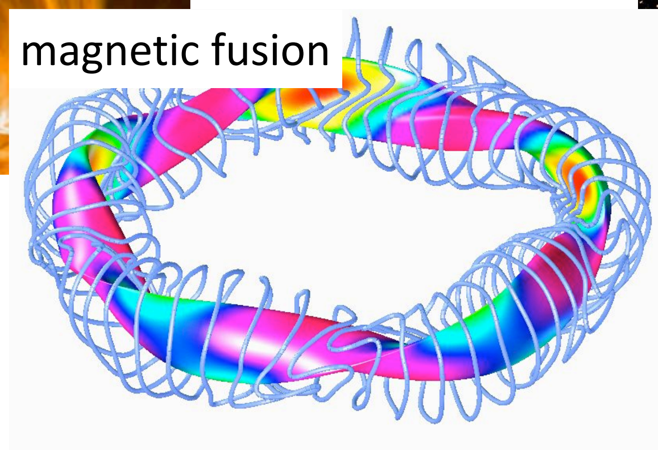
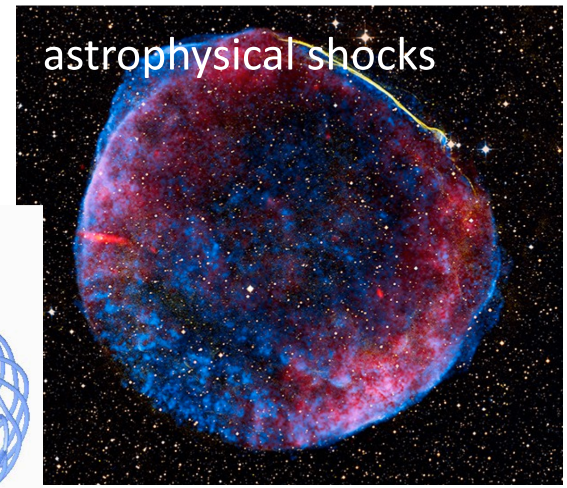
Single Particle Motion

Will Fox
PPPL and Princeton University





SULI Introductory Course in Plasma Physics
Princeton Plasma Physics Laboratory
June 11, 2018

Acknowledgement to Cami Collins (GA) for sharing a previous version of this presentation

My "Single-Researcher Motion" through Plasma Physics

NUF student

GENERAL ATOMICS

High-school Physics and math in Nepal

PhD student

MIT

UNH
Space Science Center

Research Scientist

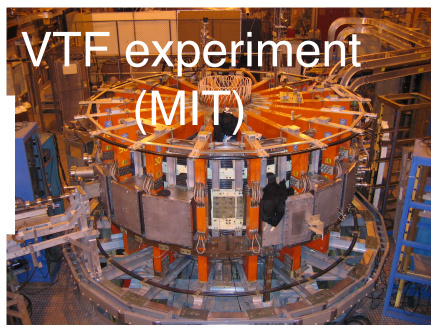
PPPL

Undergrad thesis at PPPL

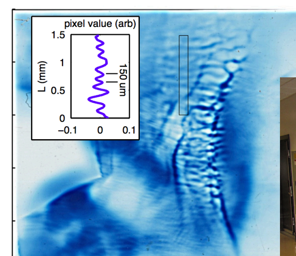
2001 **2002** **2002-2009** **2009-2013** **2013 - now**

Institute of Physics Publishing and International Atomic Energy Agency — Nuclear Fusion
Nucl. Fusion 42 (2002) 1124–1133 — PII: S0029-5515(02)38137-7

Analysis of current drive using MSE polarimetry without equilibrium reconstruction



Expts at NIF and OMEGA



Key Points to Take Away

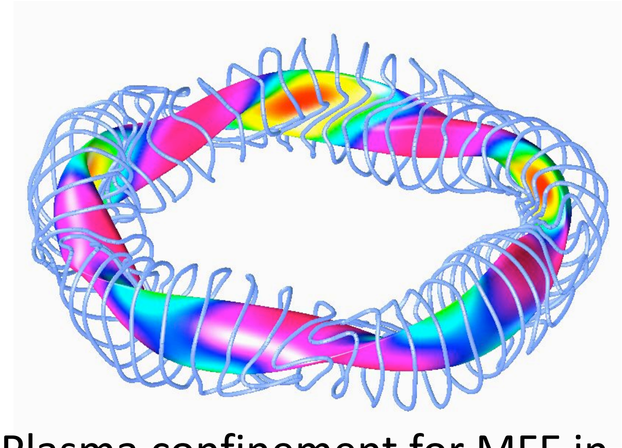
- **Motion of Single particles important to understand behavior of plasmas**
- **How magnetic fields modify single particle motion**
    - **Gyromotion about a *guiding center***
    - **Forces can cause guiding center *drift***
- **Some real life consequences:**
    - Why do tokamaks have helical B fields?
    - Why do astrophysical shocks need a magnetic field?

References

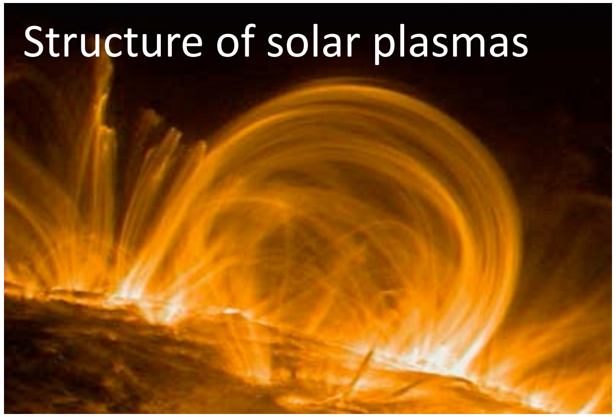
- NRL Plasma Formulary
  www.nrl.navy.mil/ppd/content/nrl-plasma-formulary

- Introduction to Plasma Physics and Controlled Fusion by F. Chen

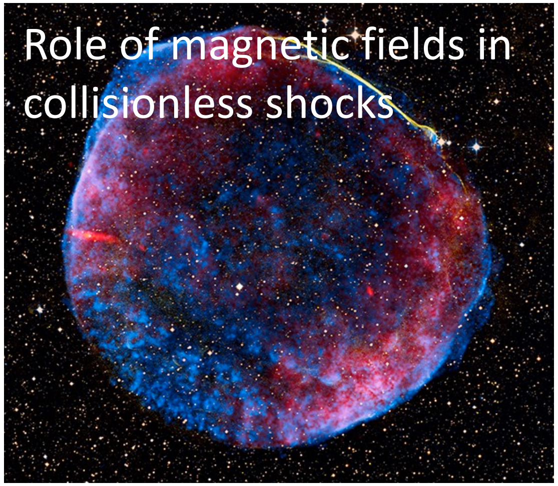
Single-particle motion is key intuition to understand many problems in plasma physics



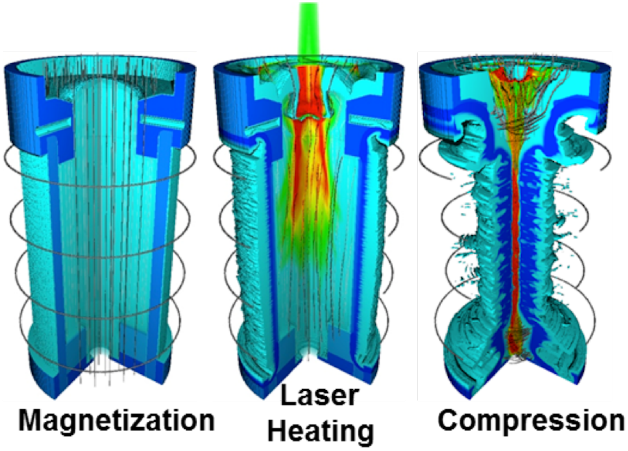
Plasma confinement for MFE in stellarators and tokamaks



Structure of solar plasmas

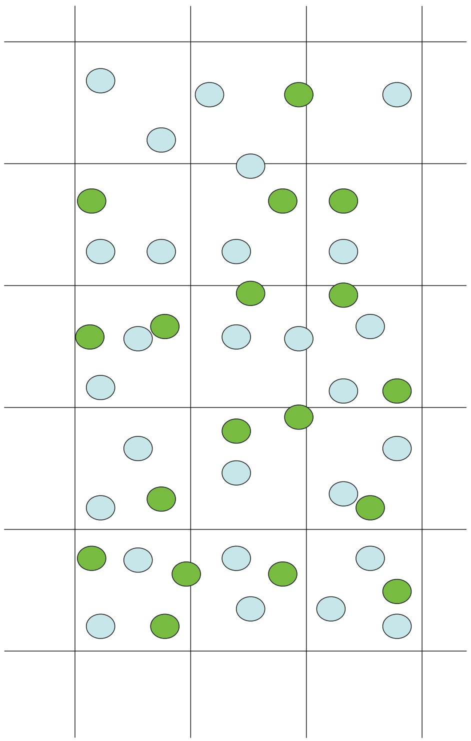


Role of magnetic fields in collisionless shocks



Magnetically-driven Inertial-confinement fusion in Z-pinches

Ideas behind single-particle physics threads all the way through to the most modern analysis and simulation techniques



- The Particle-in-Cell simulation technique

Calculate particle motion in **E** and **B** fields [**F** = m**a**]

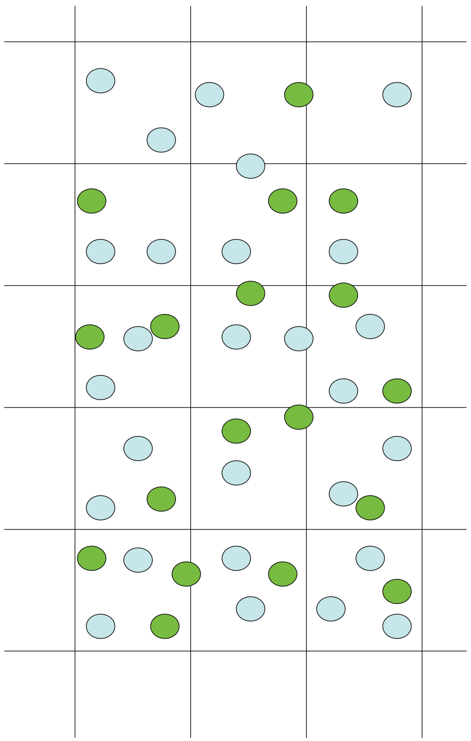
Sum over particles to calculate current **J** and charge density **ρ**

Update **E** and **B** fields on mesh [Maxwell's Equations]

- Idea allows plasma simulations that run on the world's fastest supercomputers
- Some of you may use "gyrokinetic" simulations during your internship, which take advantage of single-particle results here to speed up calculations

[K. Germaschewski, WF, et al, JCP 2016]

Ideas behind single-particle physics threads all the way through to the most modern analysis and simulation techniques



- The Particle-in-Cell simulation technique

Calculate particle motion in **E** and **B** fields [**F** = m**a**]

TODAY

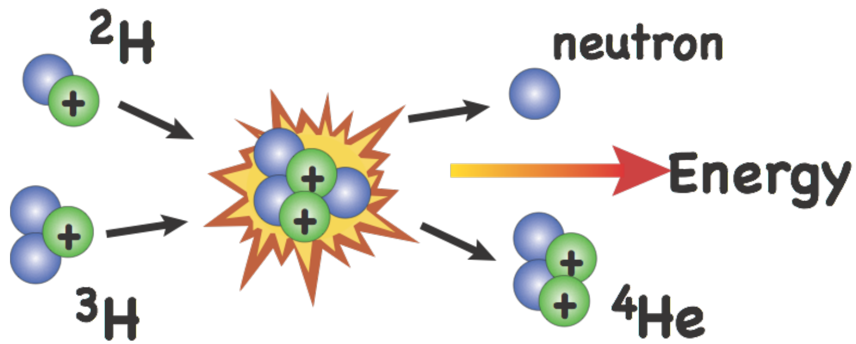
Sum over particles to calculate current **J** and charge density **ρ**

Update **E** and **B** fields on mesh Maxwell's Equations

- Idea allows plasma simulations that run on the world's fastest supercomputers
- Some of you may use "gyrokinetic" simulations during your internship, which take advantage of single-particle results here to speed up calculations

[K. Germaschewski, WF, et al, JCP 2016]

Plasma Physics is the Basis for Fusion Research



- **Sustained fusion reactions require enough particles (density) that are energetic enough (temperature) and collide often enough (confinement time).**

- **The fusion triple product is the figure of merit:**

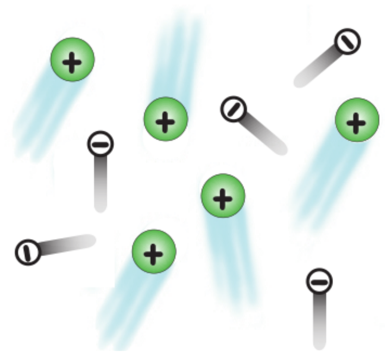
$$nT\tau_E \gtrsim 5\times 10^{21}\ \mathrm{keV\ s\ m^{-3}}$$

T~100-200 million K
n~2-3x10^{20} ions/m^3
τ ~1-2 s

D & T is a plasma at these temperatures

We can understand a lot about how fusion devices confine plasma by studying single particle motion.

Typical velocity of a 100 million K ion:

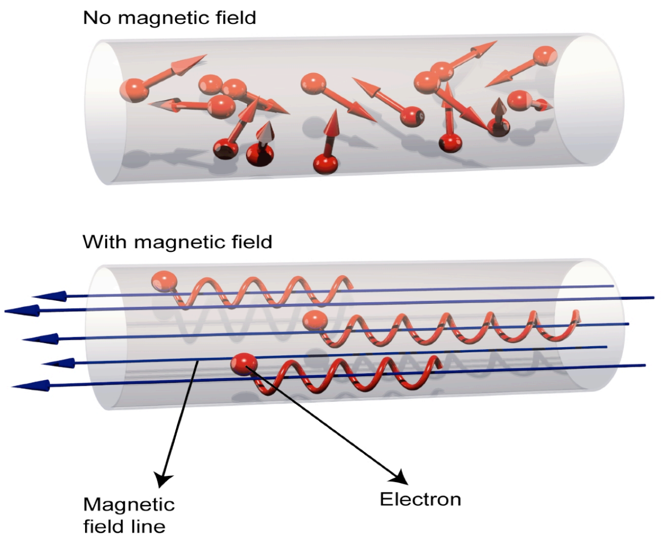


$$kT = \frac{1}{2} m v_{\mathrm{th}}^2$$

$$v_{\mathrm{th}} \sim 6 \times 10^5 \ \mathrm{m/s}$$

Even with ~10^{20} *ions/m*^{3} , the ion would travel ~10 km before colliding with another

Plasmas are effectively collisionless (a common theme)



The ITER tokamak has R_{major}=6.2 m → ~40 m circumference

The trick: use magnetic fields

Charged Particles Feel The (Lorentz) Force

- **A particle with charge (q) moving with velocity ($\mathbf{v}$) in the presence of electric and magnetic fields will experience a force:**

$$\mathbf{F} = q(\mathbf{E} + \mathbf{v} \times \mathbf{B})$$

We know from Newton's second law of motion that force causes acceleration:

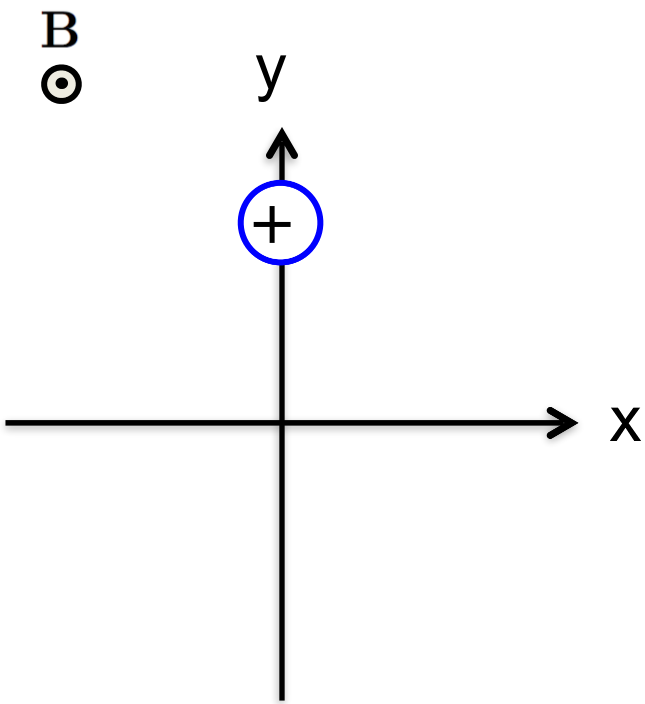
$$\mathbf{F} = m\mathbf{a}$$

A charged particle moving perpendicular to the magnetic field feels a force

How Does a Charged Particle Move in a Magnetic Field?

- **Consider the motion of a particle in a constant, uniform B field**

$$\mathbf{B} = B_z\hat{\mathbf{z}} \qquad \mathbf{E} = \mathbf{0}$$



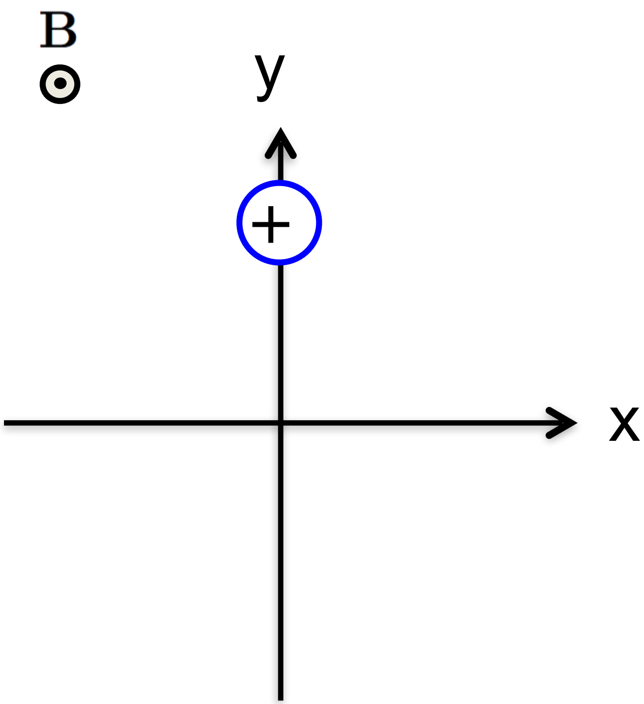
Then $$\mathbf{F} = q(\cancel{\mathbf{E}} + \mathbf{v} \times \mathbf{B})$$

So we can write

$$\frac{d\mathbf{v}}{dt} = \frac{q\mathbf{v} \times \mathbf{B}}{m}$$

Now let's do the algebra

Goal: Solve the Equations of Motion for a Charged Particle In A Magnetic Field



$$\frac{d\mathbf{v}}{dt} = \frac{q\mathbf{v} \times \mathbf{B}}{m}$$

Let's break this into components:

$$\dot{v}_x \hat{\mathbf{x}} + \dot{v}_y \hat{\mathbf{y}} + \dot{v}_z \hat{\mathbf{z}} = \frac{q v_y B_z \hat{\mathbf{x}} - q v_x B_z \hat{\mathbf{y}}}{m}$$

The 'dot' represents $\frac{d}{dt}$

Matching components:

$$\dot{v}_x = \frac{q v_y B_z}{m} \qquad \dot{v}_y = -\frac{q v_x B_z}{m}$$

$$\dot{v}_z = 0$$

Particles move freely along the field line

Take Another Time Derivative & Substitute to Obtain Differential Equations For Each Spatial Coordinate

$$\dot{v}_x = \frac{q v_y B_z}{m} \qquad \dot{v}_y = -\frac{q v_x B_z}{m}$$

$$\ddot{v}_x = \frac{q \dot{v}_y B_z}{m} \qquad \ddot{v}_y = -\frac{q \dot{v}_x B_z}{m}$$

Rewriting, we get

$$\ddot{v}_x = -\left(\frac{q B_z}{m}\right)^2 v_x \qquad \ddot{v}_y = -\left(\frac{q B_z}{m}\right)^2 v_y$$

These may remind you of the equations for a simple harmonic oscillator

Solve the Differential Equations

$$\ddot{v}_x = -\left(\frac{qB_z}{m}\right)^2 v_x \qquad \ddot{v}_y = -\left(\frac{qB_z}{m}\right)^2 v_y$$

These differential equations can be solved using sines and cosines:

$$v_x = v_\perp \cos\left(\frac{|q|B_z}{m}t + \phi_0\right) \qquad v_y = \mp v_\perp \sin\left(\frac{|q|B_z}{m}t + \phi_0\right)$$

account for positive or negative q

$$v_\perp = \sqrt{(v_x^2 + v_y^2)}$$

the magnitude of the initial velocity perpendicular to $\boldsymbol{B}$

an arbitrary phase to match the initial velocity conditions

The Result: Circular Motion About A Guiding Center

$$v_x = v_\perp \cos\left(\frac{|q|B_z}{m}t + \phi_0\right) \qquad v_y = \mp v_\perp \sin\left(\frac{|q|B_z}{m}t + \phi_0\right)$$

Integrating, we obtain

$$x = \frac{mv_\perp}{|q|B_z}\sin\left(\frac{|q|B_z}{m}t + \phi_0\right) + x_0 \qquad y = \pm\frac{mv_\perp}{|q|B_z}\cos\left(\frac{|q|B_z}{m}t + \phi_0\right) + y_0$$

- **Charged particles undergo circular orbits about a guiding center** (x_0, y_0)

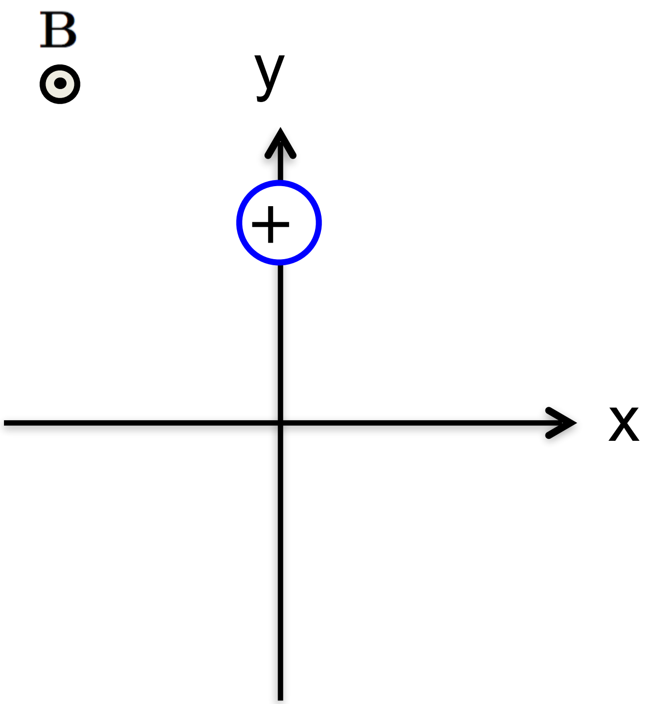
$$r_L \equiv \frac{mv_\perp}{|q|B}$$ **Larmor radius**

$$\omega_c \equiv \frac{|q|B}{m}$$ **Cyclotron frequency**

Gyromotion of a Charged Particle In A Magnetic Field

$$x = r_L \sin(\omega_c t + \phi_0) + x_0 \qquad y = \pm r_L \cos(\omega_c t + \phi_0) + y_0$$

Let's take $\phi_0 = 0$ and $x_0 = y_0 = 0$



For a positively charged particle:

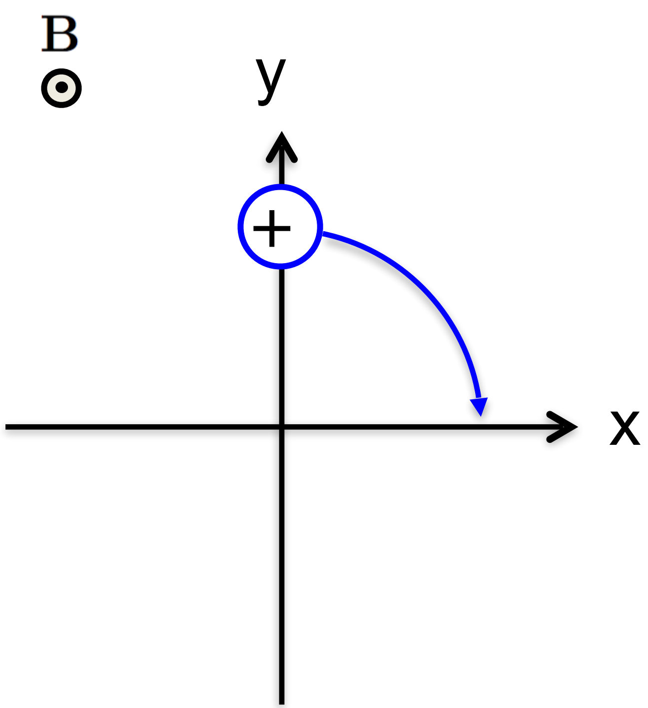
1. At $t = 0$,

$$x = 0 \qquad y = r_L$$

Gyromotion of a Charged Particle In A Magnetic Field

$$x = r_L \sin(\omega_c t + \phi_0) + x_0 \qquad y = \pm r_L \cos(\omega_c t + \phi_0) + y_0$$

Let's take $\phi_0 = 0$ and $x_0 = y_0 = 0$



For a positively charged particle:

1. At $t = 0$,

$$x = 0 \qquad y = r_L$$

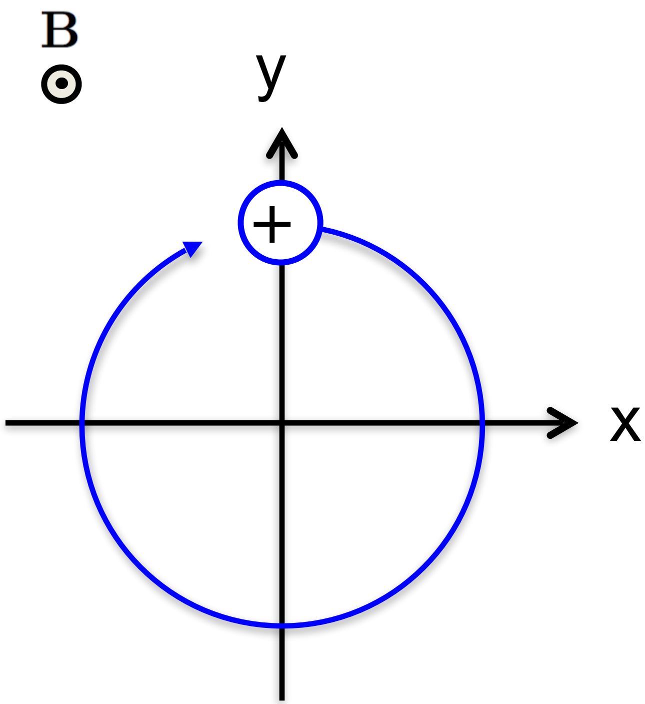
2. At $t = \frac{\pi}{2}\frac{1}{\omega_c}$,

$$x = r_L \qquad y = 0$$

Gyromotion of a Charged Particle In A Magnetic Field

$$x = r_L \sin\left(\omega_c t + \phi_0\right) + x_0 \qquad y = \pm r_L \cos\left(\omega_c t + \phi_0\right) + y_0$$

Let's take $\phi_0 = 0$ and $x_0 = y_0 = 0$



For a positively charged particle:

1. At $t = 0$,

$$x = 0 \qquad y = r_L$$

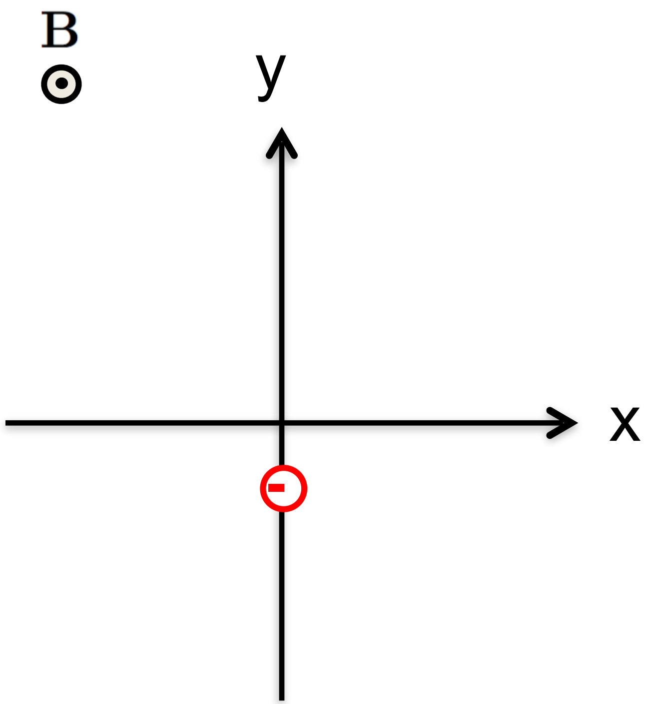
2. At $t = \frac{\pi}{2}\frac{1}{\omega_c}$,

$$x = r_L \qquad y = 0$$

Gyromotion of a Charged Particle In A Magnetic Field

$$x = r_L \sin\left(\omega_c t + \phi_0\right) + x_0 \qquad\qquad y = \pm r_L \cos\left(\omega_c t + \phi_0\right) + y_0$$

Let's take $\phi_0 = 0$ and $x_0 = y_0 = 0$



For a negatively charged particle:

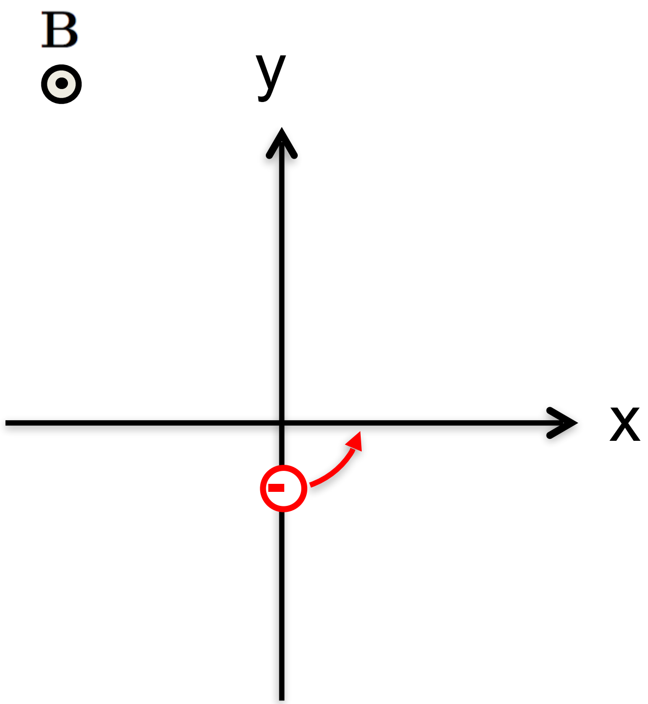
1. At $t = 0$,

$$x = 0 \qquad\qquad y = -r_L$$

Gyromotion of a Charged Particle In A Magnetic Field

$$x = r_L \sin\left(\omega_c t + \phi_0\right) + x_0 \qquad y = \pm r_L \cos\left(\omega_c t + \phi_0\right) + y_0$$

Let's take $\phi_0 = 0$ and $x_0 = y_0 = 0$



For a negatively charged particle:

1. At $t = 0$,

$$x = 0 \qquad y = -r_L$$

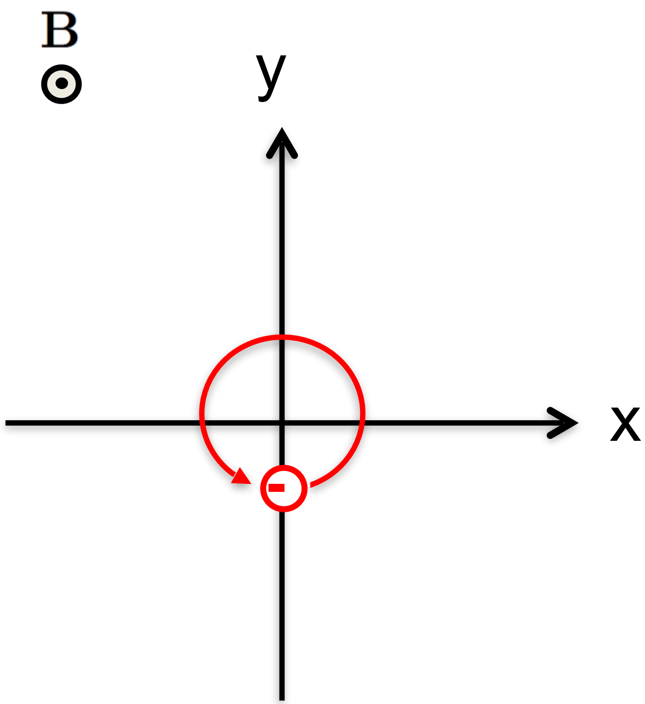
2. At $t = \frac{\pi}{2}\frac{1}{\omega_c}$,

$$x = r_L \qquad y = 0$$

Gyromotion of a Charged Particle In A Magnetic Field

$$x = r_L \sin\left(\omega_c t + \phi_0\right) + x_0 \qquad y = \pm r_L \cos\left(\omega_c t + \phi_0\right) + y_0$$

Let's take $\phi_0 = 0$ and $x_0 = y_0 = 0$



For a negatively charged particle:

1. At $t = 0$,

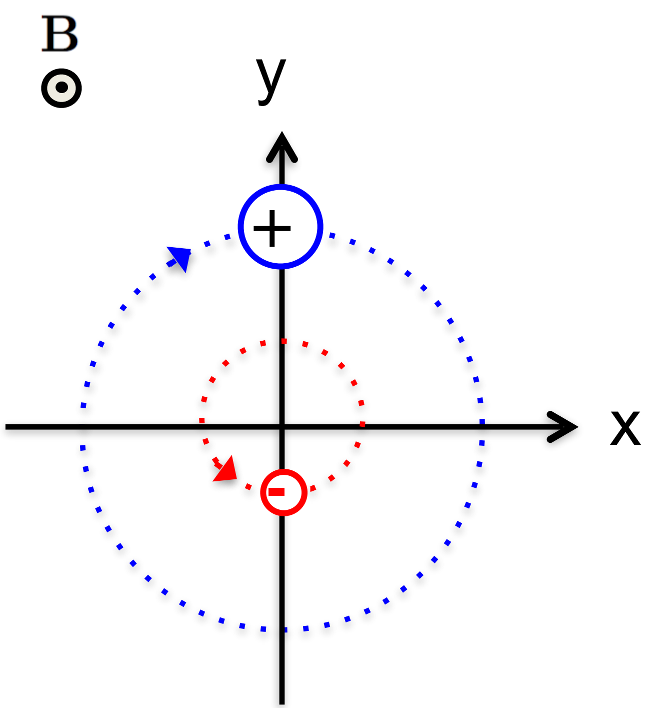
$$x = 0 \qquad y = -r_L$$

2. At $t = \frac{\pi}{2}\frac{1}{\omega_c}$,

$$x = r_L \qquad y = 0$$

Gyromotion of Ions vs. Electrons

- **The direction of gyromotion depends on the sign of the charge**
- **Ions generally have a much larger Larmor radius than electrons**



In ITER, for a typical deuterium ion with T_i=10 keV and B=5 Tesla would have

$$v_{Ti} = \sqrt{\frac{kT_i}{m_i}} \approx 700\ \mathrm{km/s}$$

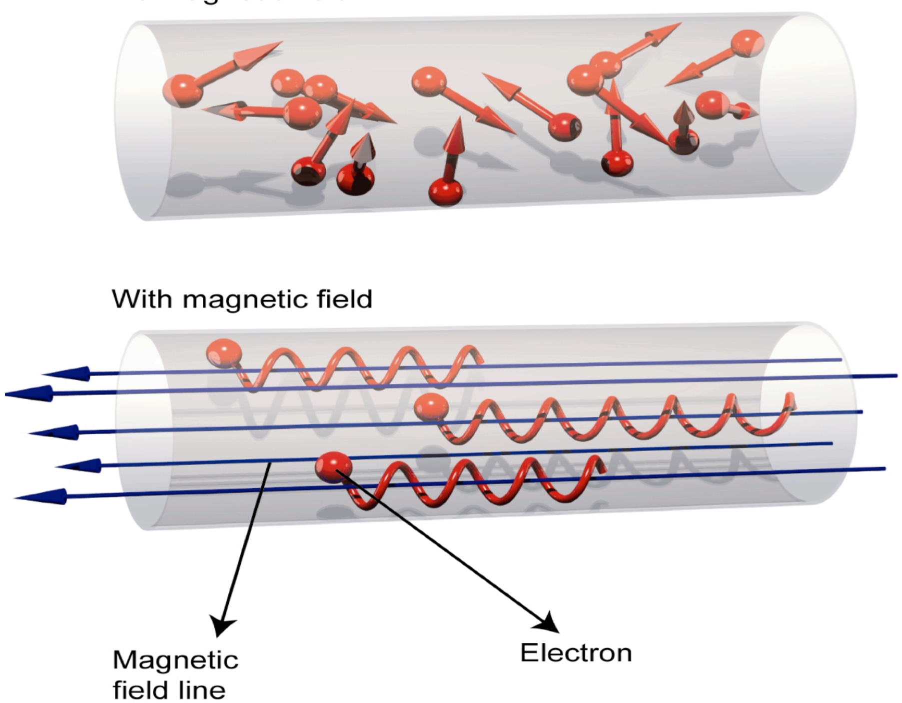
$$r_L \equiv \frac{mv_\perp}{|q|B} \approx 3\ \mathrm{mm}$$

An electron with T_e=10 keV and B=5 Tesla has

$$r_L \approx 0.05\ \mathrm{mm}$$ (60 times smaller)

Magnetic Confinement Devices Should Be Much Larger Than the Larmor Radius

$$\mathbf{r} = \left[r_L \sin\left(\omega_c t + \phi_0\right) + x_0\right]\hat{\mathbf{x}} + \left[r_L \cos\left(\omega_c t + \phi_0\right) + y_0\right]\hat{\mathbf{y}} + \left[v_{\parallel} t + z_0\right]\hat{\mathbf{z}}$$



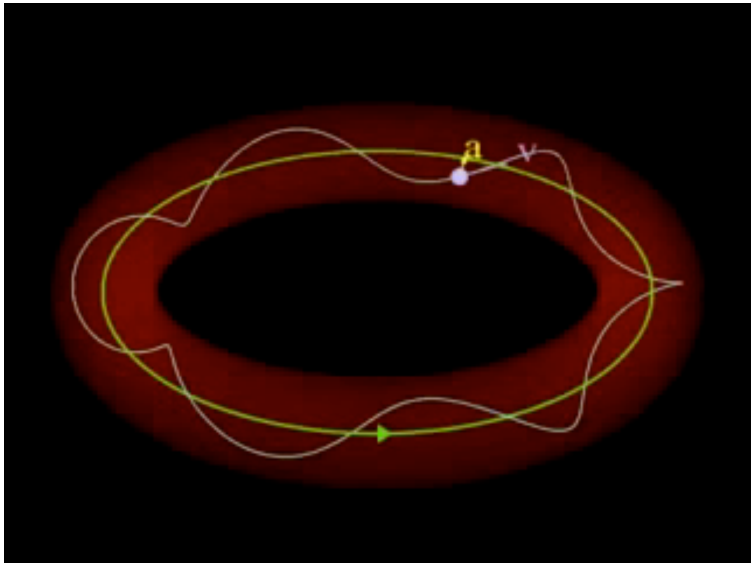
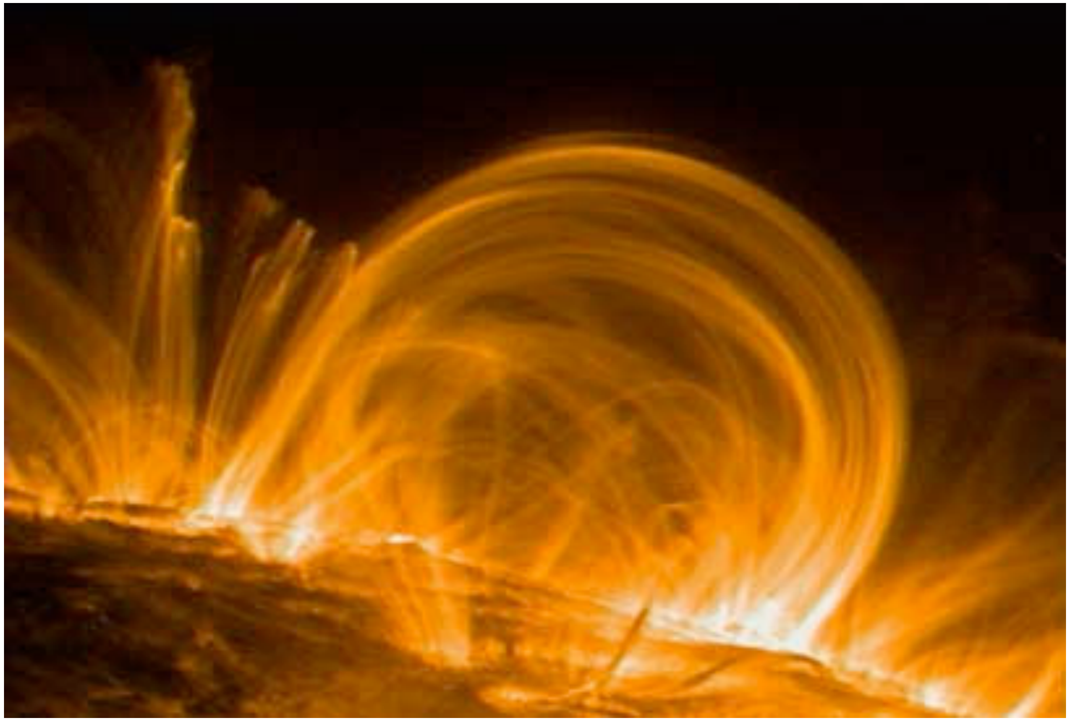


Image credit: http://iter.rma.ac.be/en/img/MagneticConfinement.jpg

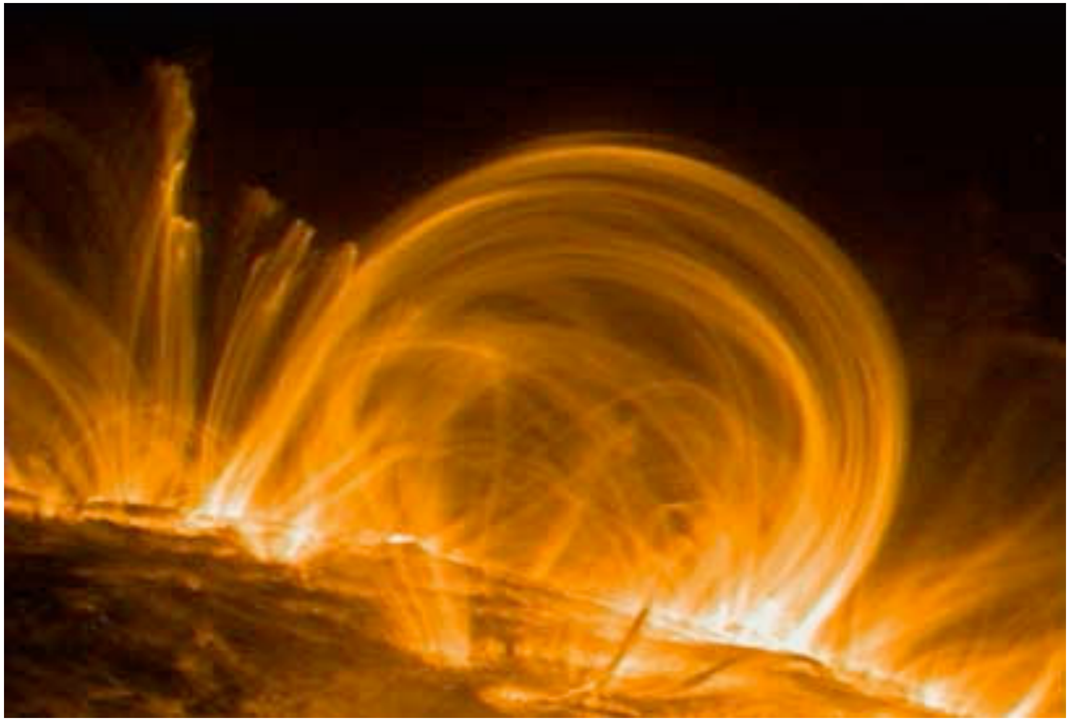
Magnetic structure of the solar corona



Solar coronal plasma
Te = 10 MK (100 eV)
B = 100 G

Which way is the magnetic field running?
Is the gradient stronger along or across the field?

Magnetic structure of the solar corona

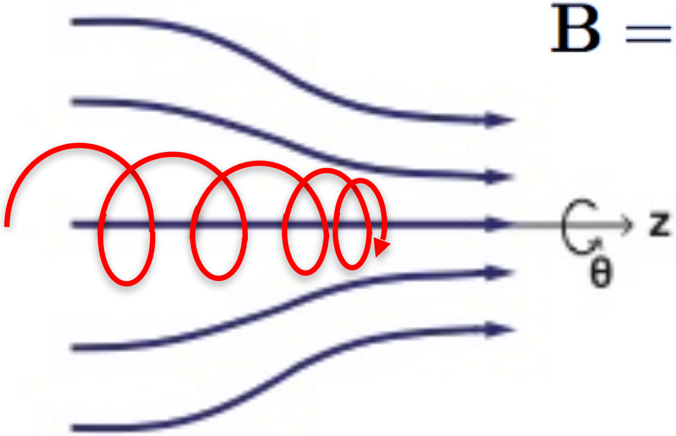


Solar coronal plasma
Te = 10 MK (100 eV)
B = 100 G

Which way is the magnetic field running?
Is the gradient stronger along or across the field?

Follow up: Shouldn't heat drain off the field lines?
What keeps the corona hot? [An active research question!!]

Magnetic Mirrors



$$\mathbf{B} = B_r\hat{\mathbf{r}} + B_z\hat{\mathbf{z}} \qquad \mathbf{F} = q(\mathbf{v}\times\mathbf{B})$$

The B_r ends up causing additional acceleration in the z direction:

$$m\frac{dv_z}{dt} = -qv_\theta B_r$$

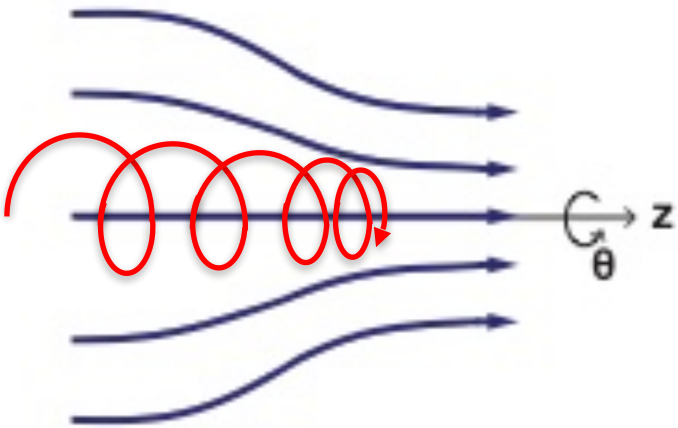
Result:
gyromotion + mirror force in the $-\hat{\mathbf{z}}$ direction

$$F_z = -\frac{mv_\perp^2}{2B}\frac{\partial B_z}{\partial z}$$

The magnetic moment is $\mu \equiv \dfrac{mv_\perp^2}{2B}$

mirror force $\mathbf{F}_{||} = -\mu\nabla_{||}B$

Magnetic Moment Is Conserved



$$\mathbf{F}_{||} = -\mu \nabla_{||} B \qquad \mu \equiv \frac{m v_\perp^2}{2B}$$

The magnetic moment is a constant of motion

$$m \frac{dv_{||}}{dt} = -\mu \frac{dB}{ds}$$

s is the coordinate along the field line

We can write $$\frac{dB}{dt} = \frac{\partial B}{\partial s}\frac{ds}{dt}$$

this is $v_{||}$

Then $$\frac{d}{dt}\left(\frac{1}{2} m v_{||}^2\right) = -\mu \frac{dB}{dt}$$

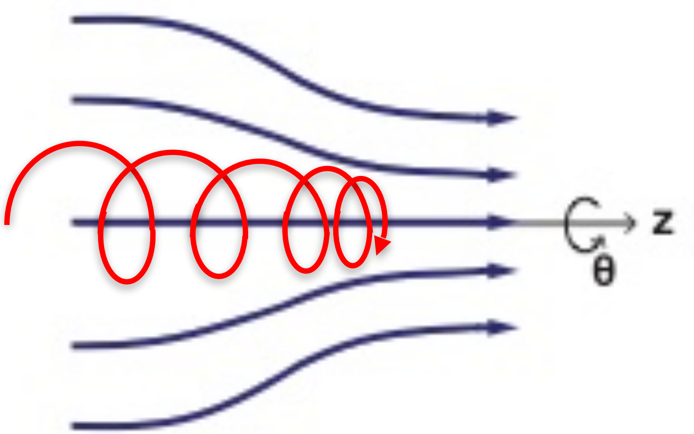
this is μB

We also have conservation of energy: $$\frac{d}{dt}\left(\frac{1}{2} m v_{||}^2 + \frac{1}{2} m v_\perp^2\right) = 0$$

$$\boxed{B \frac{d\mu}{dt} = 0}$$

μ is an "adiabatic invariant" - a deeply utilized concept for magnetized plasmas

More Insight Into Magnetic Mirrors



$$\mu \equiv \frac{mv_\perp^2}{2B} \qquad \frac{d\mu}{dt} = 0$$

1. As the particle moves to stronger B, $v_\perp$ must increase.

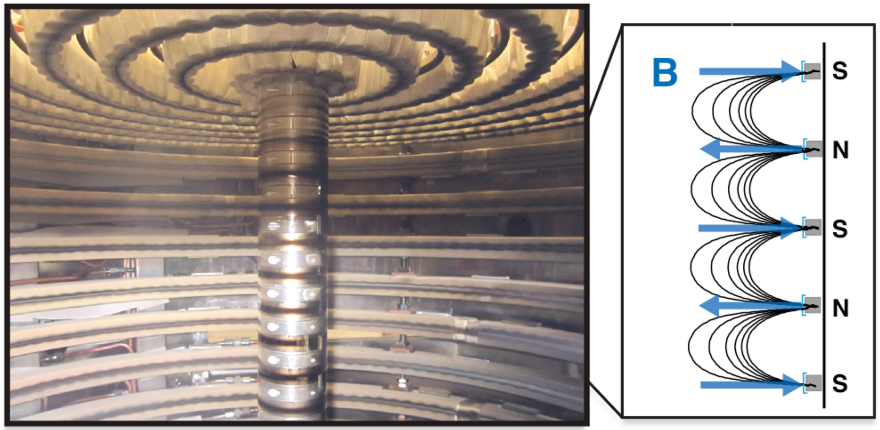
2. Since energy is conserved, $v_\parallel$ must decrease.

3. If B is strong enough, $v_\parallel \to 0$ and the particle is reflected.

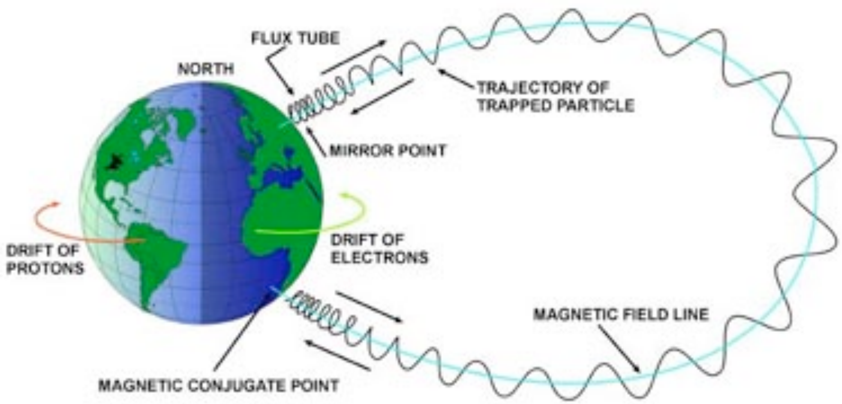
$$E_o = \frac{1}{2}mv_\parallel^2 + \frac{1}{2}mv_\perp^2 \qquad v_\parallel = \pm\sqrt{\frac{2}{m}(E_o - \mu B)}$$

The particle is reflected when $E_o \leq \mu B$

Magnetic Mirror Confinement In Action



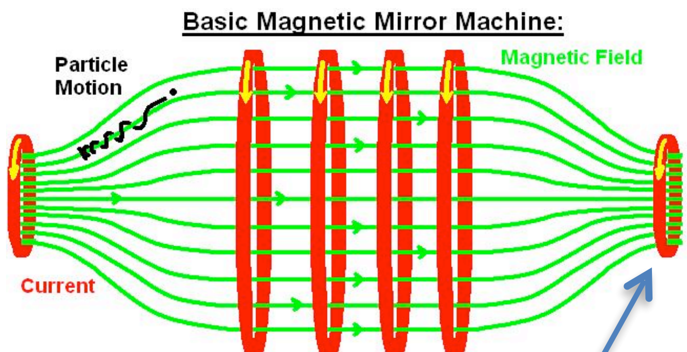
Multicusp Confinement Devices



Charged particles can be trapped by Earth's magnetic field

Early Fusion Experiments

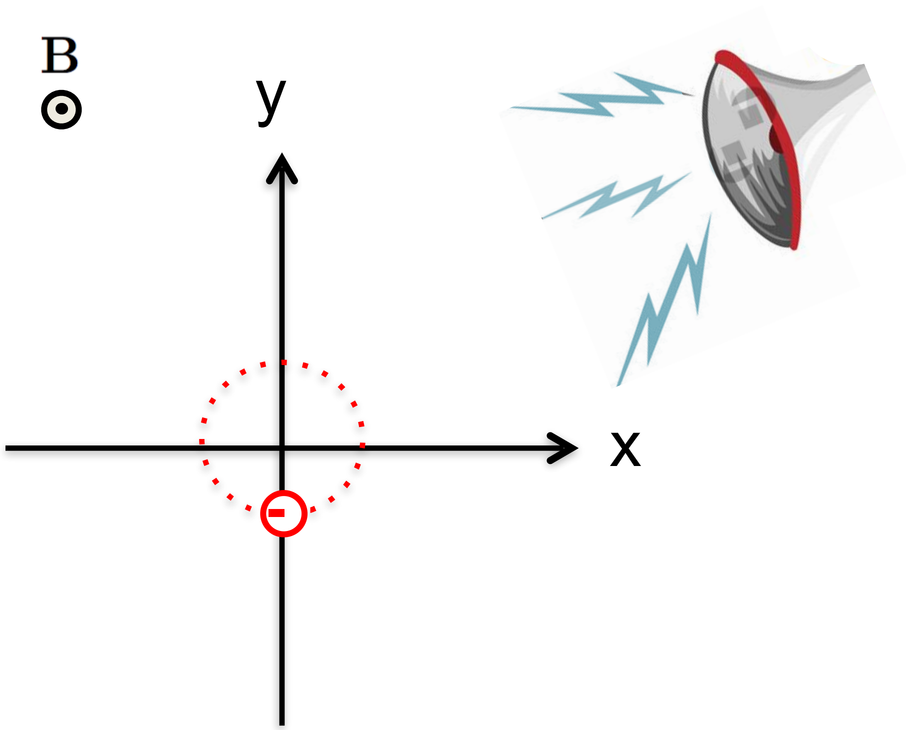
Ex: Tandem Mirror Experiment (LLNL,1980's) and other variants (Polywell devices)



Particles with enough $v_{||}$ can still escape

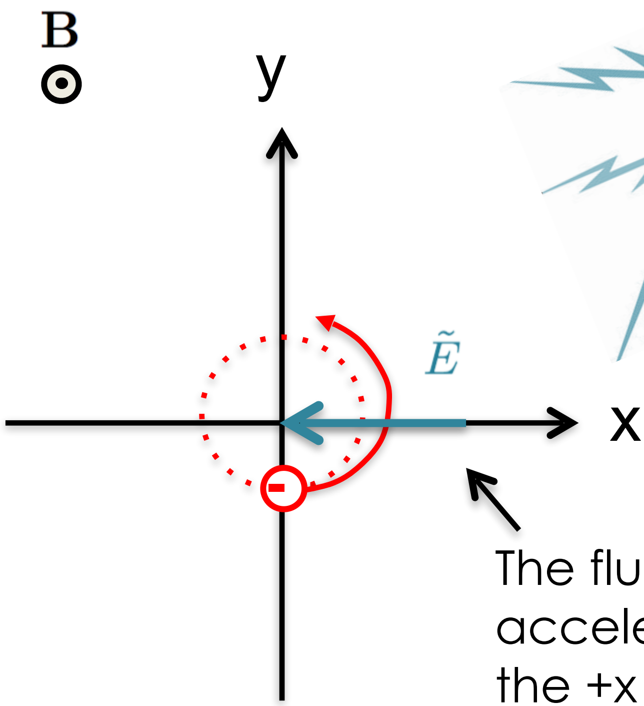
Time-varying Electric and Magnetic Fields Can Be Used To Accelerate & Heat Particles

- **A high frequency electro-magnetic field can be used to accelerate electrons or ions.**



Time-varying Electric and Magnetic Fields Can Be Used To Accelerate & Heat Particles

- **A high frequency electro-magnetic field can be used to accelerate electrons or ions.**
- **Particle gains energy as the applied electric field component oscillates at the cyclotron frequency ("in-phase" with the gyro-orbit).**



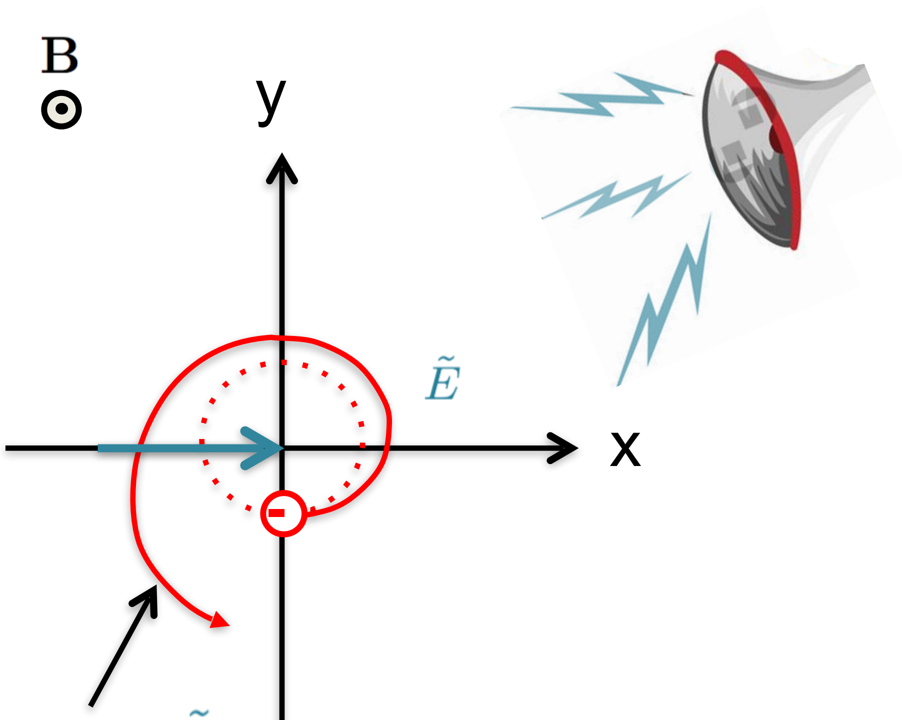
Cyclotron frequency

$$\omega_c \equiv \frac{|q|B}{m}$$

The fluctuating $\tilde{E}$ accelerates the electron in the +x direction $\tilde{\mathbf{F}} = -e\tilde{\mathbf{E}}$

Time-varying Electric and Magnetic Fields Can Be Used To Accelerate & Heat Particles

- **A high frequency electro-magnetic field can be used to accelerate electrons or ions.**
- **Particle gains energy as the applied electric field component oscillates at the cyclotron frequency ("in-phase" with the gyro-orbit).**



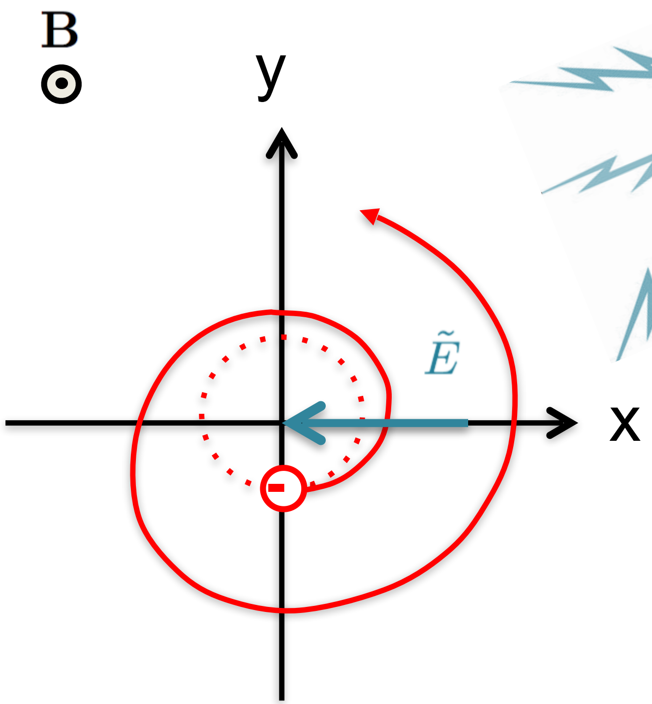
Now $\tilde{E}$ accelerates the electron in the -x direction

Cyclotron frequency

$$\omega_c \equiv \frac{|q|B}{m}$$

Time-varying Electric and Magnetic Fields Can Be Used To Accelerate & Heat Particles

- **A high frequency electro-magnetic field can be used to accelerate electrons or ions.**
- **Particle gains energy as the applied electric field component oscillates at the cyclotron frequency ("in-phase" with the gyro-orbit).**

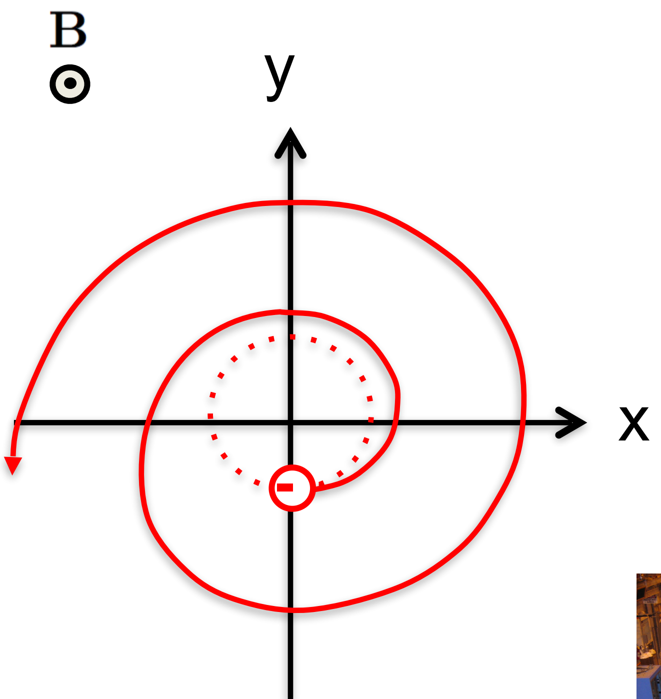


Cyclotron frequency

$$\omega_c \equiv \frac{|q|B}{m}$$

The Cyclotron Frequency is Important for Cyclotron Resonance Heating

- **A high frequency electro-magnetic field can be used to accelerate electrons or ions.**
- **Particle gains energy as the applied electric field component oscillates at the cyclotron frequency ("in-phase" with the gyro-orbit).**

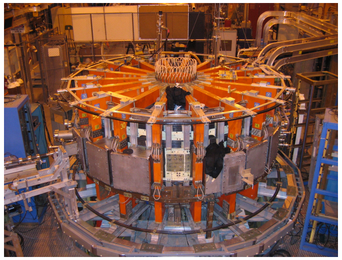


Cyclotron frequency

$$\omega_c \equiv \frac{|q|B}{m}$$

Ex: For an electron, what *B* corresponds to 2.45 Ghz (microwave oven frequency)?

$$f = \frac{\omega_{ce}}{2\pi} = 2.45 \text{ GHz} \Rightarrow B = 875 \text{ Gauss}$$

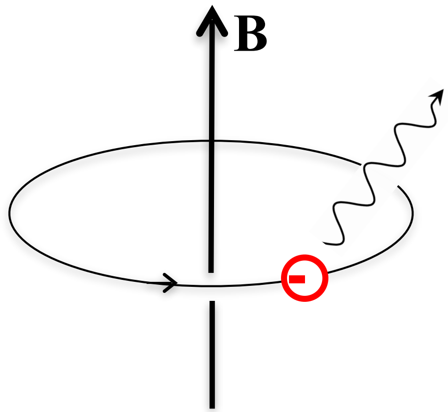


Remember VTF?
This is how we initiated our plasma!

Other Practical Applications: EM Emission from Charged Particle Acceleration

- **Electron cyclotron emission (measure T_e profiles)**

  Produced by acceleration of gyrating charged particle
  EM radiation emitted at discrete frequencies:



$$\omega = n\omega_{ce} \qquad \omega_{ce} = \frac{eB}{m_e}$$

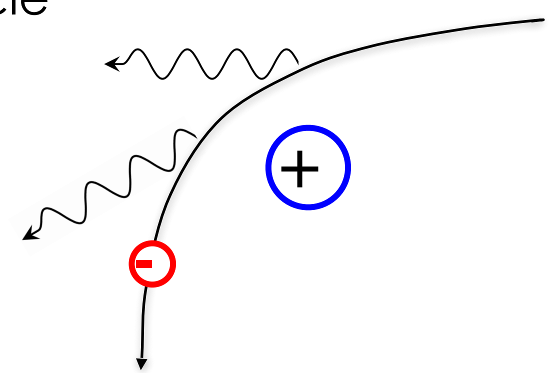
  Detected radiated power is proportional to T_e:

$$I(\omega) = \frac{\omega^2 kT_e}{8\pi^3 c^2}$$

- **Bremsstrahlung emission**

  Produced by deceleration of deflected charged particle

$$I(\omega) \propto \frac{n_e^2 Z_{eff} g}{\sqrt{T_e}} \exp\left(-\frac{\hbar\omega}{kT_e}\right)$$

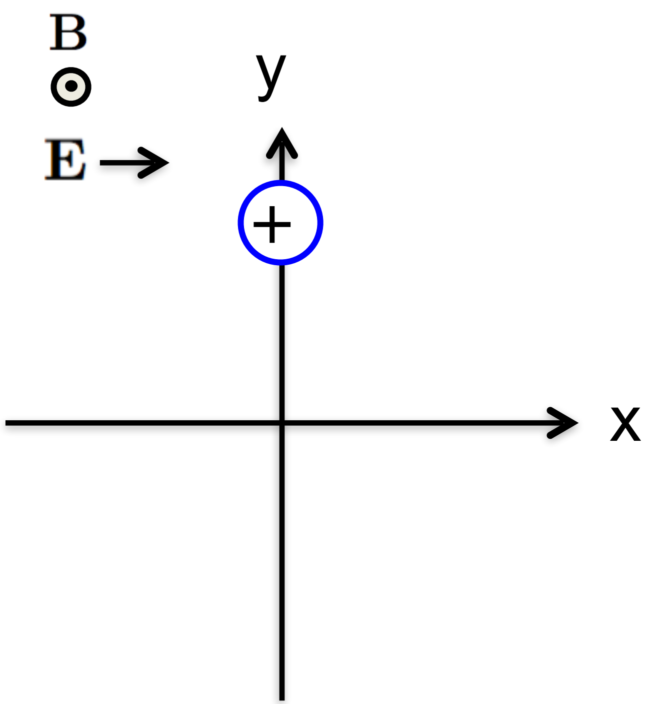


  Radiated power depends on n_e, Te, charge state Z_{eff}
  (can be used to measure Z_{eff})

Next Simplest Case to Analyze: Constant, Uniform Electric Field Perpendicular to Magnetic Field

$$\mathbf{F} = q(\mathbf{E} + \mathbf{v} \times \mathbf{B})$$

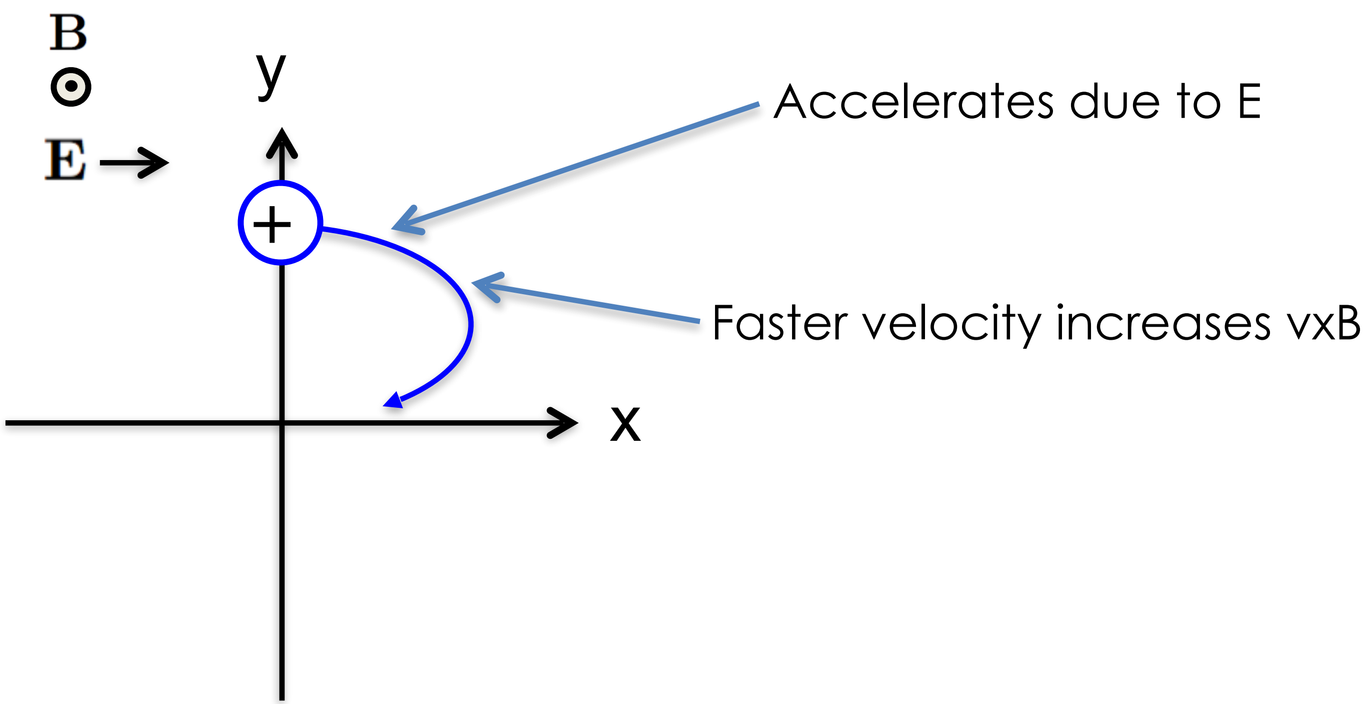
$$\mathbf{E} = E_x \hat{\mathbf{x}} \qquad \mathbf{B} = B_z \hat{\mathbf{z}}$$



Next Simplest Case to Analyze: Constant, Uniform Electric Field Perpendicular to Magnetic Field

$$\mathbf{F} = q(\mathbf{E} + \mathbf{v} \times \mathbf{B})$$

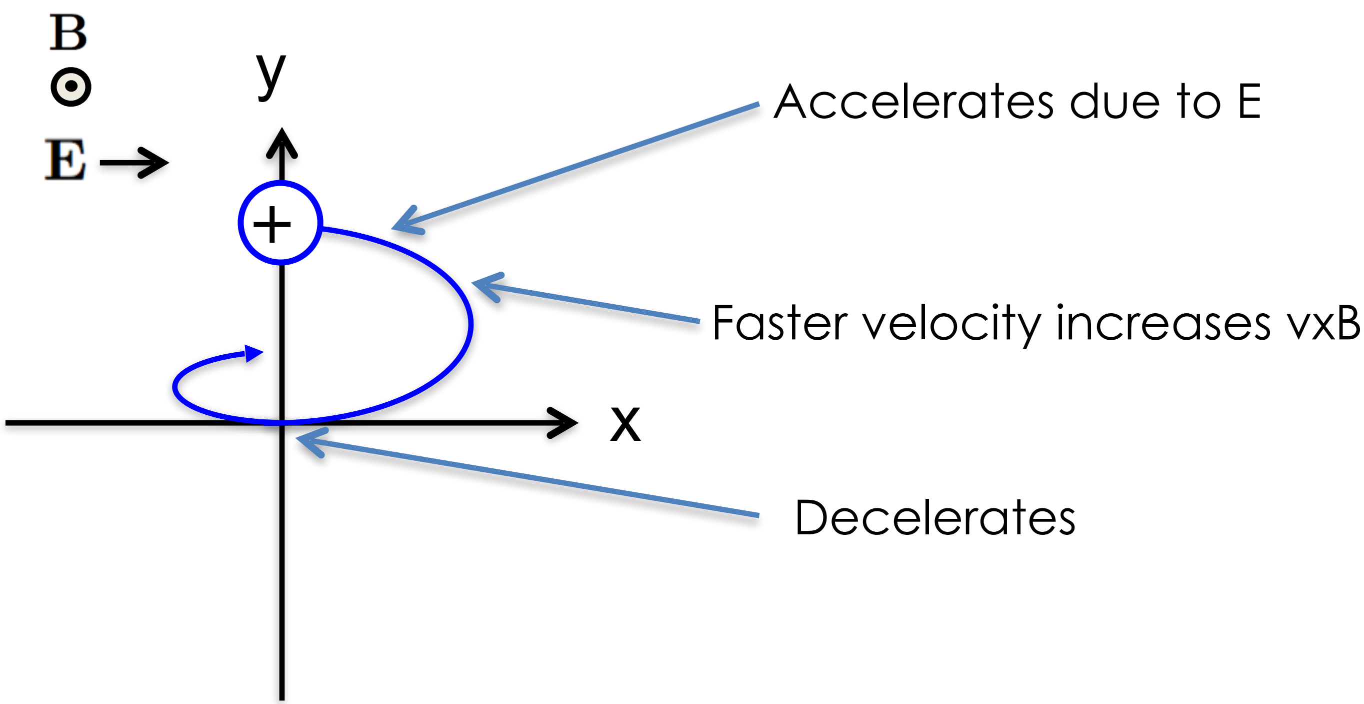
$$\mathbf{E} = E_x \hat{\mathbf{x}} \qquad \mathbf{B} = B_z \hat{\mathbf{z}}$$



Next Simplest Case to Analyze: Constant, Uniform Electric Field Perpendicular to Magnetic Field

$$\mathbf{F} = q(\mathbf{E} + \mathbf{v} \times \mathbf{B})$$

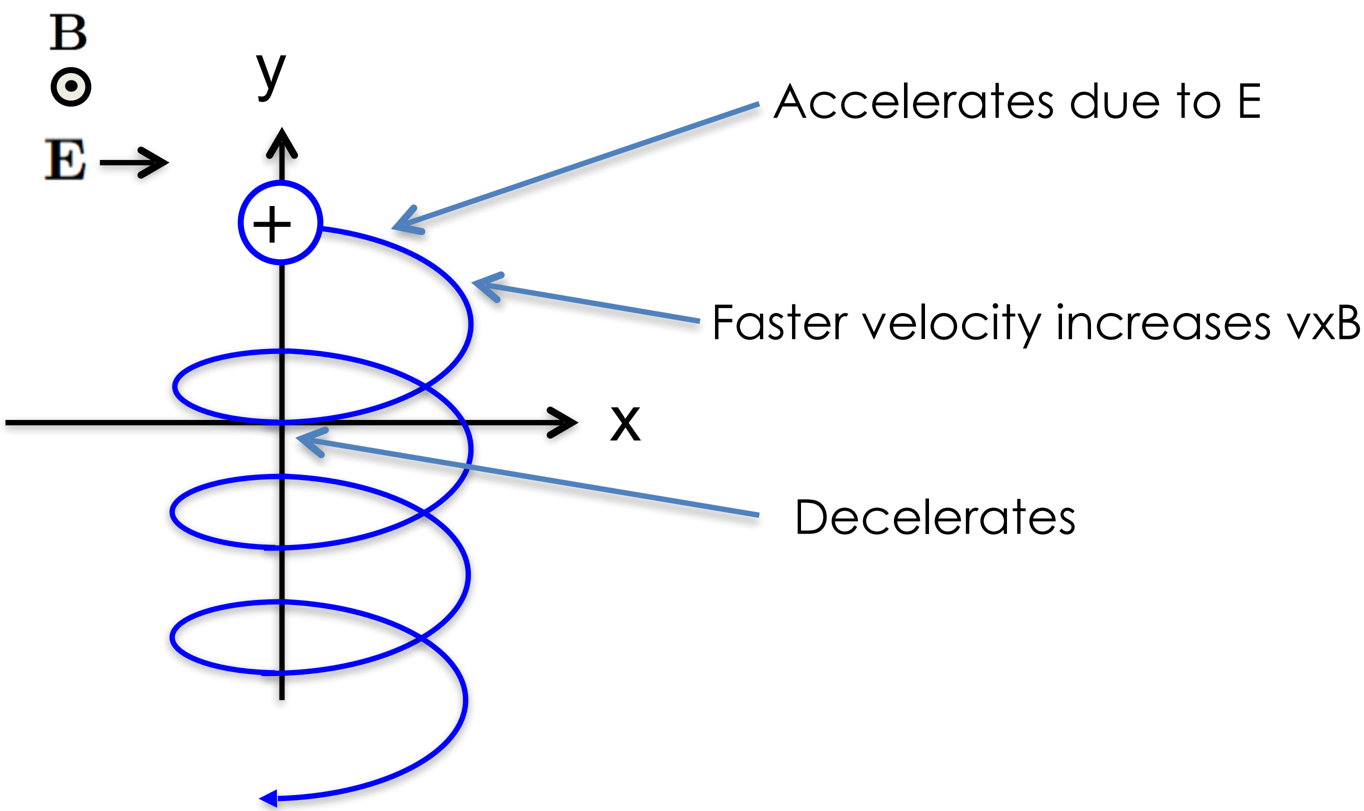
$$\mathbf{E} = E_x \hat{\mathbf{x}} \qquad \mathbf{B} = B_z \hat{\mathbf{z}}$$



Next Simplest Case to Analyze: Constant, Uniform Electric Field Perpendicular to Magnetic Field

$$\mathbf{F} = q(\mathbf{E} + \mathbf{v} \times \mathbf{B})$$

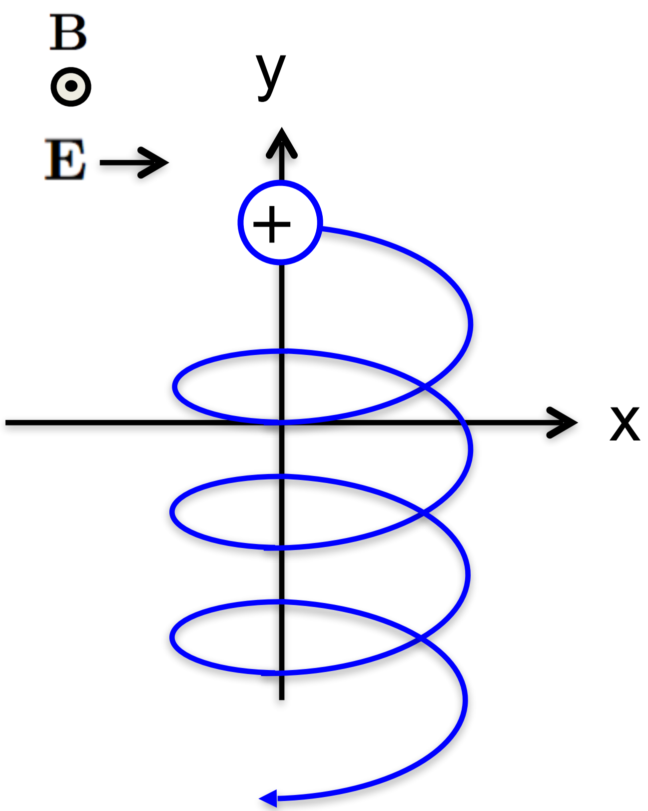
$$\mathbf{E} = E_x\hat{\mathbf{x}} \qquad \mathbf{B} = B_z\hat{\mathbf{z}}$$



Next Simplest Case to Analyze: Constant, Uniform Electric Field Perpendicular to Magnetic Field

$$\mathbf{F} = q(\mathbf{E} + \mathbf{v} \times \mathbf{B})$$

$$\mathbf{E} = E_x \hat{\mathbf{x}} \qquad \mathbf{B} = B_z \hat{\mathbf{z}}$$



$$\dot{v}_x = \frac{q}{m}\left(v_y B_z + E_x\right)$$

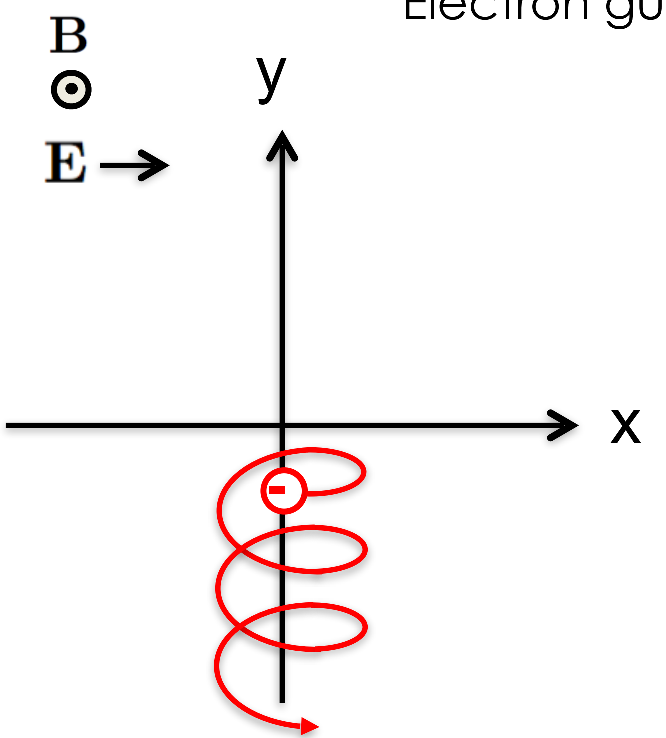
$$v_y = \mp v_\perp \sin\left(\frac{|q|B_z}{m}t + \phi_0\right) - \frac{E_x}{B_z}$$

Ion guiding center drifts in the direction $-\hat{\mathbf{y}}$

Guiding Center Drift Due to E x B

$$v_y = \mp v_\perp \sin\left(\frac{|q|B_z}{m}t + \phi_0\right) - \frac{E_x}{B_z}$$

Electron guiding center also drifts in the direction $-\hat{\mathbf{y}}$



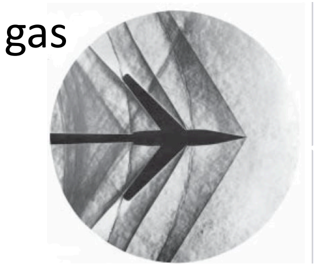
The ExB drift can be written more generally as

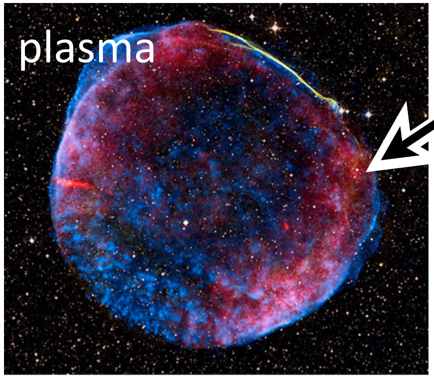
$$\mathbf{v_E} = \frac{\mathbf{E} \times \mathbf{B}}{B^2}$$

- **ExB drift is independent of charge and mass**
- **Both electrons and ions move together**

Preview for MHD lecture: Magnetic field also moves with plasma, via Faraday's law. Plasma and field can be thought of being "frozen together"

Digression: How magnetic fields mediate collisionless shocks in astrophysical plasmas





SNR1006

Collisionless SNR shocks shown to be the sites of cosmic ray acceleration. [Ackerman Science 2013]

Shocks occur where supersonic flows interact. Conversion of kinetic energy to heat

In gas, inter-particle collisions mediate the shock

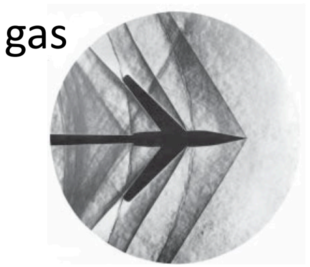
In plasma, *collisionless* shocks have shock width << mean-free-path.

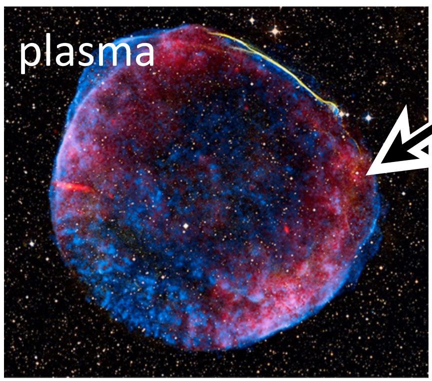
Collective electromagnetic fields are required to mediate shock in collisionless plasmas

Proposed mechanisms:

- Pile-up of pre-existing field ("magnetized shocks" … shortly)
- **Self-generation** of a turbulent magnetic field near shock by Weibel instability *

Magnetic fields mediate collisionless shocks in astrophysical plasmas





SNR1006

Shocks occur where supersonic flows interact. Conversion of kinetic energy to heat

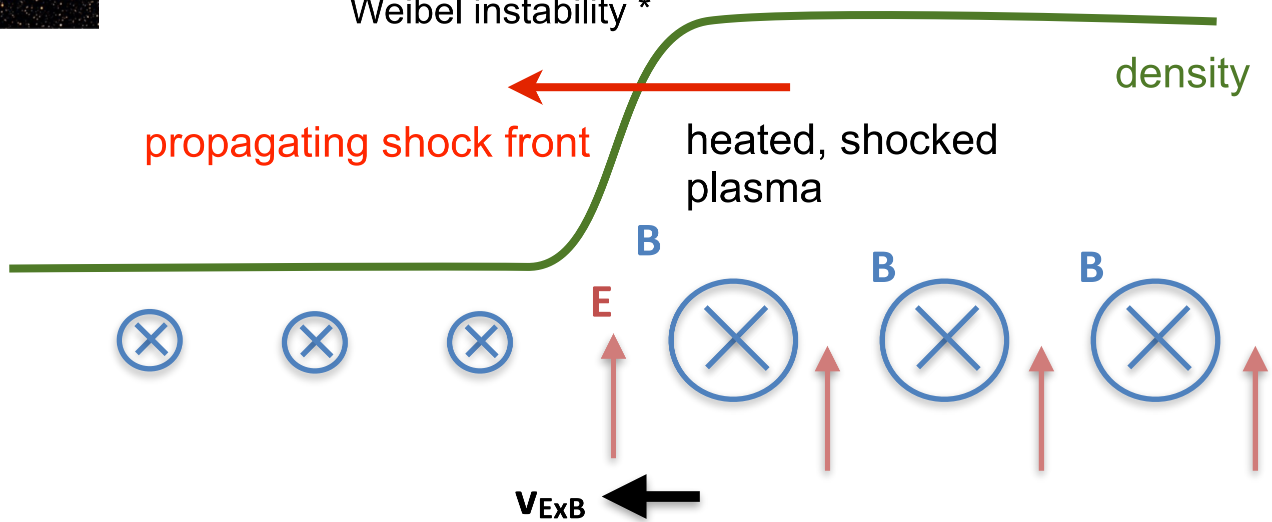
In gas, inter-particle collisions mediate the shock

In plasma, *collisionless* shocks have shock width << mean-free-path.

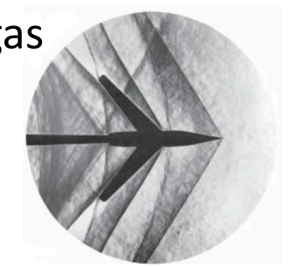
Collective electromagnetic fields are required to mediate shock in collisionless plasmas

Proposed mechanisms:

- Pile-up of pre-existing field (“magnetized shocks” … shortly)
- **Self-generation** of a turbulent magnetic field near shock by Weibel instability *



Magnetic fields mediate collisionless shocks in astrophysical plasmas

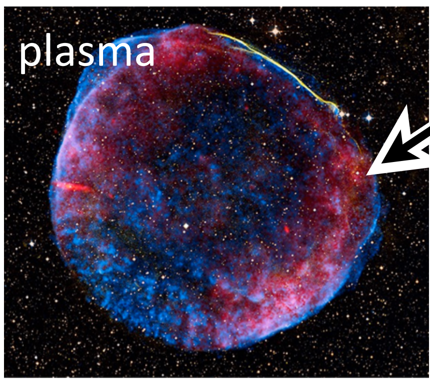


Shocks occur where supersonic flows interact. Conversion of kinetic energy to heat

In gas, inter-particle collisions mediate the shock

In plasma, *collisionless* shocks have shock width << mean-free-path.

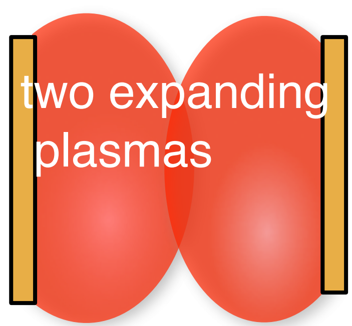
Collective electromagnetic fields are required to mediate shock in collisionless plasmas

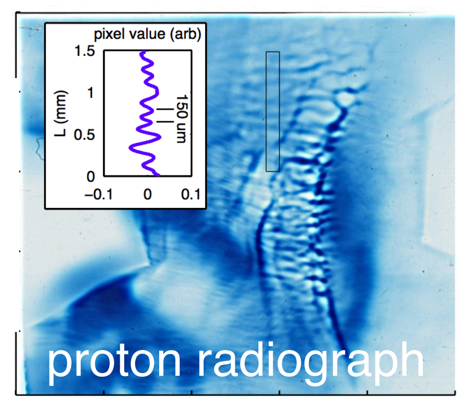


SNR1006

Proposed mechanisms:

- Pile-up of pre-existing field ("magnetized shocks" … shortly)
- **Self-generation** of a turbulent magnetic field near shock by Weibel instability *





The Weibel field-generation mechanism was verified by laboratory experiments [Fox, et al, Phys. Rev Lett (2013); Huntington et al, Nat. Phys 2015]

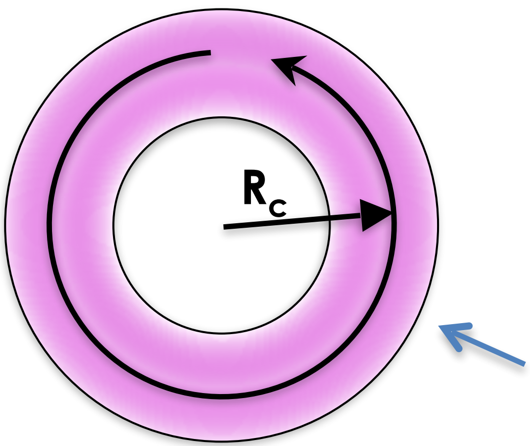
Other Forces Can Also Cause Guiding Center Drift

- **Any force perpendicular to B can cause particles to drift**

Drift due to force: $$\mathbf{v_d} = \frac{1}{q}\frac{\mathbf{F} \times \mathbf{B}}{B^2}$$

Examples of forces: $$\mathbf{F_g} = m\mathbf{g}$$ gravity

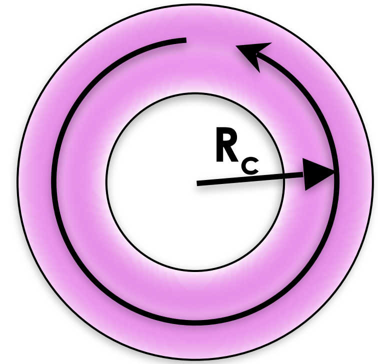
$$\mathbf{F_{cf}} = \frac{mv_{\parallel}^2}{R_c}\hat{\mathbf{r}}$$ centrifugal



- Bend the magnetic field into a donut shape
- No end losses because the field lines go around and close on themselves
- BUT a particle following a toroidal magnetic field would experience $\mathbf{F_{cf}}$

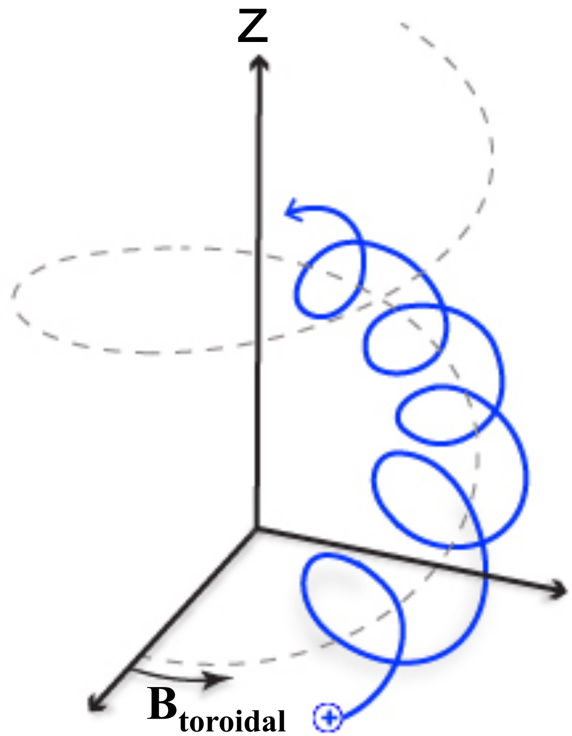
Curvature Drift Due to Bending Field Lines

The outward centrifugal force causes curvature drift



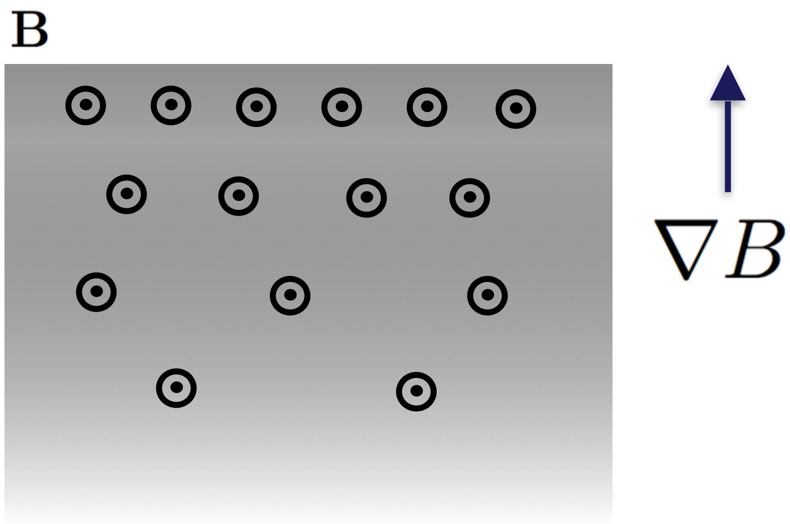
$$\mathbf{F_{cf}} = \frac{mv_\parallel^2}{R_c}\hat{\mathbf{r}}$$

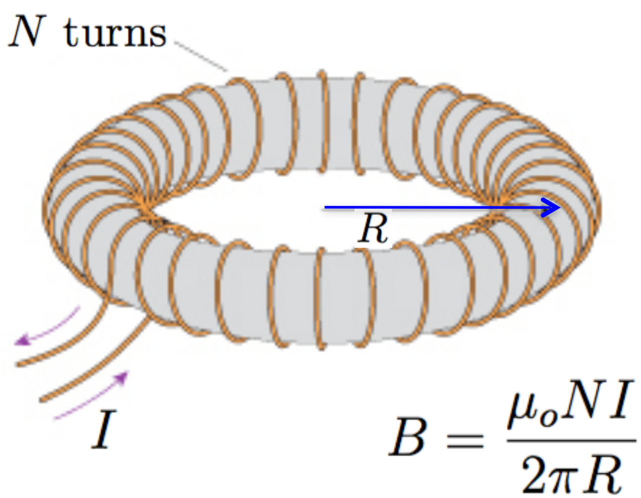
$$\mathbf{V_R} = \frac{mv_\parallel^2}{qB^2}\frac{\mathbf{R_c} \times \mathbf{B}}{R_c^2}$$



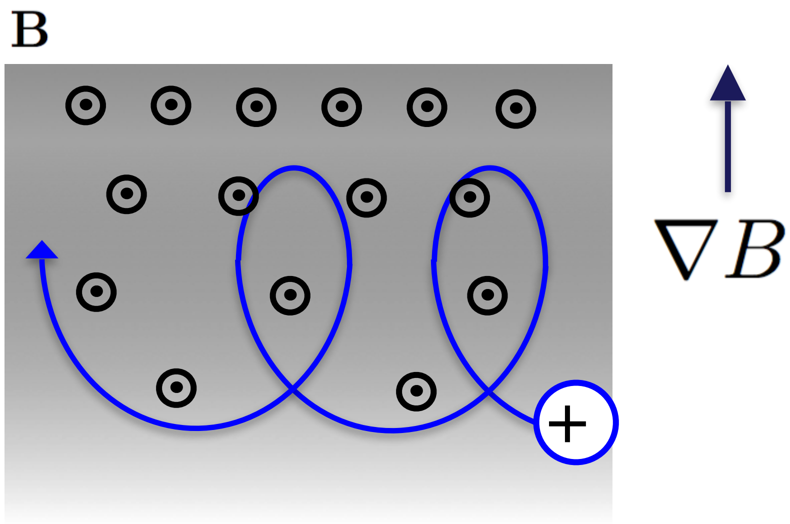
A particle moving along a curved field line will drift up or down, depending on the sign of the charge

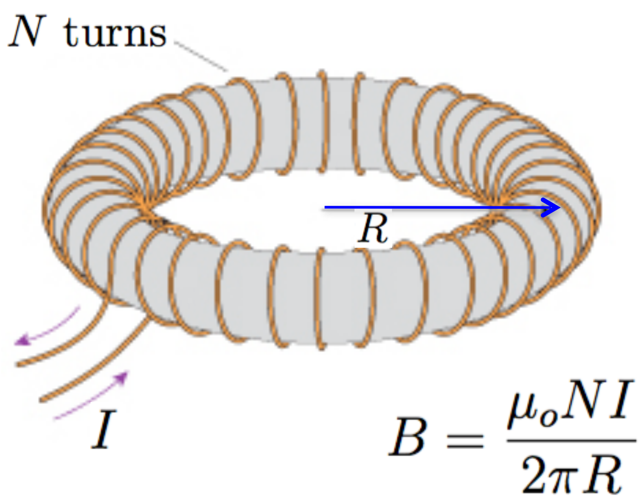
Spatially Varying Magnetic Field Strength Also Causes Drift





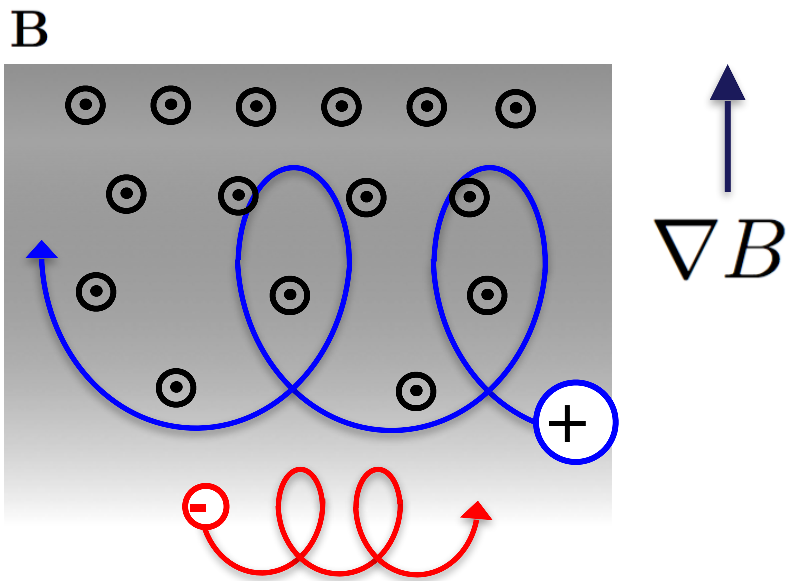
Spatially Varying Magnetic Field Strength Also Causes Drift

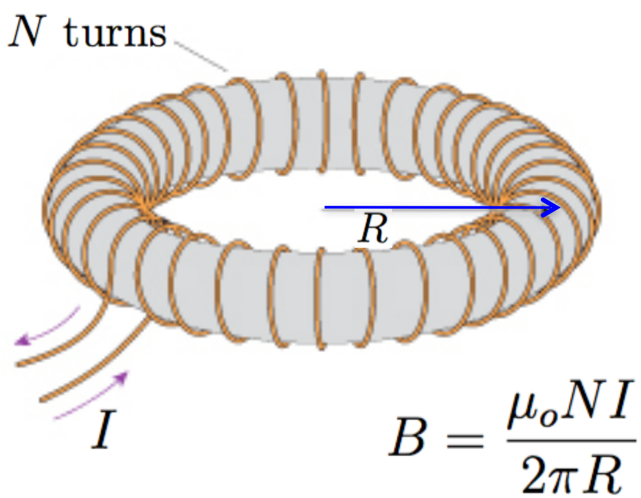




- **The gyro-radius will be larger where the field is weaker and smaller where the field is stronger**

Spatially Varying Magnetic Field Strength Also Causes Drift





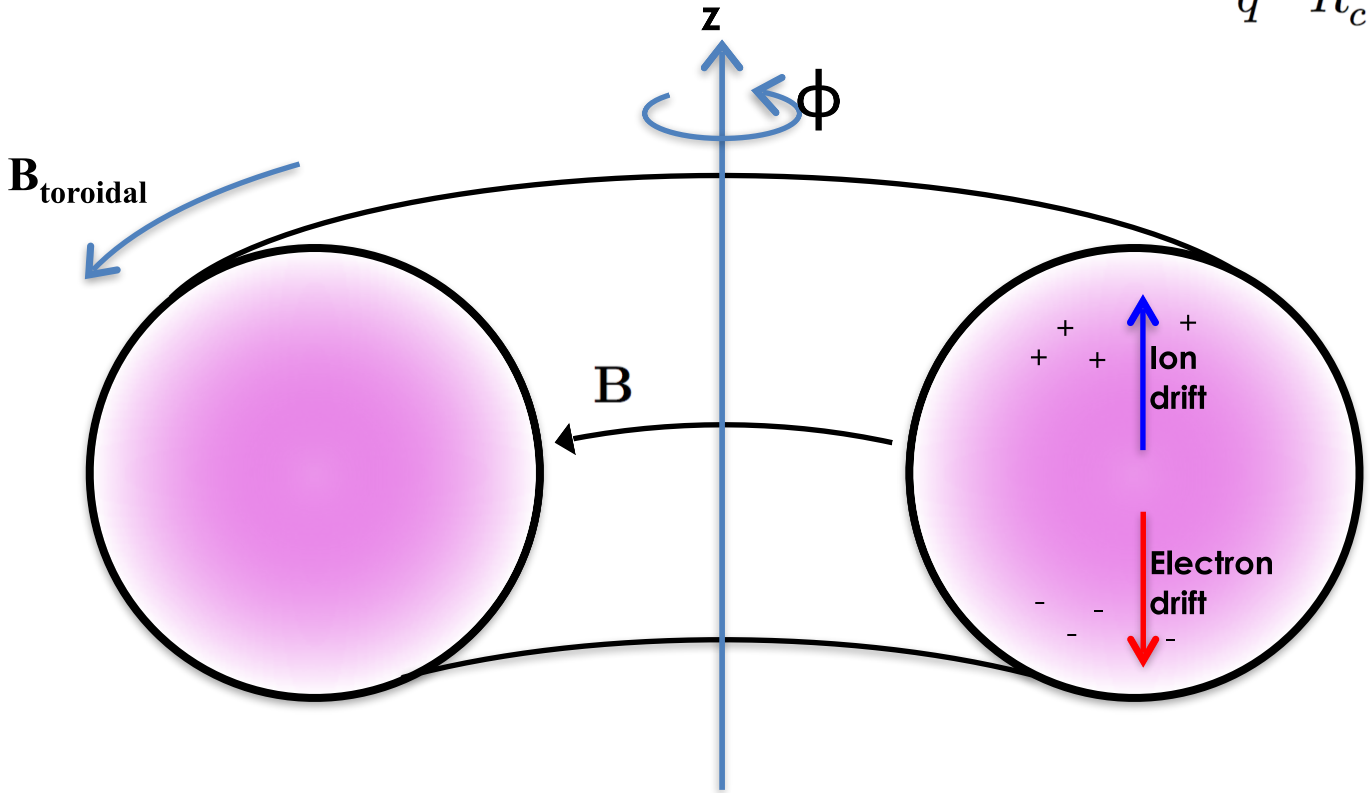
- **The gyro-radius will be larger where the field is weaker and smaller where the field is stronger**
- **The resulting drift velocity is described by:**

$$\mathbf{V}_{\nabla \mathbf{B}} = \frac{m v_\perp^2}{2qB} \frac{\mathbf{B} \times \nabla B}{B^2}$$

Application: What Happens To Charged Particles In A Purely Toroidal Magnetic Field?

- **Charged particles in a curved magnetic field will experience both ∇B and curvature drift: these effects add**

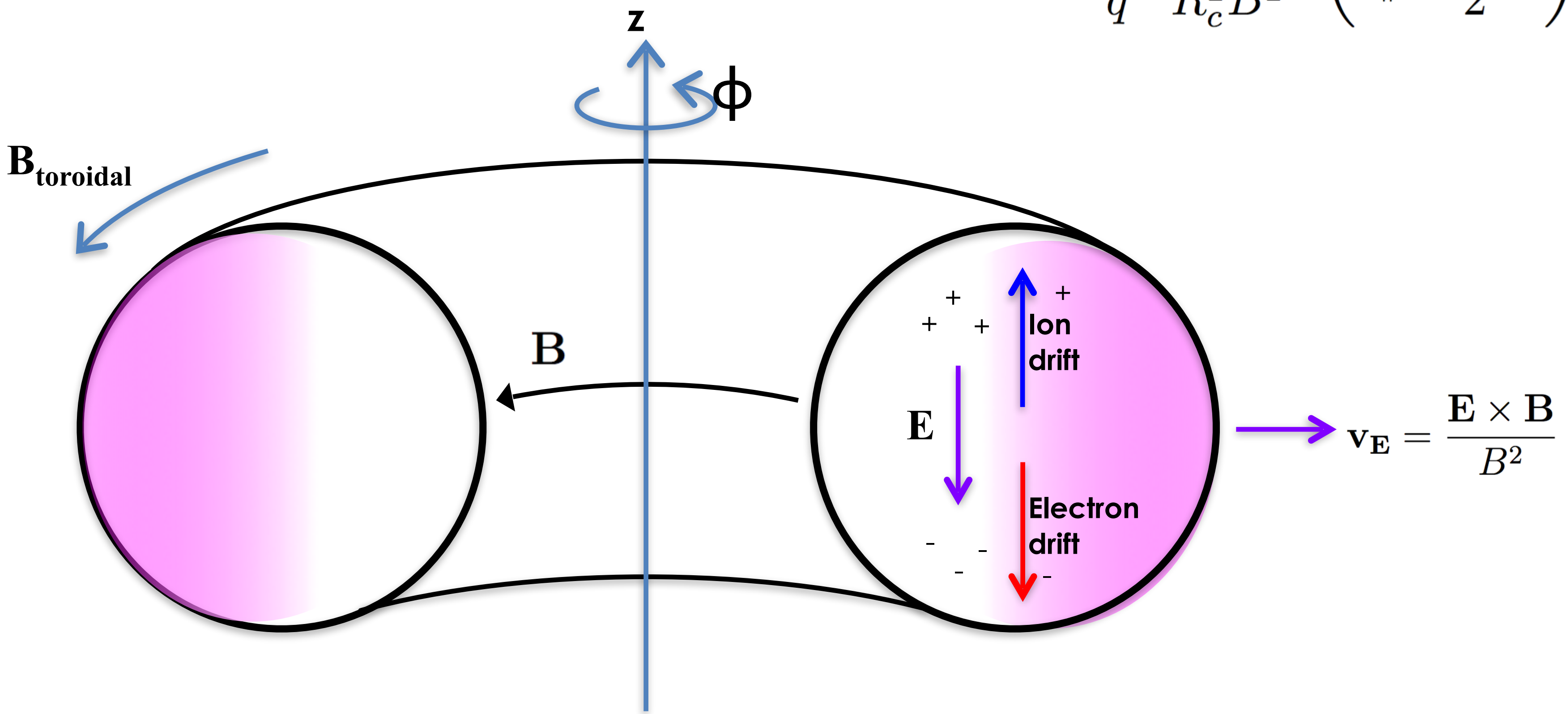
$$\mathbf{V_R} + \mathbf{V_{\nabla B}} = \frac{m}{q}\frac{\mathbf{R_c} \times \mathbf{B}}{R_c^2 B^2}\left(v_\parallel^2 + \frac{1}{2}v_\perp^2\right)$$



Charged Particles Will Drift Outward

- Charged particles in a curved magnetic field will experience both ∇B and curvature drift

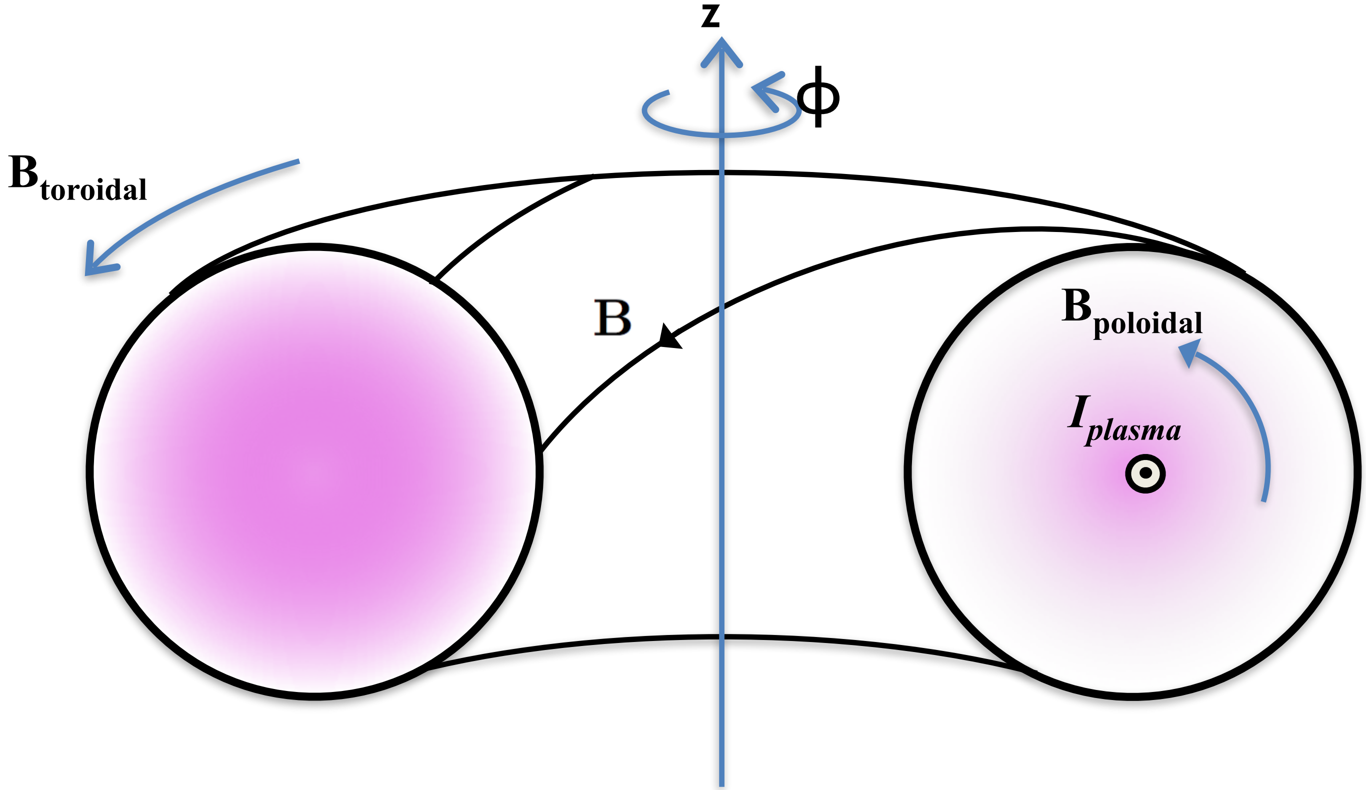
$$\mathbf{V_R} + \mathbf{V_{\nabla B}} = \frac{m}{q}\frac{\mathbf{R_c}\times\mathbf{B}}{R_c^2 B^2}\left(v_\parallel^2 + \frac{1}{2}v_\perp^2\right)$$



- This means that no matter what, particles in a torus with a purely toroidal field will drift radially out and hit the walls.

Tokamak Solution: Add Poloidal Magnetic Field

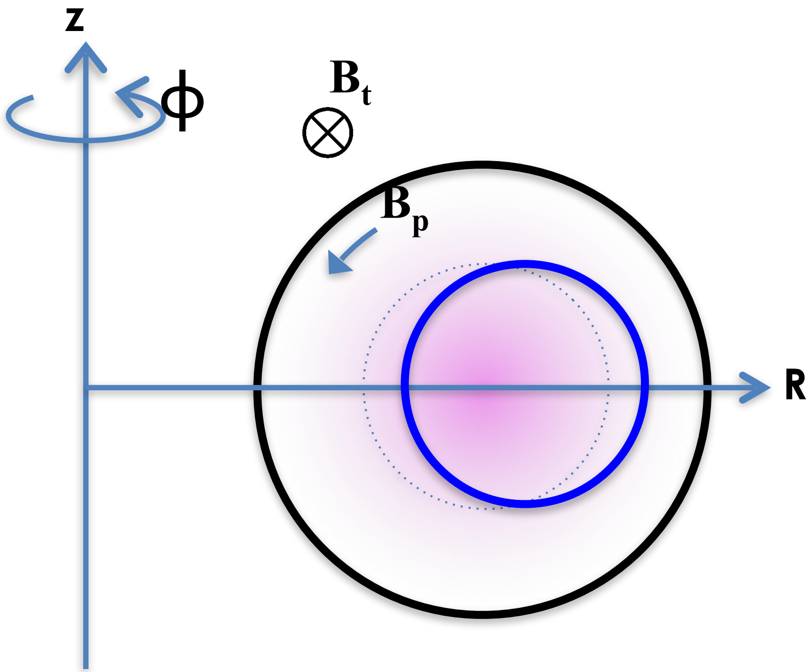
Toroidal: long way around
Poloidal: short way around
1. Use external coils to apply a toroidal magnetic field
2. Drive toroidal current in the plasma to generate a poloidal magnetic field



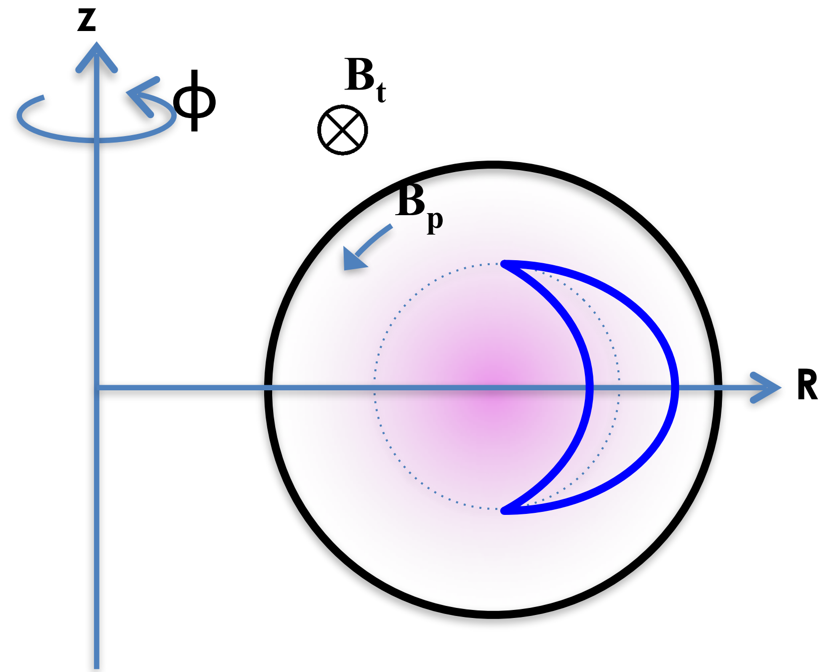
- **The resulting helical magnetic field is much better at confining charged particles.**
- **The challenge: how to drive current in plasma in steady state while keeping the plasma stable and free of disruptions?**

There Are Two Main Classes of Particle Orbits In Tokamaks

$$\mathbf{V_R} + \mathbf{V_{\nabla B}} = \frac{m}{q} \frac{\mathbf{R_c} \times \mathbf{B}}{R_c^2 B^2} \left(v_\parallel^2 + \frac{1}{2} v_\perp^2 \right)$$



Passing



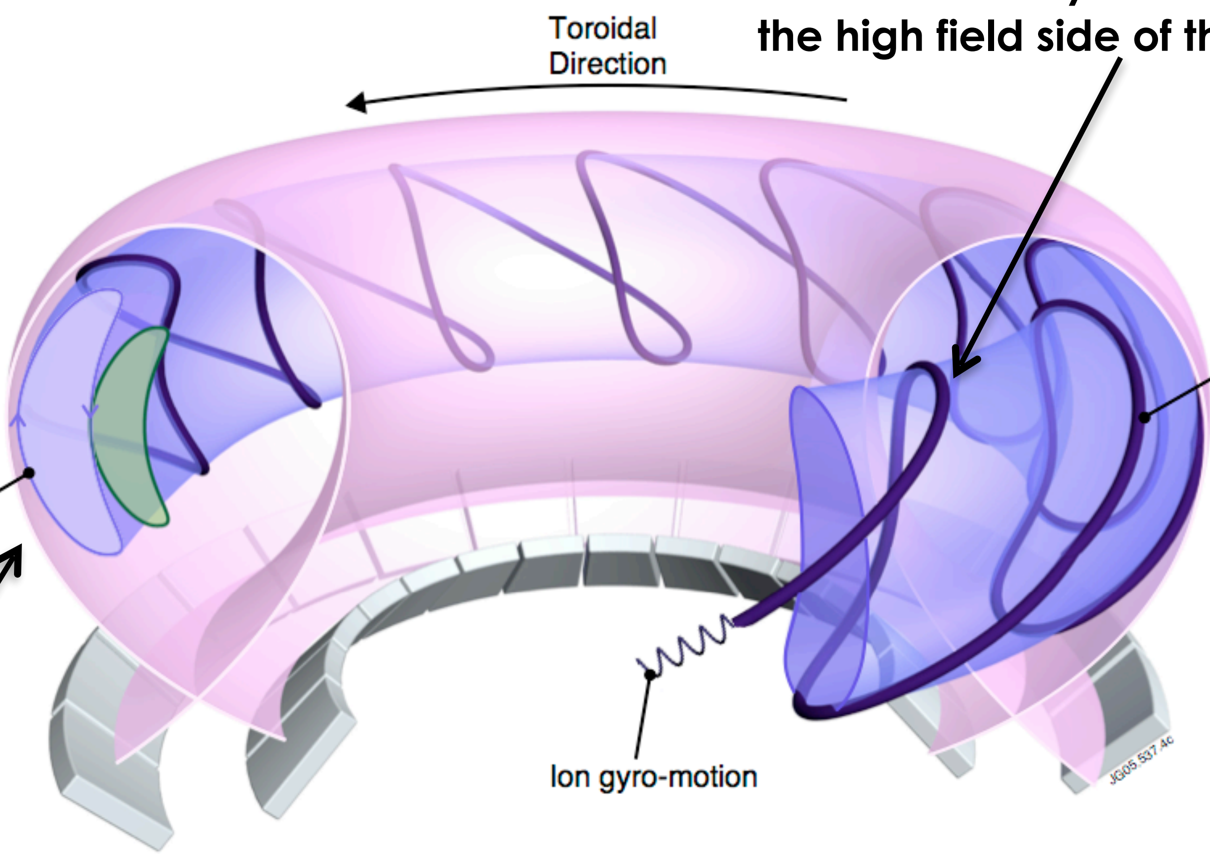
Trapped

Particles with sufficient $\mathbf{v}_{||}$ will follow the helical magnetic field around the torus

Particles with lower $\mathbf{v}_{||}$ are reflected as they encounter stronger B and therefore execute "banana" orbits as they precess around the torus B

Banana Orbits

Particles that don't have enough $v_{||}$ are reflected by the mirror force at the high field side of the tokamak

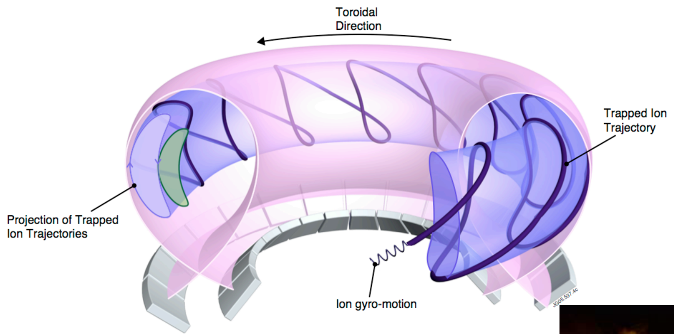


Trapped particles won't hit the wall if the banana orbit width Δr is small enough

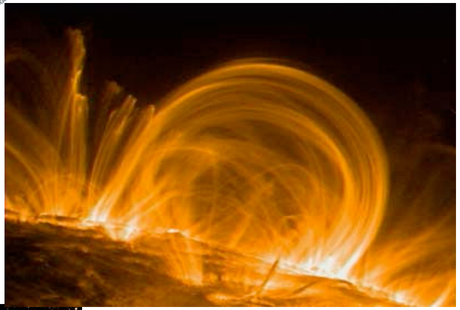
$$\Delta r = 2 r_L \frac{B_T}{B_p} \sqrt{\frac{r}{R}}$$

Image credit: euro-fusion.org

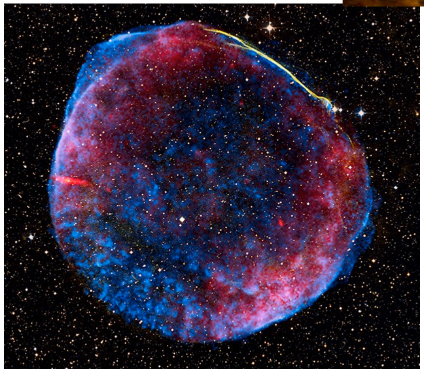
Conclusions - single particle and gyro motion in plasmas



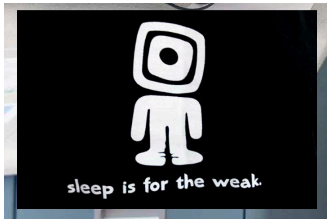
- Magnetic fields needed to *confine* hot collisionless fusion plasmas. ExB, gradB and curvature drifts



- Cross-field confinement structures solar plasmas, too



- Magnetic fields mediate collective behavior of plasmas, via drifts
- Plasmas can generate their own B fields!



- Work hard and soak it in. Have a good summer!