# Single Particle Motion

*Presented by*

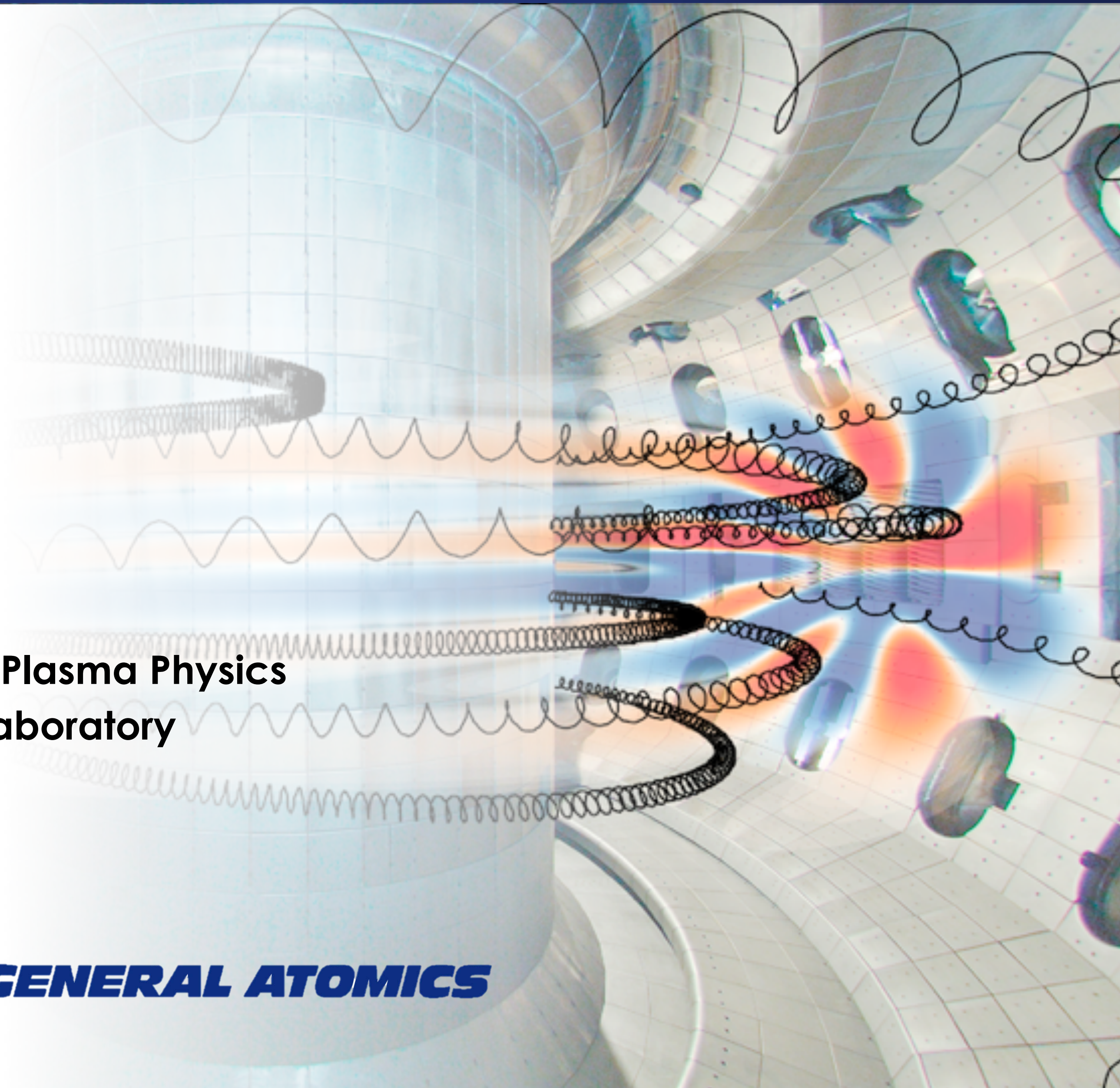
**Cami Collins**

*at the*

**SULI Introductory Course in Plasma Physics**

**Princeton Plasma Physics Laboratory**

**June 10, 2019**



DIII-D NATIONAL FUSION FACILITY SAN DIEGO

GENERAL ATOMICS

# My path in plasma physics

MONTANA STATE UNIVERSITY

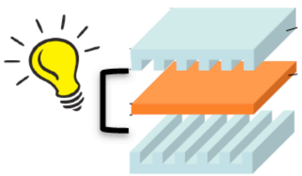
WISCONSIN
UNIVERSITY OF WISCONSIN–MADISON

UCIrvine
University of California, Irvine

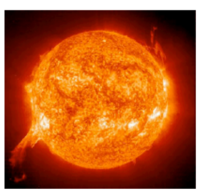
GENERAL ATOMICS

**Undergrad (2003-07)** → **Grad (2007-13)** → **Postdoc (2014-16)** → **Scientist!**

Solid Oxide Fuel Cells

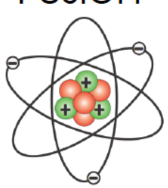


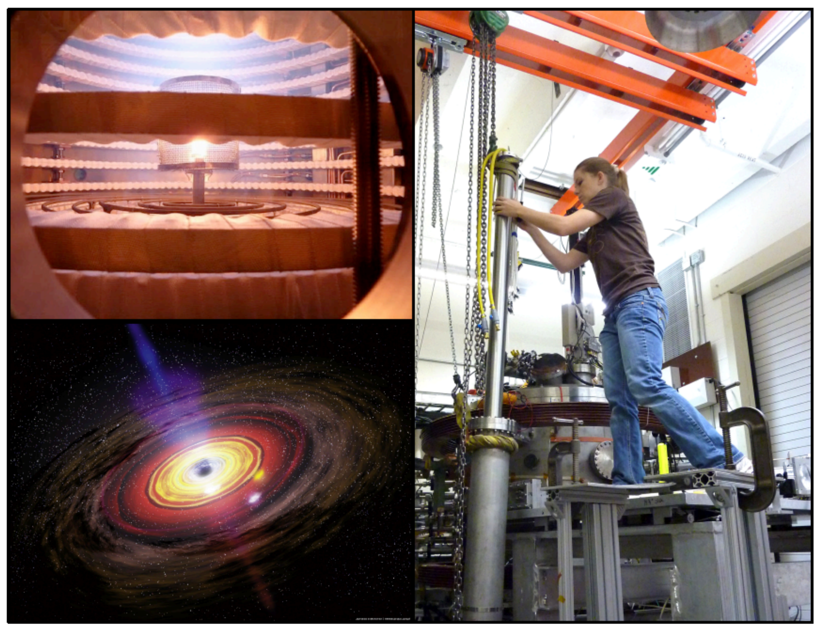
Solar Physics



**Nat. Und. Fellowship**

Fusion





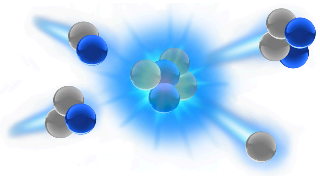
Laboratory Plasma Astrophysics

**Fusion Energy Fellowship**

Fusion Energy

# I want fusion energy to work.
## But what does that mean?

plasma physics



stability

transport

turbulence

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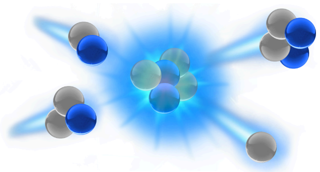

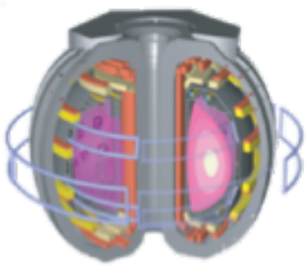

stability

transport

turbulence

shaping

ELM controlled

high bootstrap current

high confinement

high beta

steady state

efficient current drive

dissipative divertor

disruption mitigation

control

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## But what does that mean?

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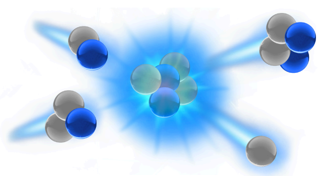
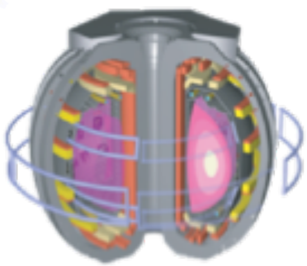
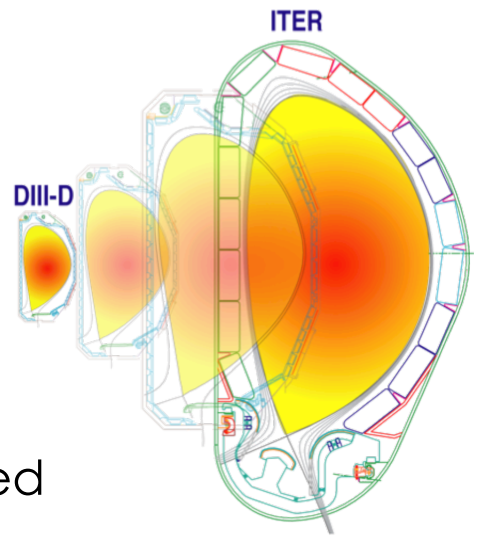
materials science





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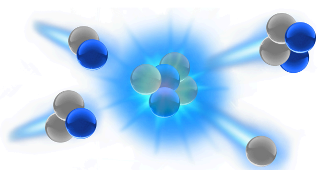

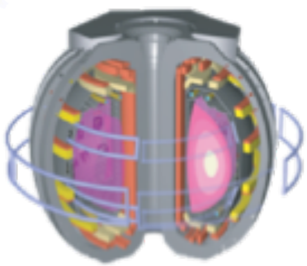

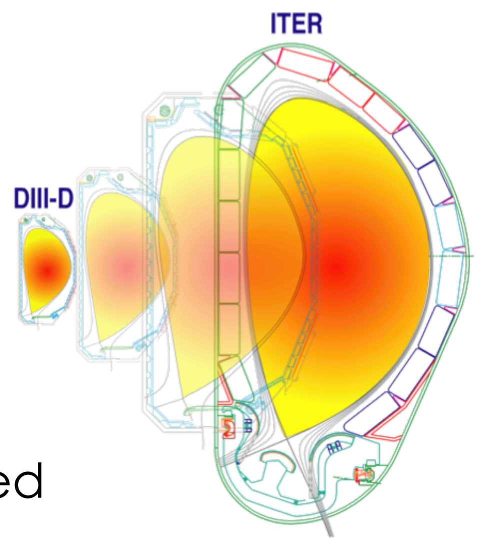


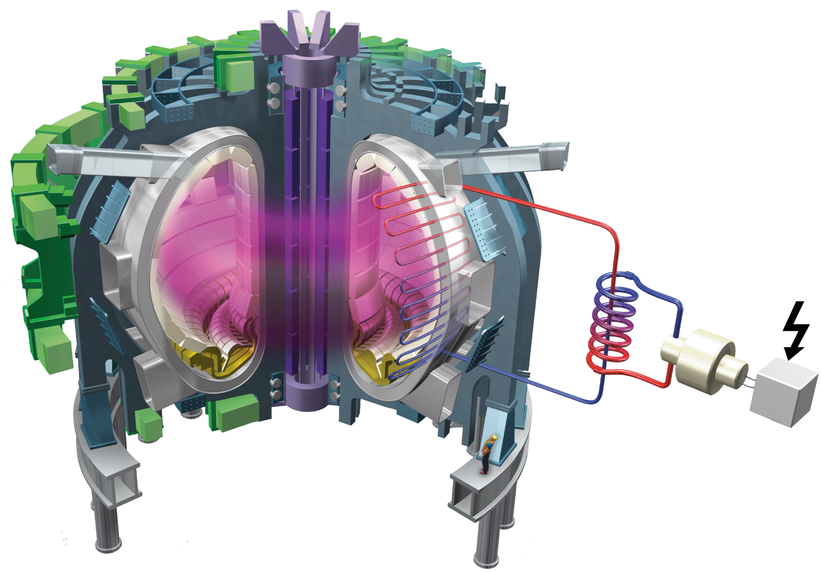

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net electricity

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heat extraction

metal walls, liquid walls?

control

tritium breeding

nuclear materials

# I want fusion energy to work.
## But what does that mean?

plasma physics

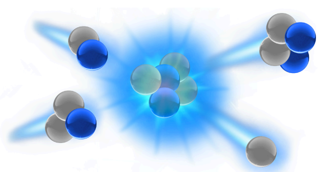
engineering

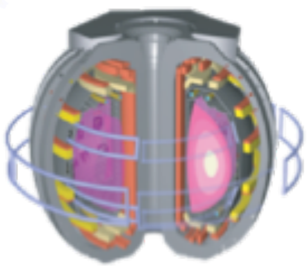
computer science

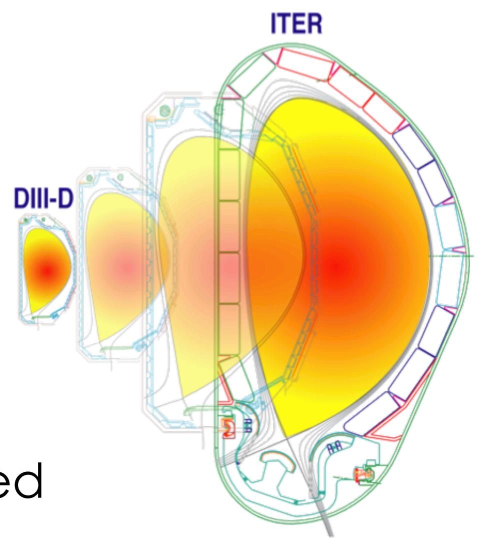
social science?

materials science

political science?





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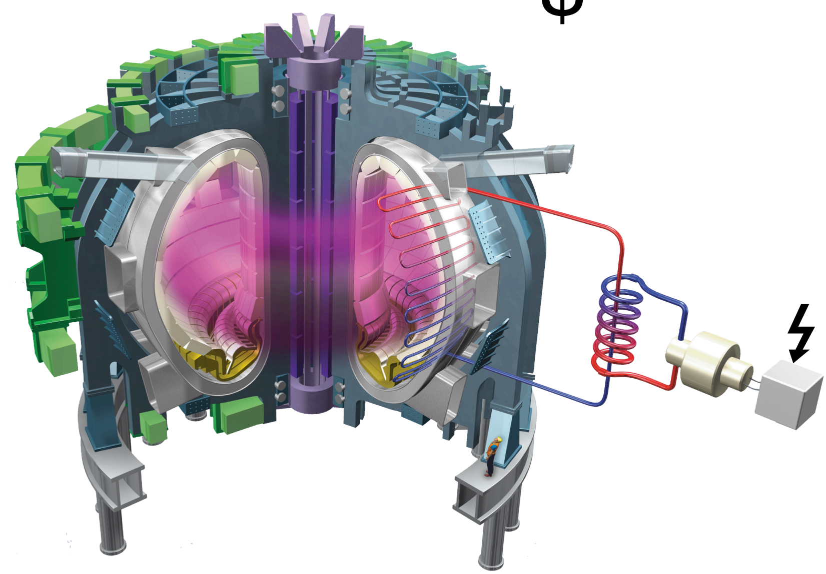

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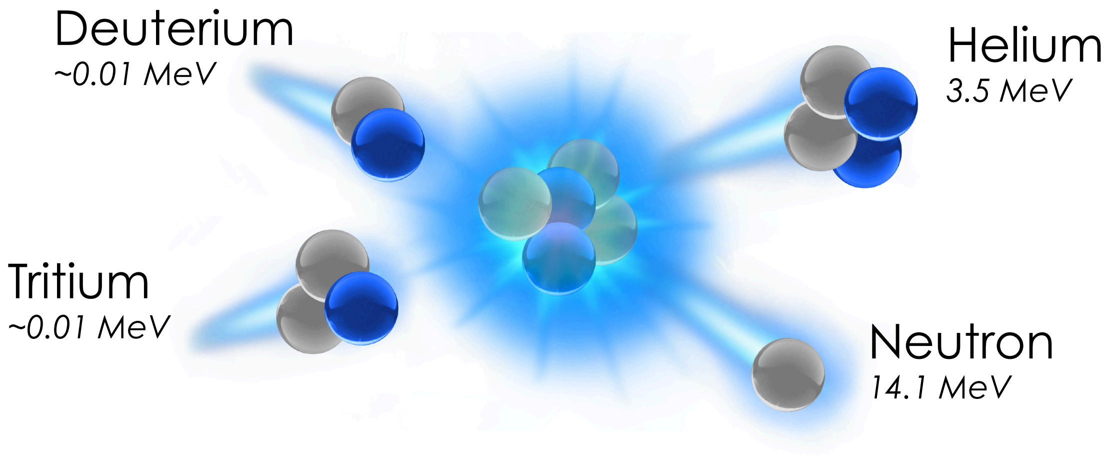
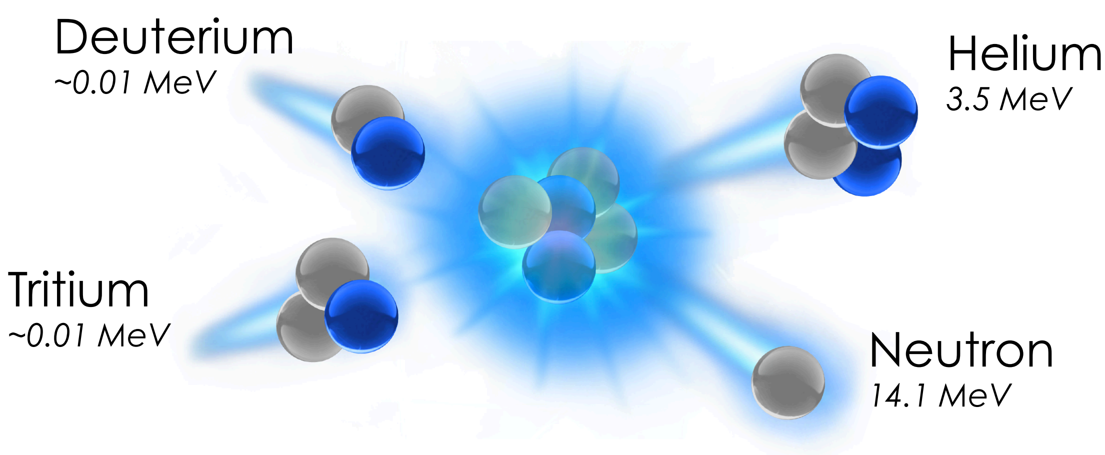
# Plasma Physics is the Basis for Fusion Research




- **Sustained fusion reactions require enough particles (density) that are energetic enough (temperature) and collide often enough (confinement time).**

# Plasma Physics is the Basis for Fusion Research




- **Sustained fusion reactions require enough particles (density) that are energetic enough (temperature) and collide often enough (confinement time).**

- **The fusion triple product is the figure of merit:**

$$nT\tau_E \gtrsim 5\times 10^{21}\ \mathrm{keV\ s\ m^{-3}}$$

$T$~*100-200 million K*

$n$~*2-3x10$^{20}$ ions/m$^3$*

$\tau$ ~*1-2 s*

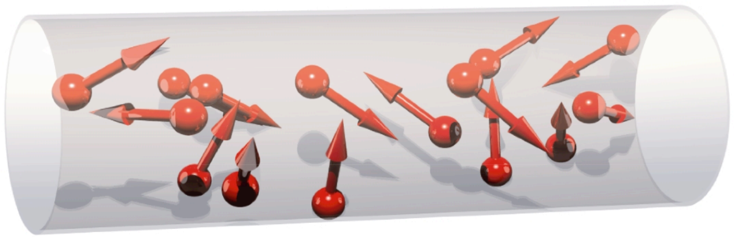
**D & T is a plasma at these temperatures**

# We can understand a lot about how fusion devices confine plasma by studying single particle motion.

**Typical velocity of a 100 million K ion:**

$$kT = \frac{1}{2} m v_{\rm th}^2$$

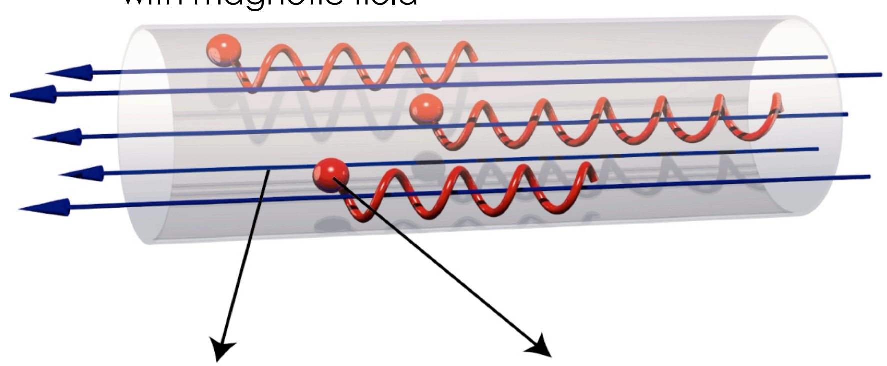
$$v_{\rm th} \sim 6 \times 10^5 \ \mathrm{m/s}$$




Even with ~$10^{20}$ *ions/m*$^3$ , the ion would travel ~10 km before colliding with another

It would be crazy to build a fusion reactor that big!

**The trick: use magnetic fields**



# Outline

- **Gyromotion about a guiding center**
- **Forces can cause guiding center drift**
- **Real life consequences:**
  – Why do tokamaks have helical B fields?
  – What is a banana orbit?
  – Why are instabilities like Alfvén Eigenmodes bad for fusion?

**References**

- NRL Plasma Formulary
  www.nrl.navy.mil/ppd/content/nrl-plasma-formulary
- Introduction to Plasma Physics and Controlled Fusion by F. Chen

# Charged Particles Feel The (Lorentz) Force

- **A particle with charge ($q$) moving with velocity ($\mathbf{v}$) in the presence of electric and magnetic fields will experience a force:**

$$\mathbf{F} = q(\mathbf{E} + \mathbf{v} \times \mathbf{B})$$

We know from Newton's second law of motion that force causes acceleration:

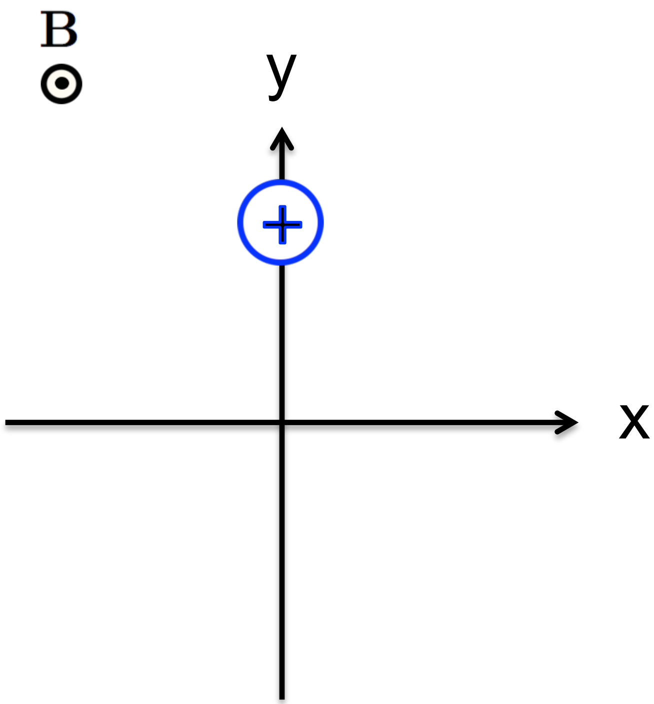
$$\mathbf{F} = m\mathbf{a}$$

A charged particle moving perpendicular to the magnetic field feels a force

# How Does a Charged Particle Move in a Magnetic Field?

- **Consider the motion of a particle in a constant, uniform B field**

$$\mathbf{B} = B_z\hat{\mathbf{z}} \qquad \mathbf{E} = \mathbf{0}$$

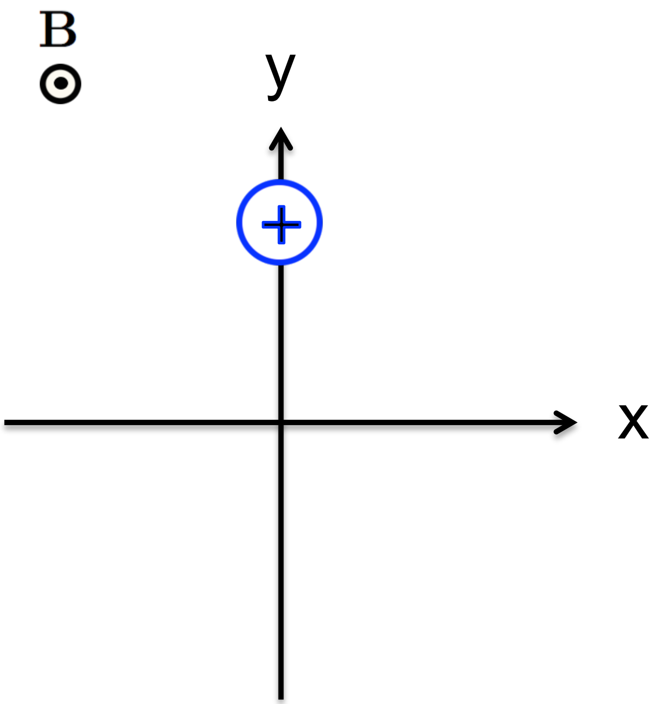



Then $$\mathbf{F} = q(\cancel{\mathbf{E}} + \mathbf{v} \times \mathbf{B})$$

So we can write

$$\frac{d\mathbf{v}}{dt} = \frac{q\mathbf{v} \times \mathbf{B}}{m}$$

# Goal: Solve the Equations of Motion for a Charged Particle In A Magnetic Field




$$\frac{d\mathbf{v}}{dt} = \frac{q\mathbf{v} \times \mathbf{B}}{m}$$

Let's break this into components:

$$\dot{v}_x\hat{\mathbf{x}} + \dot{v}_y\hat{\mathbf{y}} + \dot{v}_z\hat{\mathbf{z}} = \frac{qv_yB_z\hat{\mathbf{x}} - qv_xB_z\hat{\mathbf{y}}}{m}$$

The 'dot' represents $\frac{d}{dt}$

Matching components:

$$\dot{v}_x = \frac{qv_yB_z}{m} \qquad \dot{v}_y = -\frac{qv_xB_z}{m}$$

$$\dot{v}_z = 0$$

Particles move freely along the field line

# Take Another Time Derivative & Substitute to Obtain Differential Equations For Each Spatial Coordinate

$$\dot{v}_x = \frac{q v_y B_z}{m} \qquad \dot{v}_y = -\frac{q v_x B_z}{m}$$

$$\ddot{v}_x = \frac{q \dot{v}_y B_z}{m} \qquad \ddot{v}_y = -\frac{q \dot{v}_x B_z}{m}$$

Rewriting, we get

$$\ddot{v}_x = -\left(\frac{qB_z}{m}\right)^2 v_x \qquad \ddot{v}_y = -\left(\frac{qB_z}{m}\right)^2 v_y$$

These may remind you of the equations for a simple harmonic oscillator

# Solve the Differential Equations

$$\ddot{v}_x = -\left(\frac{qB_z}{m}\right)^2 v_x \qquad \ddot{v}_y = -\left(\frac{qB_z}{m}\right)^2 v_y$$

These differential equations can be solved using sines and cosines:

$$v_x = v_\perp \cos\left(\frac{|q|B_z}{m}t + \phi_0\right) \qquad v_y = \mp v_\perp \sin\left(\frac{|q|B_z}{m}t + \phi_0\right)$$

account for positive or negative *q*

$$v_\perp = \sqrt{(v_x^2 + v_y^2)}$$

an arbitrary phase to match the initial velocity conditions

the magnitude of the initial velocity perpendicular to ***B***

# The Result: Circular Motion About A Guiding Center

$$v_x = v_\perp \cos\left(\frac{|q|B_z}{m}t + \phi_0\right) \qquad v_y = \mp v_\perp \sin\left(\frac{|q|B_z}{m}t + \phi_0\right)$$

Integrating, we obtain

$$x = \frac{mv_\perp}{|q|B_z}\sin\left(\frac{|q|B_z}{m}t + \phi_0\right) + x_0 \qquad y = \pm\frac{mv_\perp}{|q|B_z}\cos\left(\frac{|q|B_z}{m}t + \phi_0\right) + y_0$$

- **Charged particles undergo circular orbits about a guiding center** $(x_0, y_0)$

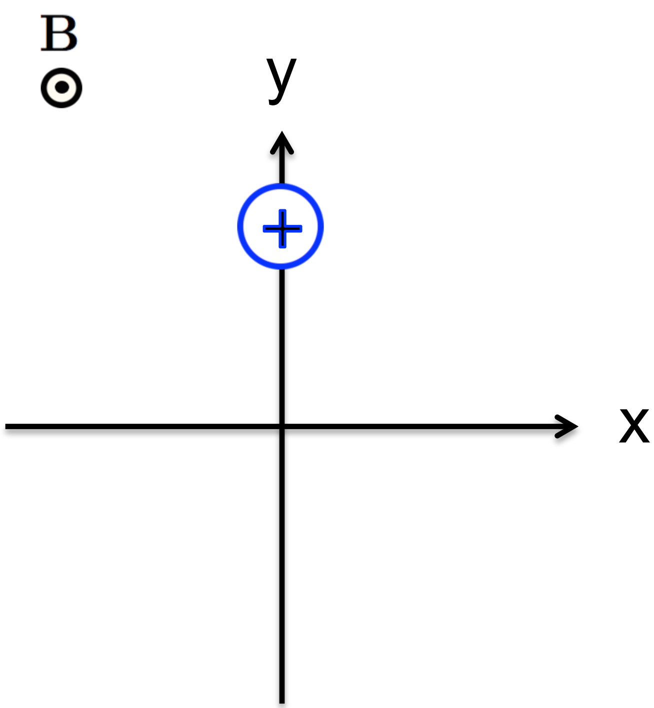
$$r_L \equiv \frac{mv_\perp}{|q|B}$$ **Larmor radius**

$$\omega_c \equiv \frac{|q|B}{m}$$ **Cyclotron frequency**

# Gyromotion of a Charged Particle In A Magnetic Field

$$x = r_L \sin(\omega_c t + \phi_0) + x_0 \qquad y = \pm r_L \cos(\omega_c t + \phi_0) + y_0$$

Let's take $\phi_0 = 0$ and $x_0 = y_0 = 0$




For a positively charged particle:

1. At $t = 0$ ,

$$x = 0 \qquad y = r_L$$

# Gyromotion of a Charged Particle In A Magnetic Field

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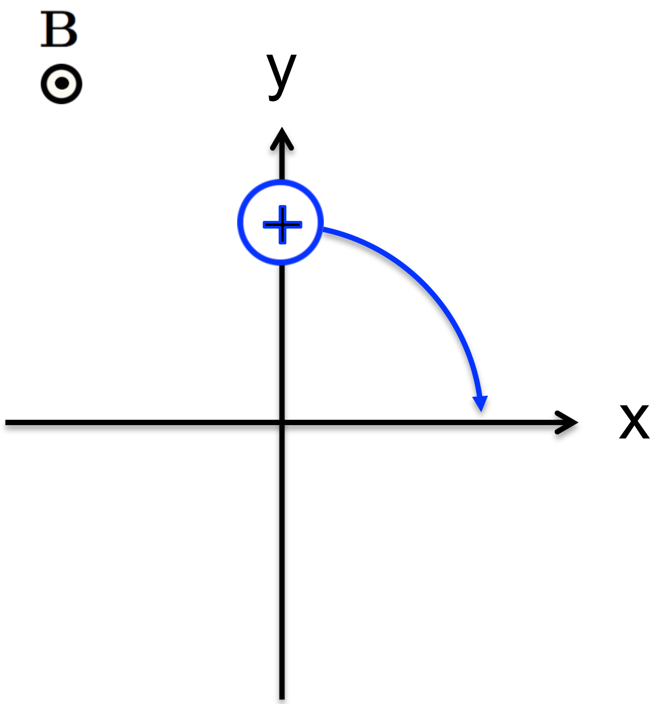


For a positively charged particle:

1. At $t = 0$,

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2. At $t = \frac{\pi}{2}\frac{1}{\omega_c}$,

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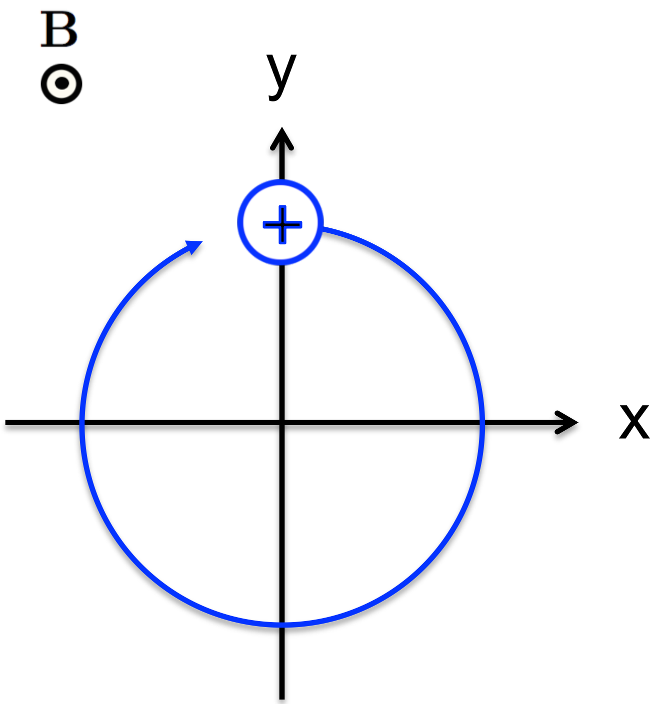


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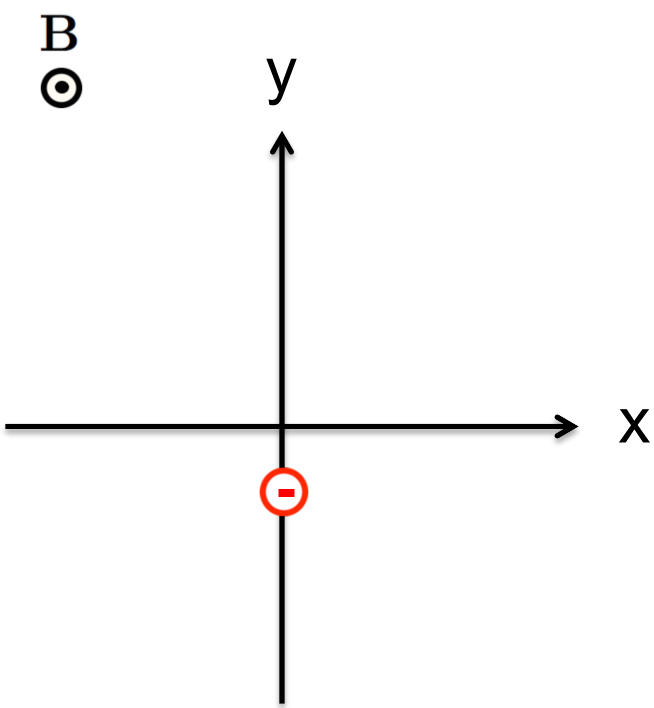


For a negatively charged particle:

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# Gyromotion of a Charged Particle In A Magnetic Field

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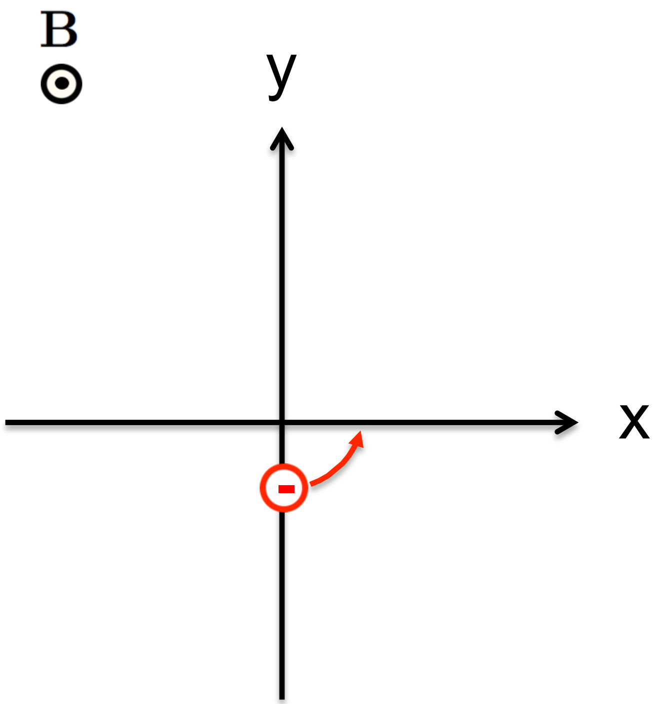


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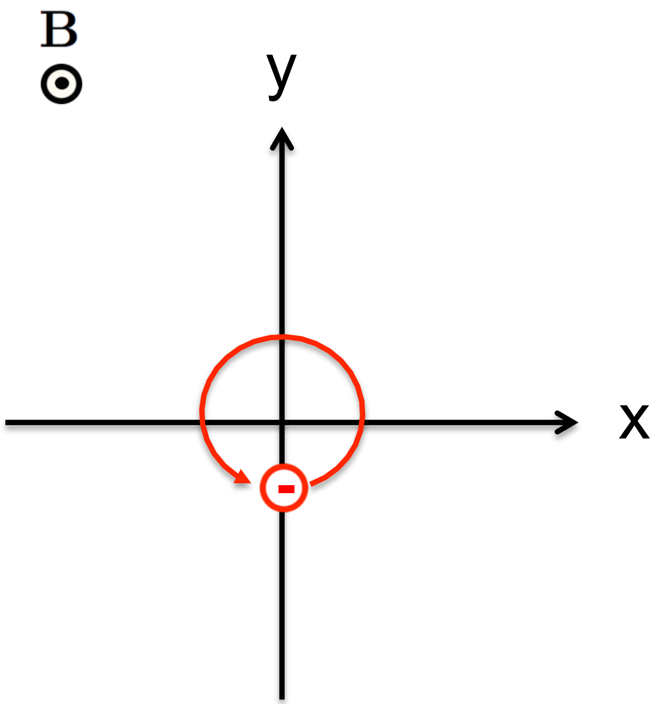


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2. At $t = \frac{\pi}{2}\frac{1}{\omega_c}$ ,

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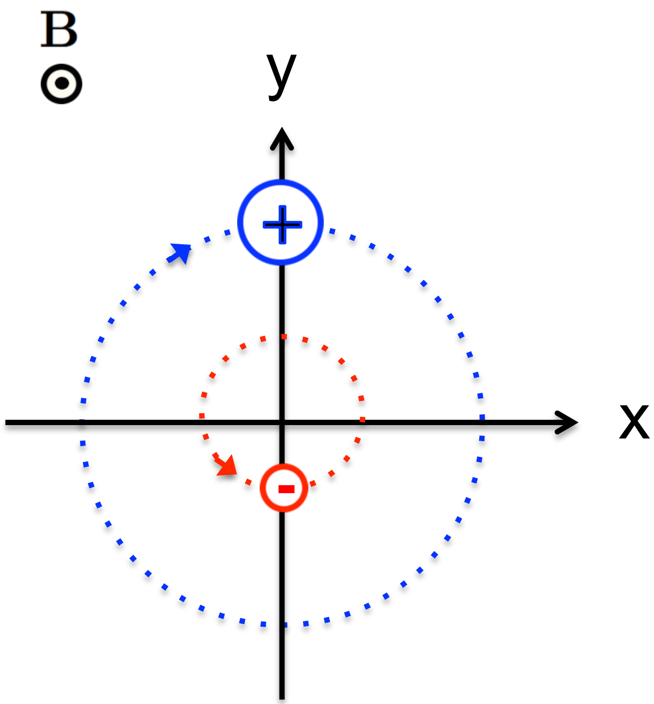
# Gyromotion of Ions vs. Electrons

- **The direction of gyromotion depends on the sign of the charge**
- **Ions generally have a much larger Larmor radius than electrons**




In ITER, a typical deuterium ion with $T_i$=10 keV and B=5 Tesla would have

$$v_{Ti} = \sqrt{\frac{kT_i}{m_i}} \approx 700 \text{ km/s}$$

$$r_L \equiv \frac{mv_\perp}{|q|B} \approx 3 \text{ mm}$$

An electron with $T_e$=10 keV and B=5 Tesla has

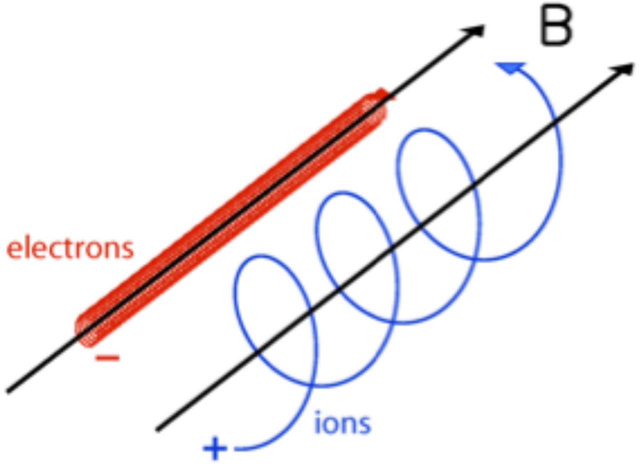
$$r_L \approx 0.05 \text{ mm}$$ (60 times smaller)

# Magnetic Confinement Devices Should Be Much Larger Than the Larmor Radius

$$\mathbf{r} = \left[r_L \sin\left(\omega_c t + \phi_0\right) + x_0\right]\hat{\mathbf{x}} + \left[r_L \cos\left(\omega_c t + \phi_0\right) + y_0\right]\hat{\mathbf{y}} + \left[v_{\|} t + z_0\right]\hat{\mathbf{z}}$$

- Particles are confined **perpendicular** to the applied magnetic field

$$\mathrm{r_L} = \frac{m v_\perp}{eB}$$




- Tokamak approach: **parallel** confinement is achieved through toroidal geometry

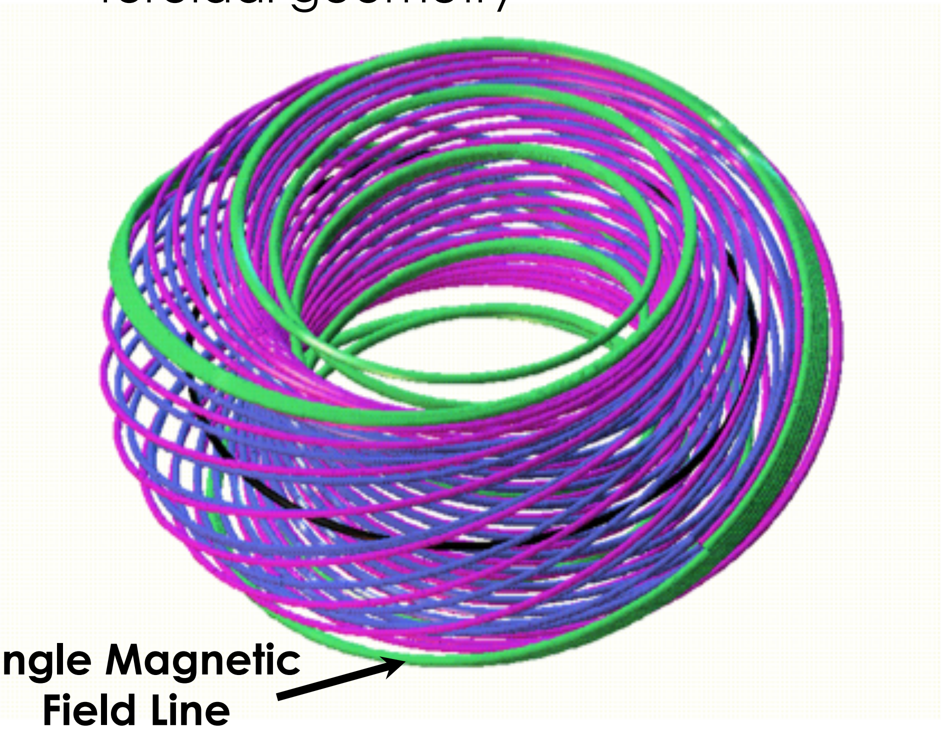
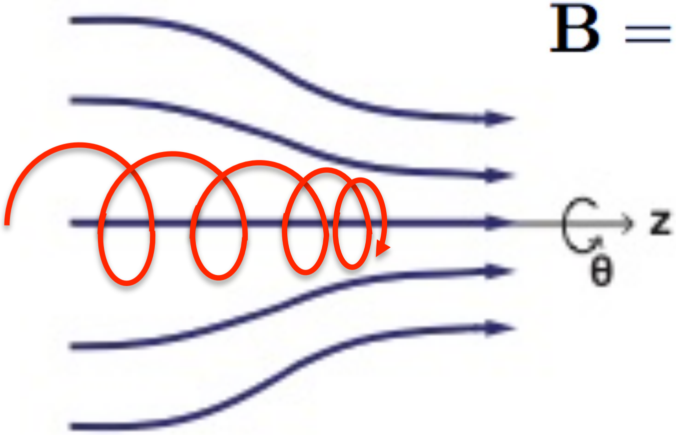





Image credit: http://iter.rma.ac.be/en/img/MagneticConfinement.jpg

# Magnetic Mirrors




$$\mathbf{B} = B_r\hat{\mathbf{r}} + B_z\hat{\mathbf{z}} \qquad \mathbf{F} = q(\mathbf{v} \times \mathbf{B})$$

The $B_r$ ends up causing additional acceleration in the z direction:

$$m\frac{dv_z}{dt} = -qv_\theta B_r$$

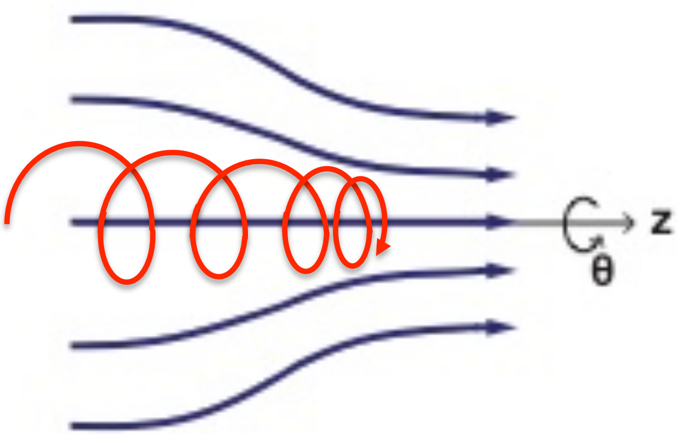
Result:
gyromotion + mirror force in the $-\hat{\mathbf{z}}$ direction

$$F_z = -\frac{mv_\perp^2}{2B}\frac{\partial B_z}{\partial z}$$

The magnetic moment is $\mu \equiv \dfrac{mv_\perp^2}{2B}$

mirror force $\mathbf{F}_{||} = -\mu\nabla_{||}B$

# Magnetic Moment Is Conserved




$$\mathbf{F}_{\|} = -\mu \nabla_{\|} B \qquad \mu \equiv \frac{m v_{\perp}^2}{2B}$$

**The magnetic moment is a constant of motion**

$$m \frac{dv_{\|}}{dt} = -\mu v_{\|} \frac{\partial B}{\partial s}$$

$s$ is the coordinate along the field line

We can write $\dfrac{dB}{dt} = \dfrac{\partial B}{\partial s} \dfrac{ds}{dt}$

this is $v_{\|}$

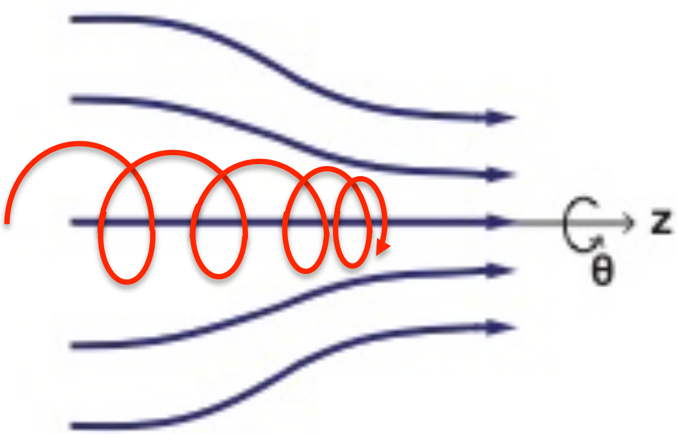
Then $\dfrac{d}{dt}\left(\dfrac{1}{2} m v_{\|}^2\right) = -\mu \dfrac{dB}{dt}$

We also have conservation of energy: $\dfrac{d}{dt}\left(\dfrac{1}{2} m v_{\|}^2 + \dfrac{1}{2} m v_{\perp}^2\right) = 0$

this is $\mu B$

$$\boxed{B \frac{d\mu}{dt} = 0}$$

# More Insight Into Magnetic Mirrors




$$\mu \equiv \frac{mv_\perp^2}{2B} \qquad \frac{d\mu}{dt} = 0$$

1. As the particle moves to stronger $B$, $v_\perp$ must increase.

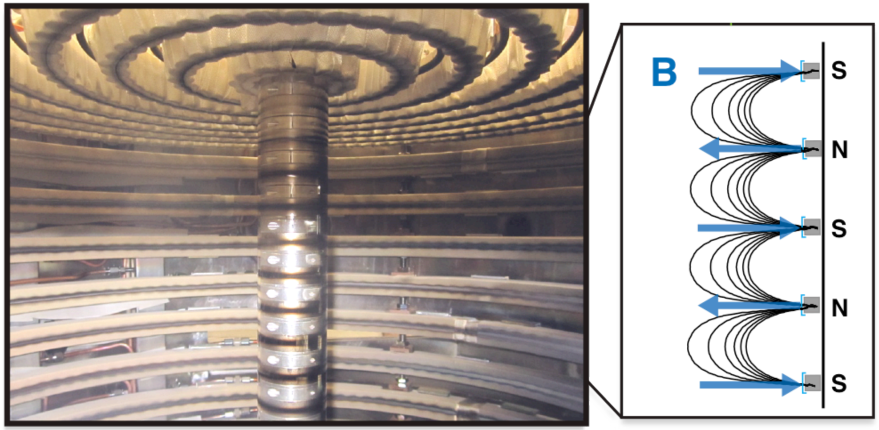
2. Since energy is conserved, $v_\parallel$ must decrease.

3. If $B$ is strong enough, $v_\parallel \to 0$ and the particle is reflected.

$$E_o = \frac{1}{2}mv_\parallel^2 + \frac{1}{2}mv_\perp^2 \qquad v_\parallel = \pm\sqrt{\frac{2}{m}(E_o - \mu B)}$$

The particle is reflected when $E_o \leq \mu B$

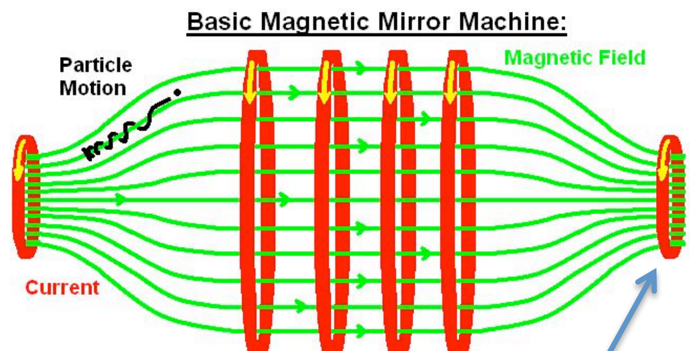
# Magnetic Mirror Confinement In Action

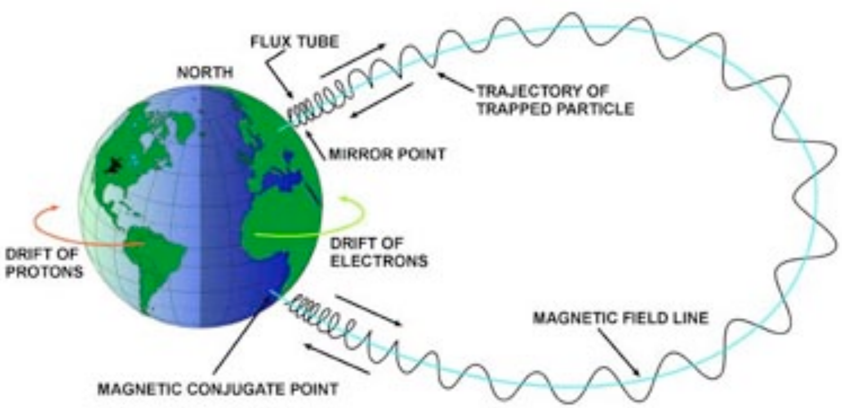



**Multicusp Confinement Devices**

### Early Fusion Experiments

Ex: Tandem Mirror Experiment (LLNL, 1980's) and other variants (Polywell devices)



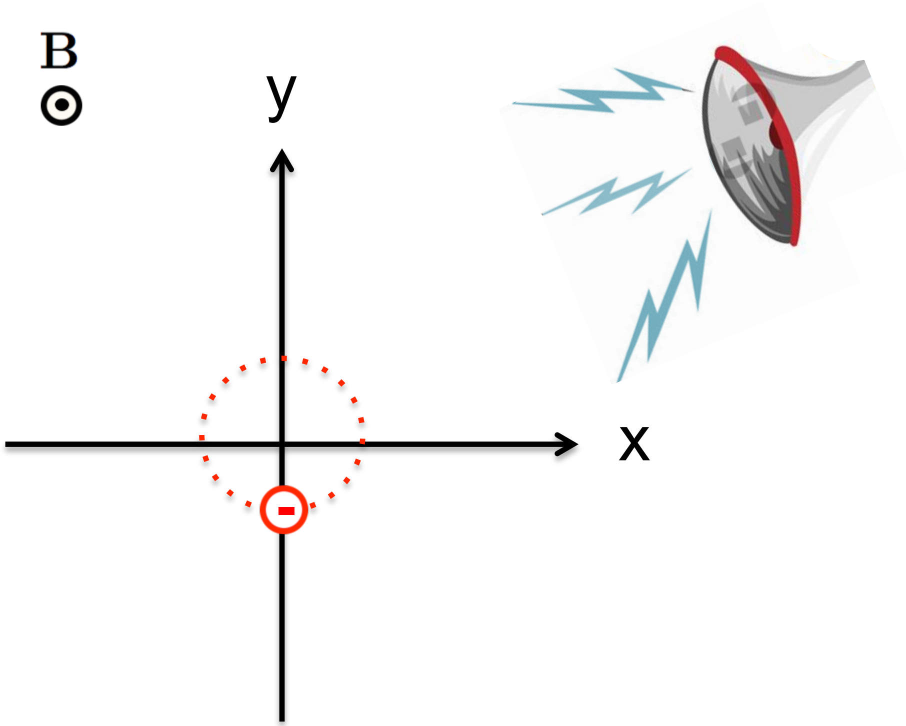




Particles with enough $v_{||}$ can still escape

**Charged particles can be trapped by Earth's magnetic field**

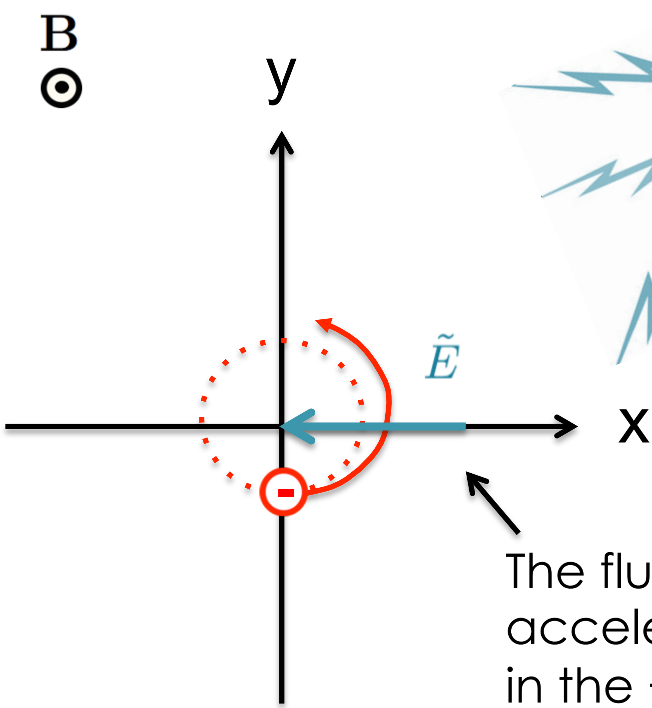
# Time-varying Electric and Magnetic Fields Can Be Used To Accelerate & Heat Particles

- **A high frequency electro-magnetic field can be used to accelerate electrons or ions.**



# Time-varying Electric and Magnetic Fields Can Be Used To Accelerate & Heat Particles

- **A high frequency electro-magnetic field can be used to accelerate electrons or ions.**
- **Particle gains energy as the applied electric field component oscillates at the cyclotron frequency ("in-phase" with the gyro-orbit).**




**Cyclotron frequency**

$$\omega_c \equiv \frac{|q|B}{m}$$

The fluctuating $\tilde{E}$
accelerates the electron
in the +x direction $\tilde{\mathbf{F}} = -e\tilde{\mathbf{E}}$

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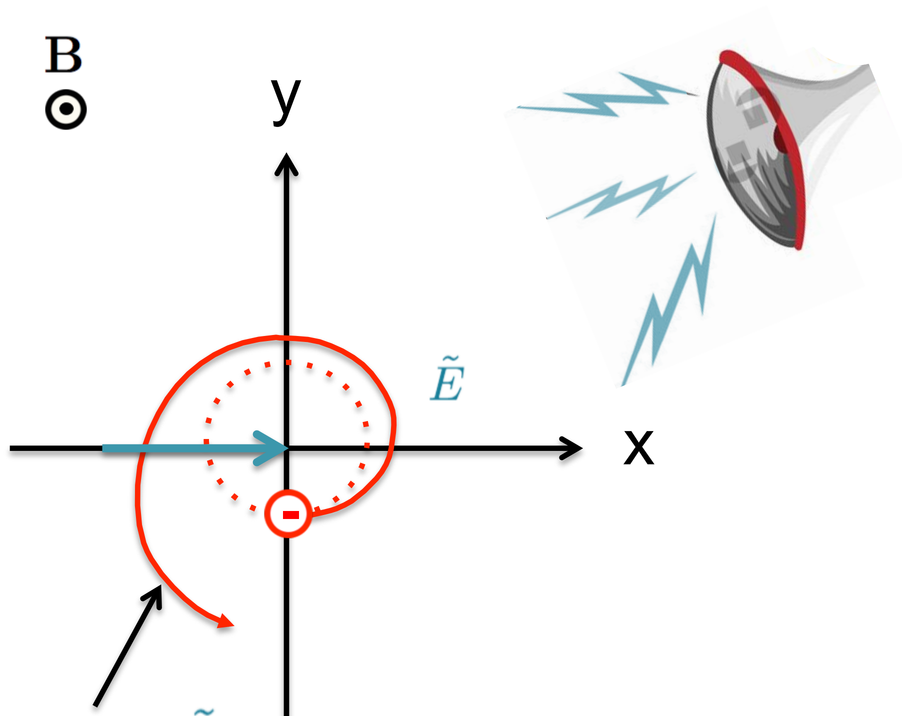


Now $\tilde{E}$ accelerates the electron in the -x direction

**Cyclotron frequency**

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# Time-varying Electric and Magnetic Fields Can Be Used To Accelerate & Heat Particles

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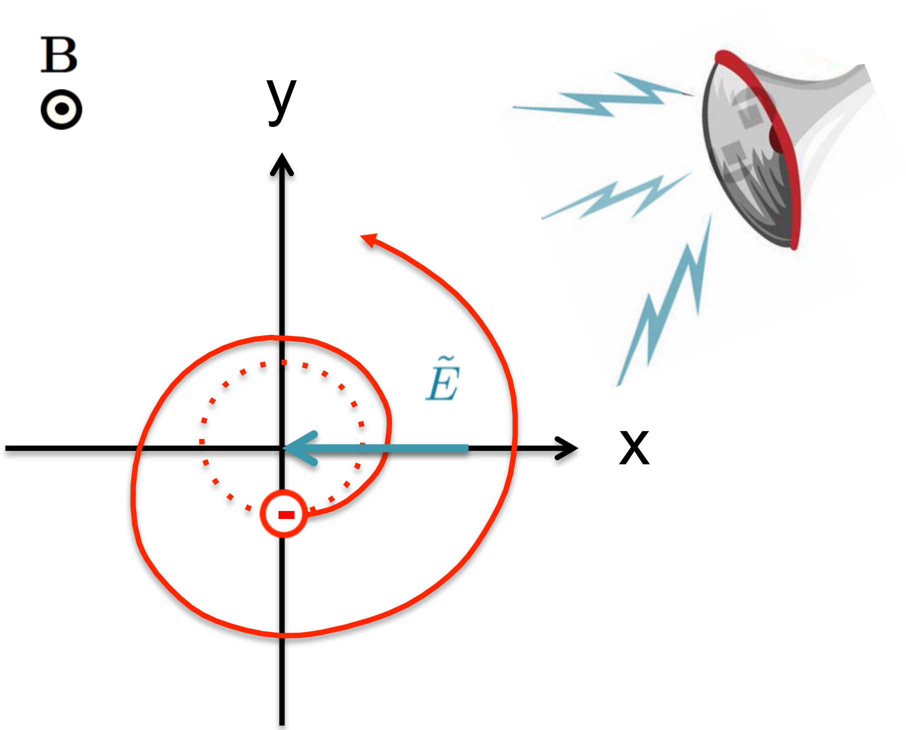


**Cyclotron frequency**

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# The Cyclotron Frequency is Important for Cyclotron Resonance Heating

- **A high frequency electro-magnetic field can be used to accelerate electrons or ions.**
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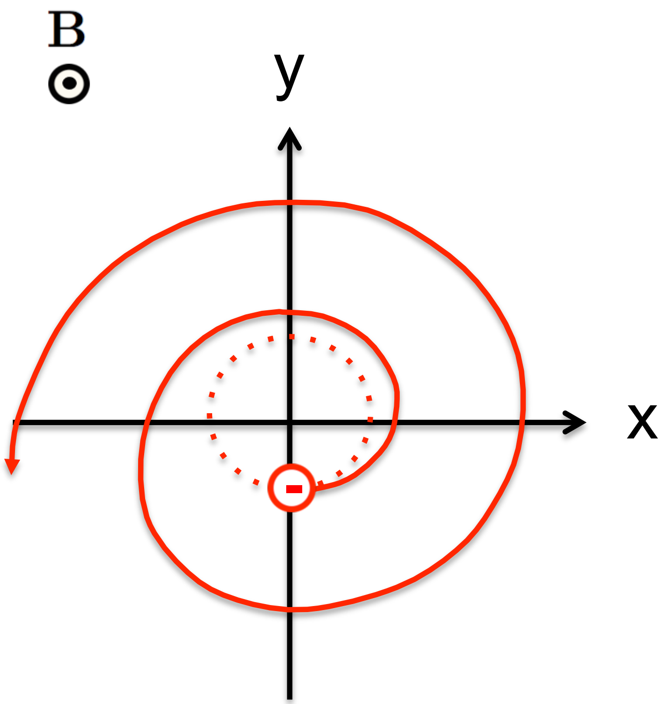



**Cyclotron frequency**

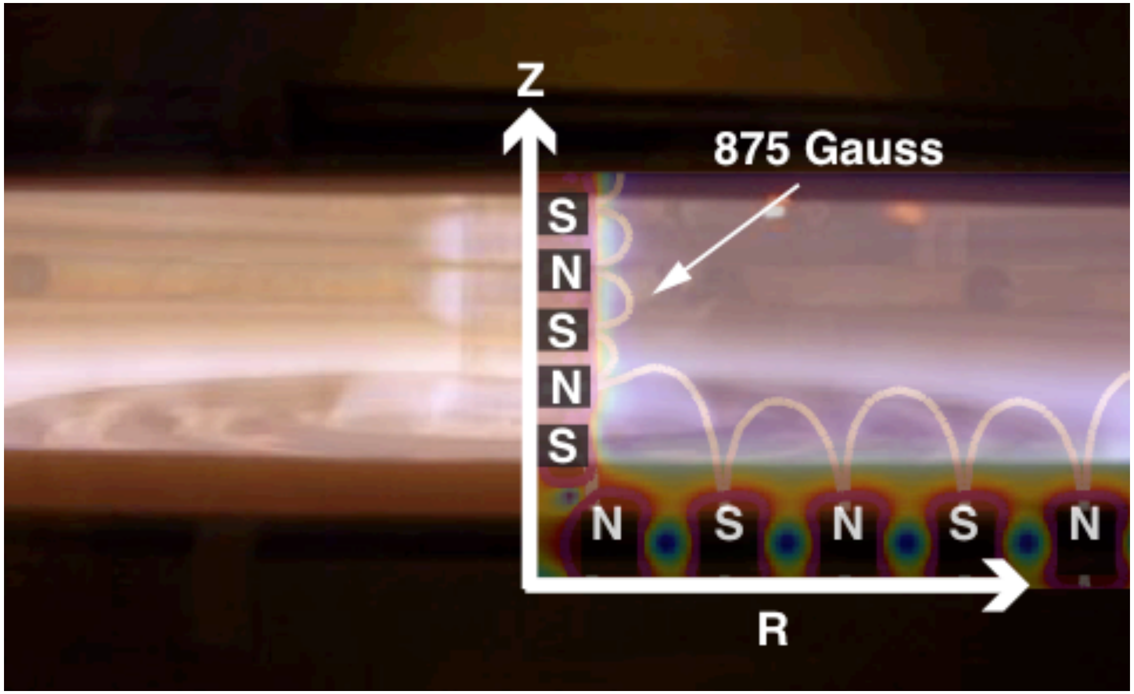
$$\omega_c \equiv \frac{|q|B}{m}$$

Ex: For an electron, what *B* corresponds to 2.45 Ghz (microwave oven frequency)?

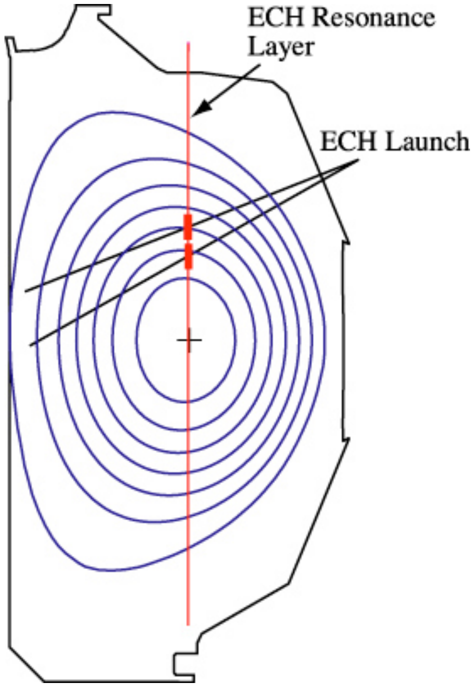
$$f = \frac{\omega_{ce}}{2\pi} = 2.45\ \mathrm{GHz} \Rightarrow B = 875\ \mathrm{Gauss}$$

# Example of Cyclotron Heating in Action

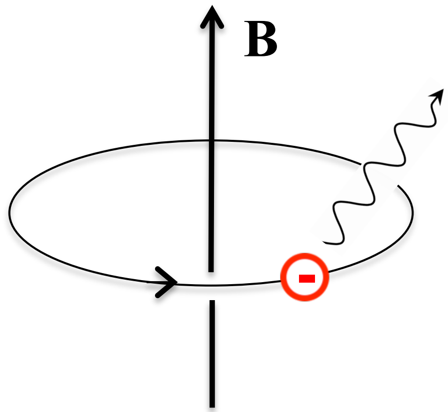
Electron Cyclotron Heating In A Plasma Experiment at UW-Madison (2.45 GHz, B=875 Gauss)




In the DIII-D tokamak, use 110 GHz second harmonic heating (B~2 Tesla)



# Other Practical Applications: EM Emission from Charged Particle Acceleration




- **Electron cyclotron emission (measure $T_e$ profiles)**
  Produced by acceleration of gyrating charged particle
  EM radiation emitted at discrete frequencies:

$$\omega = n\omega_{ce} \qquad \omega_{ce} = \frac{eB}{m_e}$$

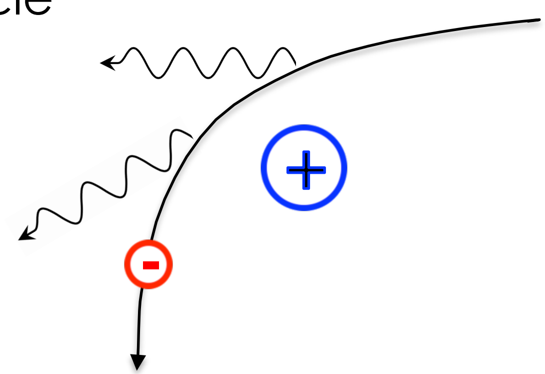
  Detected radiated power is proportional to $T_e$:

$$I(\omega) = \frac{\omega^2 k T_e}{8\pi^3 c^2}$$

- **Bremsstrahlung emission**
  Produced by deceleration of deflected charged particle

$$I(\omega) \propto \frac{n_e^2 Z_{eff} g}{\sqrt{T_e}} \exp\left(-\frac{\hbar\omega}{kT_e}\right)$$

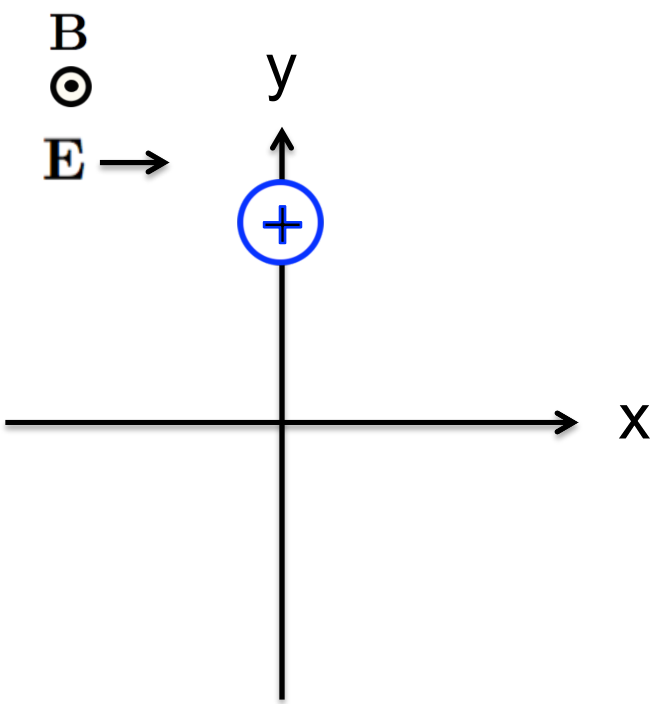
  Radiated power depends on $n_e$, Te, charge state $Z_{eff}$ (can be used to measure $Z_{eff}$)



# Next Simplest Case to Analyze: Constant, Uniform Electric Field Perpendicular to Magnetic Field

$$\mathbf{F} = q(\mathbf{E} + \mathbf{v} \times \mathbf{B})$$

$$\mathbf{E} = E_x \hat{\mathbf{x}} \qquad \mathbf{B} = B_z \hat{\mathbf{z}}$$



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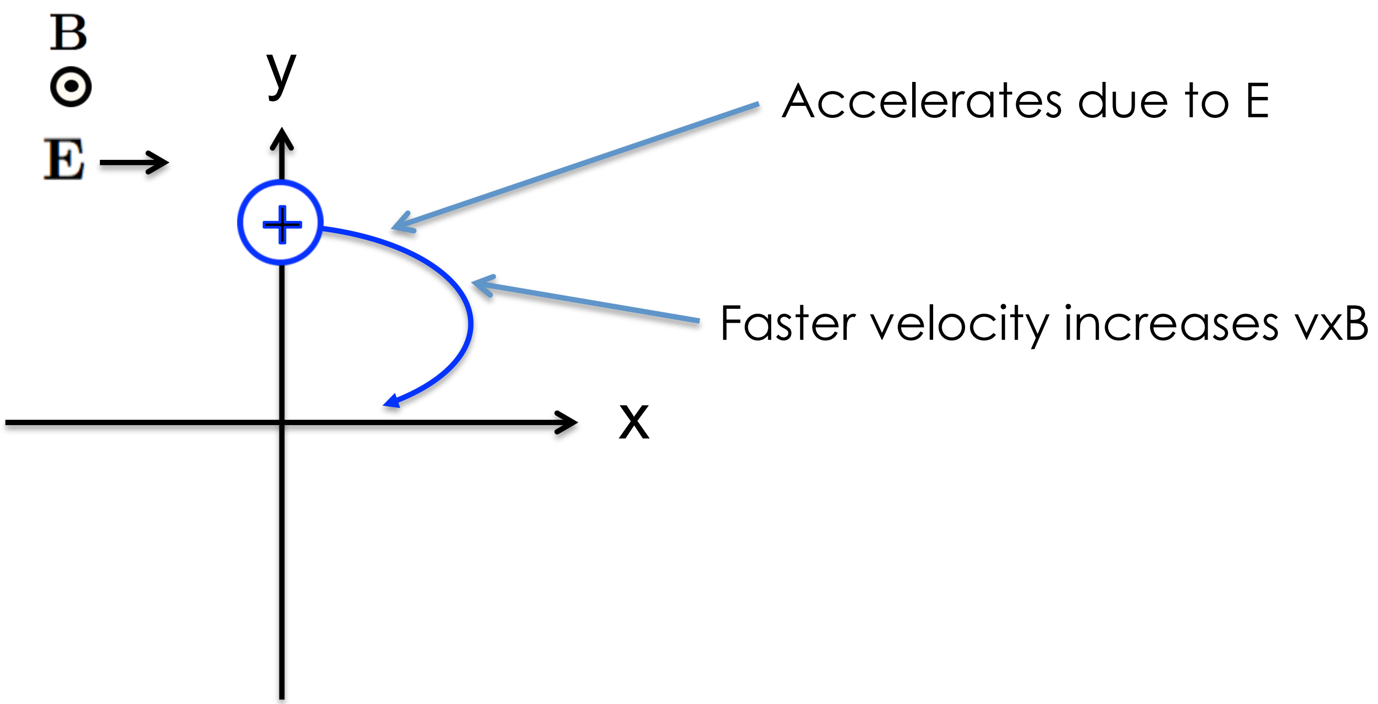

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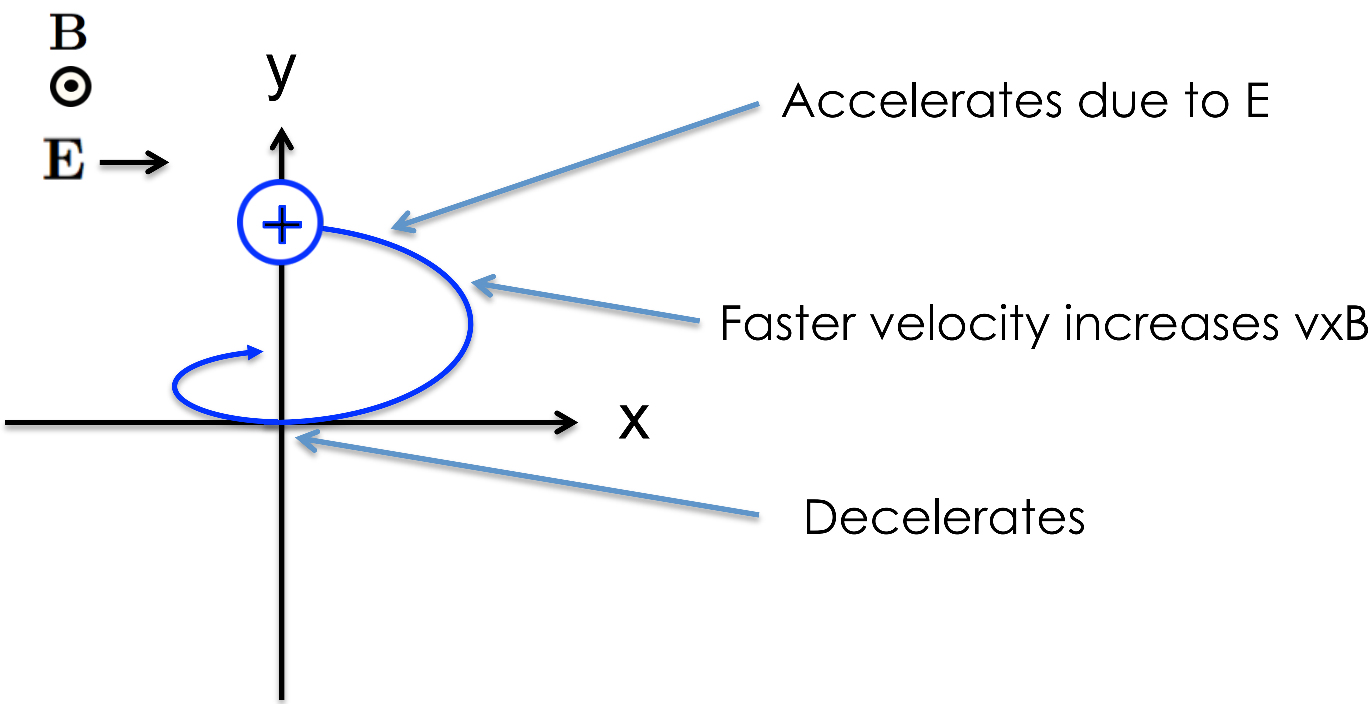

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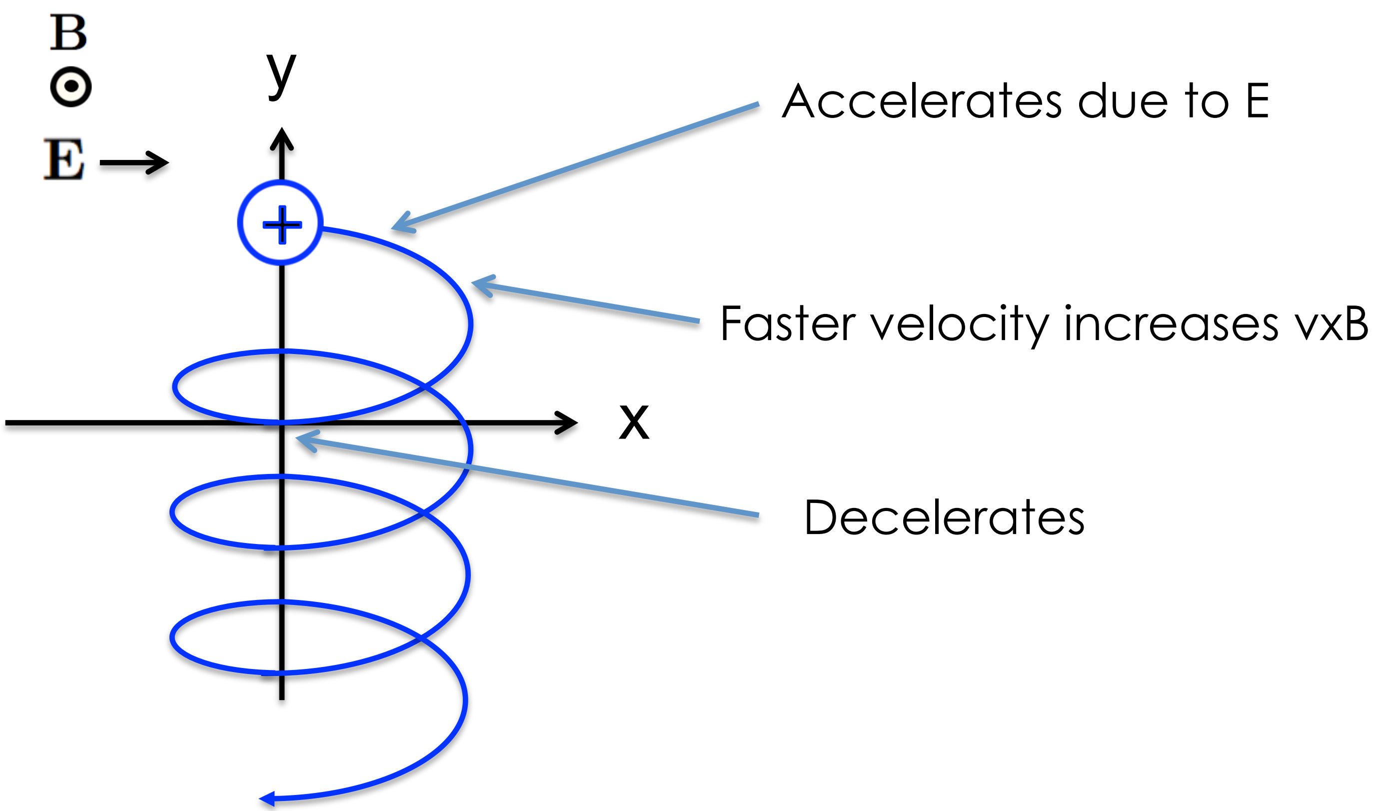

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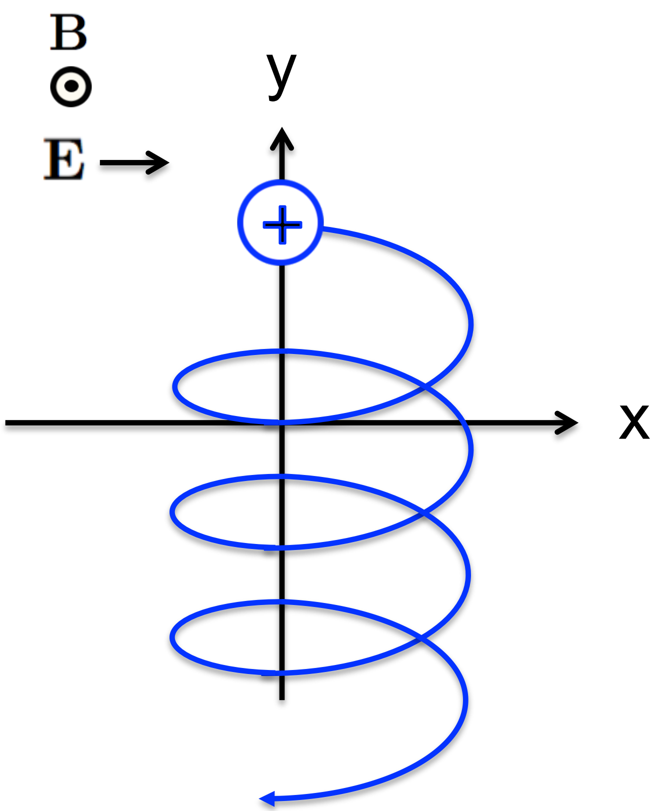



$$\dot{v}_x = \frac{q}{m}(v_y B_z + E_x)$$

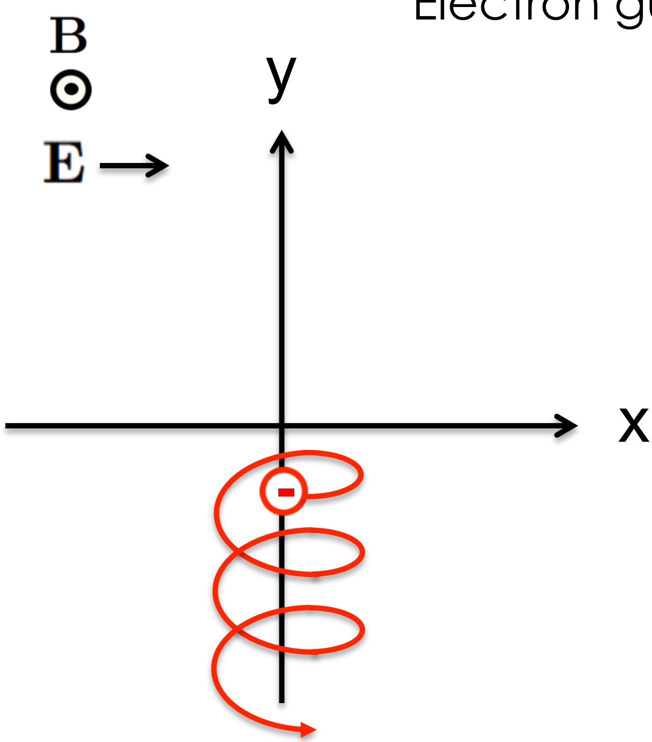
$$v_y = \mp v_\perp \sin\left(\frac{|q|B_z}{m}t + \phi_0\right) - \frac{E_x}{B_z}$$

Ion guiding center drifts in the direction $-\hat{\mathbf{y}}$

# Guiding Center Drift Due to E x B

$$v_y = \mp v_\perp \sin\left(\frac{|q|B_z}{m}t + \phi_0\right) - \frac{E_x}{B_z}$$

Electron guiding center also drifts in the direction $-\hat{\mathbf{y}}$




The ExB drift can be written more generally as

$$\mathbf{v_E} = \frac{\mathbf{E}\times\mathbf{B}}{B^2}$$

- **ExB drift is independent of charge and mass**
- **Both electrons and ions move together**

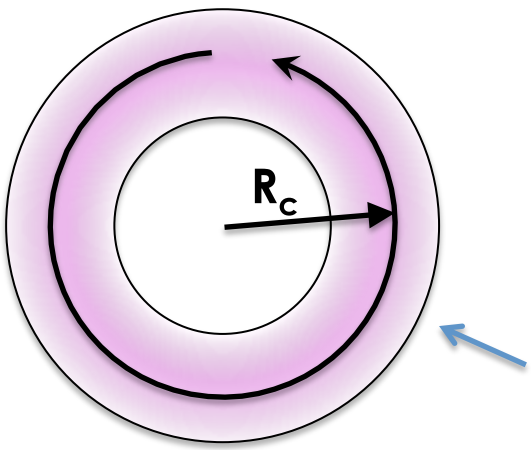
# Other Forces Can Cause Guiding Center Drift

- **Any force perpendicular to B can cause particles to drift**

Drift due to force: $$\mathbf{v_d} = \frac{1}{q}\frac{\mathbf{F}\times\mathbf{B}}{B^2}$$

Examples of forces: $$\mathbf{F_g} = m\mathbf{g}$$ gravity

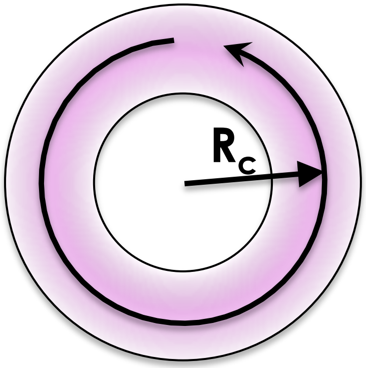
$$\mathbf{F_{cf}} = \frac{mv_\parallel^2}{R_c}\hat{\mathbf{r}}$$ centrifugal



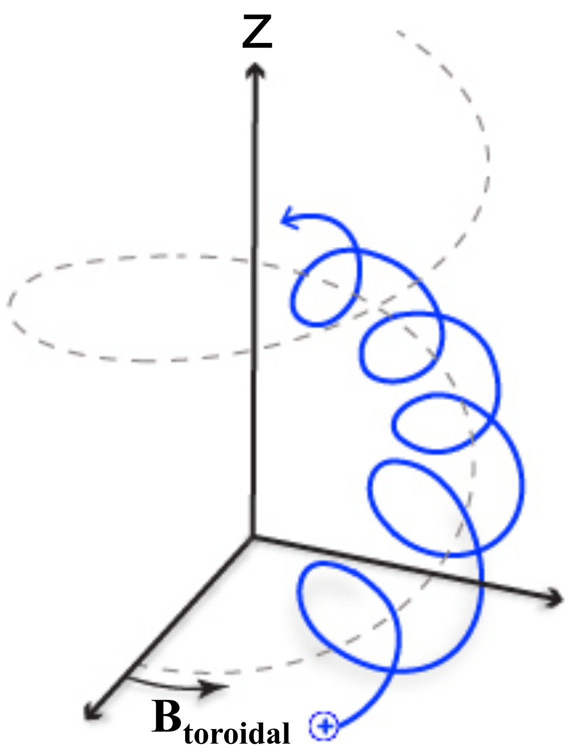

- Bend the magnetic field into a donut shape
- No end losses because the field lines go around and close on themselves
- BUT a particle following a toroidal magnetic field would experience $\mathbf{F_{cf}}$

# Curvature Drift Due to Bending Field Lines

The outward centrifugal force causes curvature drift



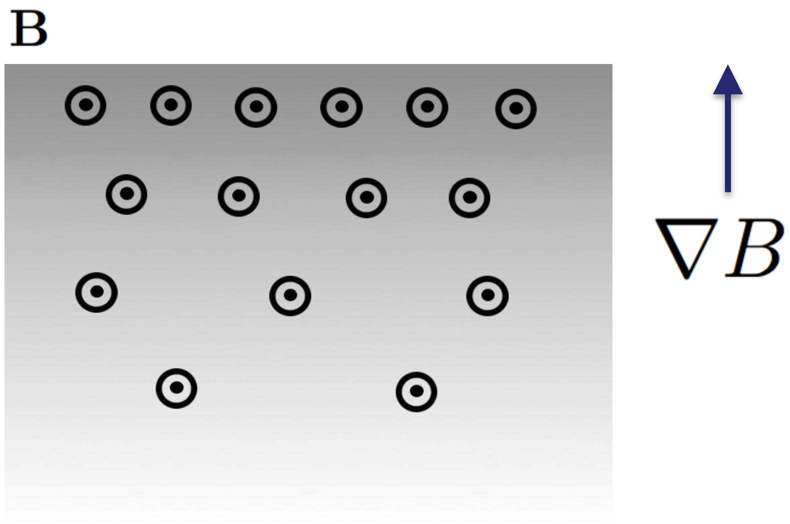

$$\mathbf{F_{cf}} = \frac{m v_{\parallel}^2}{R_c}\hat{\mathbf{r}}$$

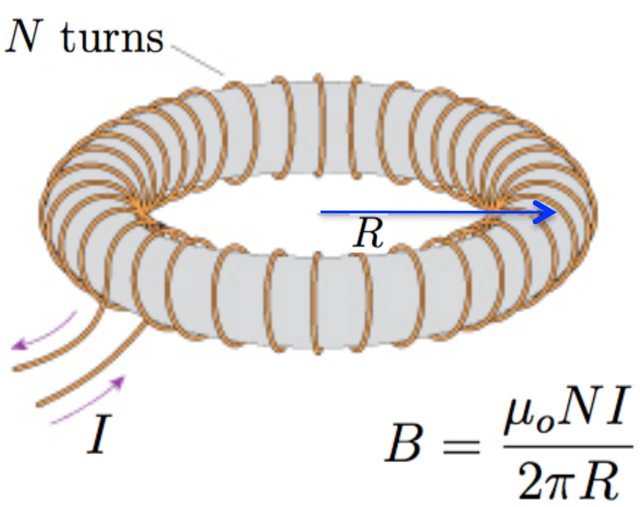



$$\mathbf{V_R} = \frac{m v_{\parallel}^2}{qB^2}\frac{\mathbf{R_c} \times \mathbf{B}}{R_c^2}$$

A particle moving along a curved field line will drift up or down, depending on the sign of the charge

# Spatially Varying Magnetic Field Strength Also Causes Drift



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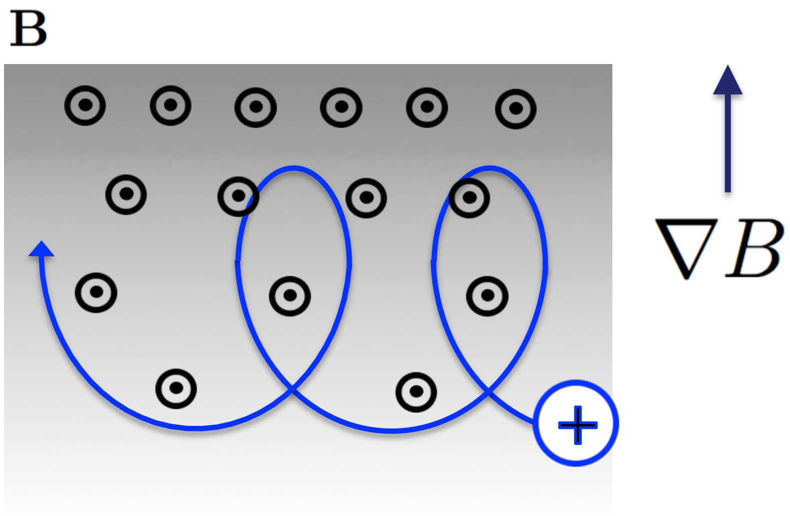
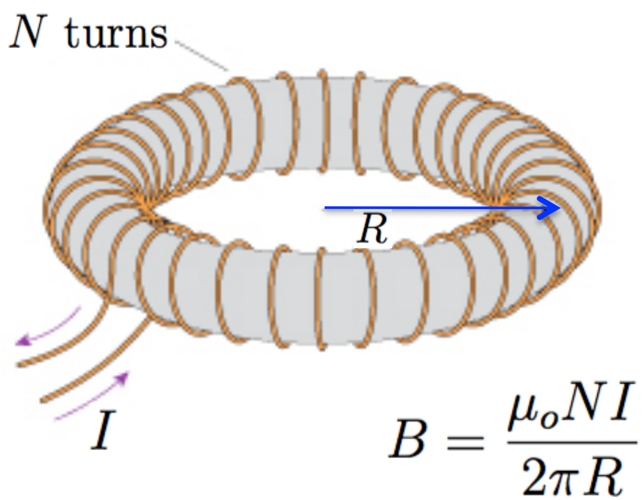
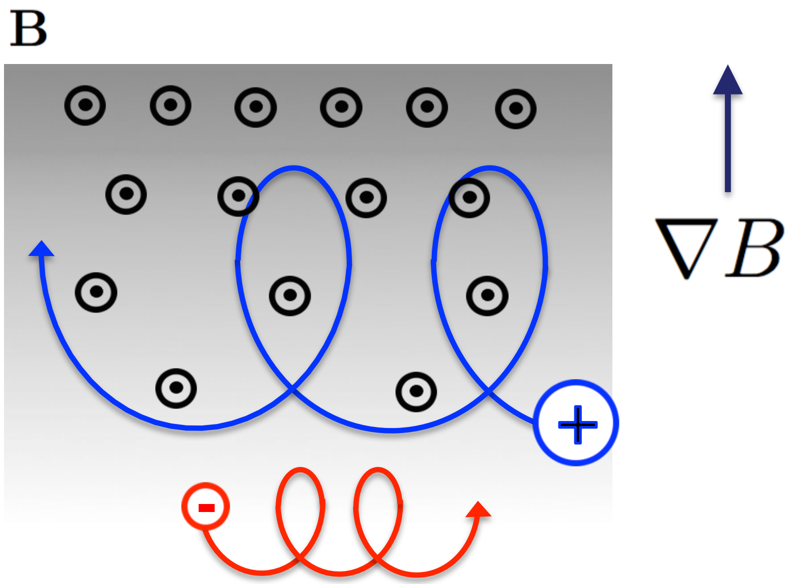
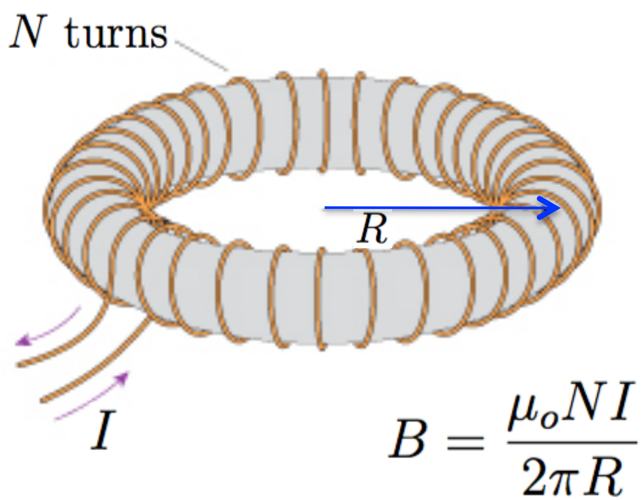



- **The gyro-radius will be larger where the field is weaker and smaller where the field is stronger**

# Spatially Varying Magnetic Field Strength Also Causes Drift



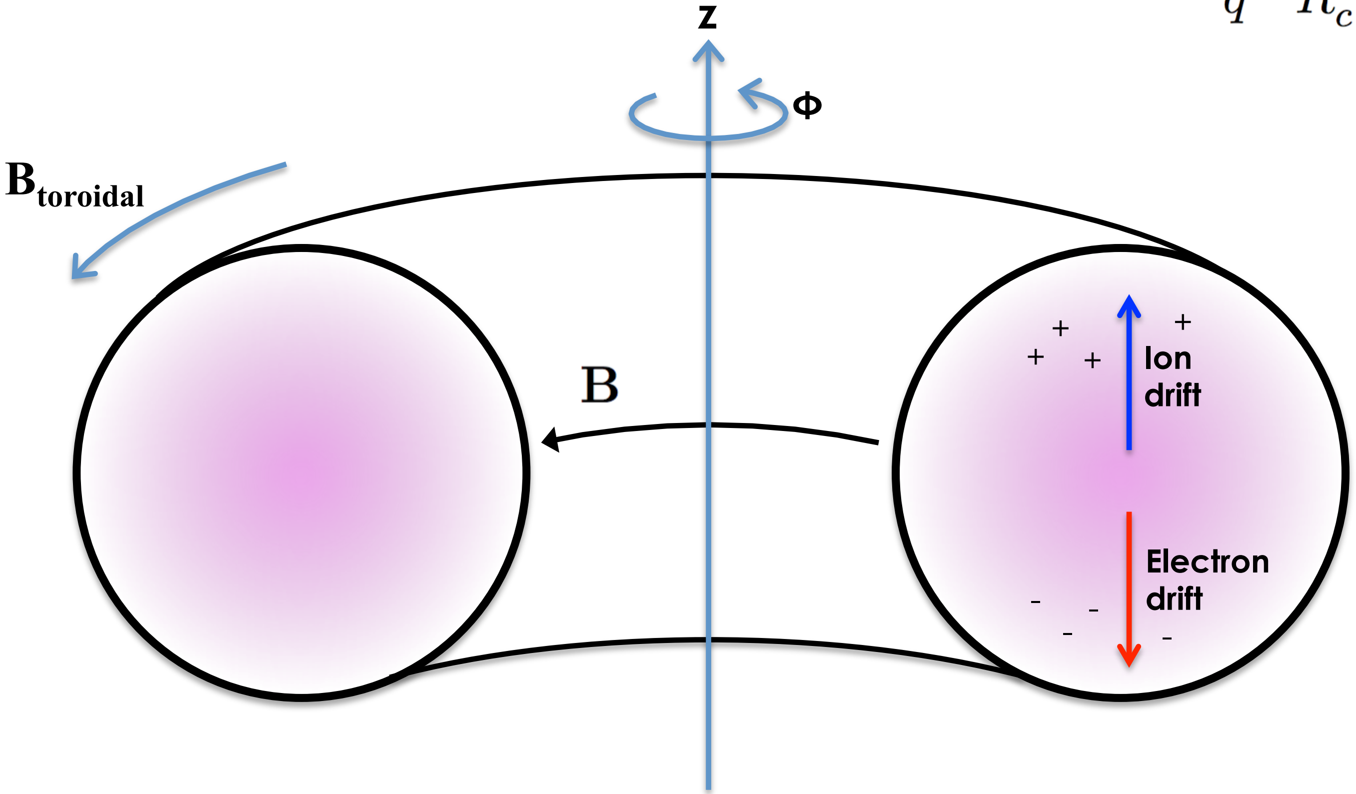

- **The gyro-radius will be larger where the field is weaker and smaller where the field is stronger**
- **The resulting drift velocity is described by:**

$$\mathbf{V}_{\nabla \mathbf{B}} = \frac{m v_\perp^2}{2qB} \frac{\mathbf{B} \times \nabla B}{B^2}$$

# What Happens To Charged Particles In A Purely Toroidal Magnetic Field?

- Charged particles in a curved magnetic field will experience both ∇B and curvature drift: these effects add

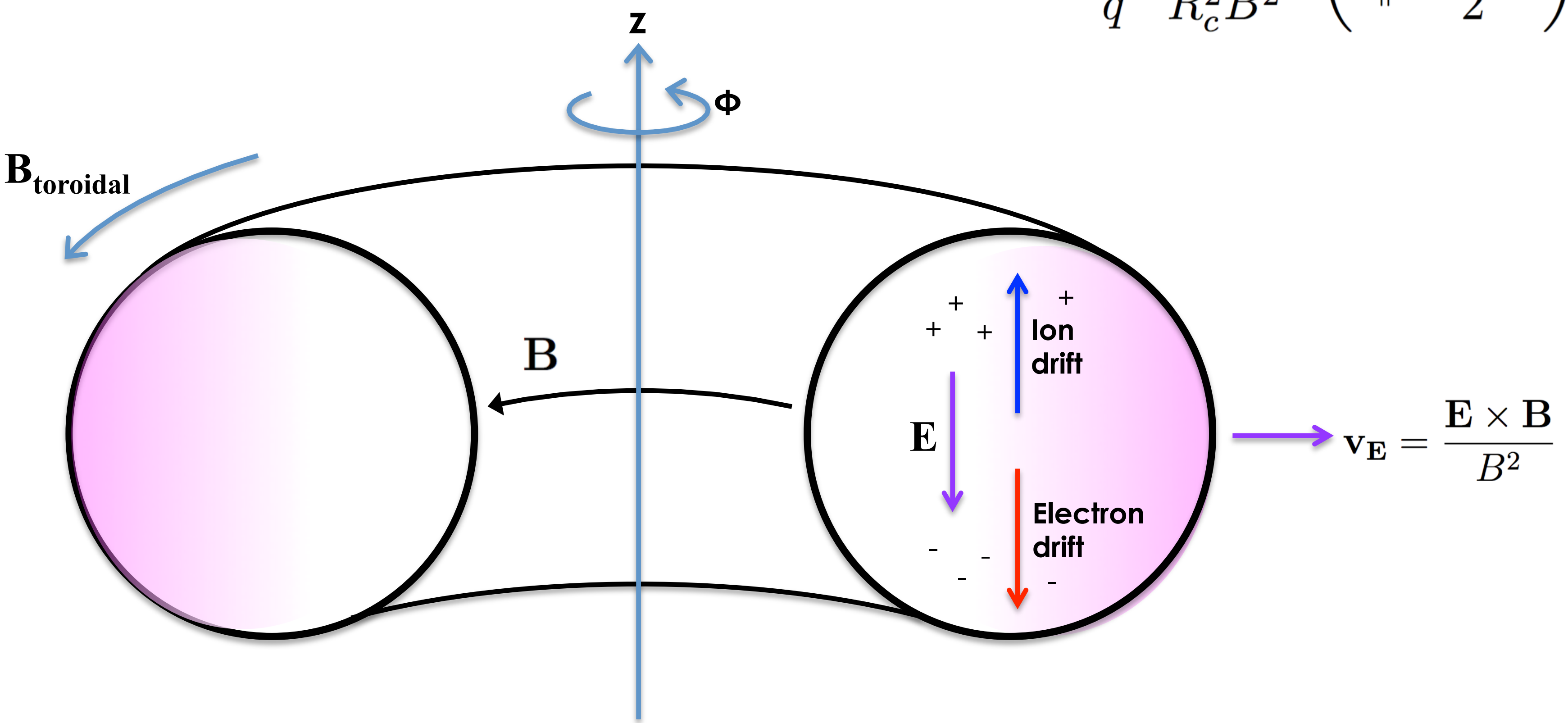
$$\mathbf{V_R} + \mathbf{V_{\nabla B}} = \frac{m}{q}\frac{\mathbf{R_c} \times \mathbf{B}}{R_c^2 B^2}\left(v_\parallel^2 + \frac{1}{2}v_\perp^2\right)$$



# Charged Particles Will Drift Outward

- Charged particles in a curved magnetic field will experience both ∇B and curvature drift

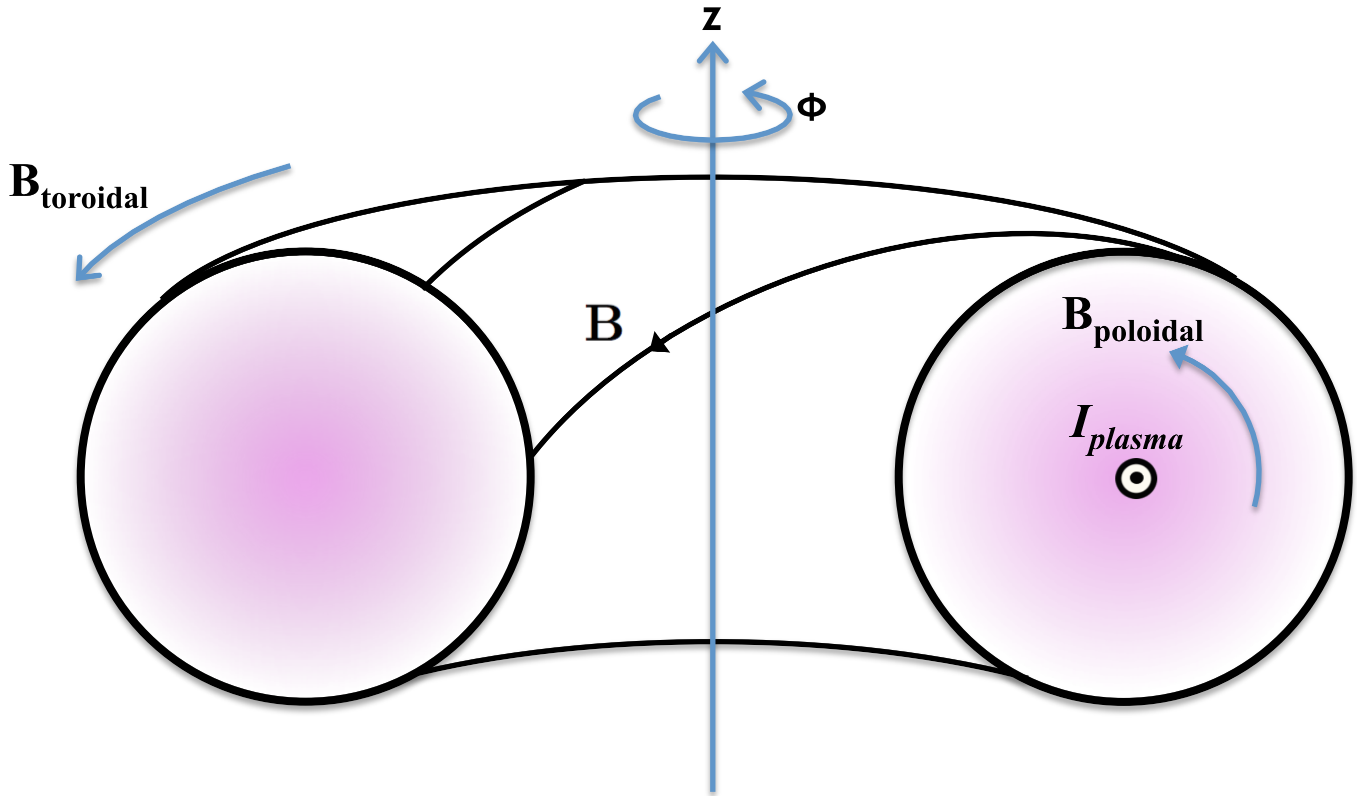
$$\mathbf{V_R} + \mathbf{V_{\nabla B}} = \frac{m}{q}\frac{\mathbf{R_c} \times \mathbf{B}}{R_c^2 B^2}\left(v_\parallel^2 + \frac{1}{2}v_\perp^2\right)$$




- This means that no matter what, particles in a torus with a purely toroidal field will drift radially out and hit the walls.

# Tokamak Solution: Add Poloidal Magnetic Field

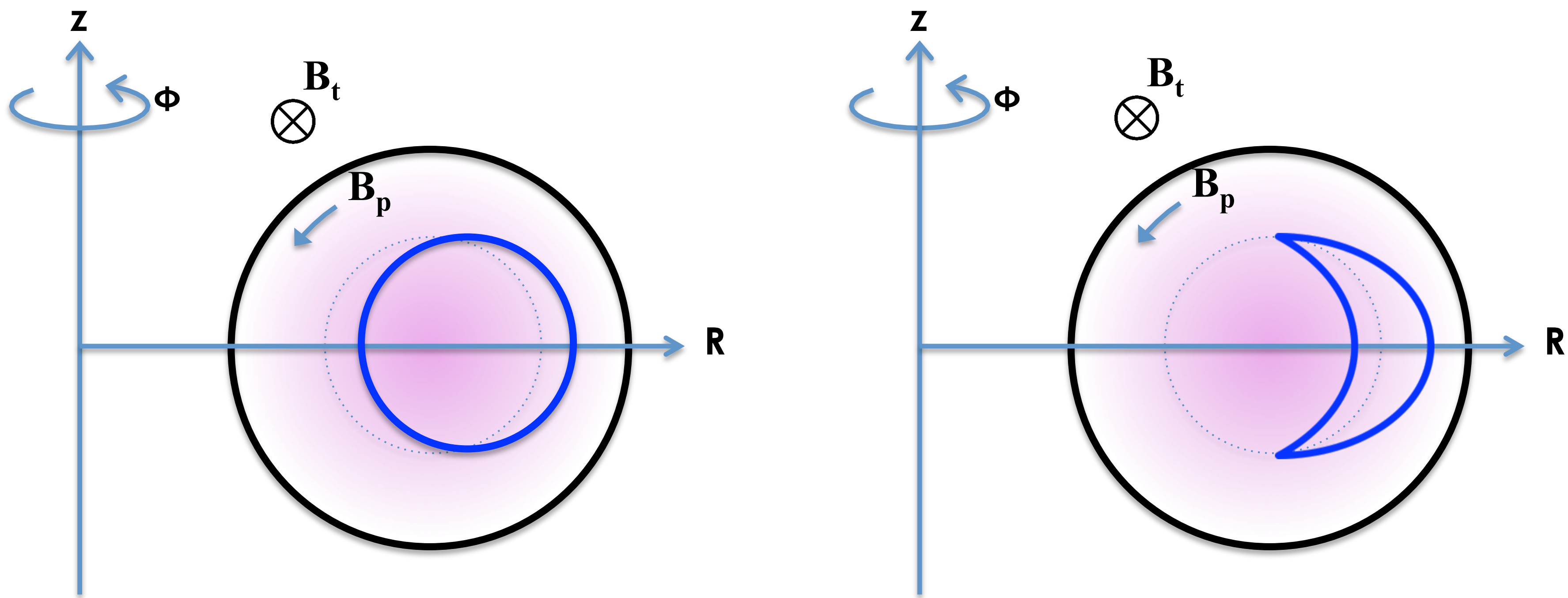
**Toroidal: long way around**
**Poloidal: short way around**
**1. Use external coils to apply a toroidal magnetic field**
**2. Drive toroidal current in the plasma to generate a poloidal magnetic field**




- **The resulting helical magnetic field is much better at confining charged particles.**
- **The challenge: how to drive current in plasma in steady state while keeping the plasma stable and free of disruptions?**

# There Are Two Main Classes of Particle Orbits In Tokamaks

$$\mathbf{V_R} + \mathbf{V_{\nabla B}} = \frac{m}{q}\frac{\mathbf{R_c} \times \mathbf{B}}{R_c^2 B^2}\left(v_\parallel^2 + \frac{1}{2}v_\perp^2\right)$$



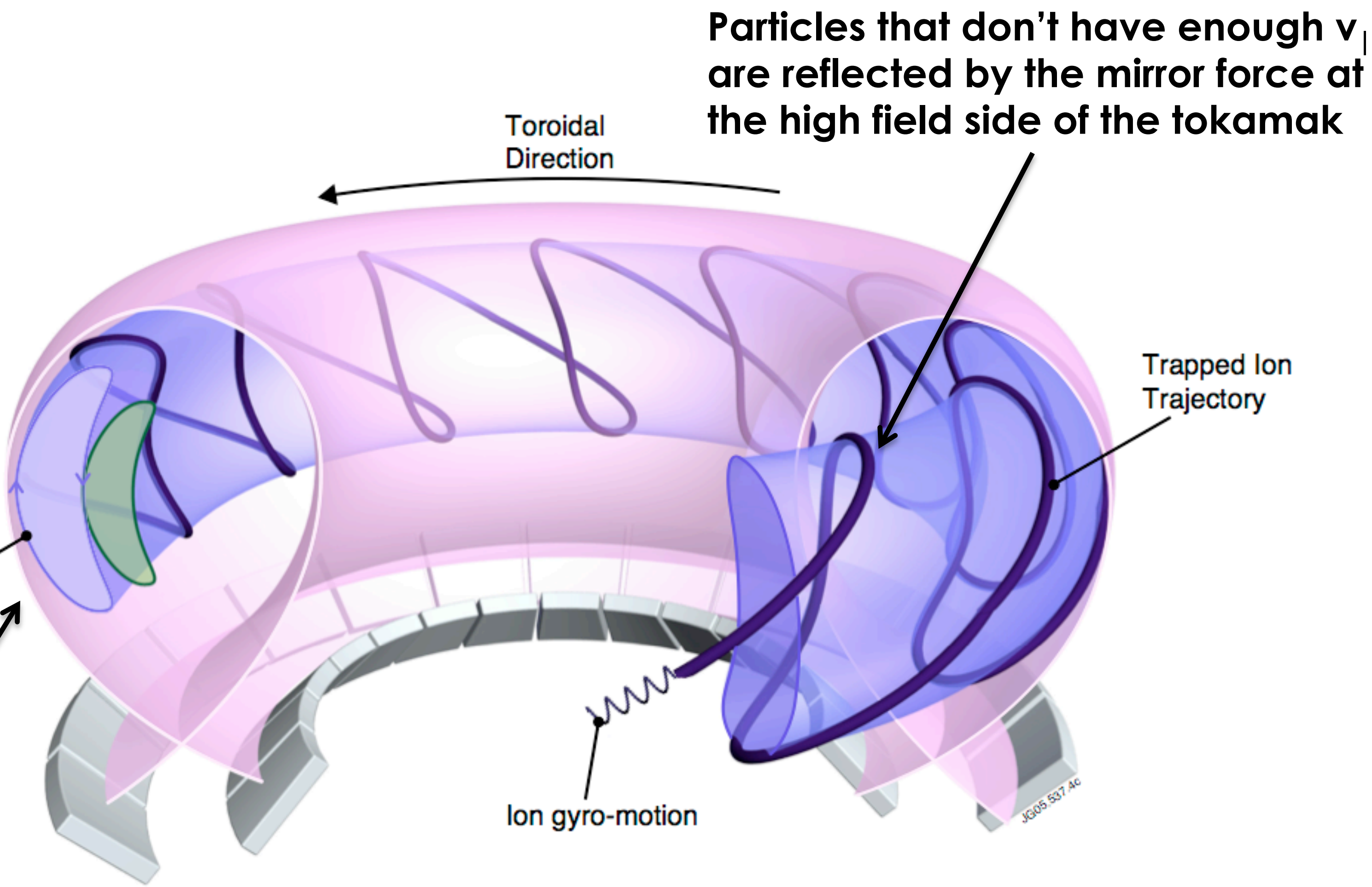

Passing

Trapped

Particles with sufficient $\mathbf{v}_{||}$ will follow the helical magnetic field around the torus

Particles with lower $\mathbf{v}_{||}$ are reflected as they encounter stronger $B$ and therefore execute "banana" orbits as they precess around the torus B

# Banana Orbits




Trapped particles won't hit the wall if the banana orbit width $\Delta r$ is small enough

$$\Delta r = 2r_L \frac{B_T}{B_p}\sqrt{\frac{r}{R}}$$

Image credit: euro-fusion.org

# Classifying Particle Orbits In Tokamaks Is Important for Understanding Basic Physics Mechanisms Like Wave-Particle Interactions

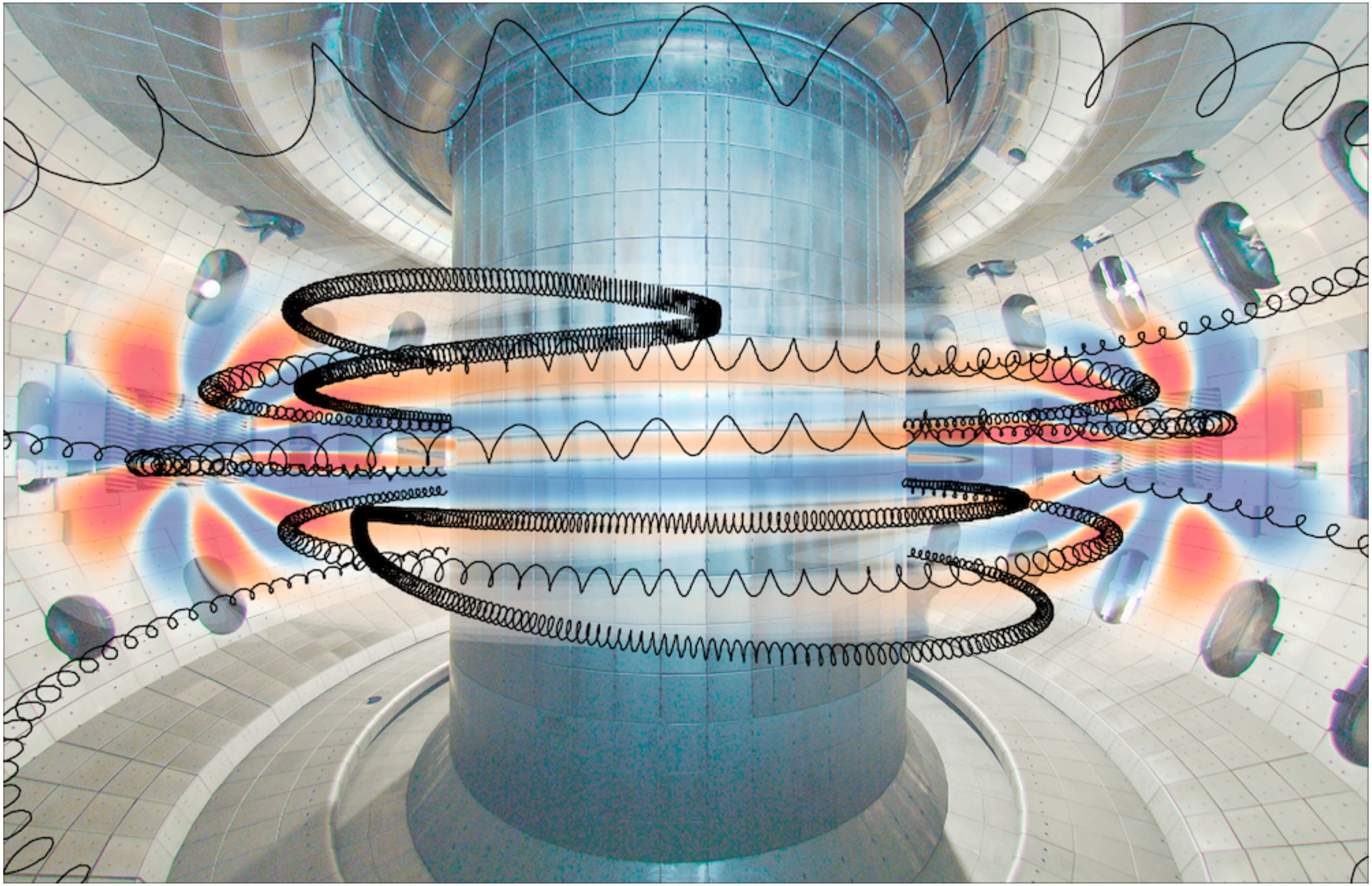


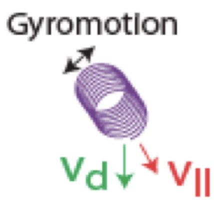
Image credit: Pace et. al.,*Physics Today* (2015)

# Particle Resonance With Alfvén Eigenmode (AE) Instabilities

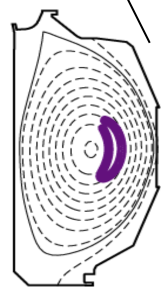
- **Occurs when the wave and particle orbit phases match after many cycles:**

$$\omega = l\omega_{ci} + p\omega_\theta + n\omega_\zeta$$

AE mode frequency




Time to complete cyclotron orbit



Time to complete poloidal orbit

Time to complete toroidal orbit

[W.W. Heidbrink, Phys. Plasmas 15 (2008)]

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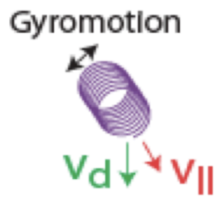


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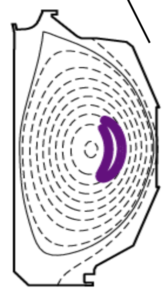
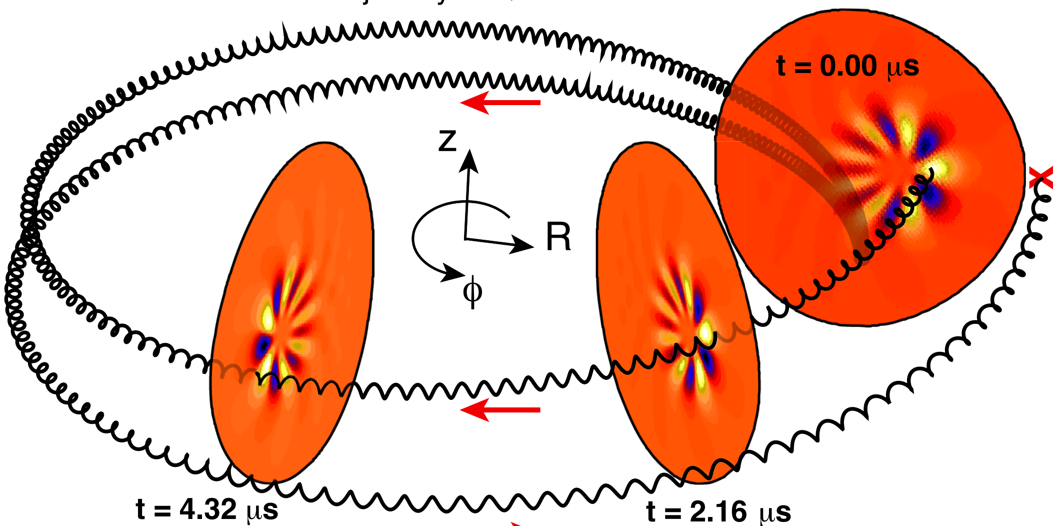


Time to complete poloidal orbit

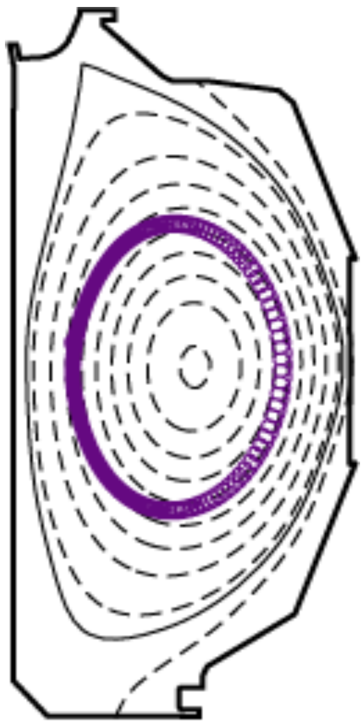
Time to complete toroidal orbit

- **Power transfer can occur as the ion stays in phase with the wave as it traverses the mode**



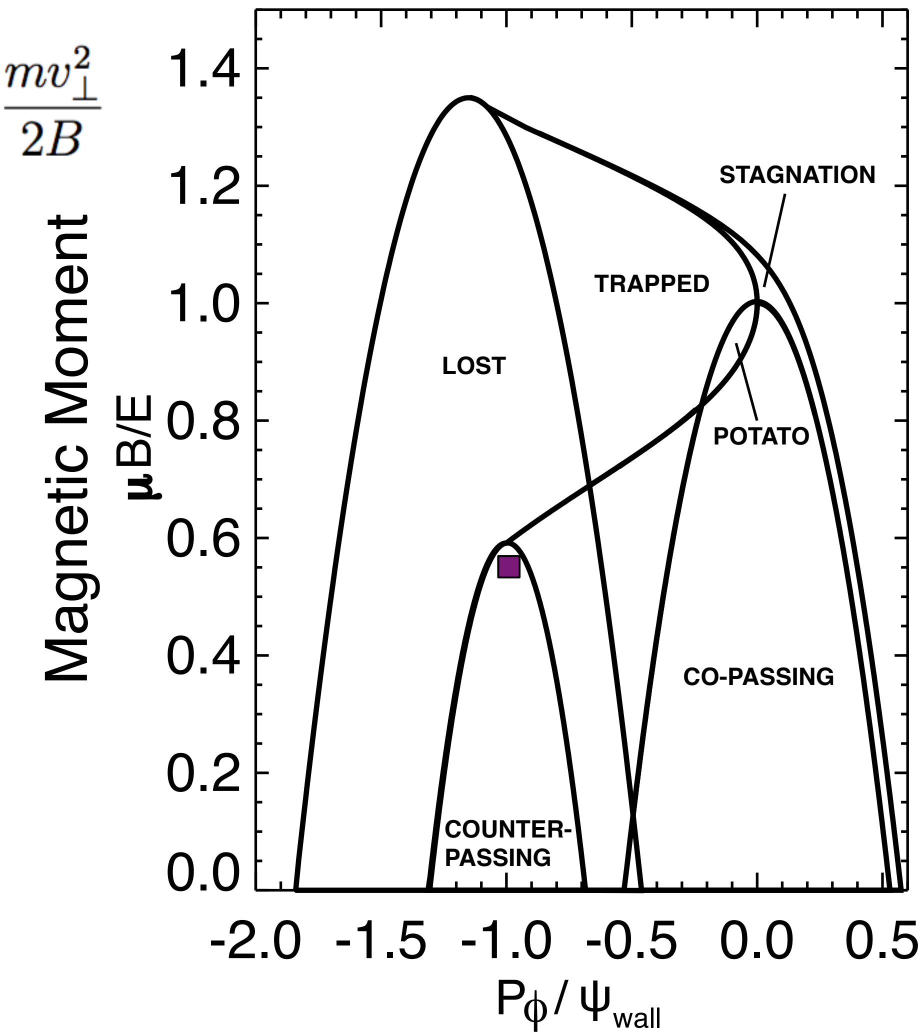

*[W.W. Heidbrink, Phys. Plasmas 15 (2008)]* *[Pace, Physics Today (Oct. 2015)]*

# The Fast Ion Distribution Function Is "Most Simply" Described Using Constants of Motion



Projection of 80 keV $D^+$ orbit in the DIII-D tokamak

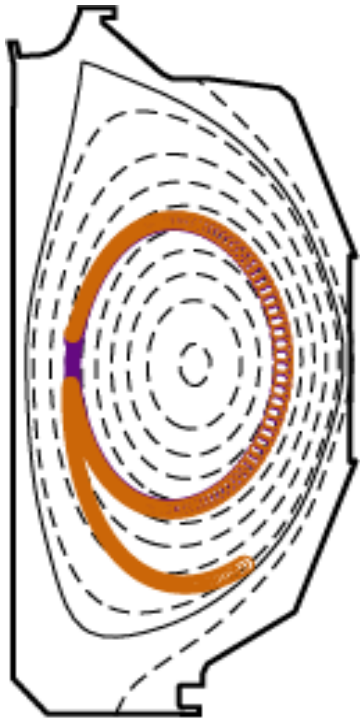
$$\mu \equiv \frac{mv_\perp^2}{2B}$$



Toroidal Canonical Angular Momentum

*[R. B. White, Theory of toroidally confined plasmas, Imperial College Press (2001)]*

# Small Changes in Particle Energy Can Cause Large Changes in Orbit Topology



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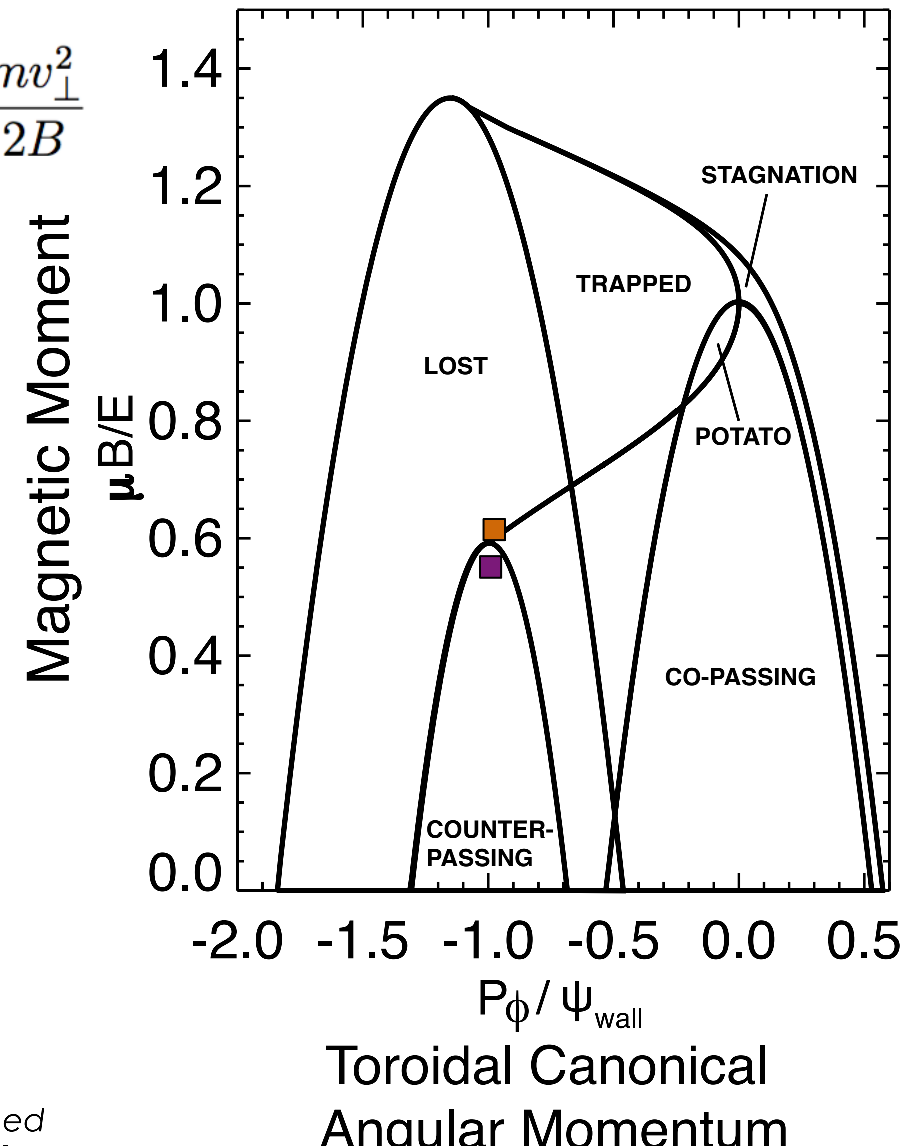
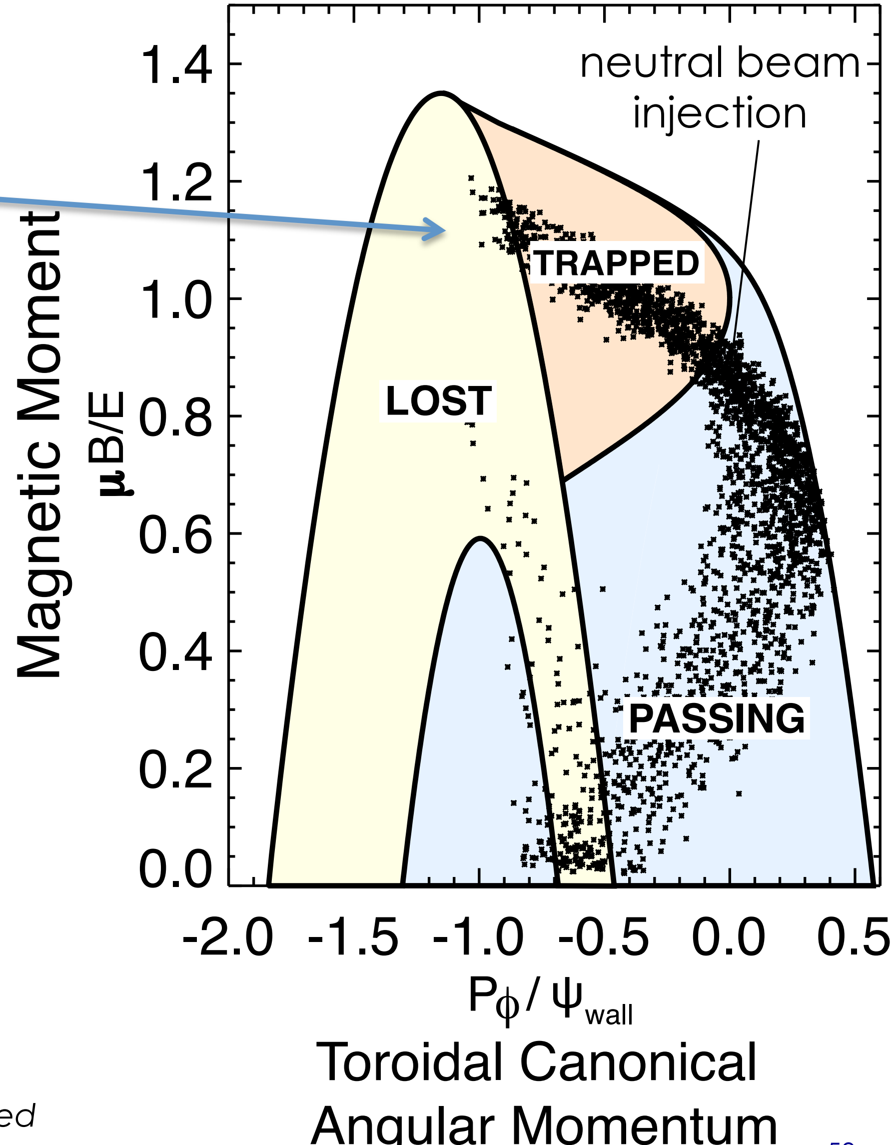



*[R. B. White, Theory of toroidally confined plasmas, Imperial College Press (2001)]*

# There Can Be Several Different Populations of Fast Ions In Fusion Devices

- Example of distribution function $F(E,\mu,P_\phi)$ for neutral beam injection, which is anisotropic and non-Maxwellian

- The distribution function for fusion products (alpha particles) is isotropic

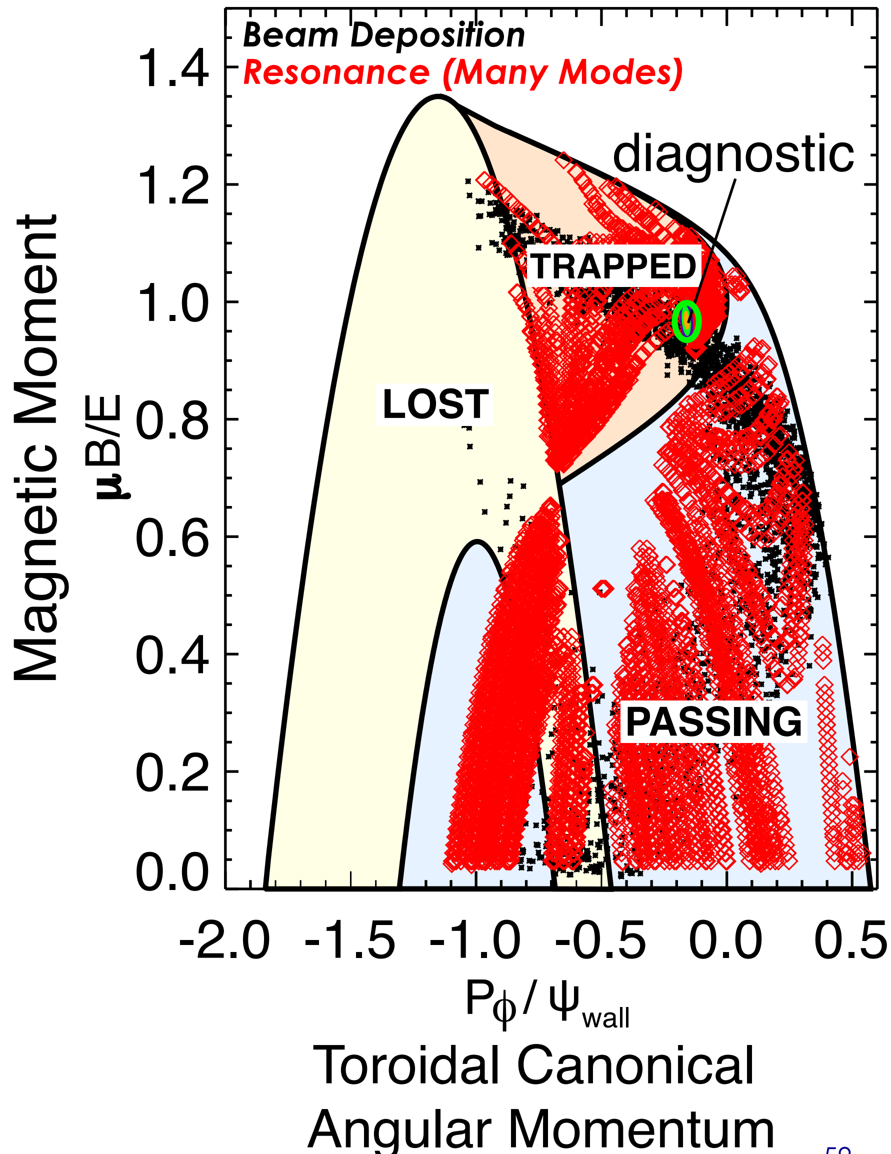



*[R. B. White, Theory of toroidally confined plasmas, Imperial College Press (2001)]*

# Active Research: Calculating the Energetic Particle Distribution Function After Transport by Instabilities

- Transport can occur if **fast ions** intersect **AE resonances** in this "phase space" plot
- In certain conditions, AEs can cause significant transport of fast ions and significantly degrade fusion performance

→ We are working on controlling/avoiding AEs



# Conclusions

- Charged particles undergo gyromotion about magnetic fields, and are free to move along the magnetic field line.
- Depending on magnetic field geometry or the presence of other forces like electric fields, particles can drift across field lines (and even leave the system→hit the walls).
- (One) challenge for fusion energy research is to confine enough charged particles that are energetic enough for long enough lengths of time to achieve sustained fusion.