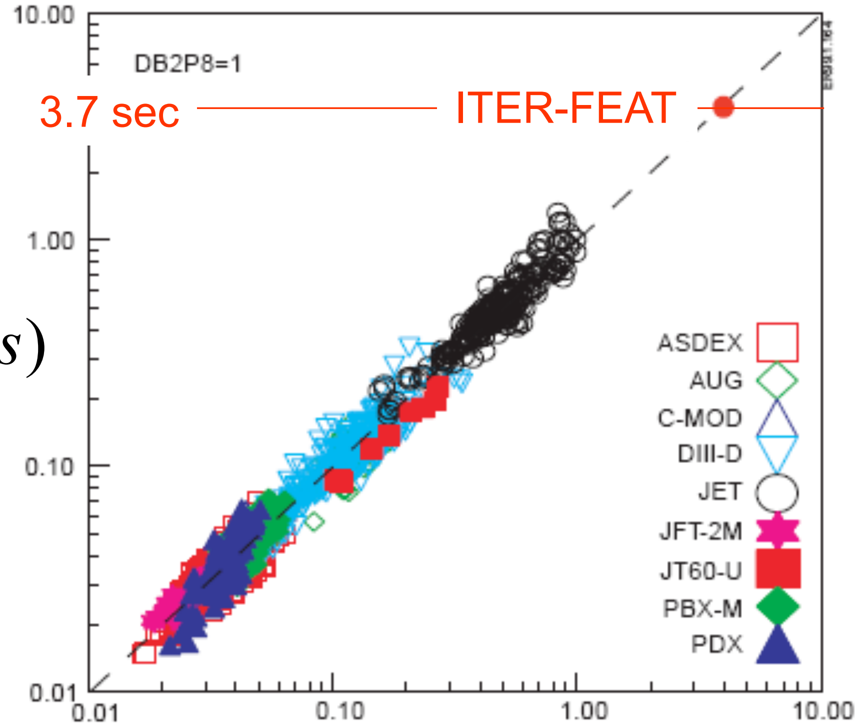
Energy Confinement Time (τ_E) Increases with Quantities Limited by MHD Considerations



$\tau_{E,th}^{exp}\,(s)$

$\tau_{E,th}^{98(y,2)}\,(s)$

τ_E = Energy / Power

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6

Current (strong) *B-field (weak)* inverse aspect ratio elongation

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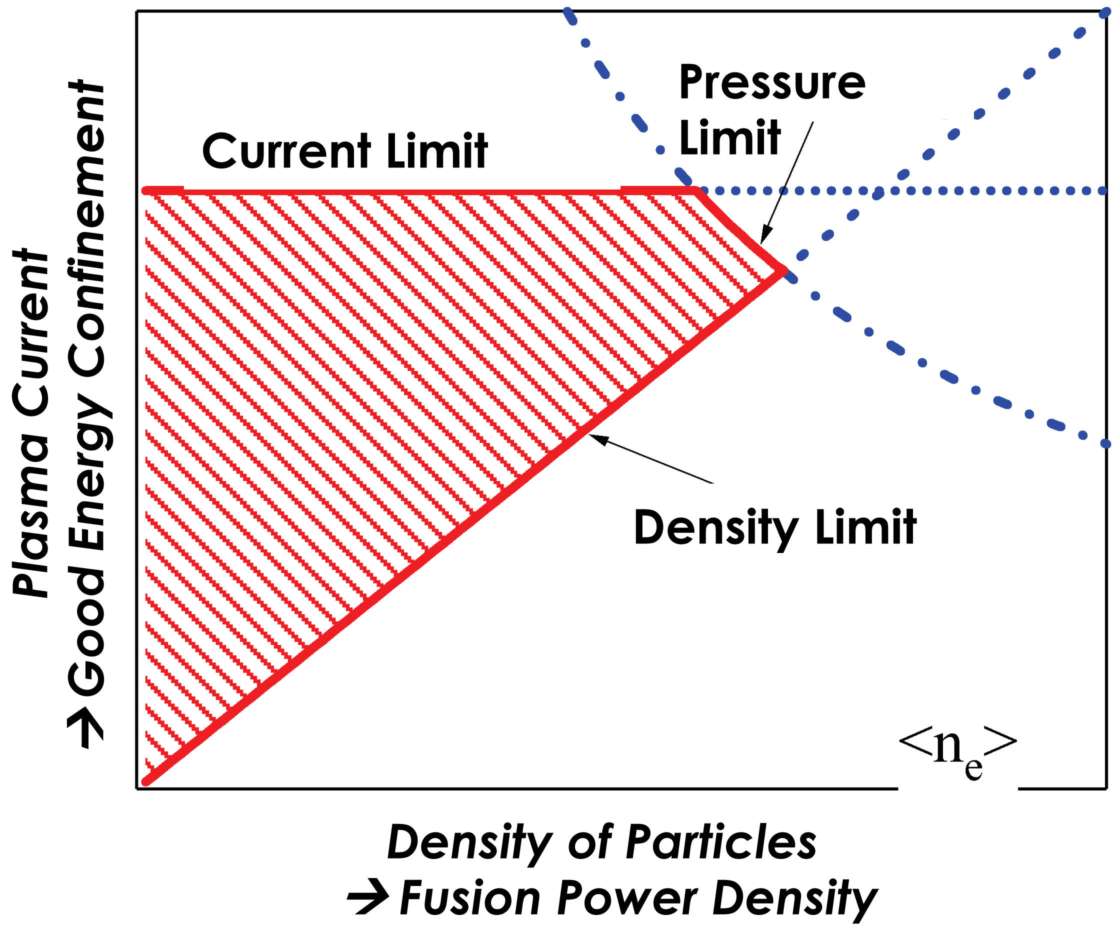
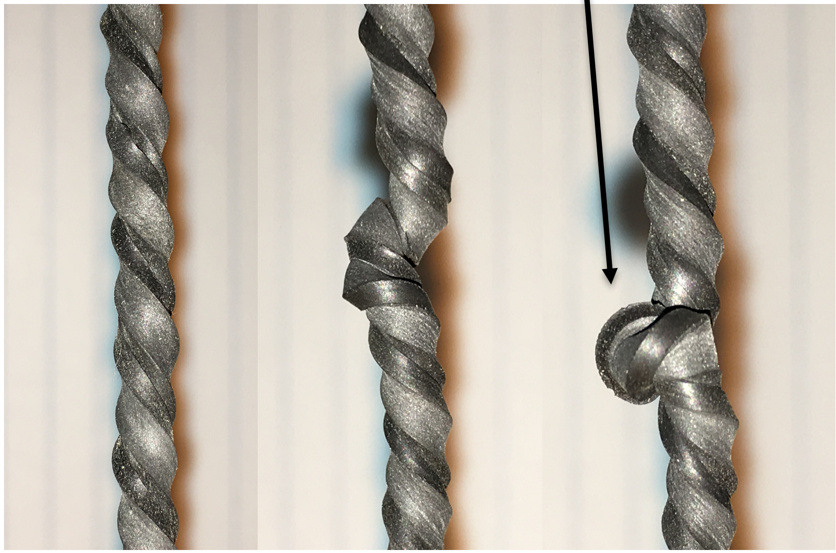
Reminder: Most Tokamak Operational Limits are Governed by MHD → Let's Start with the Current Limit



Maximum Plasma Current is Set by Kink Instabilities: Essentially a Limit on the Magnetic Field "Twist"

- **Mechanical analog is twisting an elastic band**
- **Eventually it develops a *kink***



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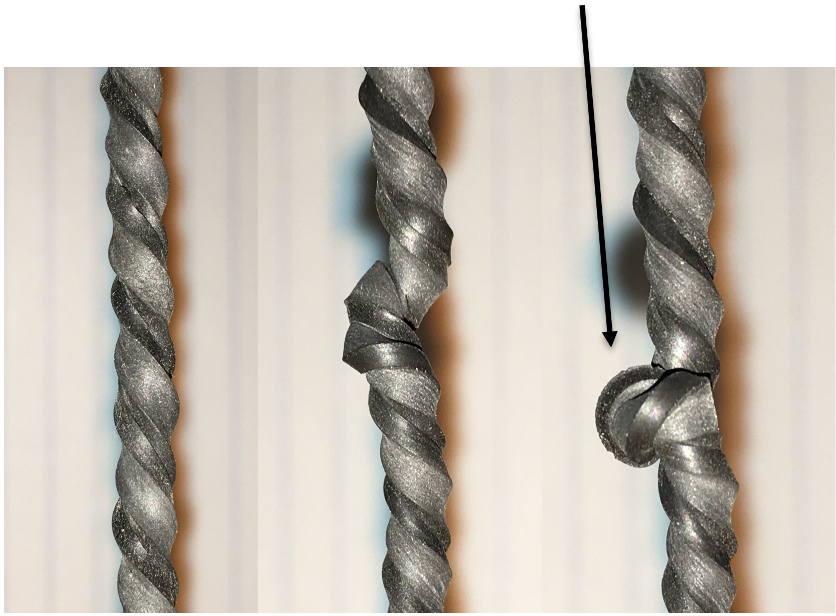
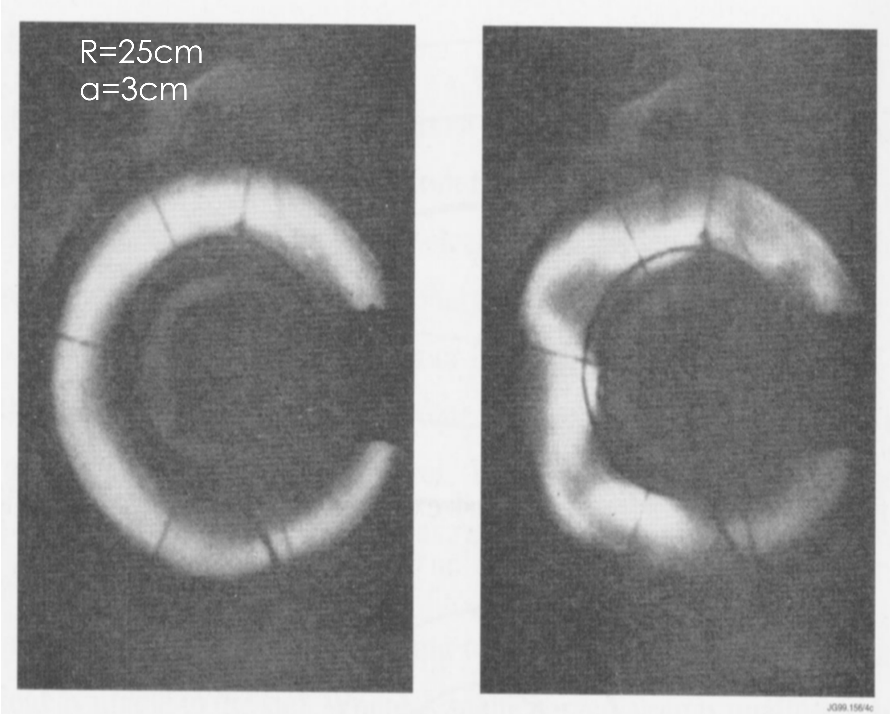


First observations in plasma of the KINK INSTABILITY:



Carruthers & Davenport, Harwell, 1950s.

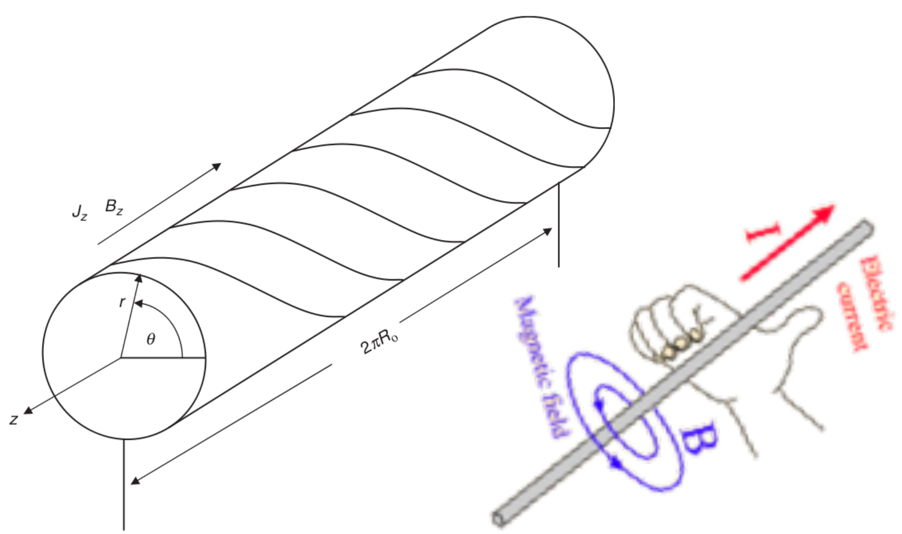
Conceptual Picture of the Kink Instability: Consider How Much "Twist" Is in the Magnetic Field

- **Stability is parametrized by a ratio of the axial (toroidal) field to the azimuthal (poloidal) field:**
  - This is called the "Safety Factor" **q**
  - Low q is "bad" for kink stability

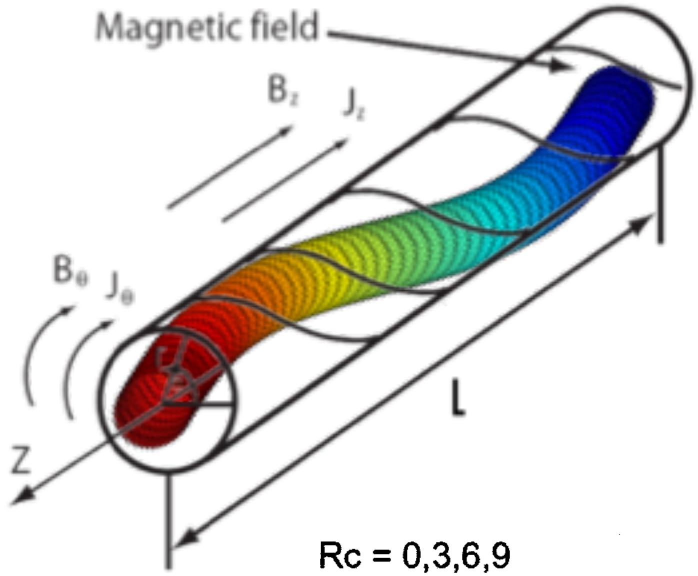
$$q_* = \frac{a}{R}\frac{B_\phi}{B_\theta}$$

- **The poloidal field arises from the axial (toroidal) current**
  - Toroidal field is stabilizing, plasma current is destabilizing

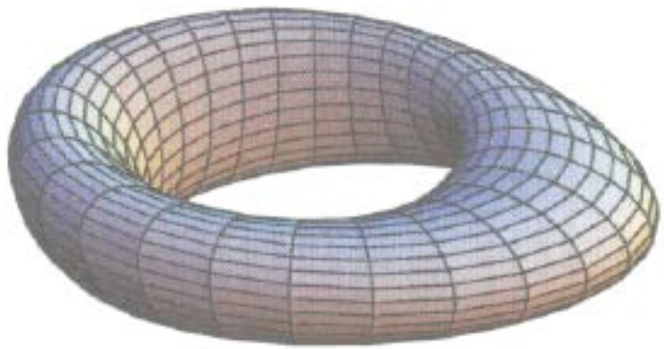
$$q_* = \frac{2\pi}{\mu_0}\frac{B_\phi}{I_P}\frac{a^2}{R}$$

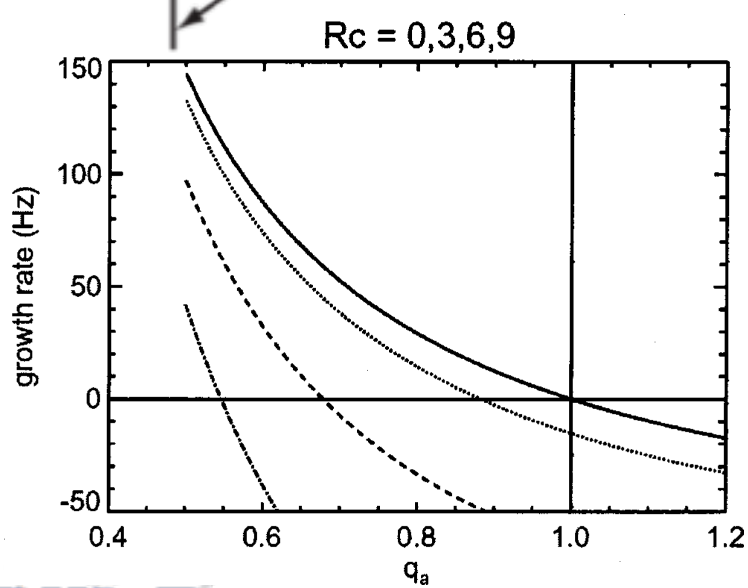


Toroidal Geometry Provides Additional Complexities ... Not a Single Limit, But Rather Regions of Instability

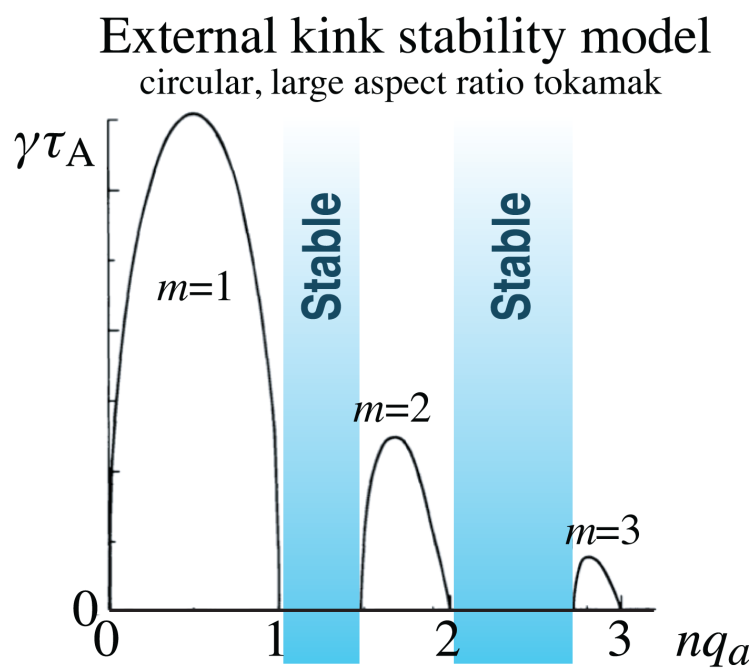


Navratil, APS-DPP '04



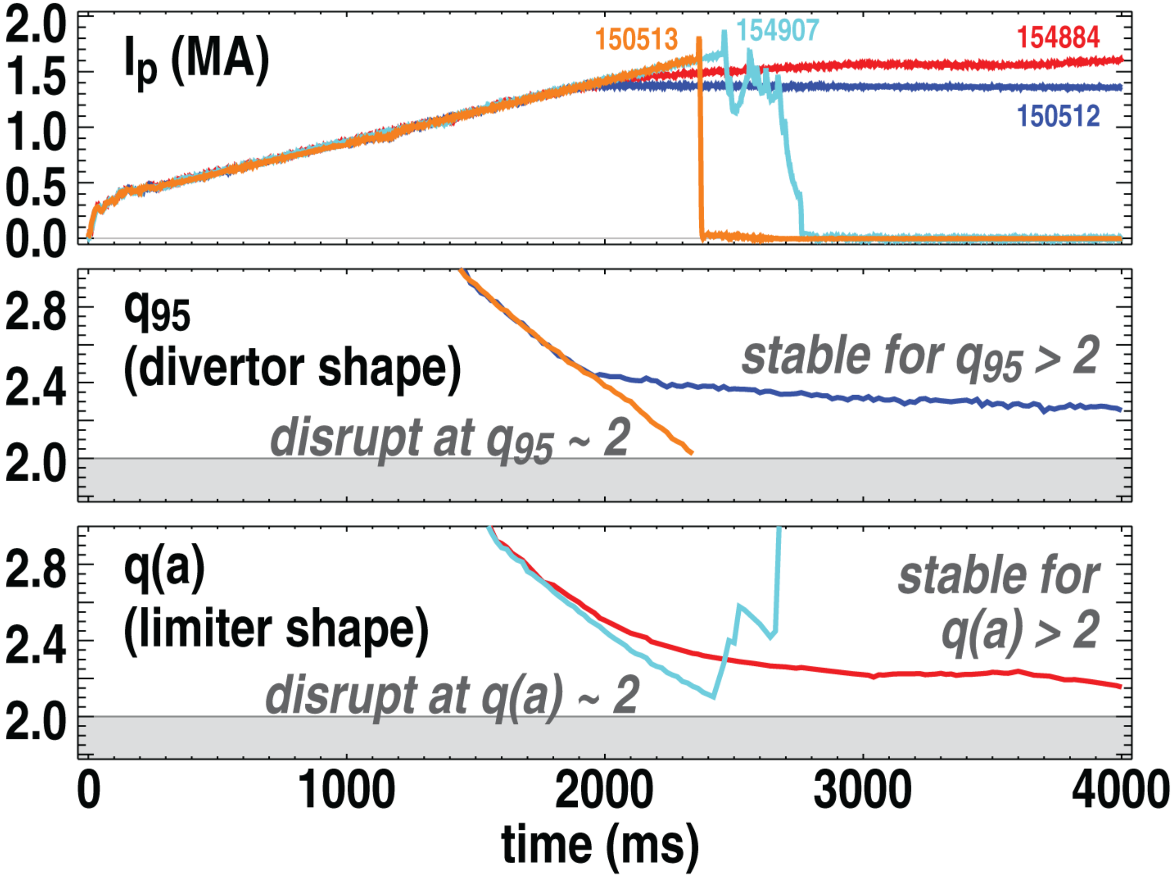


C. C. Hegna, PoP 2004

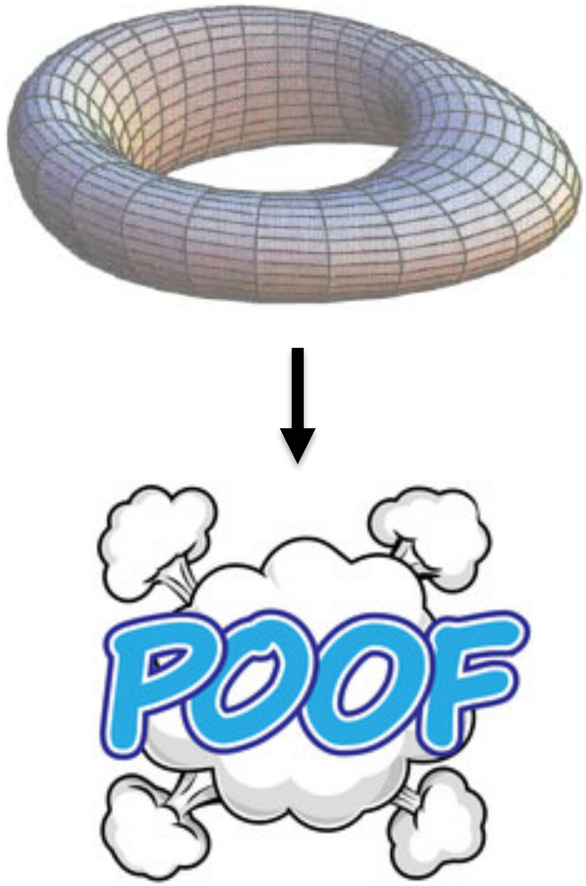


[J. A. Wesson, *Nucl. Fusion* **18** (1978) 87]

What Happens When you Cross a Stability Boundary? … Tokamak Plasmas Tend to Go Poof



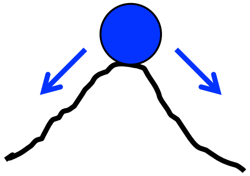
J. Hanson, APS-DPP 2013

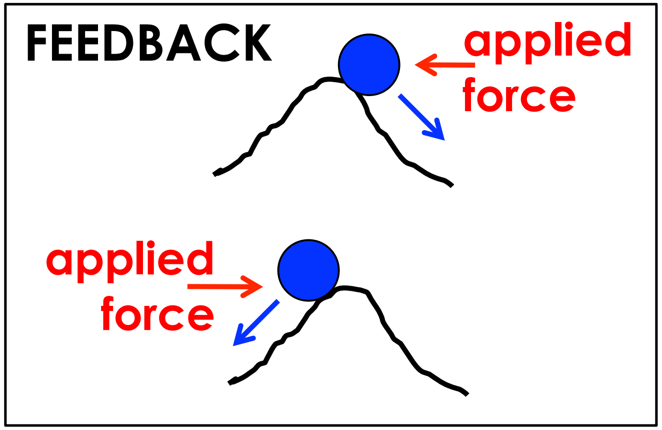


"Disruption"

Some MHD Stability Limits Can be Overcome by Active Control

- **Open-loop instability:**

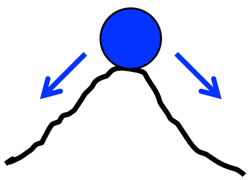


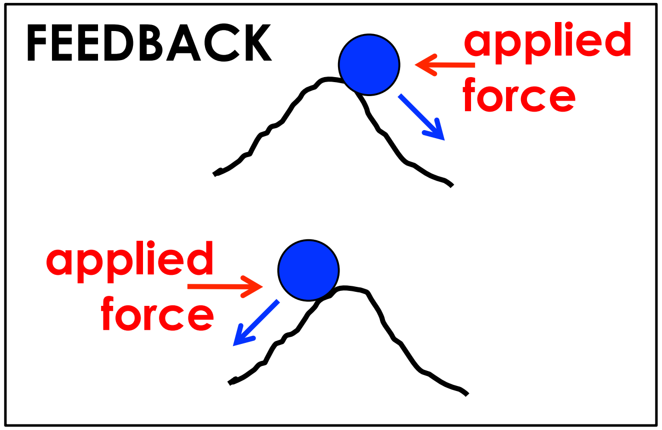


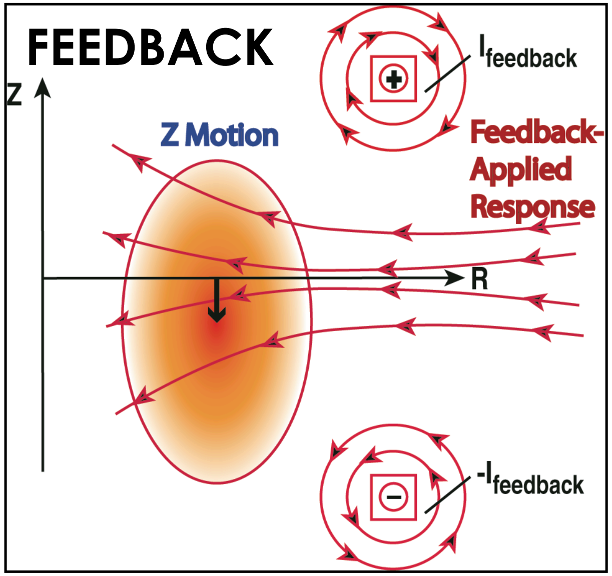
Courtesy M. Walker

Some MHD Stability Limits Can be Overcome by Active Control … Example: Elongation Control

- **Open-loop instability:**

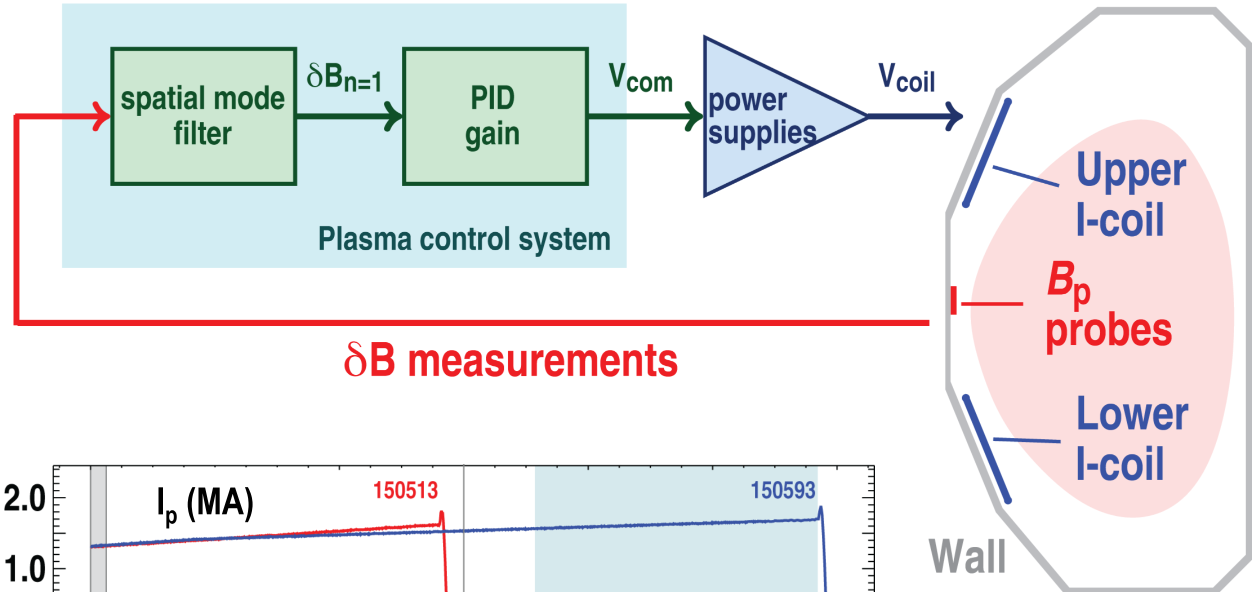


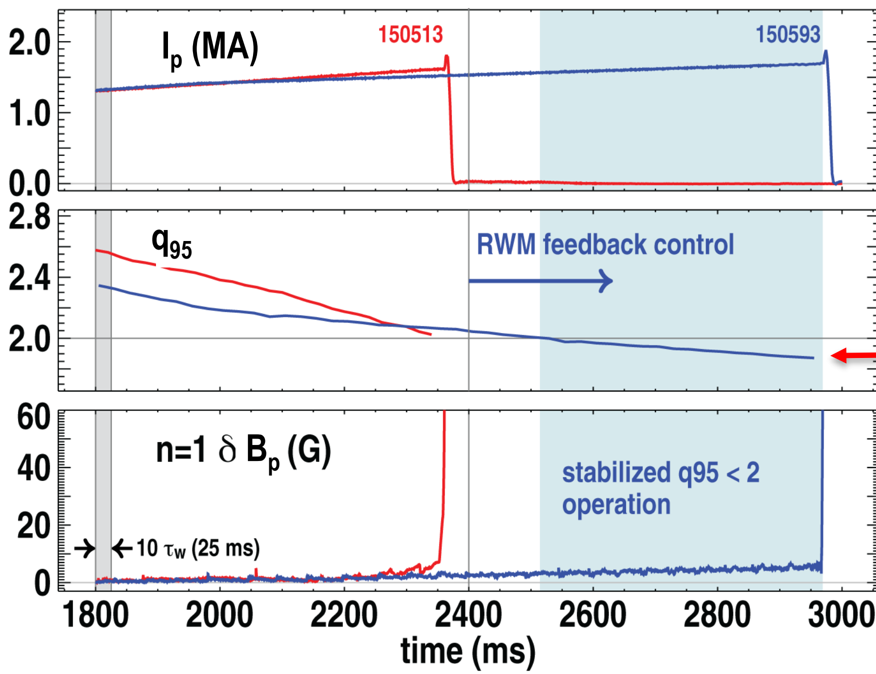




Courtesy M. Walker

2nd Example:
Active Feedback Can Overcome the Current Limit

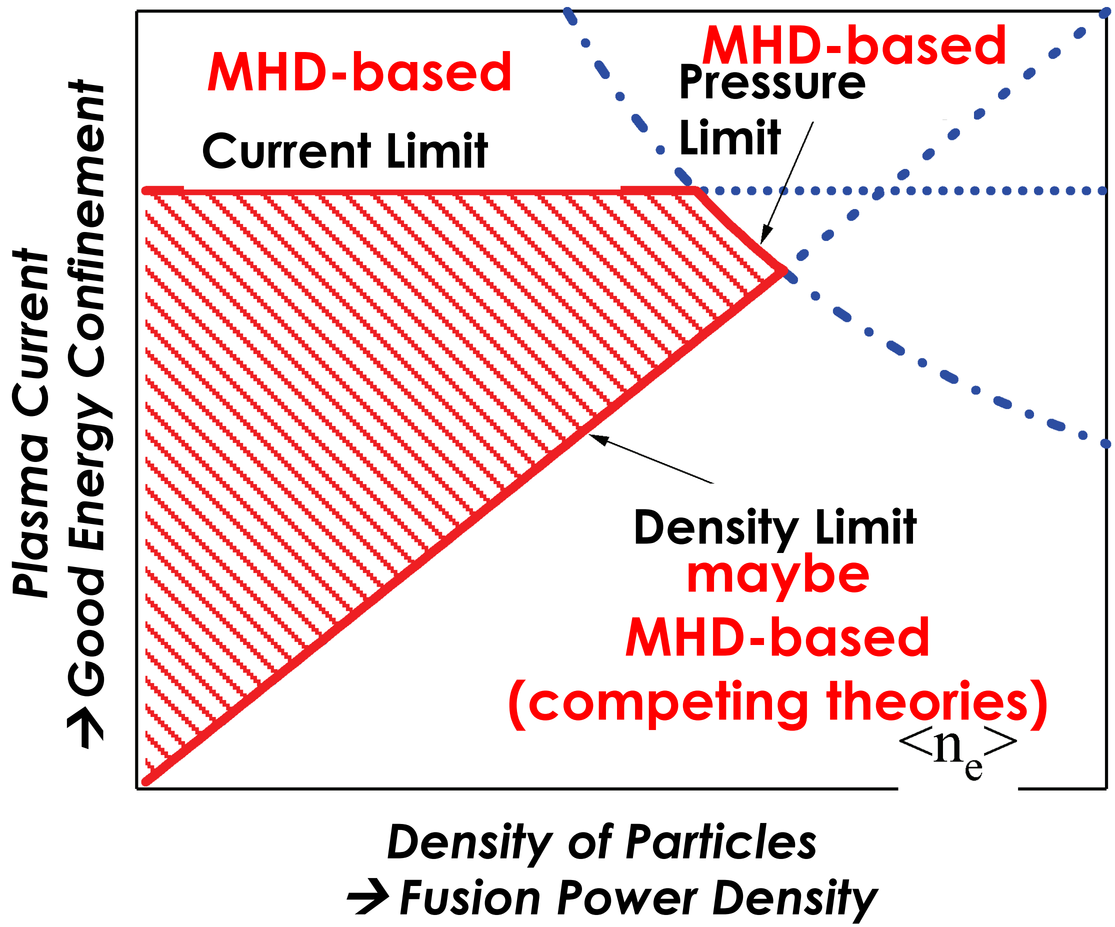




Lower q Accessed
(… to a point)

Courtesy J. Hanson

Pressure Limits are Also an Active Area of MHD Research



Pressure Limits are Parametrized by the Plasma Beta ... A Measure of Magnetic Field "Utilization"

- **Conceptually the plasma beta is as follows:**

$$\beta = \frac{\text{plasma pressure}}{\text{magnetic pressure}}$$

- **Mathematically we write it:**

$$\beta = \frac{2\mu_0 \langle p \rangle}{B^2}$$

- **Typical values of β are only few %**

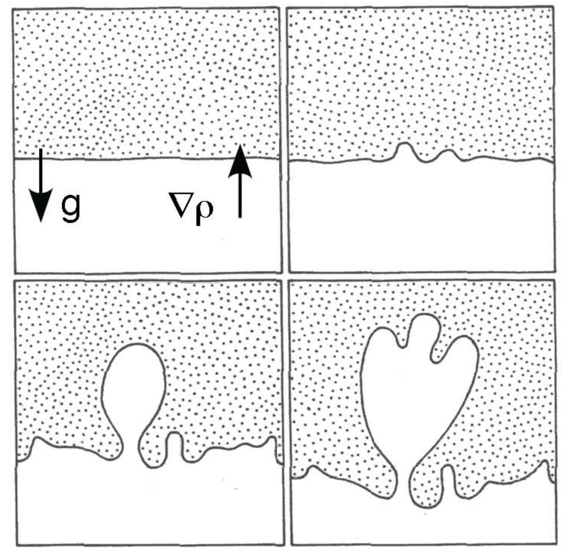
- **Low beta is more MHD stable**
  - … but lower pressure (less fusion) at constant magnetic field

- **Above a critical beta MHD instability is found**

The Pressure Limit Originates from "Interchange" Instabilities (Mixed with Kinking Component)

- **Mechanical Example**
  - Replace Gravity with Magnetic Field
  - (imperfect analogy)

Gravitational Rayleigh-Taylor



Linear

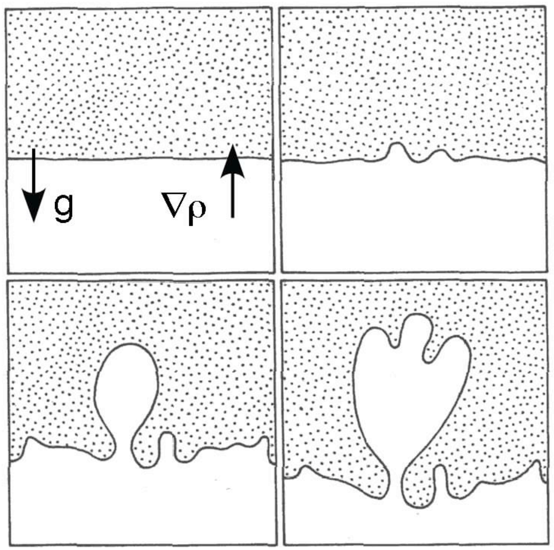
Nonlinear:
Bubbles,
fingers,
vortices,
…

The Pressure Limit Originates from "Interchange" Instabilities (Mixed with Kinking Component)

- **Mechanical Example**
  - Replace Gravity with Magnetic Field
  - (imperfect analogy)

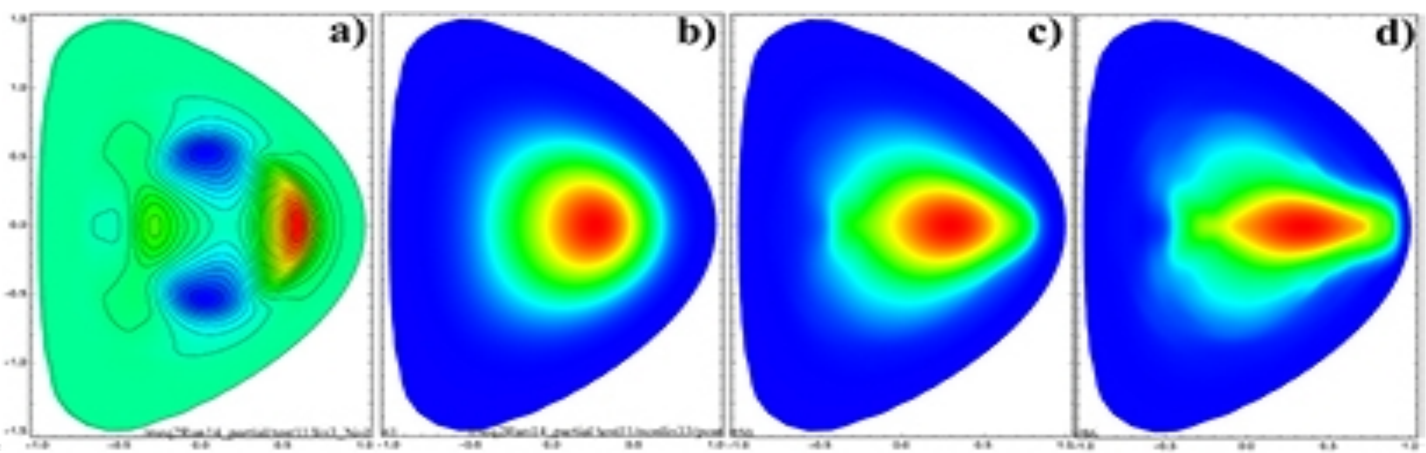
- **Tokamak Example**
  - "Bubble" is the plasma escaping
  - Called "Ballooning"

Gravitational Rayleigh-Taylor



Linear

Nonlinear: Bubbles, fingers, vortices, …



Aydemir Nucl Fusion 2017

Tokamak Pressure Limits Follow the "Normalized Beta"

- **Achievable beta is Found to rise with Normalized Current**

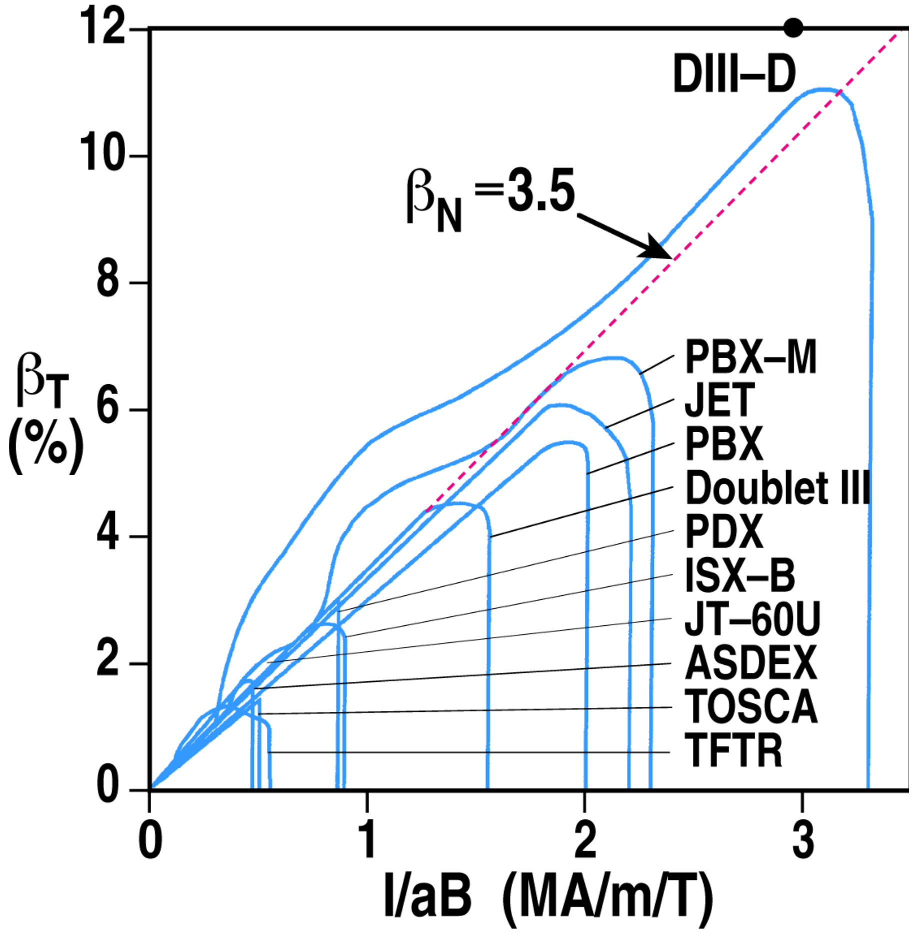
$$I_N = \frac{I_P}{aB_\phi}$$

  - ($\mu_0 I_N$ is dimensionless)

- **A consistent beta limit when normalized to I_N is found:**

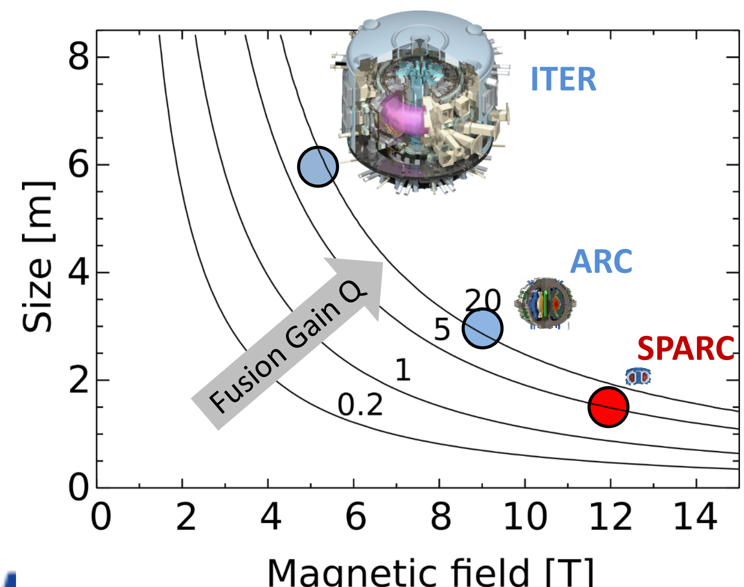
$$\beta_N = \frac{\beta}{I_N} = \frac{\beta}{\left(\frac{I_P}{aB_\phi}\right)}$$

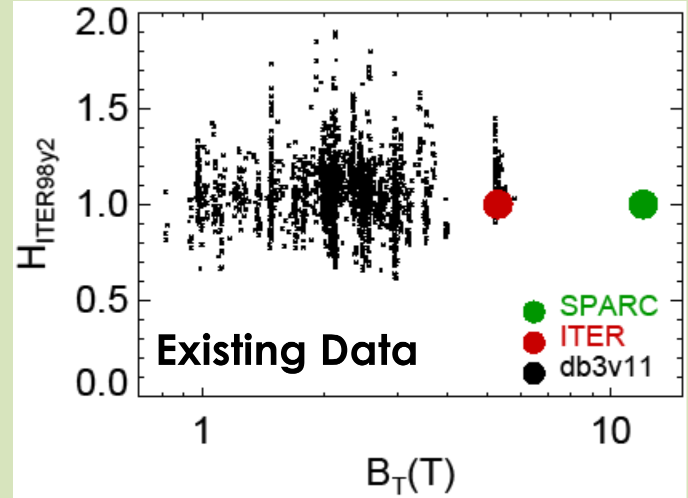
- **The critical β_N is around 3**
  - Give or take …
  - Complex calculations

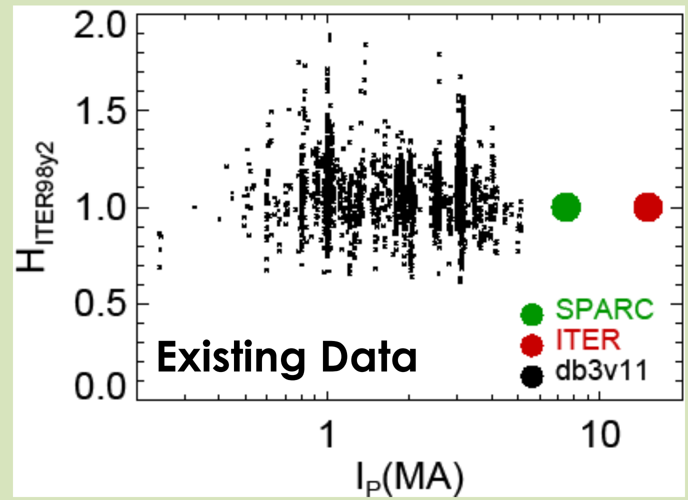


Side-note on the High-Field Breakeven Path (SPARC): High B → High I_P w/o Kinks → High τ_E → Fusion Gain

- **Pressure (p) can be higher at high magnetic field**
  - *w/ same **Beta (β)***
- **Energy confinement time (τ_E) will be higher at high current**
  - *w/ same **Safety Factor (q)***







M. Greenwald et al, NAS 2018

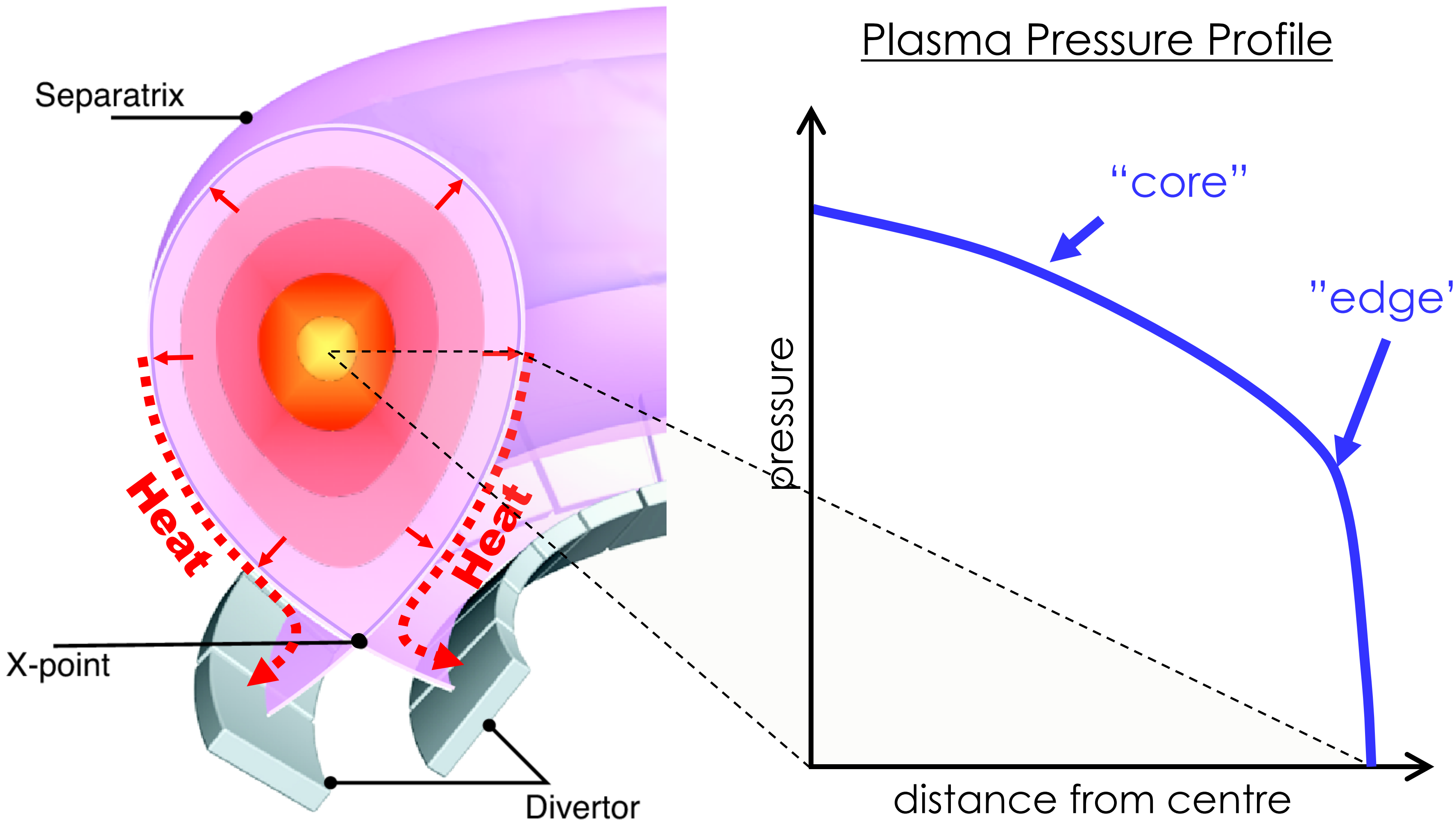
Review of Concepts – Global MHD Stability Limits

- **Fusion requires high pressure and good energy confinement**
  - MHD sets limits on both
- **Increasing current is good for energy confinement**
  - Until it is limited by the "Kink Instability"
  - Parametrized by the safety factor (**q**)
- **Increasing pressure is needed for fusion**
  - Until it is limited by "Interchange" instabilities
  - Parametrized by the normalized beta ($\boldsymbol{\beta_N}$)
- **Active feedback with magnetic coils can push back on both**
  - Expands the achievable operating space

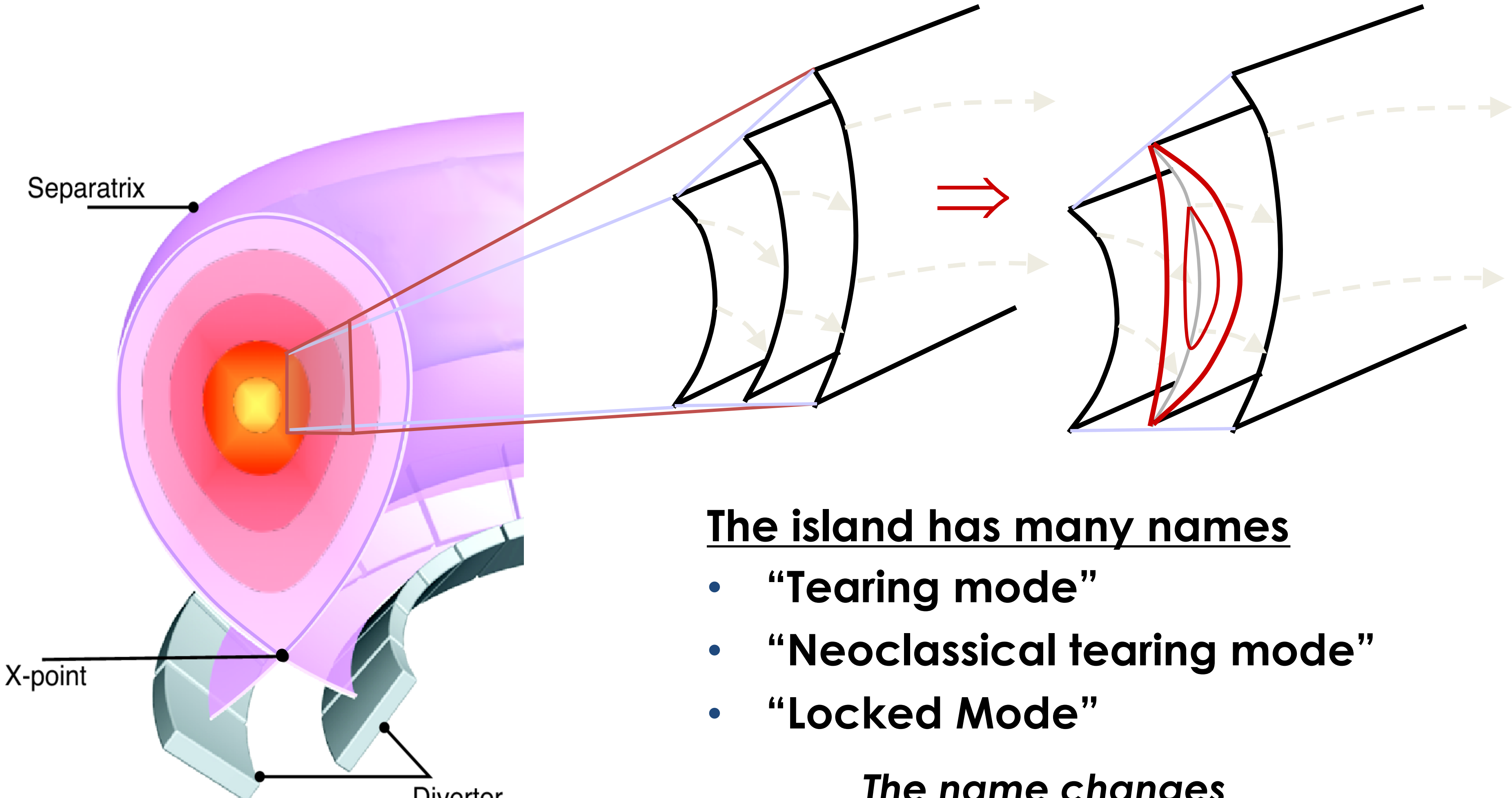
Outline of Presentation

- Pre-amble: Why the MHD model?
- Development of the MHD Equations
- MHD Equilibrium: 1-D, 2-D, 3-D Configurations
- MHD and its Relation to Global Operational Limits
- **Brief Tour of Common MHD Instabilities and Their Control**

Reality: Tokamaks Do *Not* Operate so Close to Global Limits. Yet Other ("Lesser") Instabilities Still Exist



The "Magnetic Island" is Probably the Most Common Instability the Tokamak Encounters

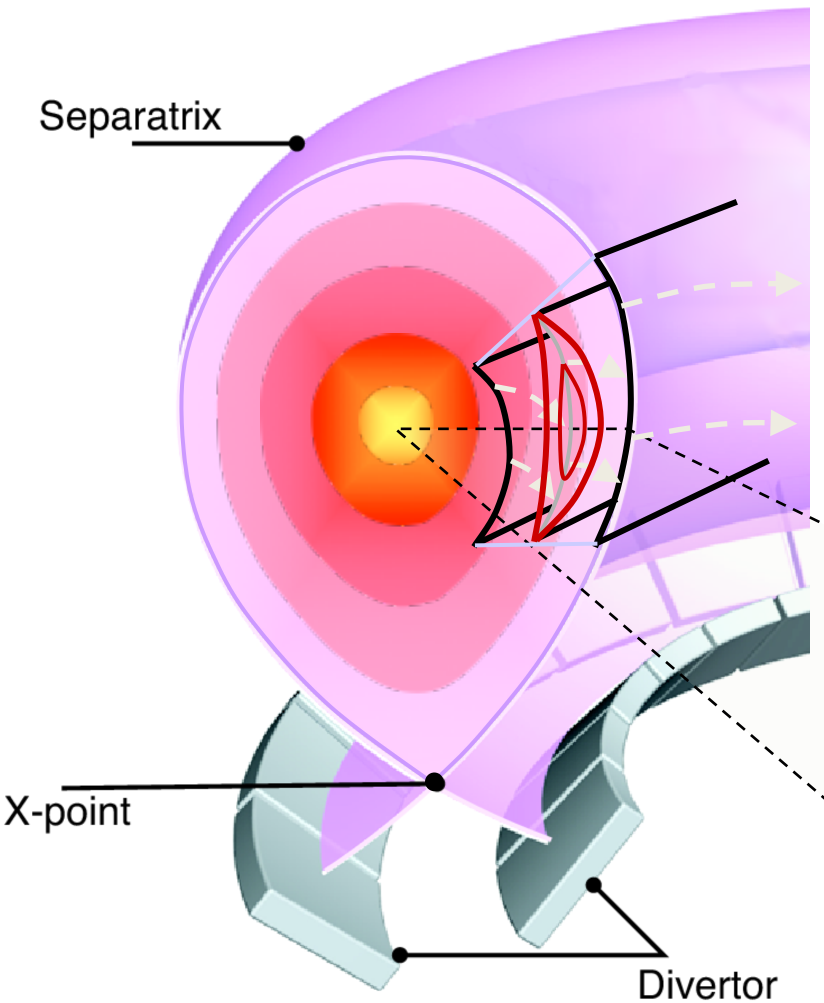


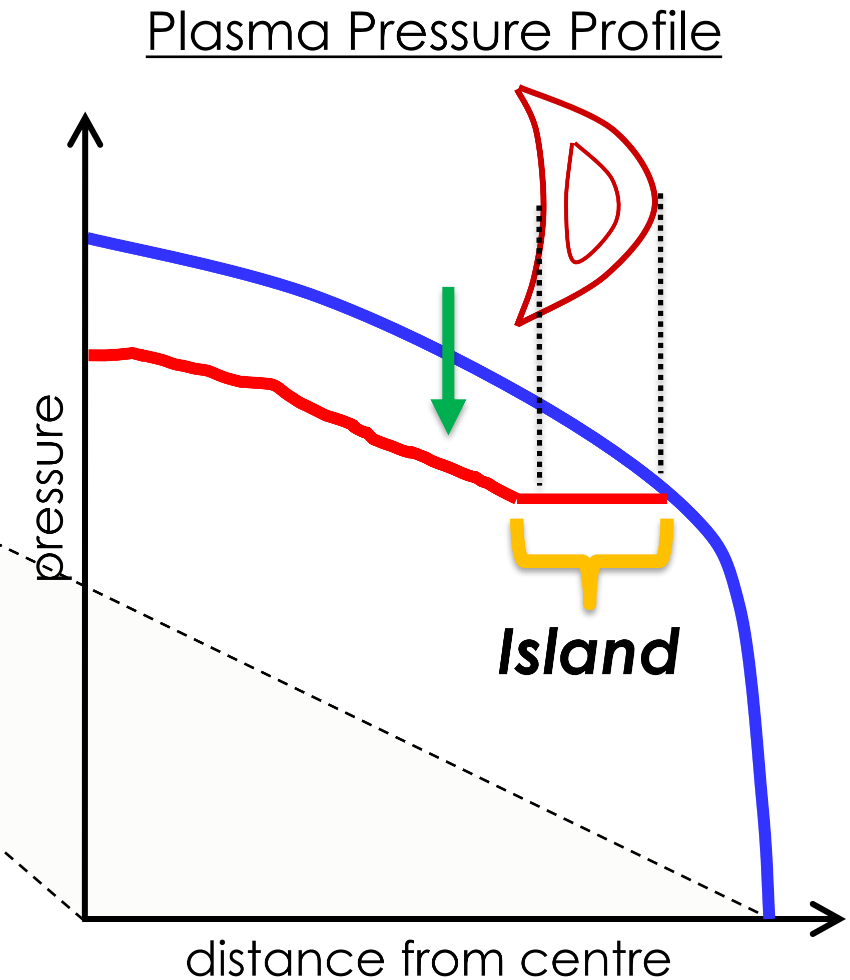
<u>The island has many names</u>

- **"Tearing mode"**
- **"Neoclassical tearing mode"**
- **"Locked Mode"**

The name changes based on its origin and dynamics

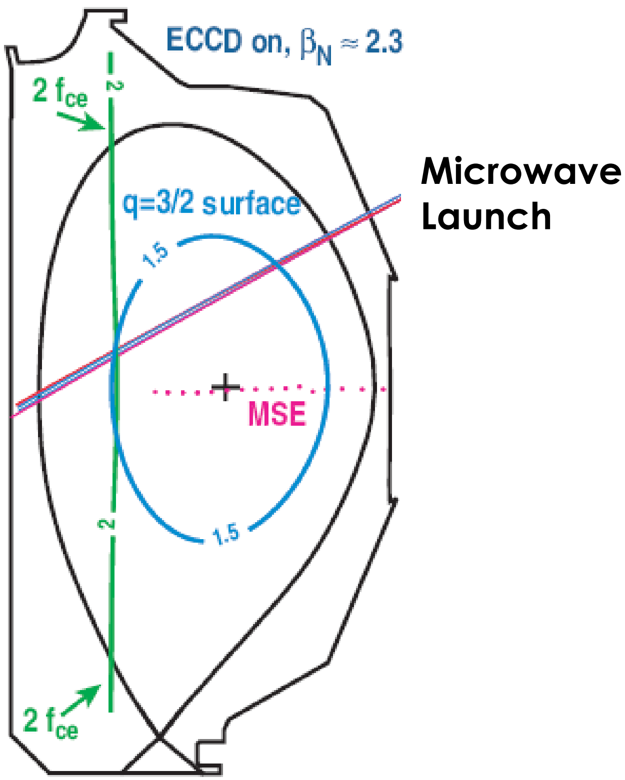
Magnetic Islands Flatten the Profile Locally
... And can Terminate the Plasma if they get Too Big

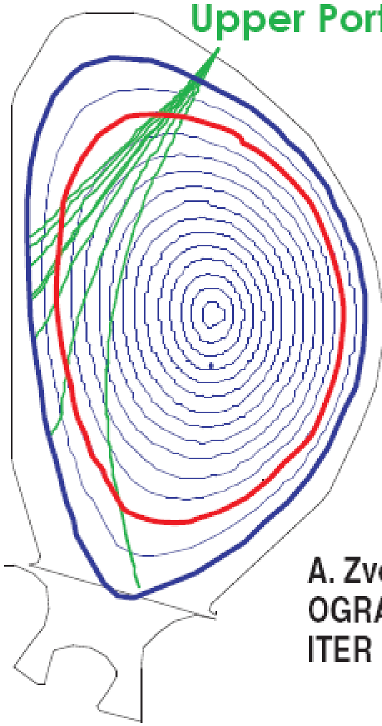




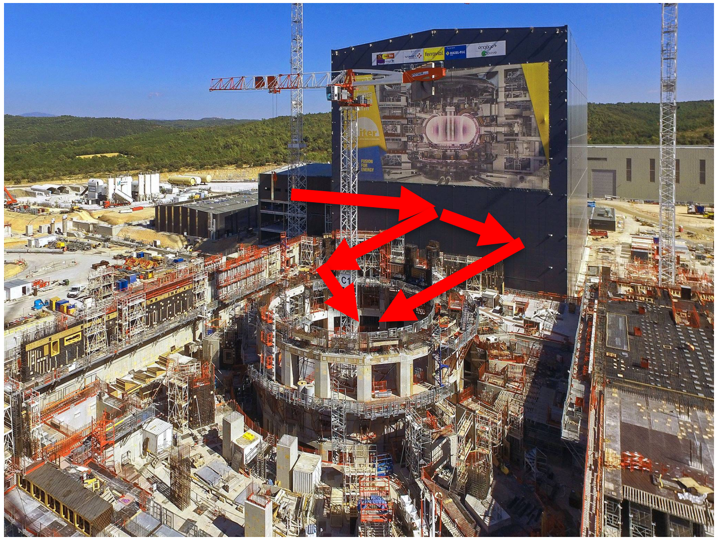
The ITER plan for NTM Control is Injection of ~ 100 GHz Microwaves to Locally "Heal" Island

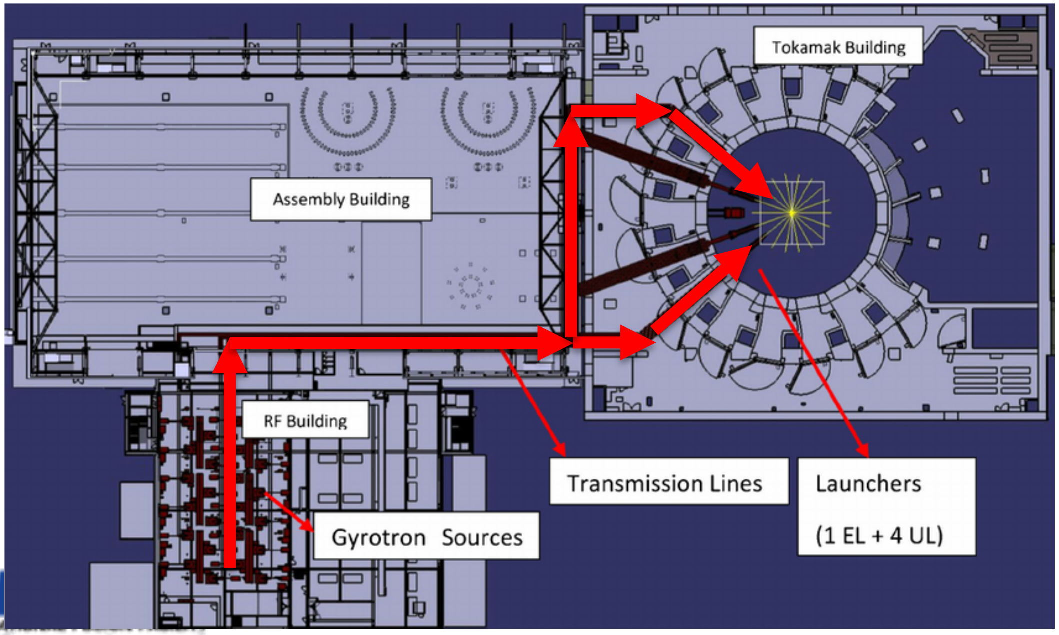
- **This technique is "routine" for many tokamak regimes**
- **Open physics questions remain regarding:**
  - How close to island do you need to aim? How much power?
  - Is it a direct or indirect effect?
  - Why does it not work in certain regimes?

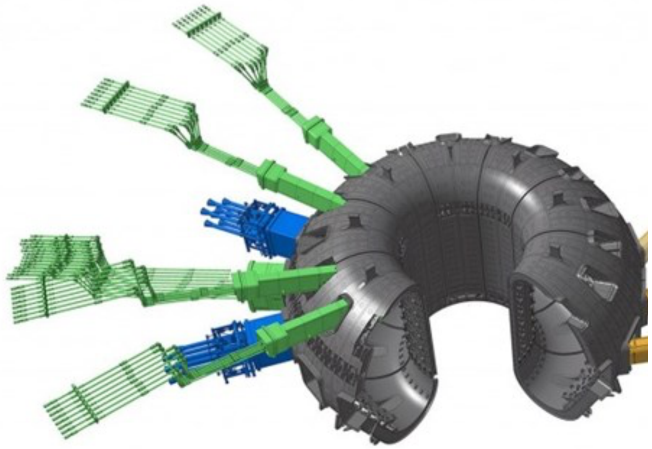




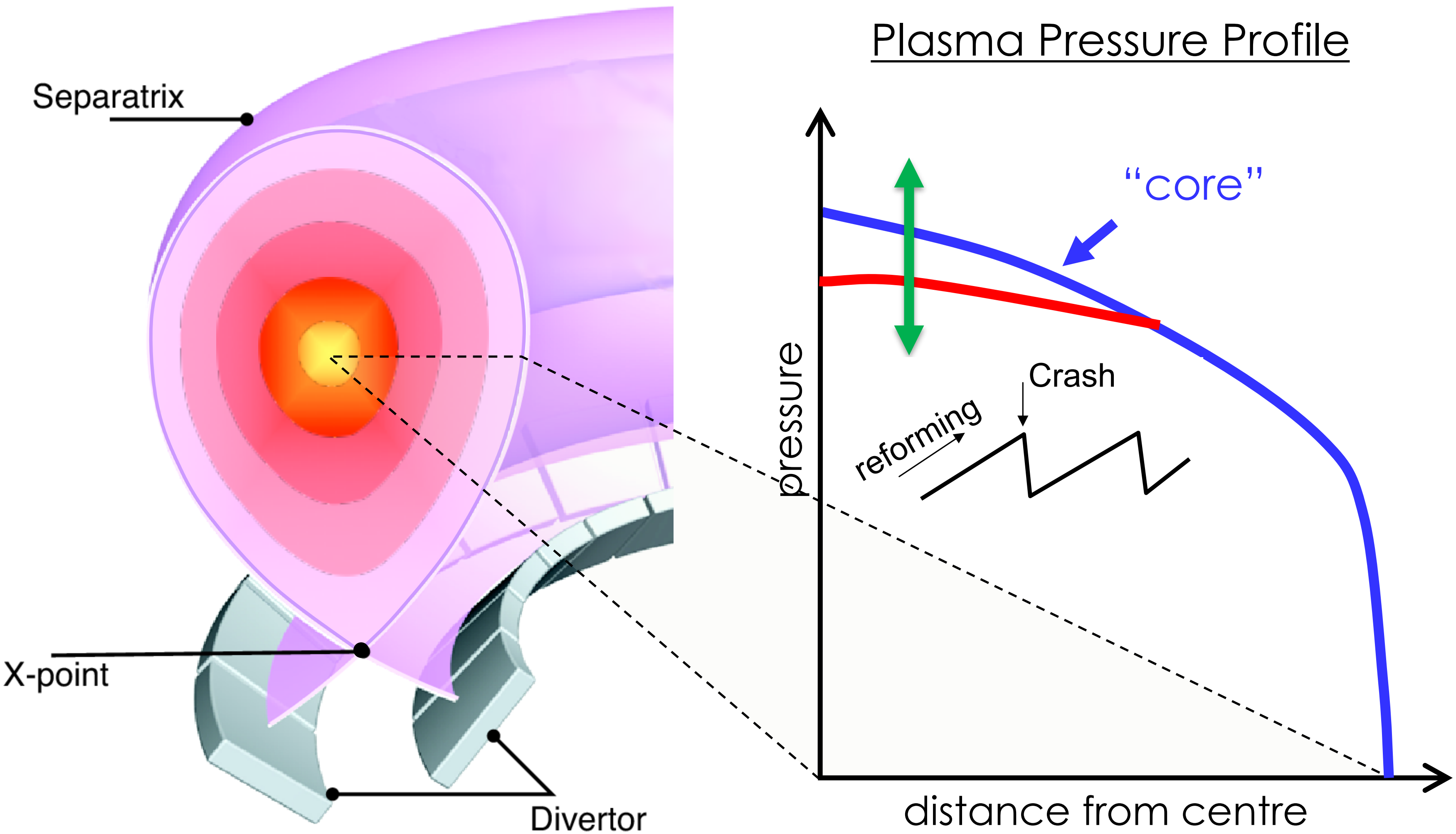
The ITER plan for NTM Control is Injection of ~ 100 GHz Microwaves to Locally "Heal" Island





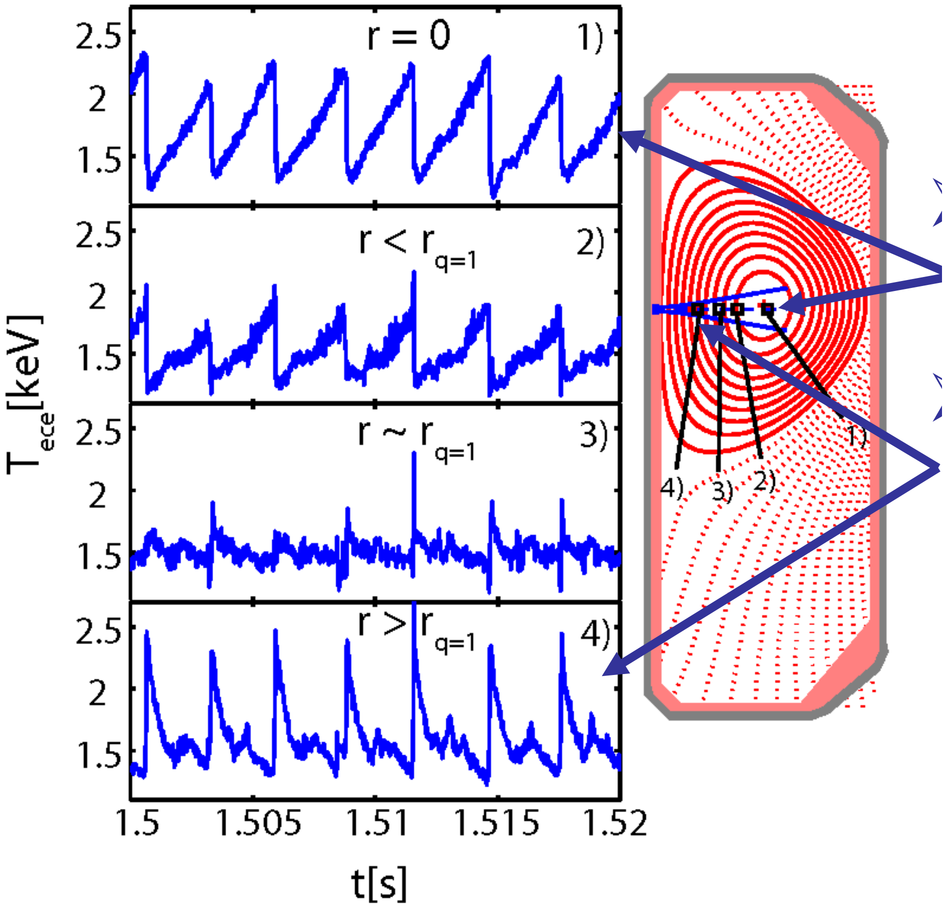


The Very Core of the Plasma Undergoes a ~ Benign ~ "Sawtooth" Relaxation In Many Tokamak Regimes



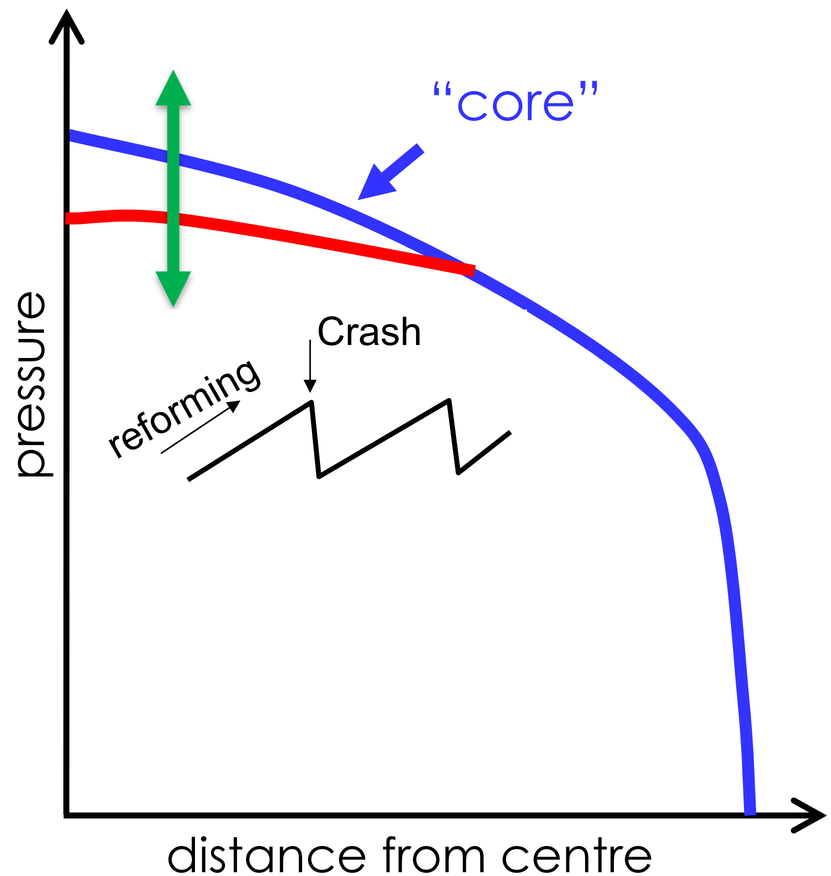
The Very Core of the Plasma Undergoes a ~ Benign ~ “Sawtooth” Relaxation In Many Tokamak Regimes

***** Name comes from shape of below***

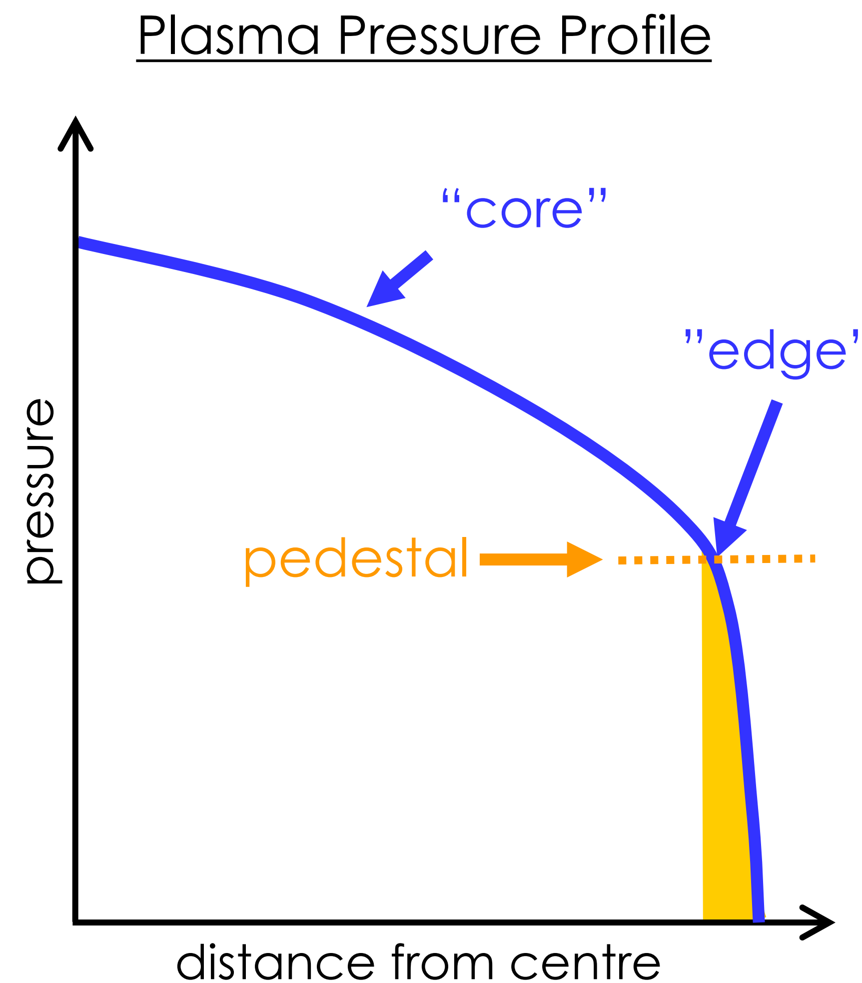


P Blanchard, PhD thesis, EPFL (2002)

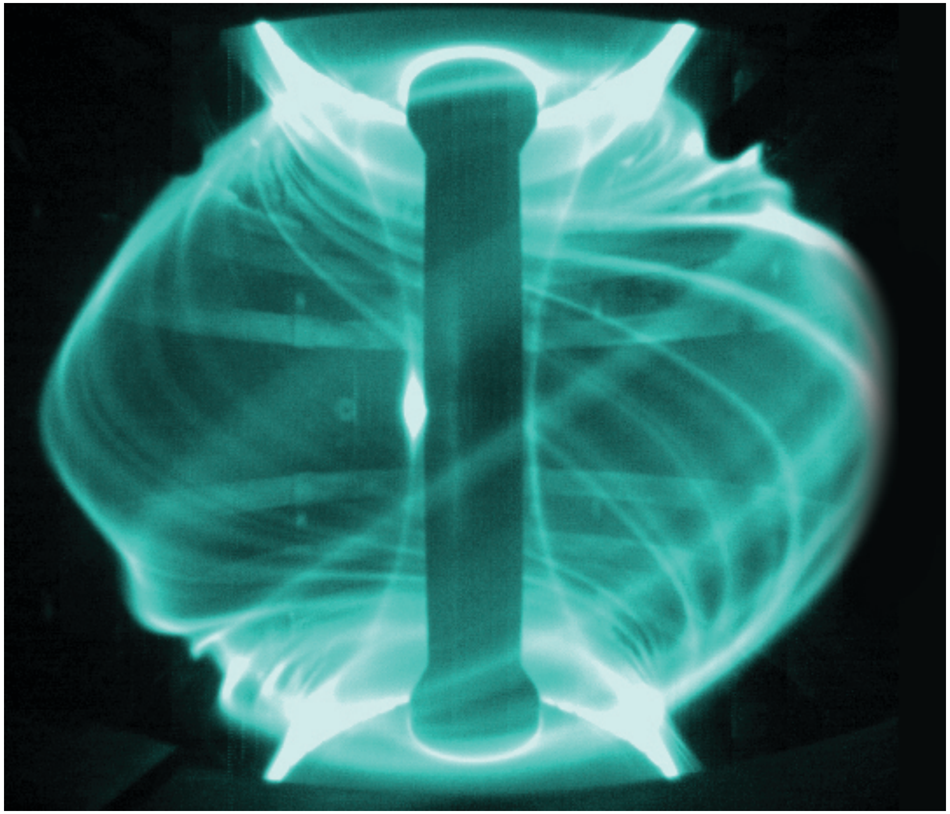
Plasma Pressure Profile



The Tokamak “Pedestal” Can be Locally MHD Unstable

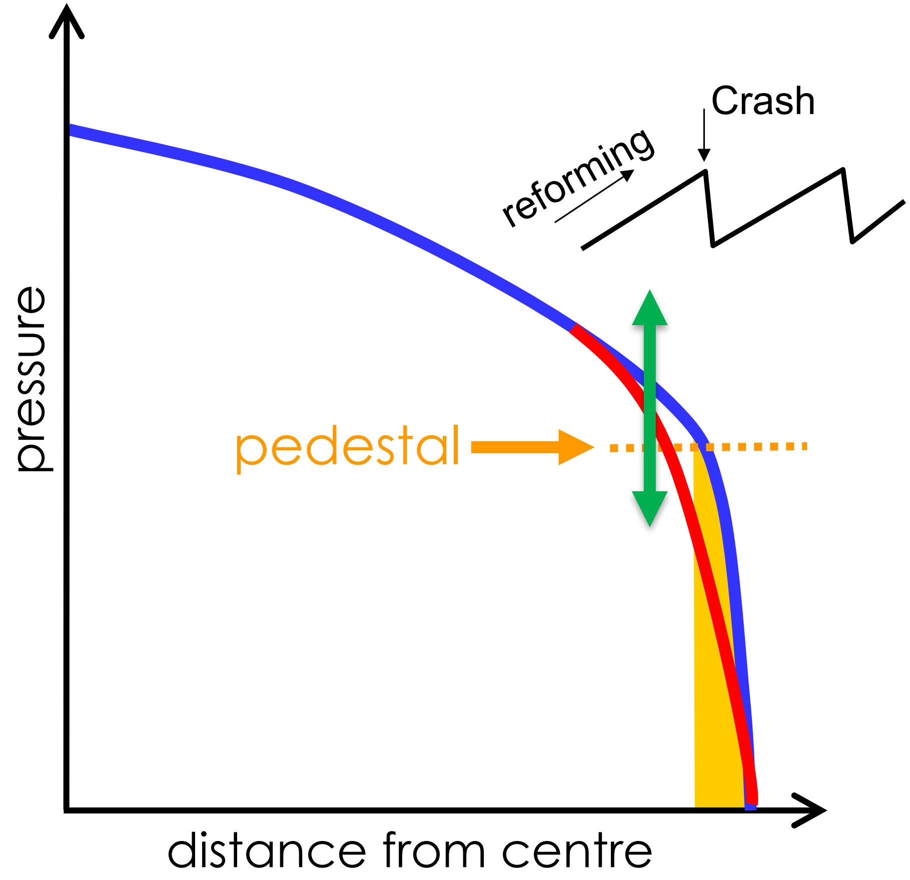


The Tokamak "Pedestal" Can be Locally MHD Unstable … Yielding an "Edge Localized Mode" (ELM)

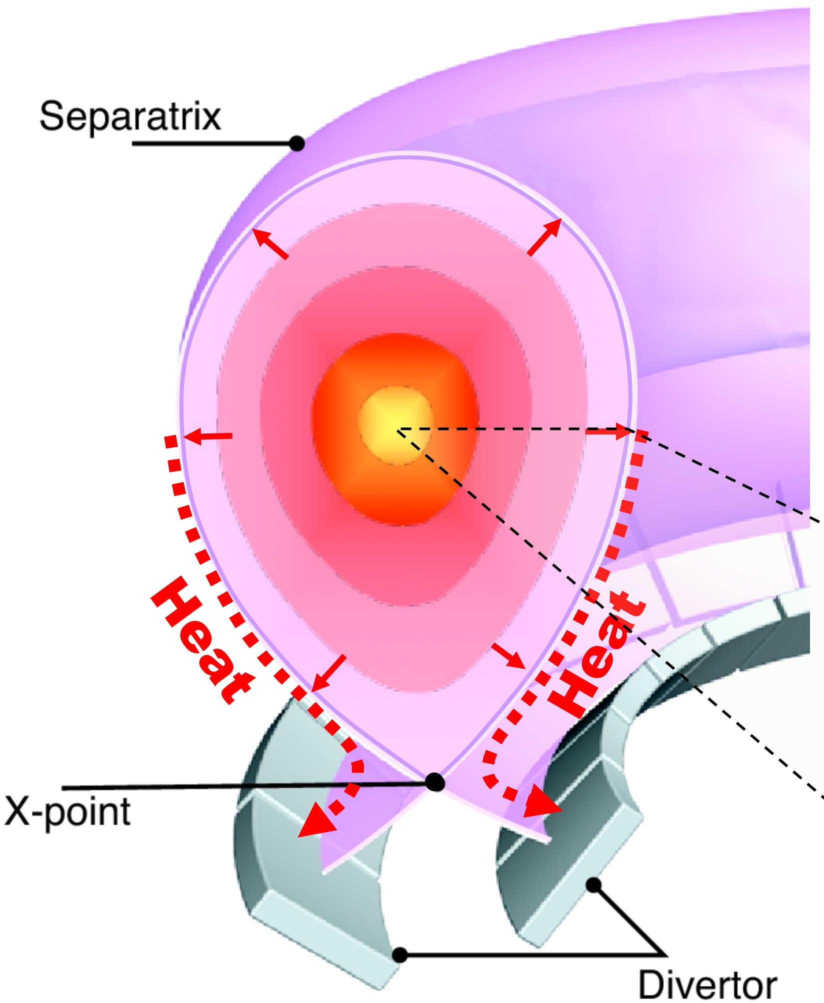


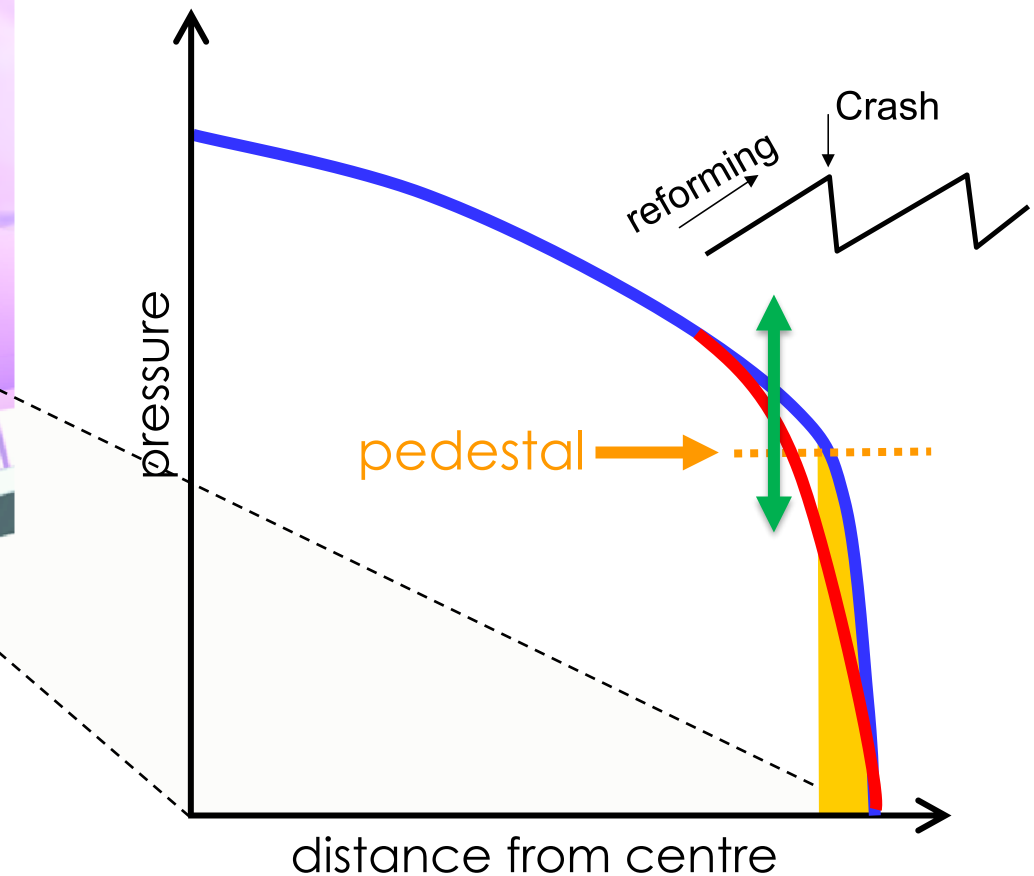
Courtesy: MAST / CCFE

Plasma Pressure Profile



Why ELMs are Bad?
Because they Dump Energy Too Quickly and Melt Stuff





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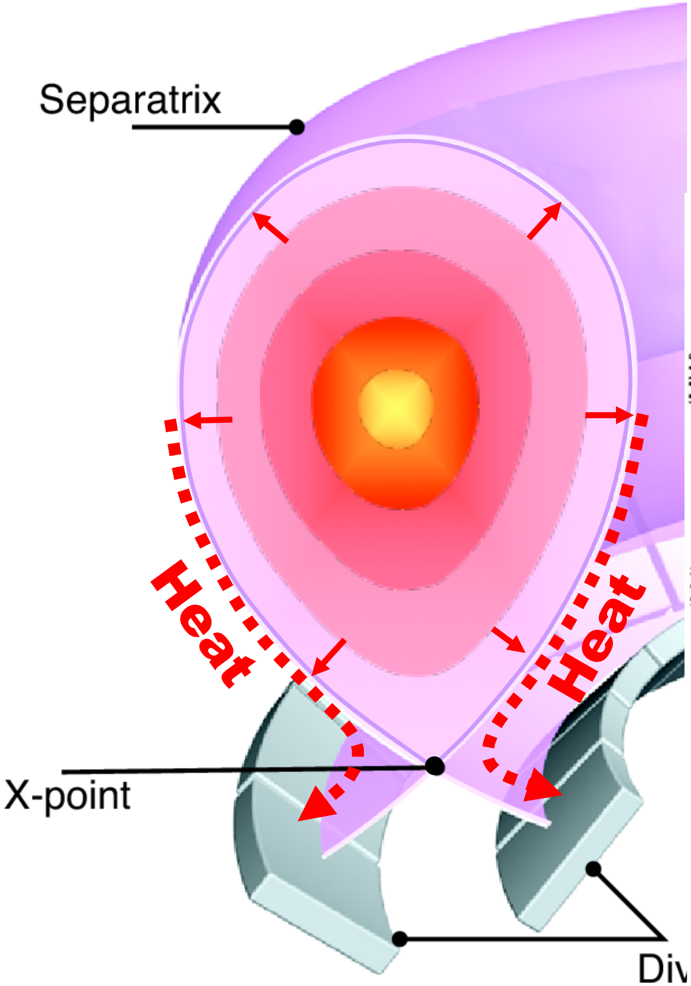
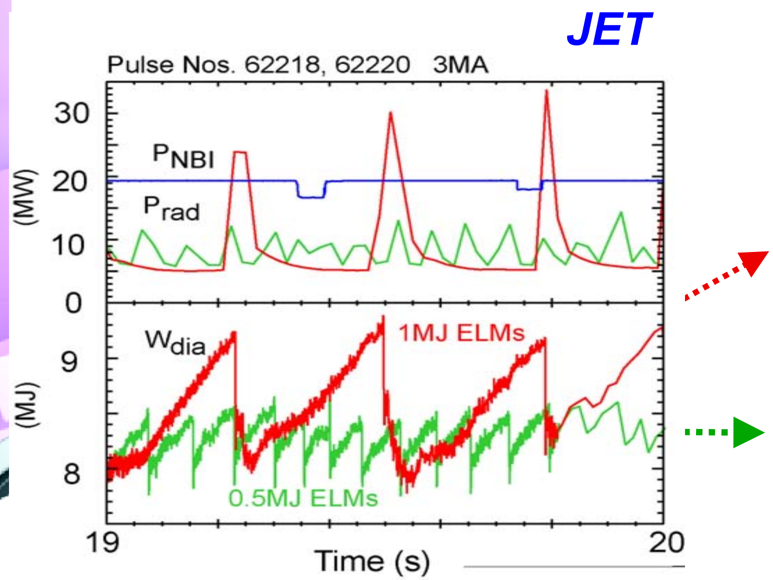
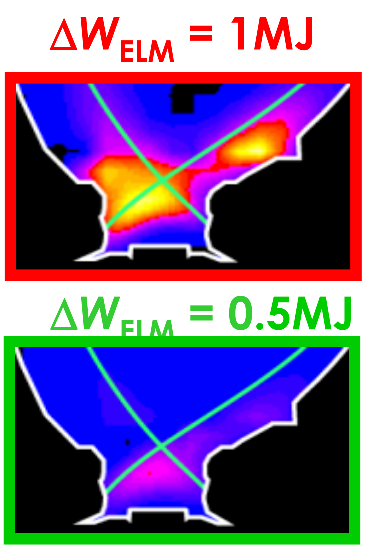






- *Small Frequent ELMs are OK*
- *Large infrequent ELMs are NOT OK*

Most Tokamak Discharges are Limit-Cycle Regulated by both Sawteeth in the Core and ELMs in the Edge

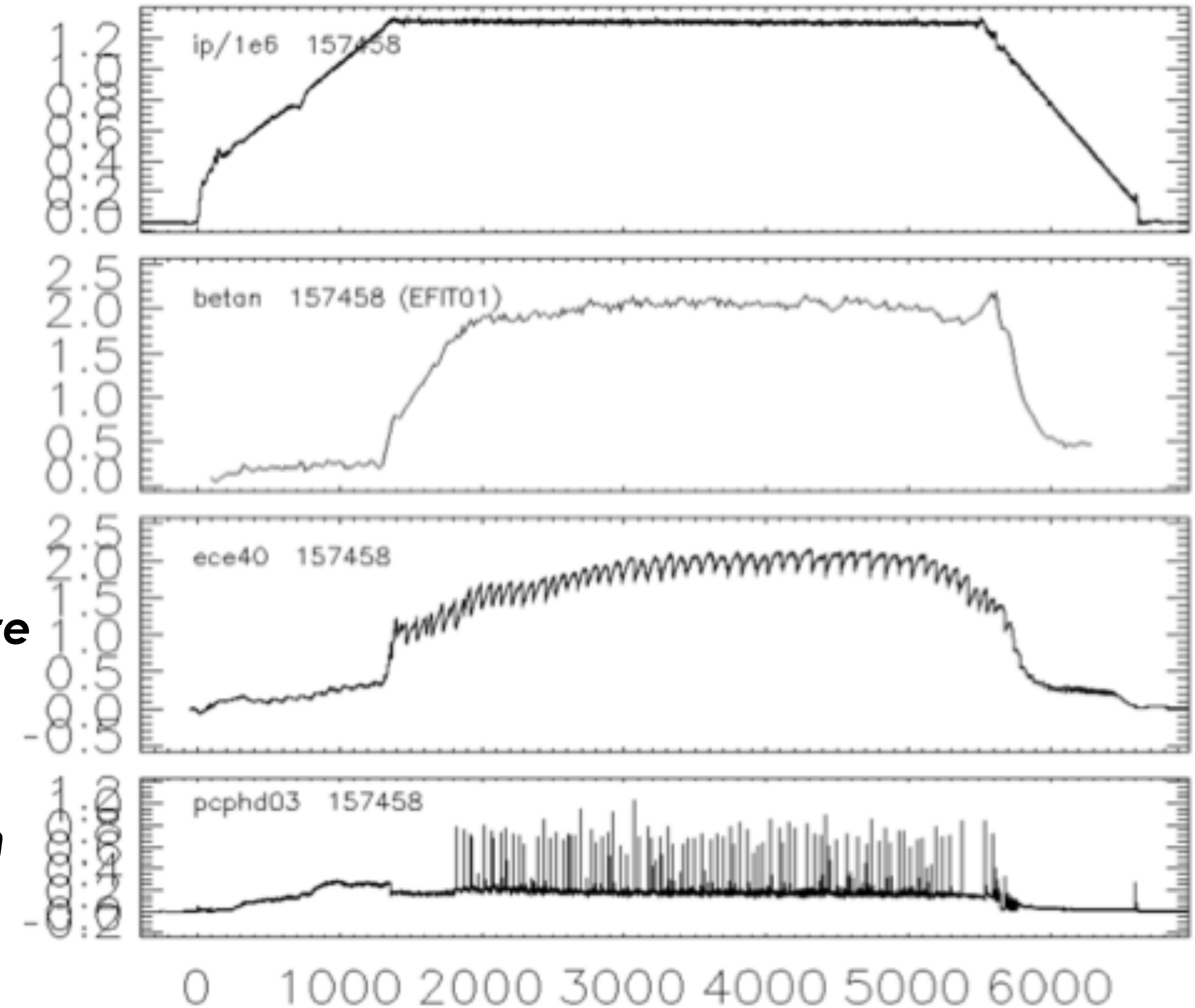
- **"Stationary" but not "Stable" per-se**

Plasma Current

Plasma Pressure

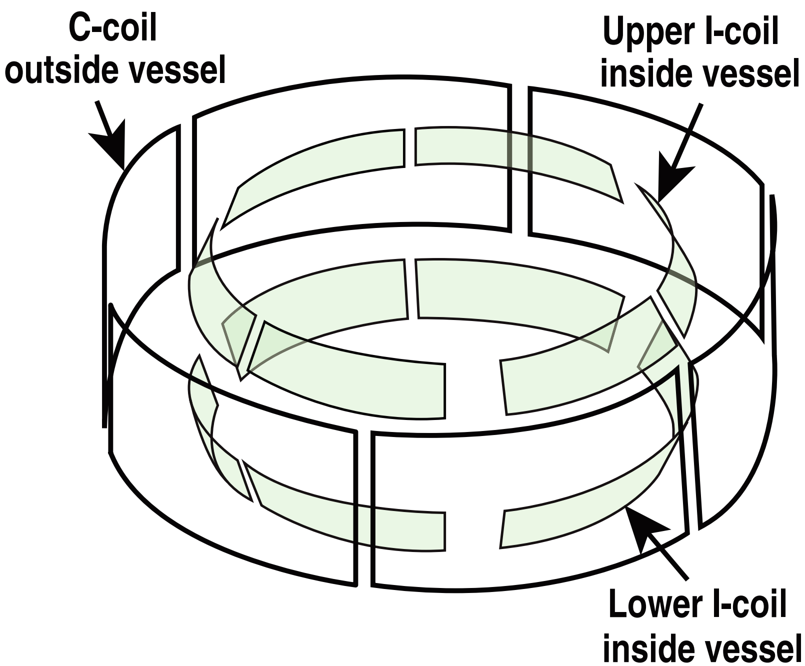
Core Temperature (Sawteeth)

Edge Vis. Emission (ELMs)

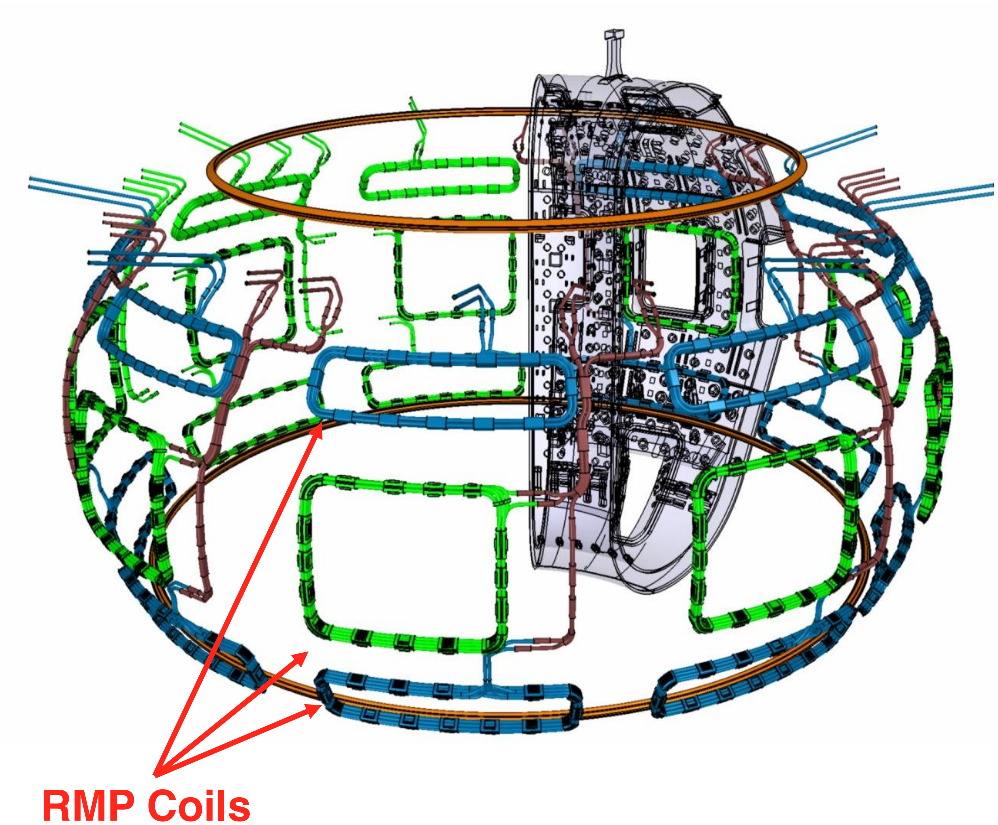


ITER Has Two Strategies for ELM Control
#1: "Resonant Magnetic Perturbation" (RMP)

DIII-D: 2 x 6 perturbation coils



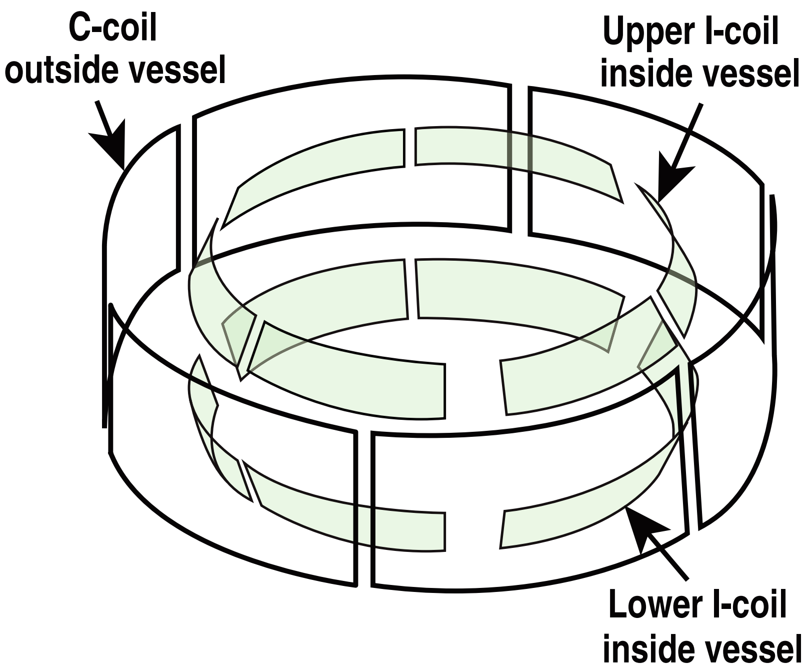
ITER: 3 x 9 perturbation coils

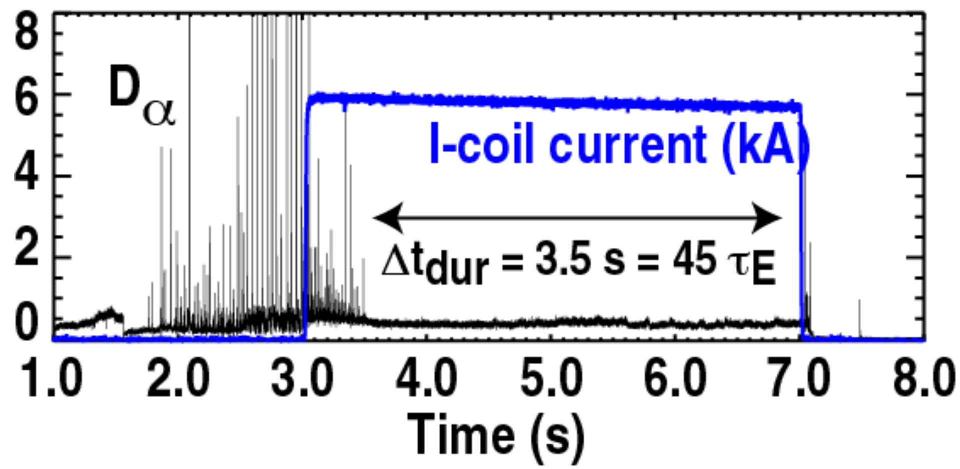


Courtesy: Park NF 2011

RMP Technique was Pioneered at DIII-D in 2000s … Since then Exported to Many Countries (& ITER)

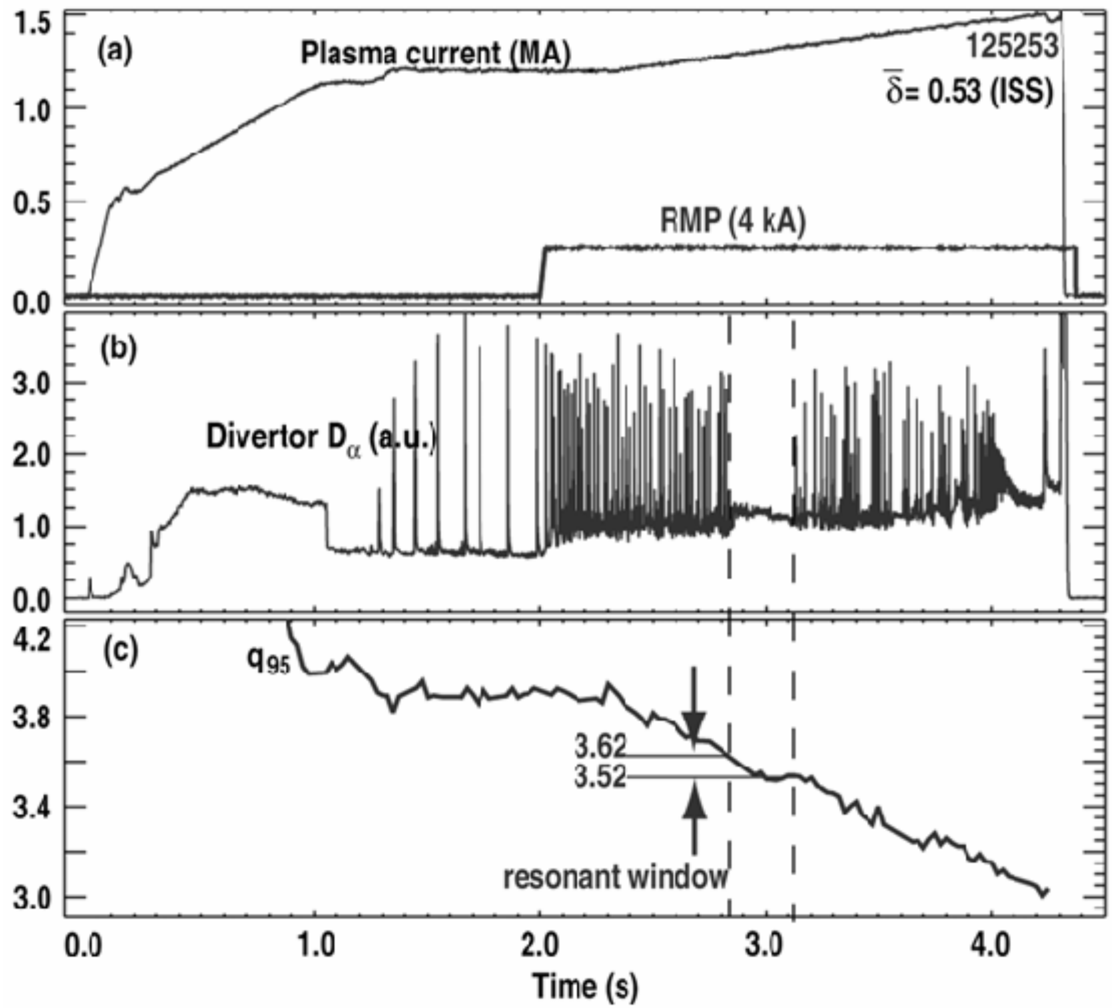
DIII-D: 2 x 6 perturbation coils





Courtesy: Park NF 2011

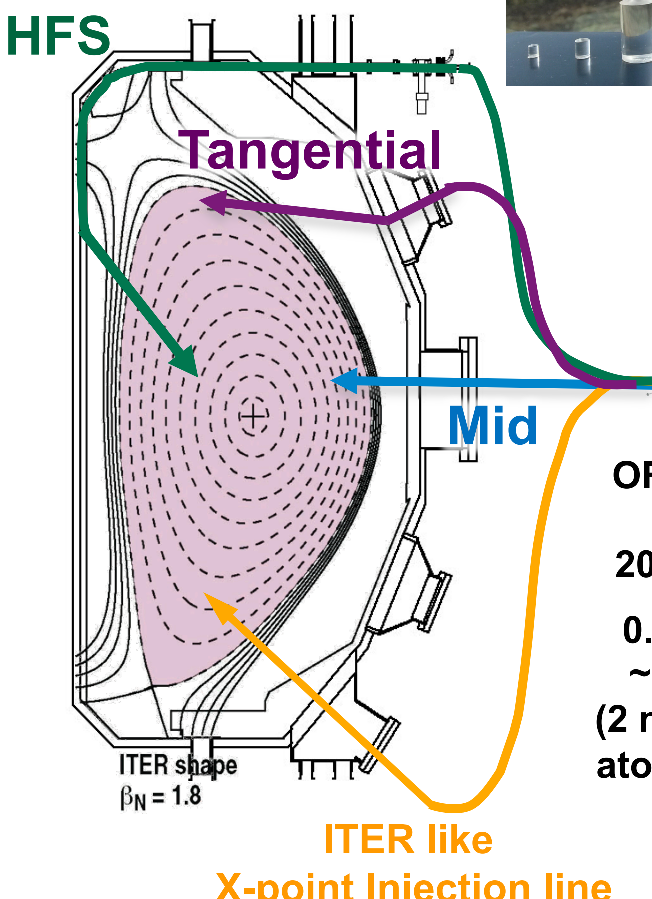
RMP-ELM Control Technique is Very Sensitive to the Edge Safety Factor (q) – "Resonant" Effect

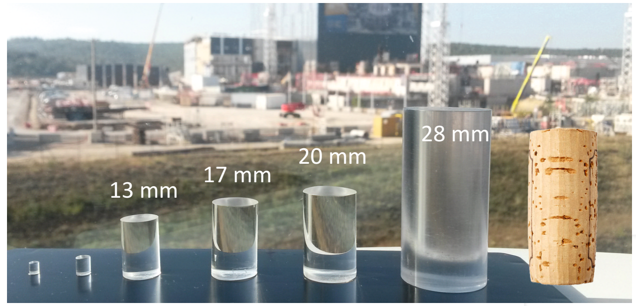


T.E. Evans, et al.,
NF **48** (2008) 024002

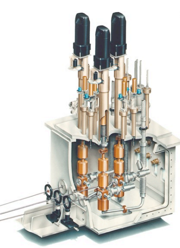
ITER Has Two Strategies for ELM Control
#2: Pellet Injection – Frequent *Direct* ELM Destabilization

DIII-D





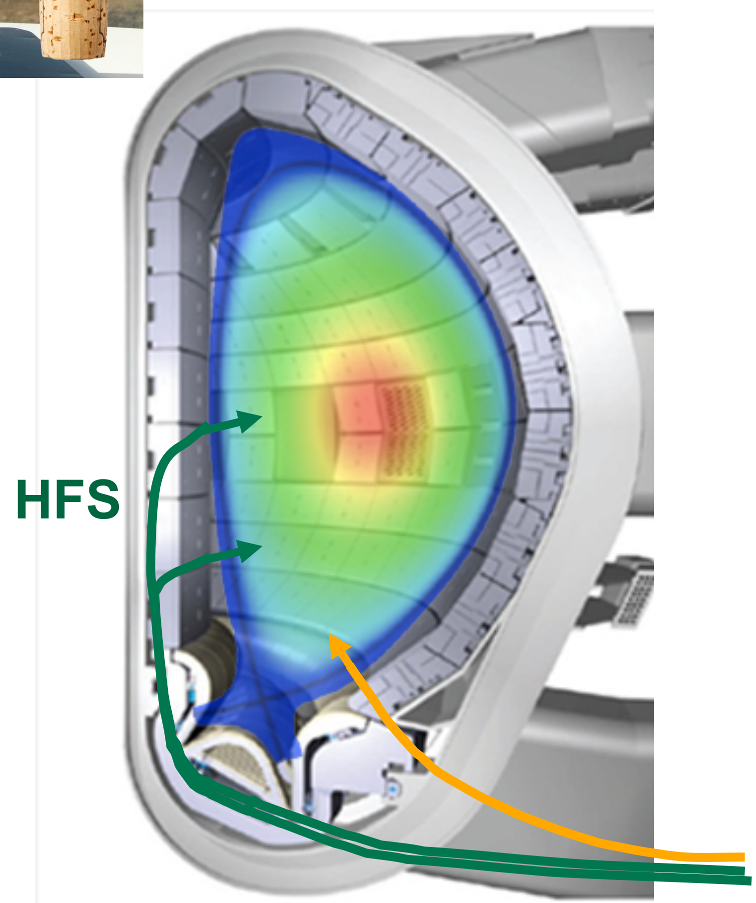
D3DPI



ORNL 3 barrel injector
20 Hz per gun

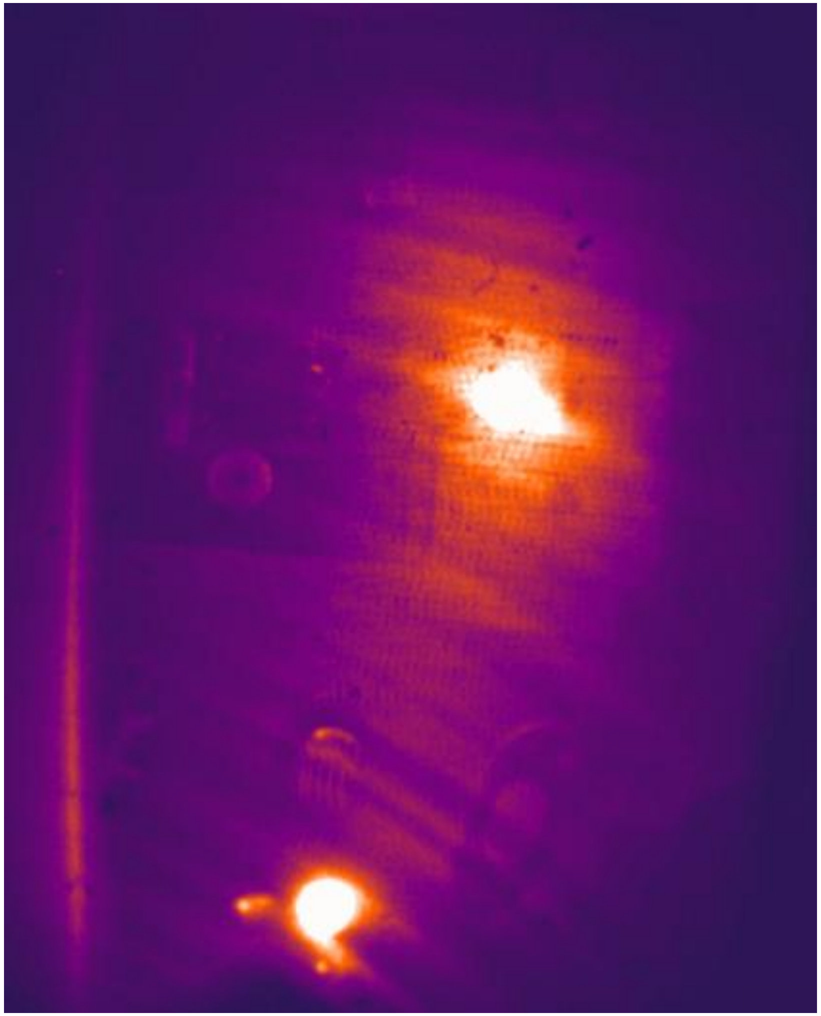
0.8mm pellets
~100-150 m/s
(2 mbar-L, 1×10^{20} atoms per pellet)

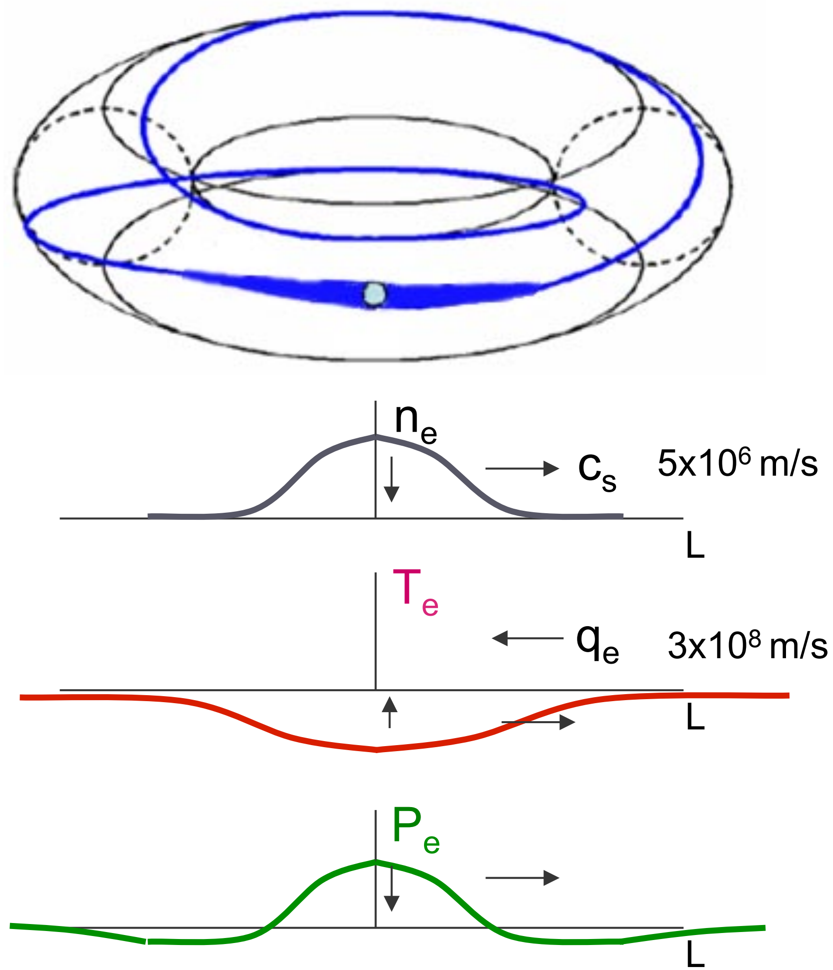
ITER



[S. Maruyama, IAEA2012]

On-Demand ELM *Destabilization* is Achieved via Locally Exceeding the Critical Pedestal Pressure





L. Baylor, APS-DPP 2014

Review of Concepts – Commonly Observed MHD Instabilities

- **A few of the most commonly encountered instabilities:**
  - Magnetic Island
  - Sawtooth
  - ELM
- **Main control tools deployed on ITER:**
  - Microwave heating
  - Resonant Magnetic Perturbations
  - Injected Pellets
- **Next-gen control tools / more stable regimes are under active study / development !!**

Outline of Presentation

- Pre-amble: Why the MHD model?
- Development of the MHD Model
- MHD Equilibrium: 1-D, 2-D, 3-D Configurations
- MHD and its Relation to Global Operational Limits
- Brief Tour of Common MHD Instabilities and Their Control

THE END

Getting involved in the DIII-D Program

- **General Atomics runs DIII-D on Behalf of the U.S. Dept of Energy**
  - You can't directly do your PhD with General Atomics

- **Your next step is picking a university *PhD / MSc* program**
  - Several programs send students to work / live @ DIII-D
  - Graduate training need not be at a "big science" facility

- **If you are interested specifically in the DIII-D Tokamak, these university programs / professors may have positions @ DIII-D:**

  - Princeton: Egemen Kolemen
  - MIT: Anne White / Miklos Porkolab
  - Columbia: Gerald Navratil
  - Auburn: David Ennis
  - Lehigh: Eugenio Schuster
  - UCLA: Troy Carter
  - UC San Diego: George Tynan
  - UC Irvine: William Heidbrink
  - UW Madison: Oliver Schmitz / Ray Fonck
  - UT Knoxville: David Donovan

Material Properties with Different Impulsive Heat Loads have been Characterized ... It's not Pretty !!

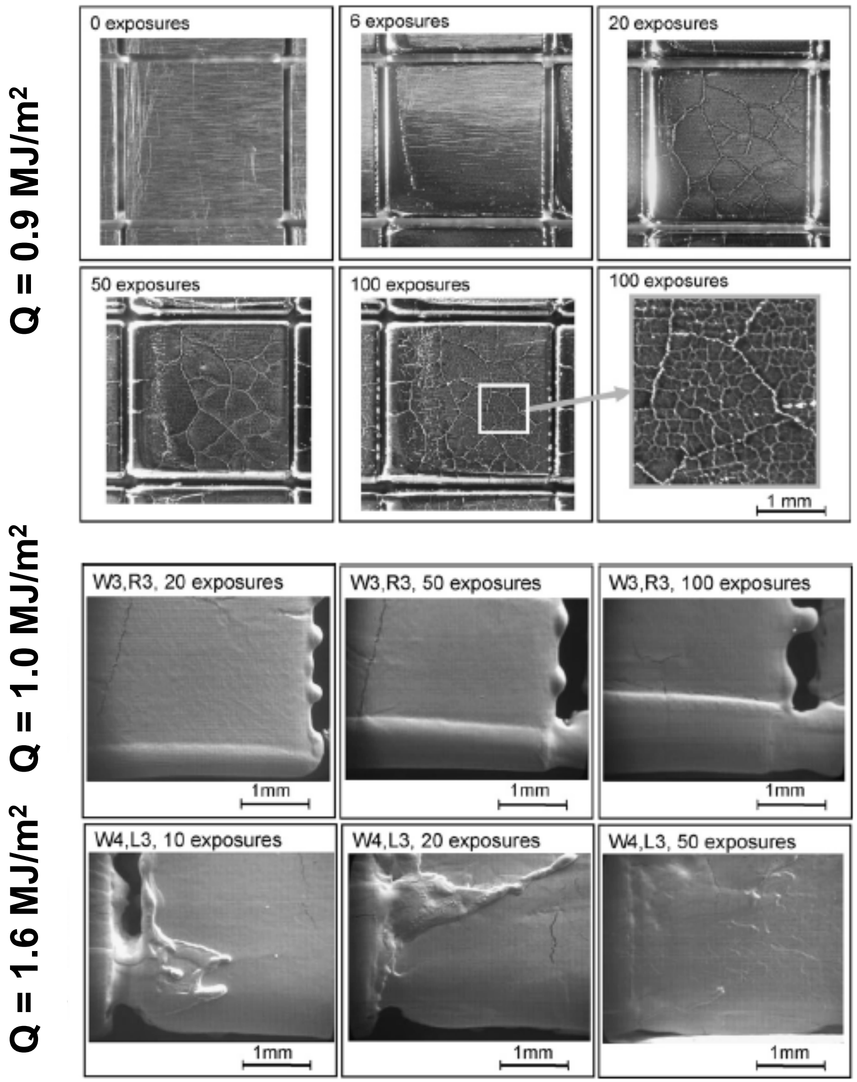
Zhitlukhin JNM 2007

ELM Simulations on QSPA
(0.1-0.6 ms, 30º to surface)

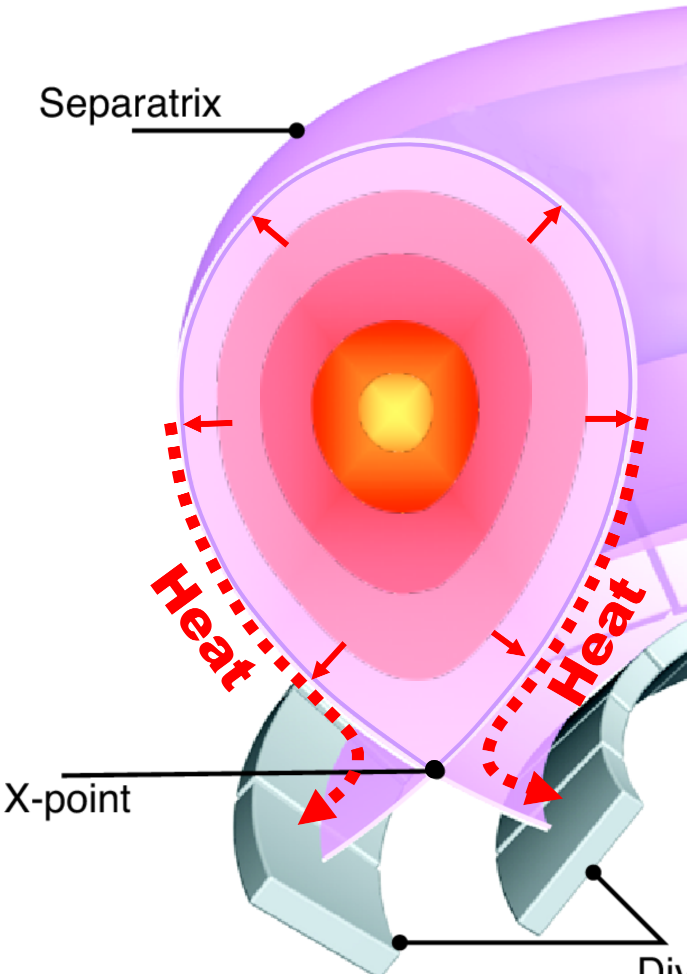
<0.4 MJ/m²
Negligible erosion

0.4-1.0 MJ/m² (JET<1.0MJ/m²)
Edge melting and surface cracking

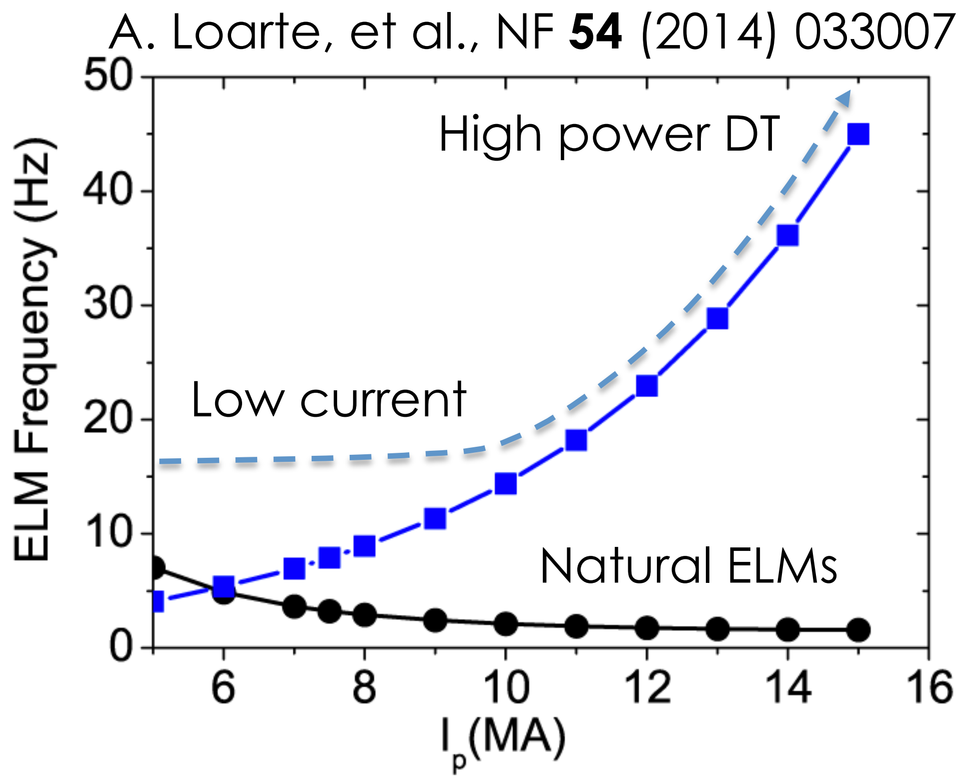
1.0-1.6 MJ/m²
Surface melting, bridge formation and droplet ejection



Why ELMs are Bad? Because they Dump Energy Too Quickly and Melt Stuff



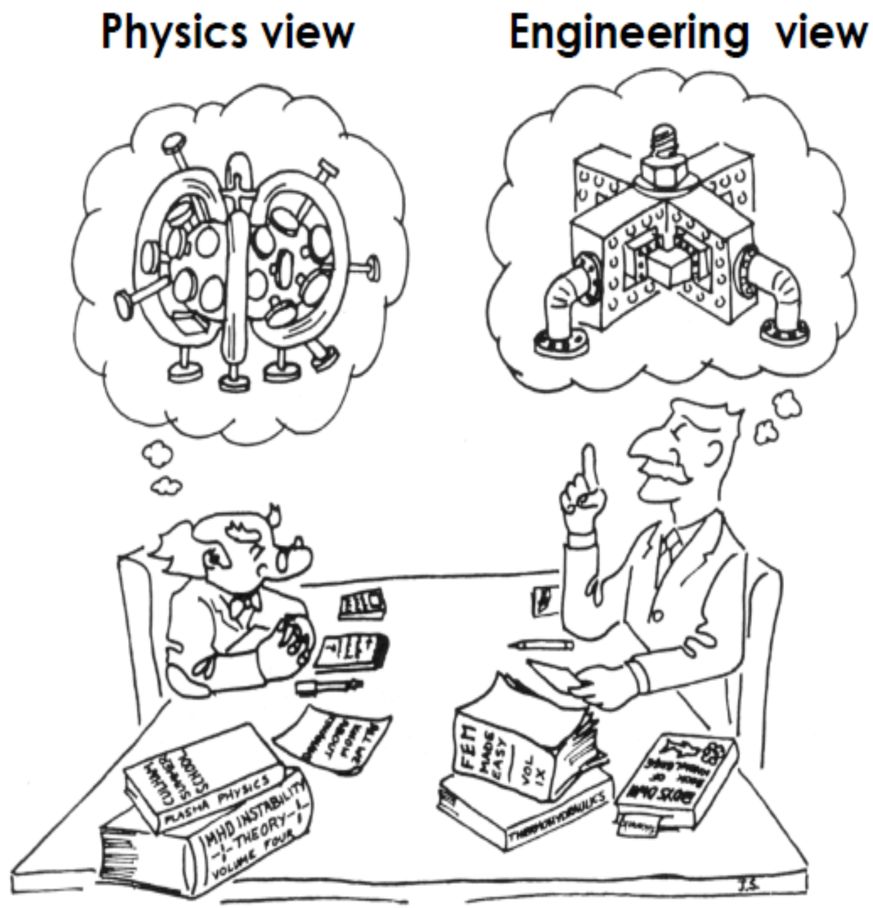
ITER ELM Control Requirement



***** Plasma Current is Bad for Stability … but good for confinement***

Physics vs Engineering

- **ss**

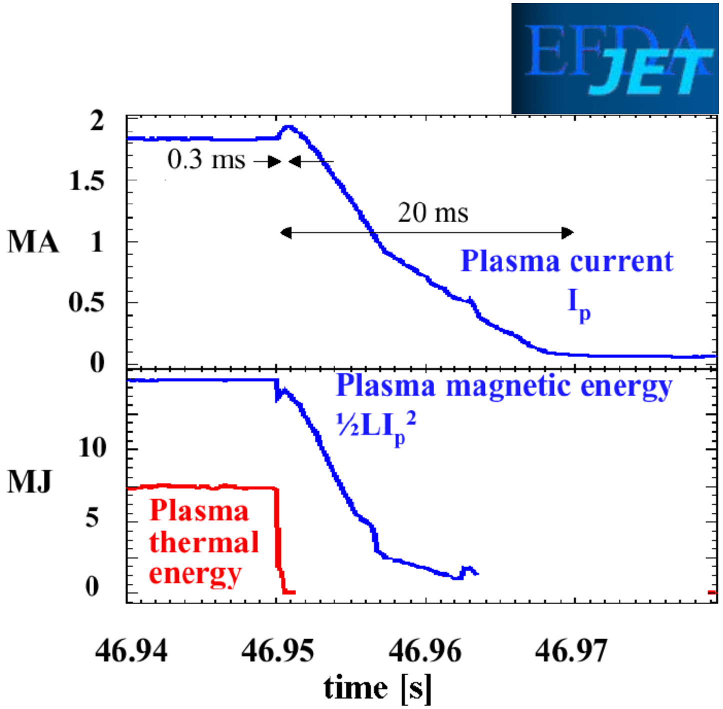


T. Todd, in R. Dendy Plasma Physics p. 448 (1993)

Disruptions

- **ss**

Disruptions occur in tokamak plasmas when unstable p(r),j(r) develop
⇒ unstable MHD modes grow
⇒ plasma confinement is destroyed (thermal quench)
⇒ plasma current vanishes (current quench)



Typical JET timescales

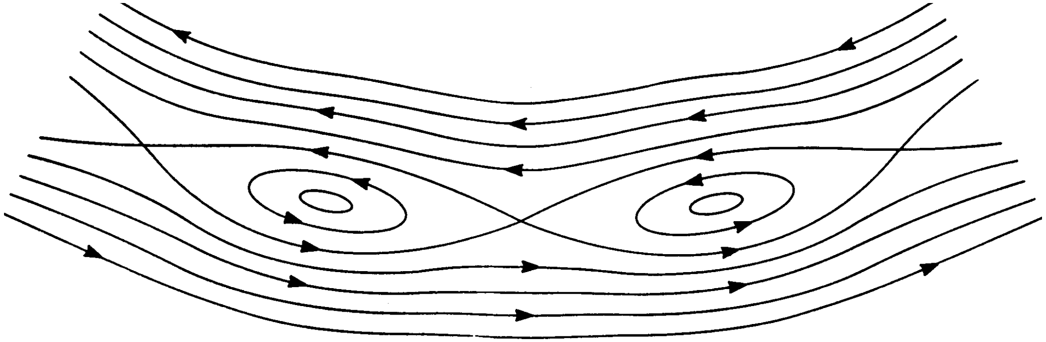
- Thermal quench < 1ms ⇒ deposits plasma thermal energy on plasma facing components (PFCs)
- Current quench > 10 ms ⇒ deposits plasma magnetic energy by radiation on PFCs & runaway electrons

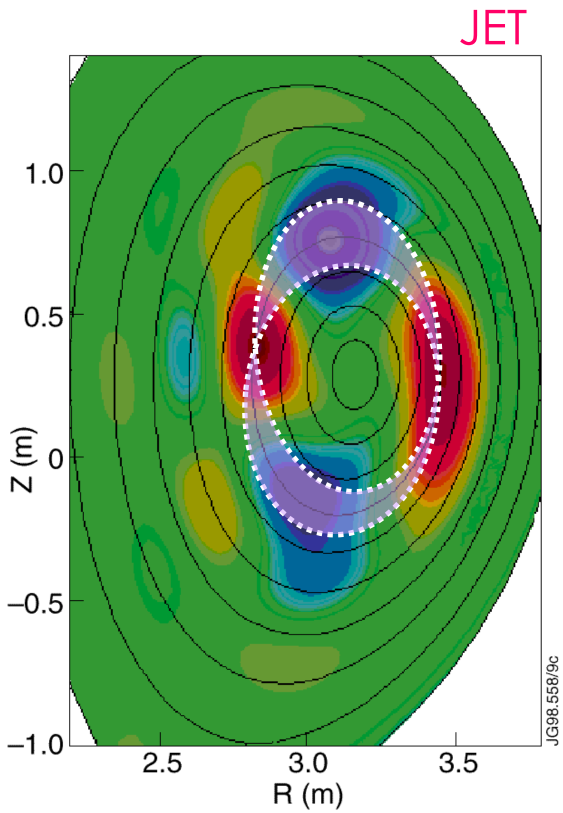
Expected values for ITER

- Thermal energy ~ 300 MJ
- Magnetic energy ~ 600 MJ
- Thermal quench time ~ 1.5 - 3 ms
- Current quench time ~ 20 - 40 ms

Tearing Instability

Good conduction about magnetic island:





Soft X ray tomography of flux tearing instability

→ flattens pressure, destroys confinement

→ requires 'rational' q flux surfaces to establish these island structures

→ q profile plays a key role in governing tearing stability