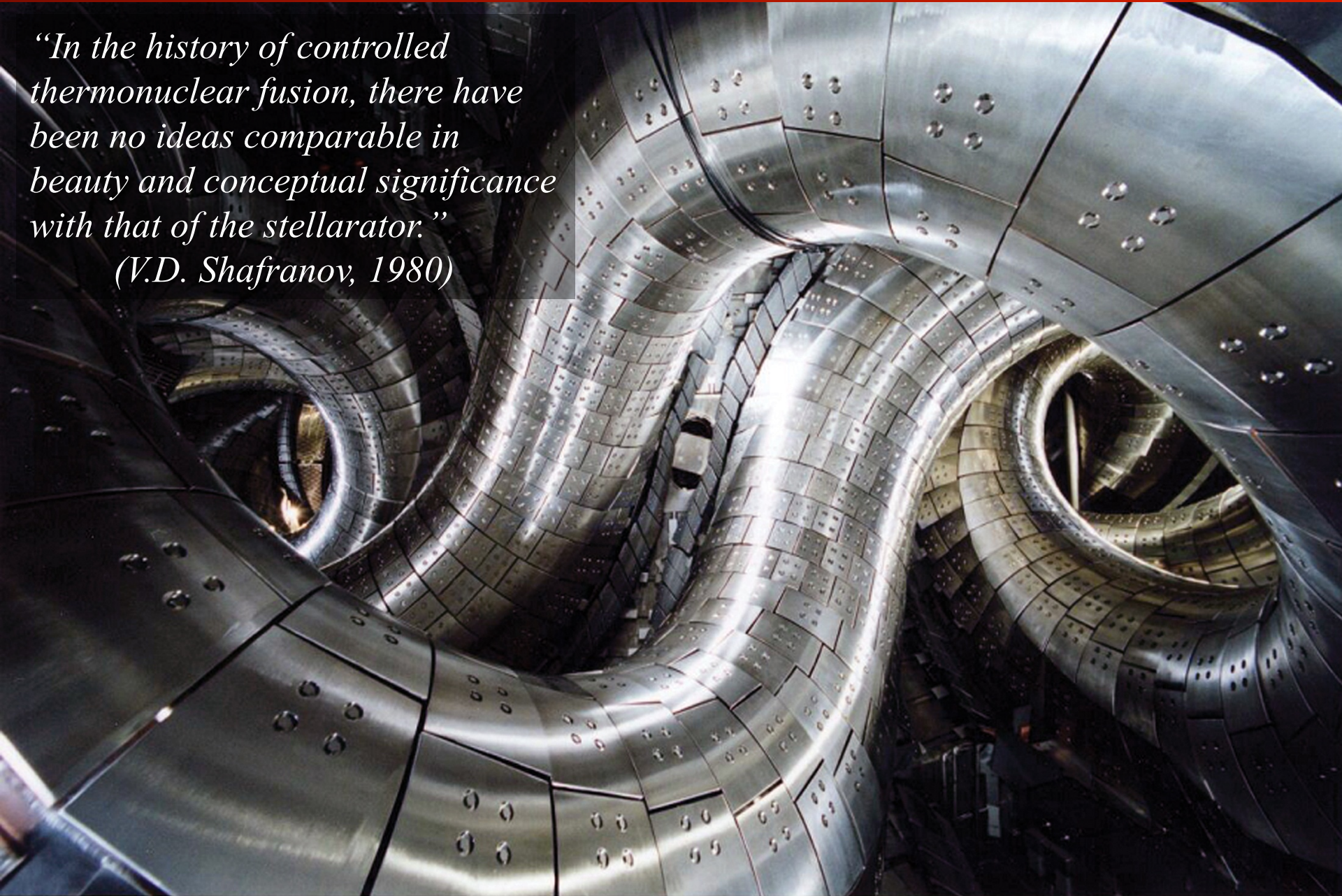


Stellarators

Matt Landreman
University of Maryland

“In the history of controlled thermonuclear fusion, there have been no ideas comparable in beauty and conceptual significance with that of the stellarator.”

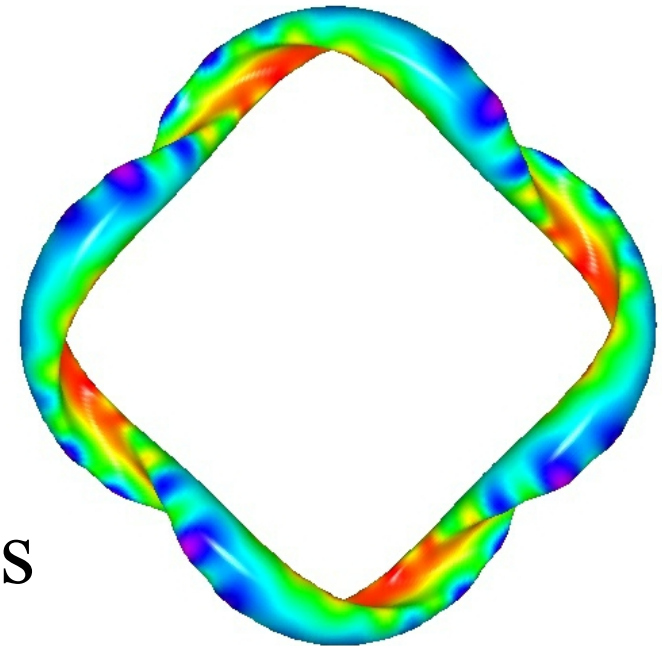
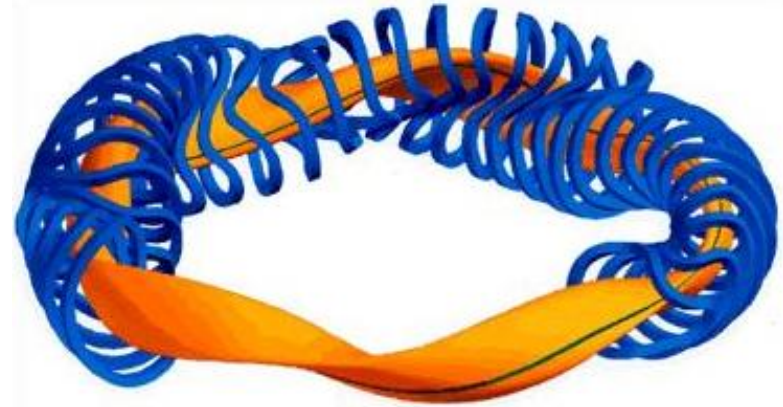
(V.D. Shafranov, 1980)



Outline

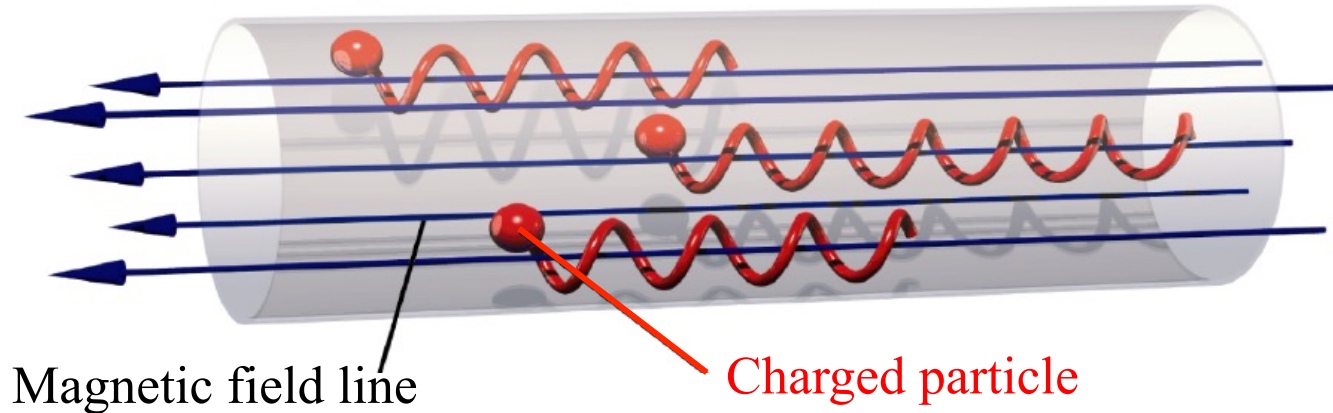
A stellarator is a *nonaxisymmetric* toroidal plasma

- Review of need for “rotational transform”
- Pros & cons vs. tokamaks
- Design principles
- History, modern experiments



Confining charged particles with a magnetic field is tricky.

Uniform straight \mathbf{B} : confinement \perp to \mathbf{B} , but end losses.



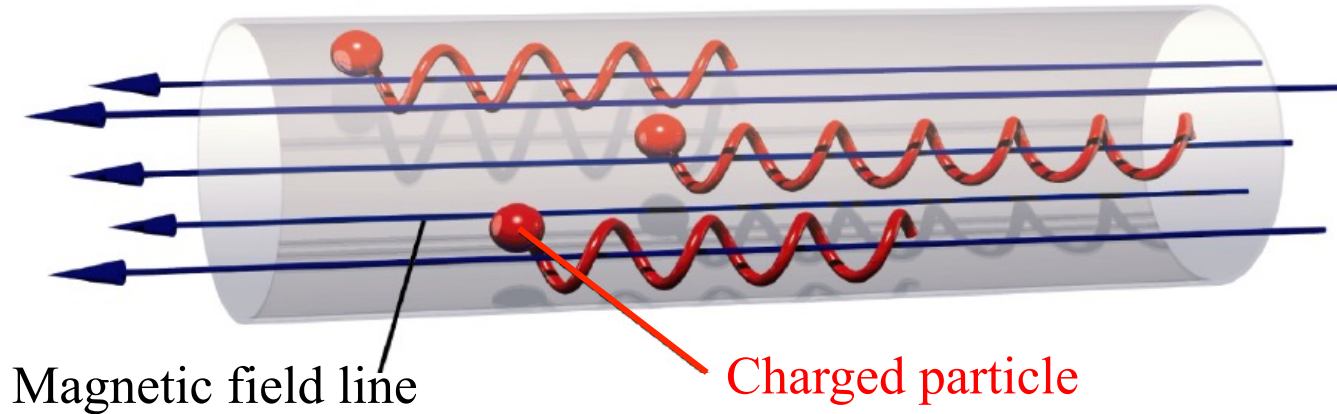
Magnetic fields are the best insulators we know –

Can support 1000x the temperature gradient of space shuttle tiles!

But, there is no confinement *along* the field...

Confining charged particles with a magnetic field is tricky.

Uniform straight \mathbf{B} : confinement \perp to \mathbf{B} , but end losses.

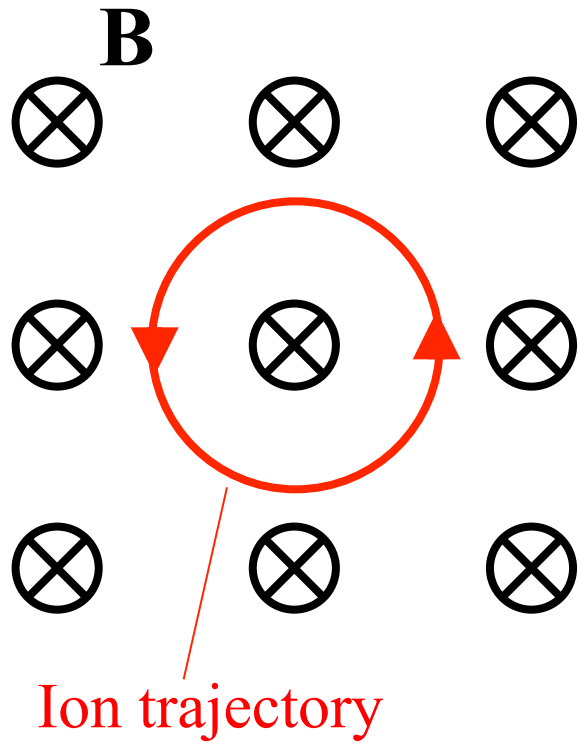


But if field lines are bent, particles drift off them due to *cross-field drifts*.



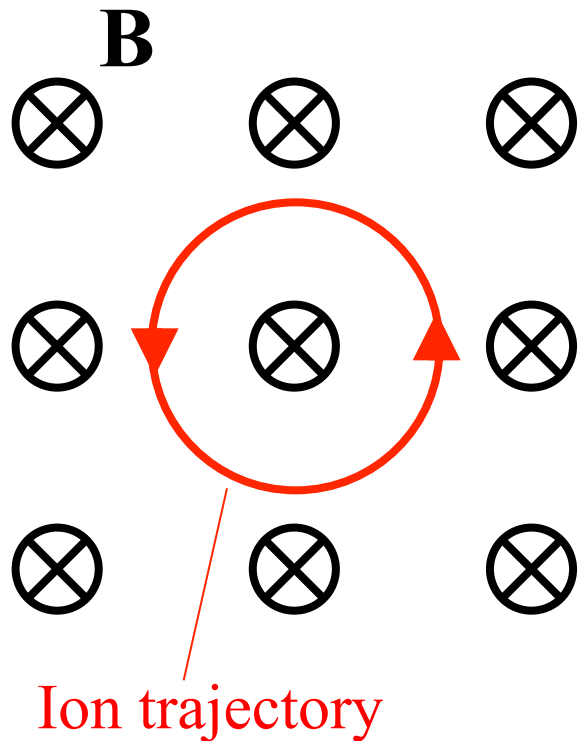
Cross-field drifts

Uniform \mathbf{B} :

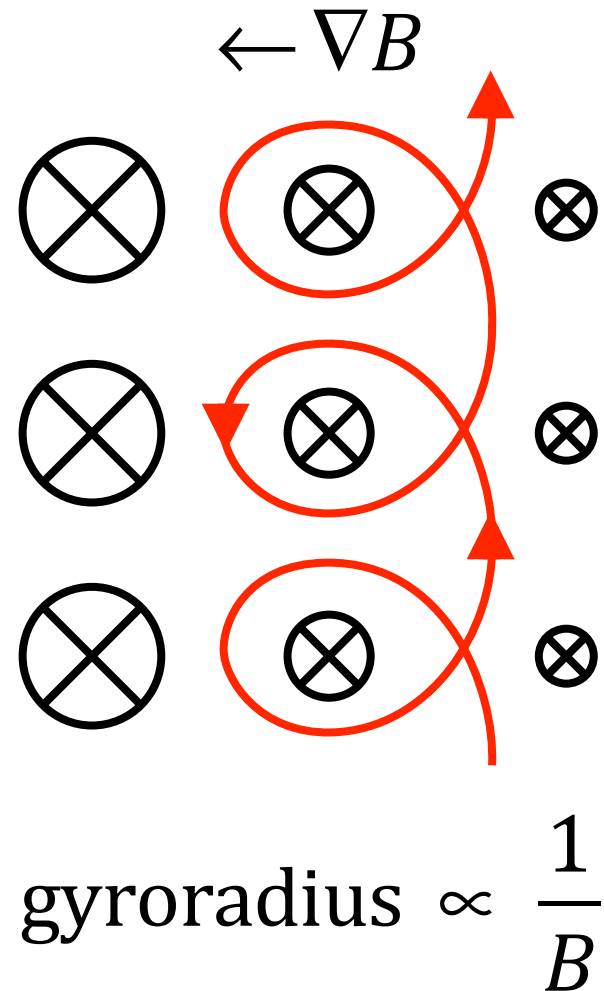


Cross-field drifts

Uniform \mathbf{B} :

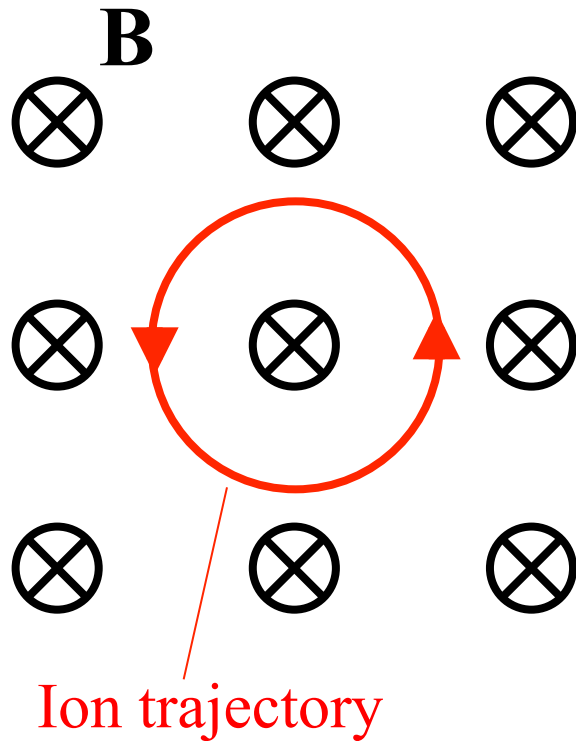


Non-uniform \mathbf{B} : suppose $\nabla B \neq 0$

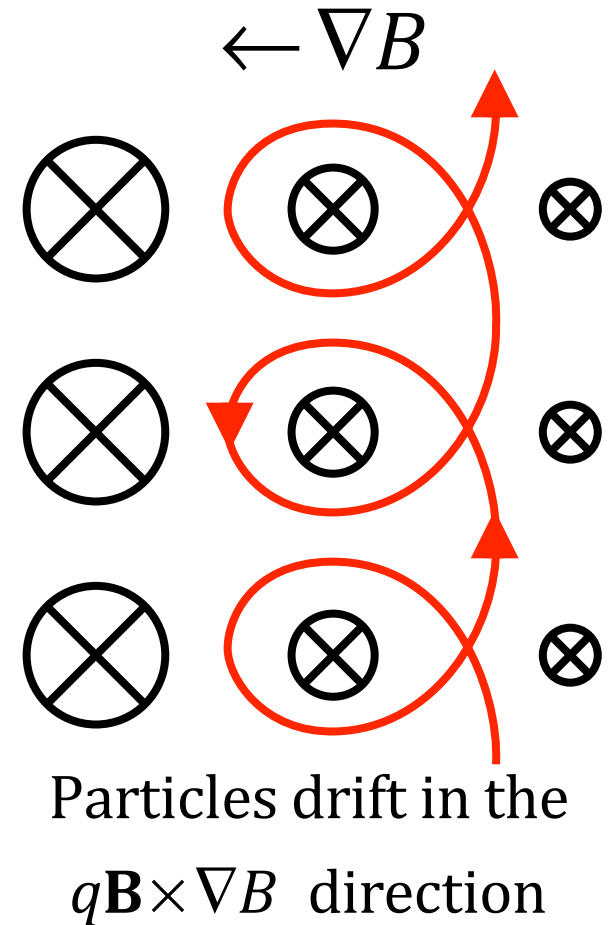


Cross-field drifts

Uniform \mathbf{B} :



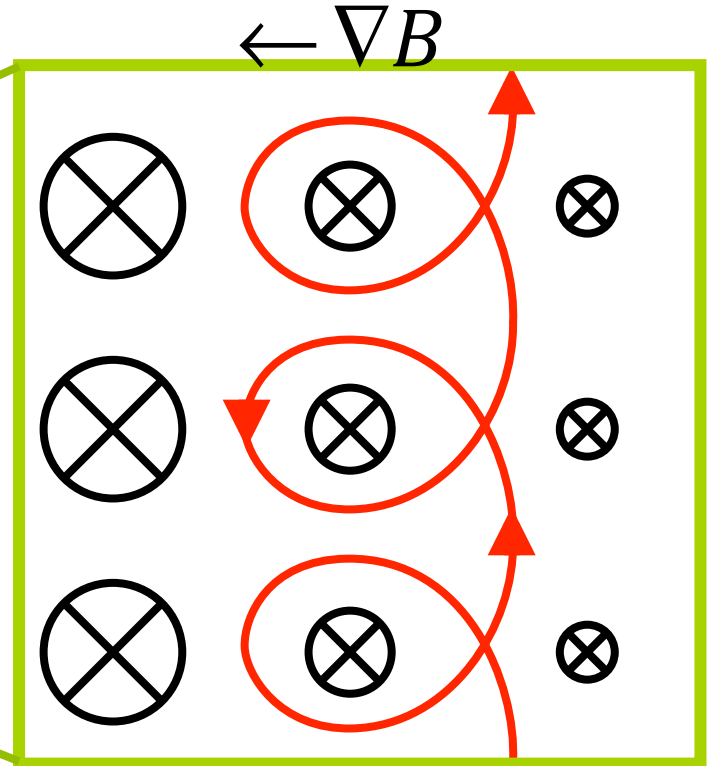
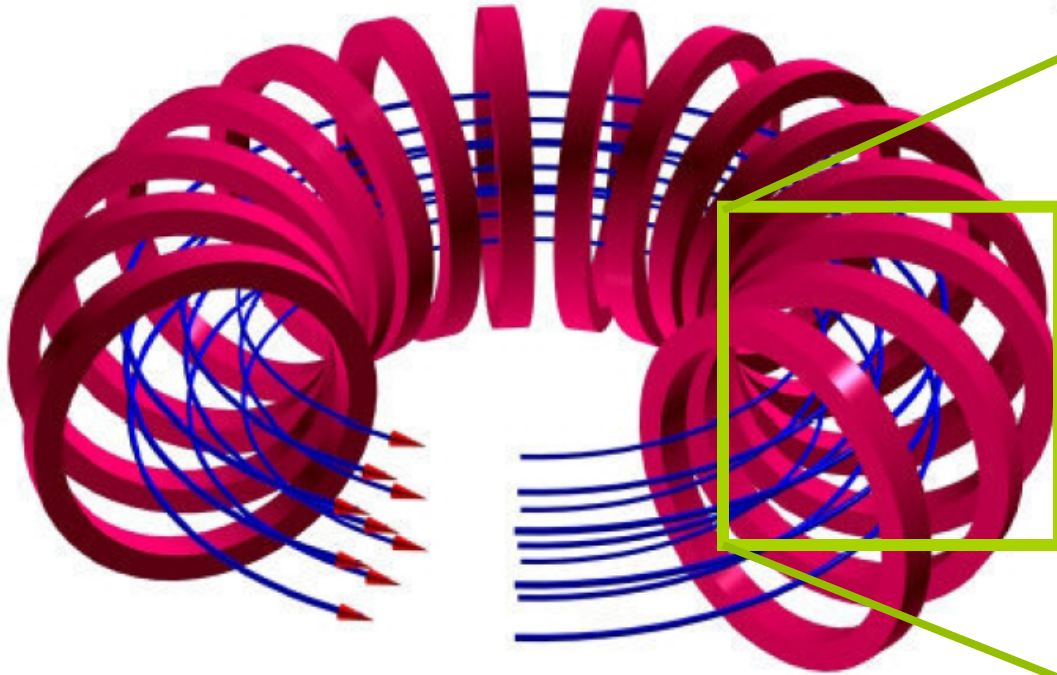
Non-uniform \mathbf{B} : suppose $\nabla B \neq 0$



Other cross-field drifts arise due to field line curvature and the \mathbf{E} from charge separation.

A purely toroidal field does not confine particles.

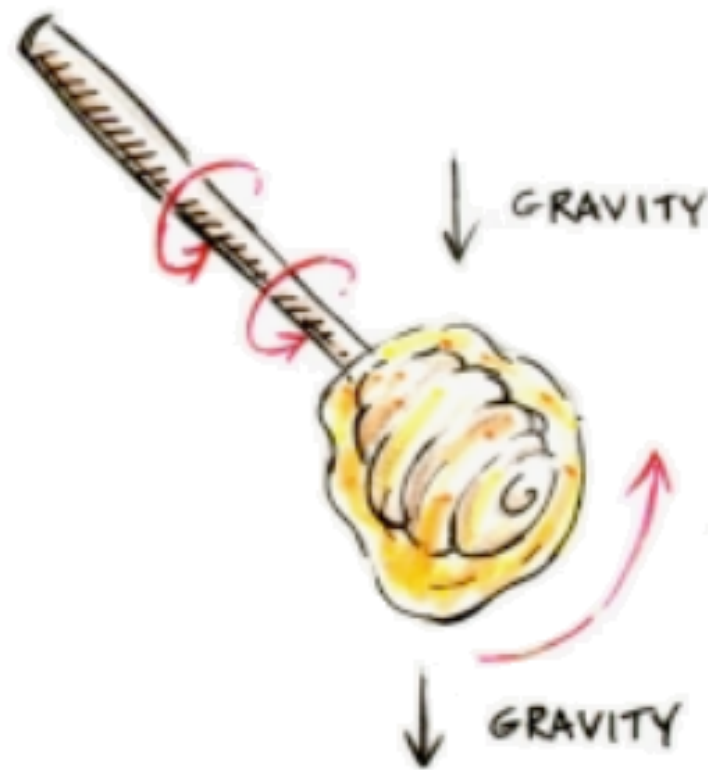
Ampere's Law: $B = \frac{\mu_0 I}{2\pi R}$ so B is larger on the inside



Ions drift up:
they are not confined!

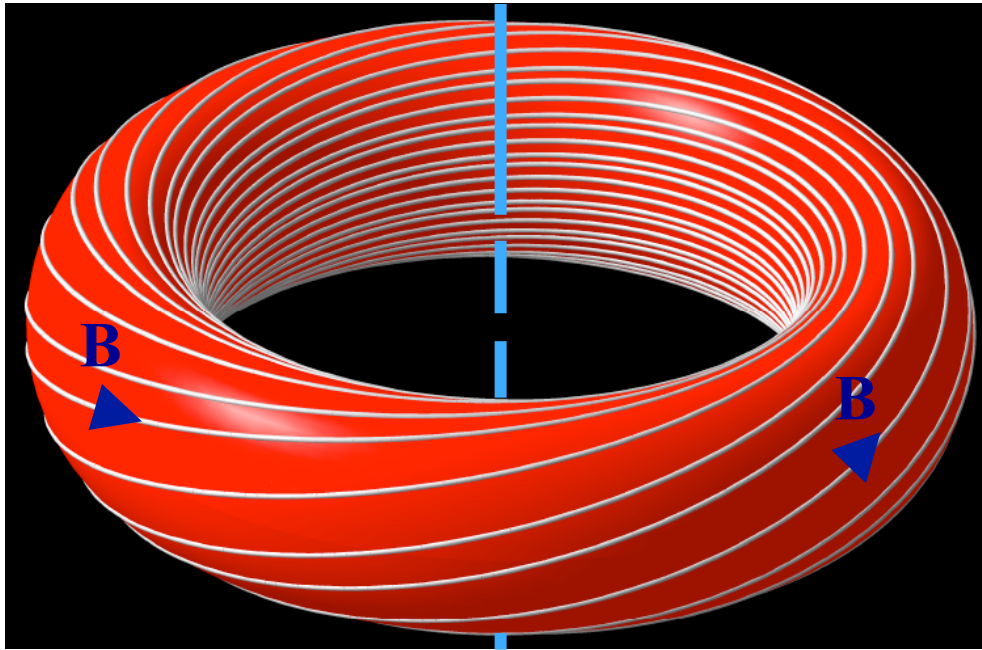
Particles drift in the
 $q\mathbf{B} \times \nabla B$ direction

How can we resolve the problem of the cross-field drifts?



*Picture from
G Hammett*

How does the tokamak resolve the problem of the cross-field drifts?

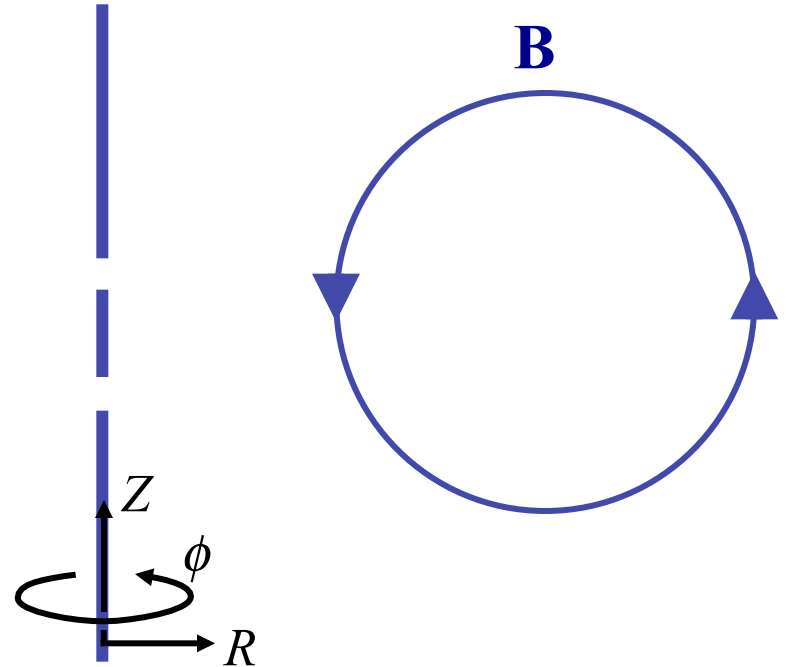
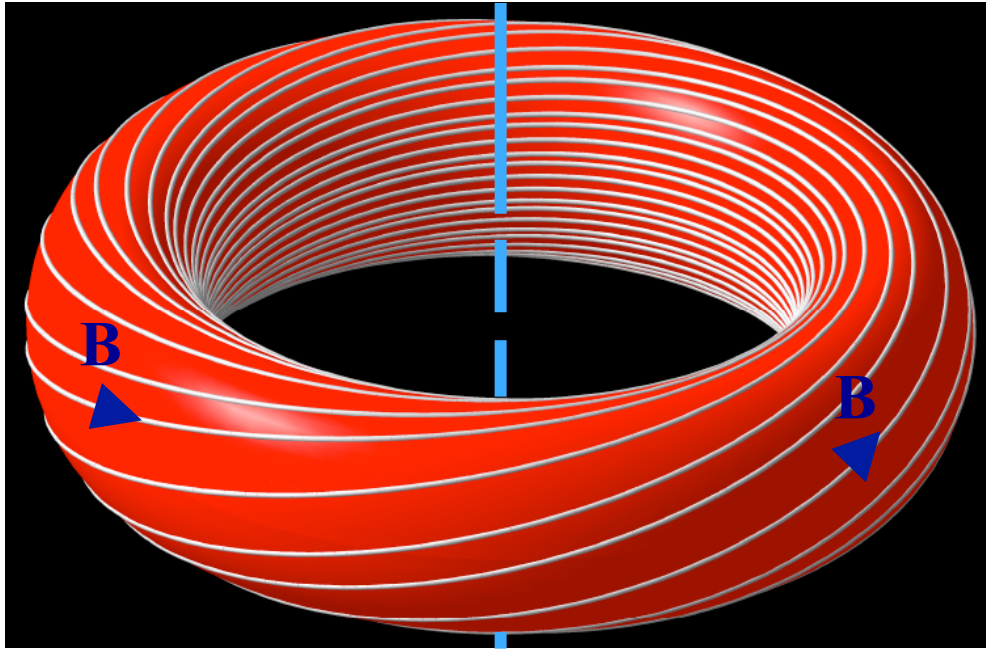


By making the field lines helical rather than toroidal.

“**Rotational transform**”: If you follow a magnetic field line around the torus once toroidally (i.e. the long way around), you come back to a different place the short way around.

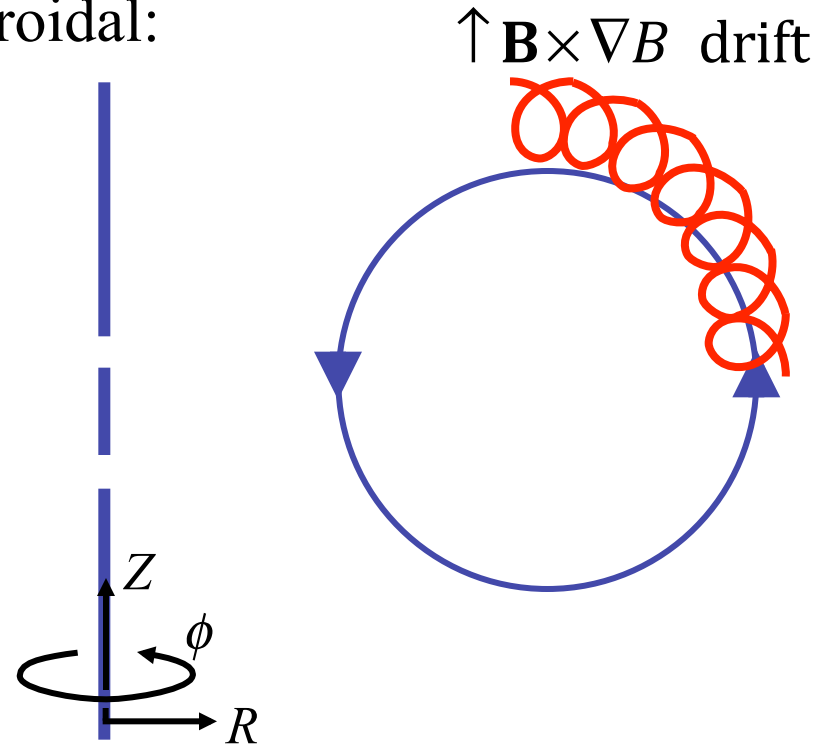
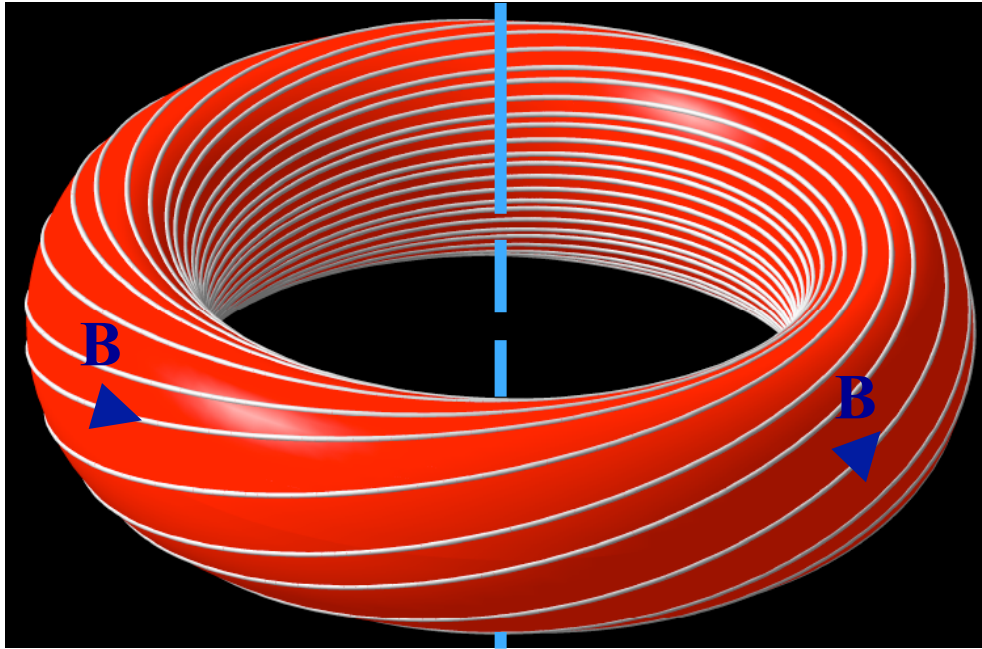
The tokamak

Make the field lines helical instead of toroidal:



The tokamak

Make the field lines helical instead of toroidal:

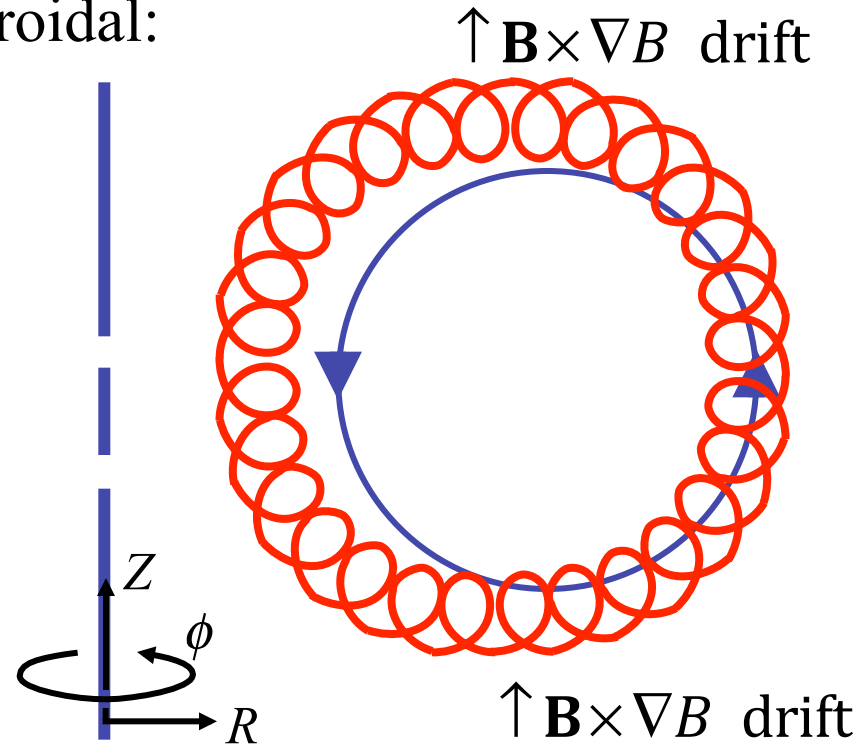
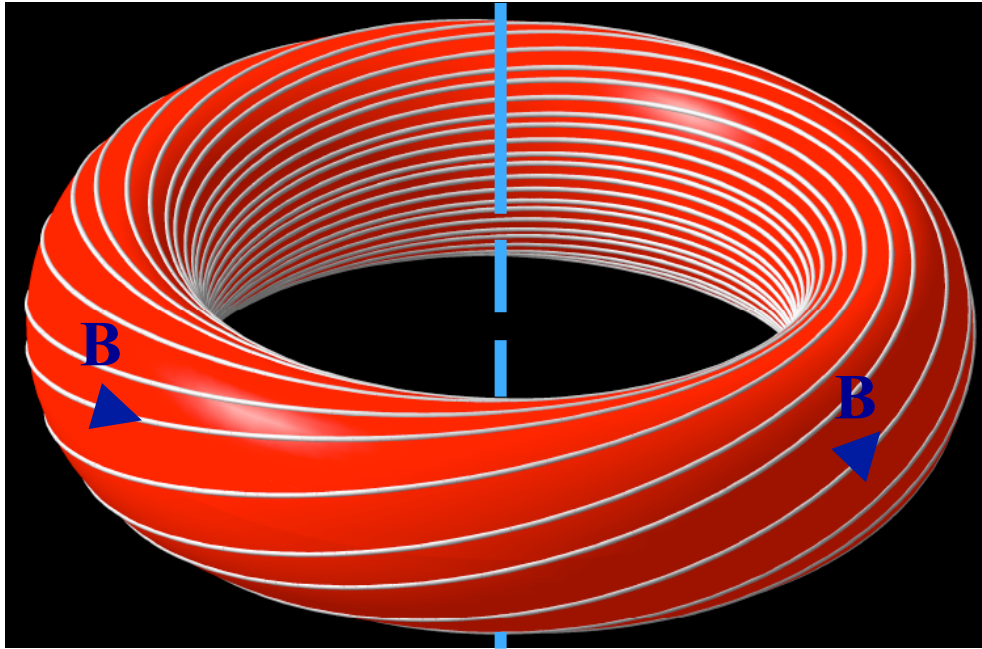


The upward cross-field drift is inward half the time and outward half the time, averaging to 0.

Now all particles are confined!

The tokamak

Make the field lines helical instead of toroidal:

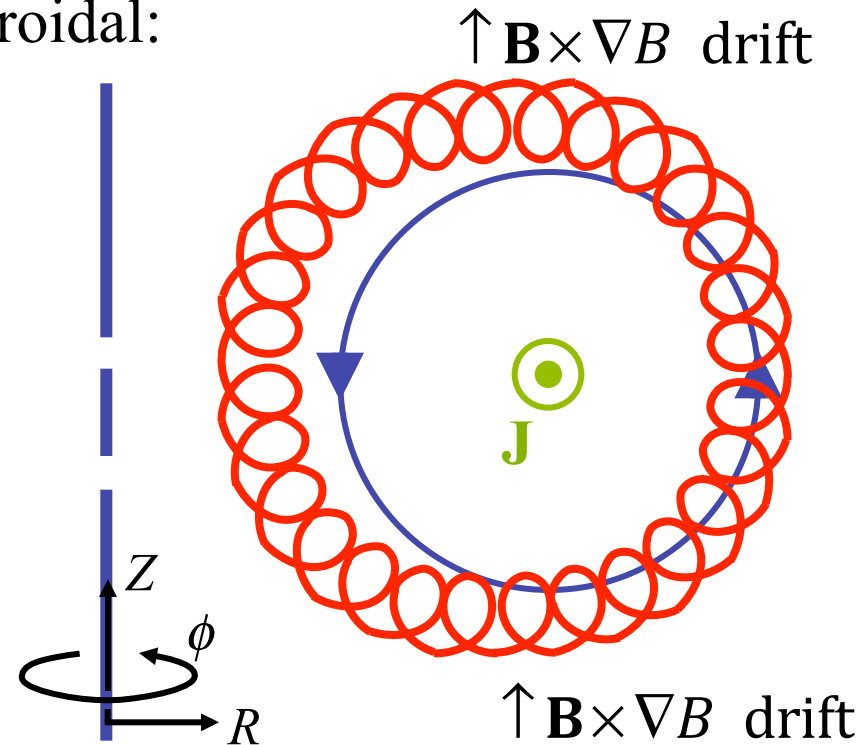
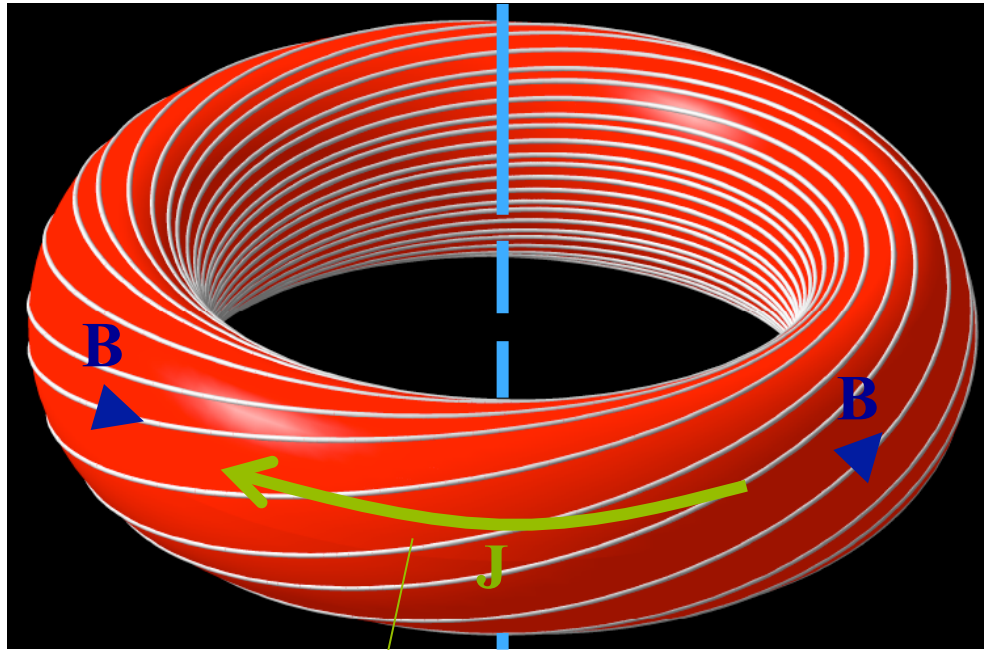


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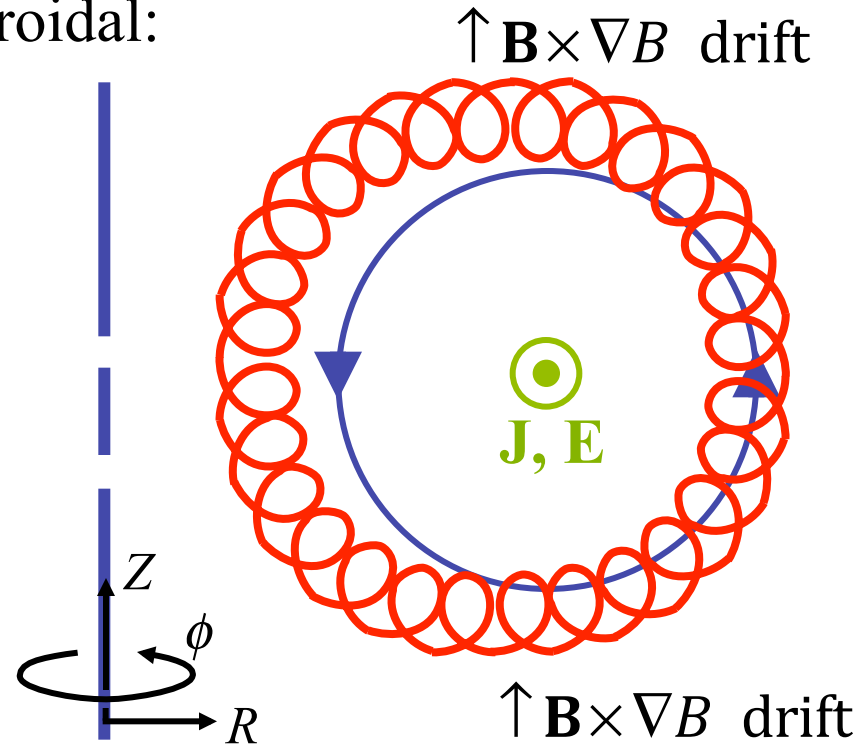
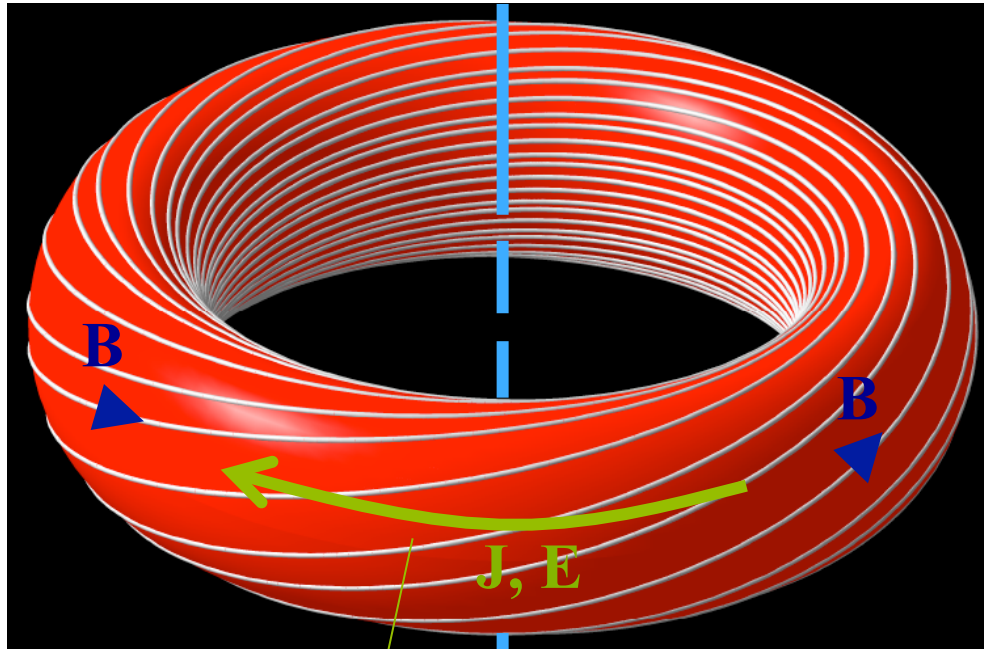
$\nabla \times \mathbf{B} = \mu_0 \mathbf{J}$, so a large current \mathbf{J} is required inside the plasma.

The upward cross-field drift is inward half the time and outward half the time, averaging to 0.

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The tokamak

Make the field lines helical instead of toroidal:

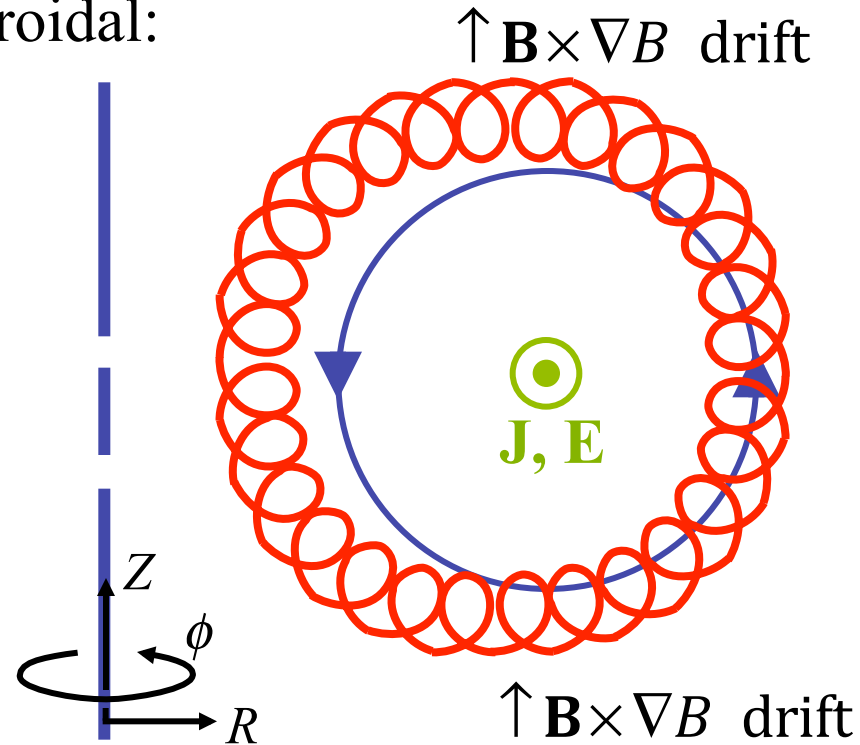
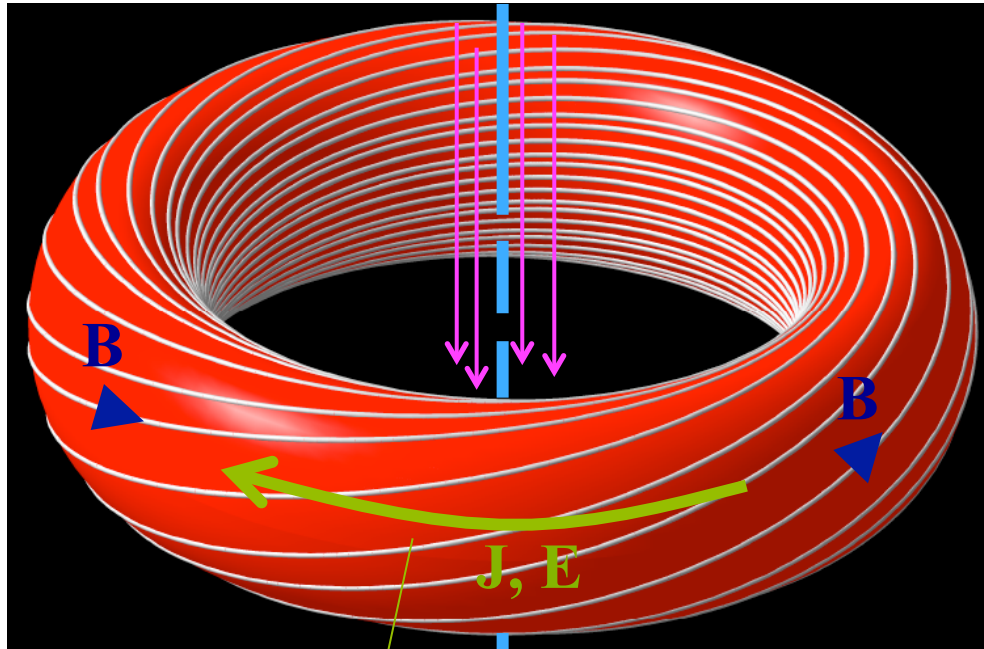


$\nabla \times \mathbf{B} = \mu_0 \mathbf{J}$, so a large current \mathbf{J} is required inside the plasma.

To make the current, you could use an electric field \mathbf{E} .

The tokamak

Make the field lines helical instead of toroidal:



$\nabla \times \mathbf{B} = \mu_0 \mathbf{J}$, so a large current \mathbf{J} is required inside the plasma.

To make the current, you could use an electric field \mathbf{E} .

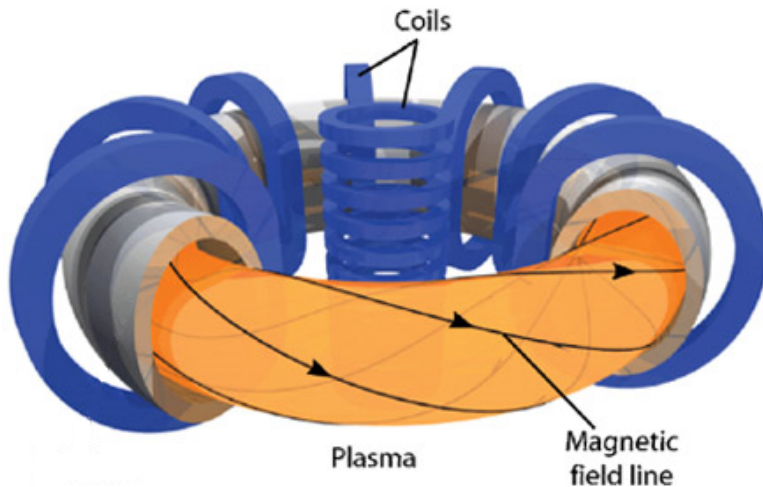
Make \mathbf{E} inductively using a transformer: $\oint \mathbf{E} \cdot d\mathbf{l} = -\frac{d\Phi_{\mathbf{B}}}{dt}$

Flux of magnetic field through the doughnut hole

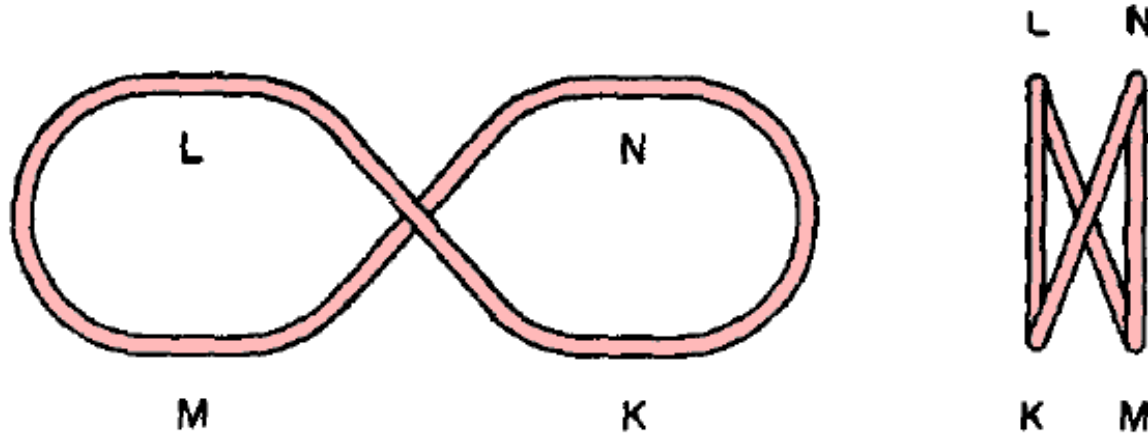
Therefore tokamaks are not naturally steady-state.*

Tokamaks

- Axisymmetric
- Large current required in the plasma.
 - Not naturally steady-state.
 - Probably needs current drive.
 - Can disrupt.

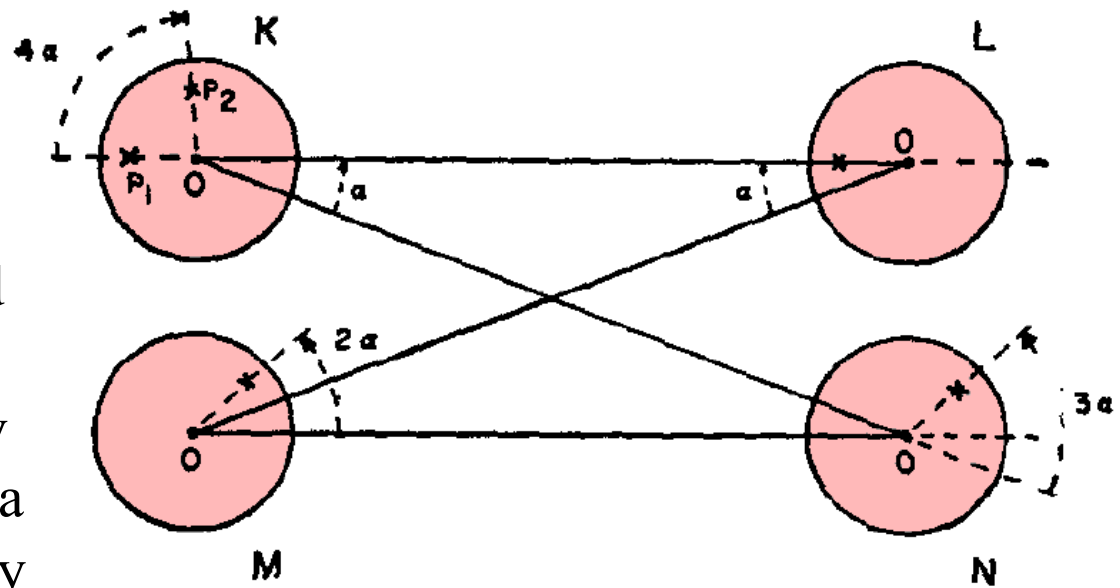


In a stellarator, rotational transform is made by breaking axisymmetry

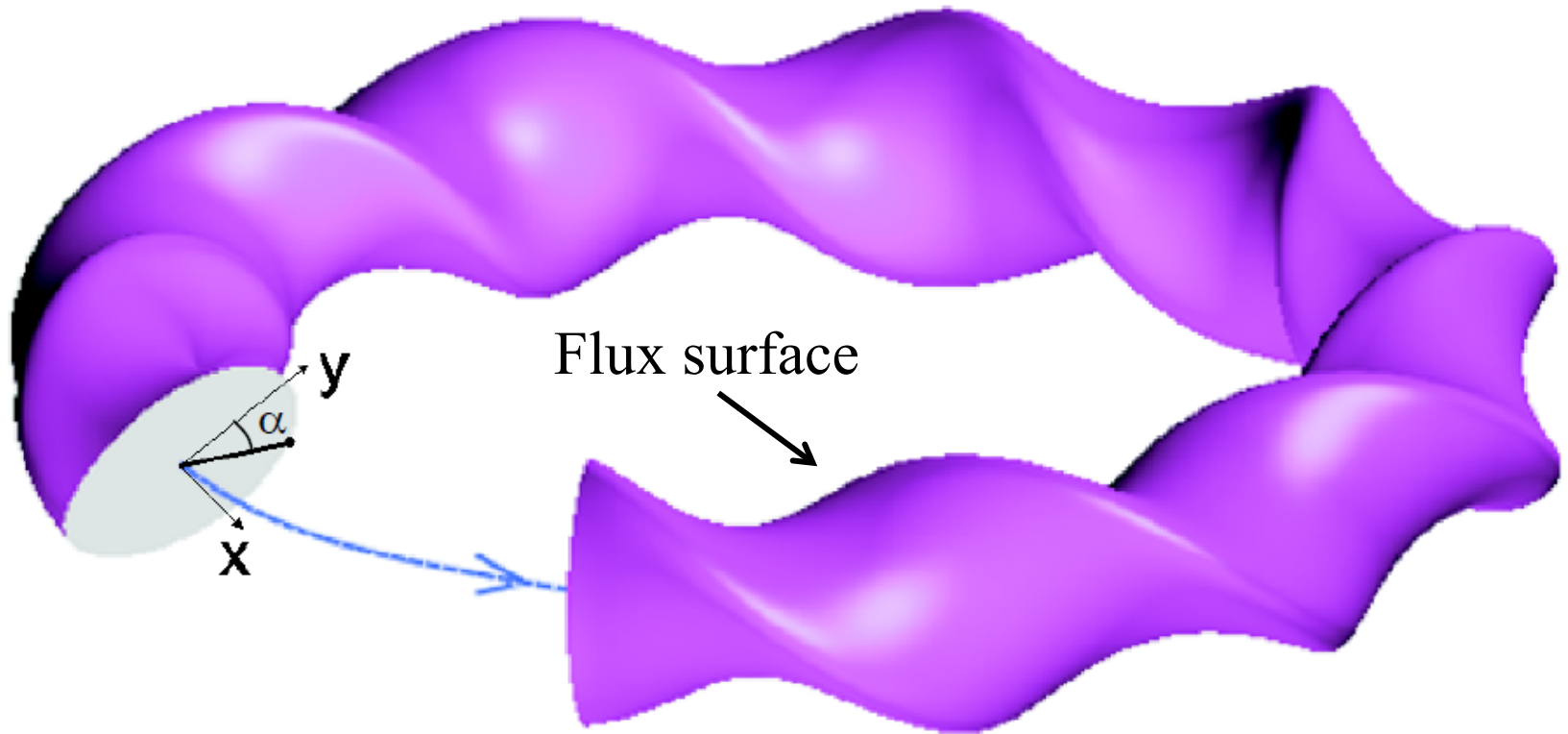


Lyman Spitzer,
Phys. Fluids 1, 253 (1958)

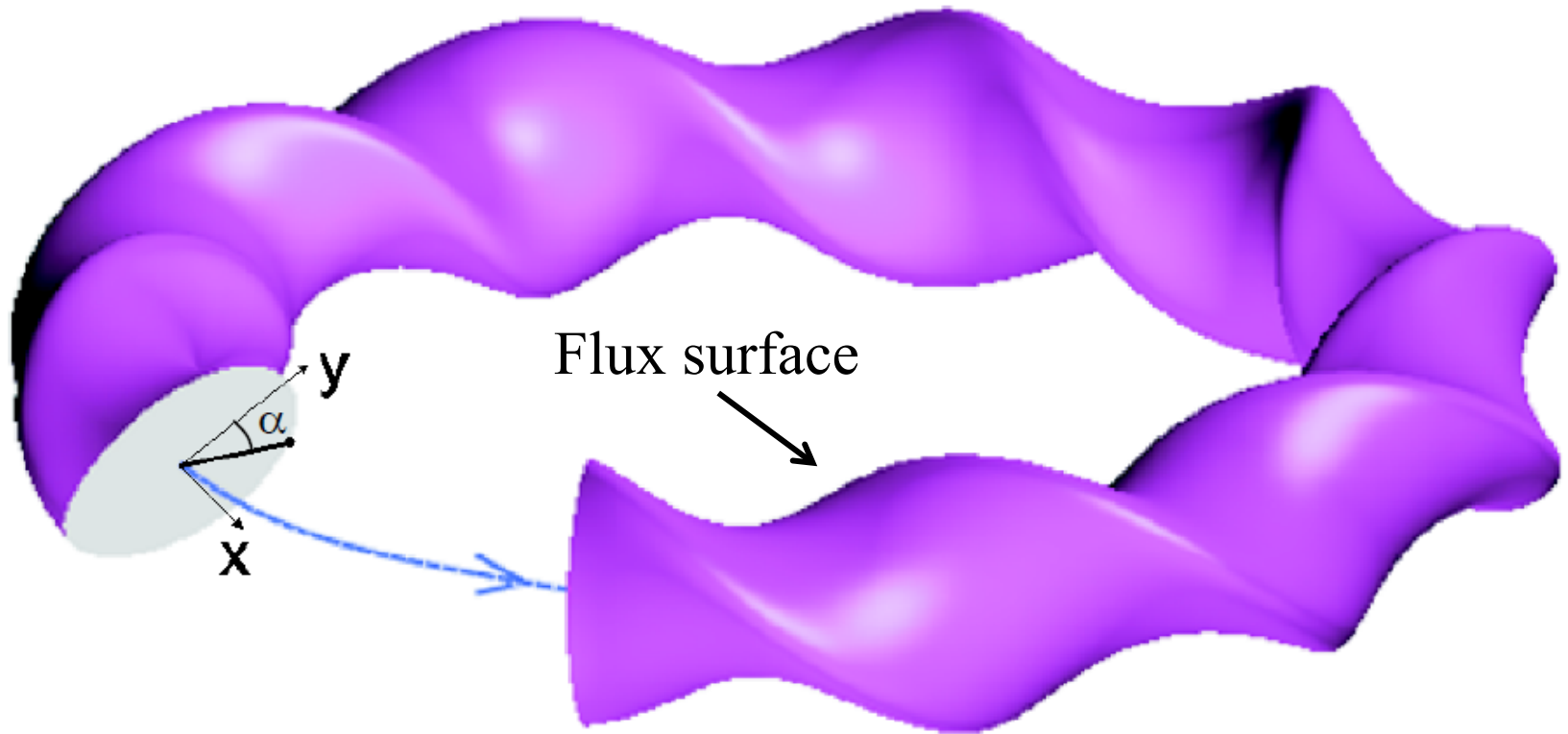
“**Rotational transform**”: If you follow a magnetic field line around the torus once toroidally (i.e. the long way around), you come back to a different place the short way around.



Rotational transform can also be produced by a rotating ellipticity



Rotational transform can also be produced by a rotating ellipticity



Equations for a magnetic equilibrium looks just like the equations for steady fluid flow:

$$(\nabla \times \mathbf{B}) \times \mathbf{B} = \nabla(\text{stuff})$$

$$(\nabla \times \mathbf{v}) \times \mathbf{v} = \nabla(\text{stuff})$$

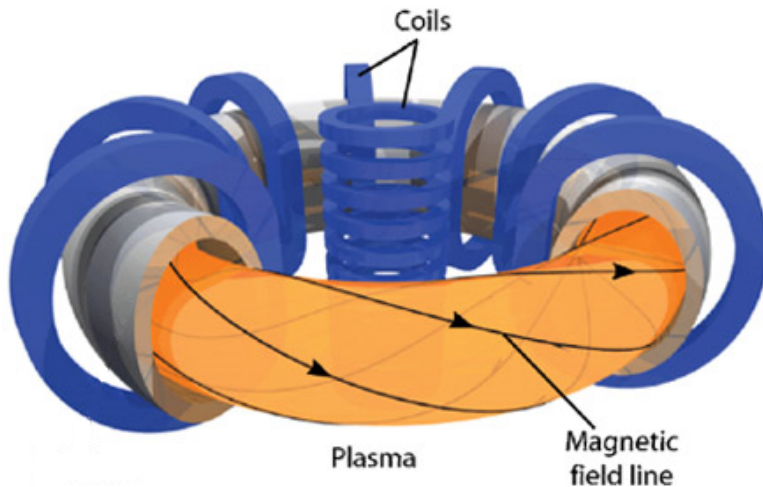
$$\nabla \cdot \mathbf{B} = 0$$

$$\nabla \cdot \mathbf{v} = 0$$

So magnetic field lines act just like streamlines of fluid flow, which here have rotational transform.

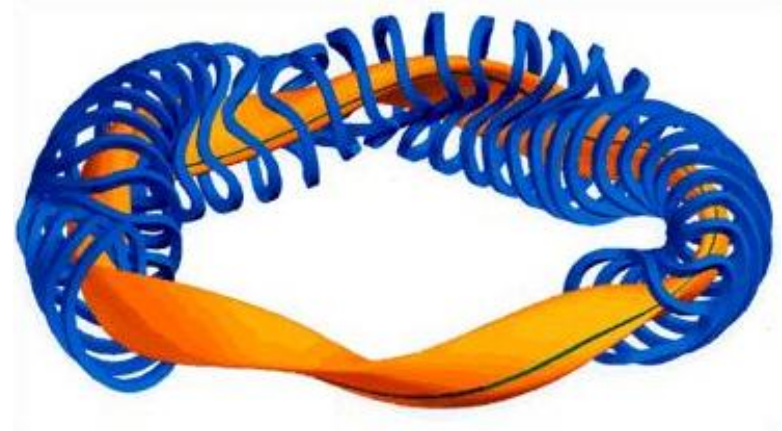
Tokamaks

- Axisymmetric
 - Large current required in the plasma.
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Stellarators

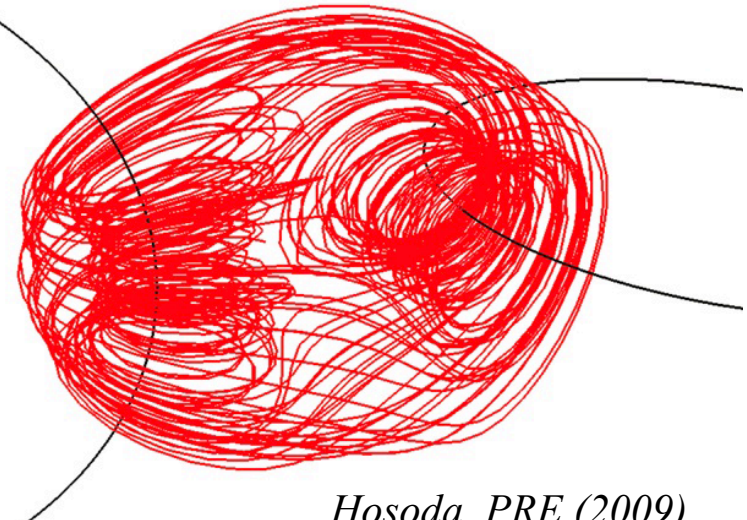
- Nonaxisymmetric
 - + No current required in the plasma.
 - + Steady-state.
 - + Less recirculating power.
 - + No disruptions.



A (solvable) challenge for stellarator design is having field lines lie on surfaces.

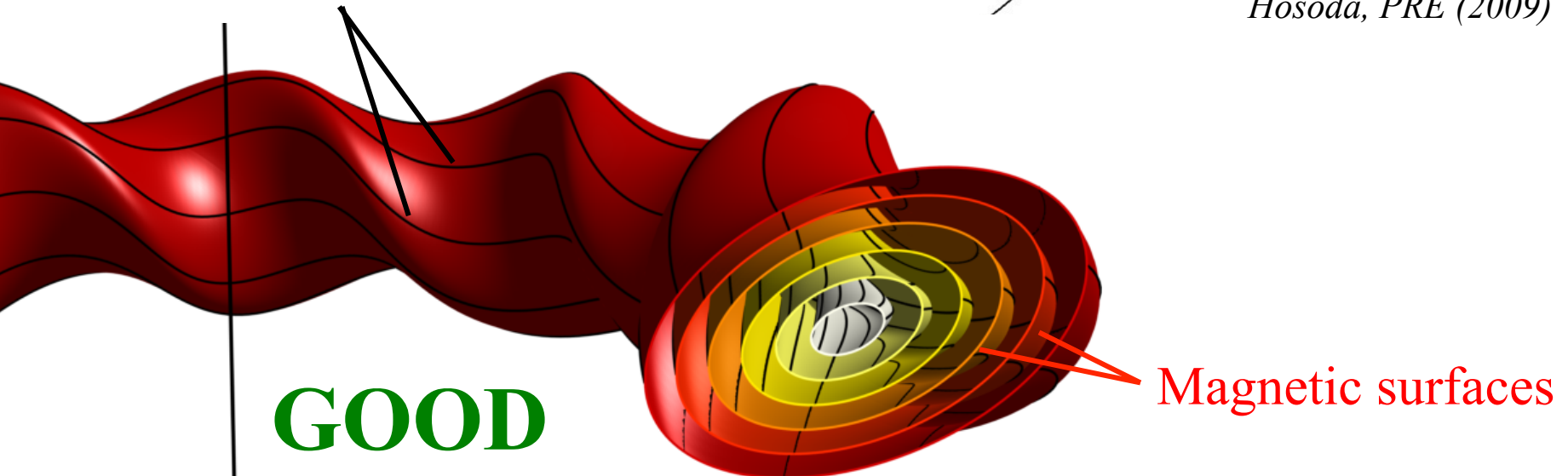
Chaotic (volume-filling) \mathbf{B} field lines would allow inside & outside to mix even without cross- \mathbf{B} drift.

BAD



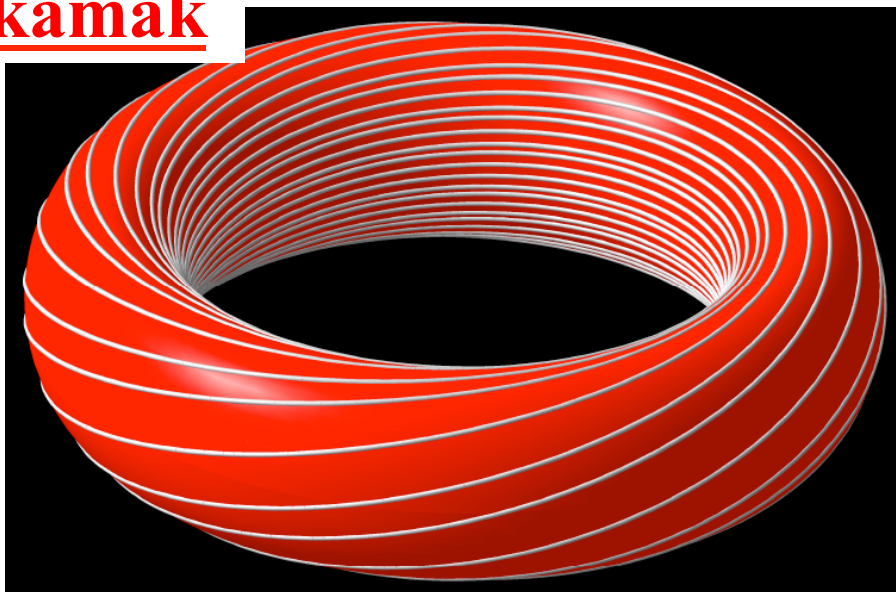
Magnetic field lines

GOOD



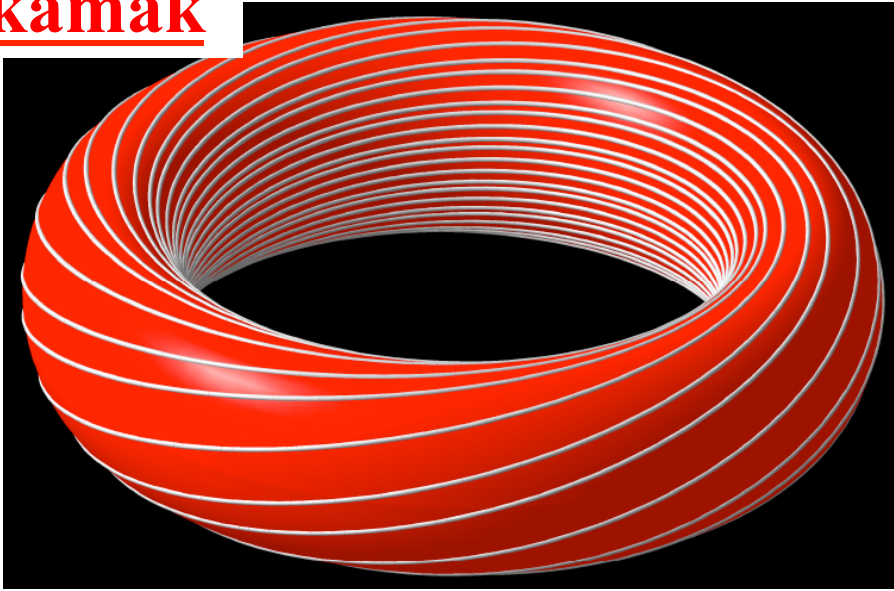
**Tokamaks are guaranteed to have
good magnetic surfaces while stellarators are not**

Tokamak

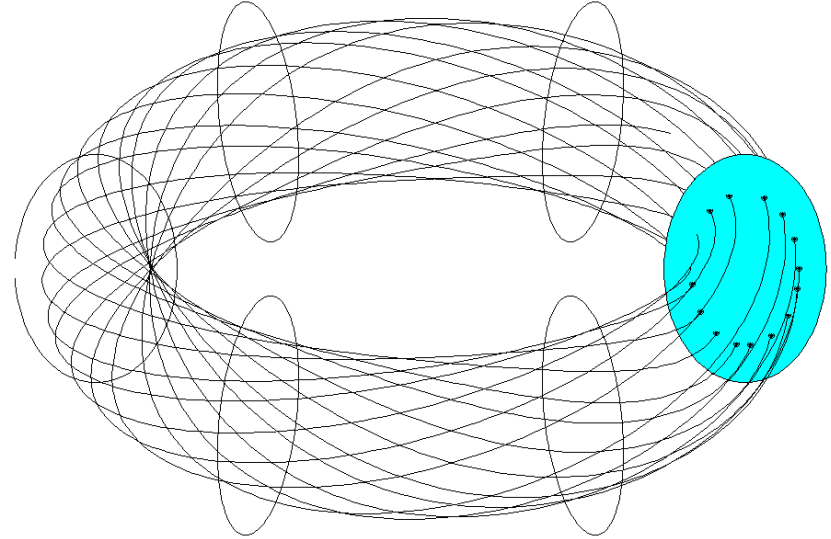


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Tokamak

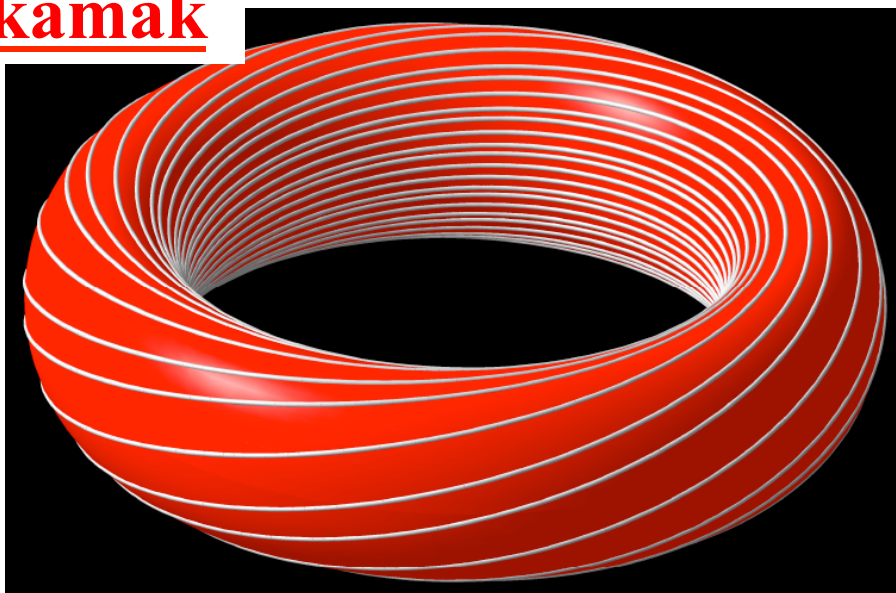


Poincare plot (a.k.a. puncture plot):

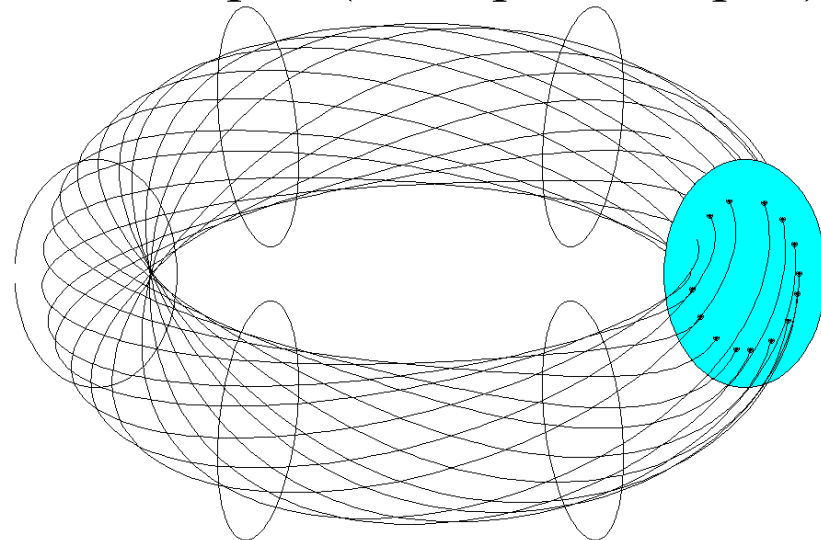


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Tokamak

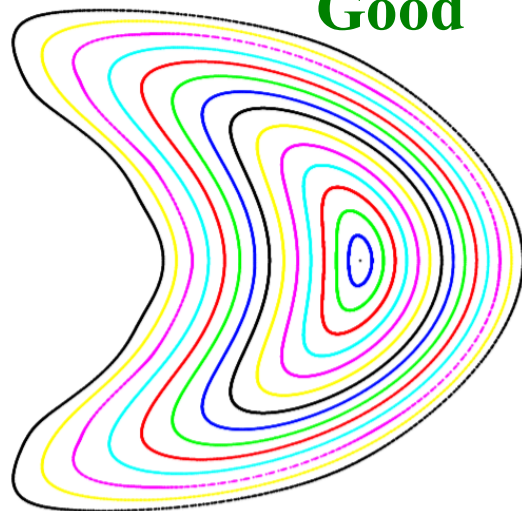


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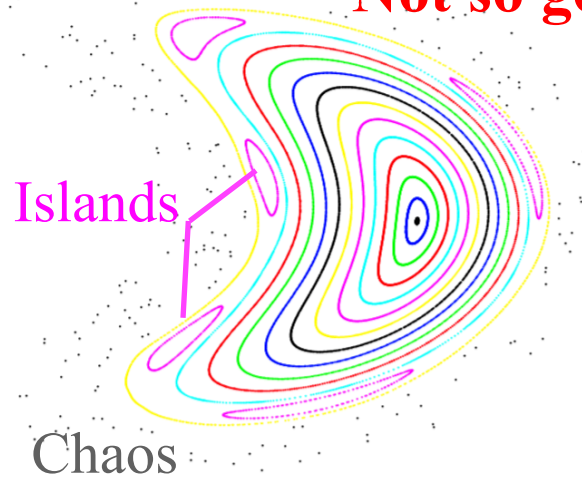
Stellarator

Good



*J P Kremer,
PhD thesis,
Columbia*

Not so good

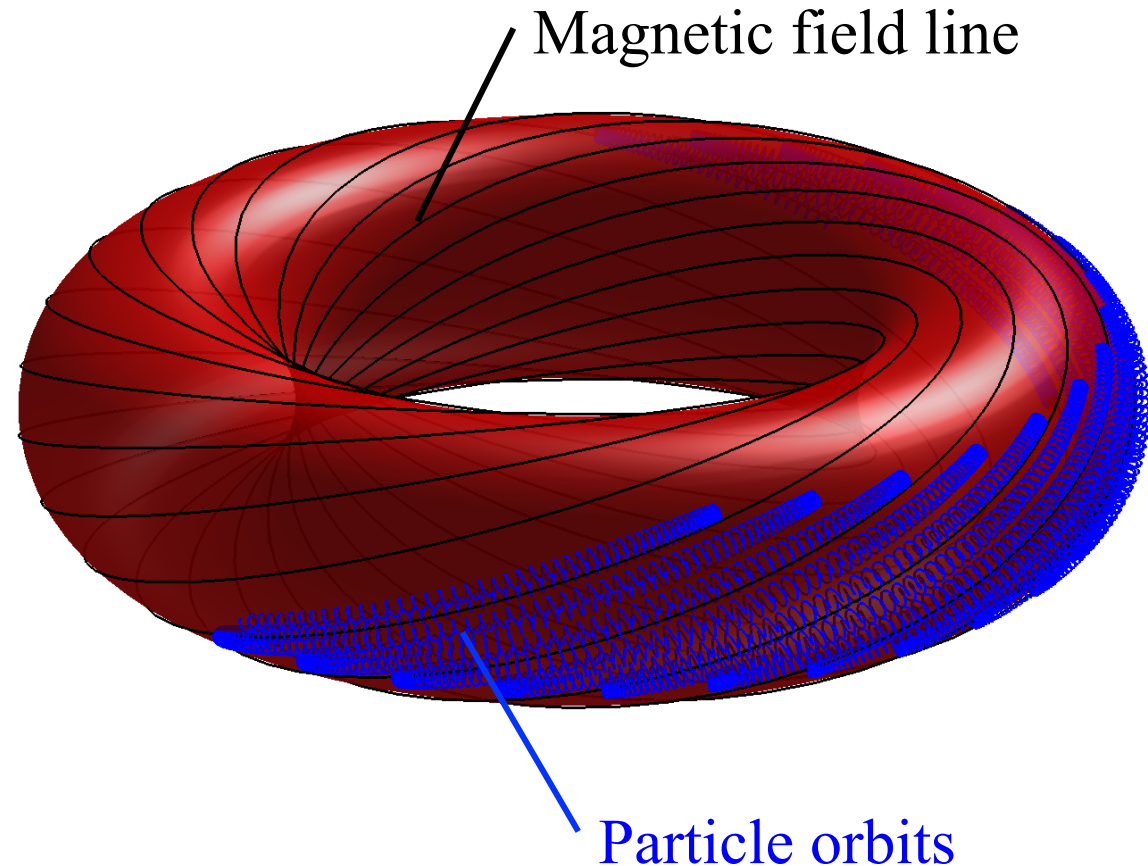


Even with rotational transform, some “trapped” particles may not be confined in stellarators.

Due to conservation of magnetic moment $\mu = \frac{mv_{\perp}^2}{2B}$,

some particles are trapped in regions of small $B = |\mathbf{B}|$.

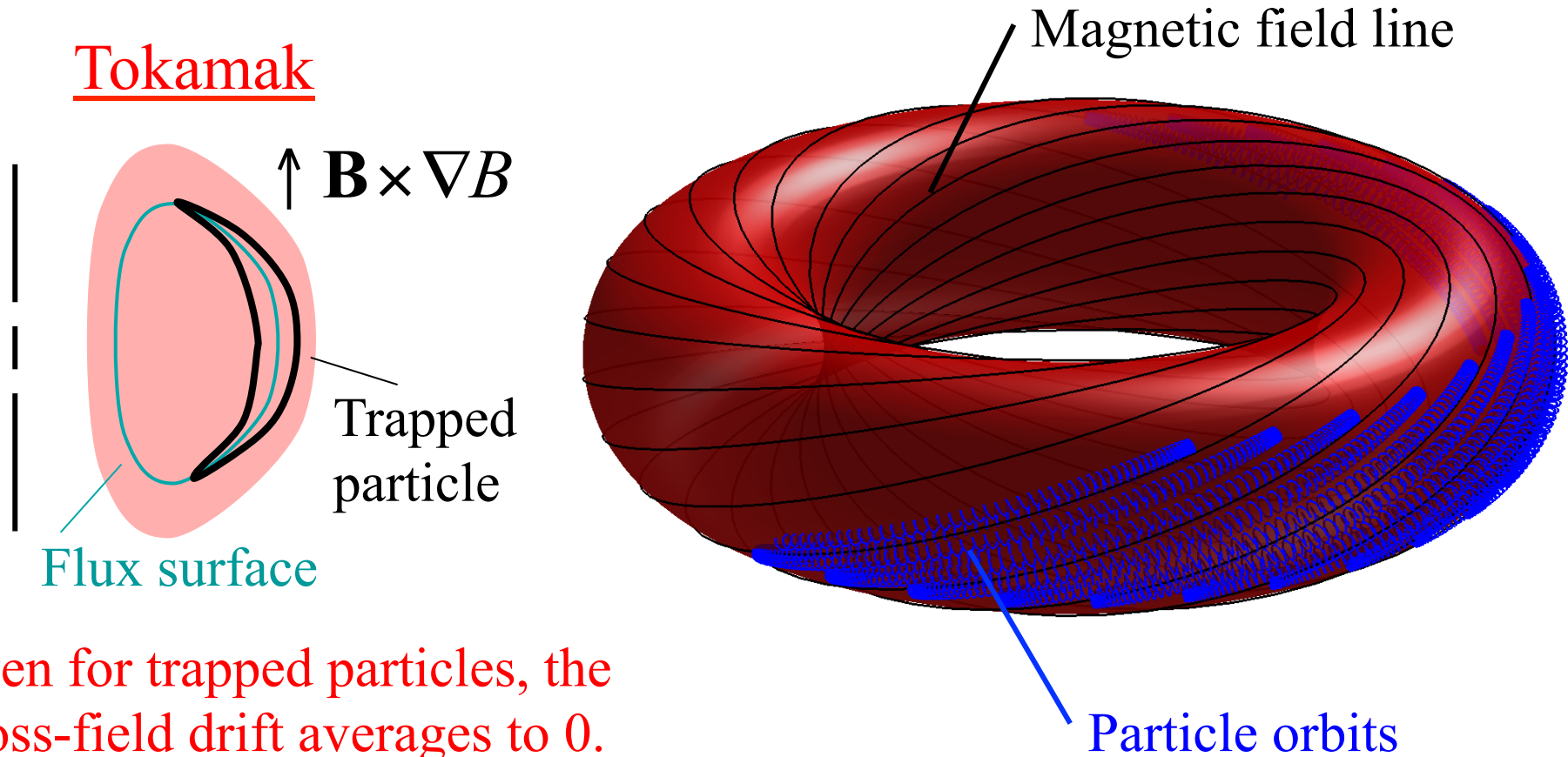
Tokamak



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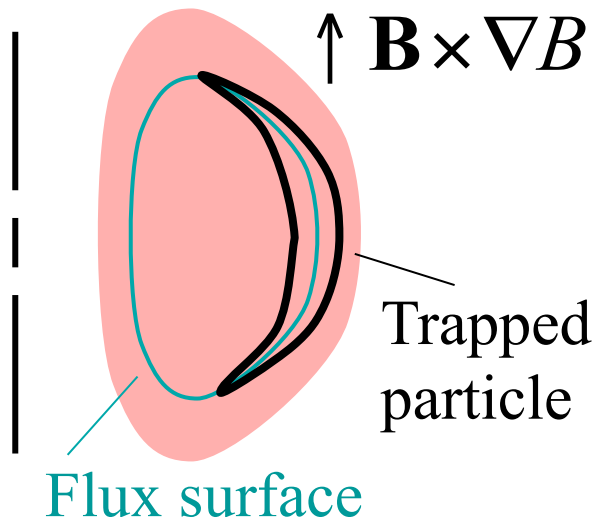


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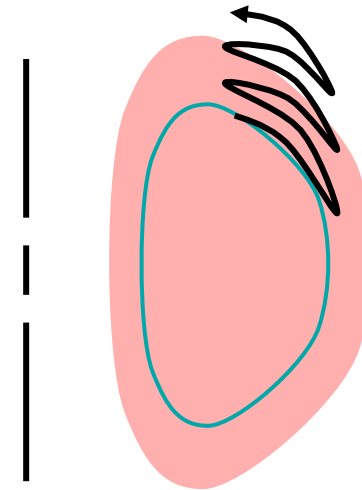
some particles are trapped in regions of small $B = |\mathbf{B}|$.

Tokamak



Even for trapped particles, the cross-field drift averages to 0.

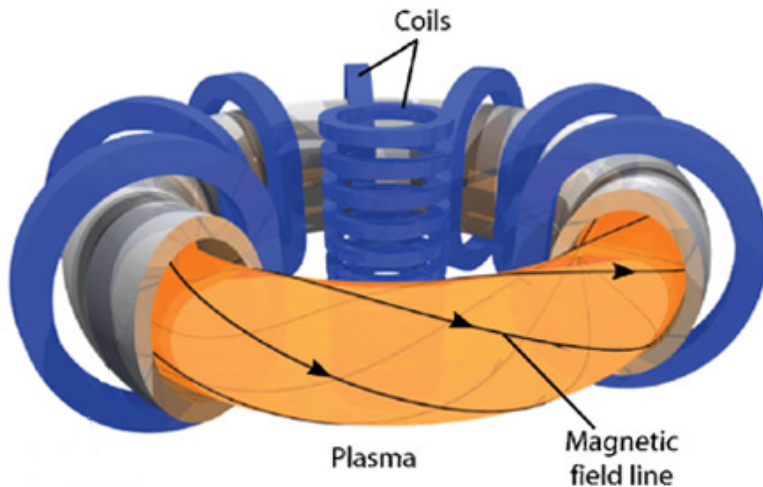
Stellarator



- Trapped particles don't “see” the rotational transform, so they may have nonzero average drift.
- Can be minimized by shaping.

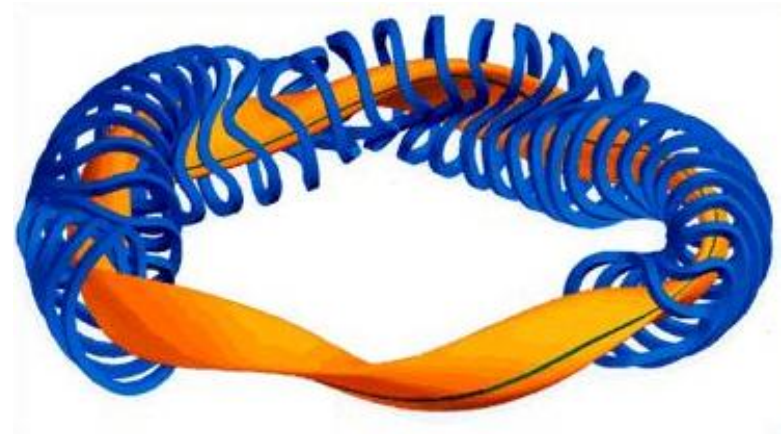
Tokamaks

- Axisymmetric
 - Large current required in the plasma.
 - Not naturally steady-state.
 - Probably needs current drive.
 - Can disrupt.



Stellarators

- Nonaxisymmetric
 - + No current required in the plasma.
 - + Steady-state.
 - + Less recirculating power.
 - + No disruptions.
 - More complicated to construct.
 - Higher heat & particle transport.

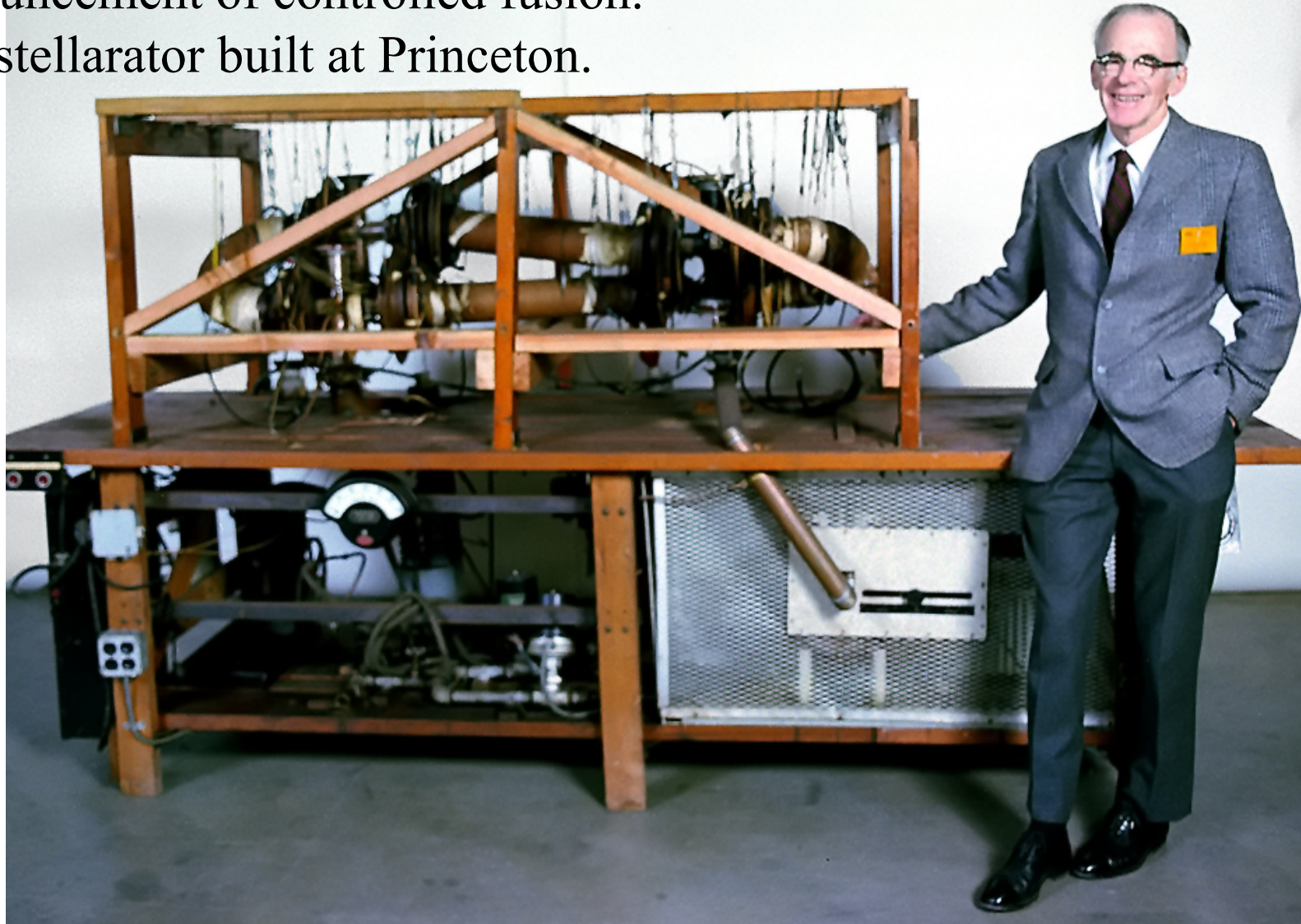


History and modern experiments

Stellarators pre-dated tokamaks...

1951: Lyman Spitzer proposes the stellarator, inspired by Argentina's announcement of controlled fusion.

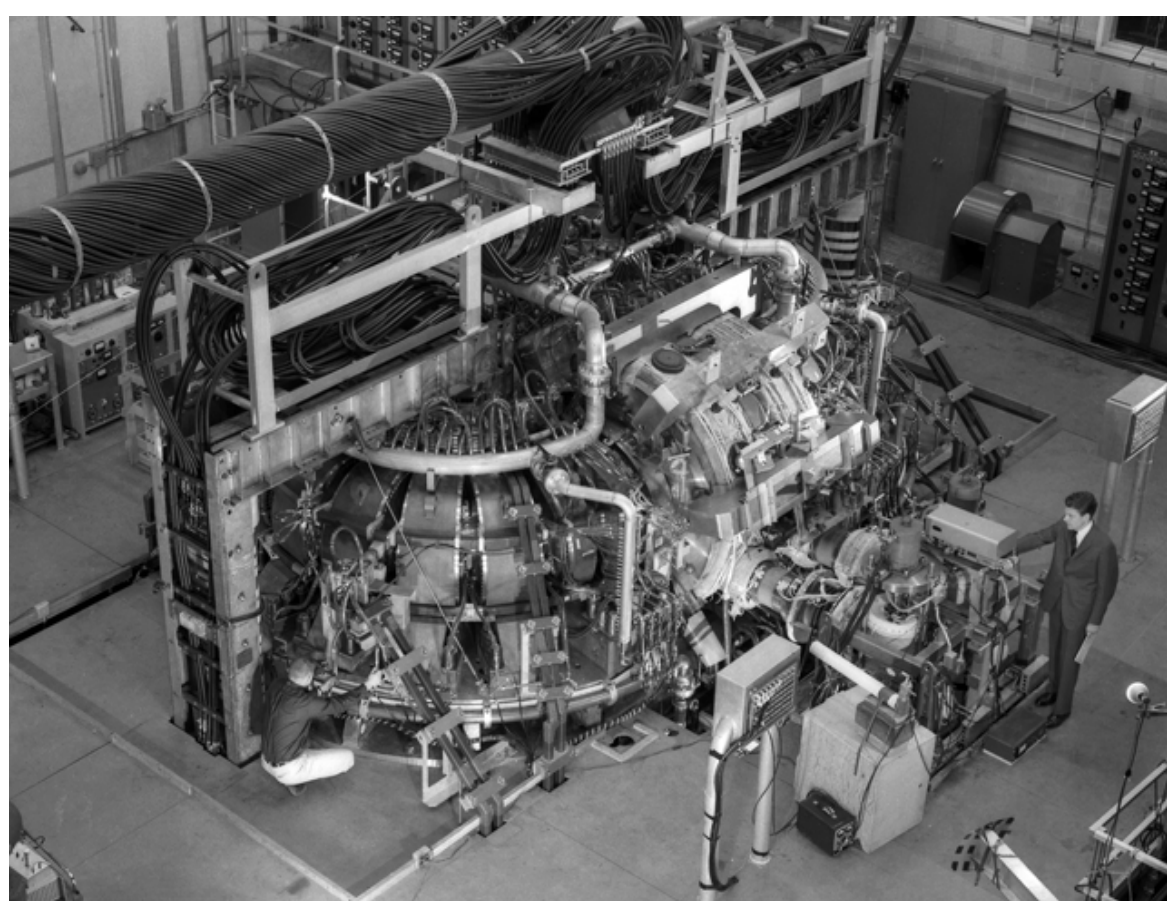
1953: First stellarator built at Princeton.



... but tokamaks left ahead

1969: Russian T-3 tokamak reports high temperatures (>100 eV),
English team confirms temperature.

Princeton C Stellarator promptly converted into a tokamak.



*(Reprinted from Nature, Vol. 224, No. 5218, pp. 488-490,
November 1, 1969)*

Measurement of the Electron Temperature by Thomson Scattering in Tokamak T3

Electron temperatures of 100 eV up to 1 keV and densities in the range $1-3 \times 10^{13}$ cm⁻³ have been measured by Thomson scattering on Tokamak T3. These results agree with those obtained by other techniques where direct comparison has been possible

by

N. J. PEACOCK, D. C. ROBINSON, M. J. FORREST

and

P. D. WILCOCK

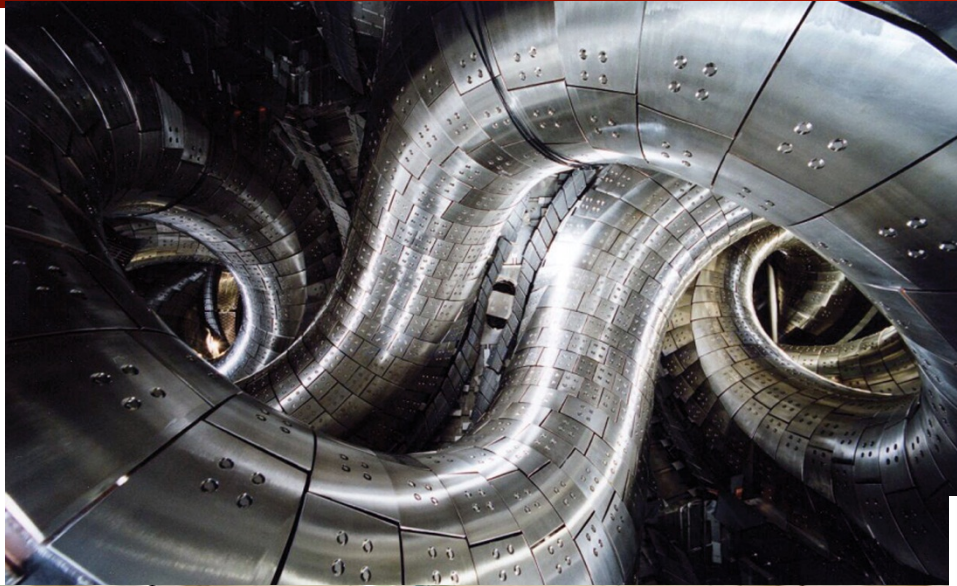
UKAEA Research Group, Culham Laboratory, Abingdon,
Berkshire

and

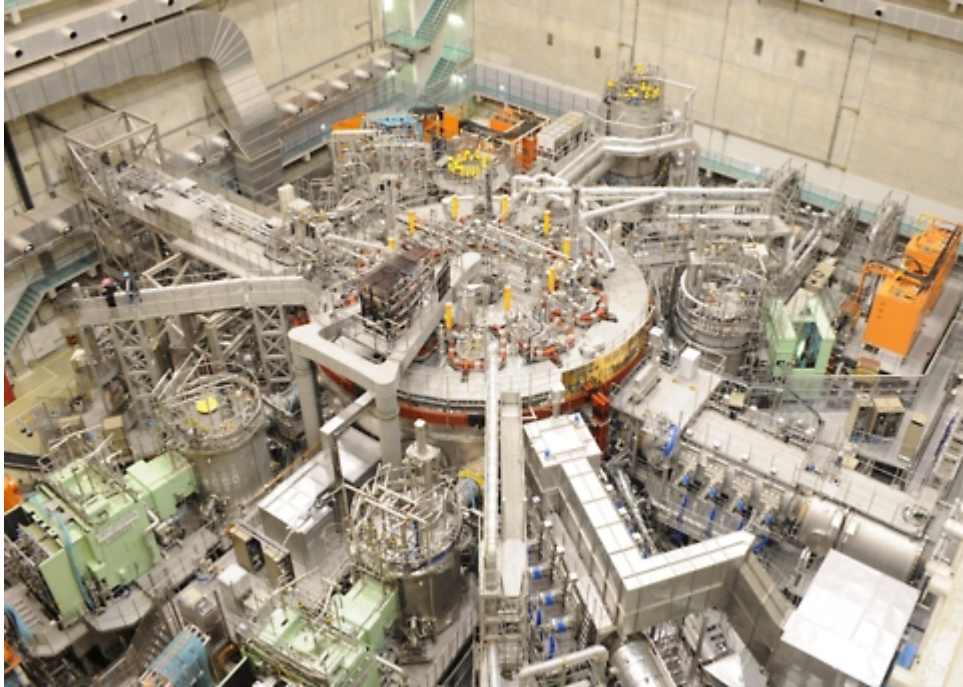
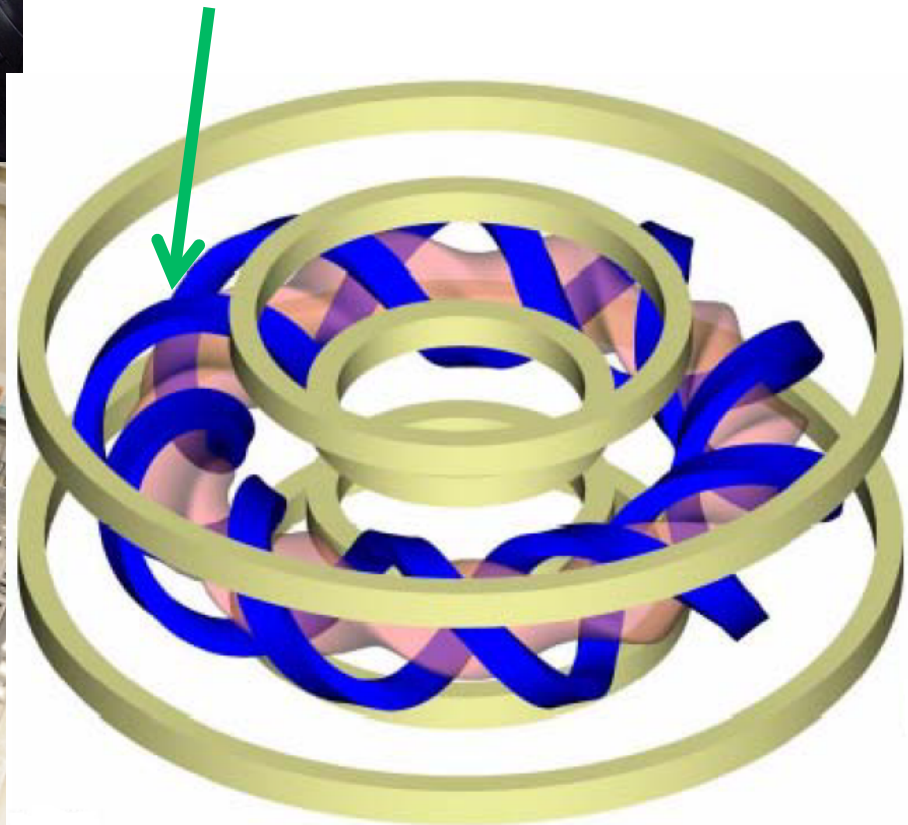
V. V. SANNIKOV

I. V. Kurchatov Institute, Moscow

Large Helical Device (LHD) in Japan



- Largest stellarator until 2016.
- Continuously wound superconducting helical coils



Modern stellarators are designed using optimization.

Find the plasma shape that minimizes $\chi_s^2 + \chi_{pr}^2 + \chi_{ht}^2 + \chi_{\alpha l}^2 + \chi_{rc}^2$

$$\chi_s^2 \propto \text{field line stochasticity}$$

$$\chi_{pr}^2 \propto 1 / (\text{maximum stable pressure})$$

$$\chi_{ht}^2 \propto \text{heat transport}$$

$$\chi_{\alpha l}^2 \propto \alpha\text{-particle loss}$$

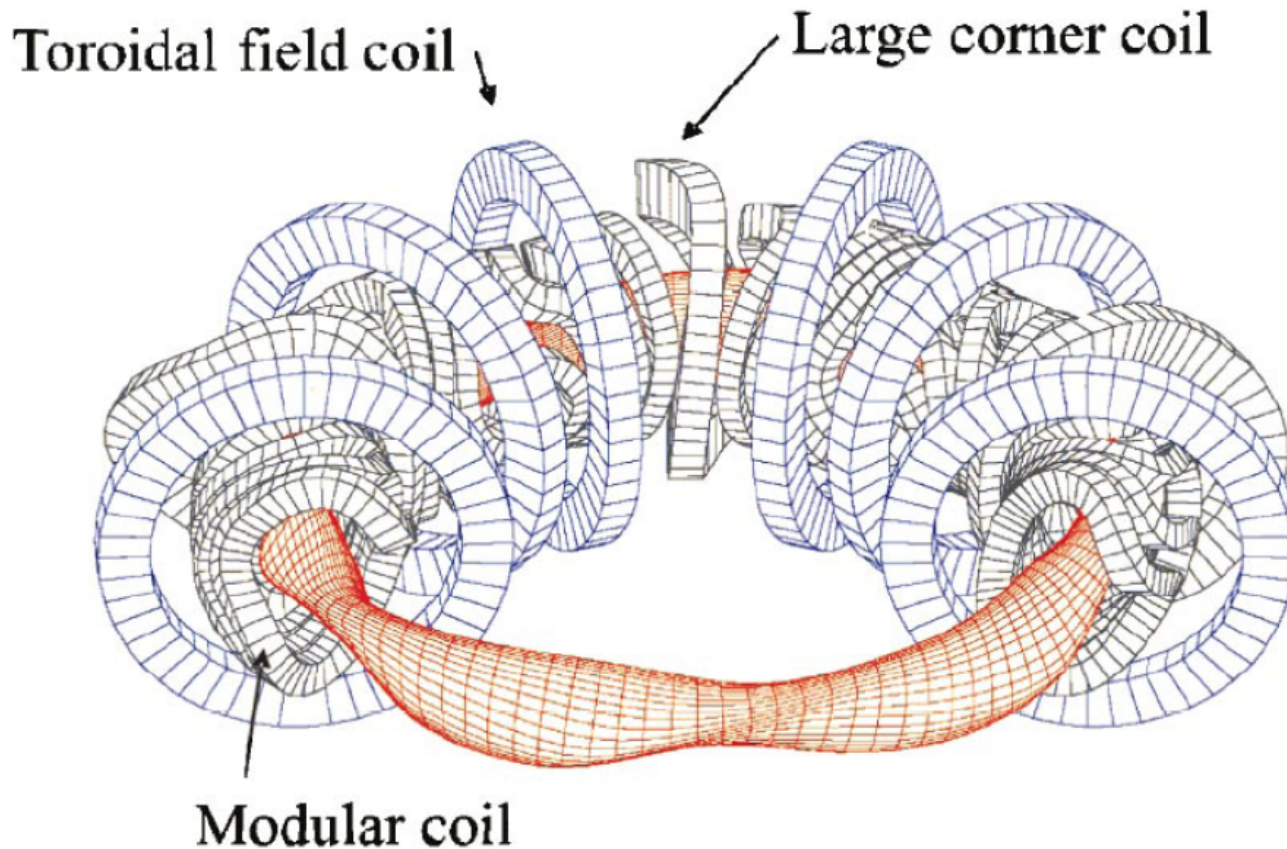
$$\chi_{rc}^2 \propto 1 / (\text{radius of curvature of coils})$$

E.g. finding the minimum of a complicated function

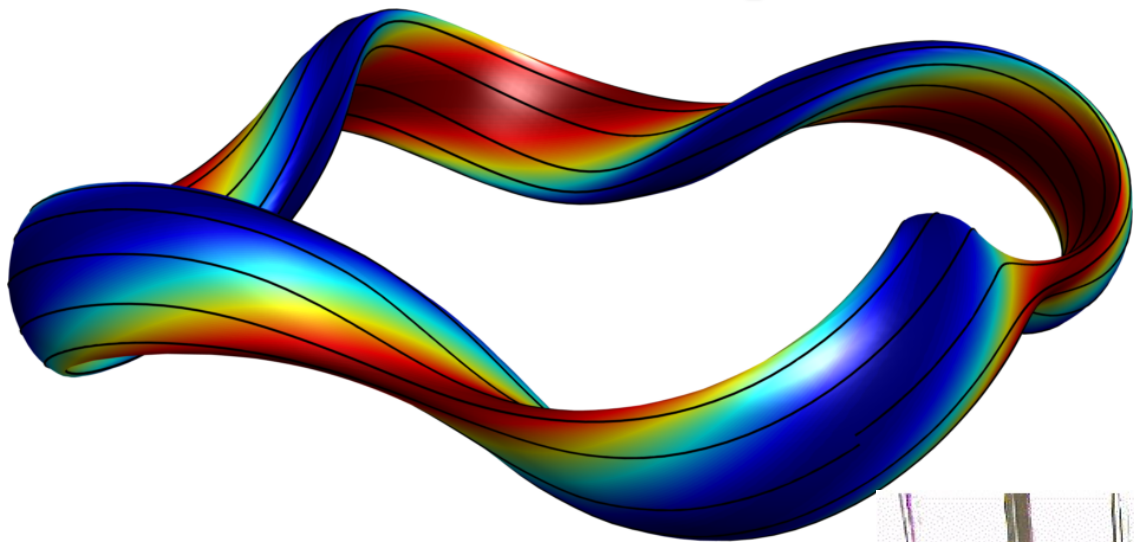
$$f(x_1, x_2, x_3, \dots, x_{50}).$$

Wendelstein 7-AS (W7-AS) in Germany

- First device with “modular coils.” Easier to assemble and maintain.
- Operated from 1988-2002.

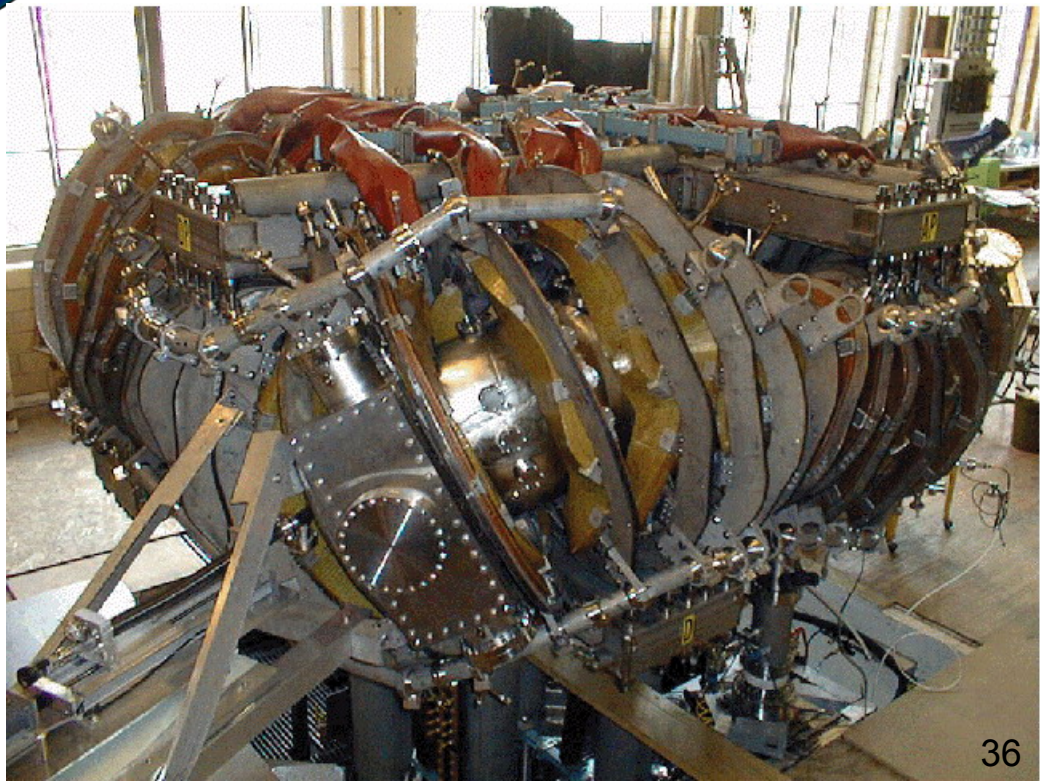
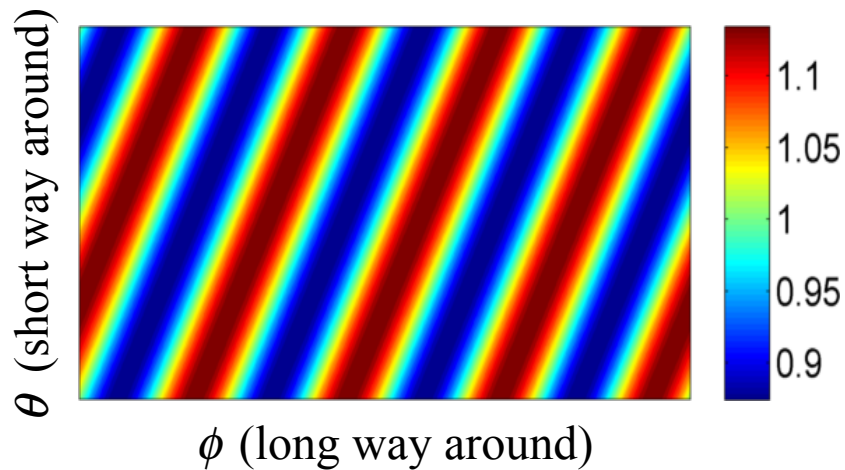


The Helicallly Symmetric Experiment (HSX) at Wisconsin explores *Quasi-symmetry*.



- Particle trajectories depend on $B=|\mathbf{B}|$ (in a special coordinate system), not the full vector \mathbf{B} .
- So symmetry in B gives tokamak-like particle orbits.

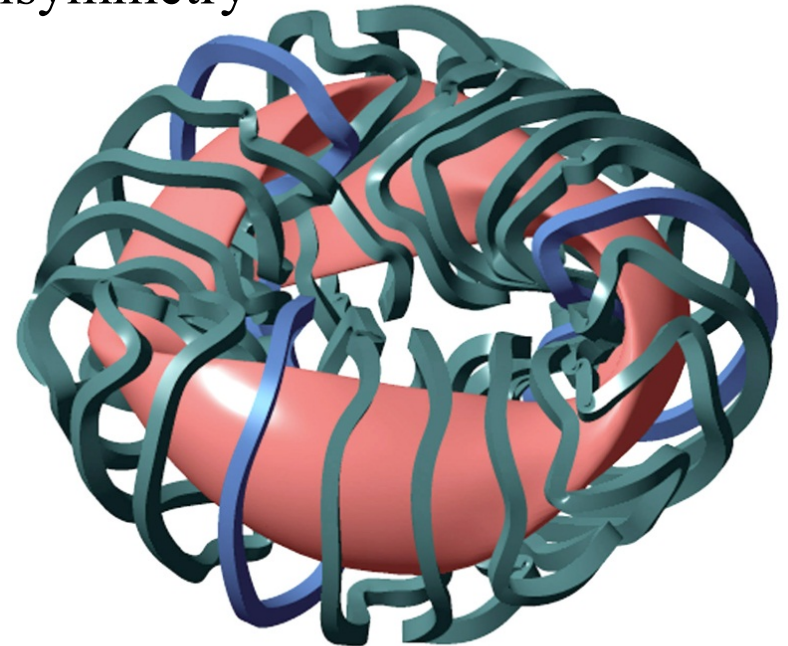
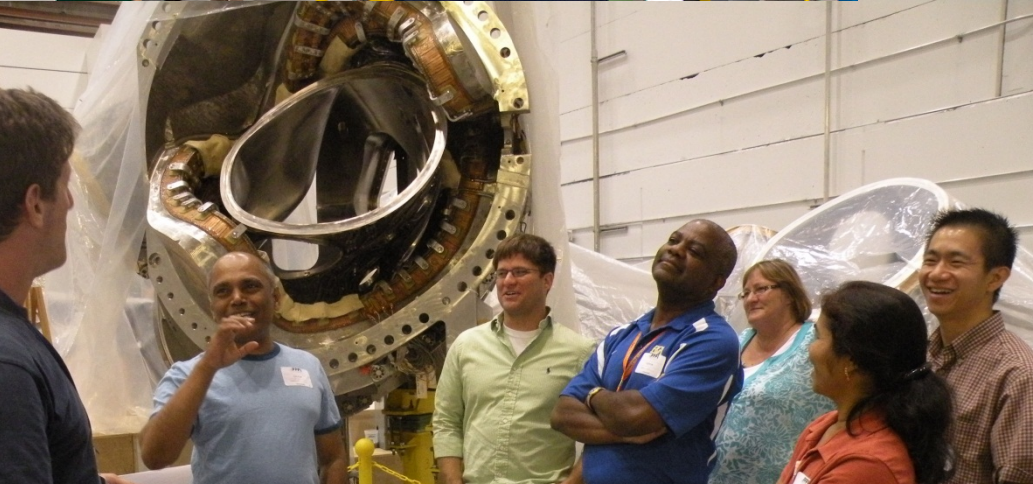
Magnetic field magnitude $|B|$ in Tesla



National Compact Stellarator Experiment (NCSX)



- Partially constructed at Princeton.
- Coils & vacuum vessel segments are completed.
- Project was terminated by Dept of Energy in 2008 due to cost overruns.
- Design principles: compactness, quasisymmetry

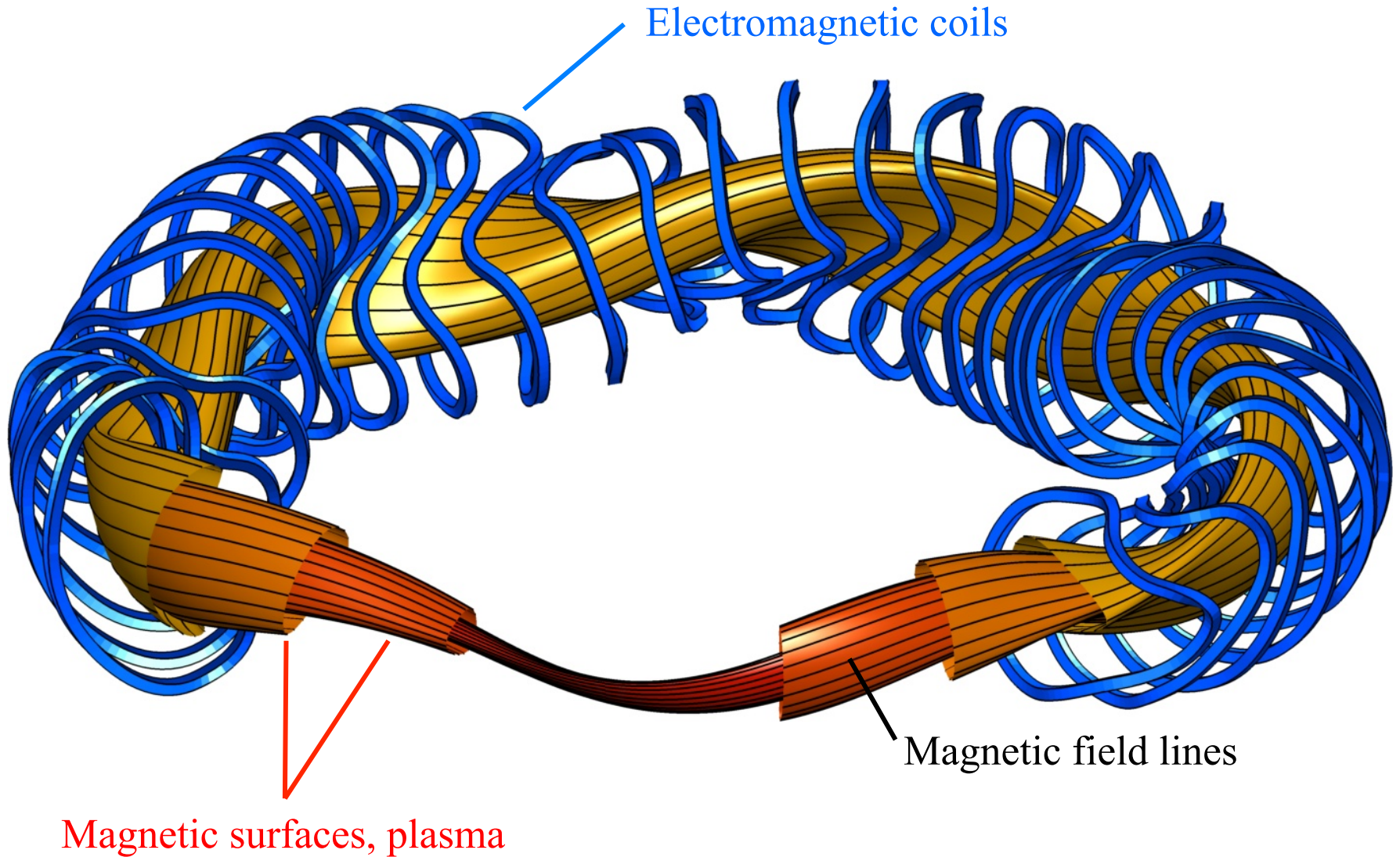


Wendelstein 7-X (W7-X) in Germany has just begun operation.

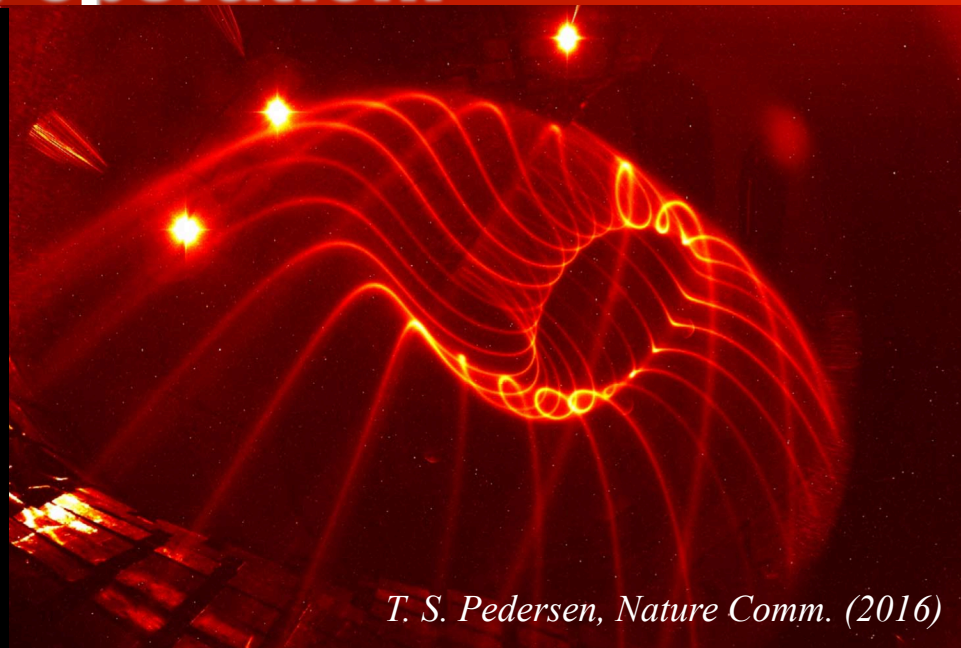
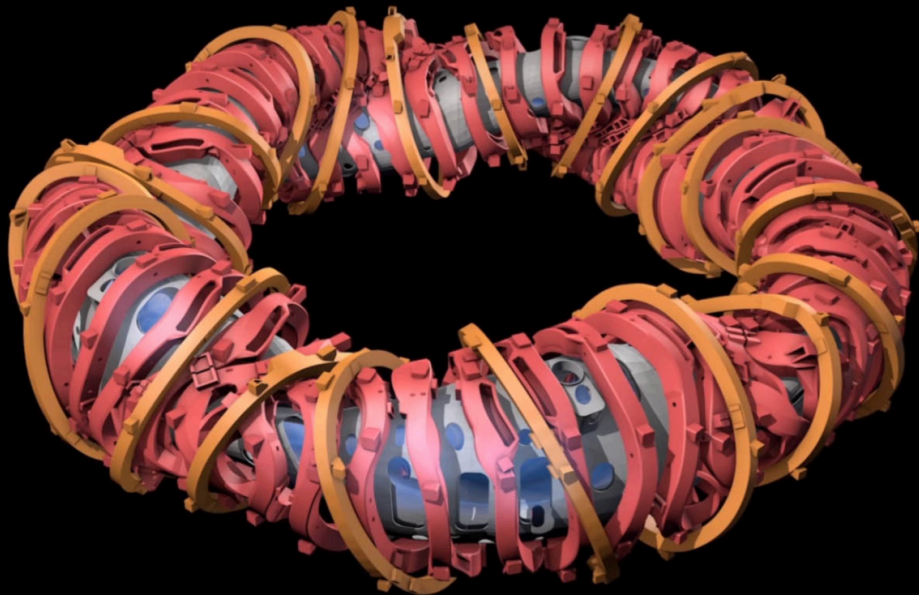
Science

Oct 21, 2015

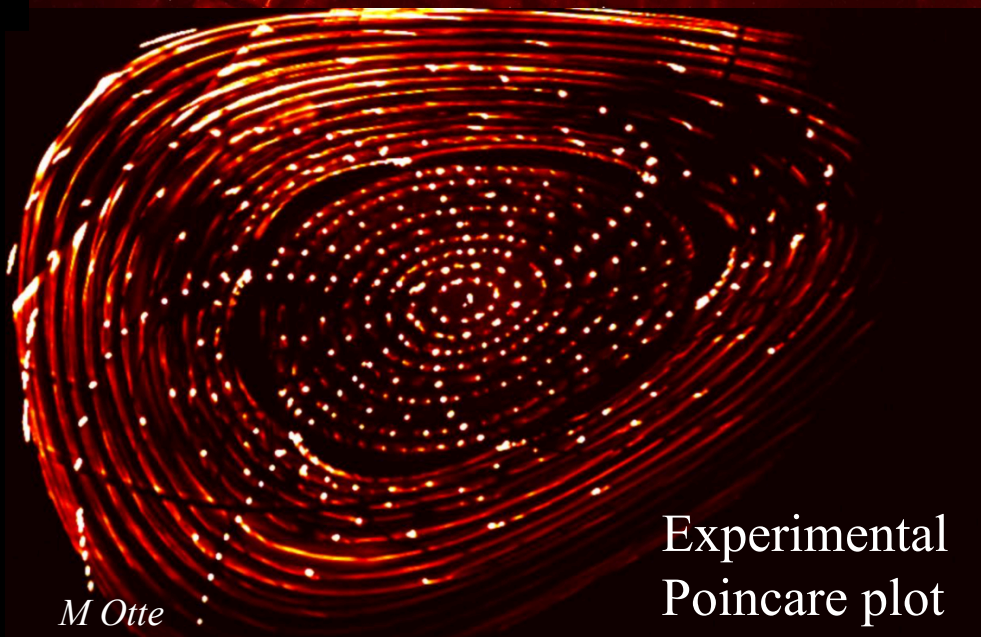
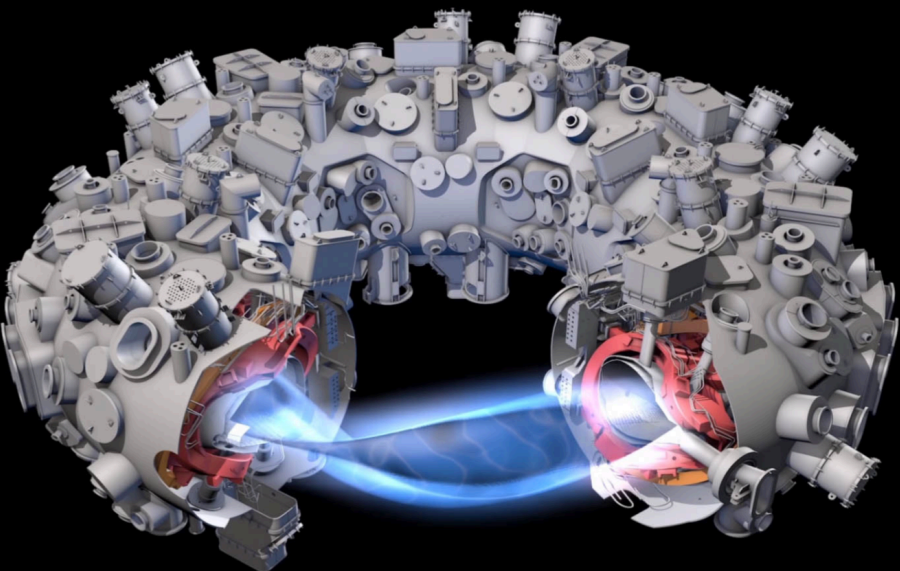
30 minute plasmas eventually



Wendelstein 7-X (W7-X) in Germany has just begun operation.



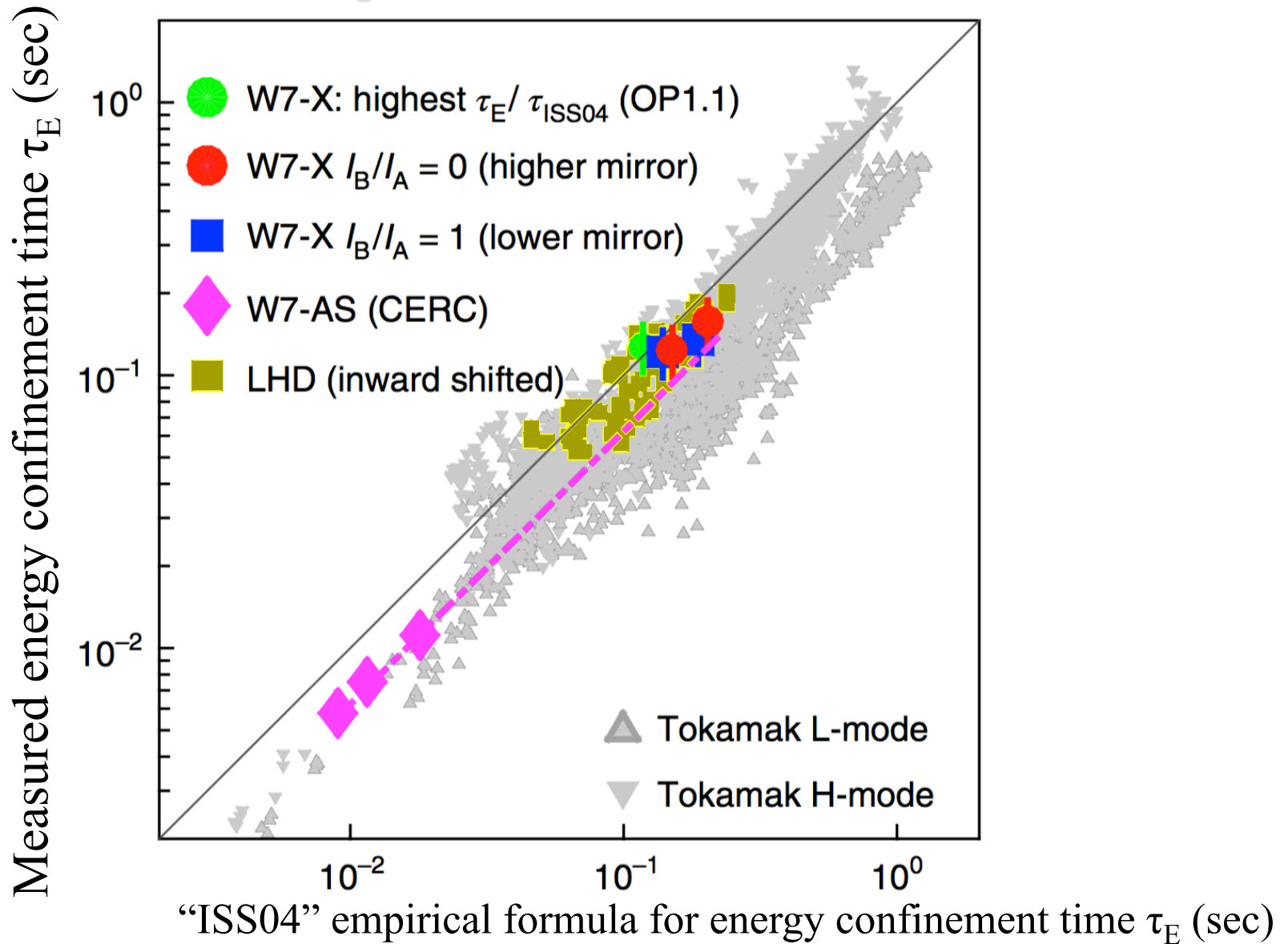
T. S. Pedersen, Nature Comm. (2016)



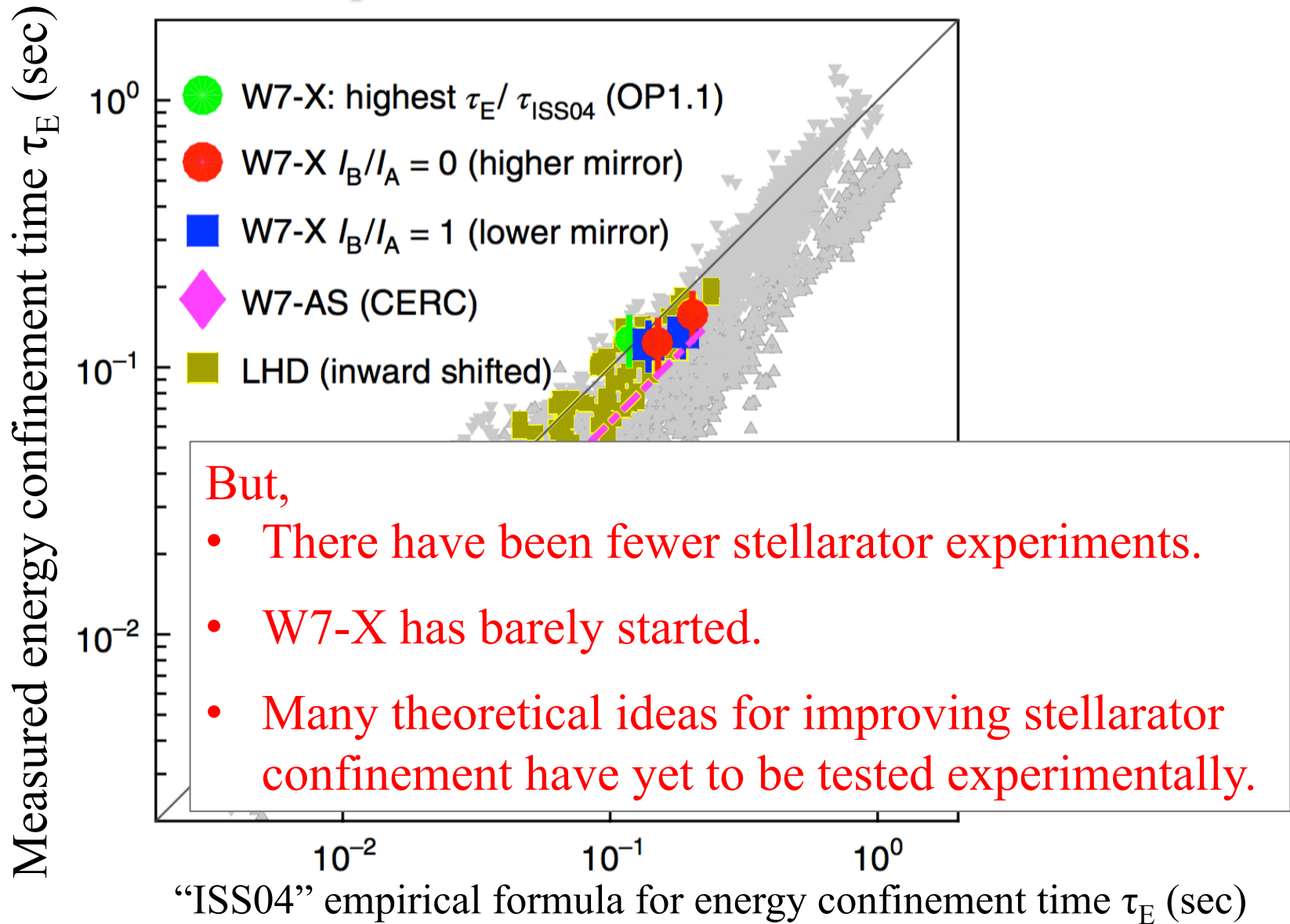
M Otte

Experimental
Poincare plot

Stellarators remain a bit behind tokamaks in performance.



Stellarators remain a bit behind tokamaks in performance.



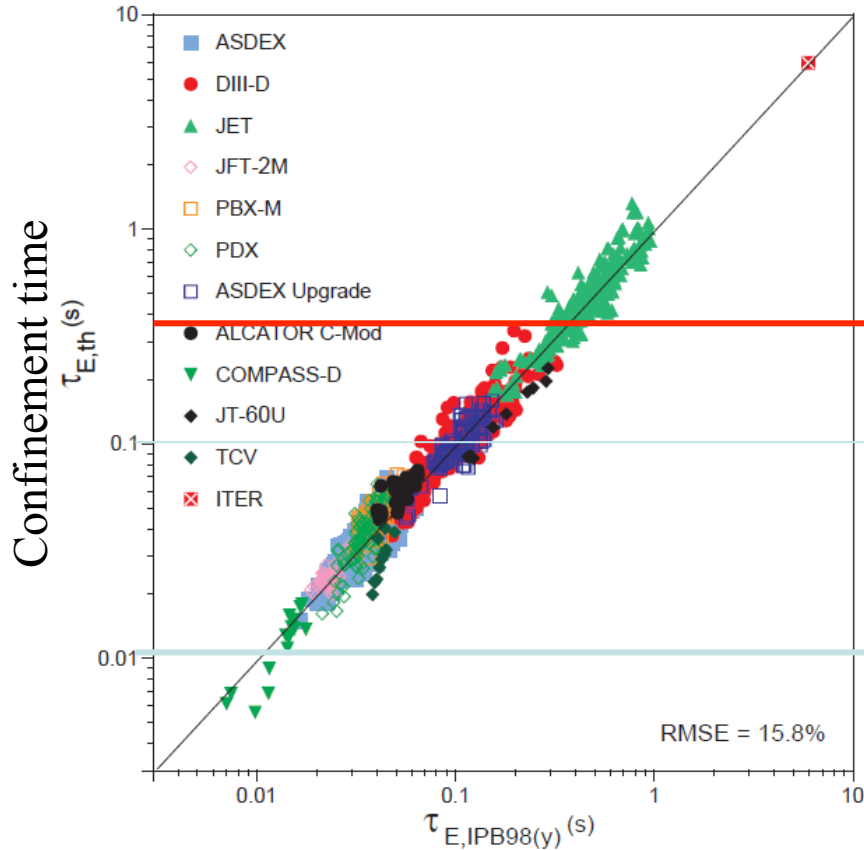
Summary

- Toroidal confinement requires rotational transform:
 - Tokamaks obtain it using a current in the plasma
 - Stellarators obtain it by nonaxisymmetric shaping.
- Unlike stellarators, tokamaks have disruptions, are intrinsically pulsed, and will require current drive.
- Historically, stellarators have had lower performance than tokamaks, but now we think we know how to design them better, and many ideas have yet to be tested experimentally.
- Many exciting stellarator experiments: HSX, CTH, CNT (US), LHD (Japan), W7-X (Germany), CFQS (China & Japan), etc.

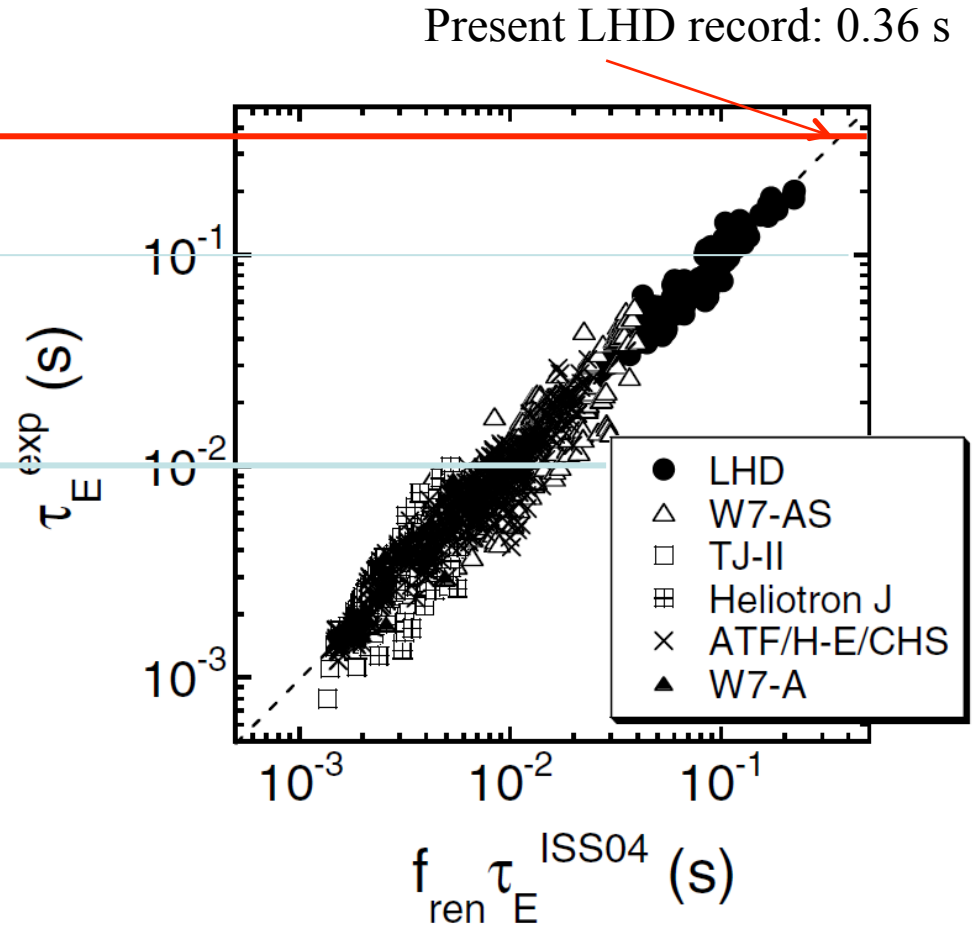
Extra slides

Performance

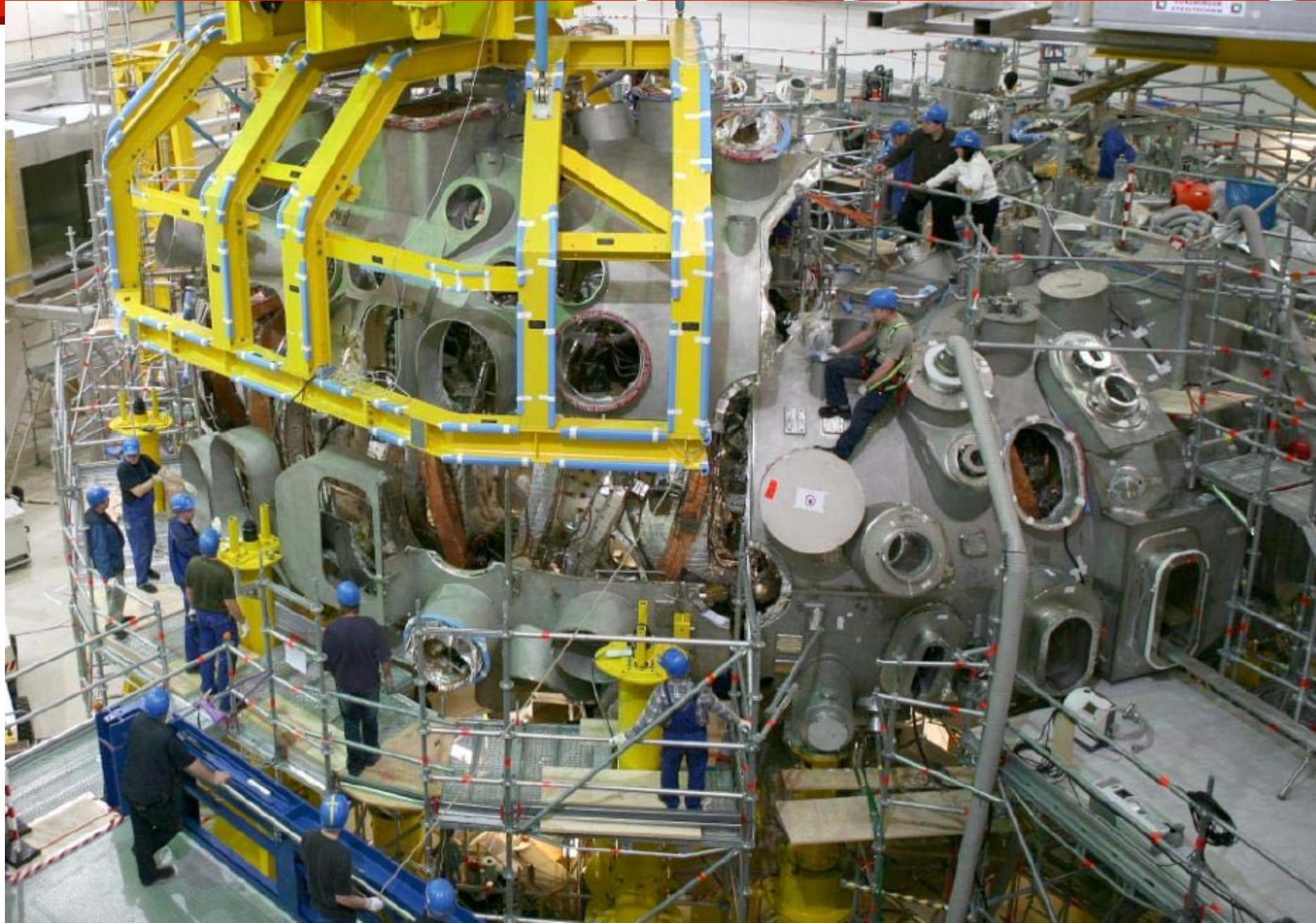
Tokamaks



Stellarators

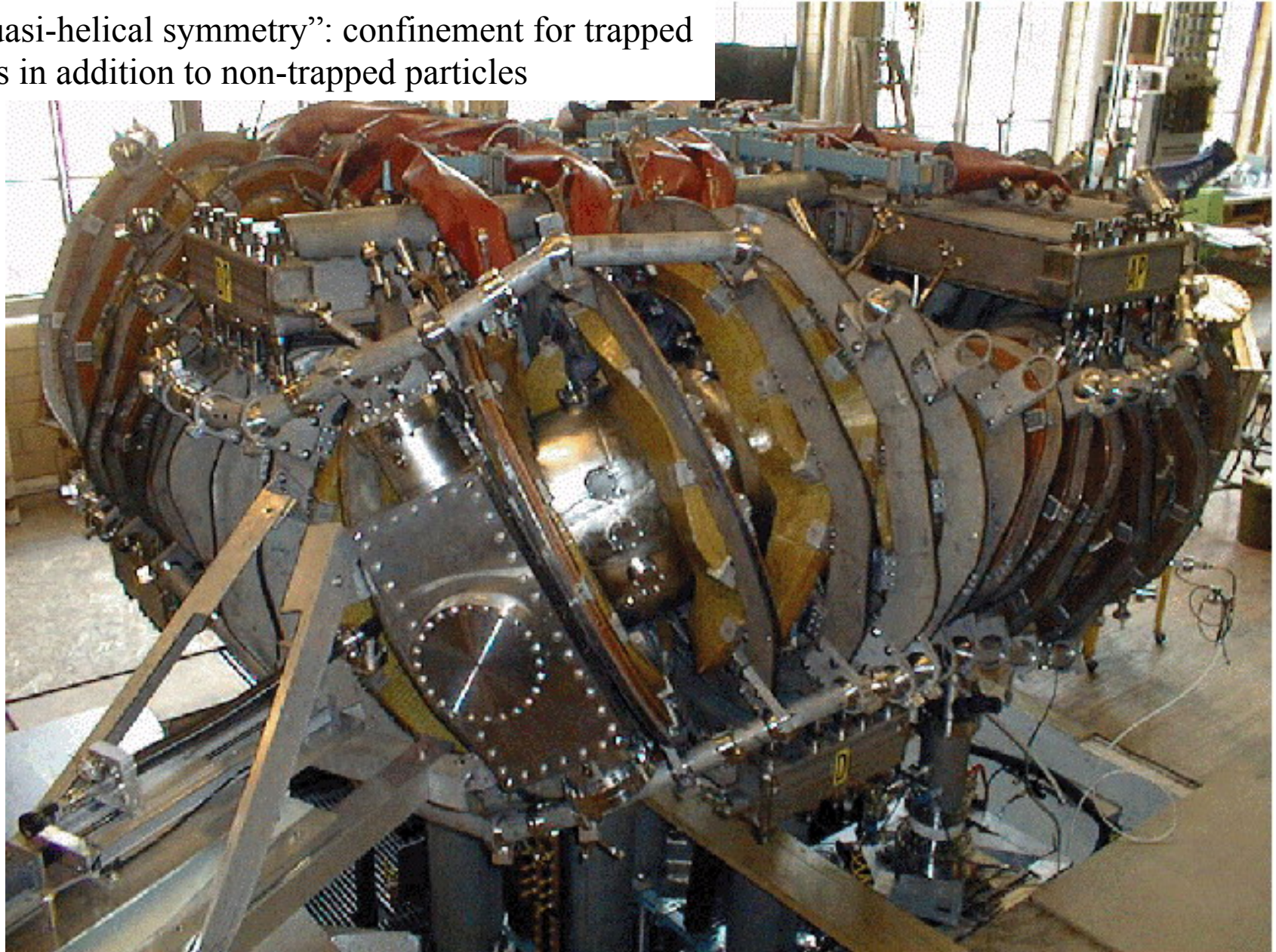


Wendelstein 7-X (W7-X) in Germany

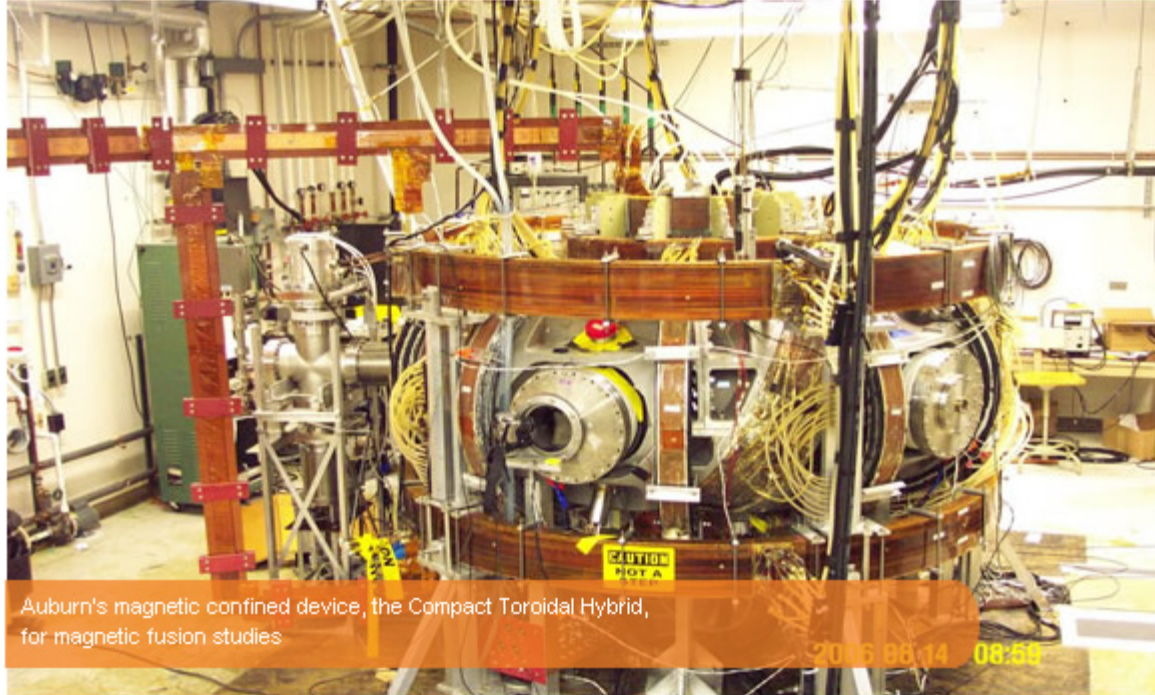


Helicallly Symmetric Experiment (HSX) at U of Wisconsin

Has “quasi-helical symmetry”: confinement for trapped particles in addition to non-trapped particles



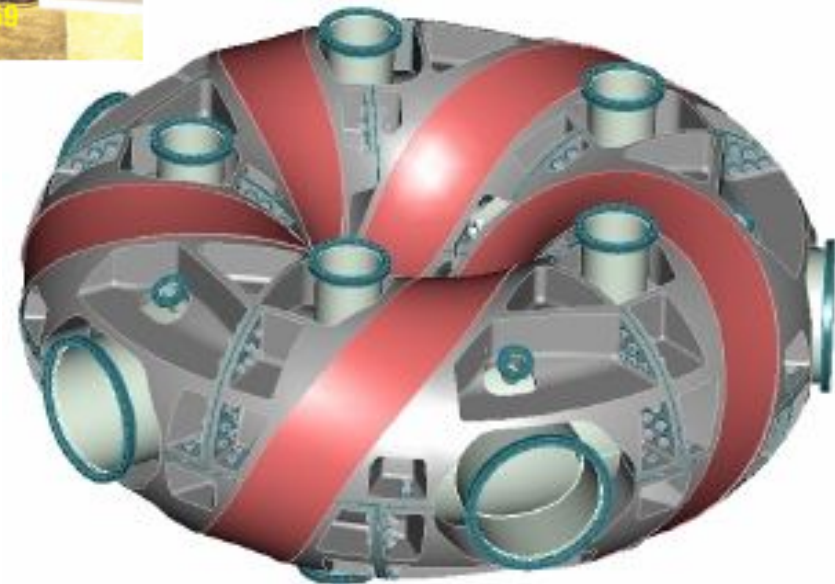
Stellarator-tokamak hybrids also may have promise



Auburn's magnetic confined device, the Compact Toroidal Hybrid, for magnetic fusion studies

2015 06 14 08:55

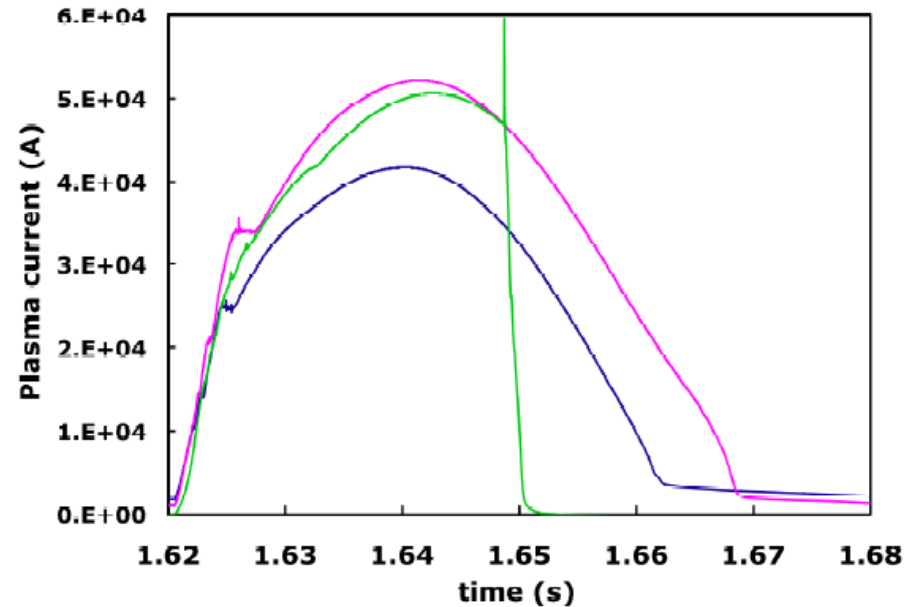
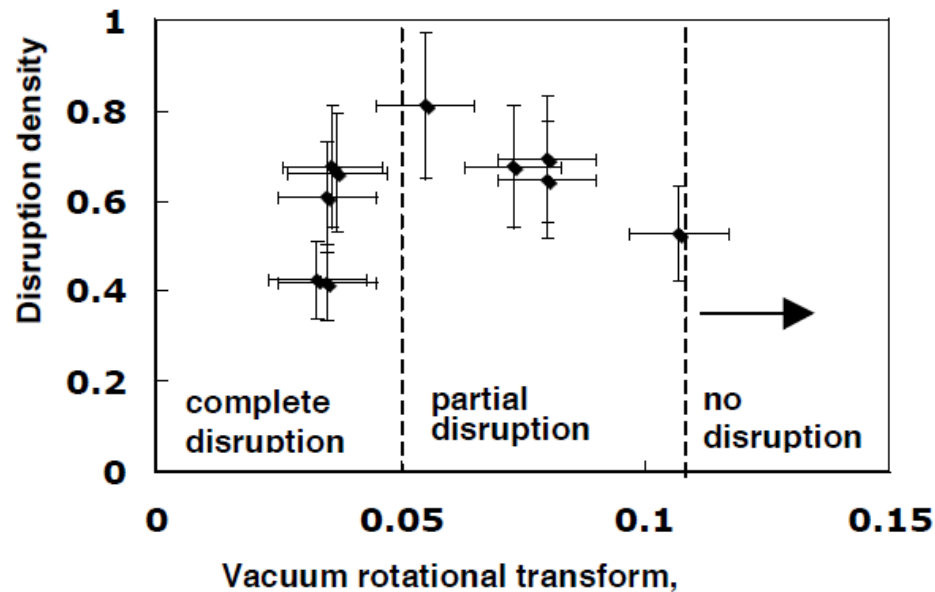
Compact Toroidal Hybrid (CTH) at Auburn University



Stellarator-tokamak hybrids also may have promise

Even a small amount of externally generated rotational transform may be enough to prevent disruptions.

Compact Toroidal Hybrid (CTH) at Auburn University



External rotational transform: — 0.18
— 0.09
— 0.03

Confining plasmas with 3D (nonaxisymmetric) magnetic fields: opportunities and challenges



Matt Landreman¹

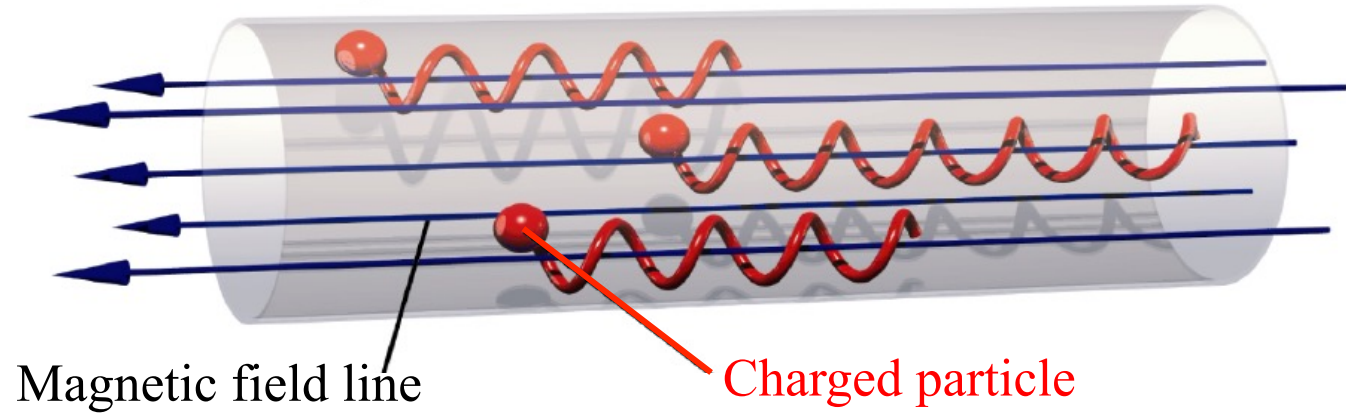
Thanks to T Antonsen¹, M Barnes², A Boozer³, W Dorland¹, P Catto⁴,
G Colyer², J Drake¹, D Ernst⁴, J Freidberg⁴, J Geiger⁵, T Görler⁵, G Hammett⁶,
A Hassam¹, P Helander⁵, E Highcock², A Mollen⁷, F Parra², G Plunk⁵,
H Smith⁵, J TenBarge¹, Y Turkin⁵

1. University of Maryland, 2. Oxford, 3. Columbia, 4. MIT,
5. Max Plank Institute for Plasma Physics, 6. Princeton, 7. Chalmers University

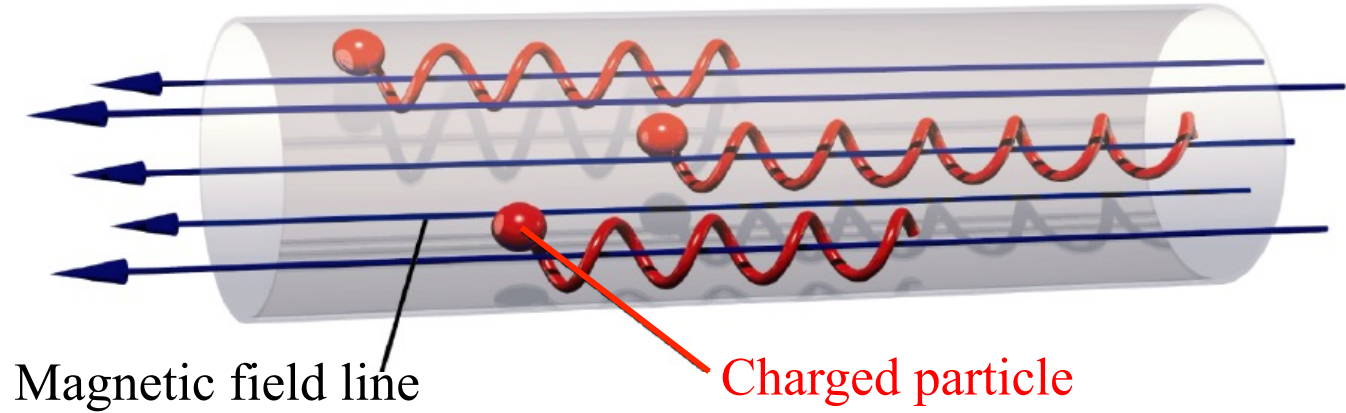
Outline

- Why use 3D shaping?
- Challenges for theory & computation
 - Impurity accumulation & transport
 - Optimization with simpler coils
 - Microinstabilities & turbulence

Uniform straight \mathbf{B} : confinement $\boxed{?}$ to \mathbf{B} , but end losses.

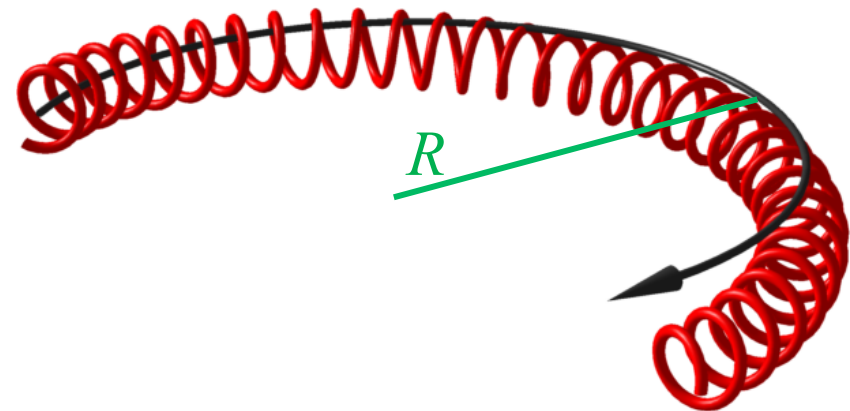
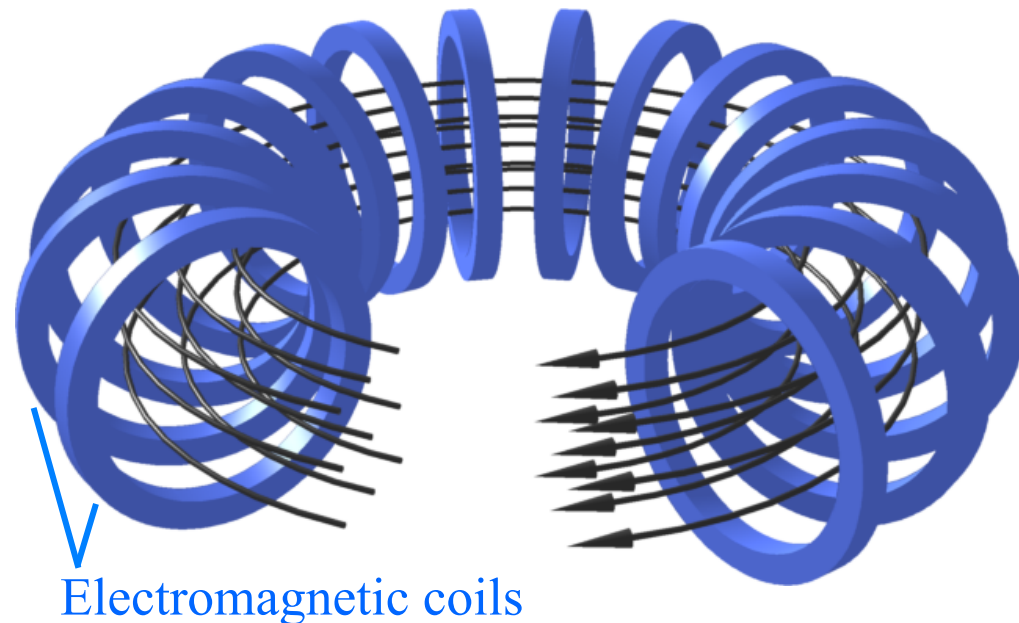


Uniform straight \mathbf{B} : confinement $\boxed{?}$ to \mathbf{B} , but end losses.



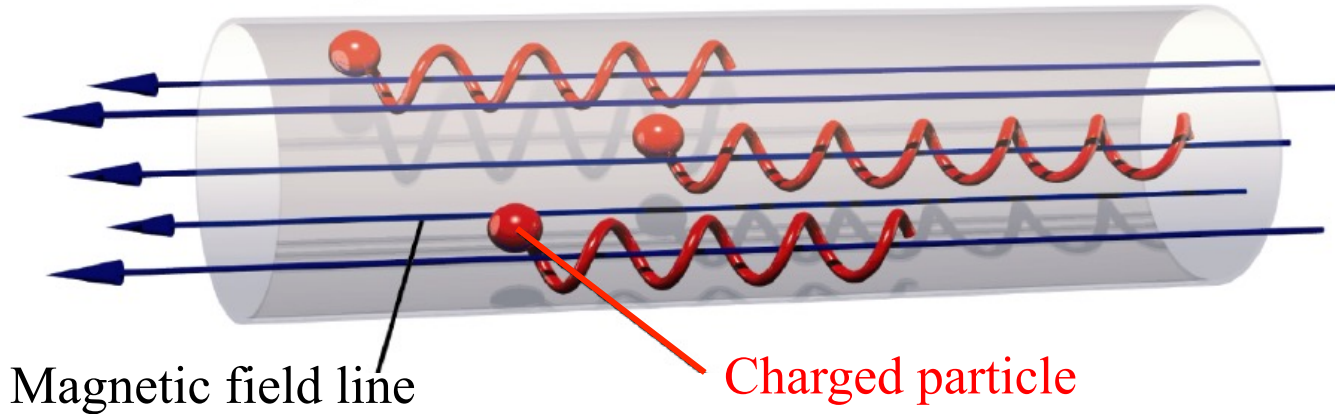
Toroidal solenoid: No confinement due to cross-field drift

$$\propto \mathbf{B} \times \nabla |\mathbf{B}| \sim 1/R$$

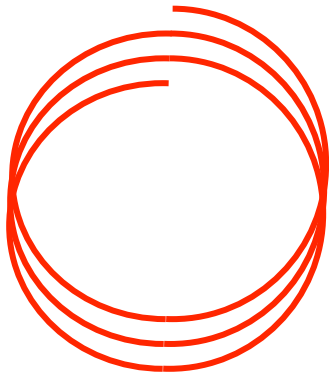


Confining charged particles with a magnetic field is tricky.

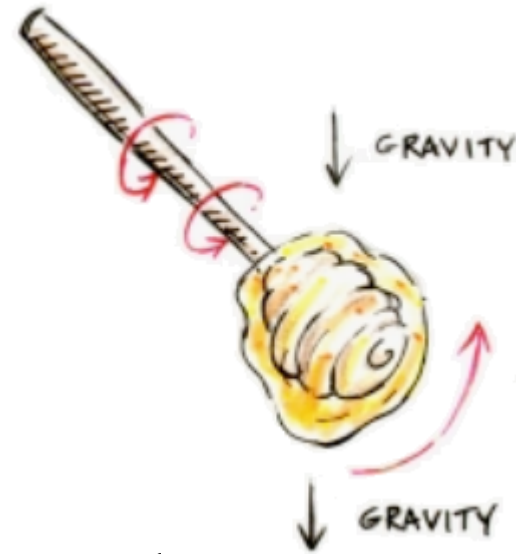
Uniform straight \mathbf{B} : confinement \perp to \mathbf{B} , but end losses.



But if field lines are bent, particles drift off them.



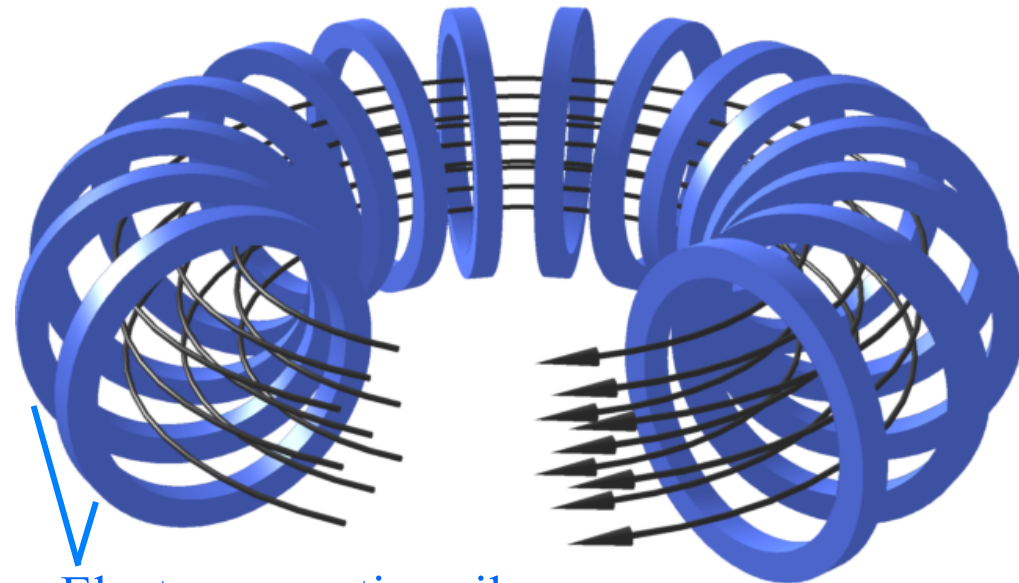
A solution: Make **B** field lines *helical*.



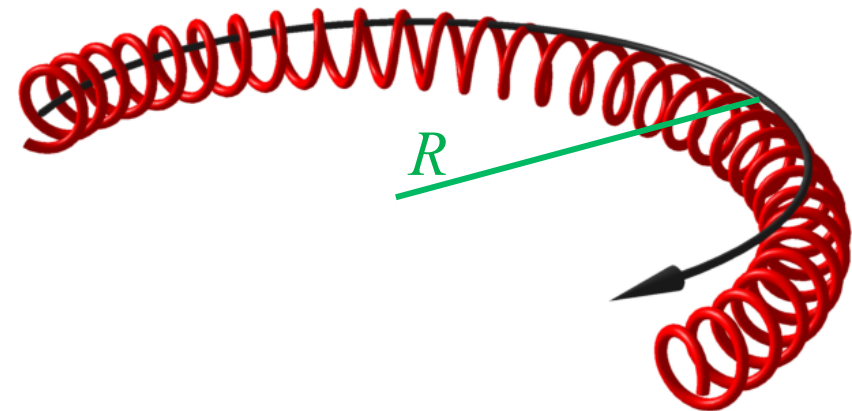
Picture from
G Hammett

Toroidal solenoid: No confinement due to cross-field drift

$$\propto \mathbf{B} \times \nabla |\mathbf{B}| \sim 1/R$$

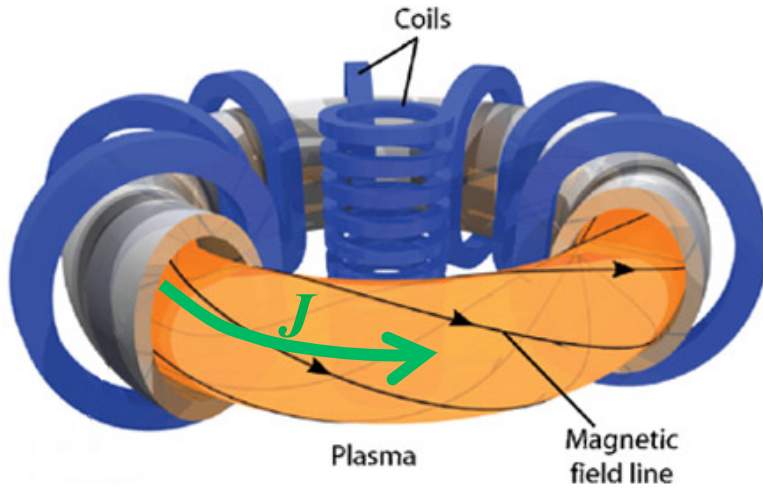


Electromagnetic coils



Tokamaks (e.g. ITER)

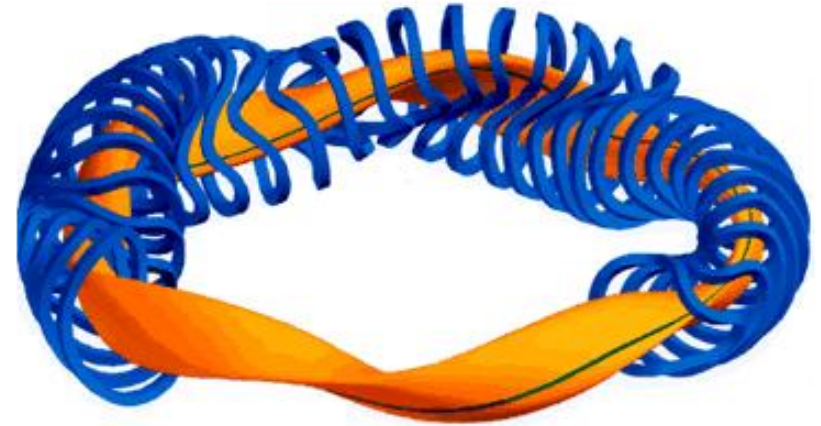
Almost axisymmetric (2D)



Large current required in the plasma.

Stellarators

Nonaxisymmetric (3D)

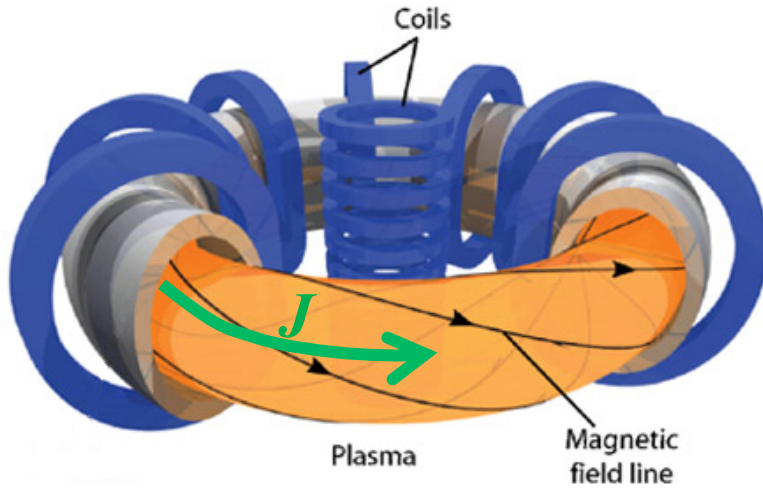


No current required in the plasma.

How can we make a helical magnetic field?

Tokamaks (e.g. ITER)

Almost axisymmetric (2D)

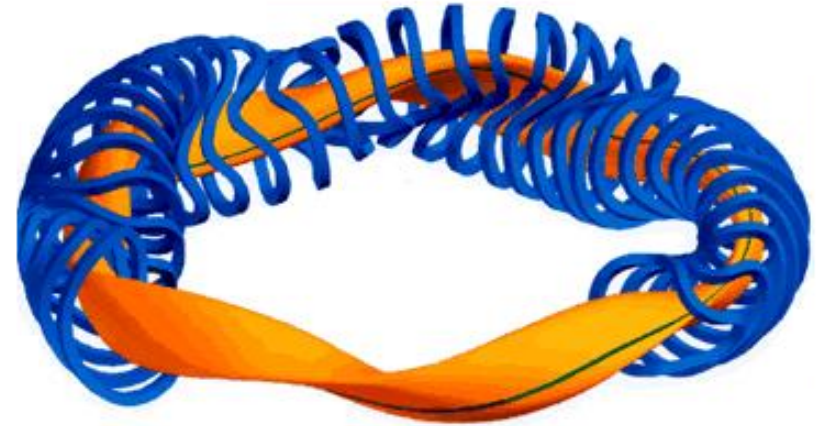


Large current required in the plasma.

– Can disrupt. Active stabilization needed.

Stellarators

Nonaxisymmetric (3D)

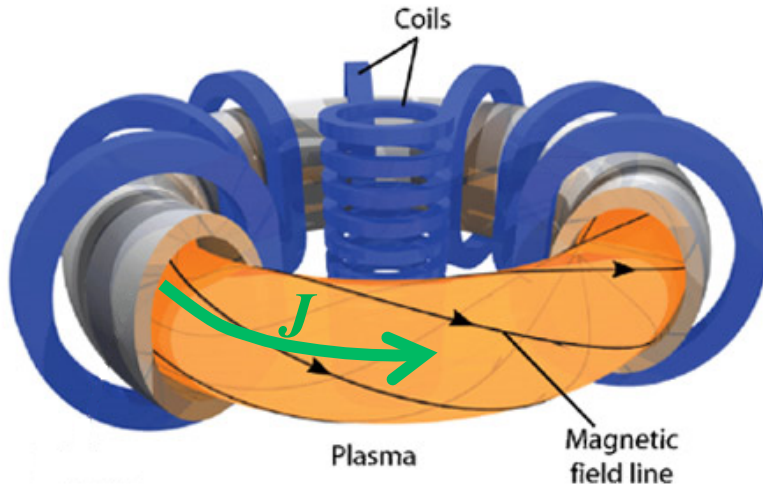


No current required in the plasma.

+ No disruptions. Passive stability.

Tokamaks (e.g. ITER)

Almost axisymmetric (2D)

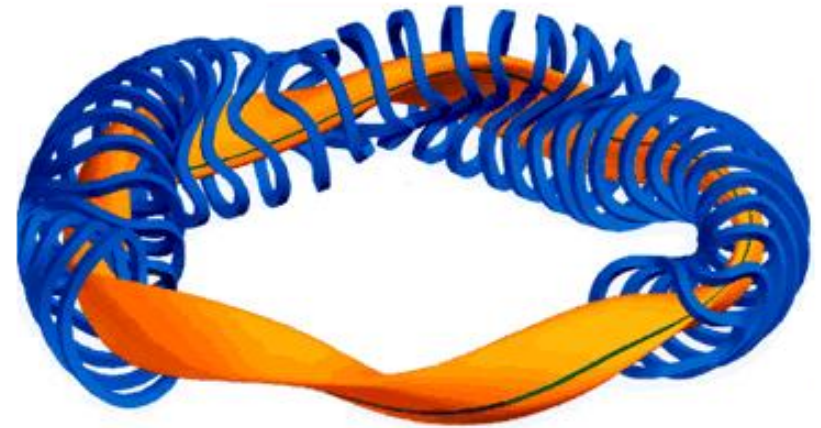


Large current required in the plasma.

- Can disrupt. Active stabilization needed.
- For steady-state, needs current drive (□ recirculating power) & careful control of $n(r)$, $T(r)$, □ n , □ T , $J(r)$.

Stellarators

Nonaxisymmetric (3D)

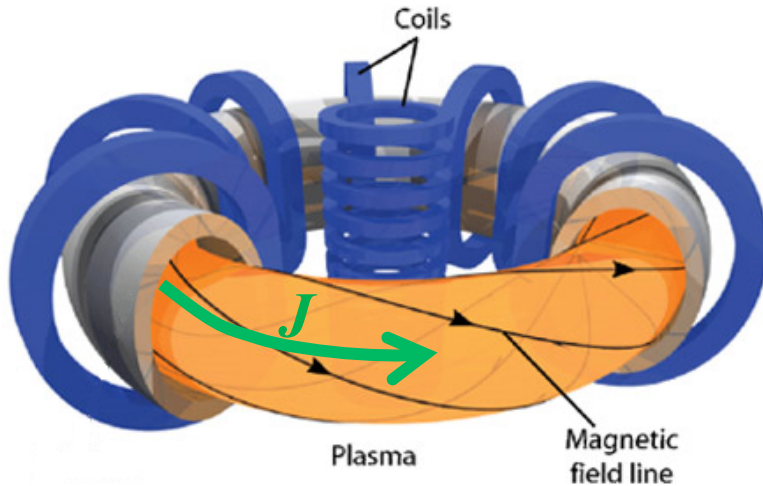


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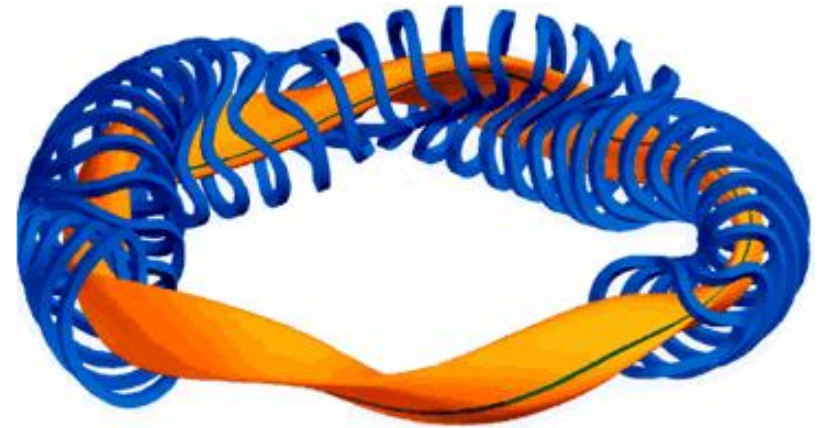


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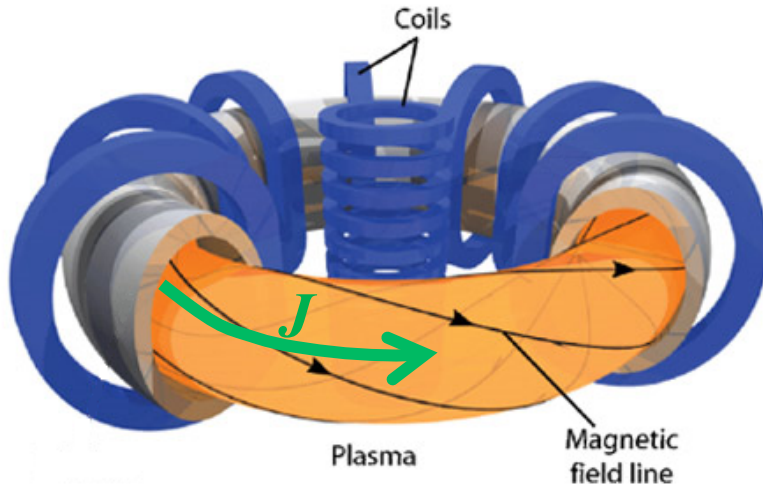


No current required in the plasma.

- + No disruptions. Passive stability.
- + Current drive & profile control unnecessary for steady-state.
- Strong impurity accumulation.

Tokamaks (e.g. ITER)

Almost axisymmetric (2D)

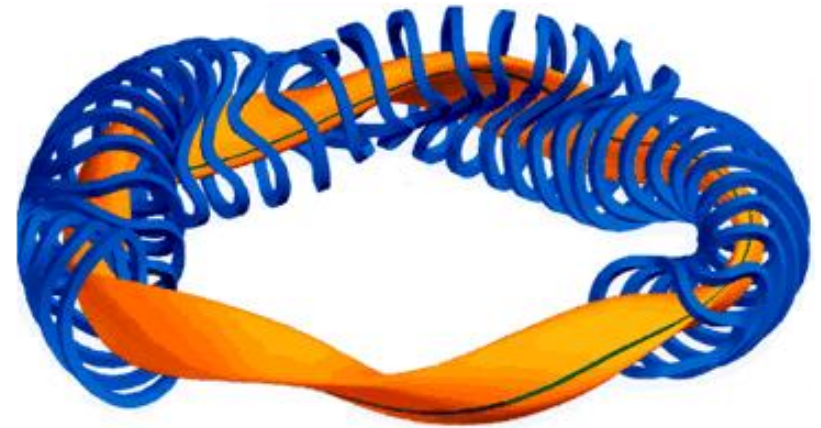


Large current required in the plasma.

- Can disrupt. Active stabilization needed.
- For steady-state, needs current drive (□ recirculating power) & careful control of $n(r)$, $T(r)$, □ n , □ T , $J(r)$.
- + Simpler to construct.

Stellarators

Nonaxisymmetric (3D)

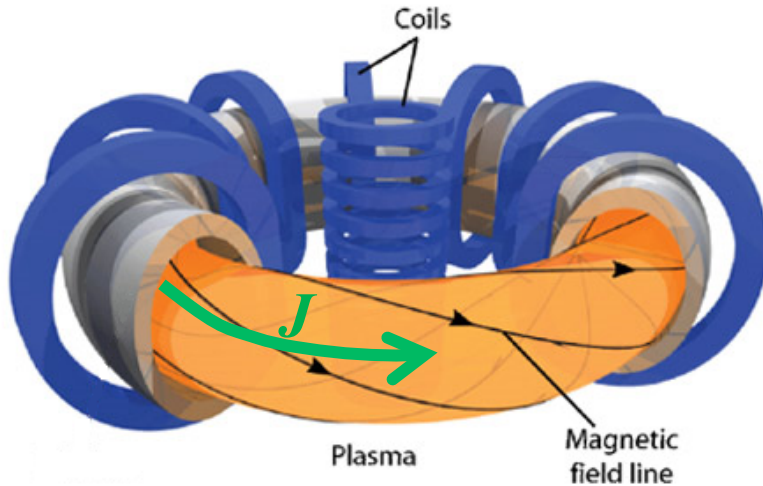


No current required in the plasma.

- + No disruptions. Passive stability.
- + Current drive & profile control unnecessary for steady-state.
- Strong impurity accumulation.
- Complicated & expensive magnets.

Tokamaks (e.g. ITER)

Almost axisymmetric (2D)

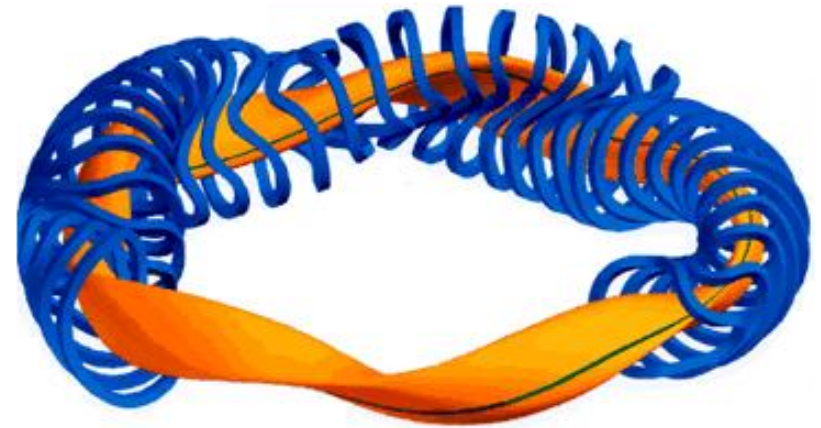


Large current required in the plasma.

- Can disrupt. Active stabilization needed.
- For steady-state, needs current drive (ⓧ recirculating power) & careful control of $n(r)$, $T(r)$, ⓧ n , ⓧ T , $J(r)$.
- + Simpler to construct.
- + Good confinement.

Stellarators

Nonaxisymmetric (3D)

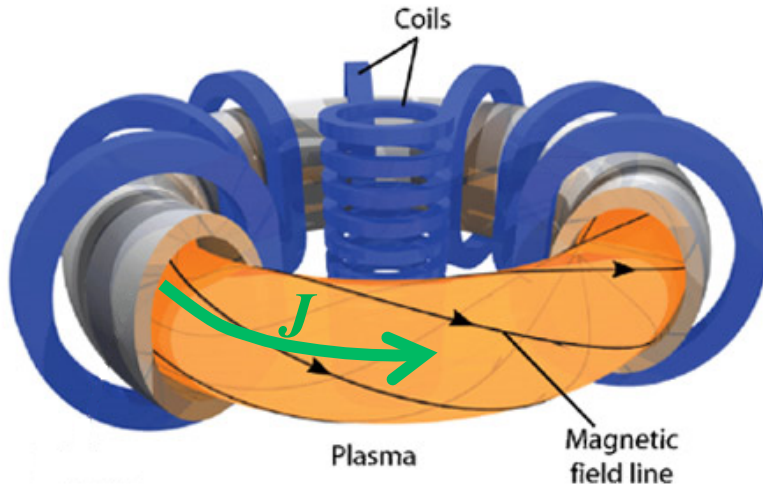


No current required in the plasma.

- + No disruptions. Passive stability.
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- Strong impurity accumulation.
- Complicated & expensive magnets.
- High heat & particle transport unless you “optimize.”

Tokamaks (e.g. ITER)

Almost axisymmetric (2D)



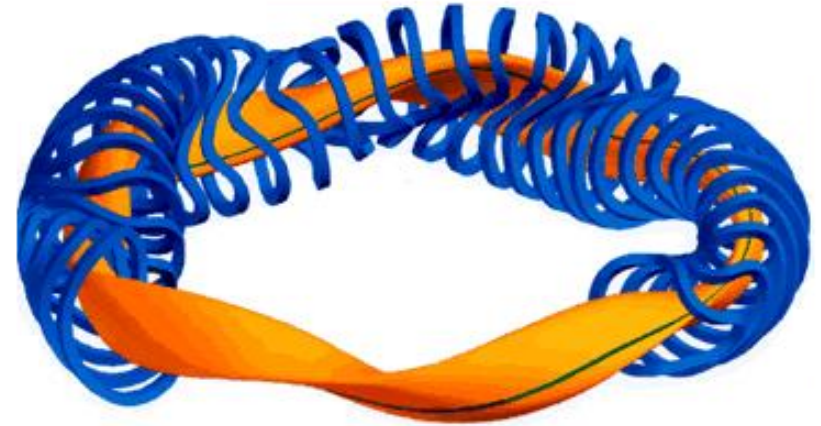
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Still 3D (ripple, intentional & self-generated perturbations))

Stellarators

Nonaxisymmetric (3D)



No current required in the plasma.

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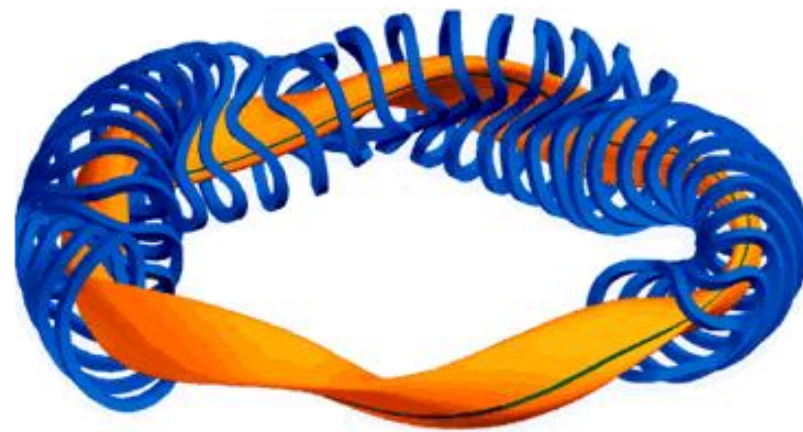
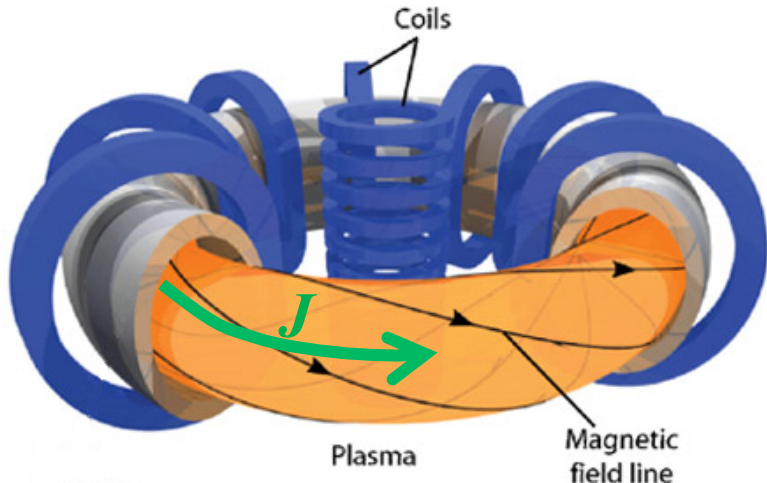
Tokamaks

Hybrids

Stellarators

Almost axisymmetric (2D)

Nonaxisymmetric (3D)



Large current required in the plasma.

- Can disrupt. Active stabilization needed.
- For steady-state, needs current drive (ⓧ recirculating power) & careful control of $n(r)$, $T(r)$, ⓧ n , ⓧ T , $J(r)$.
- + Simpler to construct.
- + Good confinement.

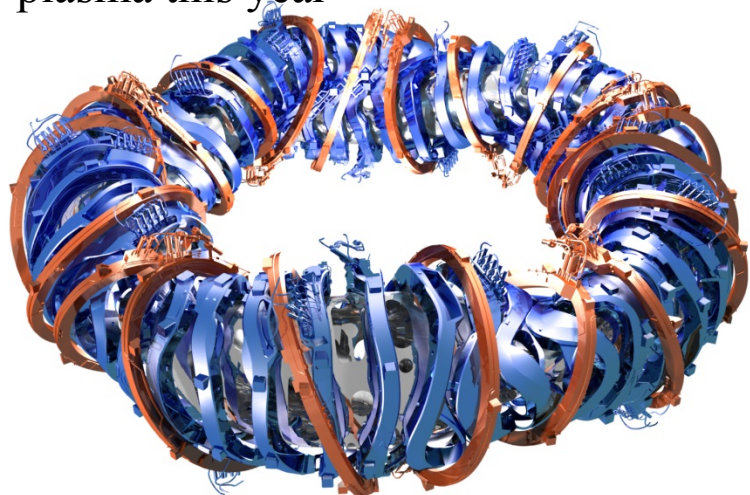
Still 3D (ripple, intentional & self-generated perturbations))

No current required in the plasma.

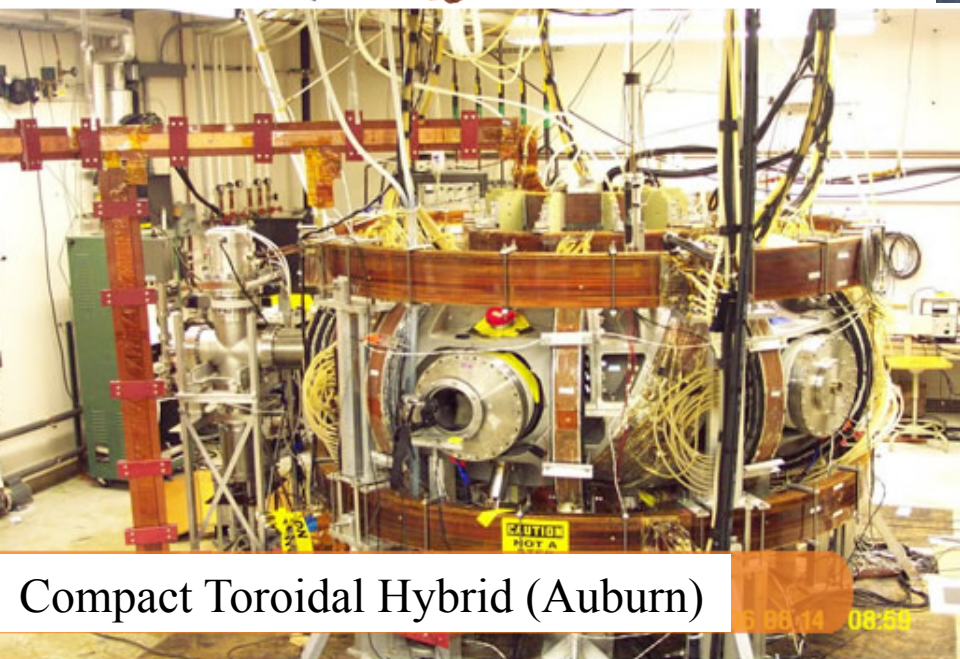
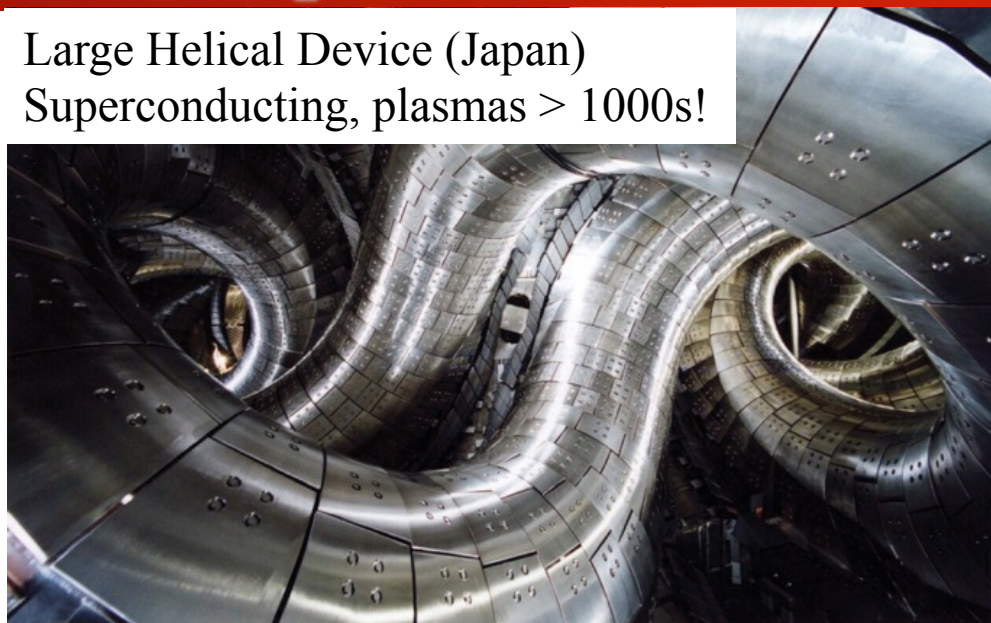
- + No disruptions. Passive stability.
- + Current drive & profile control unnecessary for steady-state.
- Strong impurity accumulation.
- Complicated & expensive magnets.
- High heat & particle transport unless you “optimize.”

Examples of 3D plasmas

W7-X (Germany), Superconducting, ~1B€,
1st plasma this year

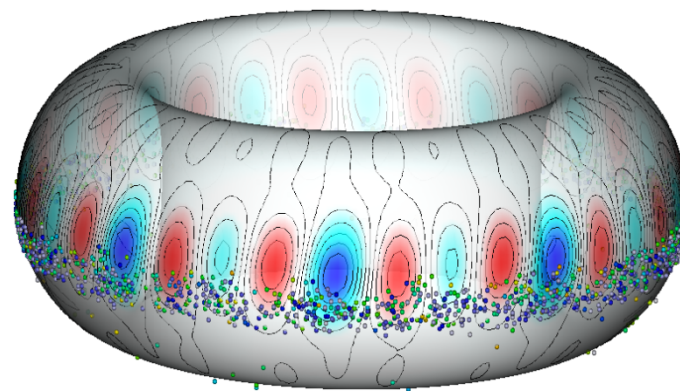


Large Helical Device (Japan)
Superconducting, plasmas > 1000s!



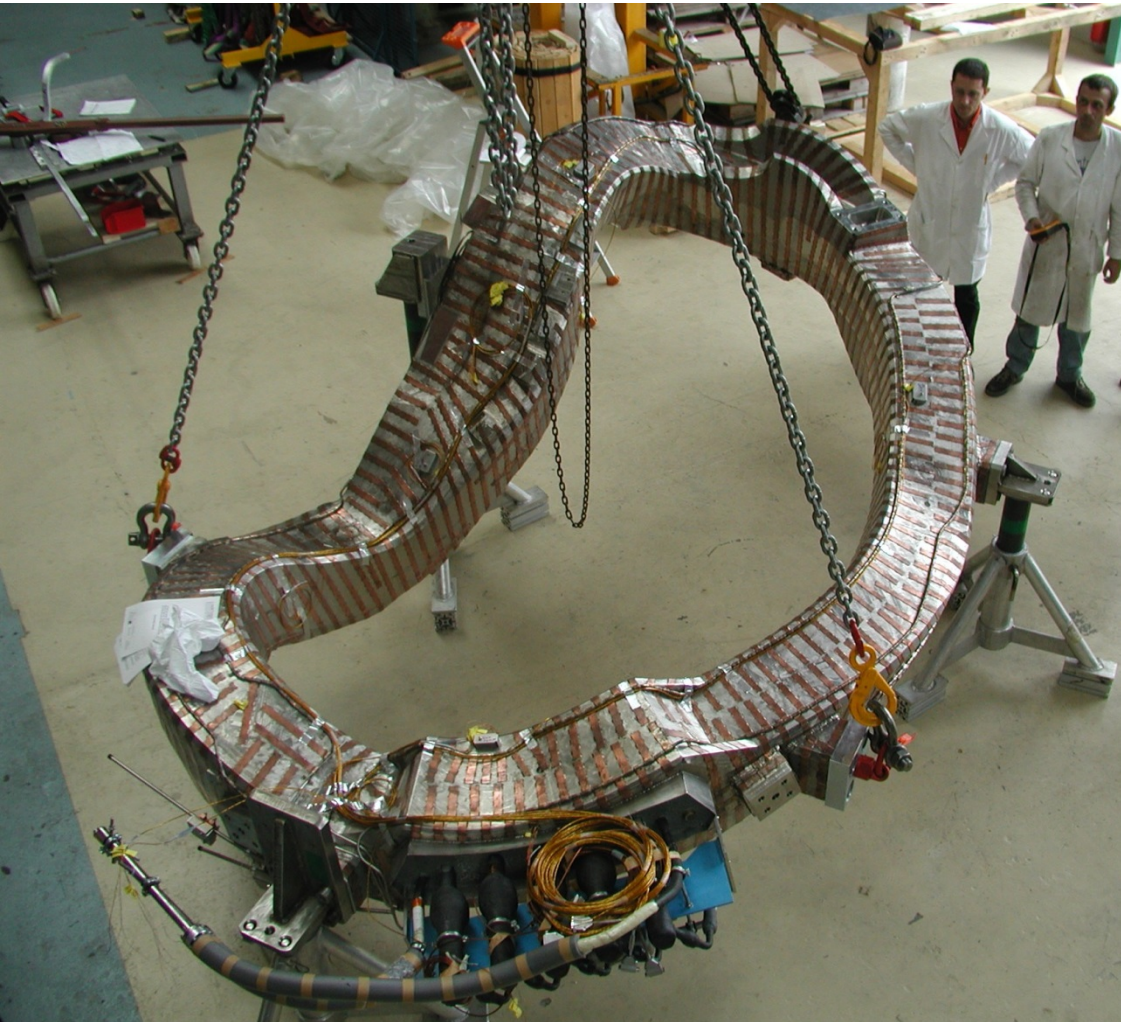
Compact Toroidal Hybrid (Auburn)

ITER & other *nominally*
axisymmetric plasmas



ITER a loss: *Spong, Phys Plasmas (2011).*

Near-optimized B fields have been created with modular coils, but are there better solutions?

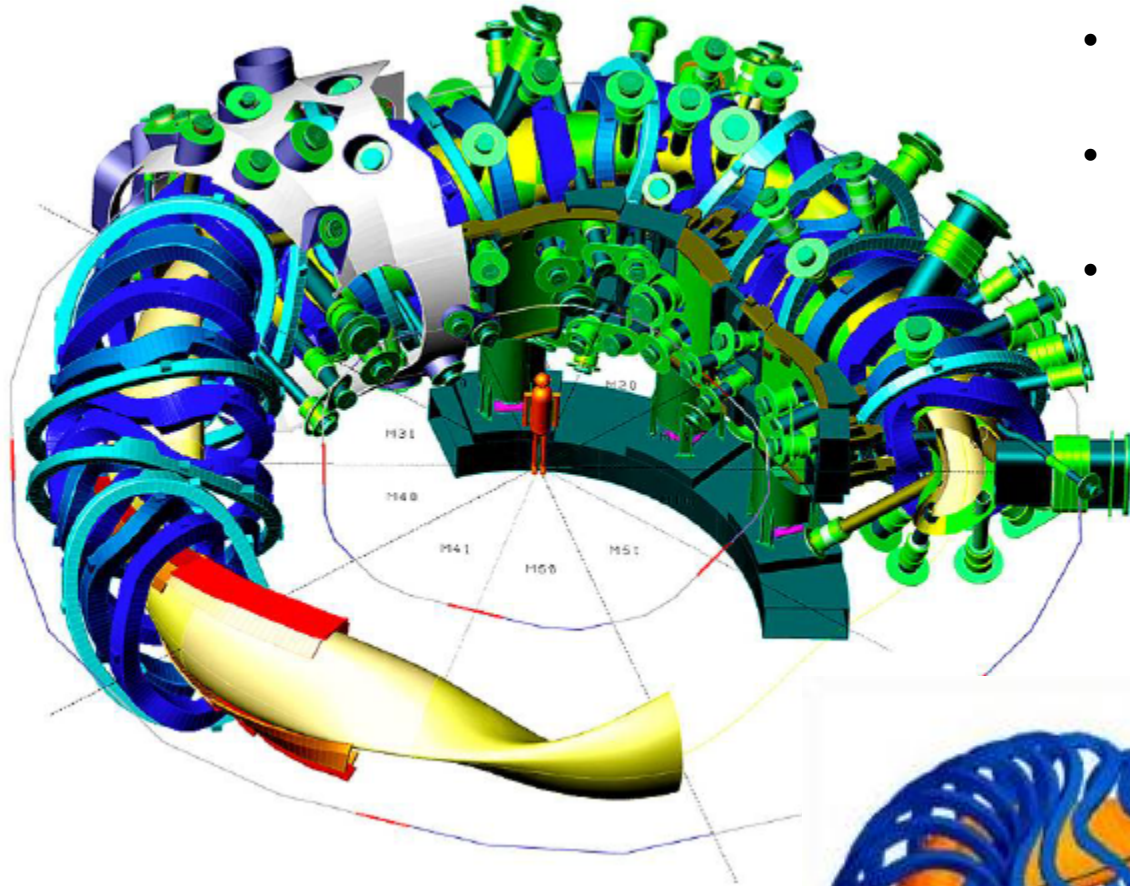


- Include wide tolerances as a design criterion?
- Superconducting monoliths?
- New high- T_c superconducting tapes?
- Continuous coils with demountable joints?

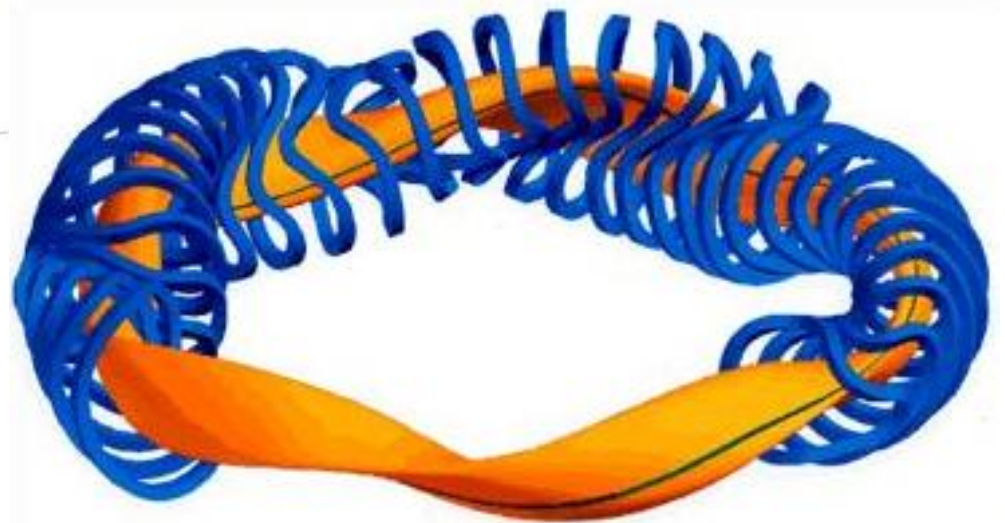
Summary

- 3D shaping can address several fusion requirements (steady-state, no disruptions, low recirculating power), and 3D effects matter in tokamaks.
- Impurity accumulation is a problem in 3D. SFINCS is a new state-of-the-art code for studying this phenomenon with the goal of reducing it.
- We know how to optimize $B = |\mathbf{B}|$, but much more can be done to achieve these fields with simpler coils.
- 3D shaping is a barely-explored knob to control plasma turbulence.

Wendelstein 7-X (W7-X) in Germany

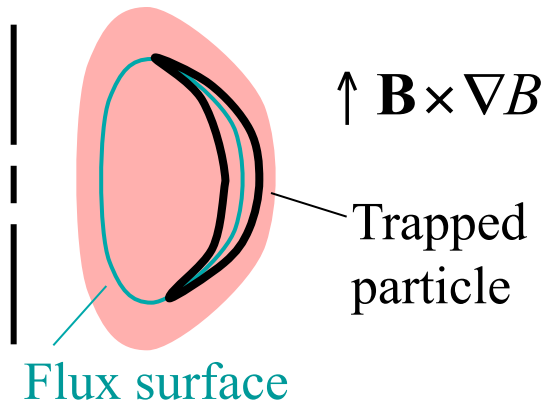


- Almost completed.
- First plasma expected in 2015.
- Modular superconducting coils.

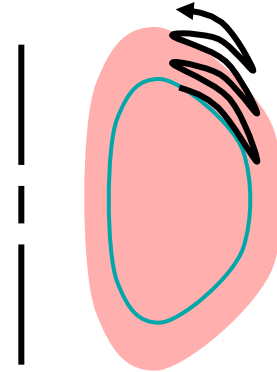


Omnigenity = no unconfined orbits.

Tokamak



Stellarator



- Unconfined particles can damage plasma-facing components.
- Reduced t_E

For a reactor, then, a stellarator must be nearly *omnigenous*:

$$0 = \Delta\psi \text{ per bounce} = \oint_{\text{bounce}} (\mathbf{v}_d \cdot \nabla\psi) dt \quad \text{for all } \mu.$$

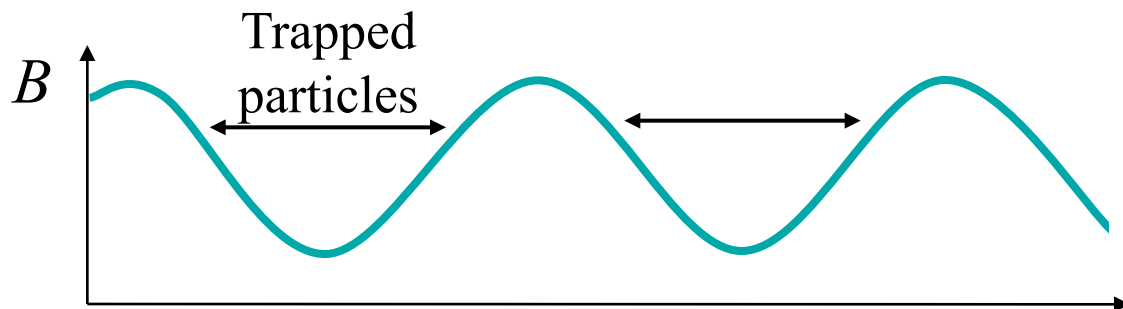
Equivalent definition: J is a flux function,

where $J = \oint_{\parallel} v_{\parallel} dl$ is the longitudinal invariant.

Another perspective

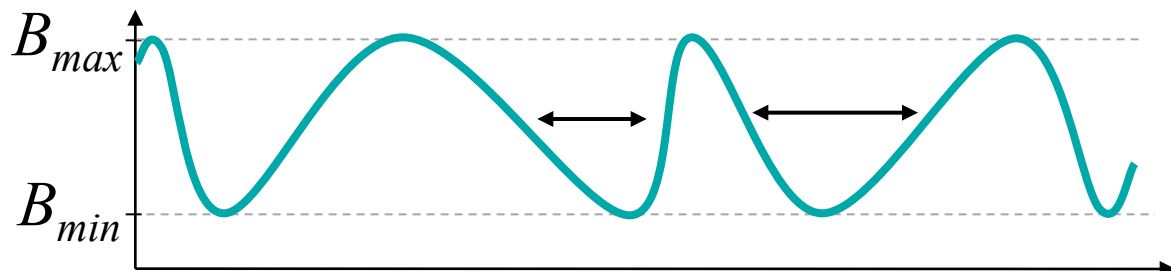
Tokamak or
quasisymmetric stellarator:

$$\Delta\psi = 0$$



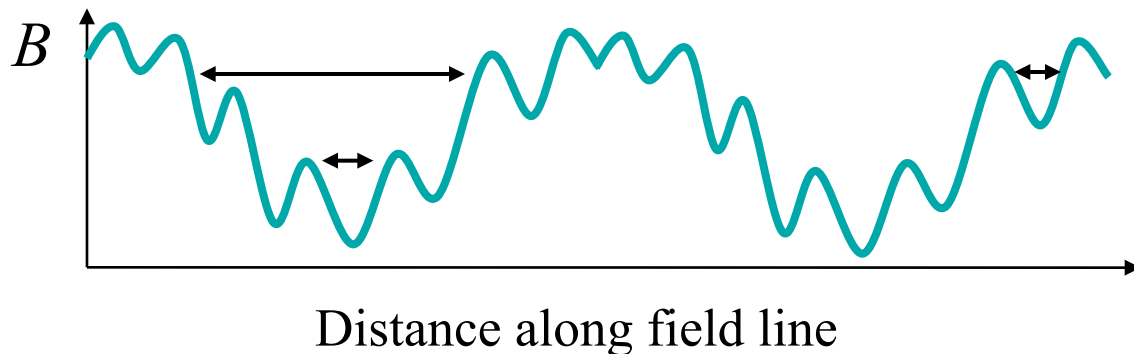
Omnigenous
stellarator:

$$\Delta\psi = 0$$



Non-optimized stellarator:

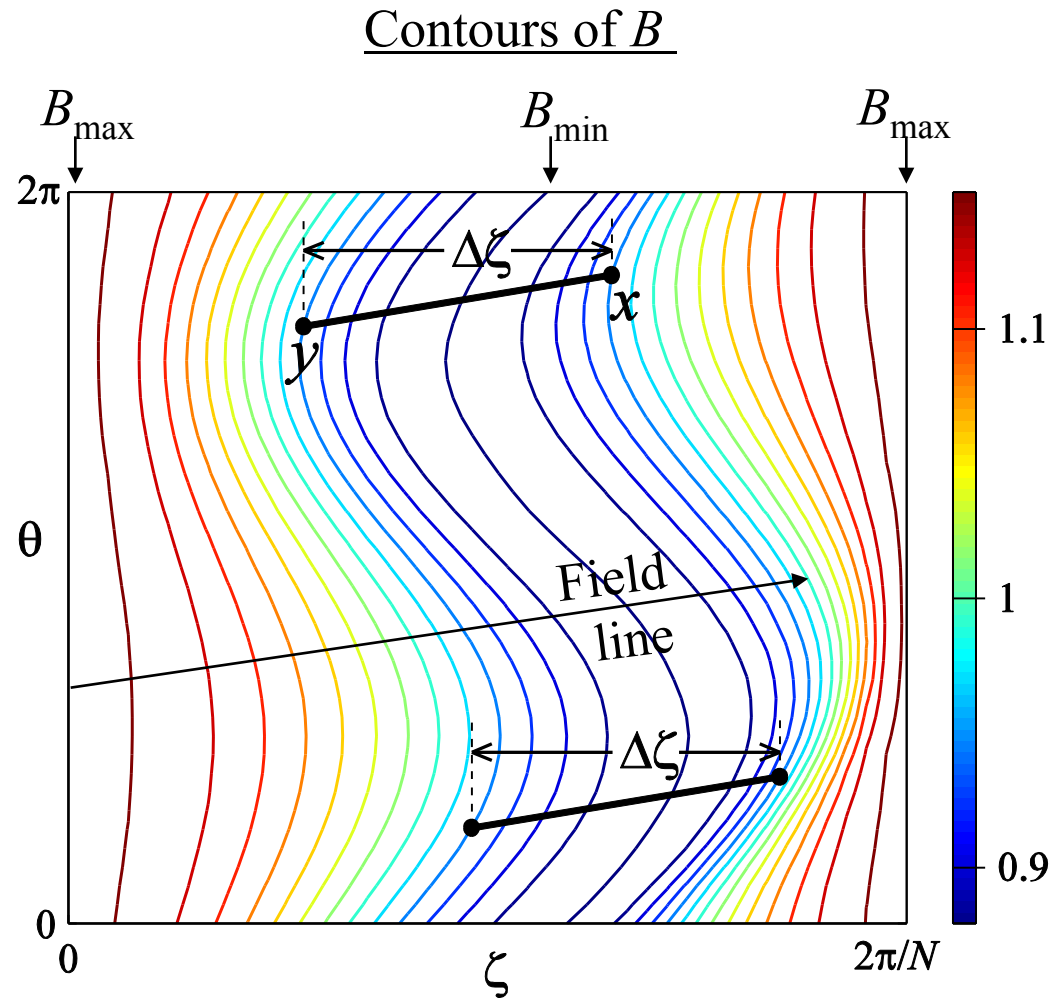
$$\Delta\psi \neq 0$$



Omnigenity constrains shape of B contours

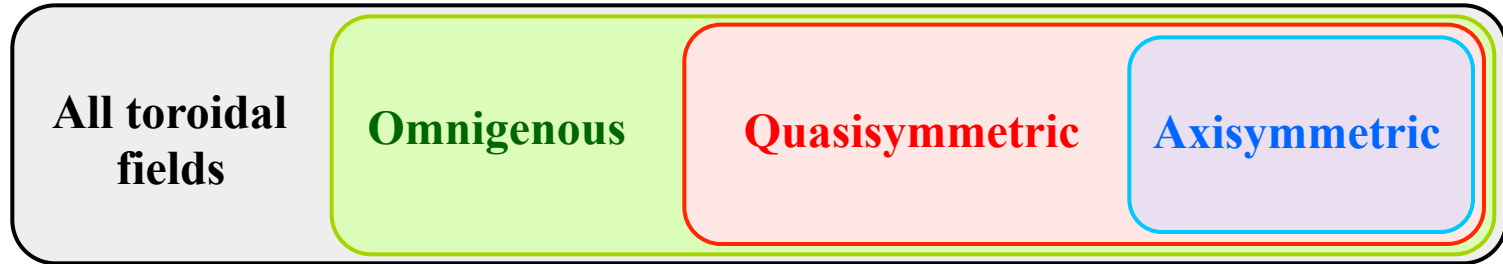
- Dz between the two points with same B on a field line is independent of field line.

(Cary & Shasharina, *PoP* 1997)

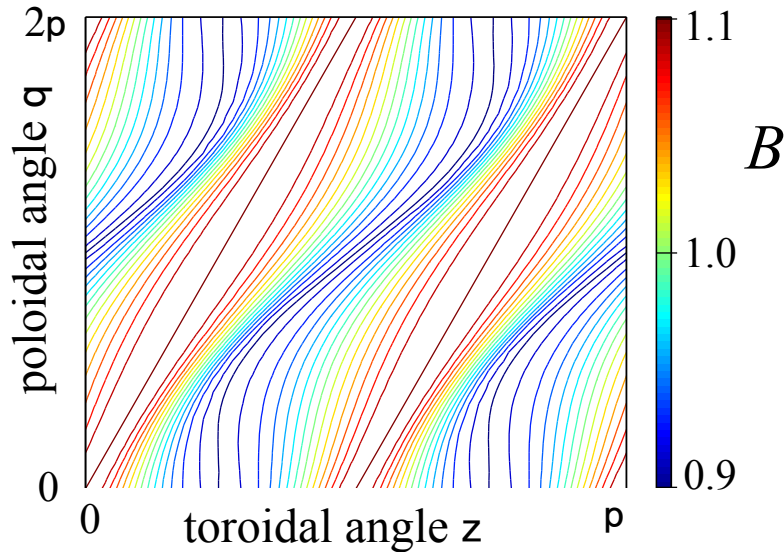


Omnigenity is more general than quasisymmetry.

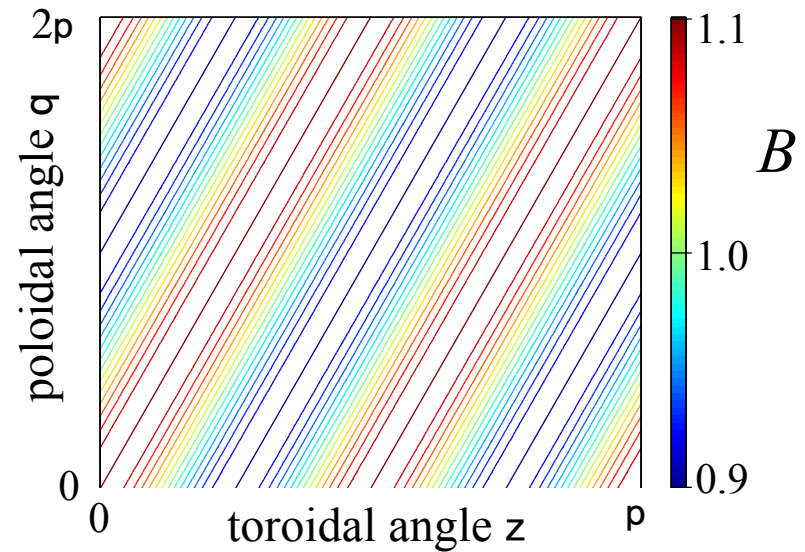
Cary & Shasharina, PoP (1997), PRL (1997)



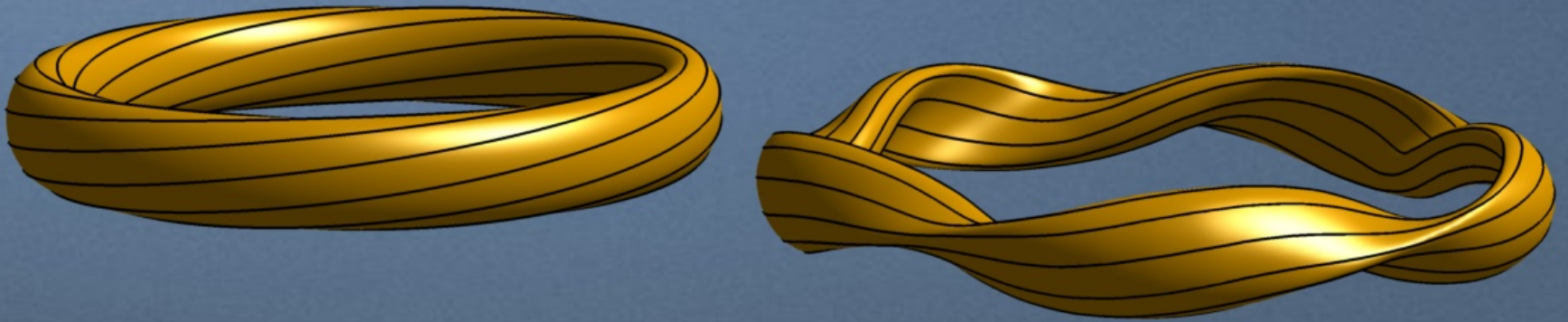
Omnigenity: B contours may be curved



Quasisymmetry: $B = B(Mq - Nz)$



Breaking axisymmetry in magnetized plasmas

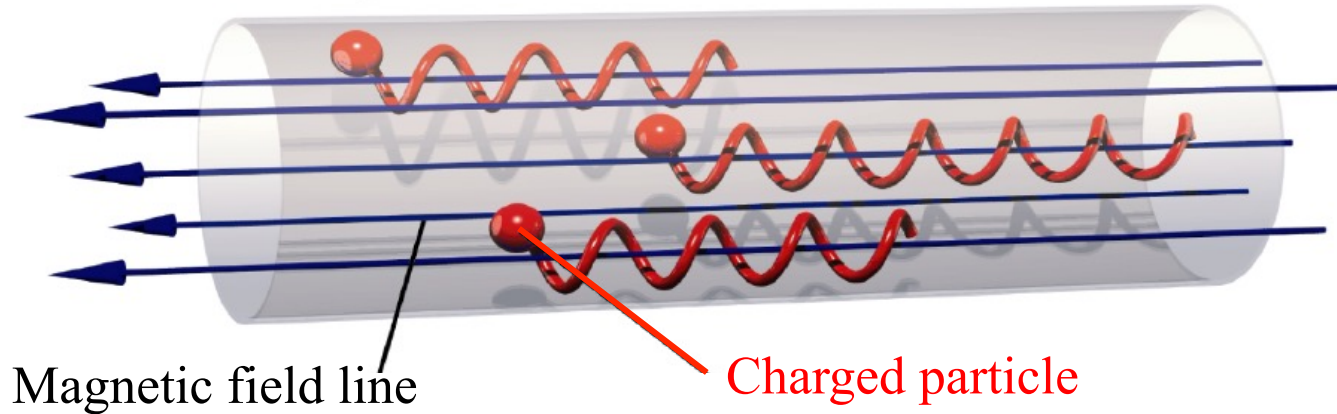


Matt Landreman
University of Maryland

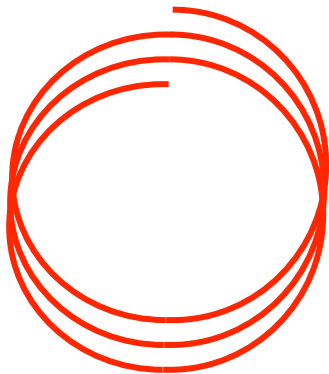
Thanks to A Mollen, H Smith, and P Helander
Max Planck Institute for Plasma Physics

Confining charged particles with a magnetic field is tricky.

Uniform straight \mathbf{B} : confinement \perp to \mathbf{B} , but end losses.



But if field lines are bent, particles drift off them.

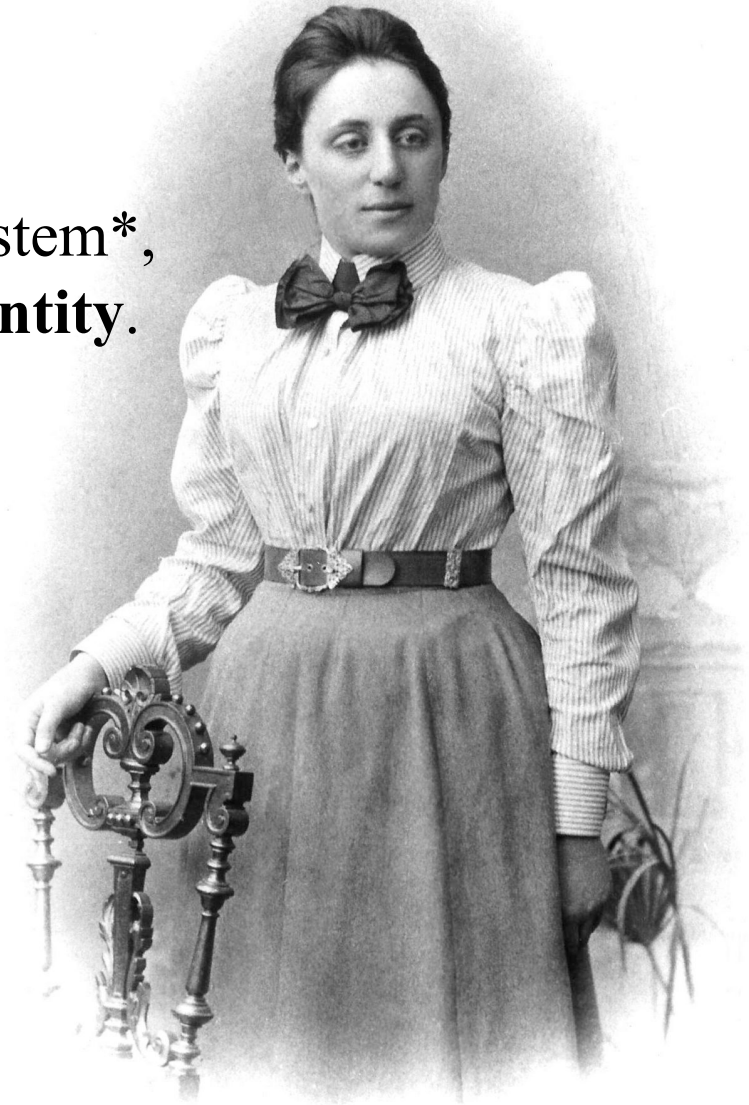


To confine particles, constrain their position with a conservation law.

Noether's theorem:

For each **continuous symmetry** of a system*, there is a corresponding **conserved quantity**.

* For this talk: Lagrangian is independent of a coordinate.



Emmy Noether (1882-1935)

Axisymmetry + Noether's Theorem is one way to achieve magnetic confinement.

Continuous rotational symmetry \Rightarrow Canonical angular momentum is conserved.

$$L_\phi = mv_\phi R + qA_\phi R = \text{constant}$$

 vector potential: $\mathbf{B} = \nabla \times \mathbf{A}$

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Strong \mathbf{B} limit $\Rightarrow |mv_\phi| \ll |qA_\phi| \Rightarrow$ Particles stuck to constant- $A_\phi R$ surfaces.

Axisymmetry + Noether's Theorem is one way to achieve magnetic confinement.

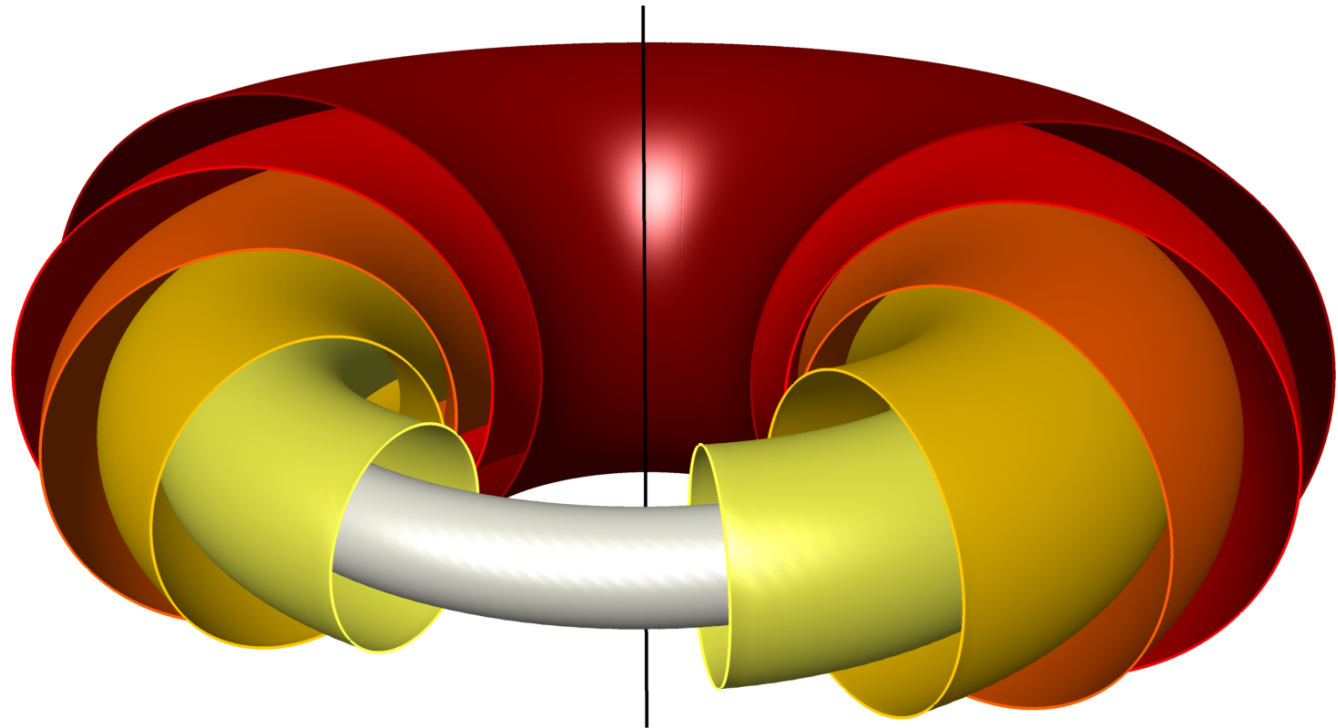
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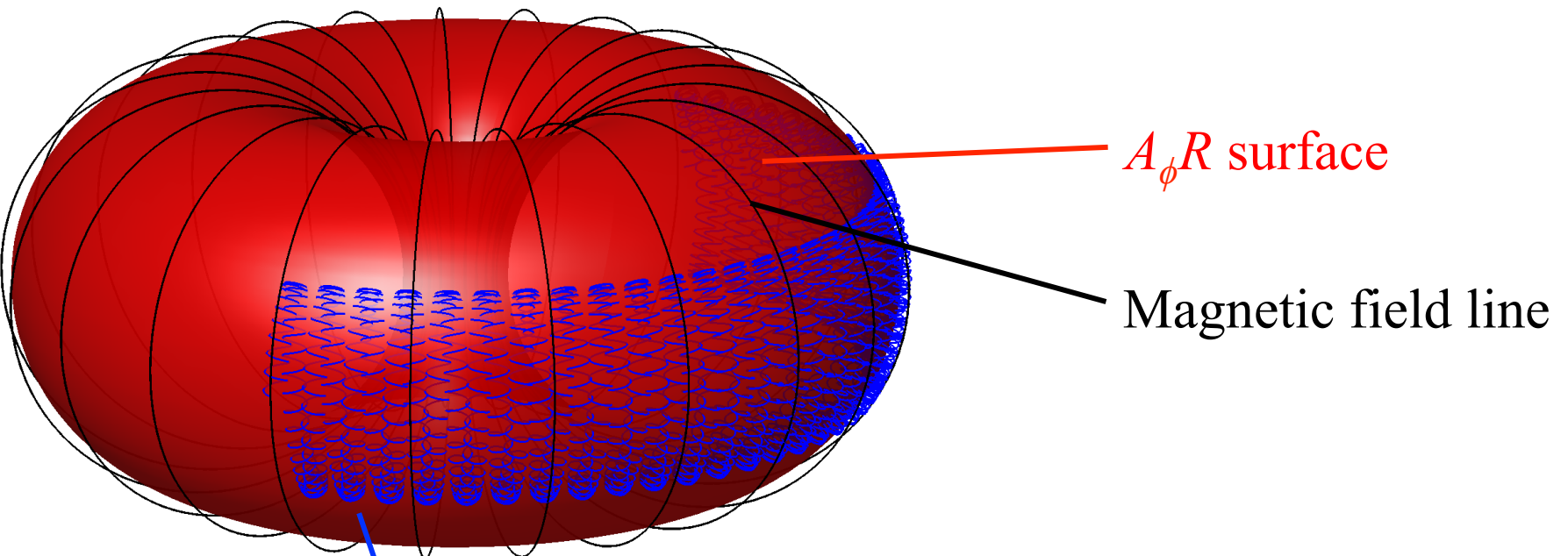
\swarrow vector potential: $\mathbf{B} = \nabla \times \mathbf{A}$

Strong \mathbf{B} limit $\Rightarrow |mv_\phi| \ll |qA_\phi| \Rightarrow$ Particles stuck to constant- $A_\phi R$ surfaces.

If $A_\phi R$ surfaces are bounded like this, then particles will be confined:



In axisymmetry, particles are confined (close) to $A_\phi R$ surfaces, despite complicated orbits.



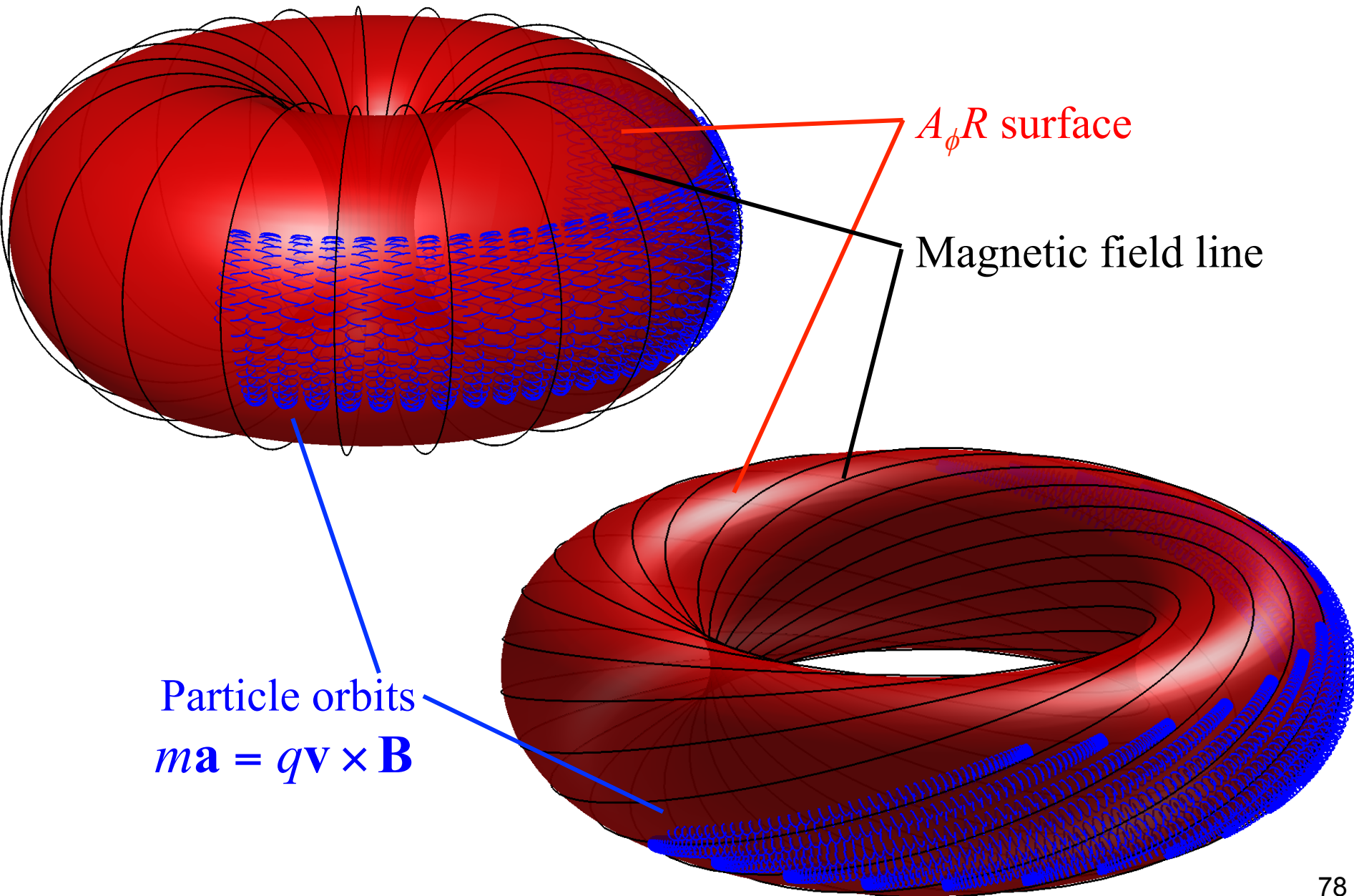
$A_\phi R$ surface

Magnetic field line

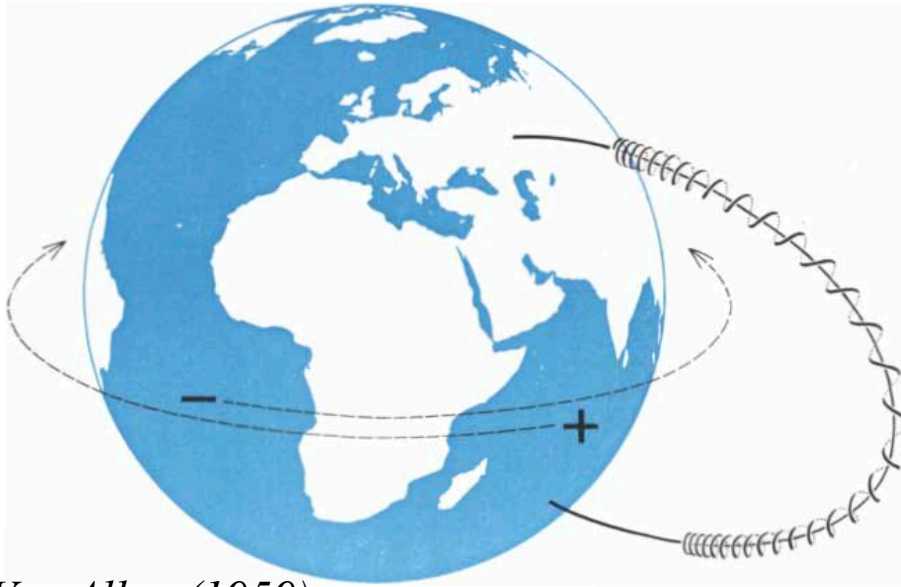
Particle orbits

$$m\mathbf{a} = q\mathbf{v} \times \mathbf{B}$$

In axisymmetry, particles are confined (close) to $A_\phi R$ surfaces, despite complicated orbits.

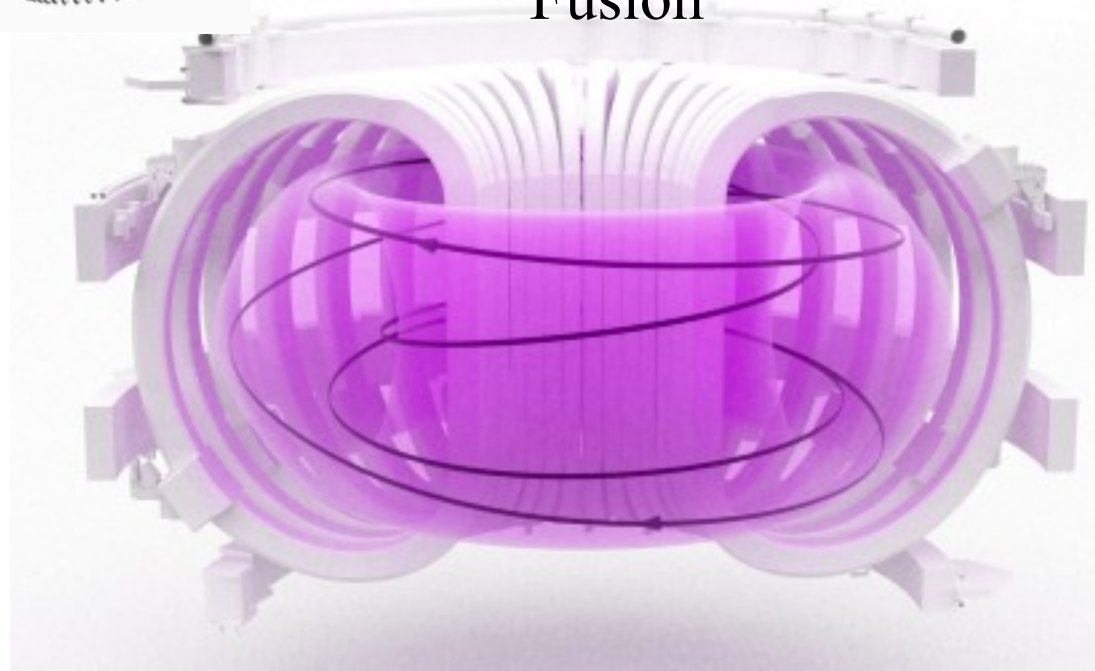


Particles are actually confined this way in nature and in the laboratory.

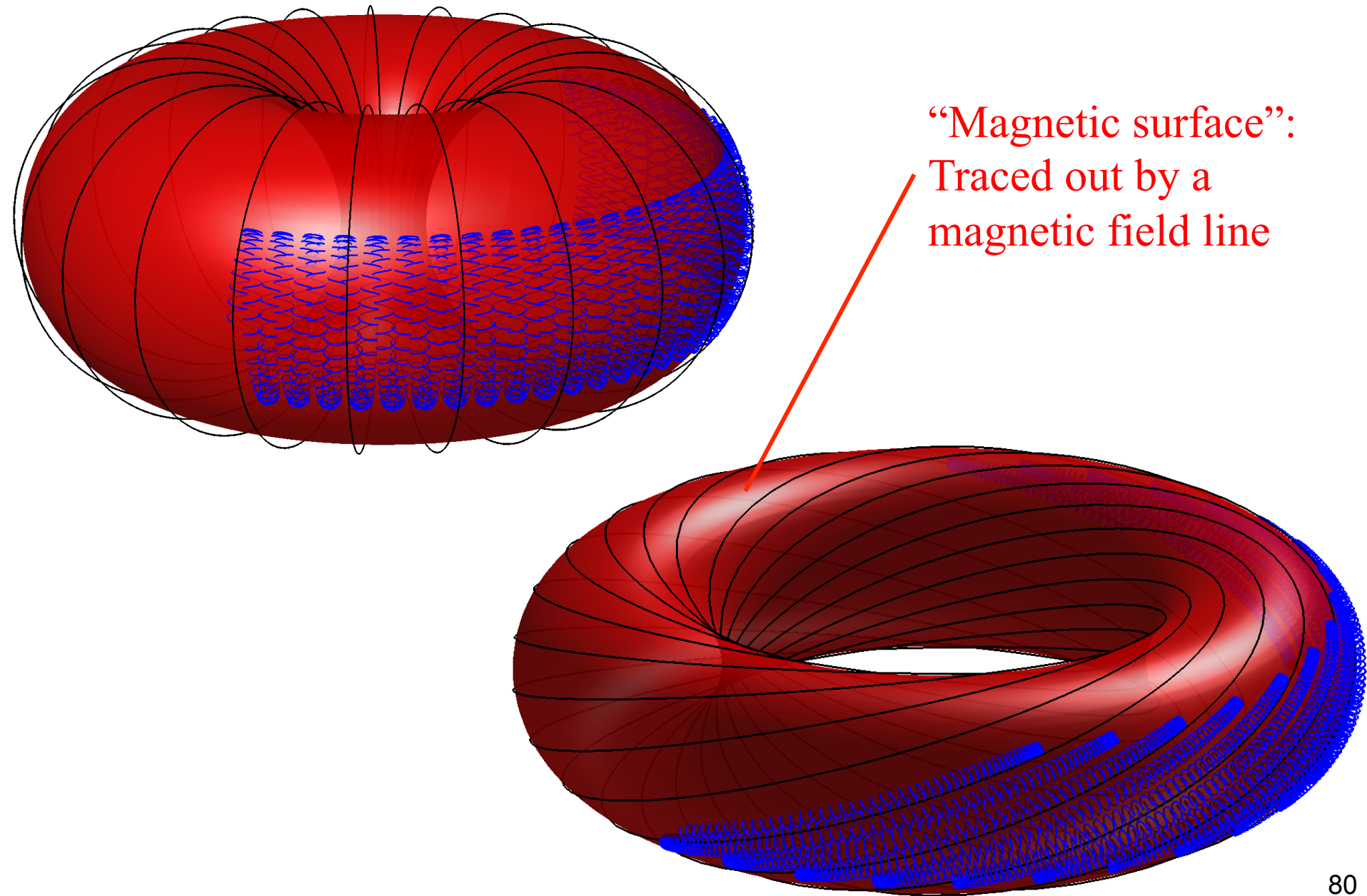


Van Allen (1959)

Fusion



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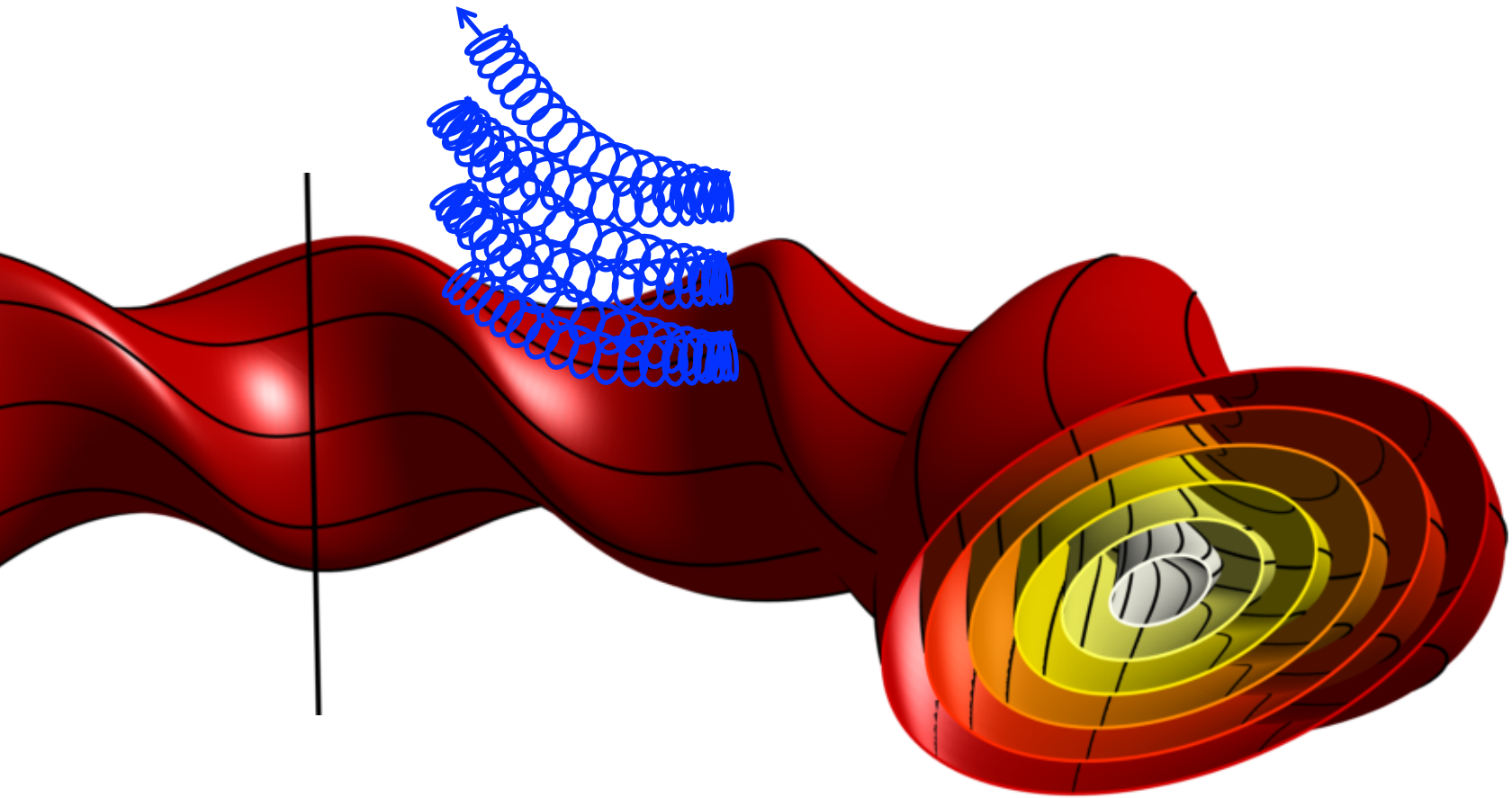


When axisymmetry is broken, usually some particle trajectories are not confined.

To confine particles in nonaxisymmetric \mathbf{B} , you better still have magnetic surfaces:

Chaotic \mathbf{B} would allow inside & outside to mix even without cross- \mathbf{B} drift.

But even with magnetic surfaces, particles need not stay on a surface.

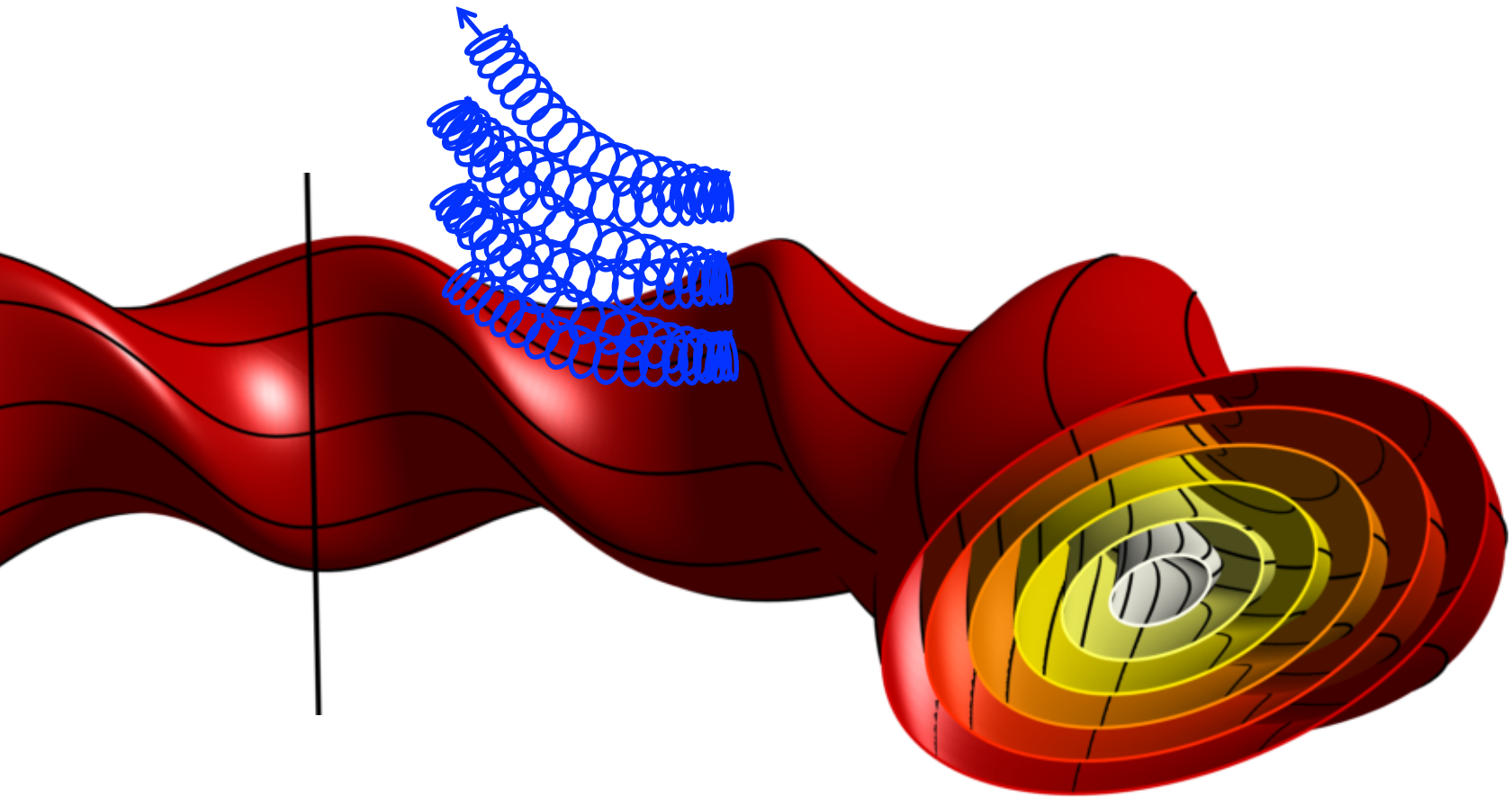


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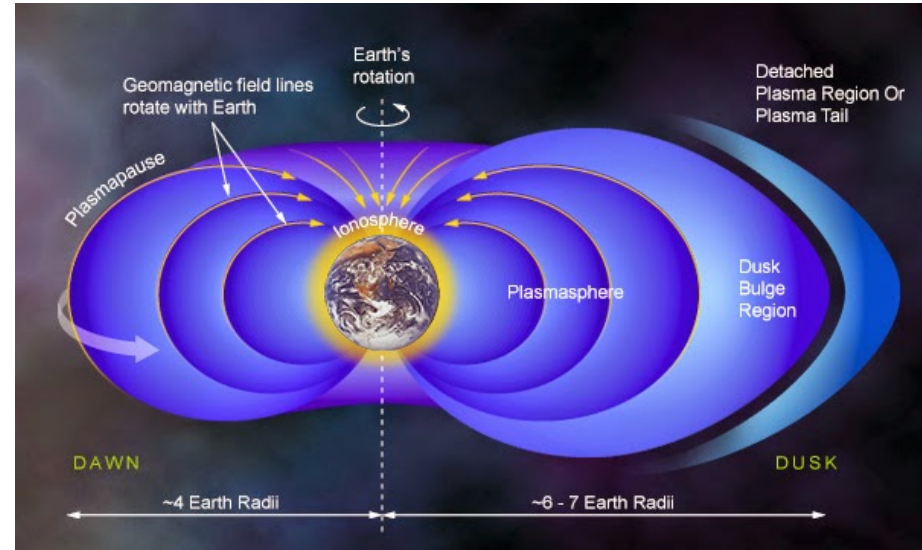
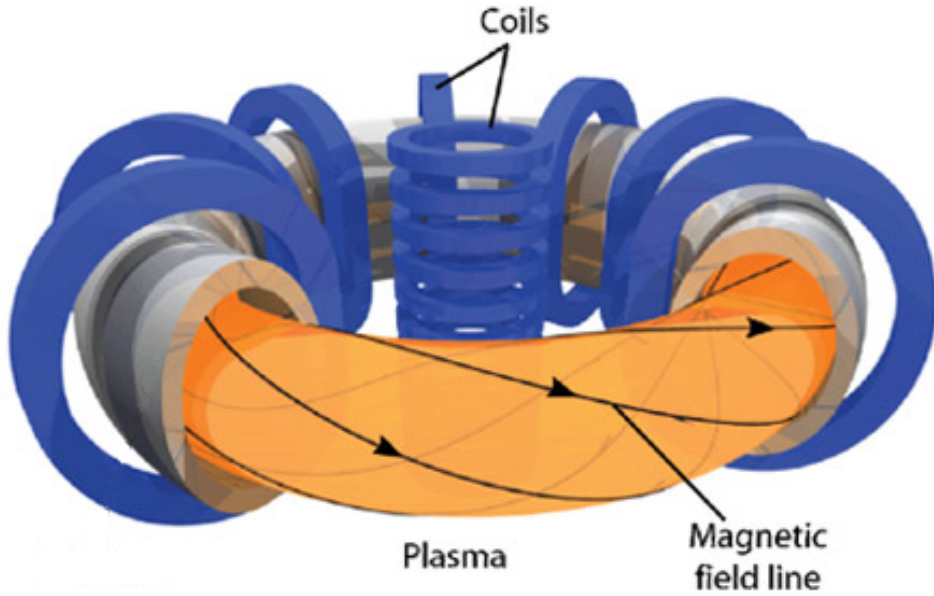
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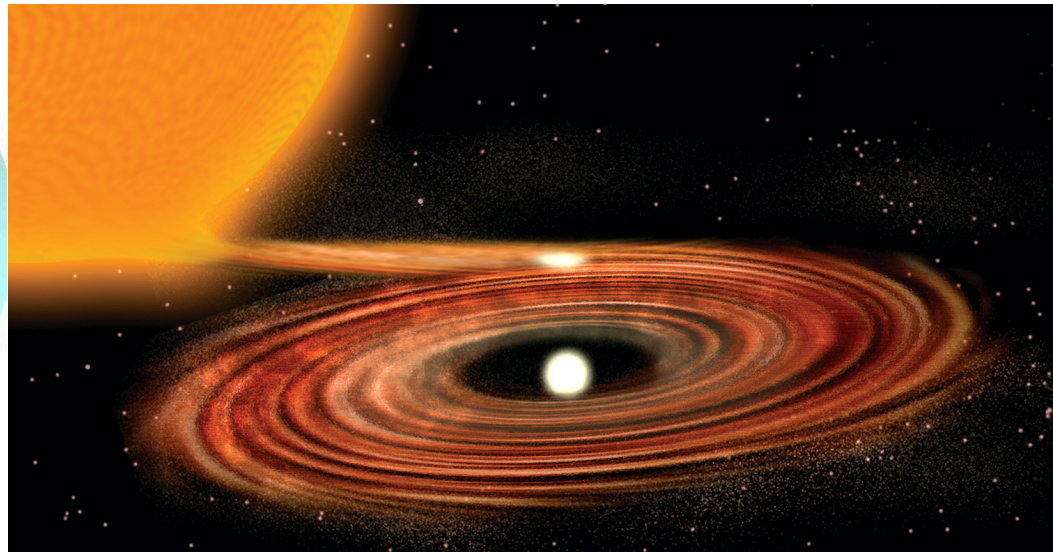
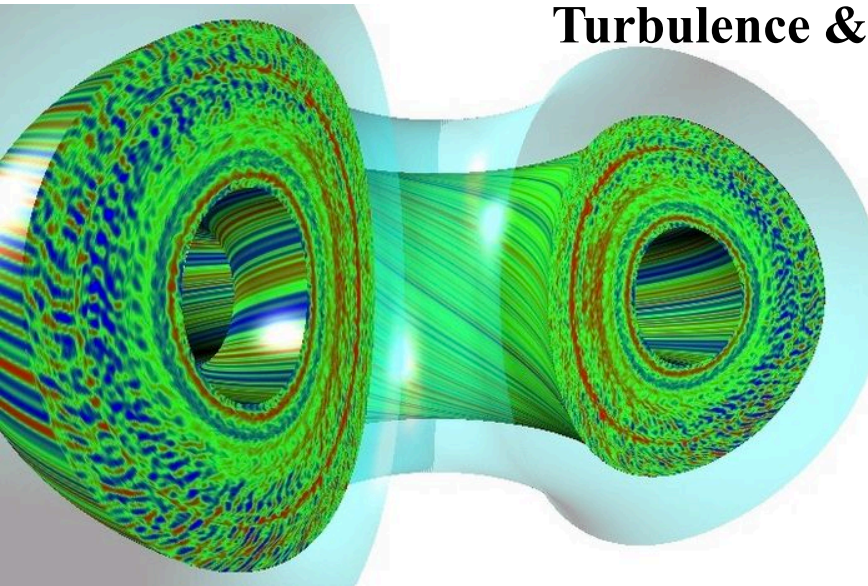


But are there *some* nonaxisymmetric shapes with good confinement?

Complication 1: No plasma is perfectly axisymmetric.

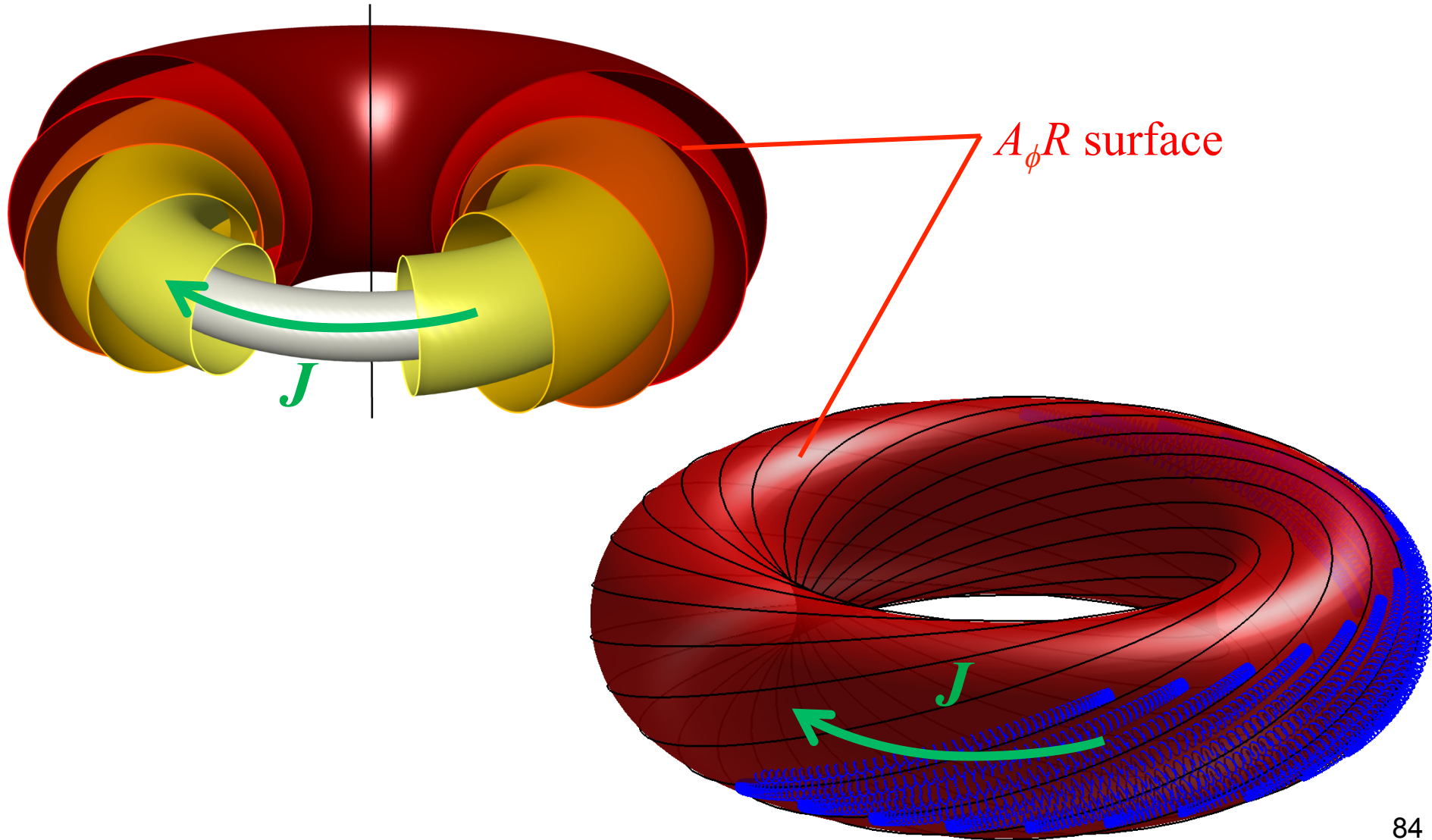


Turbulence & waves break symmetry:



Complication 2: Axisymmetric confinement requires an internal current.

$\nabla \times (\nabla \times \mathbf{A}) = \nabla \times \mathbf{B} = \mu_0 \mathbf{J}$ so nested $A_\phi R$ surfaces require a J_ϕ .



Outline

- Symmetry can give confinement.
- Hidden symmetry (“quasi-symmetry.”)
- Symmetry-breaking causes species separation: physics & simulations.
- Conclusions

Averaging over fast gyration, dynamics depend on \mathbf{B} through $|\mathbf{B}|$.

Lagrangian for particle in magnetic field: $\mathcal{L} = \frac{m}{2} |\dot{\mathbf{x}}|^2 + q\mathbf{A} \cdot \dot{\mathbf{x}}$ (Neglect \mathbf{E})

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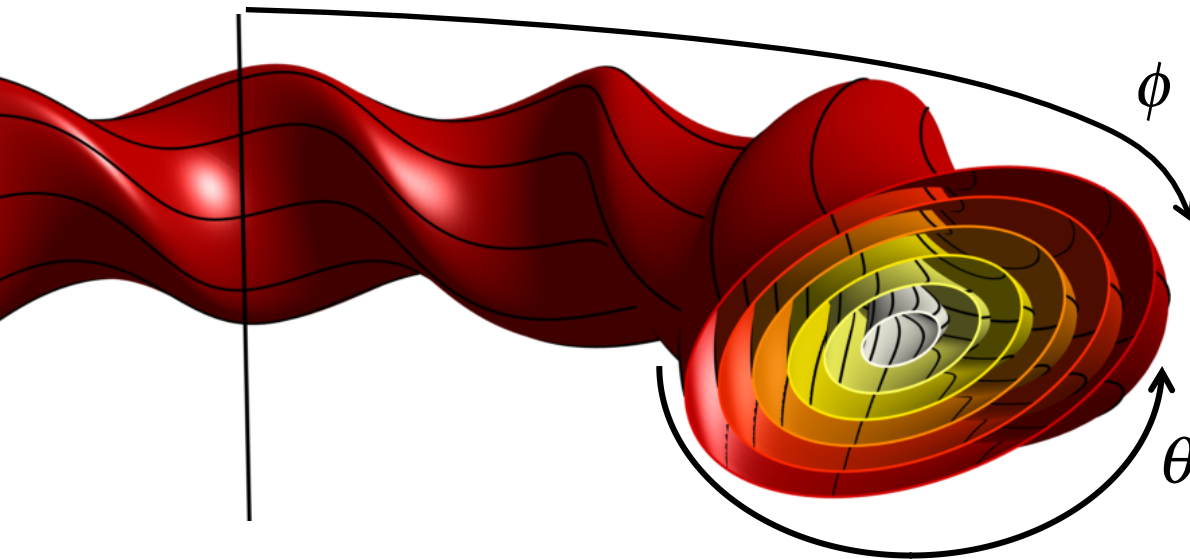
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use angle coordinates:

$$\mathcal{L} = \frac{mG^2 \dot{\phi}^2}{2B^2} - \mu B + q\psi \dot{\theta} - q\chi \dot{\phi}$$

Independent of θ and ϕ

Only depends on θ and ϕ through $B = |\mathbf{B}|$!



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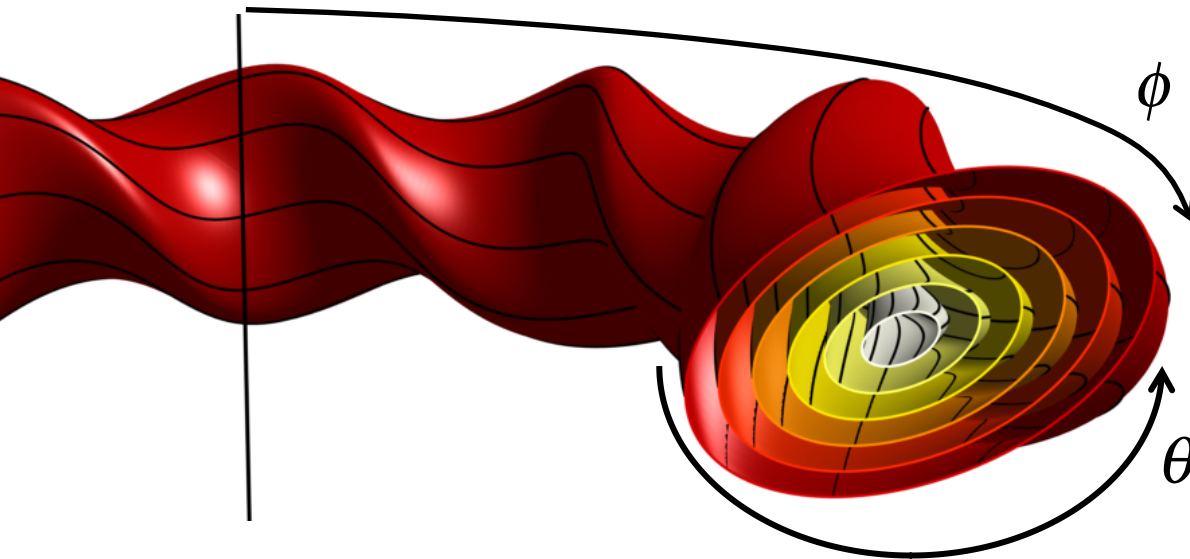
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Only depends on θ and ϕ through $B = |\mathbf{B}|$!



If $\frac{\partial |\mathbf{B}|}{\partial \phi} = 0$,

then canonical

angular momentum $\frac{\partial \mathcal{L}}{\partial \dot{\phi}}$

would be conserved.

Quasi-symmetry: $|\mathbf{B}|$ is symmetric even if \mathbf{B} is not.

- Can you actually make a non-symmetric \mathbf{B} with symmetric $|\mathbf{B}|$?

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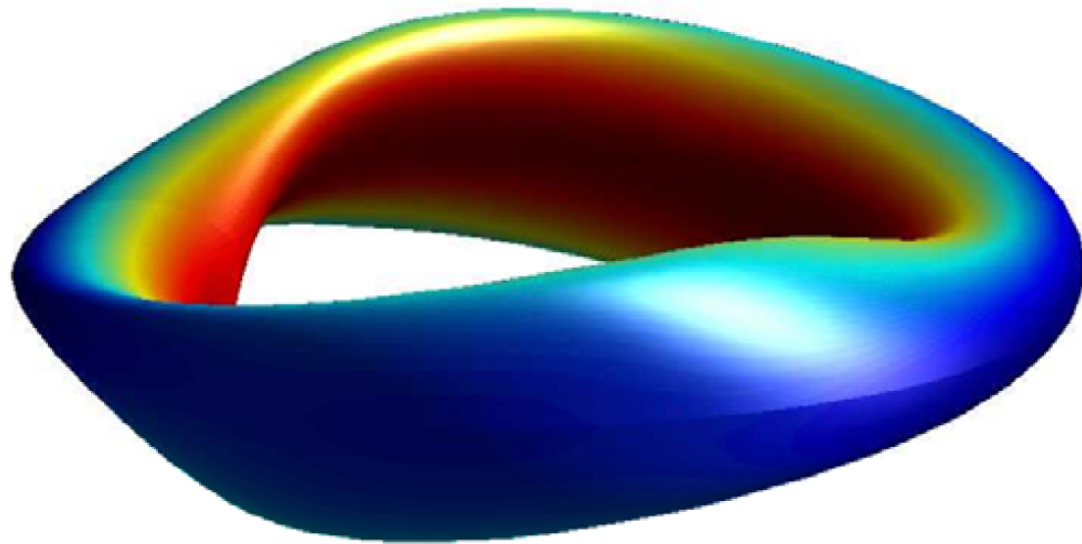
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Yes!!

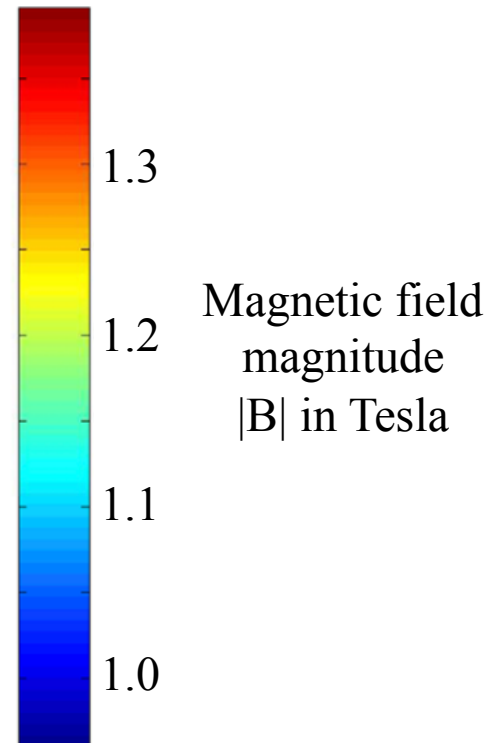
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Can you actually find such a magnetic field? **Yes!**

ESTELLE experiment: $\partial B / \partial \phi = 0$



Drevlak et al, (2013)

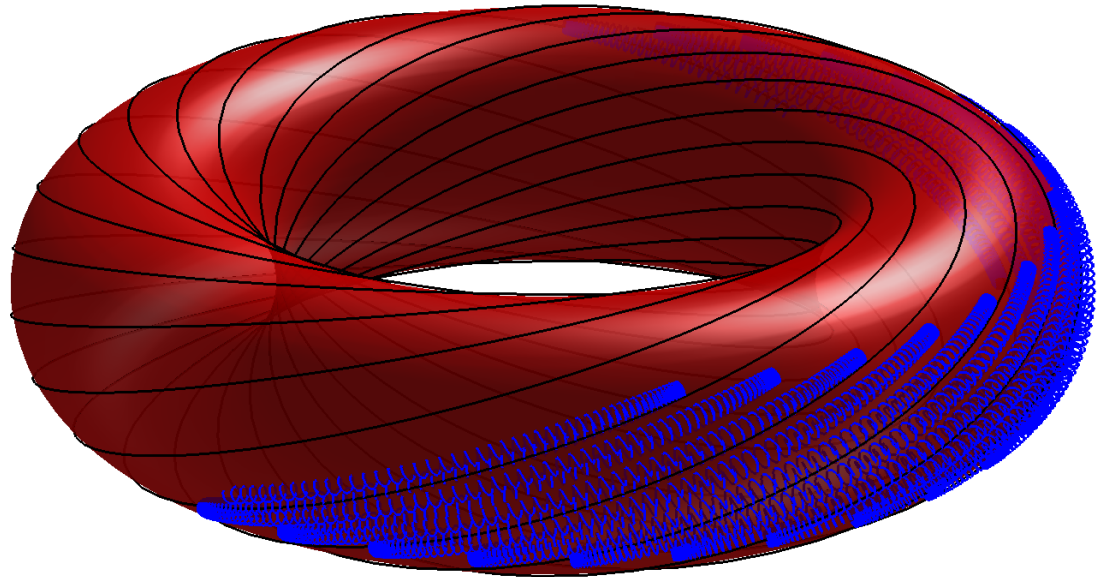


- Found using numerical optimization.
- Particle orbits are confined just as in an axisymmetric \mathbf{B} .
- No \mathbf{J} required in plasma \Rightarrow Very stable.

Tokamak vs stellarator

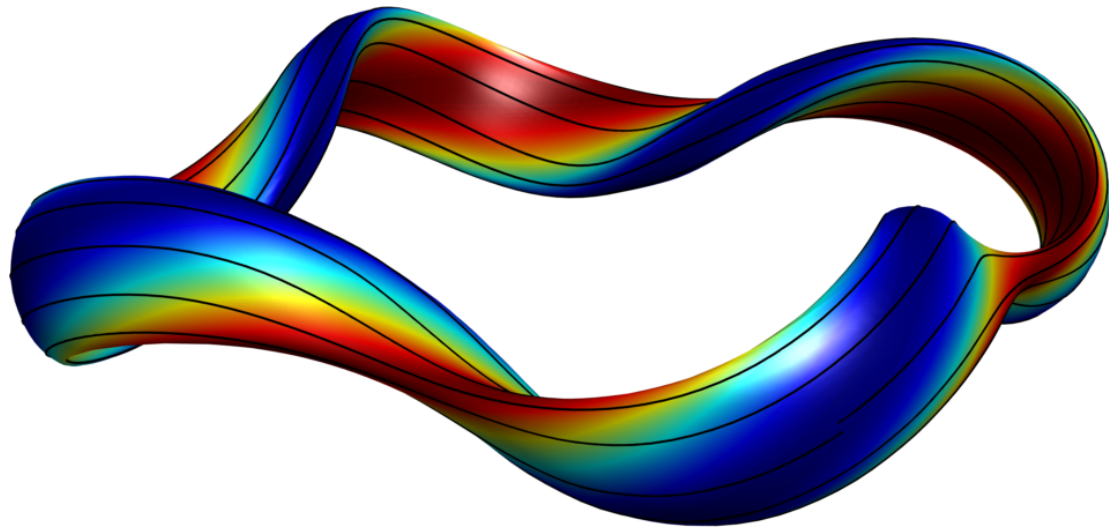
Tokamak:

- Axisymmetric
- Robust confinement
- Requires J_ϕ in plasma:
HUGE problem!



Stellarator:

- Nonaxisymmetric
- Requires careful shaping to get confinement
- No \mathbf{J} required in plasma

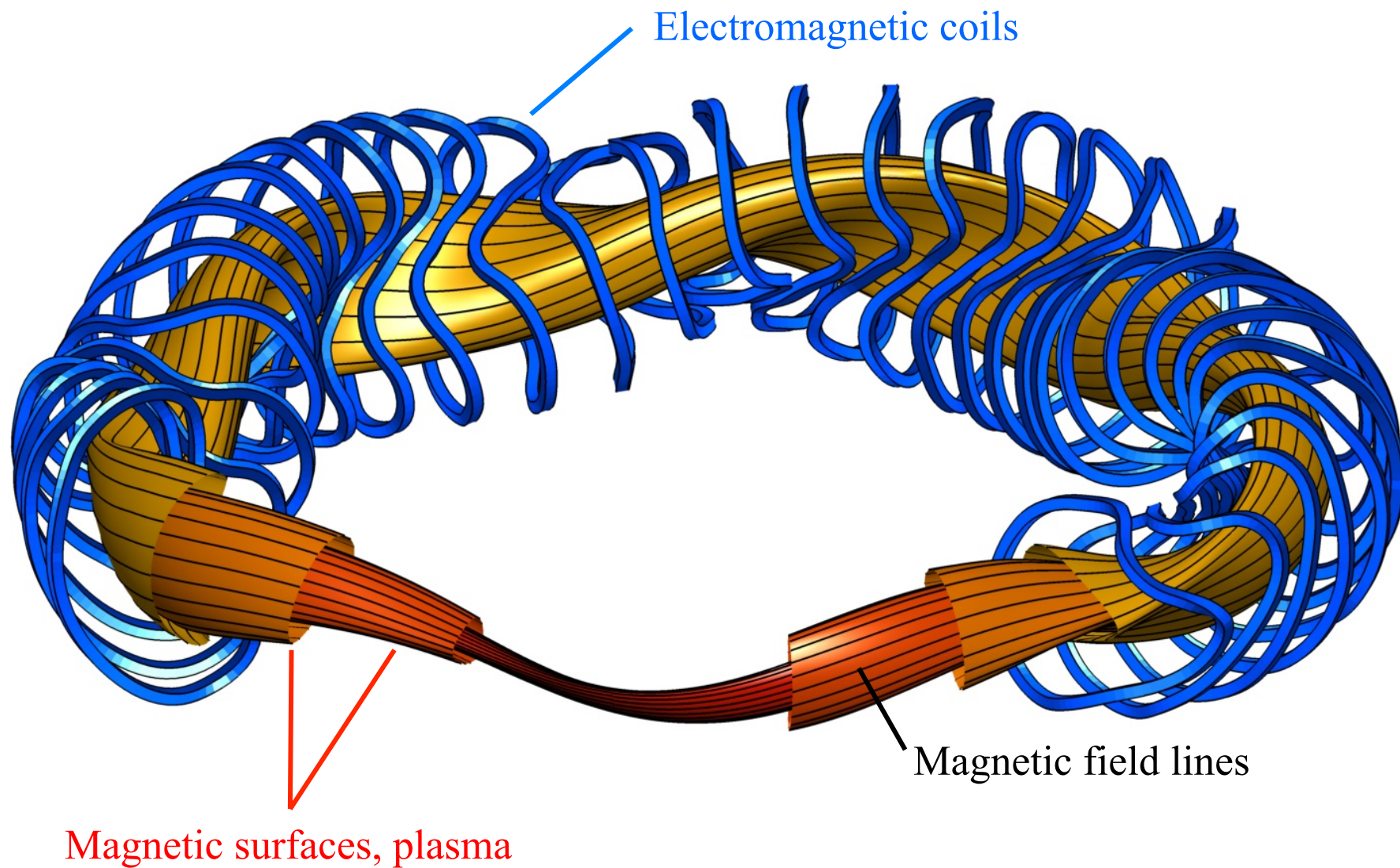


Example of very nonaxisymmetric magnetic confinement: Wendelstein 7-X (Germany)

Science

Oct 21, 2015

30 minute plasmas eventually



Example of very nonaxisymmetric magnetic confinement: W7-X (Germany)

Science

Oct 21, 2015

Good-enough particle confinement, but not perfect -
Not quasisymmetric.

