Plasma waves

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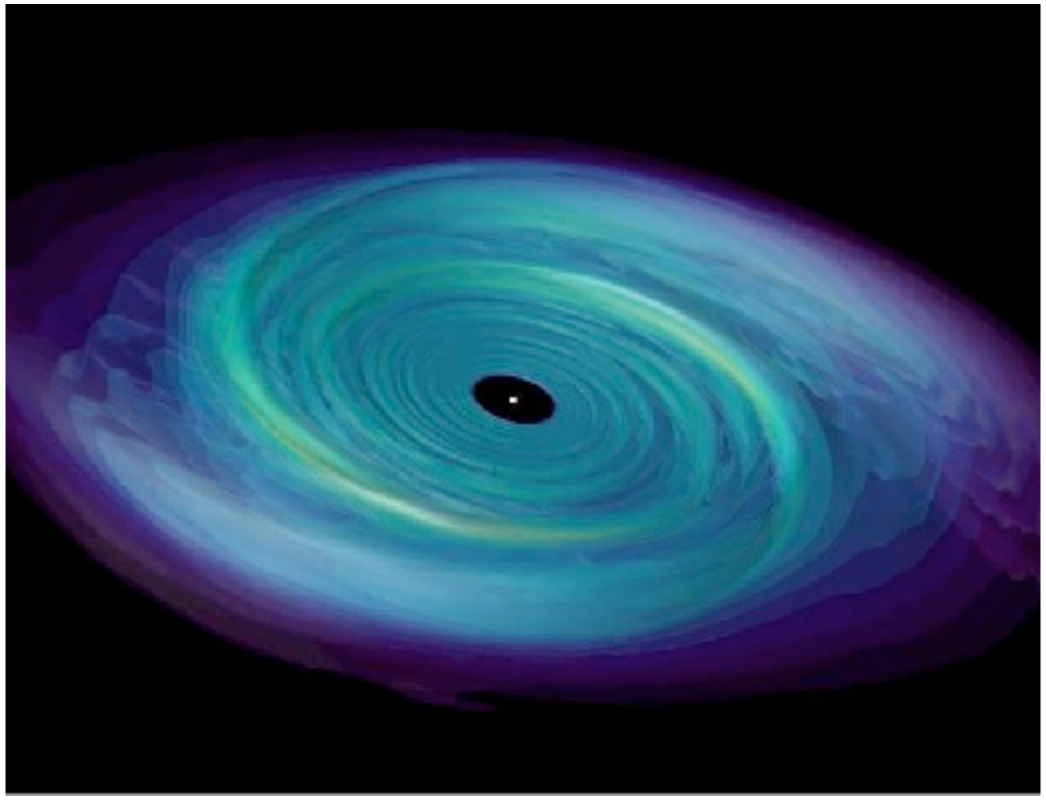
Importance of plasma waves

- Along with single particle motion, understanding of linear waves are foundation for physical intuition for behavior of plasmas
- Waves play direct role in important physical processes: RF heating in fusion plasmas, particle acceleration by waves in space plasmas, plasma turbulence in astrophysical objects



Wave equation, plasma dielectric model for linear waves

- Treat plasma as conducting medium; will lead to dielectric description (but start by treating plasma charge and currents as free)

$$\nabla \times \mathbf{B} = \mu_o \mathbf{j} + \frac{1}{c^2}\frac{\partial \mathbf{E}}{\partial t} \qquad \nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t}$$

$$\nabla \times \nabla \times \mathbf{E} + \frac{1}{c^2}\frac{\partial^2 \mathbf{E}}{\partial t^2} + \mu_o \frac{\partial \mathbf{j}}{\partial t} = 0$$

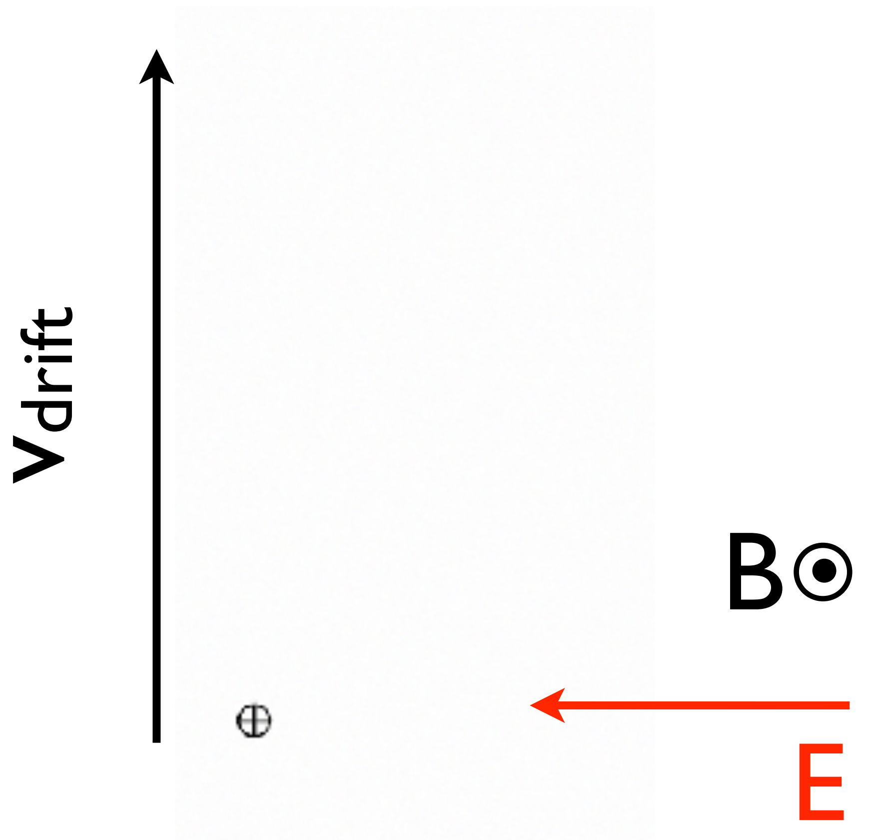
- Plasma effects buried in current, need model to relate current to E
- Model plasma as cold fluid, will find a linear, tensor conductivity

$$\mathbf{j} = \sigma \cdot \mathbf{E}$$

Important intuition: Single particle response to wave fields

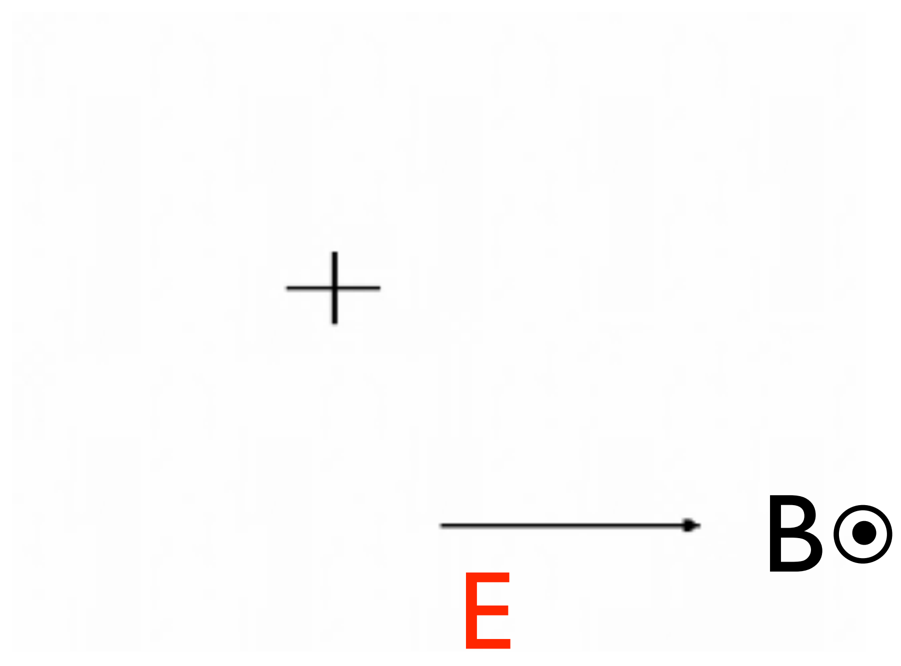
- Conductivity tensor tells us plasma response to applied electric field; useful to think about single particle orbits
- In particular for magnetized plasmas and wave electric fields that are perpendicular to B
- Two drifts matter (in uniform plasma): ExB drift and polarization drift
  - ExB drift is the dominant particle response for low frequency wave fields $\omega < \Omega_c$
  - Polarization drift is more important at higher frequencies (but can be the only source of current/plasma response even as $\omega \rightarrow 0$)

ExB and Polarization Drifts



ExB drift, DC E Field

$$\mathbf{v}_E = \frac{\mathbf{E}\times\mathbf{B}}{B^2}$$



Polarization drift, ExB drift removed

$$\mathbf{v}_p = \frac{1}{\Omega}\frac{\partial}{\partial t}\frac{\mathbf{E}_\perp}{B}$$

- No currents from ExB at low freq (ions and electrons drift the same); above ion cyclotron freq, ions primarily polarize, no ExB, can get ExB current from electrons

Model for plasma conductivity

- Use cold, two-fluid model; formally cold means:

$v_\phi \gg v_{\mathrm{th,e}}, v_{\mathrm{th,i}}$

$$\frac{\partial n_s}{\partial t} + \nabla \cdot n_s \mathbf{v}_s = 0$$

$$n_s m_s \left(\frac{\partial \mathbf{v}_s}{\partial t} + \mathbf{v}_s \cdot \nabla \mathbf{v}_s \right) = n_s q_s \left(\mathbf{E} + \mathbf{v}_s \times \mathbf{B} \right)$$

- + Maxwell's Equations

- Will compute the current that arises from the presence of a (wave) electric field; will find tensor conductivity

$$\mathbf{j} = \sum_s n_s q_s \mathbf{v}_s = \sigma \cdot \mathbf{E}$$

Model for plasma conductivity

- Assume uniform plasma, stationary (no mean flows). Assume plane wave form for all fluctuating quantities
$$f(\mathbf{r}, t) = f \exp\left(i\mathbf{k} \cdot \mathbf{r} - i\omega t\right)$$

- Linearize: assume perturbations due to waves are small: $$f = f_0 + f_1 + \ldots \quad ; \quad f_1 \ll f_o$$
$$n = n_0 + n_1 + \ldots$$
$$\mathbf{v} = \mathbf{v}_1 + \ldots$$
$$\mathbf{B} = \mathbf{B}_0 + \mathbf{B}_1$$
$$\mathbf{E} = \mathbf{E}_1 + \ldots$$

- Plug these in to our fluid equations and ignore terms higher than 1st order, e.g. ignore $\mathbf{v}_1 \times \mathbf{B}_1$ relative to $\mathbf{v}_1 \times \mathbf{B}_0$

Plasma model, cont.

$$\frac{\partial n_1}{\partial t} + n_0 \nabla \cdot \mathbf{v}_1 = 0$$

$$n_0 m \frac{\partial \mathbf{v}_1}{\partial t} = n_0 q \left(\mathbf{E}_1 + \mathbf{v}_1 \times \mathbf{B}_0\right)$$

Same as single particle EOM

Plasma model, cont.

$$\frac{\partial n_1}{\partial t} + n_0 \nabla \cdot \mathbf{v}_1 = 0$$

Same as single particle EOM

$$n_0 m \frac{\partial \mathbf{v}_1}{\partial t} = n_0 q \left(\mathbf{E}_1 + \mathbf{v}_1 \times \mathbf{B}_0 \right)$$

$$\text{Choose } \mathbf{B} = B_0 \hat{z} \ \ , \quad \mathbf{E} = \mathbf{E}_1 = E_x \hat{x} + E_z \hat{z}$$

Assume all fluctuating quantities are plane waves so that:

$$\frac{\partial \mathbf{v}_1}{\partial t} = -i\omega \mathbf{v}_1$$

Ion momentum equation becomes:

$$\begin{aligned} -i\omega v_x - \Omega_i v_y &= \frac{eE_x}{m_i} \\ \Omega_i v_x - i\omega v_y &= 0 \end{aligned} \qquad \Omega_i = \frac{eB}{m_i}$$

Plasma model, cont.

Solve for v_x, v_y:

$$v_x = \frac{-i\omega}{\Omega_i^2 - \omega^2} \frac{e}{m_i} E_x \quad \text{(polarization)}$$

$$v_y = \frac{-\Omega_i}{\Omega_i^2 - \omega^2} \frac{e}{m_i} E_x \quad \text{(E×B)}$$

For the parallel response: $v_z = \frac{ie}{\omega m_i} E_z$ (inertia-limited response)

So have all velocities as a function of wave E (equivalent for electrons), can construct conductivity from this

Plasma model, cont.

$$j_{1x} = i\omega\epsilon_o \sum_s \frac{\omega_{ps}^2}{\omega^2 - \Omega_s^2} E_{1x} - \epsilon_o \sum_s \frac{\Omega_s \omega_{ps}^2}{\omega^2 - \Omega_s^2} E_{1y}$$

$$j_{1y} = i\omega\epsilon_o \sum_s \frac{\omega_{ps}^2}{\omega^2 - \Omega_s^2} E_{1y} + \epsilon_o \sum_s \frac{\Omega_s \omega_{ps}^2}{\omega^2 - \Omega_s^2} E_{1x}$$

$$j_{1z} = i\omega\epsilon_o \sum_s \frac{\omega_{ps}^2}{\omega^2} E_{1z}$$

$$\omega_{ps}^2 = \frac{n_s q_s^2}{\epsilon_o m_s}$$

$$\sigma = i\epsilon_o \begin{pmatrix} \omega \sum_s \frac{\omega_{ps}^2}{\omega^2 - \Omega_s^2} & i \sum_s \frac{\Omega_s \omega_{ps}^2}{\omega^2 - \Omega_s^2} & 0 \\ -i \sum_s \frac{\Omega_s \omega_{ps}^2}{\omega^2 - \Omega_s^2} & \omega \sum_s \frac{\omega_{ps}^2}{\omega^2 - \Omega_s^2} & 0 \\ 0 & 0 & \omega \sum_s \frac{\omega_{ps}^2}{\omega^2} \end{pmatrix}$$

Plasma model, cont.

Back to the wave equation, rewrite with plane wave assumption:

$$-\mathbf{k} \times \mathbf{k} \times \mathbf{E} - \frac{\omega^2}{c^2}\mathbf{E} - i\omega\mu_o\sigma \cdot \mathbf{E} = 0$$

Can rewrite in the following way:

$$\mathbb{M} \cdot \mathbf{E} = 0$$

$$\mathbb{M} = (\hat{k}\hat{k} - \mathbb{I})n^2 + \epsilon \qquad n^2 = \frac{c^2k^2}{\omega^2} \text{ index of refraction}$$

$$\epsilon = \underset{\substack{\uparrow \\ \text{unit tensor}}}{\mathbb{I}} + \frac{i\sigma}{\epsilon_o\omega} \text{ dielectric tensor}$$

Cold plasma dispersion relation

Using the cold two-fluid model for σ, the dielectric tensor becomes:

$$\epsilon = \begin{pmatrix} S & -iD & 0 \\ iD & S & 0 \\ 0 & 0 & P \end{pmatrix}$$

$$S = 1 - \frac{\omega_{pi}^2}{\omega^2 - \Omega_i^2} - \frac{\omega_{pe}^2}{\omega^2 - \Omega_e^2} \quad \text{(polarization)}$$

$$D = \frac{\Omega_i \omega_{pi}^2}{\omega(\omega^2 - \Omega_i^2)} - \frac{\Omega_e \omega_{pe}^2}{\omega(\omega^2 - \Omega_e^2)} \quad \text{(E}\times\text{B response)}$$

$$P = 1 - \frac{\omega_{pi}^2}{\omega^2} - \frac{\omega_{pe}^2}{\omega^2} \quad \text{(inertial response)}$$

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Defining θ to be the angle between $\mathbf{k}$ and $\mathbf{B_o}$, the wave equation becomes:

$$\begin{pmatrix} S - n^2\cos^2\theta & -iD & n^2\sin\theta\cos\theta \\ iD & S - n^2 & 0 \\ n^2\sin\theta\cos\theta & 0 & P - n^2\sin^2\theta \end{pmatrix} \begin{pmatrix} E_x \\ E_y \\ E_z \end{pmatrix} = 0$$

$\det \mathbb{M} = 0$ provides dispersion relation for waves – allowable combinations of ω and $\mathbf{k}$

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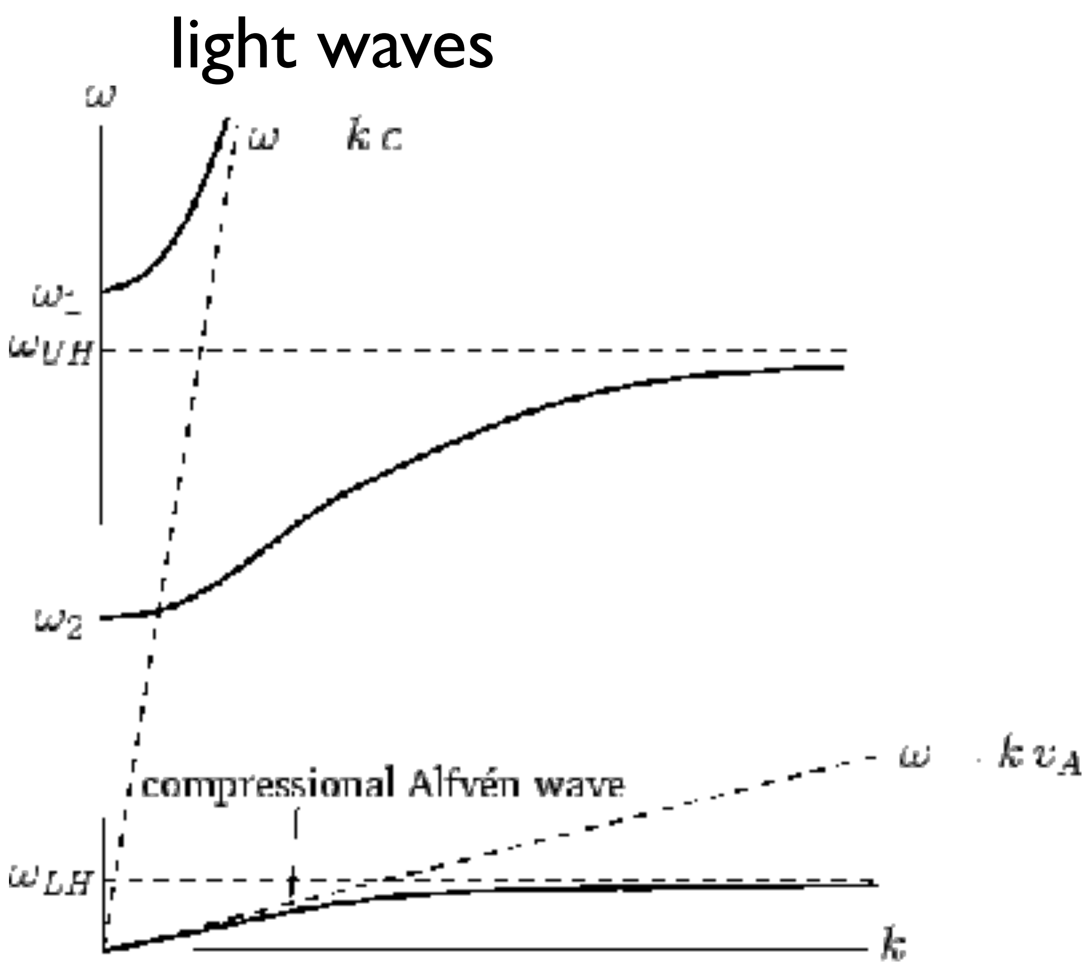
Defining θ to be the angle between $\mathbf{k}$ and $\mathbf{B_o}$, the wave equation becomes:

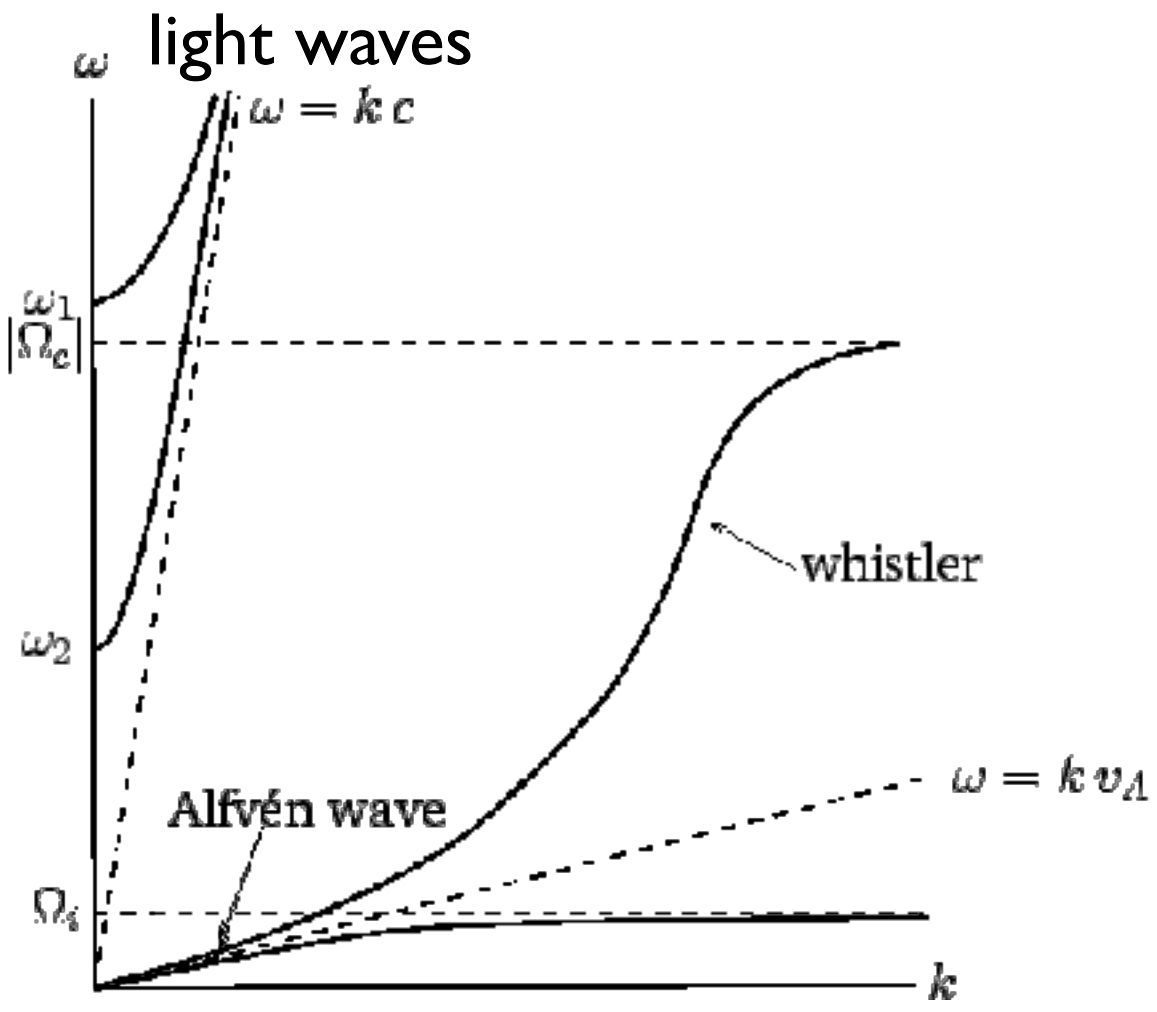
$$\begin{pmatrix} S - n_{\|}^2 & -iD & n_{\|} n_{\perp} \\ iD & S - n^2 & 0 \\ n_{\|} n_{\perp} & 0 & P - n_{\perp}^2 \end{pmatrix} \begin{pmatrix} E_x \\ E_y \\ E_z \end{pmatrix} = 0$$

$\det \mathbb{M} = 0$ provides dispersion relation for waves – allowable combinations of ω and $\mathbf{k}$

Cold plasma wave zoology

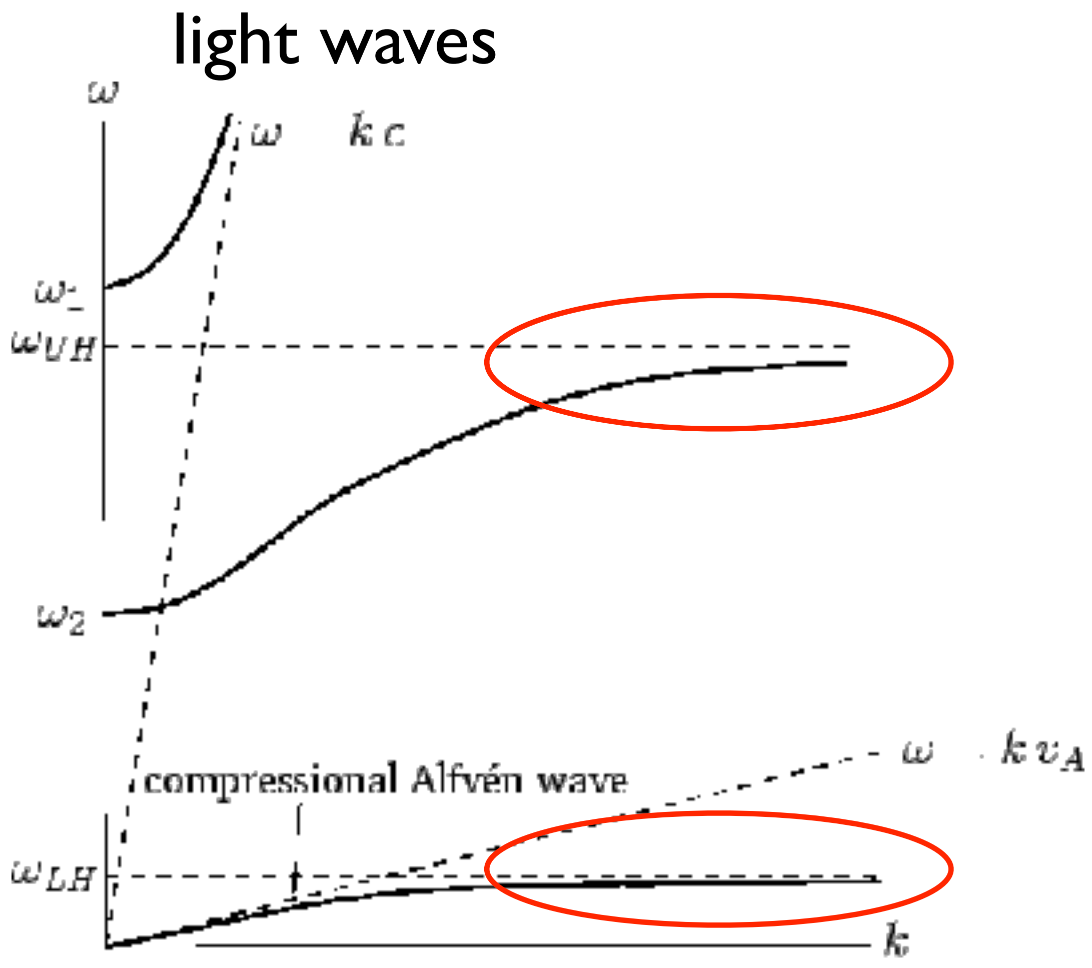


Perp. propagating ($\mathbf{k} = k_x \hat{x}$)



Parallel propagating ($\mathbf{k} = k_z \hat{z}$)

Cold plasma wave zoology



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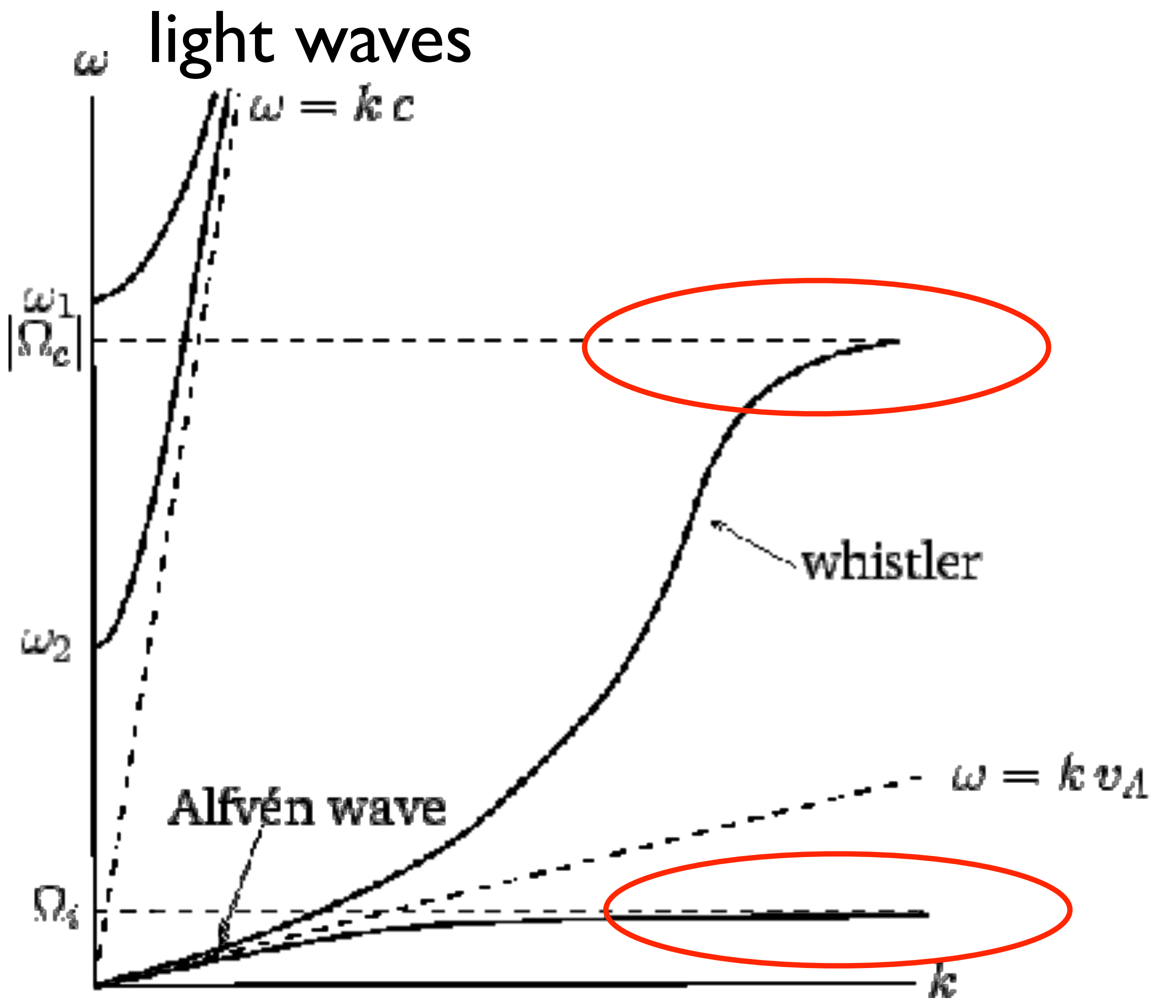
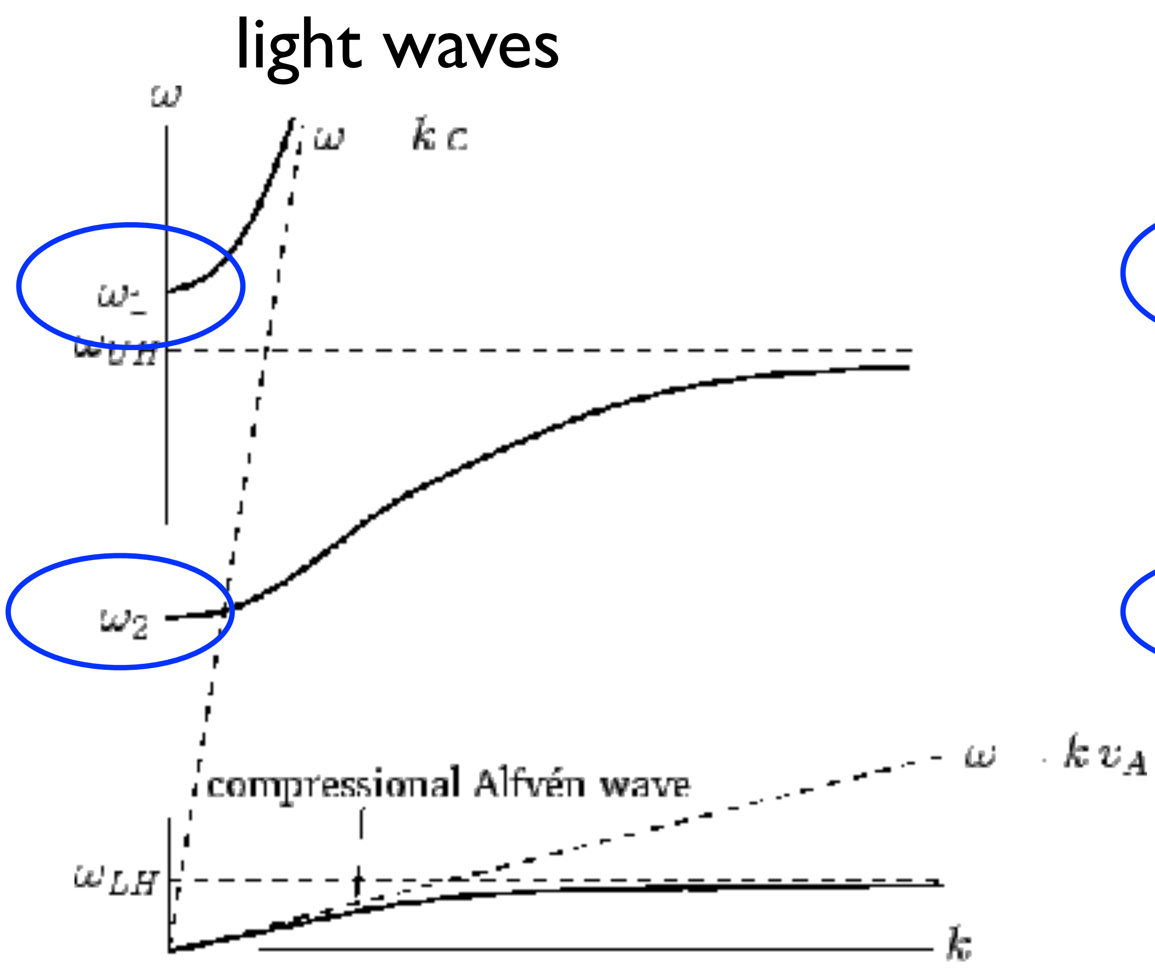


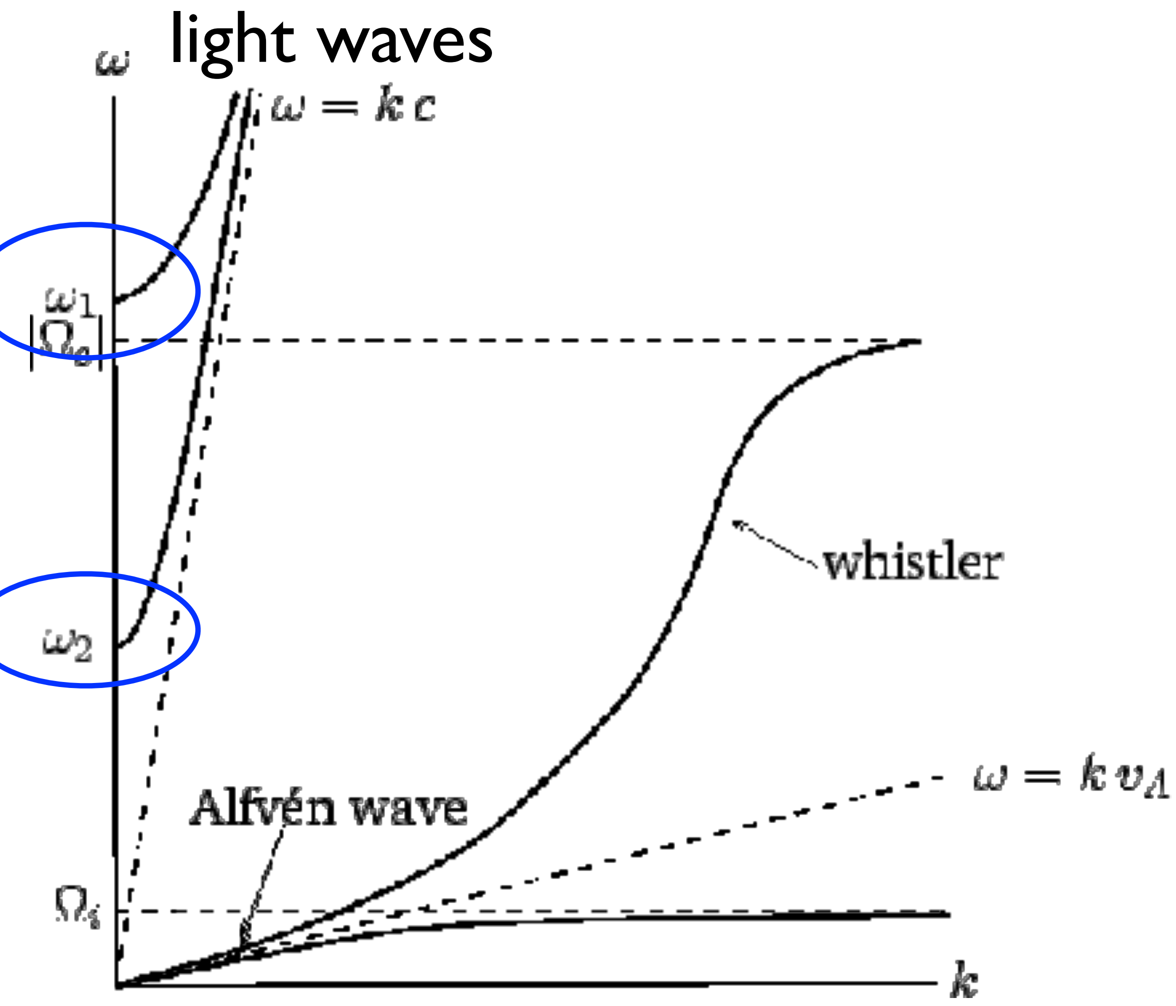
Parallel propagating $(\mathbf{k} = k_z \hat{z})$

- Resonance, $n^2 \rightarrow \infty$, when waves resonate with particle motion

Cold plasma wave zoology



Perp. propagating ($\mathbf{k} = k_x \hat{x}$)



Parallel propagating ($\mathbf{k} = k_z \hat{z}$)

- Resonance, $n^2 \rightarrow \infty$, when waves resonate with particle motion
- Cutoffs for $n^2=0$, transverse EM waves will not propagate below these frequencies (evanescent)

Plasma oscillations

- High frequency, electrostatic oscillations, happen in unmagnetized plasma or for E ∥ B_o in magnetized case
  - From our dielectric, choose k × E = 0 or θ = 0 and E = E_z (k·E ≠ 0)

$$\begin{pmatrix} S - n_{\|}^2 & -iD & 0 \\ iD & S - n_{\|}^2 & 0 \\ 0 & 0 & P \end{pmatrix} \begin{pmatrix} E_x \\ E_y \\ E_z \end{pmatrix} = 0$$

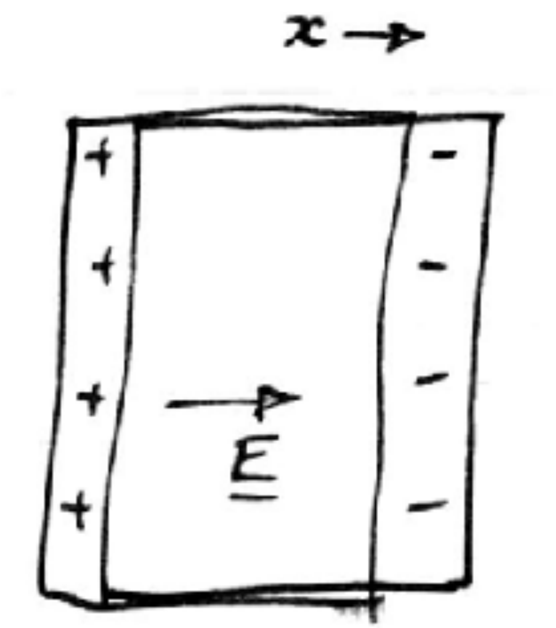
Plasma oscillations

- High frequency, electrostatic oscillations, happen in unmagnetized plasma or for E ∥ B_o in magnetized case
  - From our dielectric, choose k × E = 0 or θ = 0 and E = E_z (k·E ≠ 0)

$$P = 0$$

$$1 - \frac{\omega_{pe}^2}{\omega^2} - \cancel{\frac{\omega_{pi}^2}{\omega^2}} = 0$$

$$\omega = \omega_{pe} = \sqrt{\frac{n_e e^2}{\epsilon_o m_e}}$$

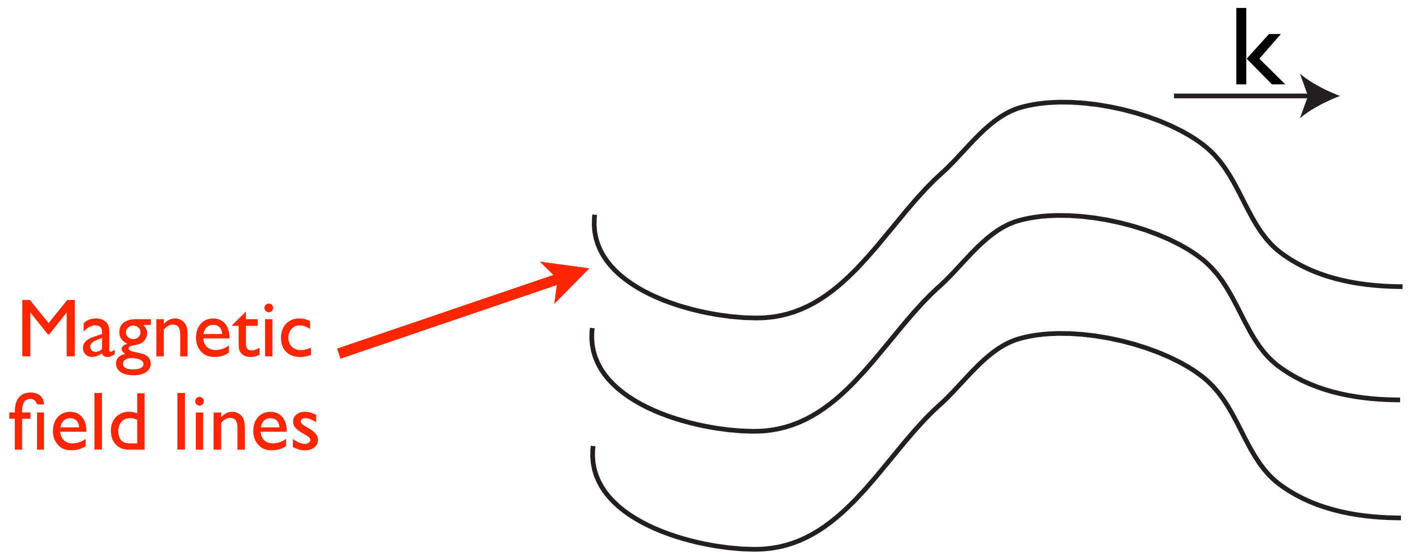


I. Hutchinson

  - Oscillation that results if you shift all the electrons in some region to the right (assuming immobile ions)
  - Highest frequency motion in a plasma (limited by electron inertia)

Low frequency waves: Alfvén waves

- For freq. much less than ion cyclotron frequency, primary waves are Alfvén waves



Shear Alfvén wave

- Primary motion: ExB motion of electrons and ions together (D→0)

- To pull this out of our cold plasma model:

$$\mathbf{k} = k_z \hat{z} \; (\theta = 0)$$

Low frequency shear Alfvén wave

- For frequency much less than ion cyclotron frequency, primary waves are Alfvén waves
- Low frequency response ($\omega \ll \Omega_i$): ExB motion of electrons and ions same, so no currents due to these drifts (D→0). Ion polarization is source of wave current. Focus on shear wave, $\mathbf{k} = k_z \hat{z} \ (\theta = 0)$

$$\begin{pmatrix} S - n_\parallel^2 & 0 & 0 \\ 0 & S - n_\parallel^2 & 0 \\ 0 & 0 & P \end{pmatrix} \begin{pmatrix} E_x \\ E_y \\ E_z \end{pmatrix} \approx 0$$

Solution:

$$n_\parallel^2 = S = 1 - \frac{\omega_{pi}^2}{\omega^2 - \Omega_i^2} - \frac{\omega_{pe}^2}{\omega^2 - \Omega_e^2}$$

Low frequency shear Alfvén wave

$$n_\parallel^2 = S = \cancel{1} - \frac{\omega_{pi}^2}{\cancel{\omega^2} - \Omega_i^2} - \frac{\omega_{pe}^2}{\cancel{\omega^2} - \Omega_e^2}$$

$$n_\parallel^2 \approx \frac{\omega_{pi}^2}{\Omega_i^2} + \frac{\omega_{pe}^2}{\Omega_e^2}$$

$$n_\parallel^2 = \frac{c^2}{v_A^2} + \cancel{\frac{c^2}{v_A^2}\frac{m_e}{m_i}}$$

$$n_\parallel^2 \approx \frac{c^2}{v_A^2}$$

$$\frac{c^2 k_\parallel^2}{\omega^2} = \frac{c^2}{v_A^2}$$

$$\boxed{\omega^2 = k_\parallel^2 v_A^2}$$

$$\frac{\omega_{pi}^2}{\Omega_i^2} = \frac{\dfrac{q^2 n_i^2}{\epsilon_o m_i}}{\dfrac{q^2 B^2}{m_i^2}} = \frac{c^2 \mu_o n_i m_i}{B^2} = \frac{c^2}{v_A^2}$$

$$v_A^2 = \frac{B^2}{\mu_o n_i m_i}$$

Alfvén waves from MHD

$$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \vec{v}) = 0$$

Continuity

$$\rho \left(\frac{\partial \vec{v}}{\partial t} + \vec{v} \cdot \nabla \vec{v} \right) = -\nabla p + \vec{j} \times \vec{B}$$

Momentum

$$\vec{E} + \vec{v} \times \vec{B} = 0$$

Ohm's Law
(electron momentum)

$$\frac{d}{dt} \left(\frac{p}{\rho^{\gamma}} \right) = 0$$

Pressure closure (adiabatic)

+ Maxwell's Equations

- Linearizing this system reveals four waves: fast and slow magnetosonic waves, the shear Alfvén wave, and the entropy wave

Oblique Alfvén waves in the cold plasma model: Inertial Alfvén waves

$$\begin{pmatrix} S - n_{\|}^2 & 0 & n_{\|} n_{\perp} \\ 0 & S - n^2 & 0 \\ n_{\|} n_{\perp} & 0 & P - n_{\perp}^2 \end{pmatrix} \begin{pmatrix} E_x \\ E_y \\ E_z \end{pmatrix} \approx 0$$

- Shear wave: $\mathbf{k}_{\perp} \parallel \mathbf{E}_{\perp} \quad , \quad E_y = 0$

$$P\left(k_{\|}^2 - \frac{\omega^2}{c^2} S\right) = -k_{\perp}^2 S$$

$$S \approx \frac{c^2}{v_A^2}$$

$$P = 1 - \frac{\omega_{pi}^2}{\omega^2} - \frac{\omega_{pe}^2}{\omega^2} \approx -\frac{\omega_{pe}^2}{\omega^2}$$

$$-\frac{\omega_{pe}^2}{\omega^2}\left(k_{\|}^2 - \frac{\omega^2}{v_A^2}\right) = -k_{\perp}^2 \frac{c^2}{v_A^2}$$

Oblique Alfvén waves in the cold plasma model: Inertial Alfvén waves

$$-\frac{\omega_{pe}^2}{\omega^2}\left(k_\parallel^2 - \frac{\omega^2}{v_A^2}\right) = -k_\perp^2 \frac{c^2}{v_A^2}$$

$$k_\parallel^2 - \frac{\omega^2}{v_A^2} = k_\perp^2 \frac{\omega^2}{v_A^2}\frac{c^2}{\omega_{pe}^2}$$

$$\frac{\omega^2}{v_A^2}\left(1 + k_\perp^2 \frac{c^2}{\omega_{pe}^2}\right) = k_\parallel^2$$

Inertial Alfvén wave

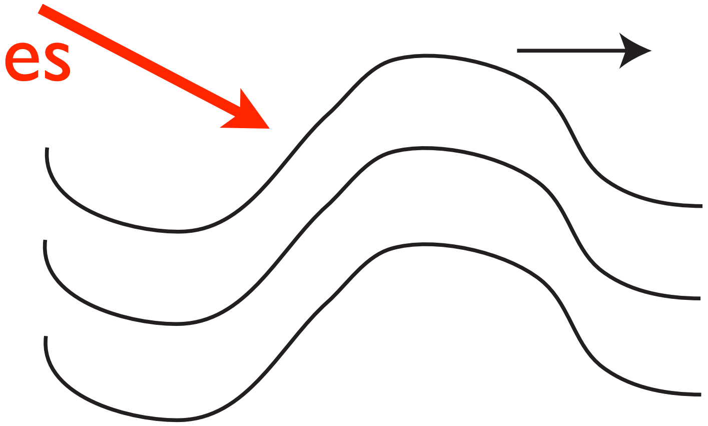
$$\boxed{\omega^2 = \frac{k_\parallel^2 v_A^2}{1 + k_\perp^2 \delta_e^2}}$$

$$\delta_e^2 = \frac{c^2}{\omega_{pe}^2} \quad \text{electron skin depth}$$

MHD Waves

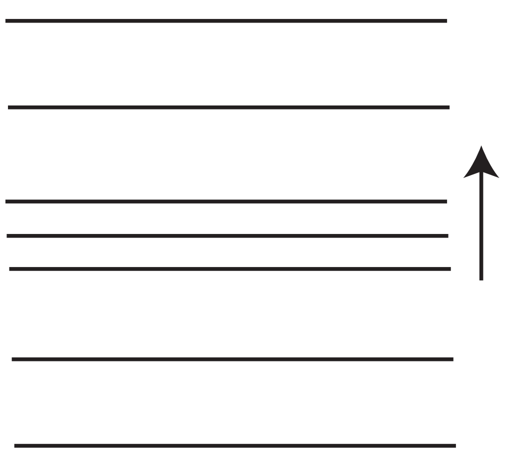
- For freq. much less than ion cyclotron frequency, primary waves are Alfvén waves

Magnetic field lines

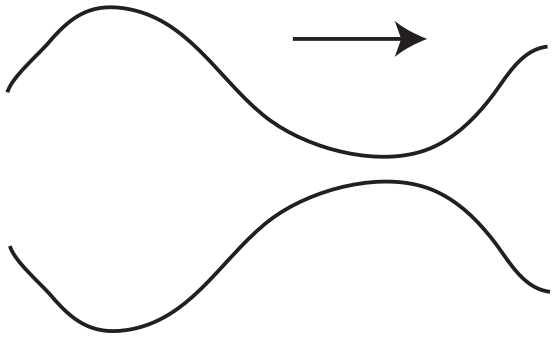


Shear Alfvén wave

$$\omega^2 = k_\parallel^2 v_A^2$$



Compressional
Alfvén wave
(fast magnetosonic)



Slow magnetosonic

$$\omega^2 = \frac{k^2}{2}\left(c_s^2 + v_A^2 \pm \sqrt{c_s^4 + v_A^4 - 2c_s^2 v_A^2 \cos 2\theta}\right)$$

sound wave response (in fast/slow modes)
not in our cold two-fluid model

Shear wave dispersion derivation

$$\rho\left(\frac{\partial\vec{v}}{\partial t}+\vec{v}\cdot\nabla\vec{v}\right)=-\nabla p+\vec{j}\times\vec{B}=-\nabla\left(p+\frac{B^2}{2\mu_o}\right)+\frac{\vec{B}\cdot\nabla\vec{B}}{\mu_o}$$

magnetic pressure

magnetic tension

$$\frac{\partial\vec{B}}{\partial t}=-\nabla\times\vec{E}=\nabla\times\vec{v}\times\vec{B}$$

- We are looking for the shear wave, so we'll make appropriate assumptions:

$\vec{k}\cdot\delta\vec{v}=0$	incompressible motion
$\vec{B}=B_o\hat{z}+\delta B\hat{x}$	no field line compression, linearly polarized
$\delta B,\delta v\propto\exp(i\vec{k}\cdot\vec{r}-i\omega t)$	plane waves
$\delta p=0$	follows from the first assumption, adiabatic assumption

Shear wave dispersion derivation, cont

$$\rho\left(\frac{\partial\vec{v}}{\partial t}+\cancel{\vec{v}\cdot\nabla\vec{v}}\right)=-\nabla\left(\cancel{p}+\cancel{\frac{B^2}{2\mu_o}}\right)+\frac{\vec{B}\cdot\nabla\vec{B}}{\mu_o}$$

$$-i\omega\rho\delta\vec{v}=\frac{ik_{\|}B_o\delta B}{\mu_o}\hat{x}$$

$$\frac{\partial\vec{B}}{\partial t}=\nabla\times\vec{v}\times\vec{B}$$

$$-i\omega\delta B=ik_{\|}\delta vB_o$$

- Combine these two to get:

$$\boxed{\omega^2=k_{\|}^2\frac{B^2}{\mu_o\rho}=k_{\|}^2v_A^2}$$

Currents in MHD AW

$$\vec{j} = \frac{1}{\mu_o}\nabla\times\vec{B} = \frac{1}{\mu_o}i\vec{k}\times(\delta B\hat{x}) \qquad \delta\vec{E} = -\delta\vec{v}\times B_o\hat{z} = -\delta v B_o\hat{y}$$

$$\delta\vec{E} = \frac{k_\parallel B_o^2}{\omega\rho\mu_o}\delta B\hat{y}$$

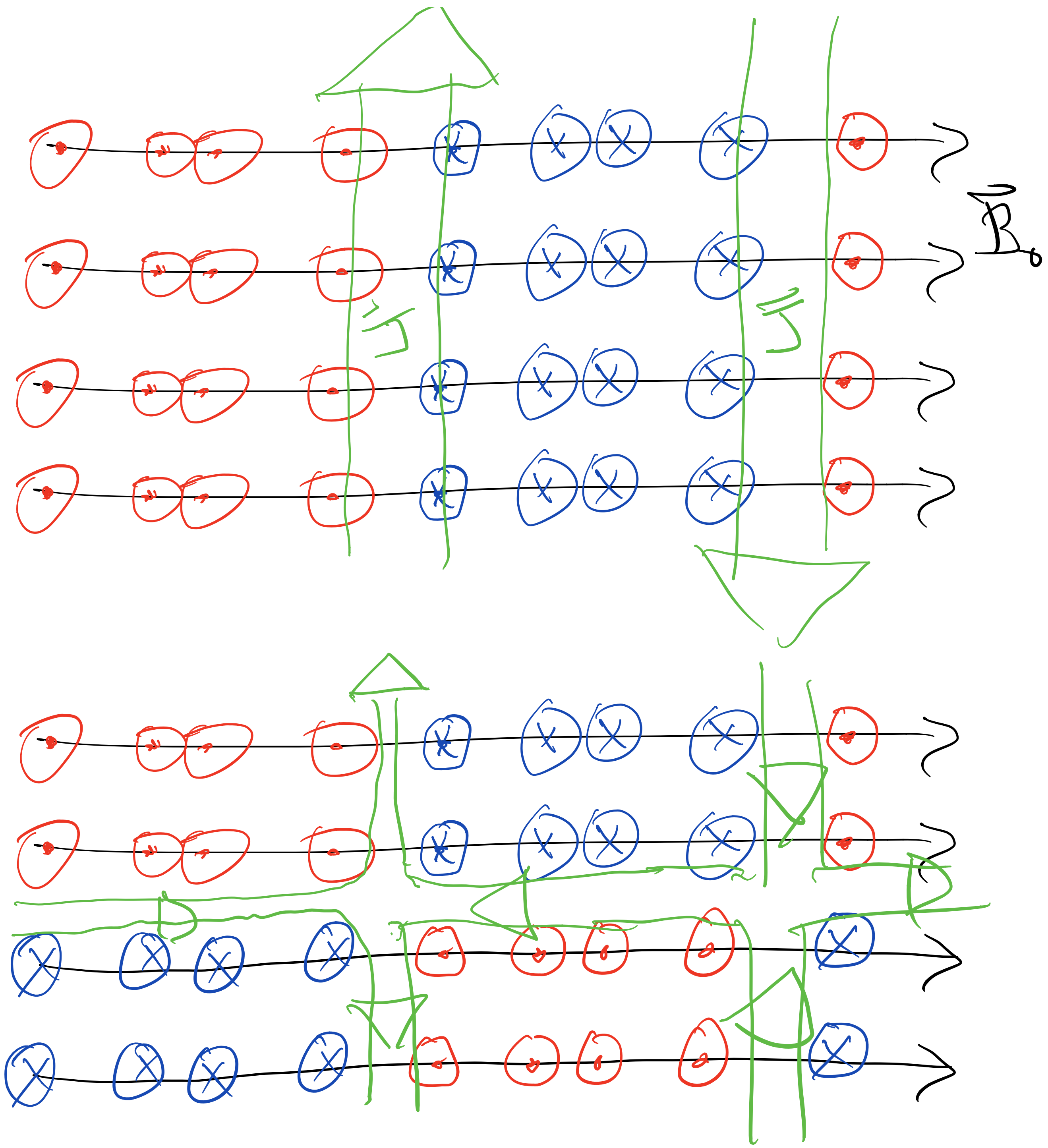
$$\boxed{\vec{j} = -\frac{i\omega ne}{\Omega_i}\frac{\delta\vec{E}}{B_o} - \frac{ik_y\delta B}{\mu_o}\hat{z}}$$

Polarization current

- Current in $k_\perp=0$ AW is entirely due to ion polarization current: no field aligned current
- As $k_\perp$ is introduced, current closes along the field (inductively driven)

Finite k⊥ introduces parallel current, electric field

- With finite k⊥, wave currents must close along the field: introduce parallel electric field to push the current.

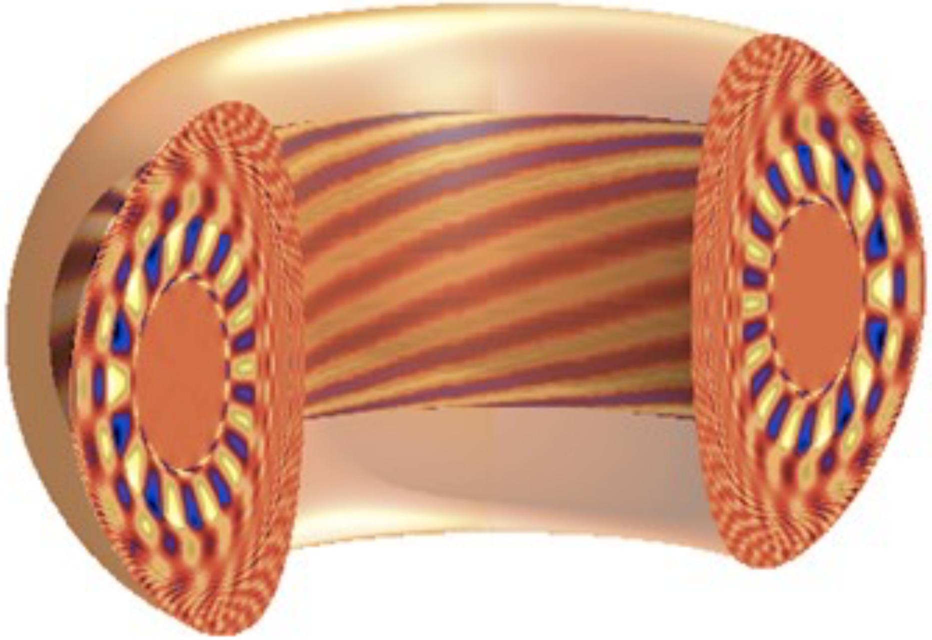


Finite $k_\perp$ introduces parallel current, electric field

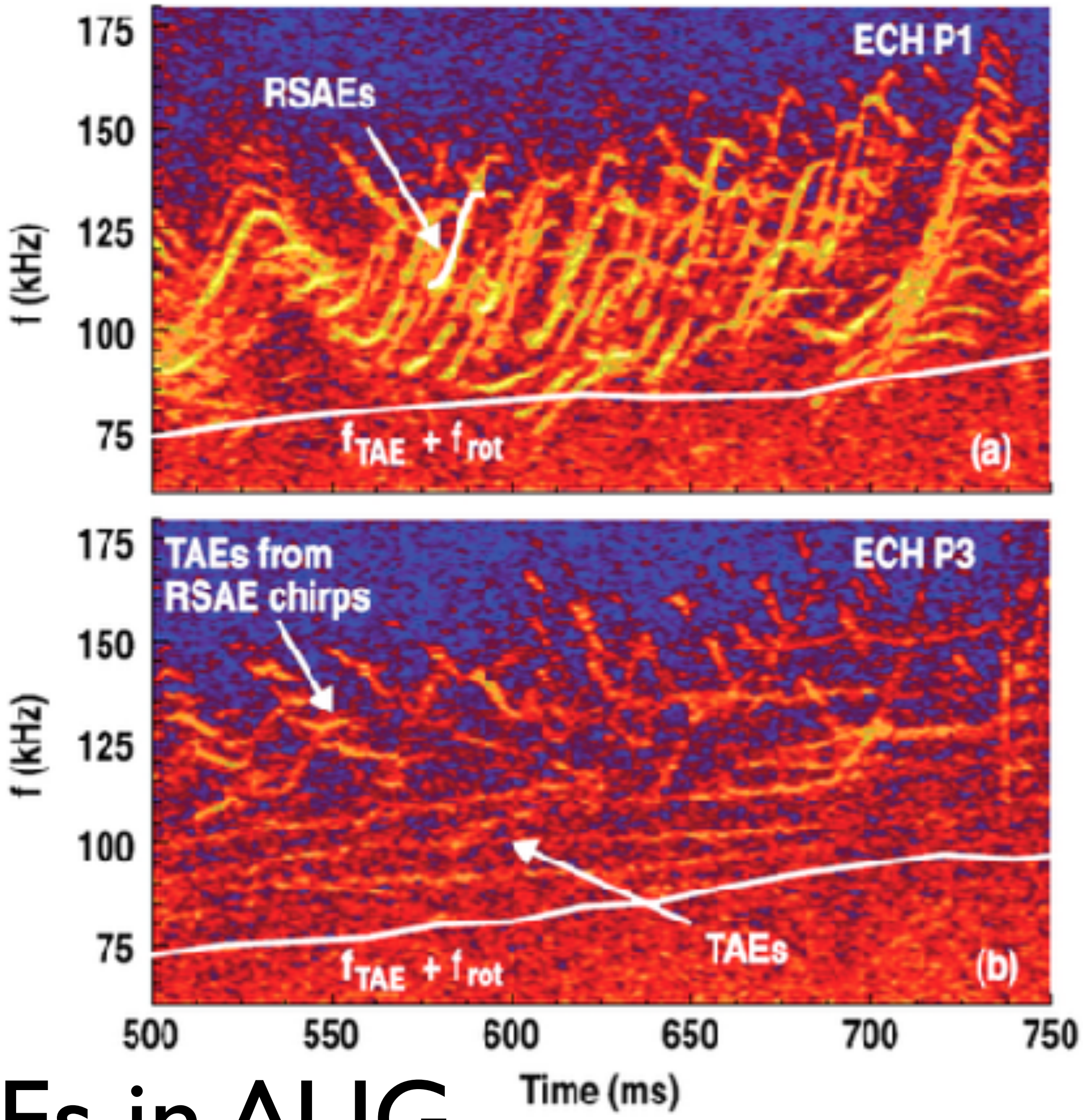
- With finite $k_\perp$, wave currents must close along the field: introduce parallel electric field to push the current.
- Parallel particle response is not simple! Easy to find departures from MHD and cold two-fluid theories — kinetic effects, wave particle resonance, etc.
  - Ions carry current across field, electrons carry parallel current (ion parallel response/current important at higher β)
  - Electron parallel response introduces dispersion and damping to Alfvén wave with finite $k_\perp$ (no $j \cdot E$ with ion polarization current (out of phase, although ion viscosity can change this), get finite $j \cdot E$ with parallel currents)
- **Why do we care?: dissipation in Alfvénic turbulence, mode conversion of AEs to oblique AWs (KAWs) in tokamaks, auroral electron acceleration by oblique AWs in magnetosphere**

Why should we care about Alfvén waves?

- Fusion plasmas: Alfvén eigenmodes, ICRF

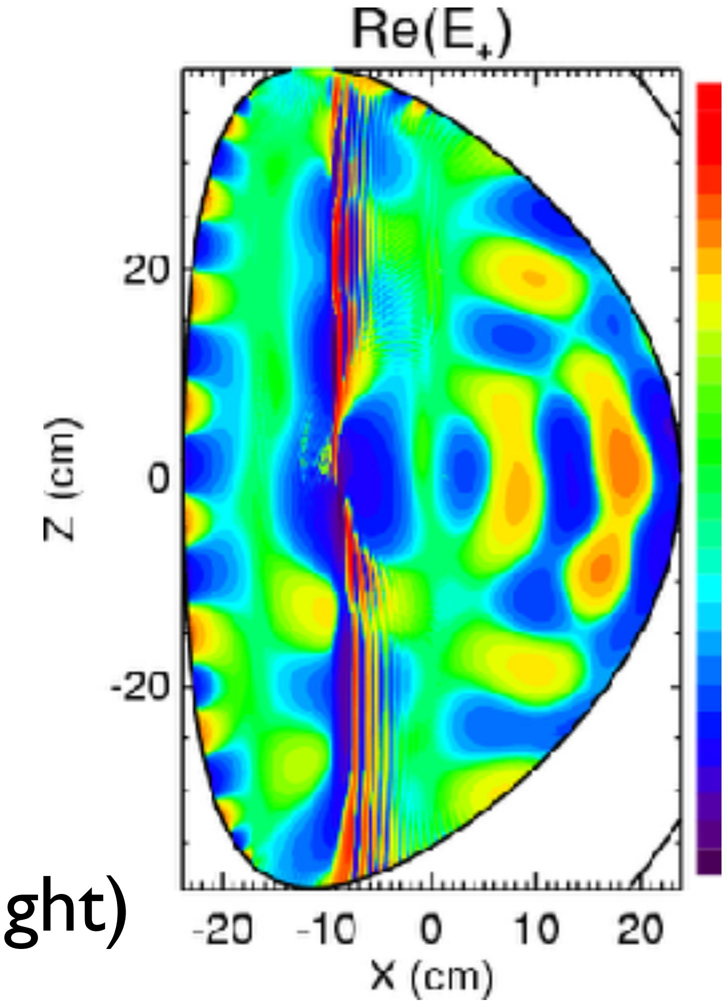


AE in ITER (Van Zeeland)



AEs in AUG

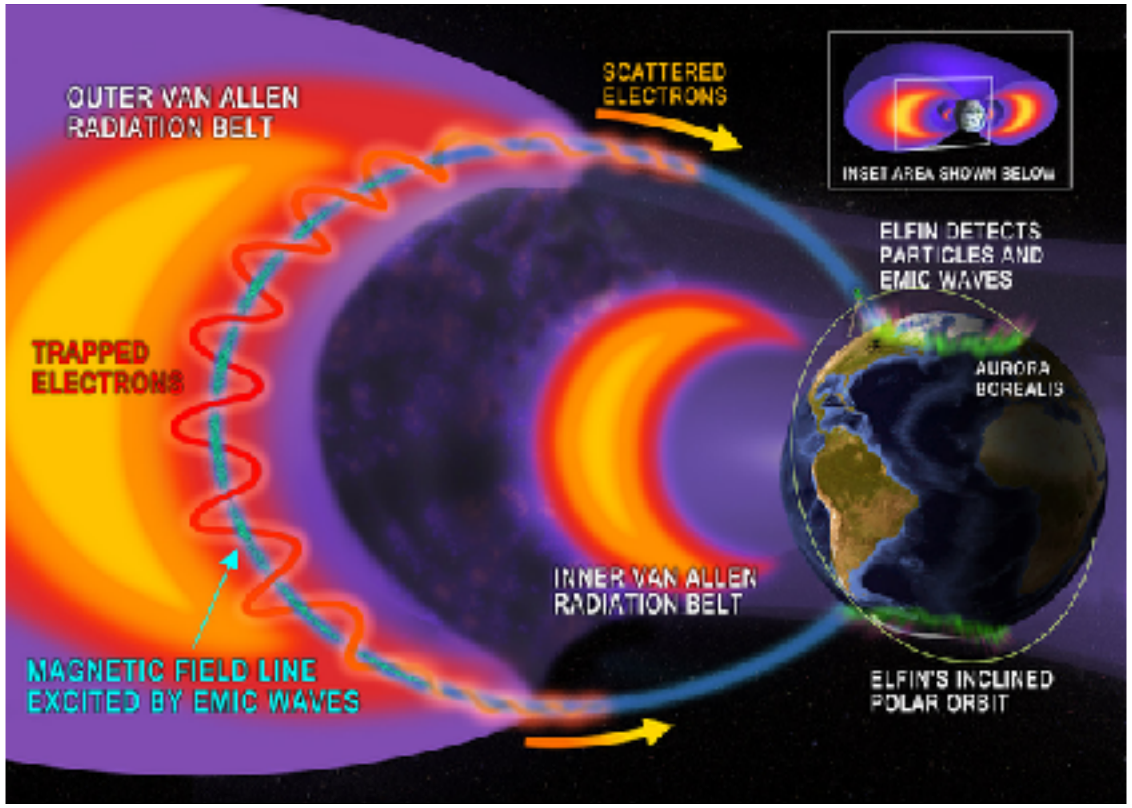
Sharapov, et al., PPCF 2017



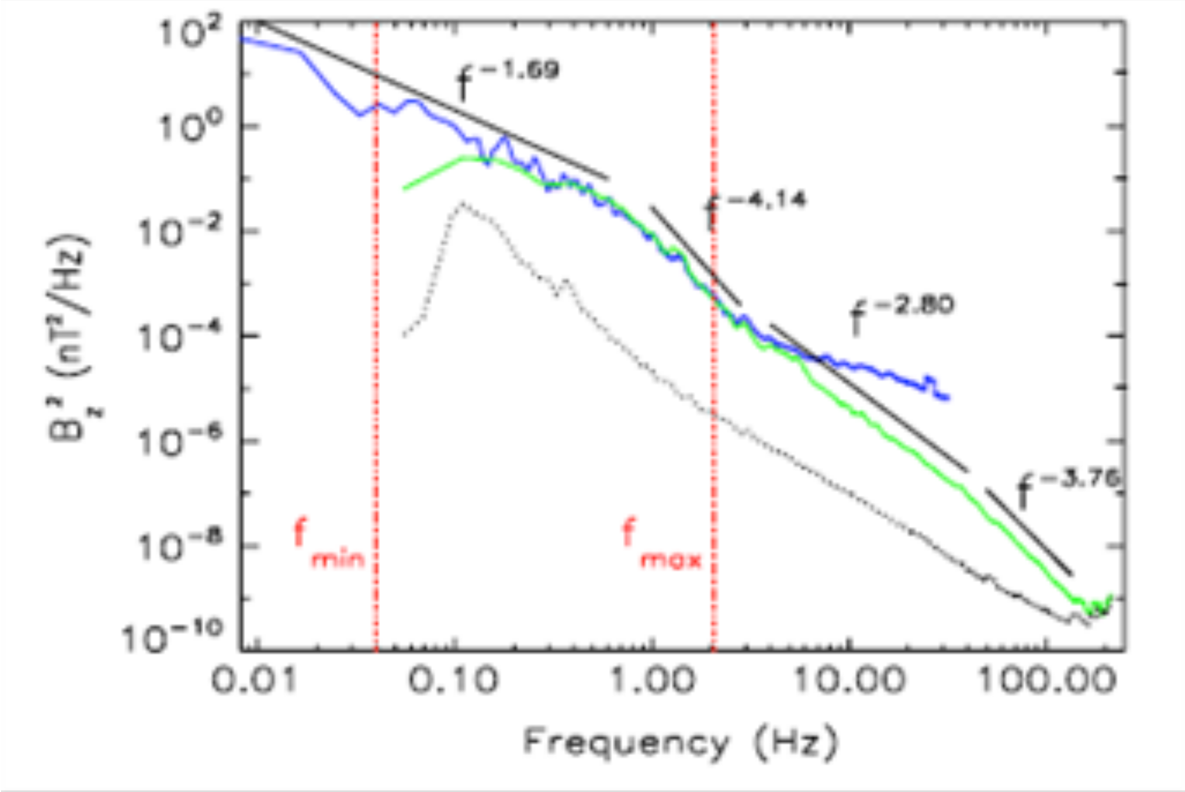
FW/ICRH (Wright)

Why should we care about Alfvén waves?

- Space/Astro plasmas:



Radiation Belts



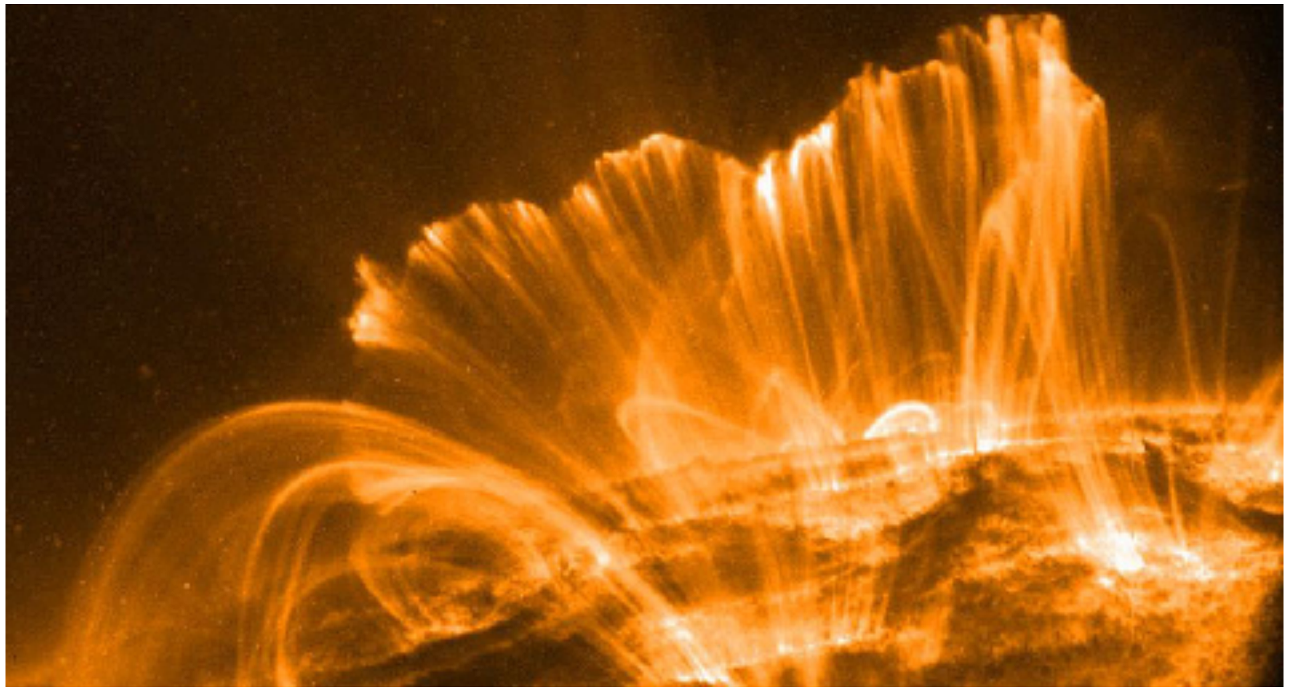
Sahraoui, et al. PRL 2010

Solar wind turbulence

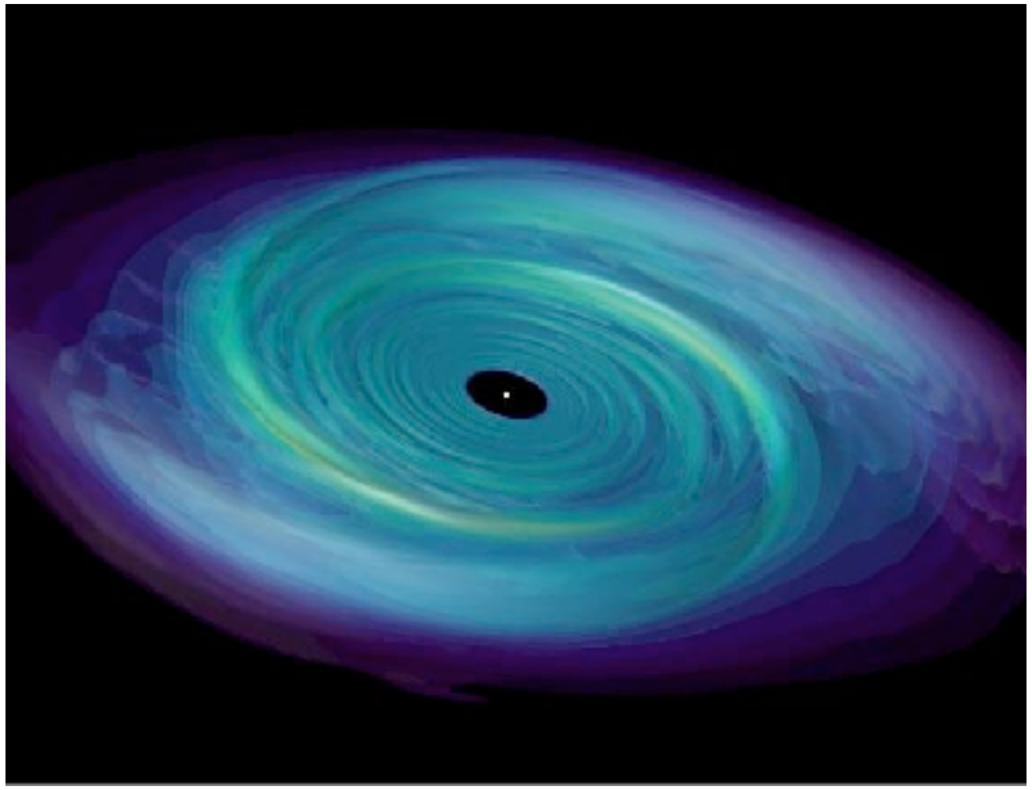
Aurora

Why should we care about Alfvén waves?

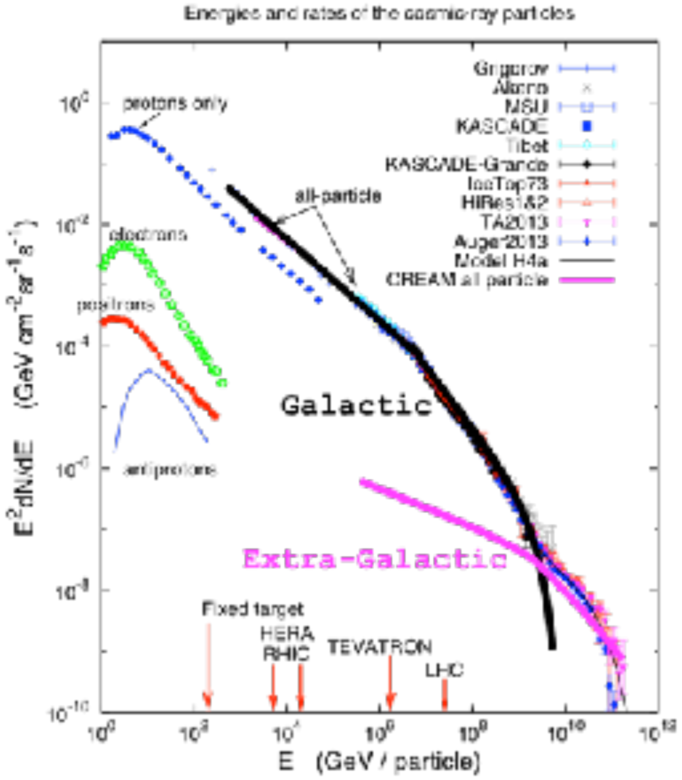
- Space/astro plasmas:



Coronal heating



Accretion disk turbulence



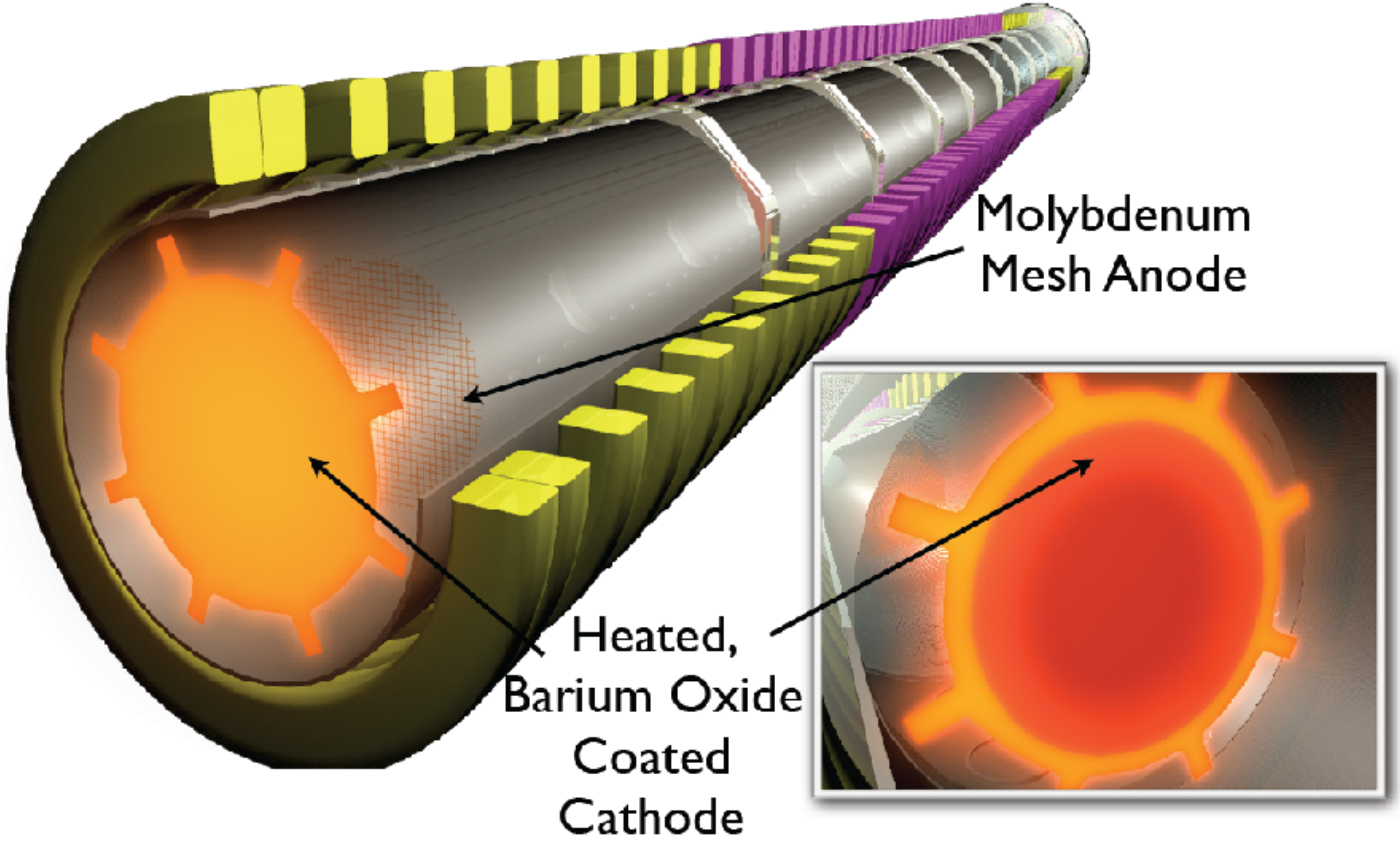
Cosmic ray scattering

Shocks

The LArge Plasma Device (LAPD)

- Solenoidal magnetic field, cathode discharge plasma (BaO and LaB_6)
- BaO Cathode: $n \sim 10^{12}$ cm^{-3}, $T_e \sim$ 5-10 eV, $T_i \lesssim$ 1 eV
- LaB_6 Cathode: $n \sim 5 \times 10^{13}$ cm^{-3}, $T_e \sim$ 10-15 eV, $T_i \sim$ 6-10 eV
- B up to 2.5kG (with control of axial field profile)
- BaO: Large plasma size, ~20m long, D~60cm
- High repetition rate: 1 Hz
- US NSF/DOE User Facility for fundamental plasma physics

LAPD Plasma source

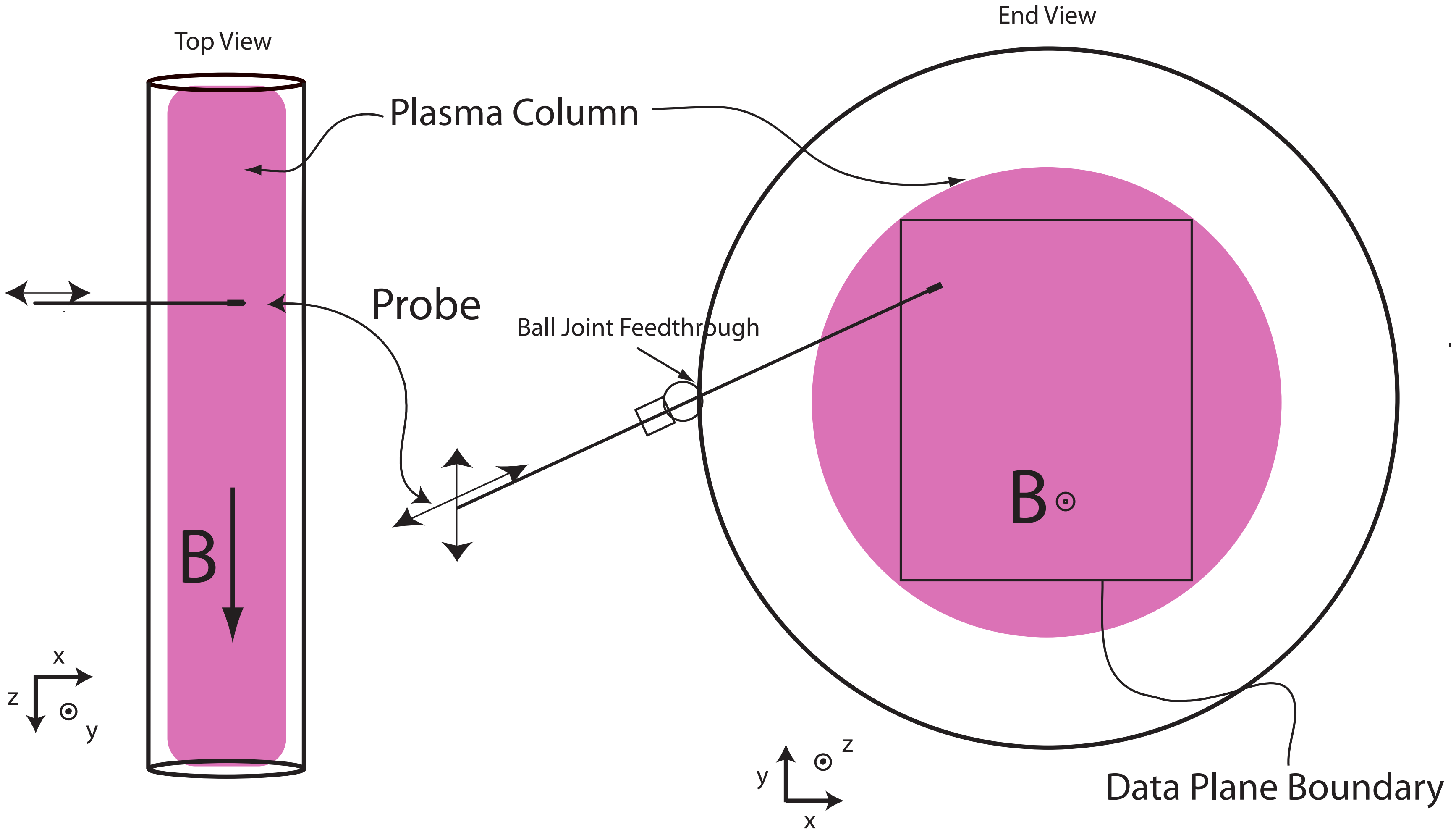


Measurement methodology in LAPD

- Use physical probes to measure local density, temperature, potential, magnetic field, flow
    - e.g. Langmuir probe: electrode biased to collect current from the plasma (determine density, temperature, potential)

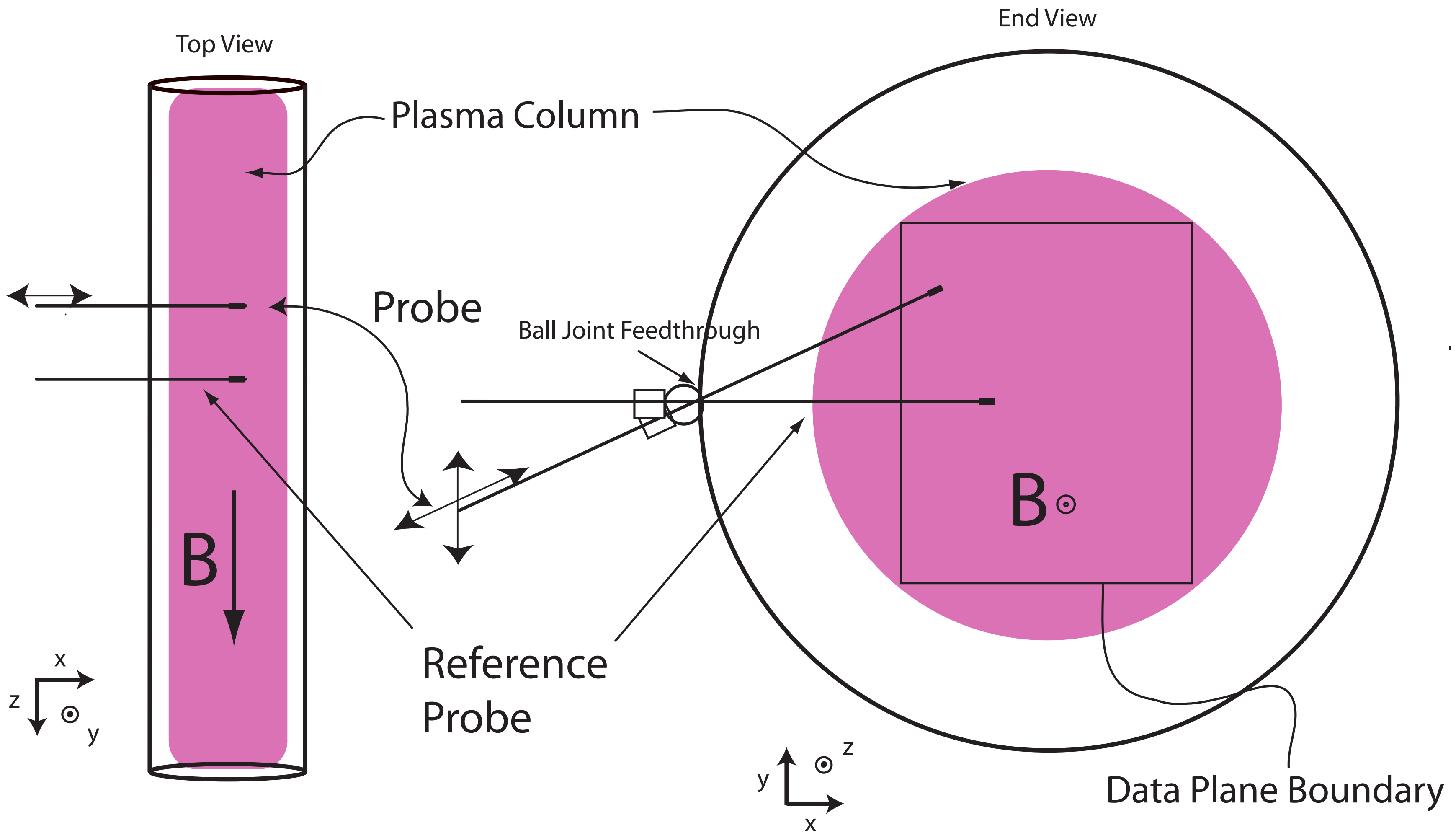
Measurement methodology in LAPD

- Use physical probes to measure local density, temperature, potential, magnetic field, flow
    - e.g. Langmuir probe: electrode biased to collect current from the plasma (determine density, temperature, potential)
- Use high rep rate (1Hz) to make detailed spatial measurements of average quantities over many discharges

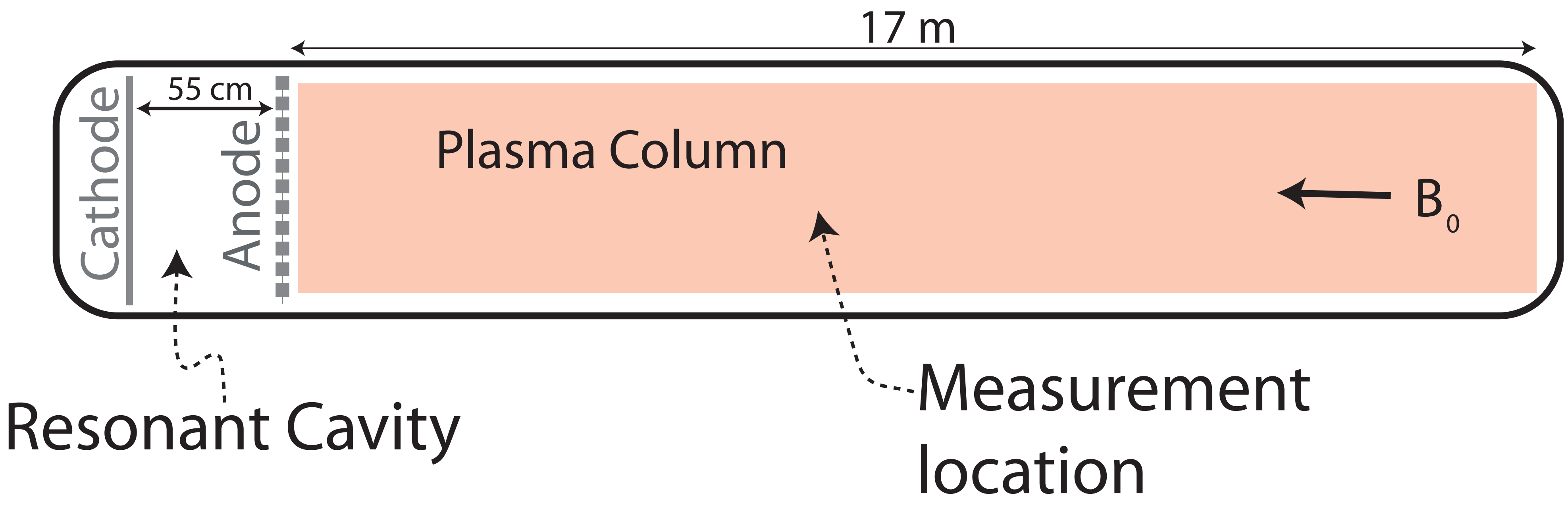


Measurement methodology in LAPD

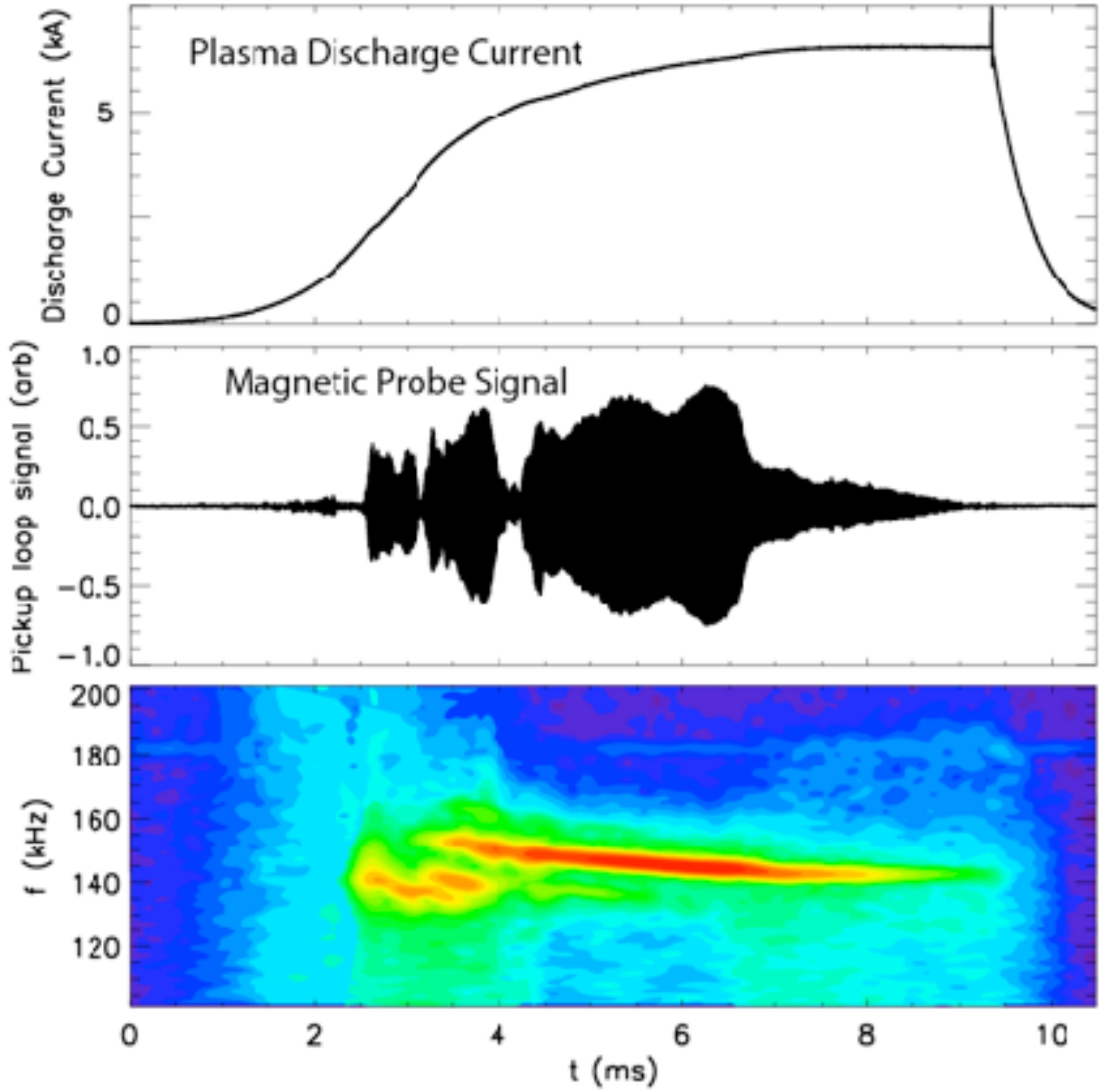
- Use physical probes to measure local density, temperature, potential, magnetic field, flow
  - e.g. Langmuir probe: electrode biased to collect current from the plasma (determine density, temperature, potential)
- Use correlation techniques to make detailed statistical measurements of turbulence (structure, etc)



Example data: Alfvén wave MASER



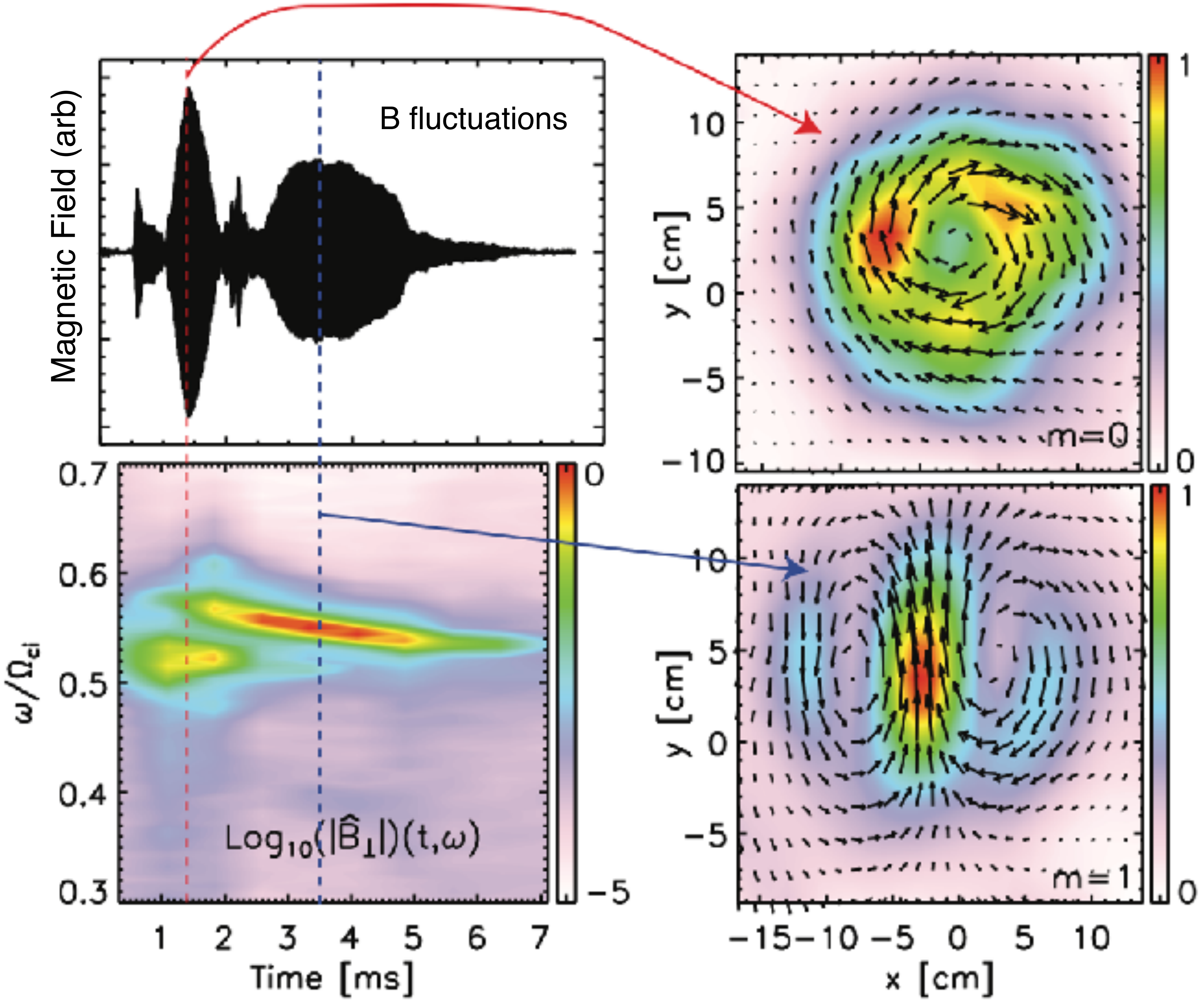
- Plasma source acts as resonant cavity for shear Alfvén waves
- Driven spontaneously by discharge current (thought to be inverse Landau damping on return current electrons)
- Alfvén wave “MASER”



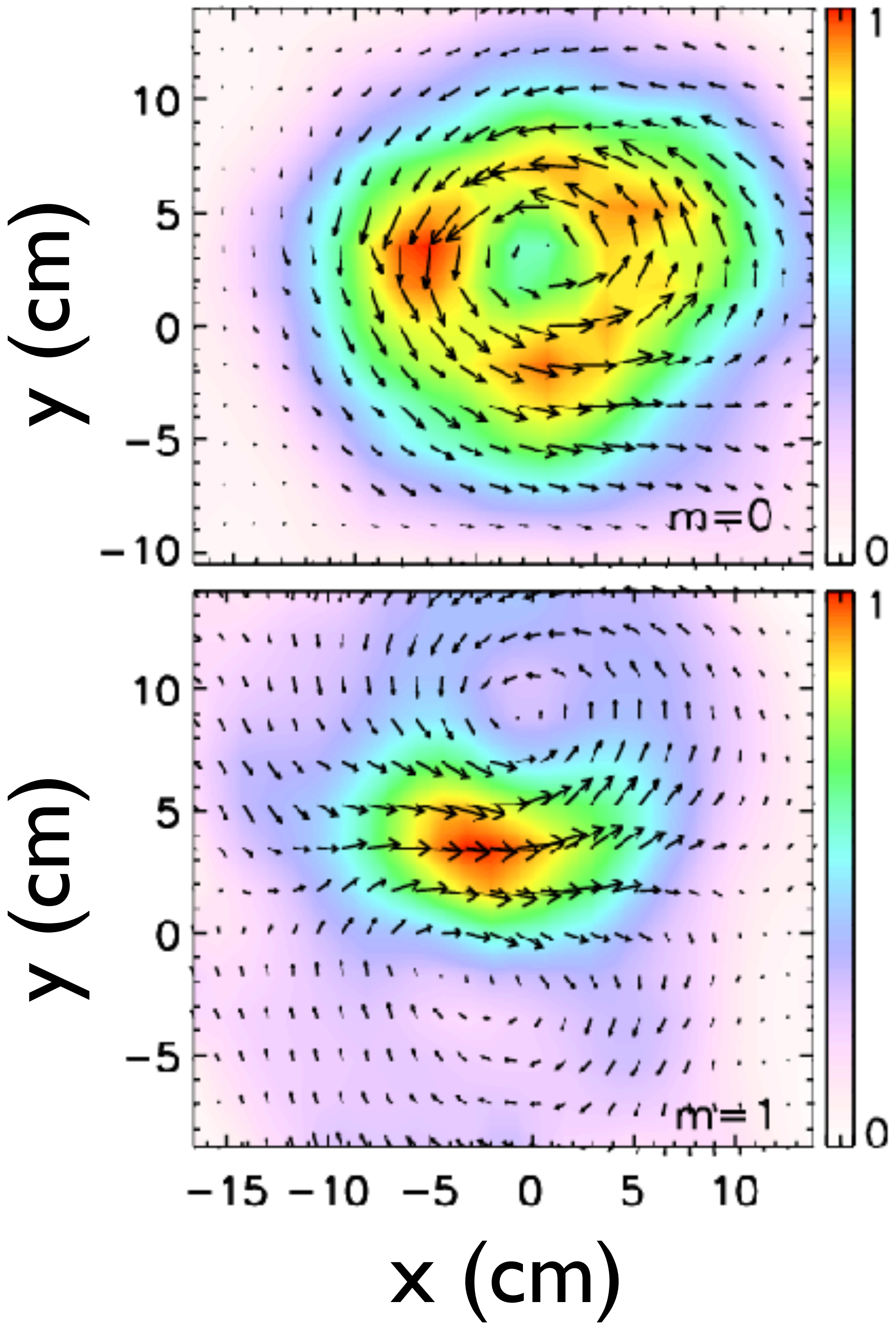
Maggs, Morales, PRL 91, 035004 (2003)

Maggs, Morales, Carter, PoP 12, 013103 (2005)

Cylindrical Alfvén eigenmodes in LAPD (Alfvén wave maser)

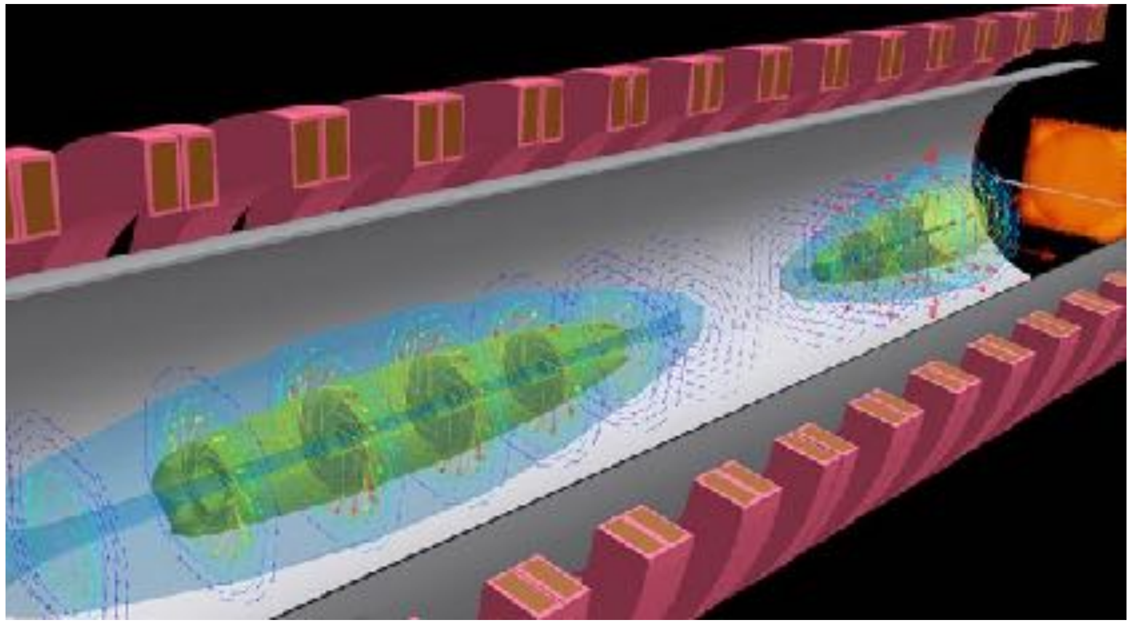


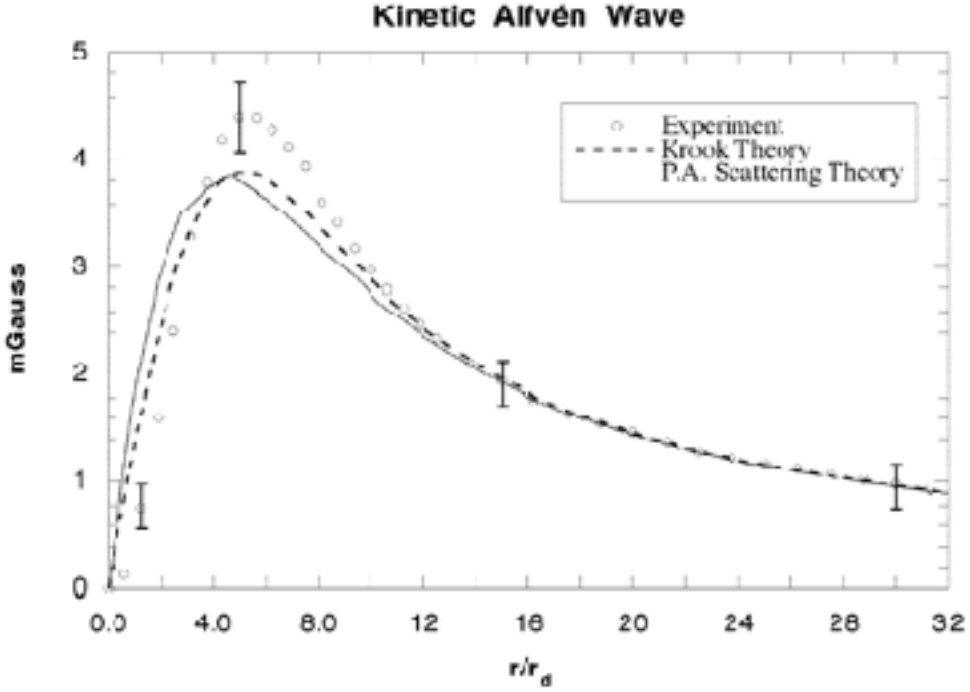
- Source region (cathode/ anode) acts as cavity for shear Alfvén waves
- Get spontaneous emission of AWs when discharge current exceeds threshold
- See m=0, m=1 cylindrical eigenmodes


y (cm)
10
5
0
−5
−10
m=0
1
0
y (cm)
10
5
0
−5
m=1
1
0
−15
−10
−5
0
5
10
x (cm)

Antenna-launched Alfvén waves in LAPD

- Below: 3D AW pattern from a small antenna (comparable to skin depth, sound gyroradius), comparison to theory



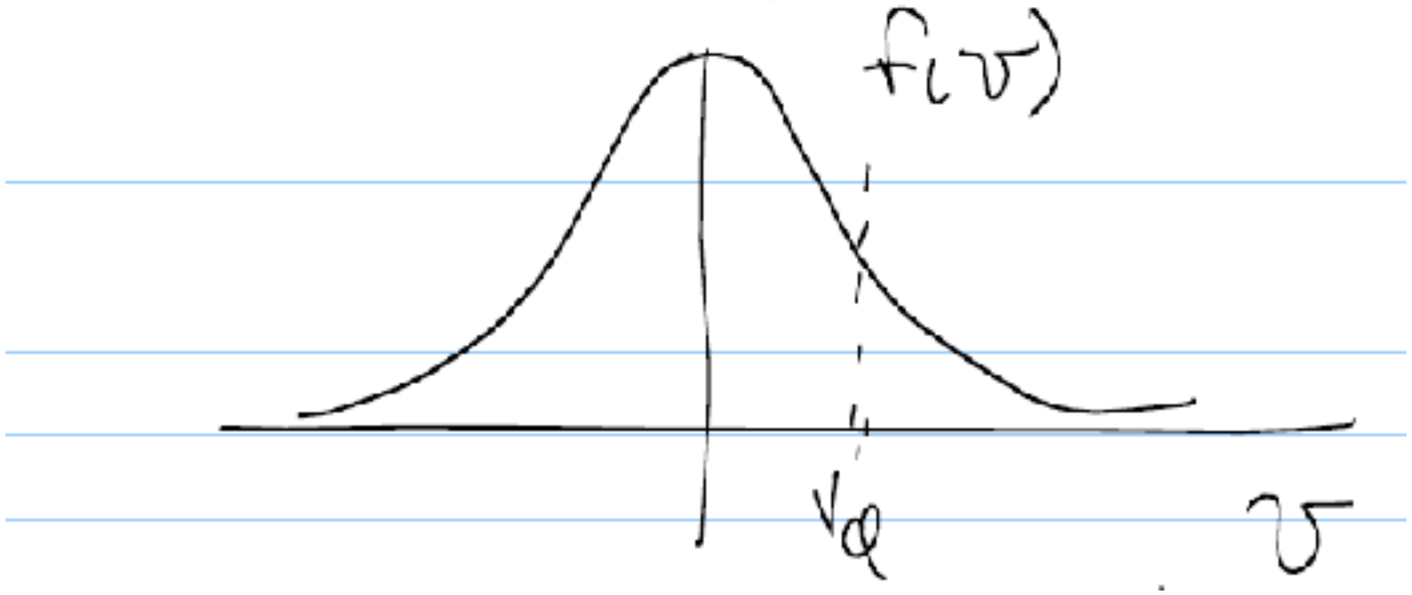


- Topics studied by UCLA group: Radiation from small source, resonance cones, field line resonances, wave reflection, conversion from KAW to IAW on density gradient… [Gekelman, Maggs, Morales, Vincena, et al]

Review: Gekelman, et al., PoP 18, 055501, (2011)

Collisionless damping: Landau Damping

- Warm plasma: particles have finite thermal speed, some particles might have speed comparable to wave phase speed, can resonate

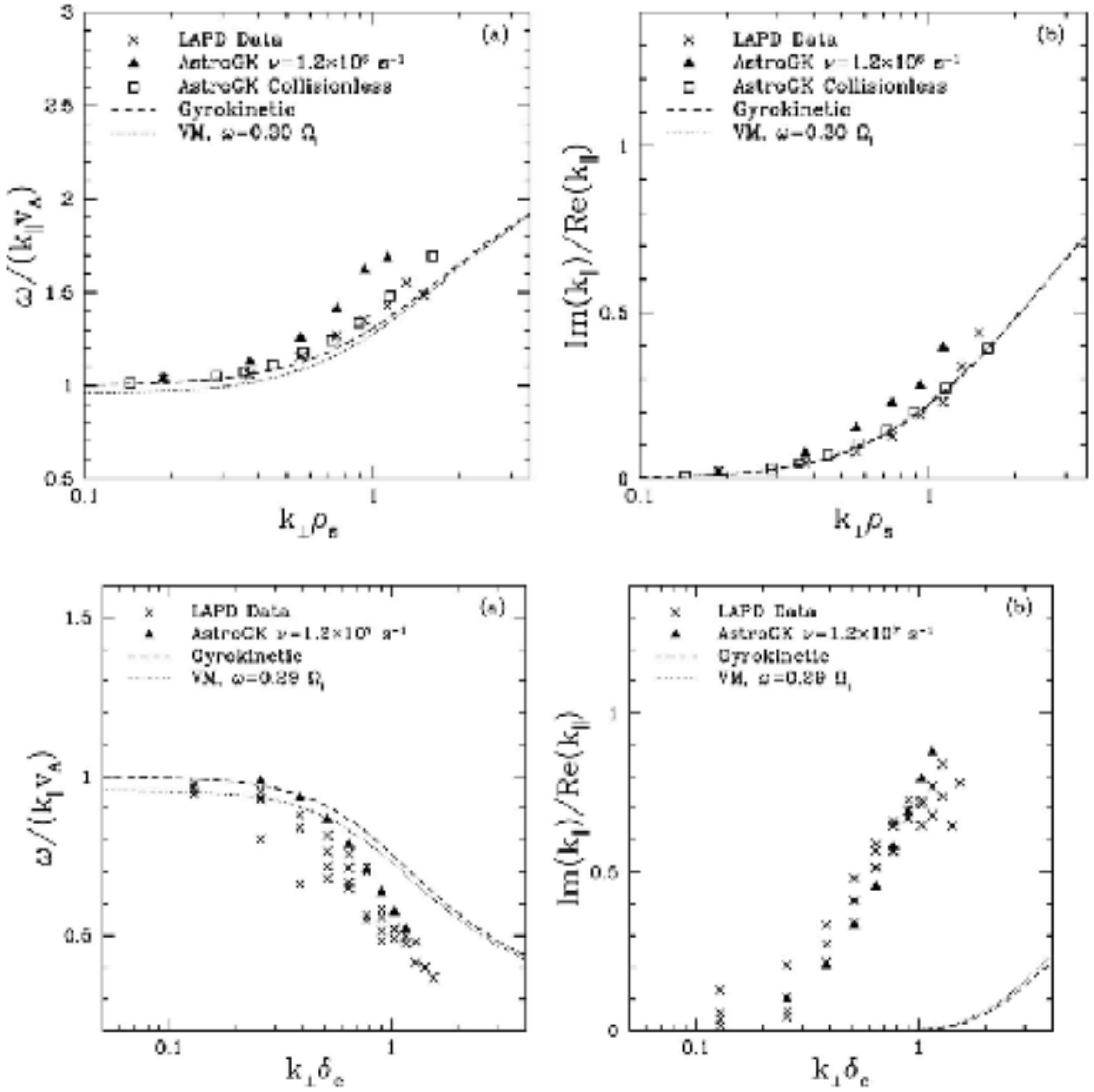


Resonant particles ride with the wave, see "DC" E-field, can exchange energy

- Particles slightly slower than wave get accelerated, take energy from wave; particles moving slightly faster are decelerated, give energy back to wave
- Can have wave damping or growth: $\gamma \propto \left.\frac{\partial f}{\partial v}\right|_{v=v_\phi}$

Collisionless damping of Alfvén waves

- Detailed measurements of dispersion and damping by U. Iowa group, compared to solar wind turbulence code AstroGK



Kletzing, et al, PRL 104, 095001 (2010)

Nielson, Howes, et al, PoP **17**, 022105 (2010)

- Ideal MHD AW does not have $E_{||}$, can not Landau damp

- AW with large $k_\perp$ (violating MHD assump.) develops $E_{||}$, can model with generalized Ohm's law:

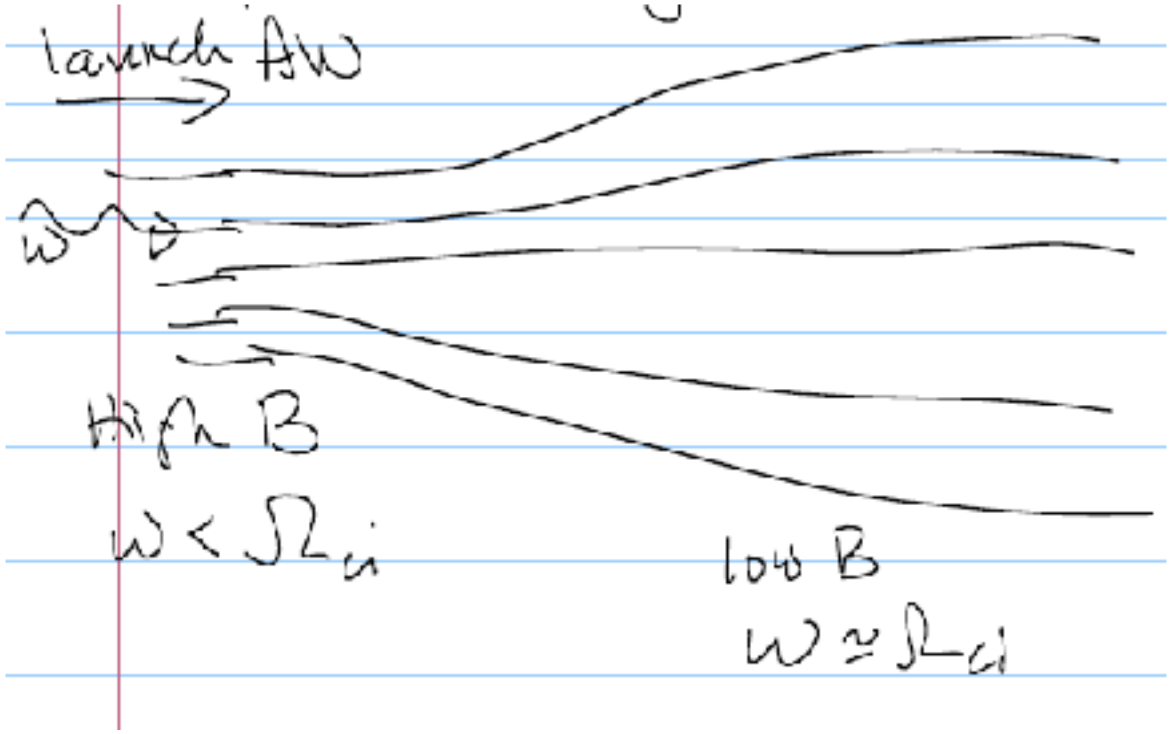
$$\vec{E} + \left(V - \frac{\vec{j}}{ne}\right) \times \vec{B} + \frac{1}{ne}\nabla p_e = \frac{m_e}{e^2 n}\frac{\partial \vec{j}}{\partial t}$$

- Get dispersive kinetic Alfvén wave, which can Landau damp

Cyclotron resonance and heating

- Another collisionless damping mechanism: waves with freq. near the cyclotron frequency can resonate with particle gyration; with finite T get absorption

- Used as heating mechanism for fusion plasmas. One of earliest ideas: heating by SAW in magnetic "beach"



- Launch SAW into decreasing field (e.g. from end of mirror machine)

- Waves absorbed where

$$\omega \approx \Omega_i$$

- Modern schemes use fast wave/compressional AW to heat ions (no parallel access to tokamak core)

Compressional Alfvén Waves/Fast Waves

Low frequency, cold plasma wave eqn

$$\begin{pmatrix} S-n_{\|}^2 & 0 & n_{\|}n_{\perp} \\ 0 & S-n^2 & 0 \\ n_{\|}n_{\perp} & 0 & P-n_{\perp}^2 \end{pmatrix} \begin{pmatrix} E_x \\ E_y \\ E_z \end{pmatrix} \approx 0$$

- Compressional wave

$$\mathbf{k}_{\perp} \perp E_{\perp} \rightarrow \text{since } k_{\perp} = k_x \quad ; \quad E_x = 0, E_y \neq 0$$

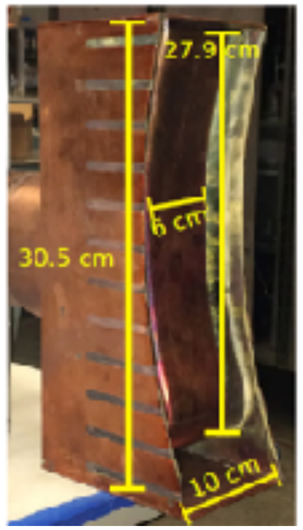
$$k_{\perp} \cdot v_{E\times B} \neq 0$$

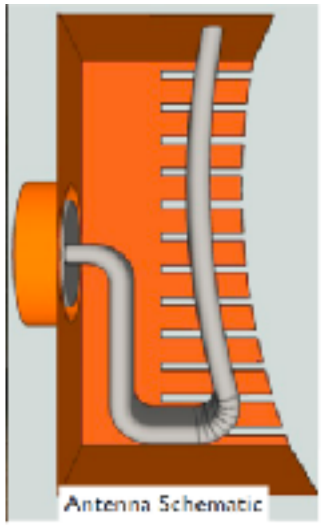
- Dispersion relation from center line in wave eqn:

$$n^2 = S \approx \frac{c^2}{v_A^2}$$

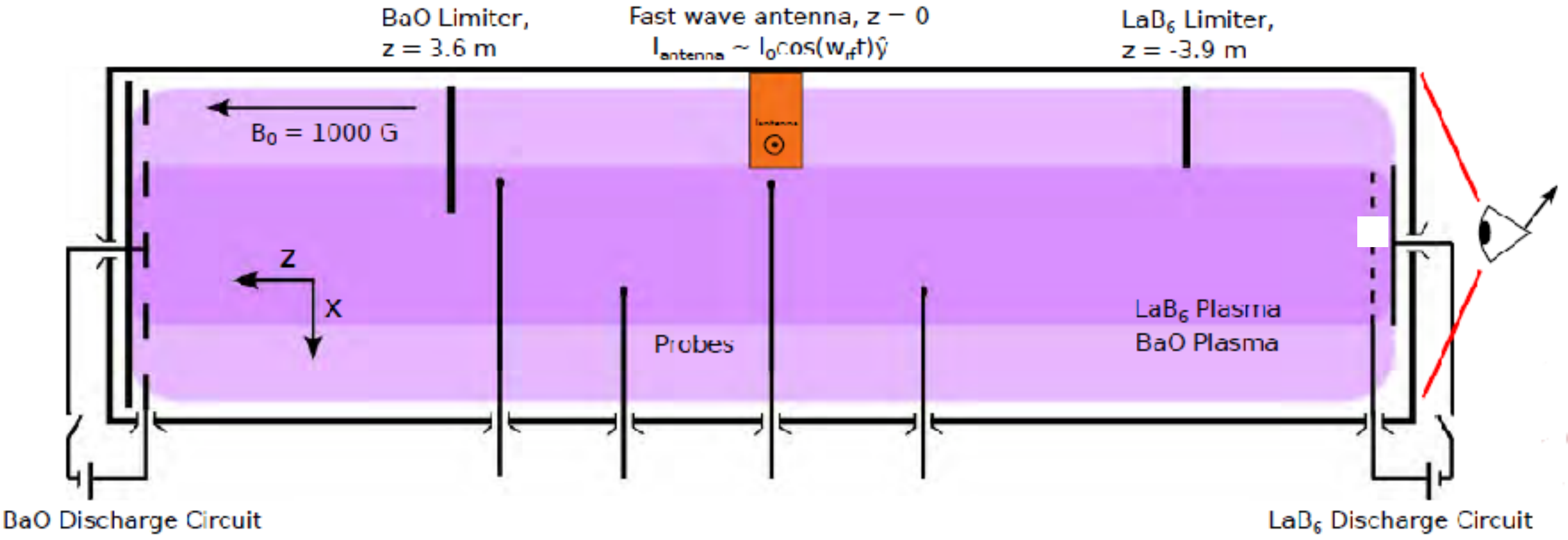
$$\omega^2 = k^2 v_A^2$$

LAPD ICRF system: generation of large amplitude fast waves in LAPD





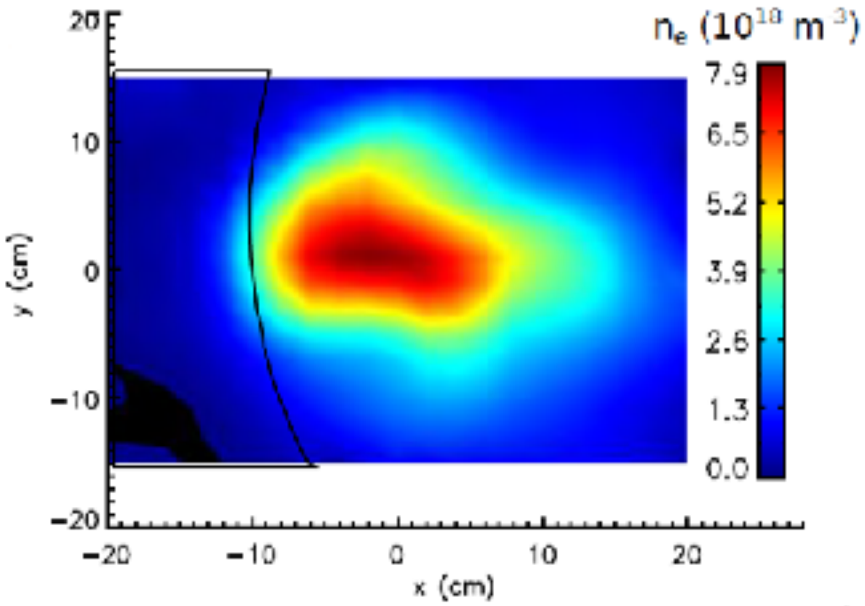
- Single strap fast wave antenna inserted up to edge of high density core (generated by smaller LaB6 cathode)
- Antenna can be tilted to any angle with respect to background field



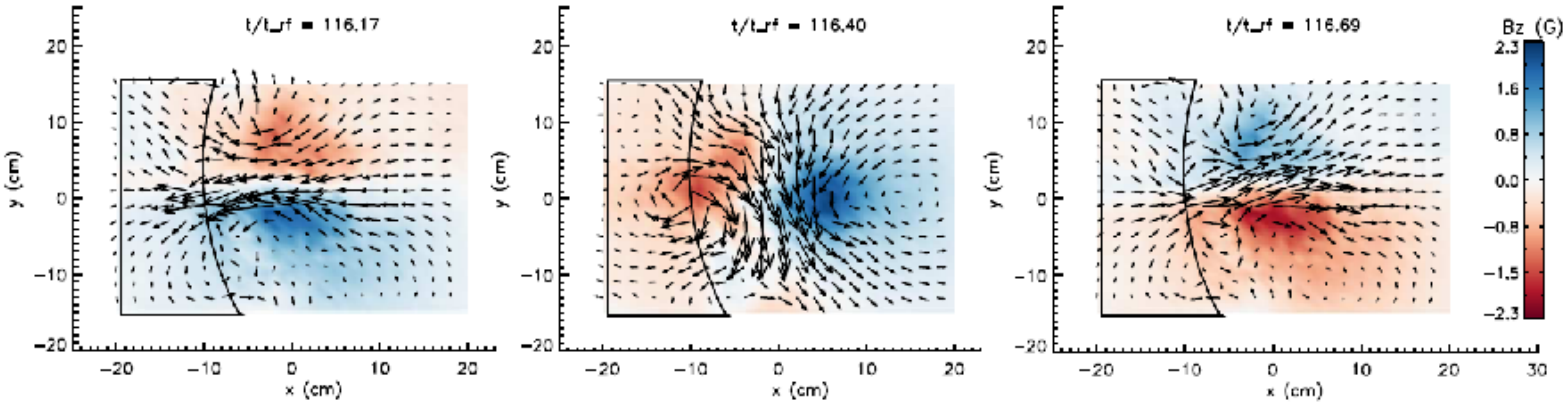
Large amplitude fast waves excited using antenna

- ~100kW of coupled power ($\delta B \sim 10$G)
- $m=1$ mode structure observed

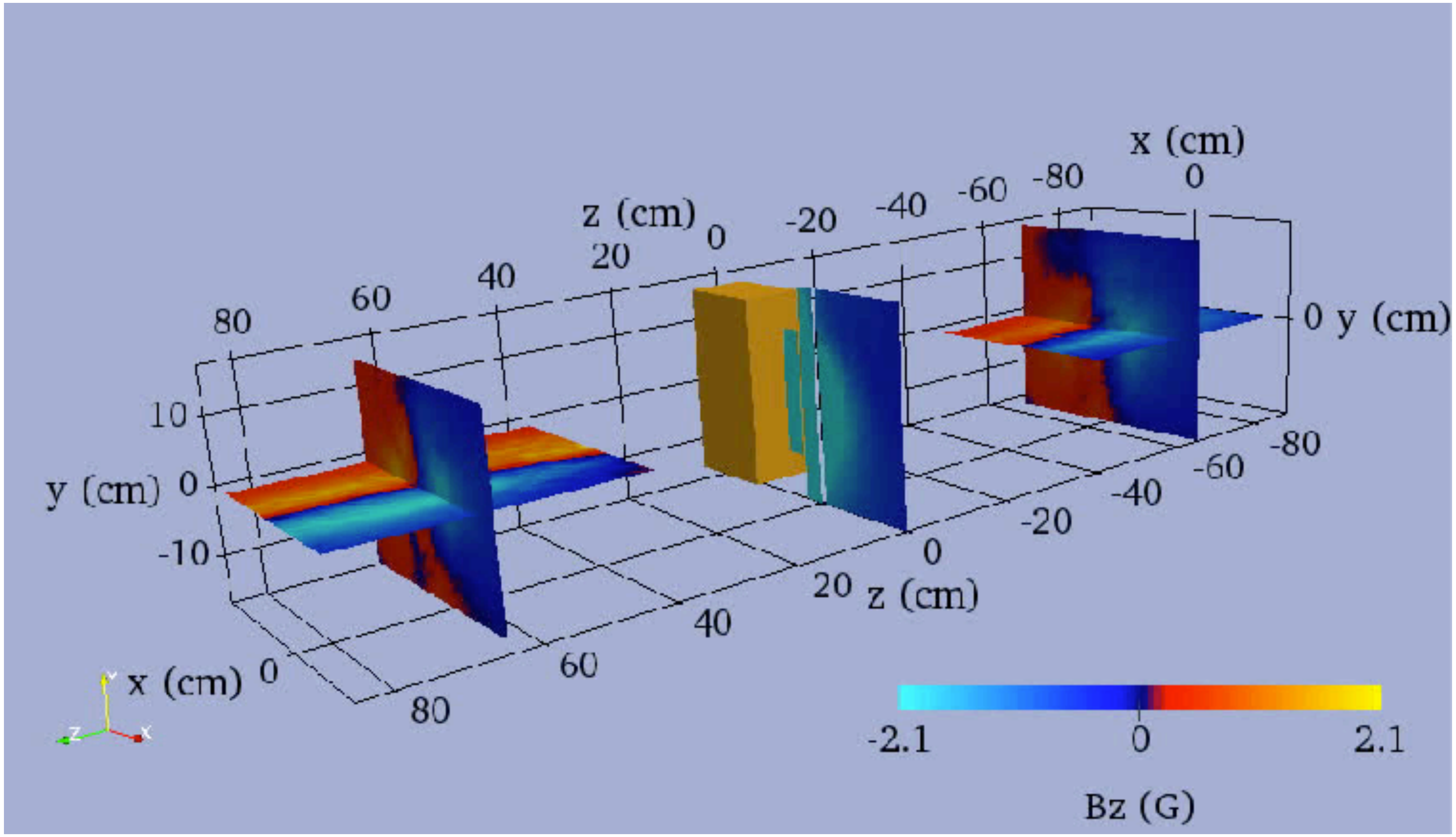
density profile



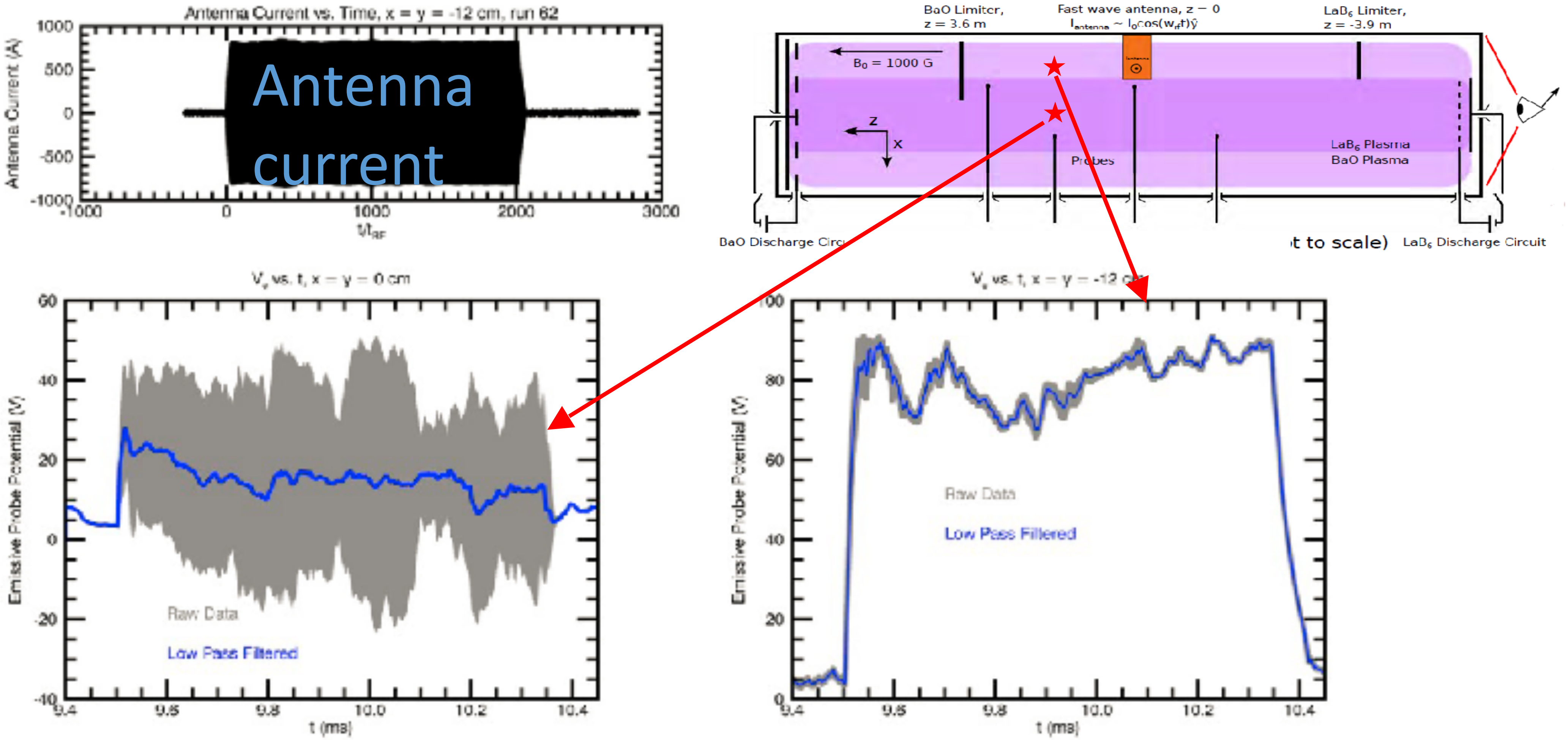
magnetic fluctuations



3D wave magnetic field measurements: fast/compressional AWs in LAPD



Plasma potential elevated by RF: RF sheath rectification observed

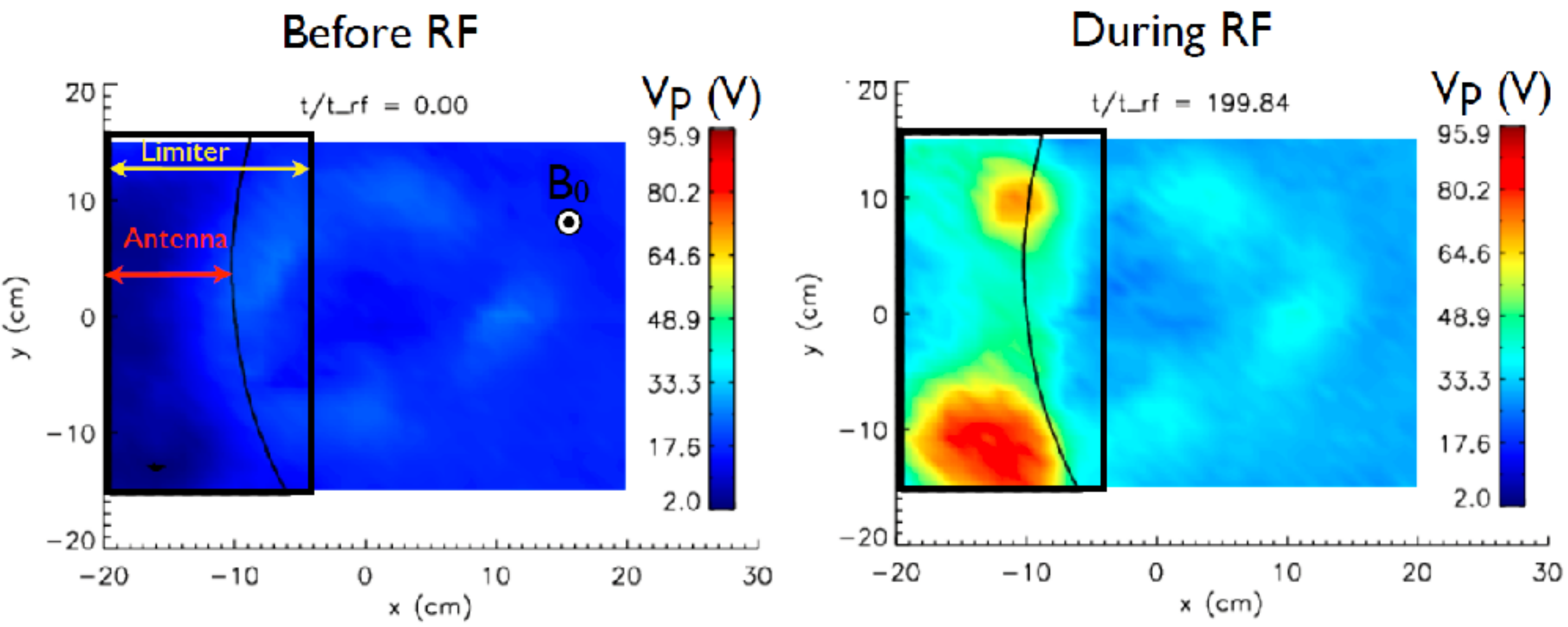


Plasma potential in core 10-15 V increase during RF

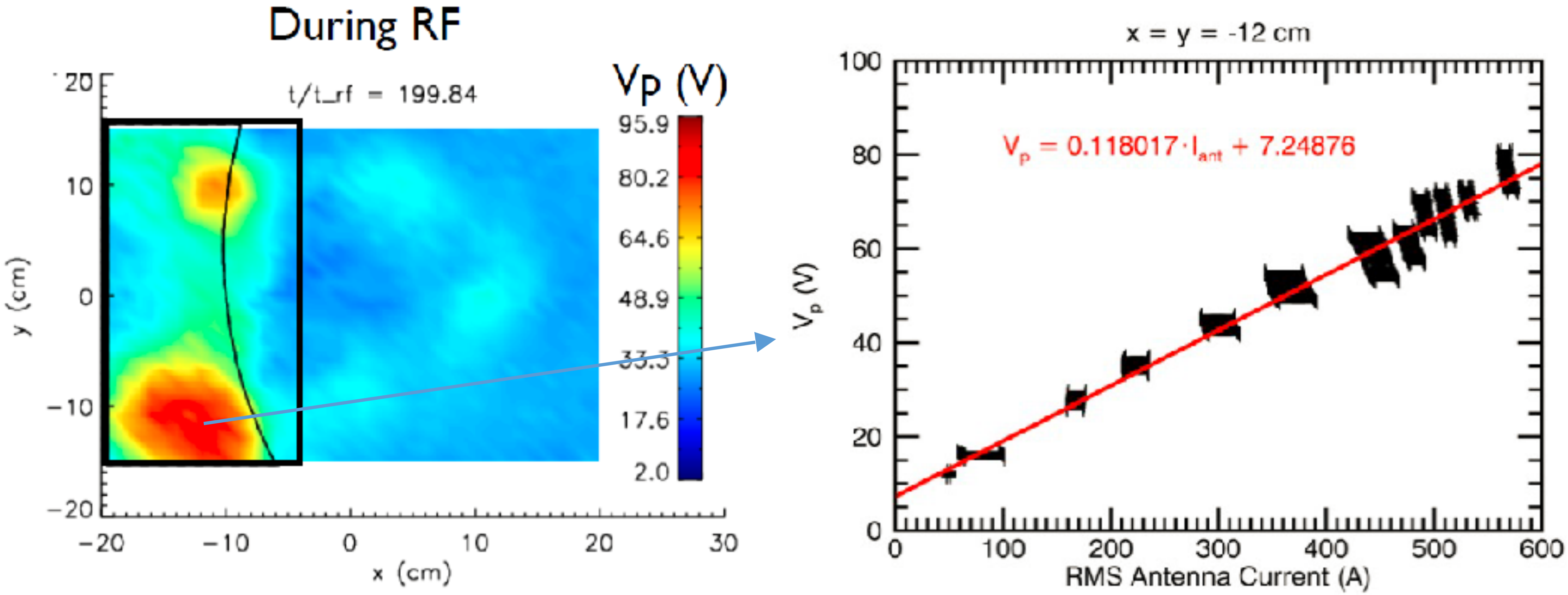
Plasma potential on field line connected to antenna, ~ 100 V increase during RF

Martin, et al., PRL 119, 205002 (2017)

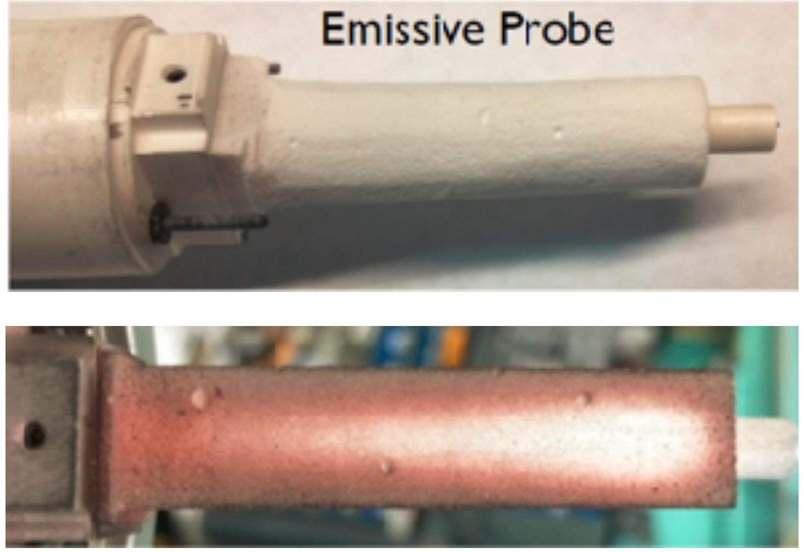
RF sheaths observed on antenna structure

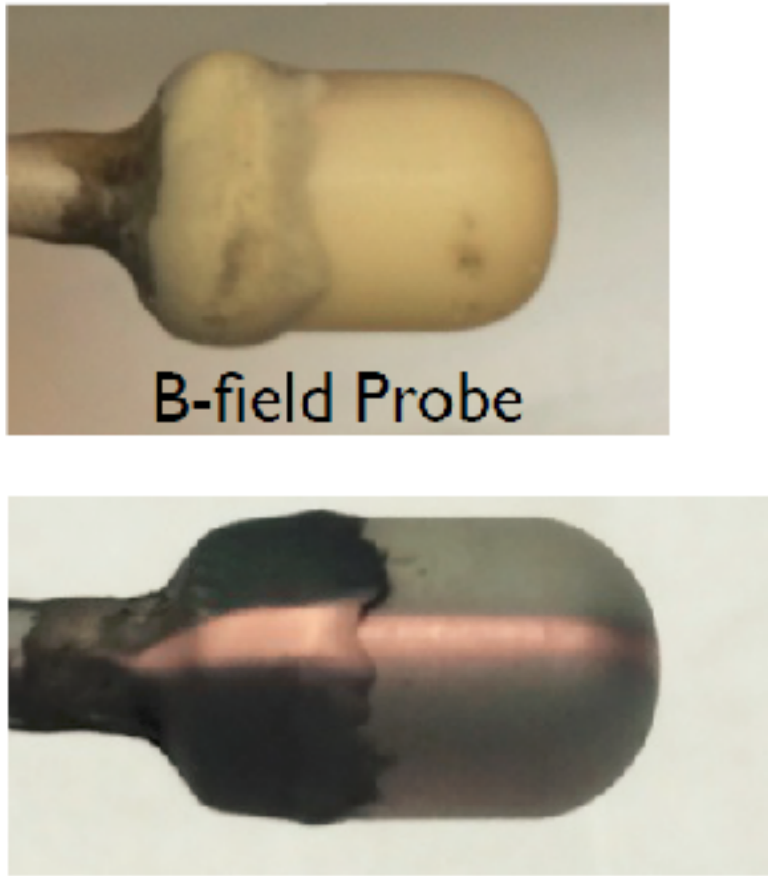


RF Sheath potential scales linearly with antenna current



Sputtering due to RF Rectification





- Probes are coated with copper after experimental run
- Coating likely originates from copper antenna structure

Higher frequency: whistler waves

Primarily parallel propagating mode with $\Omega_i < \omega < \Omega_e$

Electrons $E \times B$, ions polarize, get finite S and D

Can show in this limit:

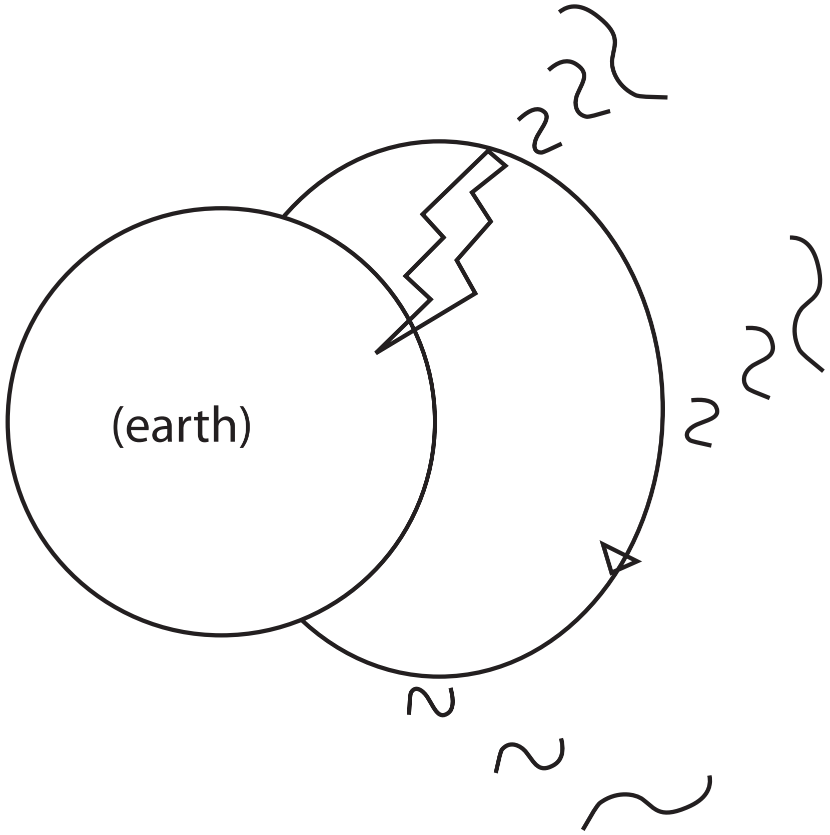
$$\omega \approx \frac{k^2 v_A^2}{\Omega_i}$$

the group velocity of this mode:

$$v_g = \frac{\partial \omega}{\partial k} \approx \frac{2c\sqrt{\omega \Omega_e}}{\omega_{pe}}$$

- Dispersive waves: phase/group velocity depends on frequency of wave

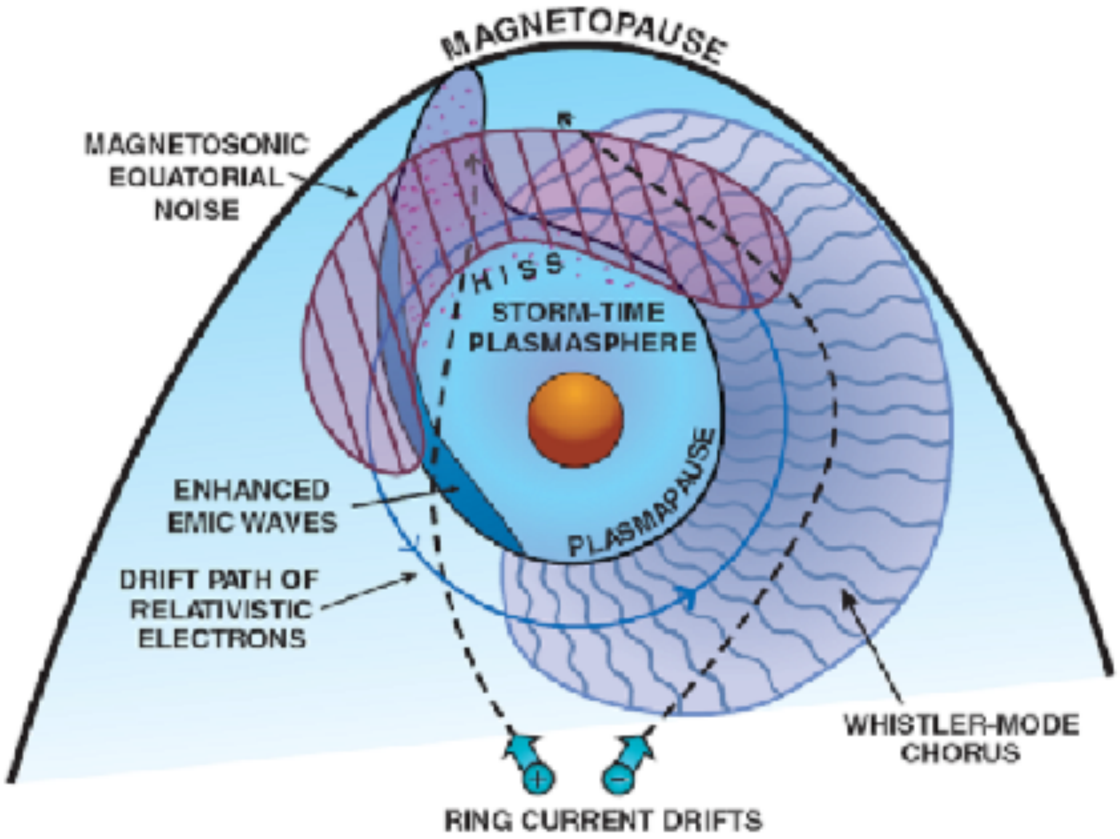
Whistlers in the magnetosphere

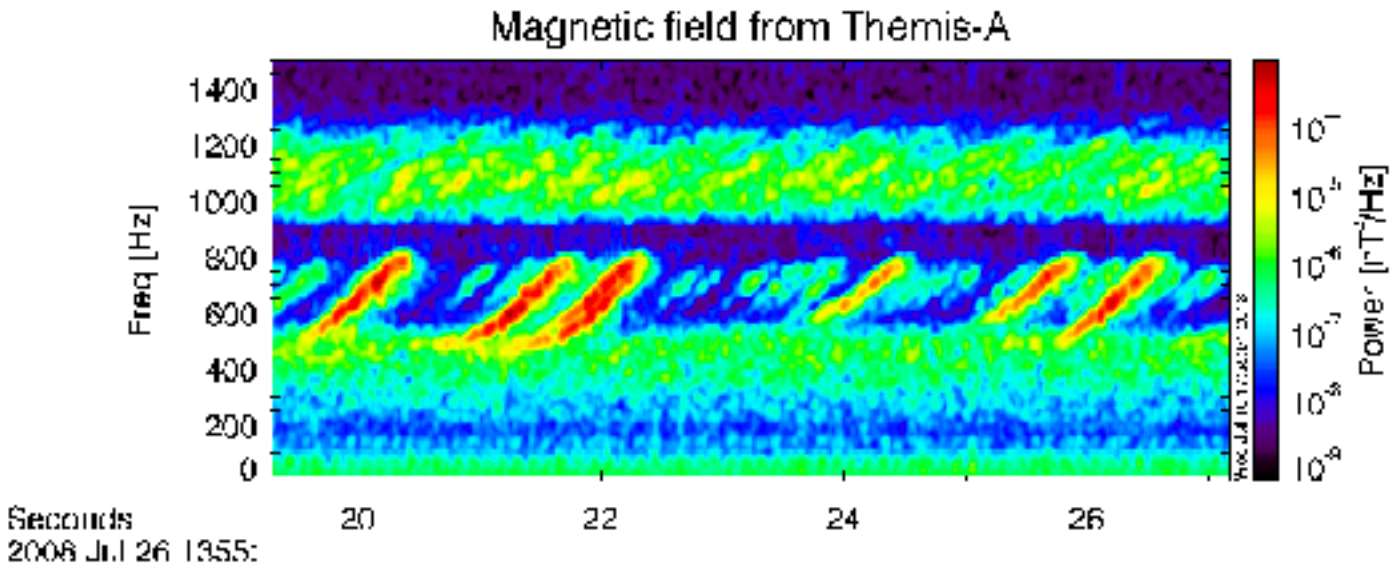


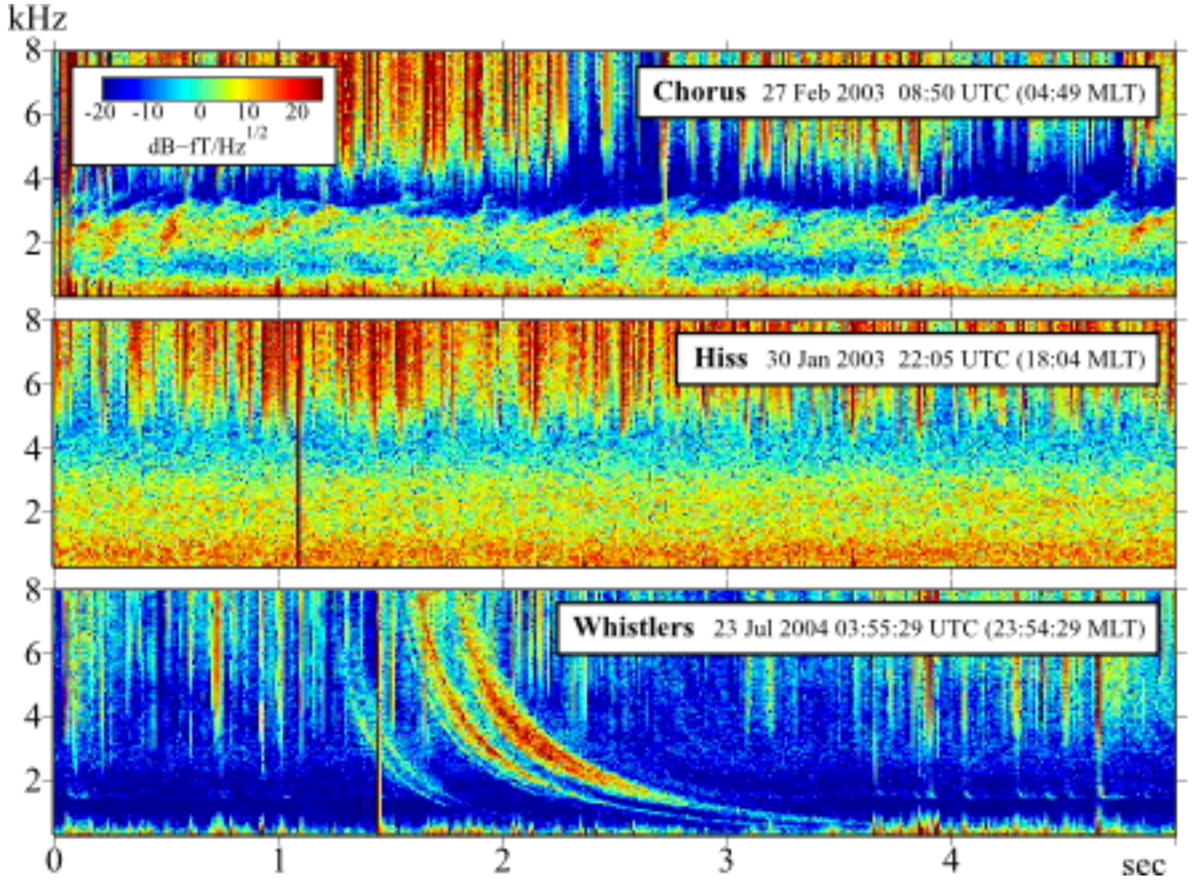
- Lightning strikes excite broad range of RF waves in magnetosphere
- Some whistler waves are born at strike site, propagate along earth's dipole field

- Because of dispersion, higher frequency waves go faster than lower frequency: higher freq at front of wave packet
- Whistler in magnetosphere are in audible range of frequencies: picked up by radio/telephone operators in WWI/II; chirp downward in frequency (hence "whistler")

Whistler waves in the Earth's radiation belts

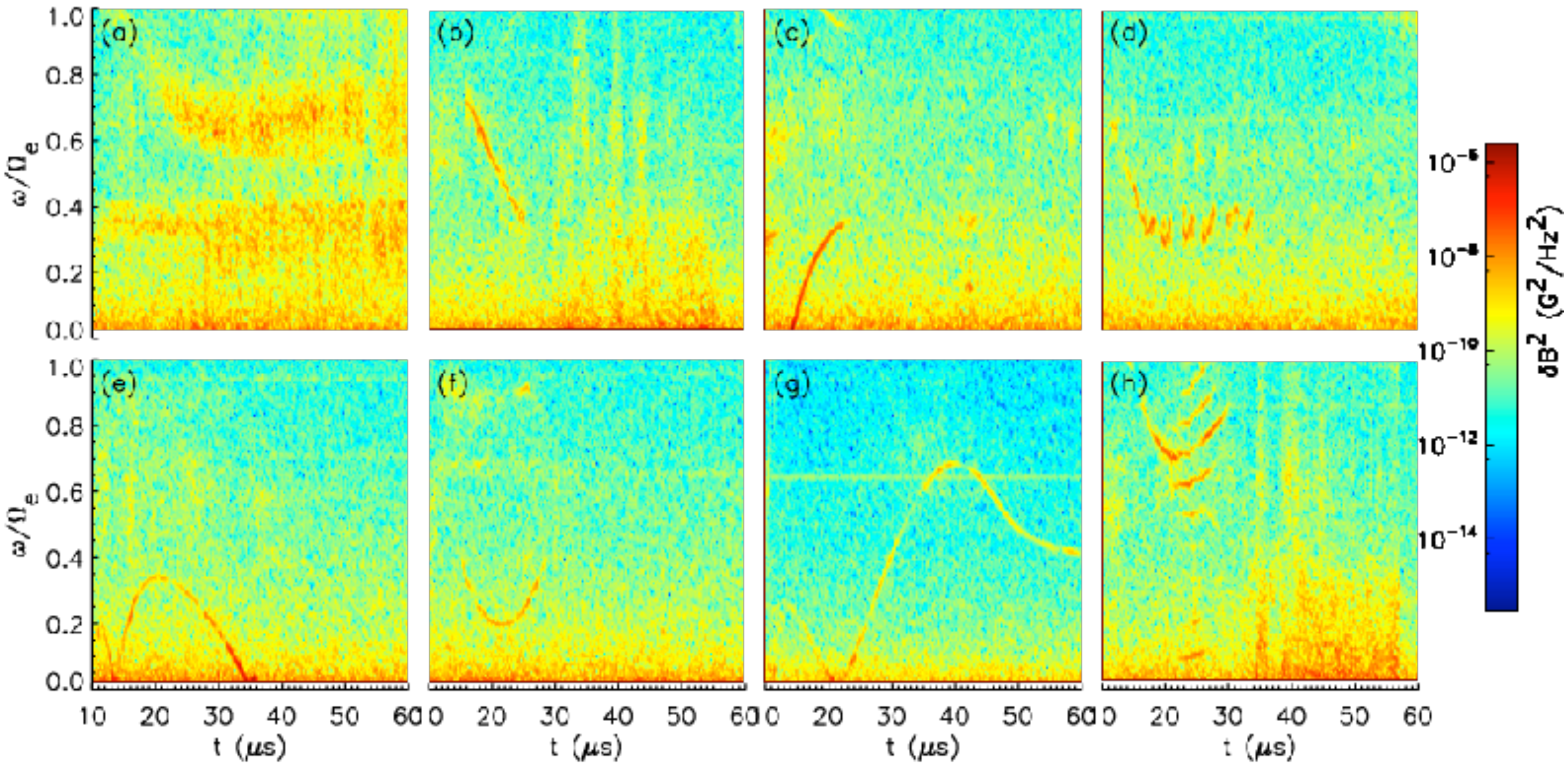






- Whistler "Chorus": frequency chirping whistler modes excited by energetic electrons trapped in the radiation belts, can lead to de-trapping of these particles (similar to *AEs in tokamaks)

Chirping whistlers in the lab (LAPD, of course)



- Electron beam (~3 keV) in LAPD. Variety of behaviors observed ("hiss", up/down chirping, "hooks") depending on beam energy, pitch angle, plasma parameters

Van Compernolle, et al., Phys. Rev. Lett. 114, 245002 (2015)
Van Compernolle, et al., Plasma Phys. Contr. Fusion 59, 014016 (2017)

Even higher frequency: modified light waves

For $\omega > \omega_{\mathrm{pe}}$, have modified light waves, no plasma normal modes (too fast for electrons to keep up!)

Example: "O-mode" transverse waves, $\mathbf{k} = k\hat{x}$, $\mathbf{E} = E_z\hat{z}$

$$n^2 = P = 1 - \frac{\omega_{pi}^2}{\omega^2} - \frac{\omega_{pe}^2}{\omega^2} \approx 1 - \frac{\omega_{pe}^2}{\omega^2}$$

$$\omega = \frac{kc}{\sqrt{1 - \frac{\omega_{pe}^2}{\omega^2}}}$$

- Wave cutoff below plasma frequency (evanescent)
- Phase velocity faster than c (wavelength longer in plasma)

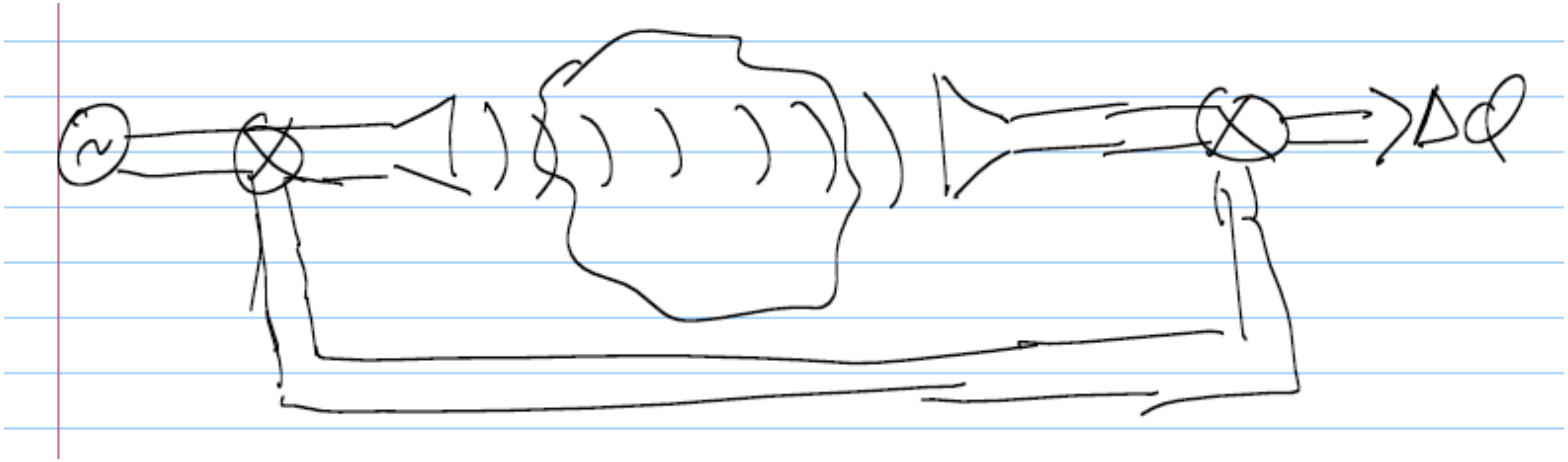
Even higher frequency: modified light waves

$$v_\phi = \frac{\omega}{k} = \frac{c}{\sqrt{1 - \frac{\omega_{pe}^2}{\omega^2}}}$$

$$v_g = \frac{\partial \omega}{\partial k} = c\sqrt{1 - \frac{\omega_{pe}^2}{\omega^2}}$$

- Wave cutoff below plasma frequency (evanescent)
- Phase velocity faster than c (wavelength longer in plasma); group speed less than c

Waves as diagnostics: interferometer



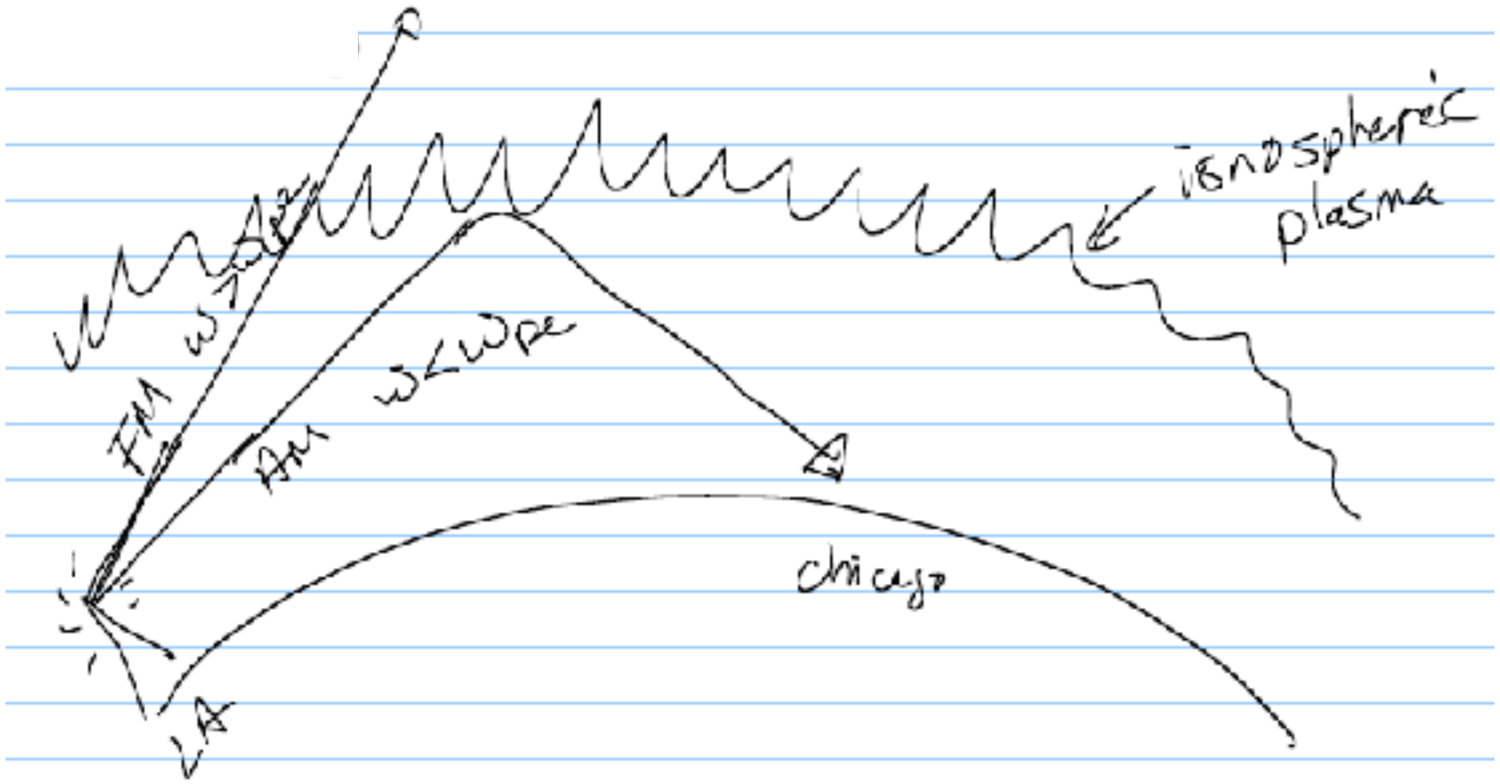
- Phase speed of EM wave in plasma depends on density, compare wave sent through plasma with wave in vacuum: plasma wave will advance phase faster, develop phase shift w/ respect to vacuum wave

$$\Delta\phi \propto \int n_e dl$$

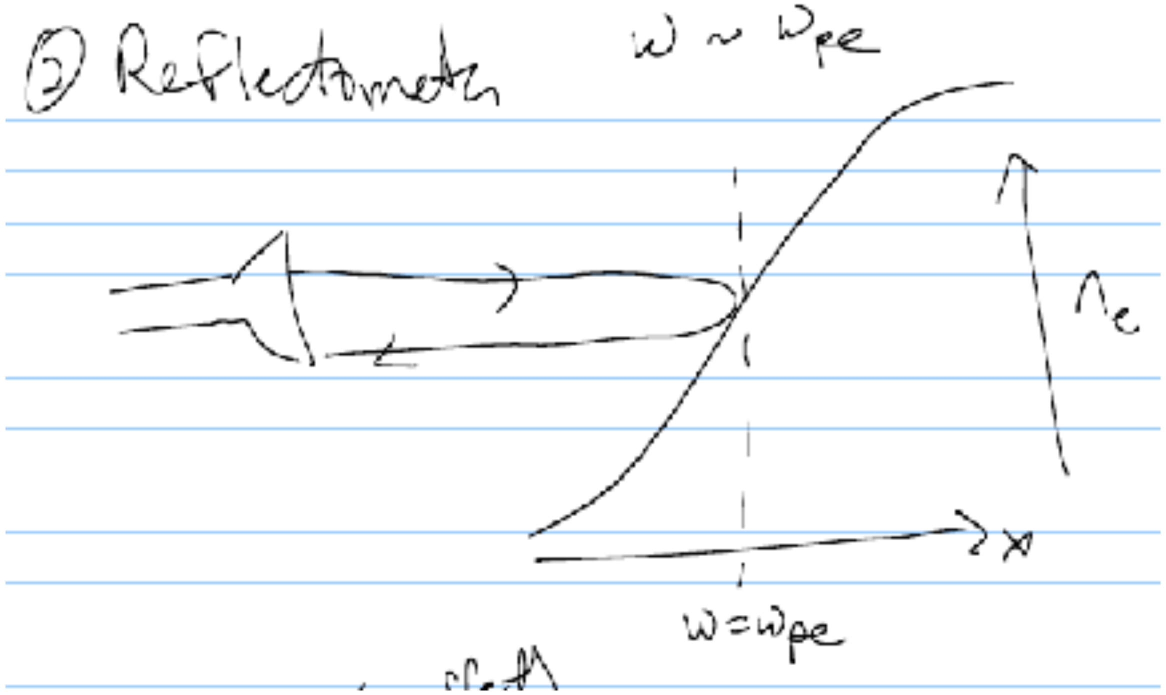
- Standard lab diagnostic, also used in astronomy

Reflection of light waves by plasma

- Transverse EM waves are cutoff for frequencies below the plasma freq. - incident waves are reflected (similar concept can be applied to reflection from conductors/ metals)
- Natural example: AM radio wave reflection from ionosphere



Reflection as plasma diagnostic

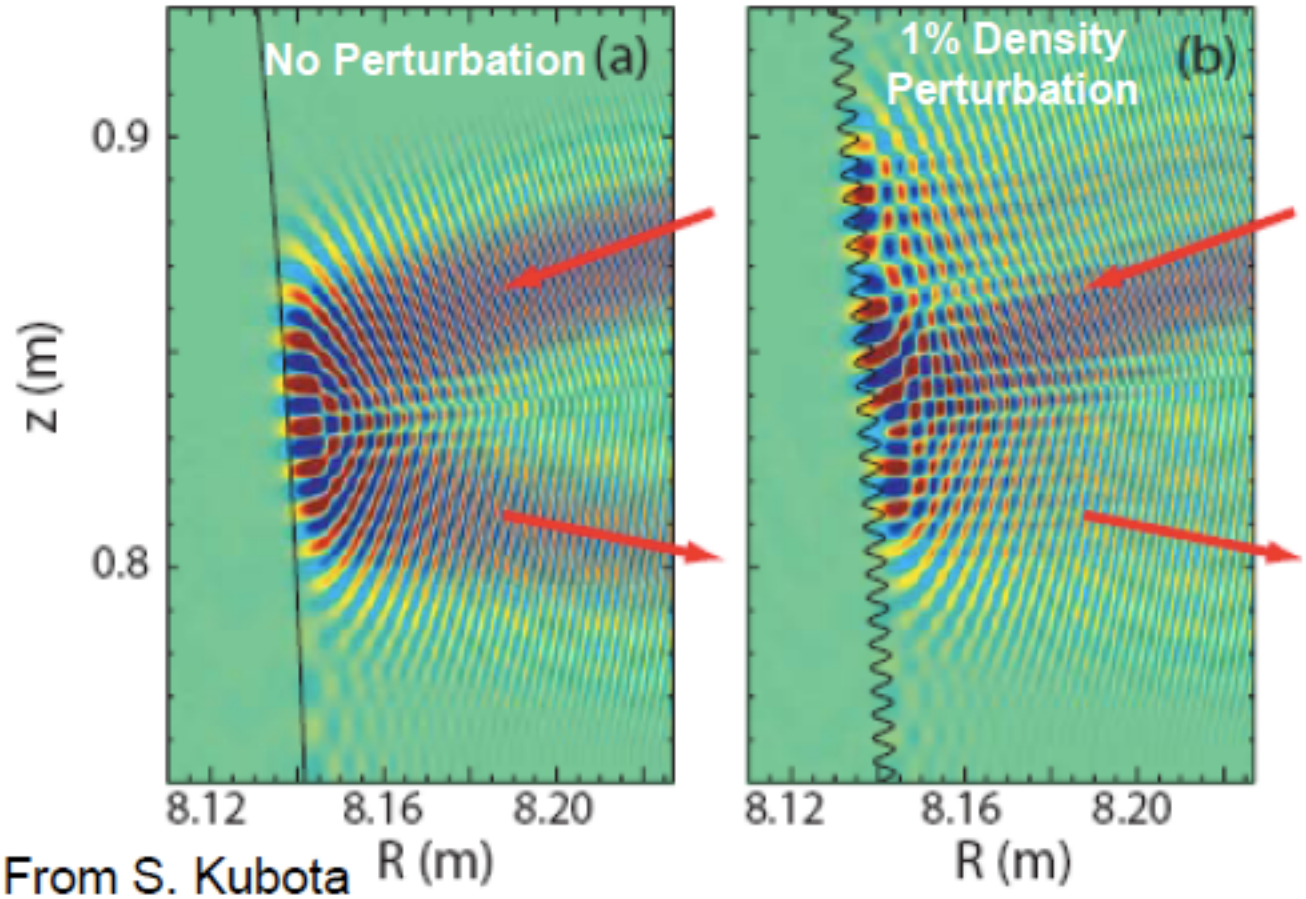


Propagate EM wave (microwave range) into plasma, up dens. grad.

Reflects at cutoff, radar ranging of cutoff surface location (scan freq for profile)

Right: simulation of microwave propagation to cutoff in tokamak

Can also measure turbulence: both through scattering & modulation of reflection



From S. Kubota

Alternate dielectric formulation: circularly polarized waves

- Previous dielectric formulation based on linearly polarized basis, some physical insight can be gained by considering a circularly polarized basis (wrt B)

$$\hat{r}, \hat{l} = \frac{\hat{x} \pm i\hat{y}}{\sqrt{2}} \quad \epsilon = \left(\begin{array}{ccc} R & 0 & 0 \\ 0 & L & 0 \\ 0 & 0 & P \end{array} \right)$$

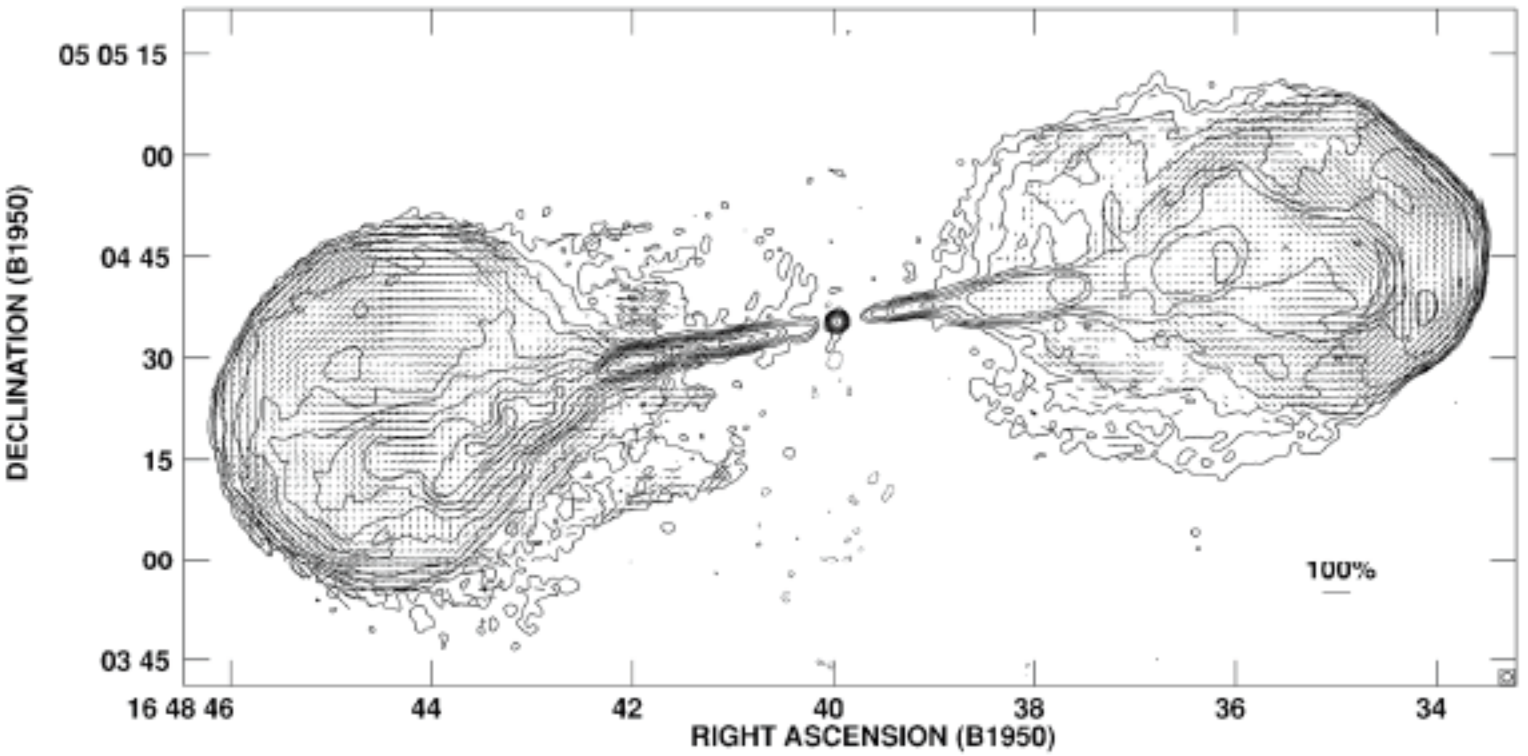
$$R = 1 - \frac{\omega_{pi}^2}{\omega^2} \frac{\omega}{\omega - \Omega_i} - \frac{\omega_{pe}^2}{\omega^2} \frac{\omega}{\omega - \Omega_e}$$

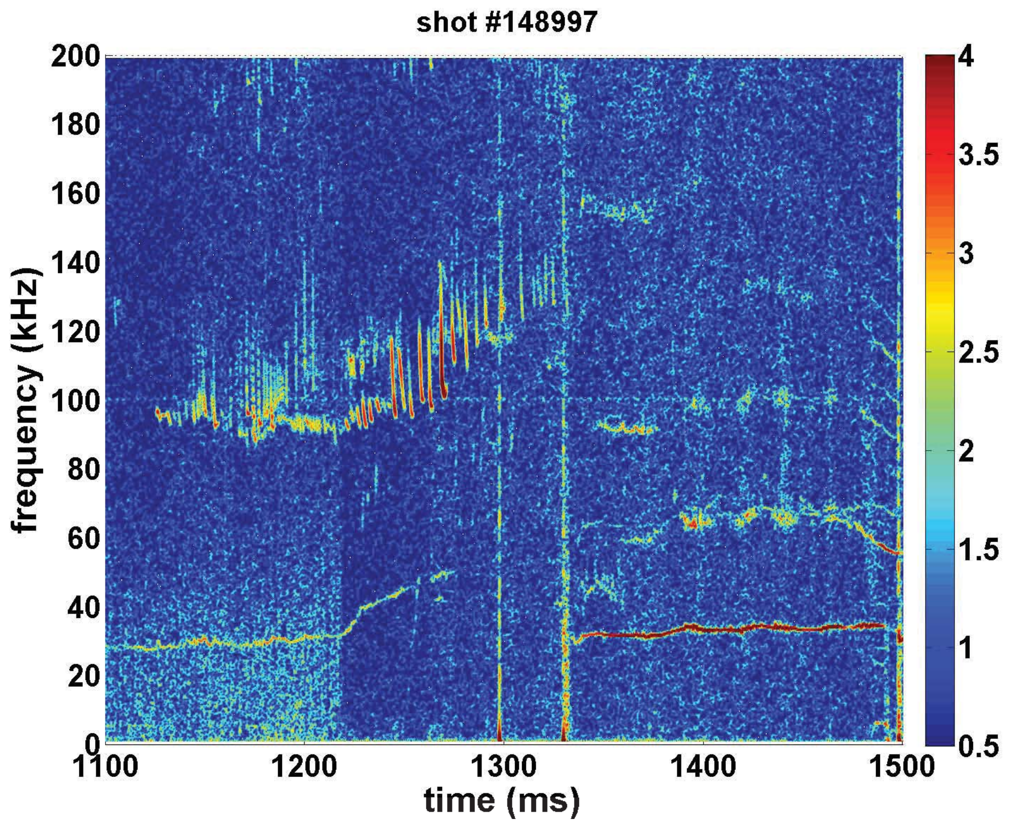
$$L = 1 - \frac{\omega_{pi}^2}{\omega^2} \frac{\omega}{\omega + \Omega_i} - \frac{\omega_{pe}^2}{\omega^2} \frac{\omega}{\omega + \Omega_e}$$

- Physical insight: ions gyrate LH, electrons RH. LHP waves have stronger ion interaction (and cyclotron resonance), RHP waves interact with electrons

- Same set of waves contained in either basis, some are easier to extract from CP

Faraday Rotation





- RHP and LHP components of an EM wave acquire different phase velocity in plasma, polarization rotates while propagating along B
- Measure line-of-sight B in lab plasmas, astrophysical plasmas (above, left: astrophysical jet; above, right: RSAEs and TMs in DIII-D)

Instabilities in magnetized plasmas

- Many plasmas of interest are not in thermal equilibrium: e.g. tokamak plasmas with strong pressure gradients or radiation belts plasmas with energetic particles (not Maxwellian distribution)

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- Departures from thermal equilibrium represent "free energy" that can be transferred from the plasma to EM waves

Instabilities in magnetized plasmas

- Many plasmas of interest are not in thermal equilibrium: e.g. tokamak plasmas with strong pressure gradients or radiation belts plasmas with energetic particles (not Maxwellian distribution)
- Departures from thermal equilibrium represent "free energy" that can be transferred from the plasma to EM waves
- These waves tend to relax the non-equilibrium feature that drives them (takes away the free energy) — leads to, for example, transport of heat, particles and momentum across B (flattens the pressure gradient)

Instabilities and turbulence driven by thermal energy gradients

- Perturbations that mix hot core plasma and cold edge plasma can release free energy (and therefore grow)

Instabilities and turbulence driven by thermal energy gradients

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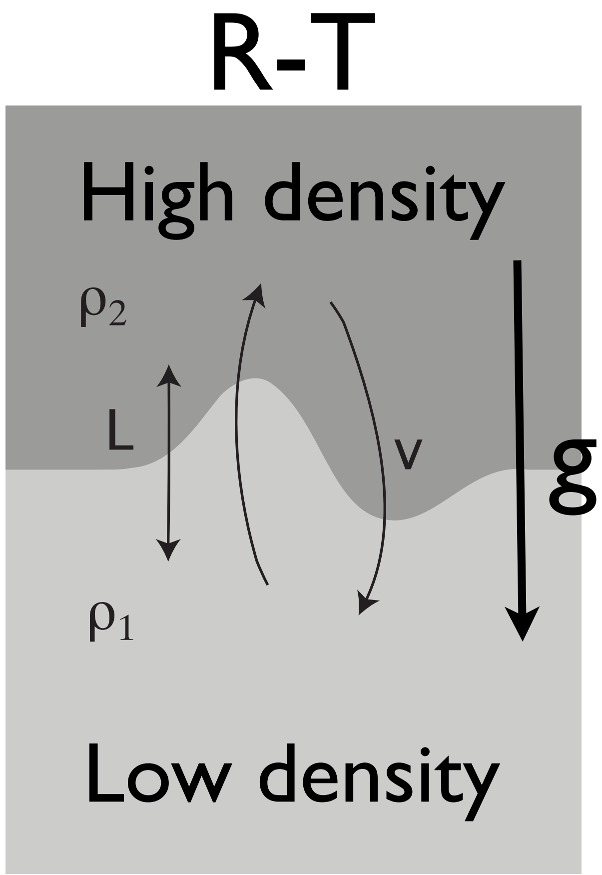
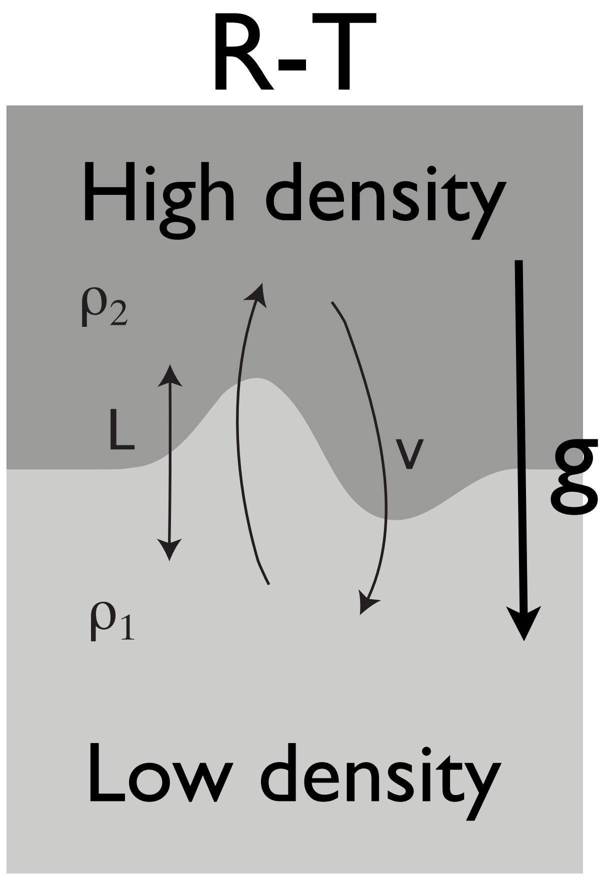
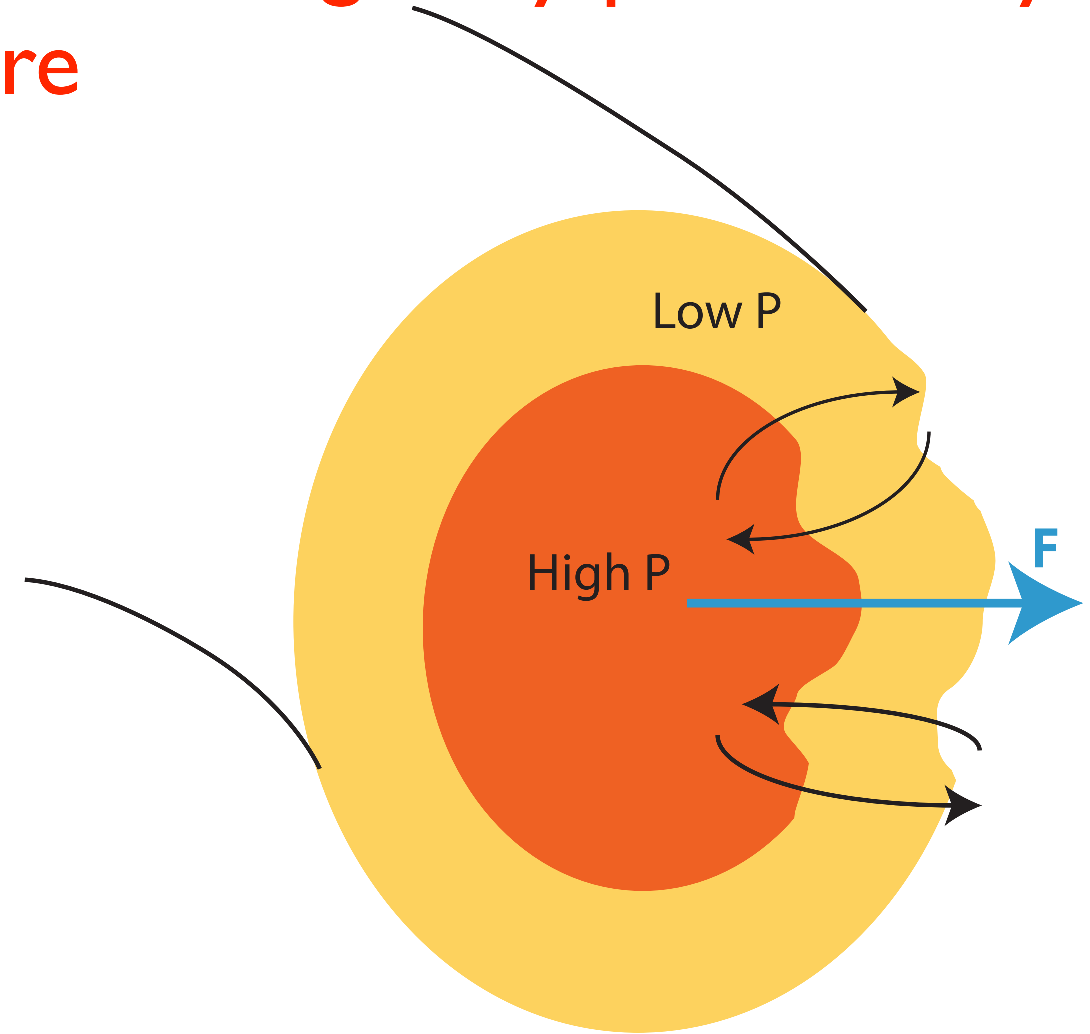
- Interchange drive is important (analogous to Rayleigh-Taylor)



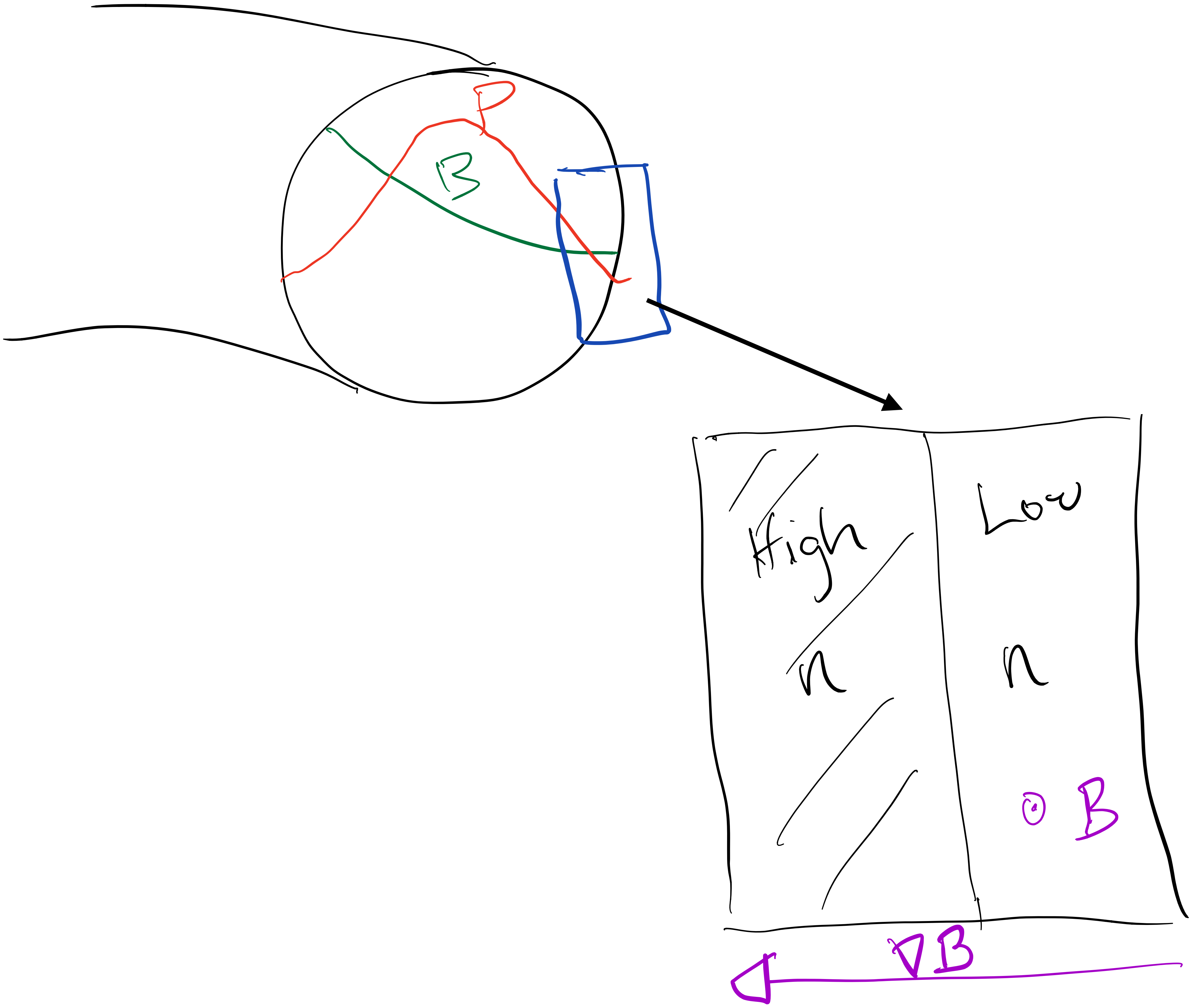
Instabilities and turbulence driven by thermal energy gradients

- Perturbations that mix hot core plasma and cold edge plasma can release free energy (and therefore grow)
- Interchange drive is important (analogous to Rayleigh-Taylor). In tokamak, effective gravity provided by magnetic field gradient/curvature

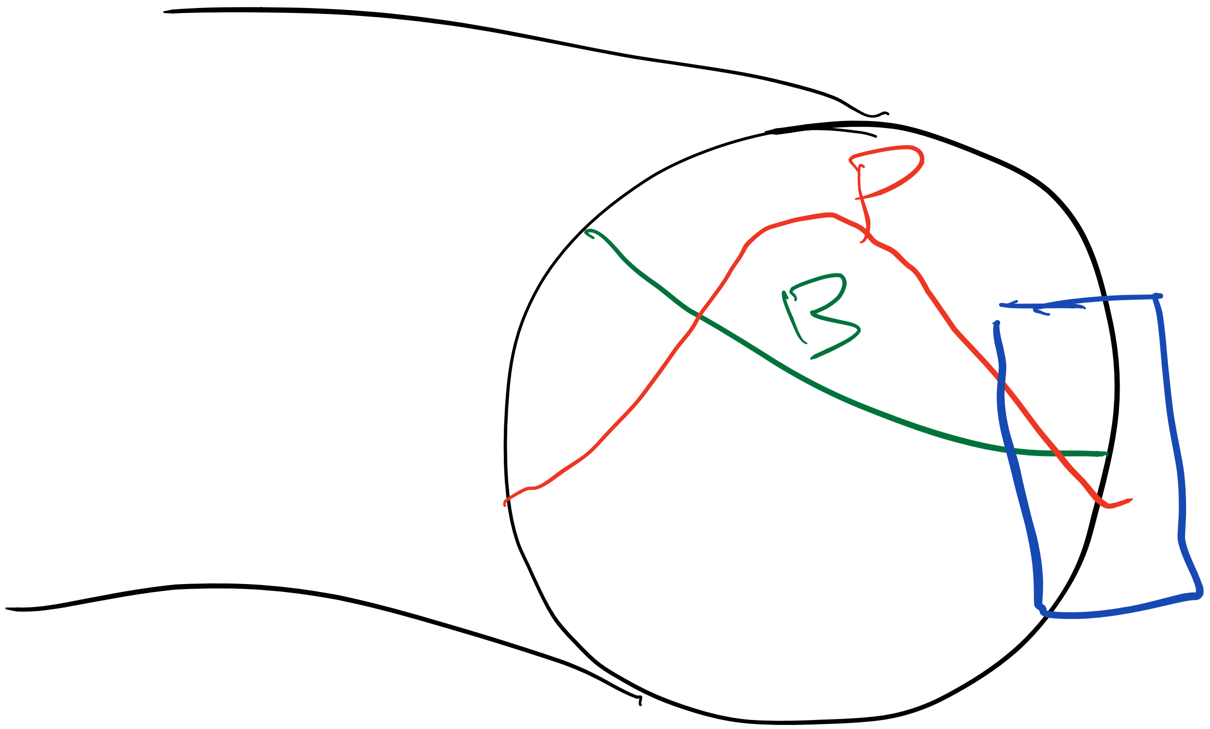




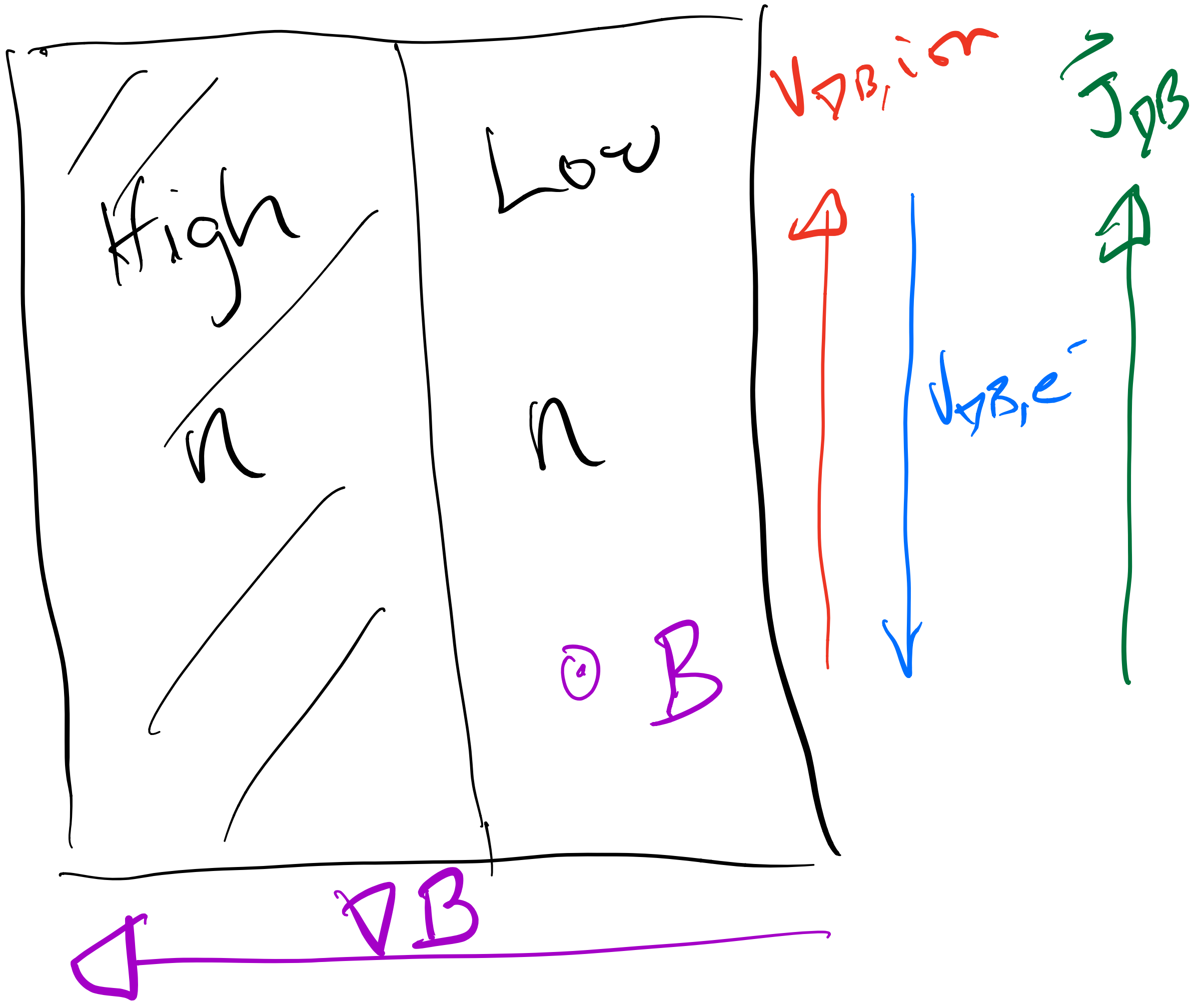
Single particle picture of interchange instability



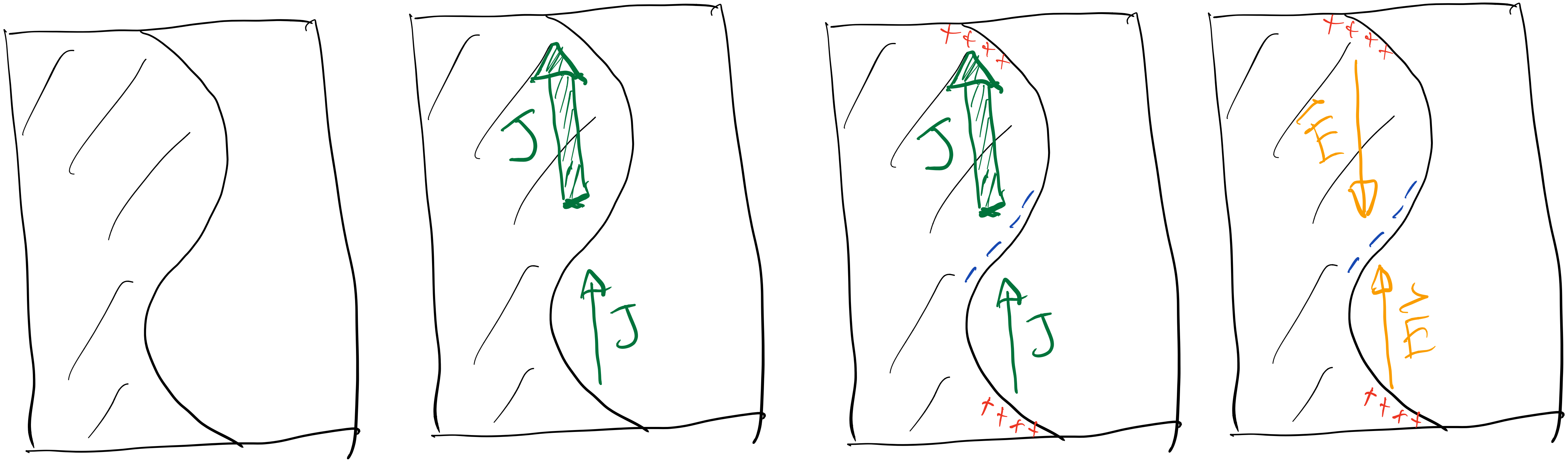
Single particle picture of interchange instability



due to ∇B, get currents due to ∇B drifts

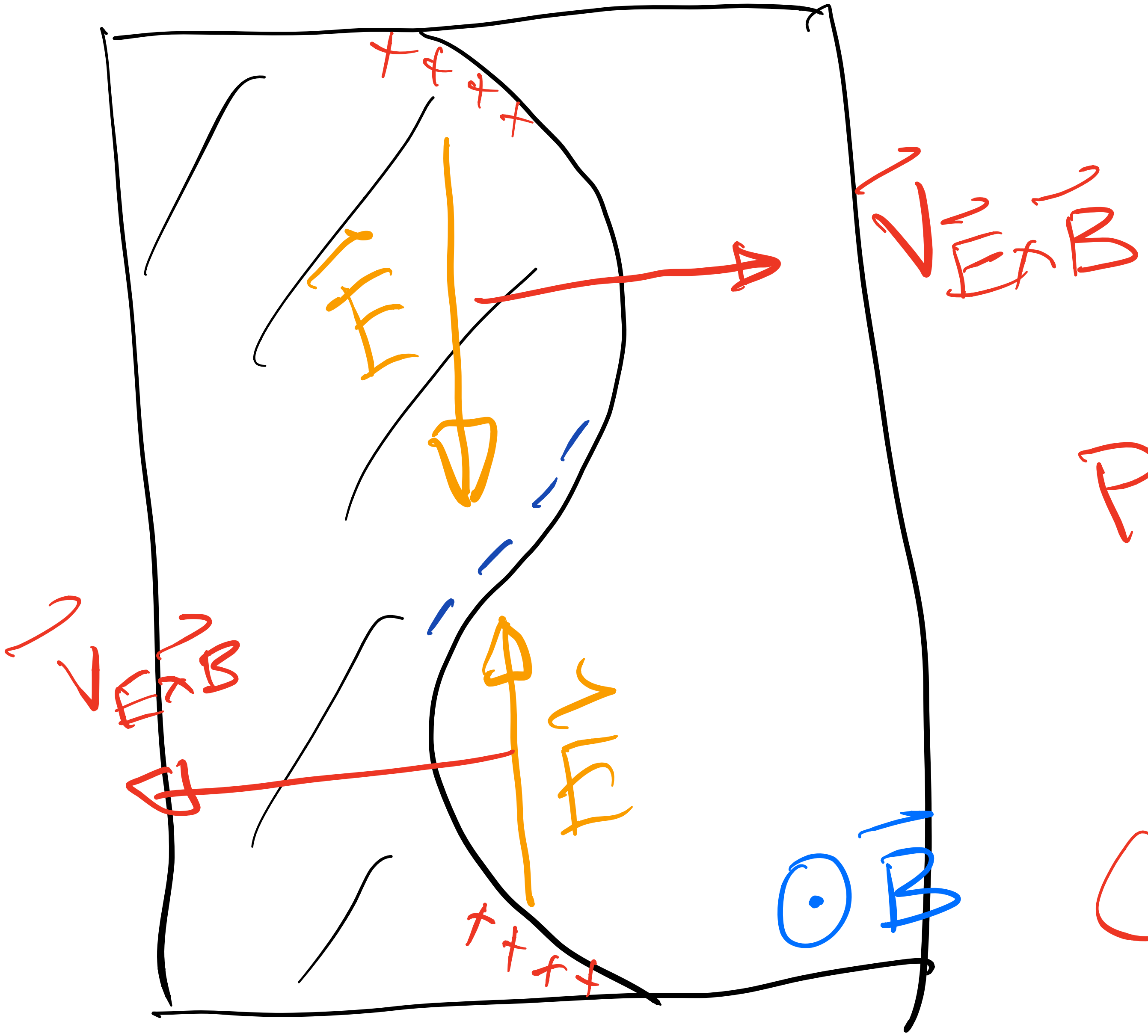


Single particle picture of interchange instability



- Imagine perturbing density as above (can arise spontaneously due to very small thermal fluctuations)
- Causes Grad-B currents to lead to charging of surface of perturbation, leading to E-field

Single particle picture of interchange instability



Perturbation grows!

(not on "inboard side")