Waves in plasmas

Saskia Mordijck

What are Waves?

Regular waves in water

http://hema.ipfw.edu/Geopics/Framesrc/Water/waves.html

Waves are a **periodic perturbation** that transfers **energy** can be described in some circumstances by a **linear approximation**. Waves occur around us. One example is the surfaces waves in the ocean

What are Waves?

Regular waves in water

http://hema.ipfw.edu/Geop

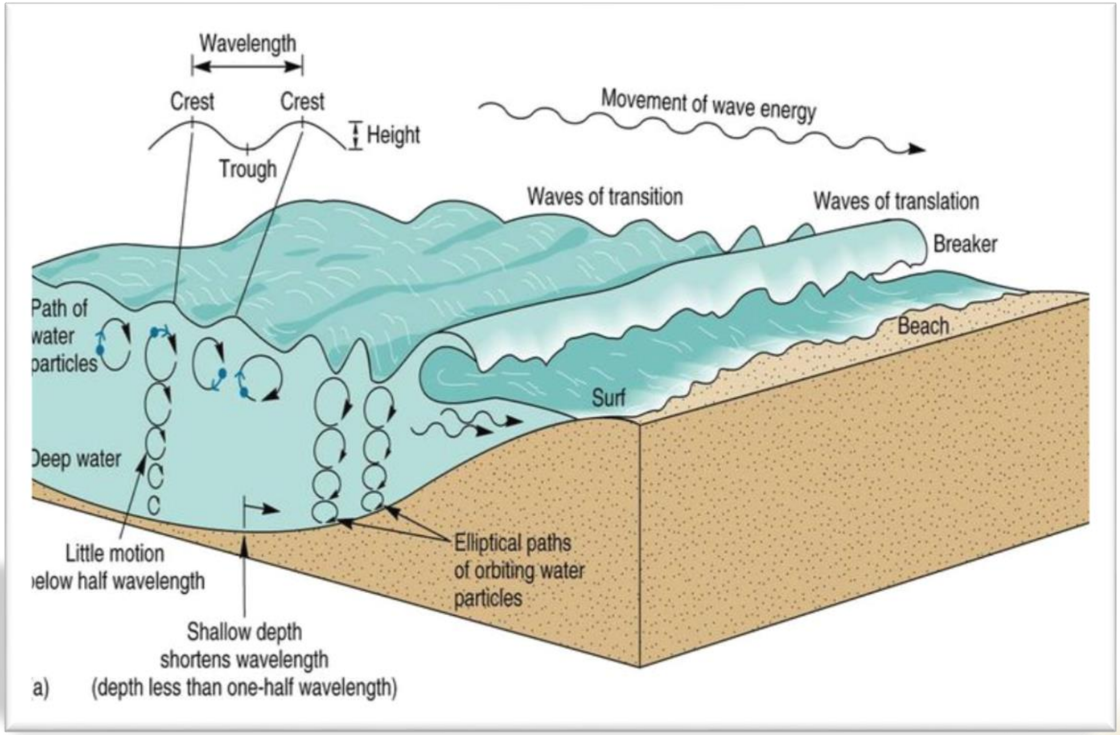
Waves that crash onto shore

http://blogs.oregonstate.edu/seagrantscholars/2016/01/26/waves/

What are Waves?

The wave characteristics can change based on its surroundings

We use the **dispersion relationship** to describe the relation between the wavelength and the frequency of the wave.



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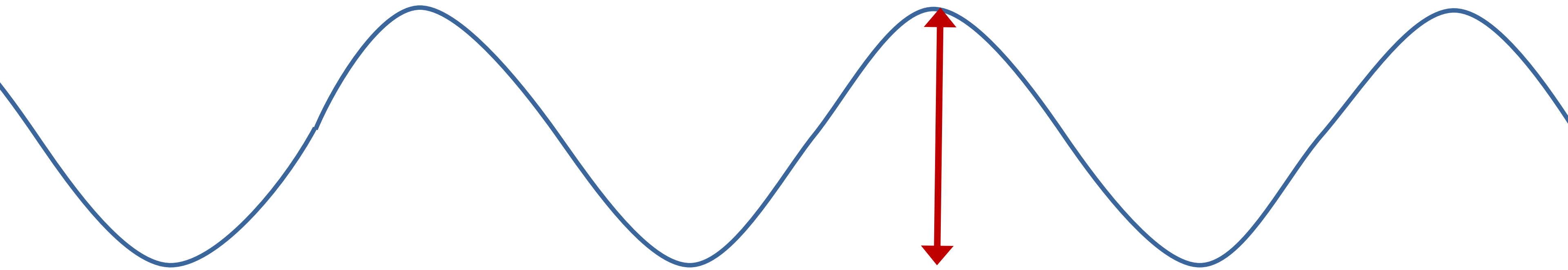
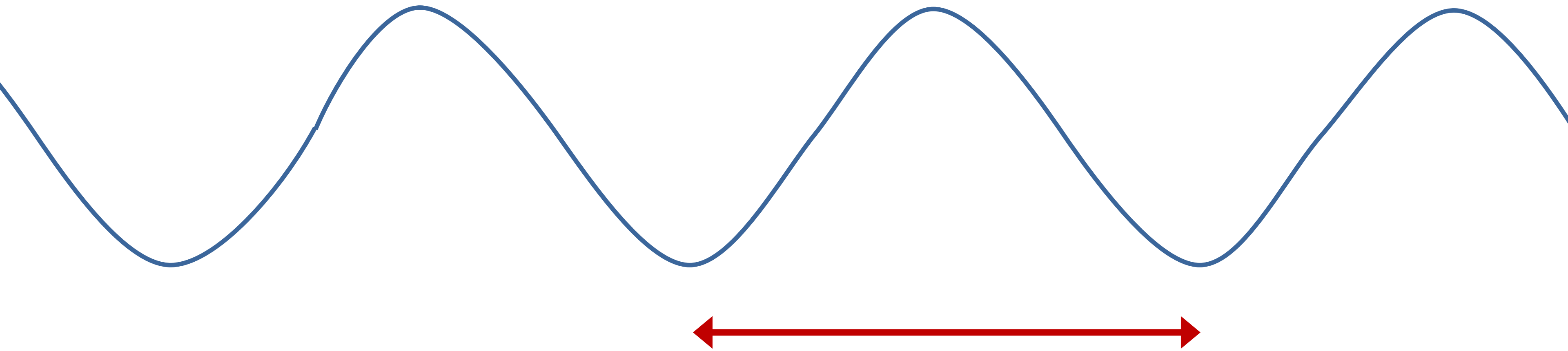
How do we describe waves mathematically



Amplitude A

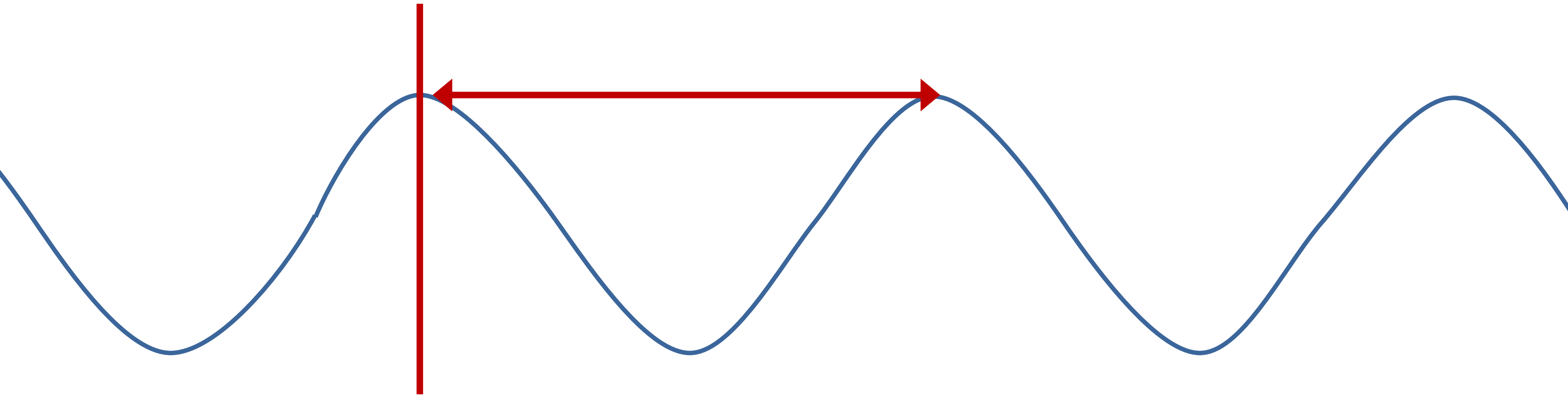
How do we describe waves mathematically



Wavelength λ

Wavenumber $k = 2\pi/\lambda$

How do we describe waves mathematically



Period T

Wavenumber k = 2π/λ

Angular frequency $\omega = 2\pi/T = 2\pi f$

Phase velocity $v_p = \omega/k$

Summary of waves in plasmas

- Plasma wave (no B-field necessary) in a cold plasma
  - Ions are stationary
- Wave-particle interaction
- Plasma wave in non-cold plasma
- MHD waves (B-field)
  - Perpendicular to the B-field
  - Parallel to the B-field

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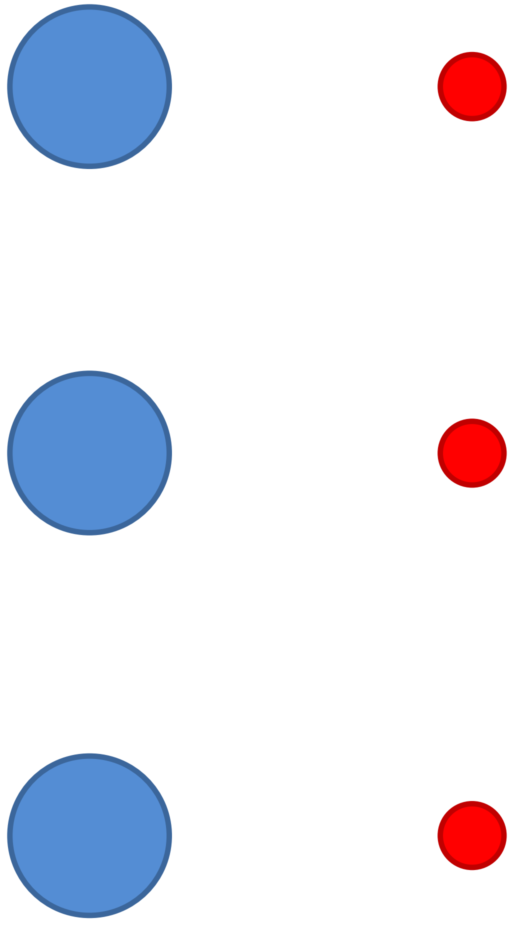
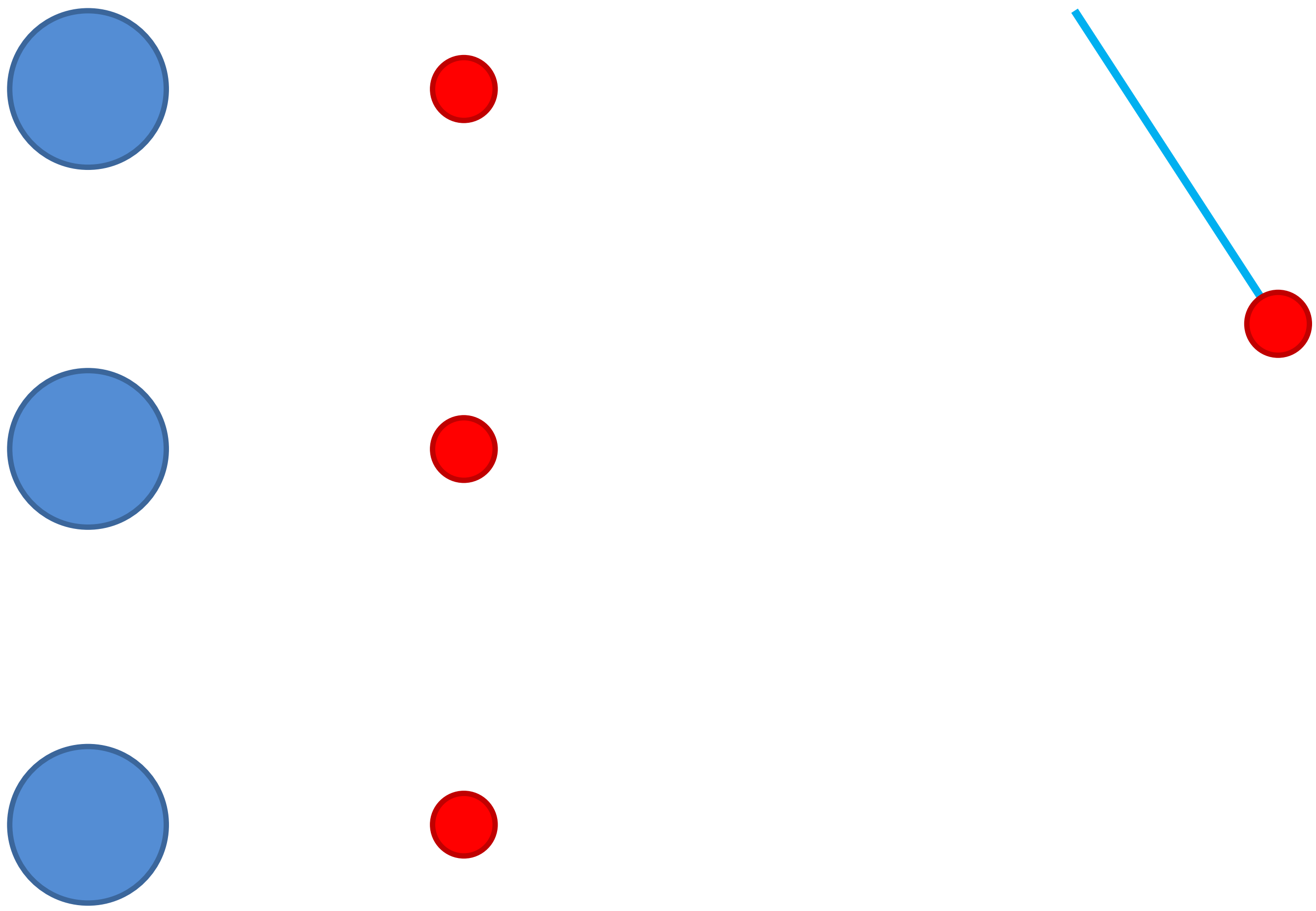
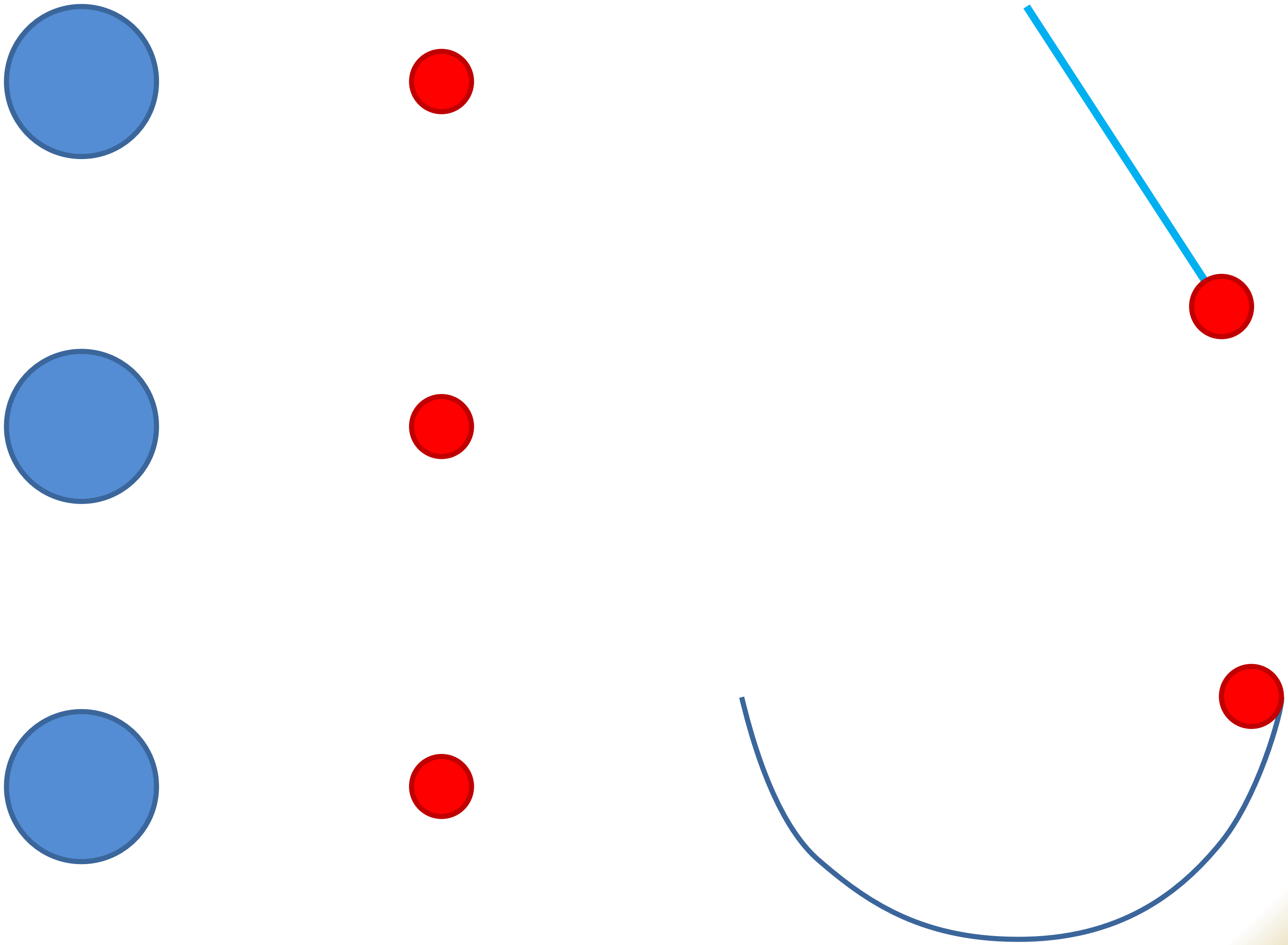
The plasma wave in a cold plasma



The plasma wave : Similar to pendulum motion



The plasma wave : Similar to ball stuck in a valley



The plasma wave: Starting from force balance

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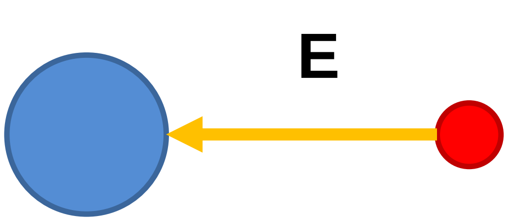

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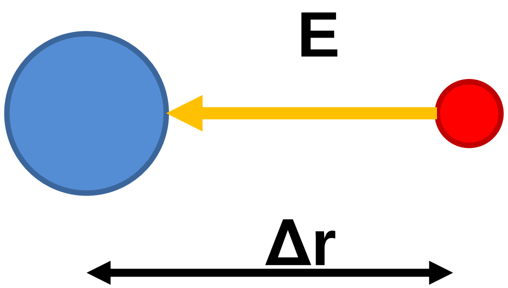

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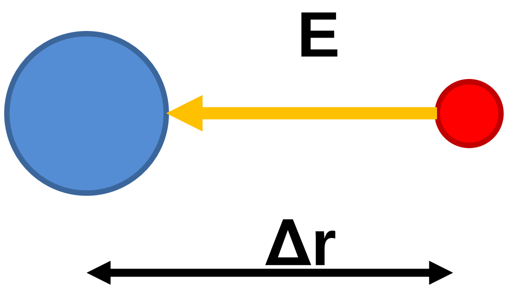

$$m_e \frac{dv}{dt} = -eE$$

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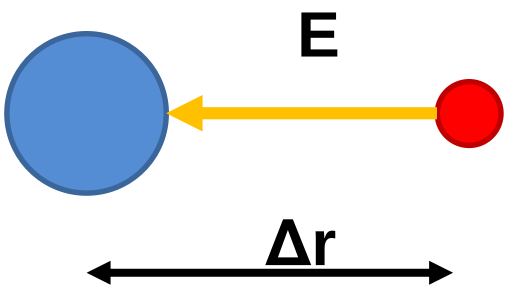

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$$E \sim \frac{q}{\Delta r^2} = \frac{en_e}{\Delta r^2}$$

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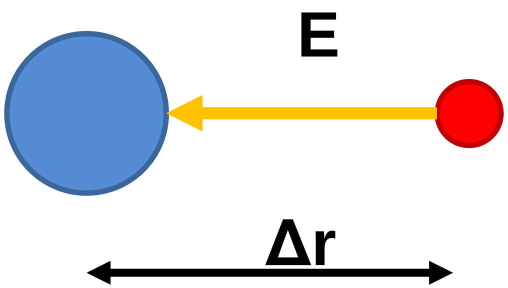

$$m_e \frac{dv}{dt} = -eE$$

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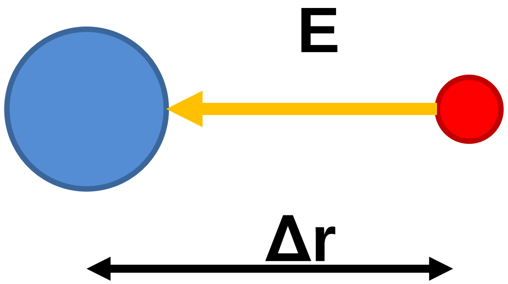
$$\frac{d^2 r}{dt^2} = -\frac{e^2 n_e}{m_e} \frac{1}{\Delta r^2}$$

The plasma wave: Derivation similar to the pendulum principle



Simple Harmonic Oscillator:

$$\frac{d^2r}{dt^2} = -\omega r$$

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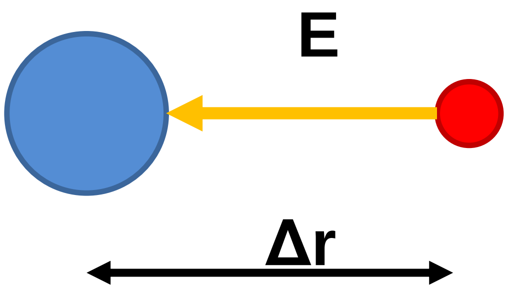

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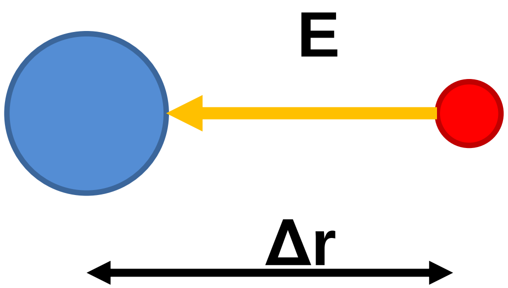
If we assume that Δr is close to 1 then:

$$\frac{1}{\Delta r^2} \sim \Delta r$$

The plasma wave: Plasma frequency



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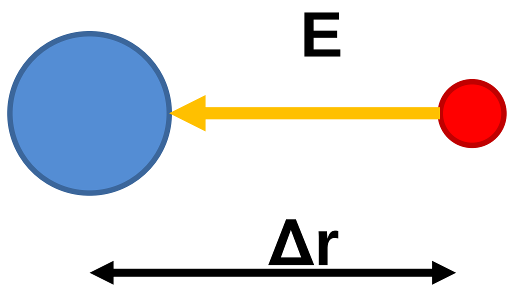

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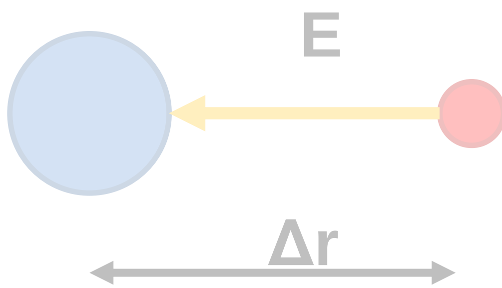

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Hand-wavy derivation at best that can not capture more complex physics of a non-cold plasma

More complete picture of plasma wave in a cold plasma

More complete picture of plasma wave in a cold plasma

Equation of motion

$$nm\left(\frac{\partial \boldsymbol{v}}{\partial t} + \boldsymbol{v}\nabla\boldsymbol{v}\right) = n\left(q\boldsymbol{E} + \frac{\boldsymbol{v}}{c} \times \boldsymbol{B}\right)$$

More complete picture of plasma wave in a cold plasma

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Continuity equation

$$\frac{\partial n}{\partial t} + \boldsymbol{\nabla} \cdot (n\boldsymbol{v}) = 0$$

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Starting point: Unmagnetized, Cold plasma

More complete picture of plasma wave in a cold plasma

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Linearize and loose second order

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More complete picture of plasma wave in a cold plasma: linearized equations in 1D

Continuity equation

$$\frac{\partial \tilde{n}}{\partial t} = -n_0 \frac{\partial \widetilde{\boldsymbol{v}}}{\partial r}$$

Equation of motion

$$m \frac{\partial \widetilde{\boldsymbol{v}}}{\partial t} = q \widetilde{\boldsymbol{E}}$$

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Maxwell equations linearized

$$\nabla \cdot \widetilde{\boldsymbol{E}} = 4\pi q \tilde{n}$$

$$\nabla \times \widetilde{\boldsymbol{E}} = -\frac{1}{c}\frac{\partial \widetilde{\boldsymbol{B}}}{\partial t}$$

$$\nabla \cdot \widetilde{\boldsymbol{B}} = 0$$

$$\nabla \times \widetilde{\boldsymbol{B}} = \frac{4\pi}{c}\tilde{\boldsymbol{J}} + \frac{1}{c}\frac{\partial \widetilde{\boldsymbol{E}}}{\partial t}$$

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Use Fourier decomposition:

$$\widetilde{\boldsymbol{A}} = \widetilde{\boldsymbol{A}} \exp(i(\boldsymbol{k} \cdot \boldsymbol{x} - \omega t))$$

More complete picture of plasma wave in a cold plasma: linearized equations in 1D

Continuity equation

$$-i\omega\tilde{n} = -n_0 i\boldsymbol{k}\cdot\widetilde{\boldsymbol{v}}$$

Equation of motion

$$-i\omega m\widetilde{\boldsymbol{v}} = q\widetilde{\boldsymbol{E}}$$

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$$i\boldsymbol{k}\cdot\widetilde{\boldsymbol{E}} = 4\pi q\tilde{n} \qquad i\boldsymbol{k}\times\widetilde{\boldsymbol{E}} = \frac{1}{c}i\omega\widetilde{\boldsymbol{B}}$$

$$i\boldsymbol{k}\cdot\widetilde{\boldsymbol{B}} = 0 \qquad i\boldsymbol{k}\times\widetilde{\boldsymbol{B}} = \frac{4\pi}{c}\tilde{\boldsymbol{J}} - \frac{i\omega}{c}\widetilde{\boldsymbol{E}} \qquad \tilde{\boldsymbol{J}} = qn_0\widetilde{\boldsymbol{v}}$$

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Equation of motion

Maxwell equations linearized

$$\omega_p^2 = \frac{4\pi n_0 q^2}{m}$$

$$i\boldsymbol{k}\cdot\tilde{\boldsymbol{E}} = 4\pi q\tilde{n}$$

Electro-Magnetic waves

$$i\boldsymbol{k}\cdot\tilde{\boldsymbol{B}} = 0$$

$$\cancel{\boldsymbol{k}(\boldsymbol{k}\cdot\tilde{\boldsymbol{E}})} - \boldsymbol{k}^2\tilde{\boldsymbol{E}} = \underbrace{\frac{4\pi n_0 q^2}{mc^2}}_{\frac{\omega_p^2}{c^2}}\tilde{\boldsymbol{E}} - \frac{\omega^2}{c^2}\tilde{\boldsymbol{E}}$$

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More complete picture of plasma wave in a cold plasma: linearized equations in 1D

$$-\boldsymbol{k}^{2}\,\widetilde{\boldsymbol{E}} = \frac{\omega_p^2}{c^2}\widetilde{\boldsymbol{E}} - \frac{\omega^2}{c^2}\widetilde{\boldsymbol{E}}$$

More complete picture of plasma wave in a cold plasma: linearized equations in 1D

$$-\boldsymbol{k}^{2}\,\widetilde{\boldsymbol{E}} = \frac{\omega_p^2}{c^2}\widetilde{\boldsymbol{E}} - \frac{\omega^2}{c^2}\widetilde{\boldsymbol{E}} \Rightarrow k^2 = \frac{\omega^2}{c^2} - \frac{\omega_p^2}{c^2}$$

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Dispersion relationship

$$\omega^2 = \omega_p^2 + c^2 k^2$$

More complete picture of plasma wave in a cold plasma: linearized equations in 1D

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Dispersion relationship

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Plasma frequency

$$\omega_p^2 = \frac{4\pi n_0 q^2}{m}$$

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Dispersion relationship

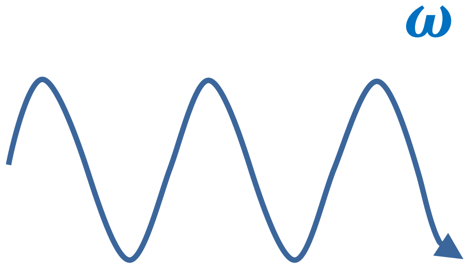
$$\omega^2 = \omega_p^2 + c^2k^2$$

Plasma frequency

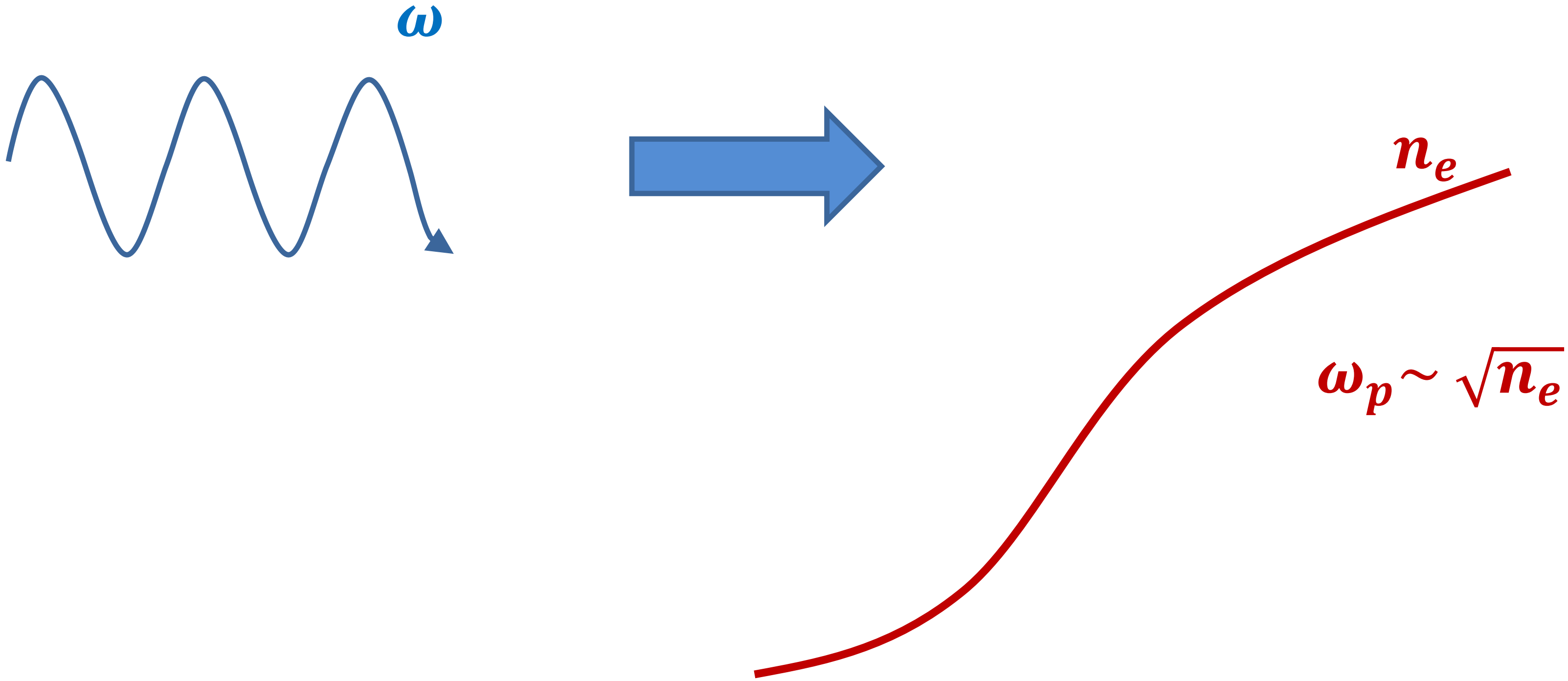
This can be used to measure density

$$\omega_p^2 = \frac{4\pi n_0 q^2}{m}$$

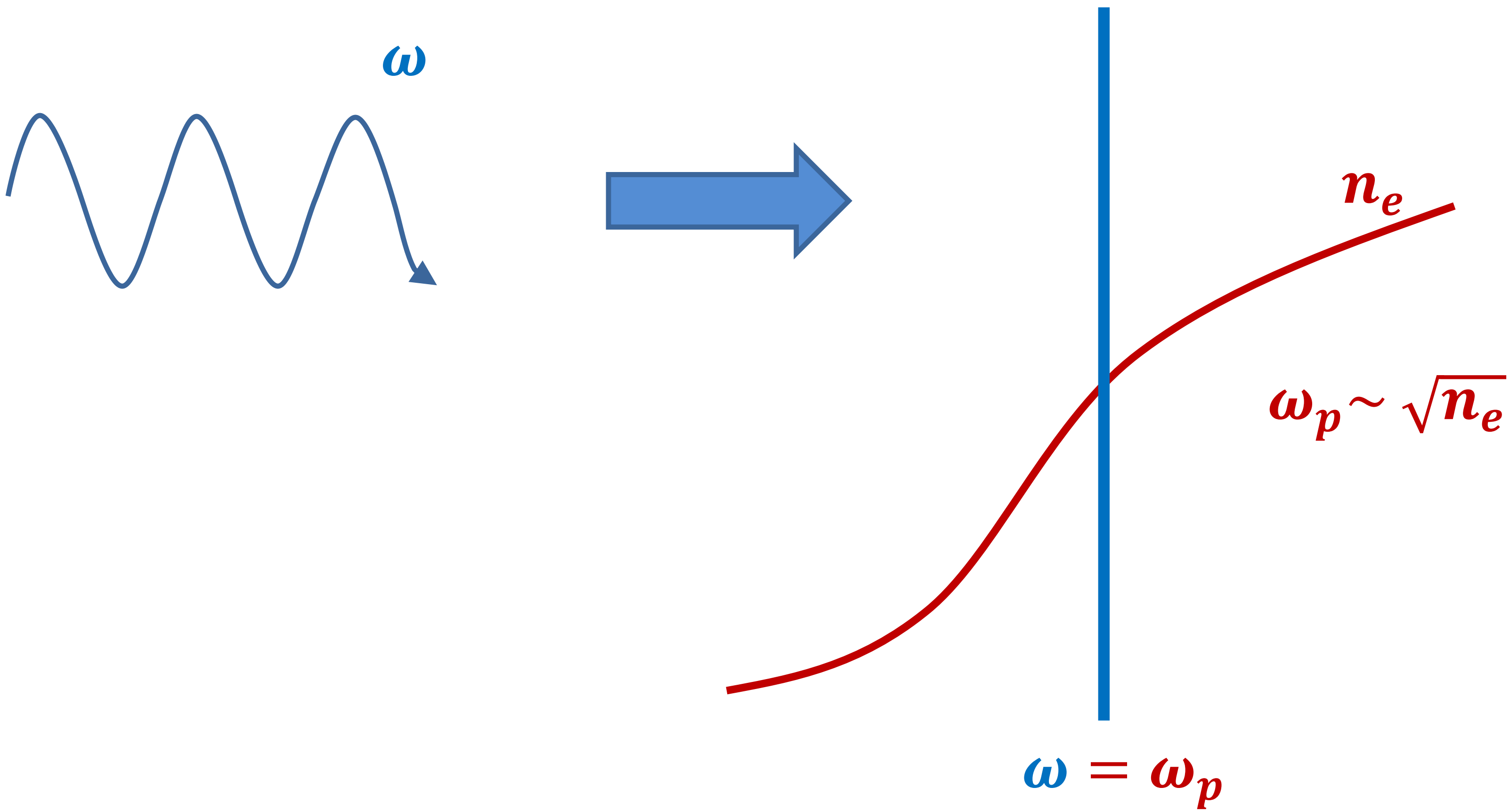
Density cut-off: waves to probe density



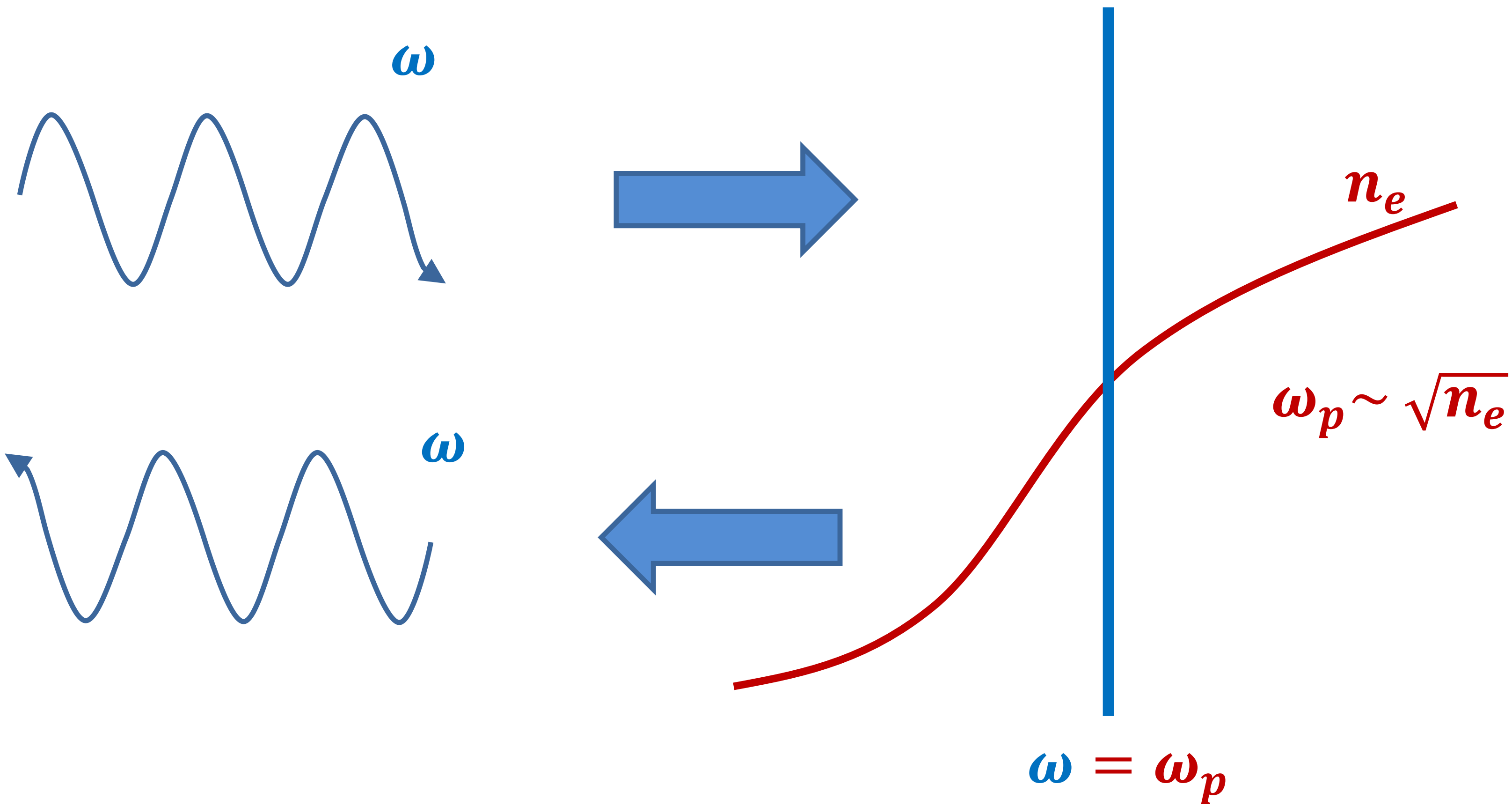
Density cut-off: waves to probe density



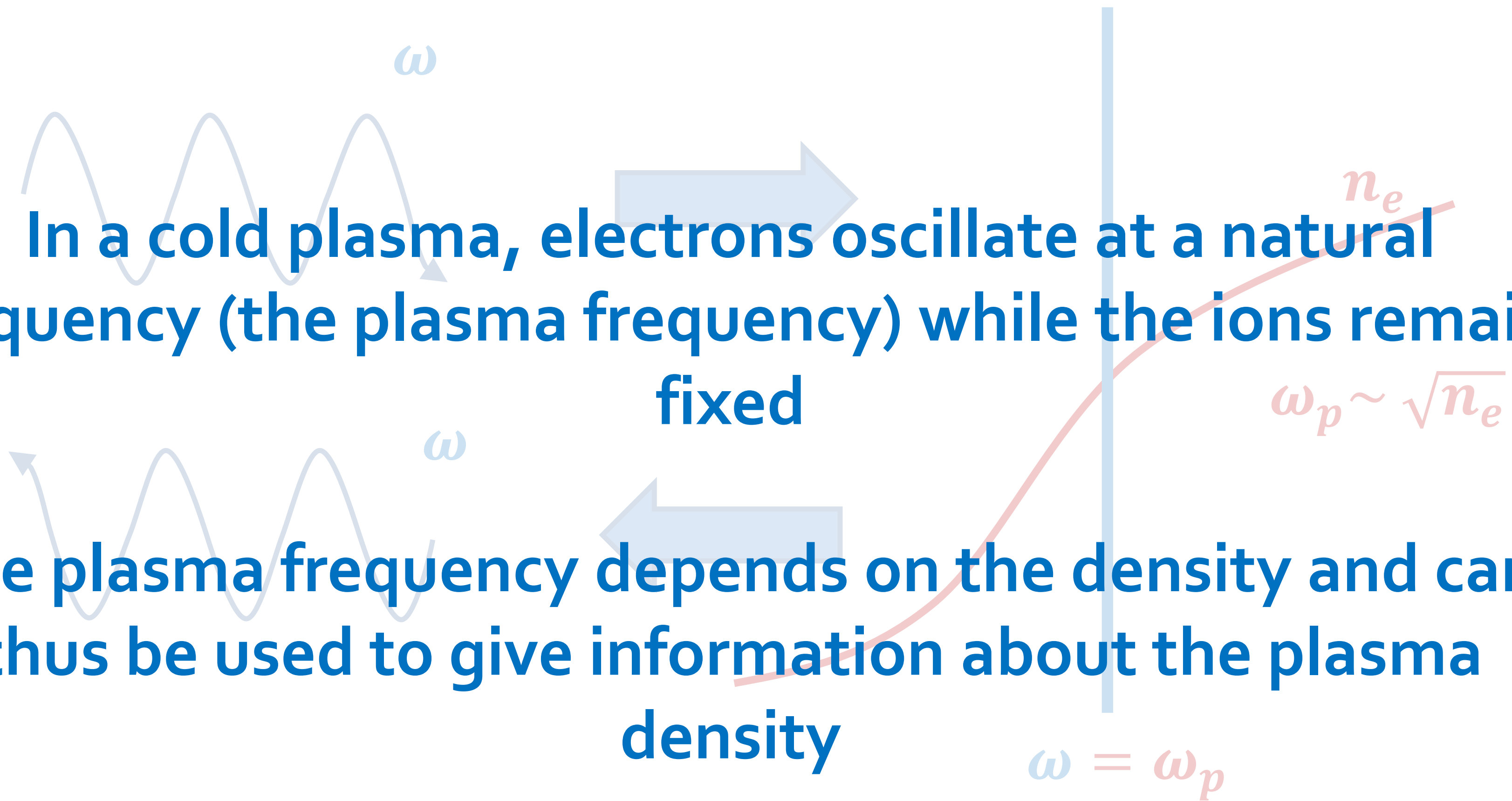
Density cut-off: waves to probe density



Density cut-off: waves to probe density



Summary of the plasma wave



In a cold plasma, electrons oscillate at a natural frequency (the plasma frequency) while the ions remain fixed

The plasma frequency depends on the density and can thus be used to give information about the plasma density

Summary of waves in plasmas

- Plasma wave (no B-field necessary) in a cold plasma
  - Ions are stationary
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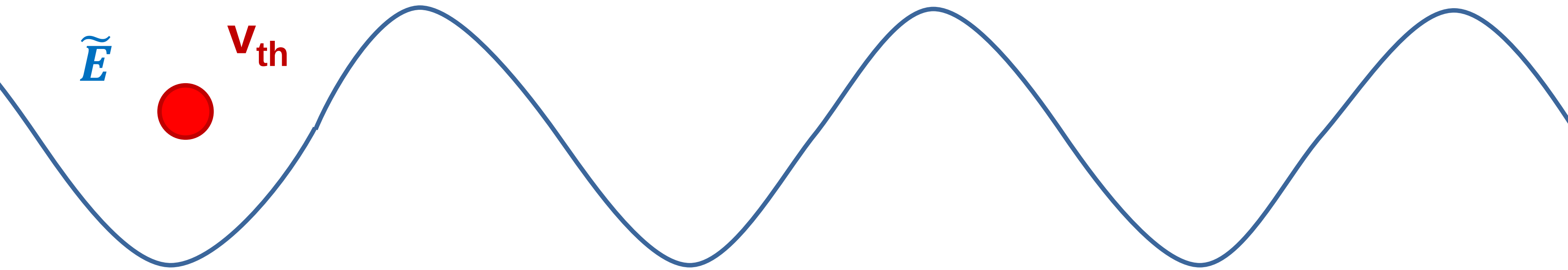
  - Perpendicular to the B-field
  - Parallel to the B-field

Interaction of waves and particles : no interaction

$$v_p = \frac{\omega}{k} < v_{th}$$

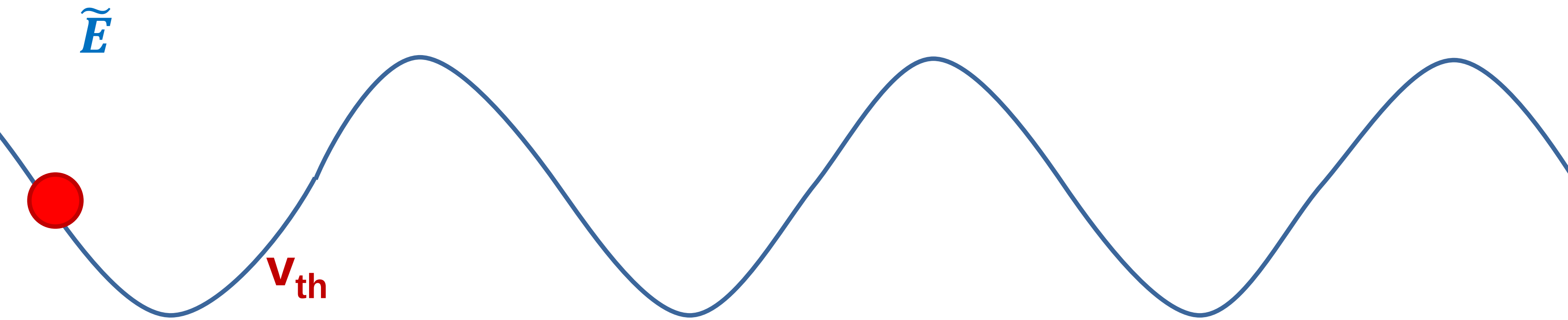


Particle is too fast to see the Electric field

$$-\boldsymbol{k}^2\,\tilde{\boldsymbol{E}} = \frac{\omega_p^2}{c^2}\tilde{\boldsymbol{E}} - \frac{\omega^2}{c^2}\tilde{\boldsymbol{E}}$$

Interaction of waves and particles : trapped particle

$$v_p = \frac{\omega}{k} > v_{th}$$

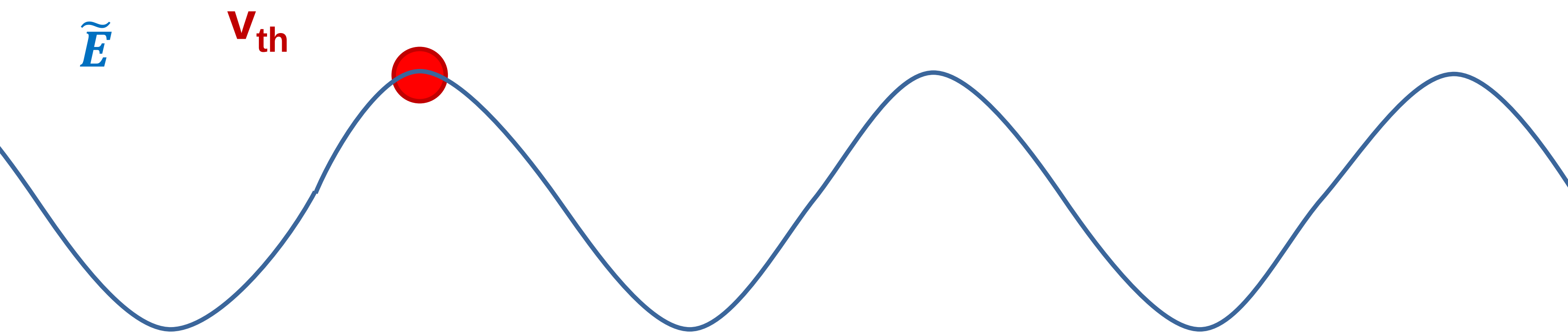


Particle is too slow and gets trapped

$$-\boldsymbol{k}^2\,\widetilde{\boldsymbol{E}} = \frac{\omega_p^2}{c^2}\widetilde{\boldsymbol{E}} - \frac{\omega^2}{c^2}\widetilde{\boldsymbol{E}}$$

Interaction of waves and particles : resonance

$$v_p = \frac{\omega}{k} \sim v_{th}$$

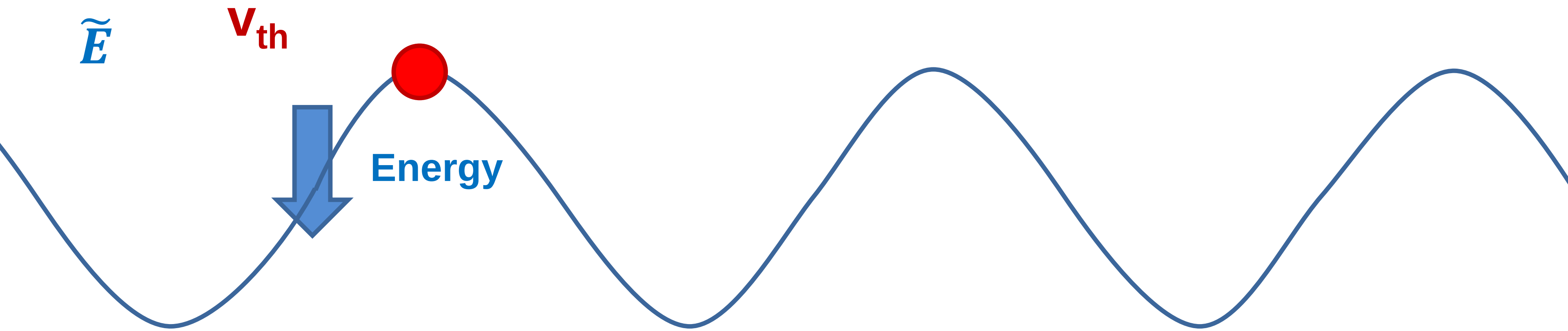


Particle has the right speed
and gets accelerated by the electric field

$$-\boldsymbol{k}^2\,\widetilde{\boldsymbol{E}} = \frac{\omega_p^2}{c^2}\widetilde{\boldsymbol{E}} - \frac{\omega^2}{c^2}\widetilde{\boldsymbol{E}}$$

Interaction of waves and particles : resonant particles can give energy to the wave

$$v_p = \frac{\omega}{k} \sim v_{th}$$



Particle has the right speed
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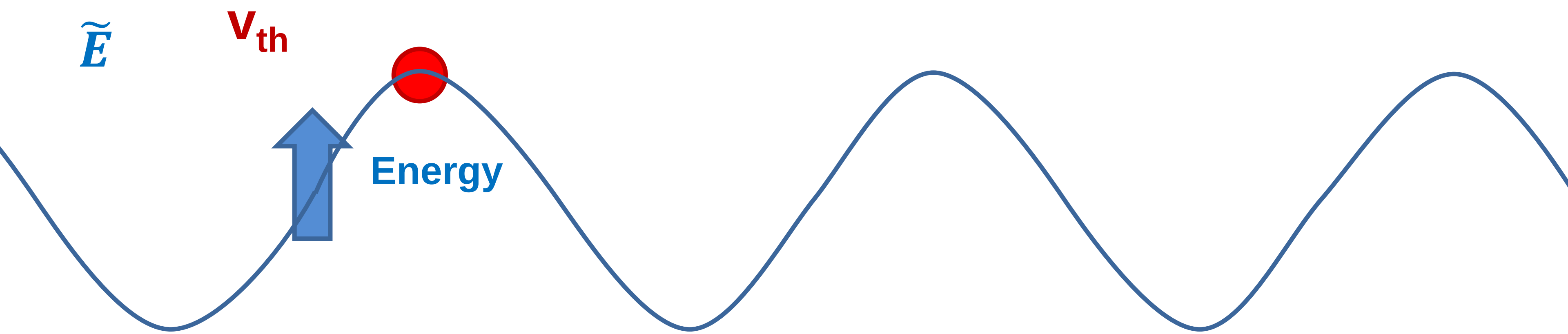
$$-\boldsymbol{k}^2\, \widetilde{\boldsymbol{E}} = \frac{\omega_p^2}{c^2}\widetilde{\boldsymbol{E}} - \frac{\omega^2}{c^2}\widetilde{\boldsymbol{E}}$$

Interaction of waves and particles : the wave can also give energy to the particles

$$v_p = \frac{\omega}{k} \sim v_{th}$$

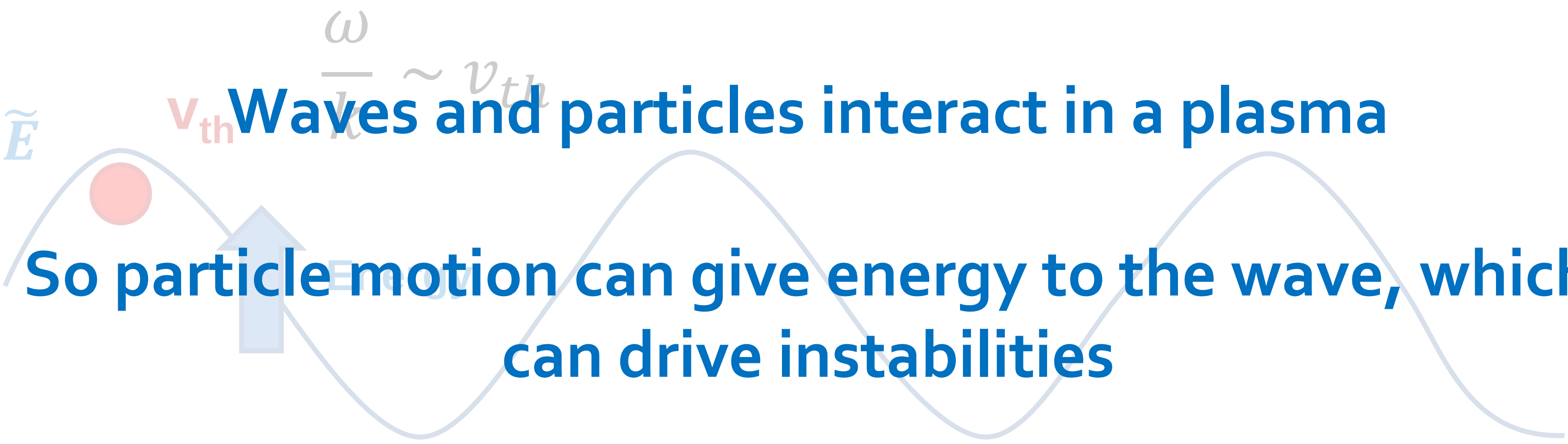


Particle has the right speed
and gets accelerated by the electric field

$$-\boldsymbol{k}^2\,\widetilde{\boldsymbol{E}} = \frac{\omega_p^2}{c^2}\widetilde{\boldsymbol{E}} - \frac{\omega^2}{c^2}\widetilde{\boldsymbol{E}}$$

Summary wave particle interaction

Waves and particles interact in a plasma



So particle motion can give energy to the wave, which can drive instabilities

On the other hand, waves can be used to give energy to particles (see next talk)

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Plasma wave in a non-cold plasma: linearized equations in 1D

Continuity equation

$$\frac{\partial \tilde{n}}{\partial t} = -n_0 \frac{\partial \widetilde{\boldsymbol{v}}}{\partial r}$$

Equation of motion

$$m \frac{\partial \widetilde{\boldsymbol{v}}}{\partial t} = q \widetilde{\boldsymbol{E}}$$

Plasma wave in a non-cold plasma: linearized equations in 1D

Continuity equation

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$$m \frac{\partial \widetilde{\boldsymbol{v}}}{\partial t} = q\widetilde{\boldsymbol{E}} + \frac{\nabla \tilde{p}}{n_0}$$

Isothermal

$$\tilde{p} = \tilde{n} T$$

Plasma wave in a non-cold plasma: linearized equations in 1D

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Isothermal

$$\tilde{p} = \tilde{n} T$$

Maxwell equations linearized

$$\nabla \cdot \widetilde{\boldsymbol{E}} = 4\pi q \tilde{n}$$

$$\nabla \times \widetilde{\boldsymbol{E}} = -\frac{1}{c} \frac{\partial \widetilde{\boldsymbol{B}}}{\partial t}$$

$$\nabla \cdot \widetilde{\boldsymbol{B}} = 0$$

$$\nabla \times \widetilde{\boldsymbol{B}} = \frac{4\pi}{c} \tilde{\boldsymbol{J}} + \frac{1}{c} \frac{\partial \widetilde{\boldsymbol{E}}}{\partial t}$$

$$\tilde{\boldsymbol{J}} = q n_0 \widetilde{\boldsymbol{v}}$$

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Use Fourier decomposition:

$$\tilde{\boldsymbol{A}} = \tilde{\boldsymbol{A}} \exp(i(\boldsymbol{k} \cdot \boldsymbol{x} - \omega t))$$

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Plasma wave in a non-cold plasma: linearized equations in 1D

Continuity equation

$$\frac{\partial \tilde{n}}{\partial t} = -n_0 \frac{\partial \widetilde{\boldsymbol{v}}}{\partial r}$$

$$\frac{\partial^2 \tilde{n}}{\partial t^2} = -\frac{n_0 q}{m} \nabla \cdot \widetilde{\boldsymbol{E}} + \frac{n_0 \nabla^2 \tilde{p}}{n_0 m}$$

Equation of motion

$$m \frac{\partial \widetilde{\boldsymbol{v}}}{\partial t} = q \widetilde{\boldsymbol{E}} + \frac{\nabla \tilde{p}}{n_0}$$

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Plasma wave in a non-cold plasma: linearized equations in 1D

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$$\frac{\partial^2 \tilde{n}}{\partial t^2} = -\omega_p^2 \tilde{n} + v_{th}^2 \nabla^2 \tilde{n}$$

Plasma wave in a non-cold plasma: linearized equations in 1D

Continuity equation

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Equation of motion

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Isothermal

$$\tilde{p} = \tilde{n} T$$

$$\frac{\partial^2 \tilde{n}}{\partial t^2} = -\frac{n_0 q}{m} \nabla \cdot \tilde{\boldsymbol{E}} + \frac{n_0 \nabla^2 \tilde{p}}{n_0 m}$$

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$$\frac{\partial^2 \tilde{n}}{\partial t^2} = -\frac{4\pi n_0 q^2 \tilde{n}}{m} + \frac{T \nabla^2 \tilde{n}}{m}$$

Plasma oscillation

Thermalization

$$\frac{\partial^2 \tilde{n}}{\partial t^2} = -\omega_p^2 \tilde{n} + v_{th}^2 \nabla^2 \tilde{n}$$

Plasma wave in a non-cold plasma: linearized equations in 1D

$\omega \gg \omega_p$ ⇨ Plasma oscillations are too fast to be screened

$\omega \ll \omega_p$ ⇨ Plasma response is trying to screen test charge

Plasma oscillation Thermalization

$$\frac{\partial^2 \tilde{n}}{\partial t^2} = -\omega_p^2 \tilde{n} + v_{th}^2 \nabla^2 \tilde{n}$$

Plasma wave in a non-cold plasma: linearized equations in 1D

$\omega \gg \omega_p$ → Plasma oscillations are too fast to be screened

$\omega \ll \omega_p$ → Plasma response is trying to screen test charge

Plasma oscillation   Thermalization

$$\frac{\partial^2 \tilde{n}}{\partial t^2} = -\omega_p^2 \tilde{n} + v_{th}^2 \nabla^2 \tilde{n}$$

Including a non-zero temperature changes the dispersion relationship

Now particles can be screened, depending on the frequency of the wave with respect to the plasma wave frequency

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  - Parallel to the B-field

Simple MHD waves : B-field is included

	$\nabla \cdot v = 0$	$\nabla \cdot v \neq 0$	
$\boldsymbol{k} = k\hat{\boldsymbol{x}}$	Shear Alfven	Acoustic	Parallel propagation
$\boldsymbol{k} = k\hat{\boldsymbol{z}}$		Magnetosonic	Perpendicular propagation

The shear Alfven wave

Wave propagation in direction of magnetic field and incompressible

Assumption: $\boldsymbol{k} = k\hat{\boldsymbol{z}}$ $\quad\quad \boldsymbol{\nabla} \cdot \boldsymbol{v} = 0$

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Linearized equations:

$$\rho_0 \frac{\partial \widetilde{\boldsymbol{v}}}{\partial t} = \nabla \left(\tilde{p} + \frac{\widetilde{B}^2}{8\pi} \right) + \boldsymbol{B_0} \cdot \frac{\nabla \widetilde{\boldsymbol{B}}}{4\pi} \qquad \frac{\partial \widetilde{\boldsymbol{B}}}{\partial t} = -\boldsymbol{B_0} \cdot \nabla \widetilde{\boldsymbol{v}}$$

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$$\nabla \cdot \left(\rho_0 \frac{\partial \widetilde{\boldsymbol{v}}}{\partial t} = \nabla\left(\tilde{p} + \frac{\tilde{B}^2}{8\pi}\right) + \boldsymbol{B_0} \cdot \frac{\nabla \widetilde{\boldsymbol{B}}}{4\pi}\right)$$

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$$\nabla\cdot\left(\rho_0 \frac{\partial \widetilde{\boldsymbol{v}}}{\partial t} = \nabla\left(\tilde{p} + \frac{\tilde{B}^2}{8\pi}\right) + \boldsymbol{B_0}\cdot\frac{\nabla\widetilde{\boldsymbol{B}}}{4\pi}\right)$$

$$\rho_0 \frac{\partial\, \nabla\cdot\widetilde{\boldsymbol{v}}}{\partial t} = \nabla^2\left(\tilde{p} + \frac{\tilde{B}^2}{8\pi}\right) + \boldsymbol{B_0}\cdot\frac{\nabla\nabla\cdot\widetilde{\boldsymbol{B}}}{4\pi}$$

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Wave propagation in direction of magnetic field and incompressible

Assumption: $\boldsymbol{k} = k\hat{\boldsymbol{z}}$ $\qquad \boldsymbol{\nabla} \cdot \boldsymbol{v} = 0$ $\qquad \tilde{p} + \dfrac{\tilde{B}^2}{8\pi} = 0$

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Wave propagation in direction of magnetic field and incompressible

Assumption: $\boldsymbol{k} = k\hat{\boldsymbol{z}}$ $\quad \boldsymbol{\nabla} \cdot \boldsymbol{v} = 0$ $\quad \tilde{p} + \frac{\tilde{B}^2}{8\pi} = 0$

Linearized equations in 1D:

$$\rho_0 \frac{\partial \tilde{v}}{\partial t} = \cancel{\nabla\left(\tilde{p} + \frac{\tilde{B}^2}{8\pi}\right)} + \frac{B_0}{4\pi}\frac{\partial \tilde{B}}{\partial z} \qquad \frac{\partial \tilde{B}}{\partial t} = -B_0 \frac{\partial \tilde{v}}{\partial z}$$

The shear Alfven wave

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$$\frac{\partial^2 \tilde{v}}{\partial t^2} = \underbrace{\frac{B_0^2}{4\pi\rho_0}}_{v_A^2} \frac{\partial^2 \tilde{v}}{\partial z^2}$$

The shear Alfven wave

Wave propagation in direction of magnetic field and incompressible

Assumption: $\boldsymbol{k} = k\hat{\boldsymbol{z}}$ $\quad \nabla \cdot \boldsymbol{v} = 0$ $\quad \tilde{p} + \frac{\tilde{B}^2}{8\pi} = 0$

The wave is the result of Magnetic tension:

$$\frac{B/4\pi}{\rho_0/B}$$

tension-in-line

mass-per-line

It is a little bit like ‘plucking’: $B_0 \perp \tilde{v}$

The Acoustic wave

Wave propagation in direction of magnetic field and compressible

Assumption: $\boldsymbol{k} = k\hat{\boldsymbol{z}}$ $\boldsymbol{\nabla} \cdot \boldsymbol{v} \neq 0$

The Acoustic wave

Wave propagation in direction of magnetic field and compressible

Assumption: $\boldsymbol{k} = k\hat{\boldsymbol{z}}$ $\qquad \boldsymbol{\nabla} \cdot \boldsymbol{v} \neq 0$

Linearized equations (and jumping a few steps):

$$\rho_0 \frac{\partial \tilde{\boldsymbol{v}}}{\partial t} = \frac{\partial \tilde{p}}{\partial z} - \frac{\partial}{\partial z} B_0 \tilde{B}/4\pi + \frac{\partial}{\partial z} B_0 \tilde{B}/4\pi \qquad \frac{\partial \tilde{B}_z}{\partial t} = \frac{B_0 \partial \tilde{v}}{\partial z} - \frac{B_0 \partial \tilde{v}}{\partial z}$$

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Link pressure to density and use continuity equation + math:

$$\frac{\partial^2 \tilde{\rho}}{\partial t^2} = \frac{\gamma p_0}{\rho_0} \frac{\partial^2 \tilde{\rho}}{\partial z^2}$$

The Acoustic wave

Wave propagation in direction of magnetic field and compressible

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The Magnetosonic wave (or compressionable Alfven wave)

Wave propagation perpendicular to the magnetic field and compressible

Assumption: $\boldsymbol{k} = k\hat{\boldsymbol{x}}$ $\quad \nabla \cdot \boldsymbol{v} \neq 0$

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$$\frac{\partial \tilde{\boldsymbol{v}}}{\partial t} = \frac{1}{\rho_0} \nabla \left(\tilde{p} + \frac{\tilde{B}^2}{8\pi} \right) + \frac{\boldsymbol{B_0}}{\rho_0} \cdot \frac{\nabla \tilde{\boldsymbol{B}}}{4\pi} \qquad \frac{1}{\rho_0} \frac{\partial \tilde{\boldsymbol{B}}}{\partial t} = -\frac{\boldsymbol{B_0}}{\rho_0} \cdot \nabla \tilde{\boldsymbol{v}}$$

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$$\frac{1}{\rho_0} \frac{\partial \tilde{\boldsymbol{B}}}{\partial t} = -\cancel{\frac{\boldsymbol{B_0}}{\rho_0}} \cdot \nabla \tilde{\boldsymbol{v}}$$

$$\frac{d\boldsymbol{B}/\rho}{dt} = 0 \qquad \tilde{\boldsymbol{B}} = \frac{\boldsymbol{B_0}\tilde{\rho}}{\rho_0}$$

The Magnetosonic wave (or compressionable Alfven wave)

Wave propagation perpendicular to the magnetic field and compressible

Assumption: $\boldsymbol{k} = k\hat{\boldsymbol{x}}$ $\quad \boldsymbol{\nabla}\cdot\boldsymbol{v} \neq 0$

Linearized equations (and jumping a few steps):

$$\frac{\partial\widetilde{\boldsymbol{v}}}{\partial t} = \frac{1}{\rho_0}\nabla\left(\tilde{p} + \frac{\tilde{B}^2}{8\pi}\right) + \cancel{\frac{\boldsymbol{B_0}}{\rho_0}\cdot\frac{\nabla\widetilde{\boldsymbol{B}}}{4\pi}}$$

$$\frac{1}{\rho_0}\frac{\partial\widetilde{\boldsymbol{B}}}{\partial t} = -\cancel{\frac{\boldsymbol{B_0}}{\rho_0}}\cdot\nabla\widetilde{\boldsymbol{v}}$$

$$\frac{d\boldsymbol{B}/\rho}{dt} = 0 \qquad \widetilde{\boldsymbol{B}} = \frac{\boldsymbol{B_0}\tilde{\rho}}{\rho_0}$$

Thermal Magnetic

$$\frac{\partial\widetilde{\boldsymbol{v}}}{\partial t} = \frac{1}{\rho_0}\nabla(\tilde{p}_T + \tilde{p}_B)$$

The Magnetosonic wave (or compressionable Alfven wave)

Wave propagation perpendicular to the magnetic field and compressible

Assumption: $\boldsymbol{k} = k\hat{\boldsymbol{x}}$ $\quad \boldsymbol{\nabla} \cdot \boldsymbol{v} \neq 0$

Linearized equations (and jumping a few steps):

$$\frac{\partial \tilde{\boldsymbol{v}}}{\partial t} = \frac{1}{\rho_0} \nabla \left(\tilde{p} + \frac{\tilde{B}^2}{8\pi} \right) + \cancel{\frac{\boldsymbol{B_0}}{\rho_0} \cdot \frac{\nabla \tilde{\boldsymbol{B}}}{4\pi}}$$

$$\frac{1}{\rho_0} \frac{\partial \tilde{\boldsymbol{B}}}{\partial t} = -\cancel{\frac{\boldsymbol{B_0}}{\rho_0}} \cdot \nabla \tilde{\boldsymbol{v}}$$

$$\frac{d\boldsymbol{B}/\rho}{dt} = 0 \qquad \tilde{\boldsymbol{B}} = \frac{\boldsymbol{B_0} \tilde{\rho}}{\rho_0}$$

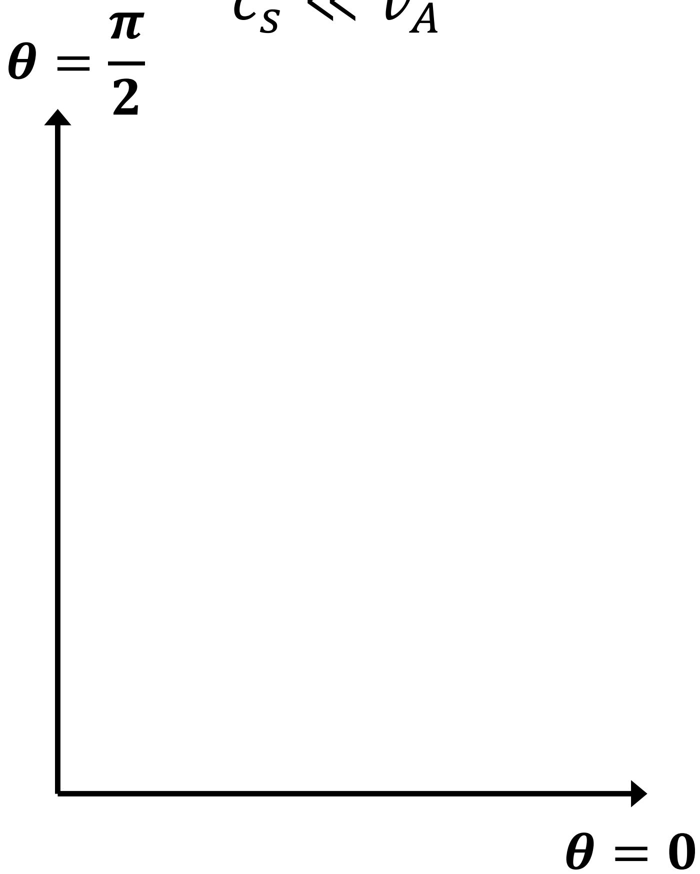
$$\frac{\partial \tilde{\boldsymbol{v}}}{\partial t} = \frac{1}{\rho_0} \nabla \left(\overset{\text{Thermal}}{\tilde{p}_T} + \overset{\text{Magnetic}}{\tilde{p}_B} \right) \Rightarrow \frac{\partial^2 \tilde{\rho}}{\partial t^2} = \left(\overbrace{\frac{\gamma p_0}{\rho_0}}^{c_s^2} + \overbrace{\frac{B_0^2}{4\pi \rho_0}}^{v_A^2} \right) \frac{\partial^2 \tilde{\rho}}{\partial z^2}$$

Simple MHD waves : B-field is included

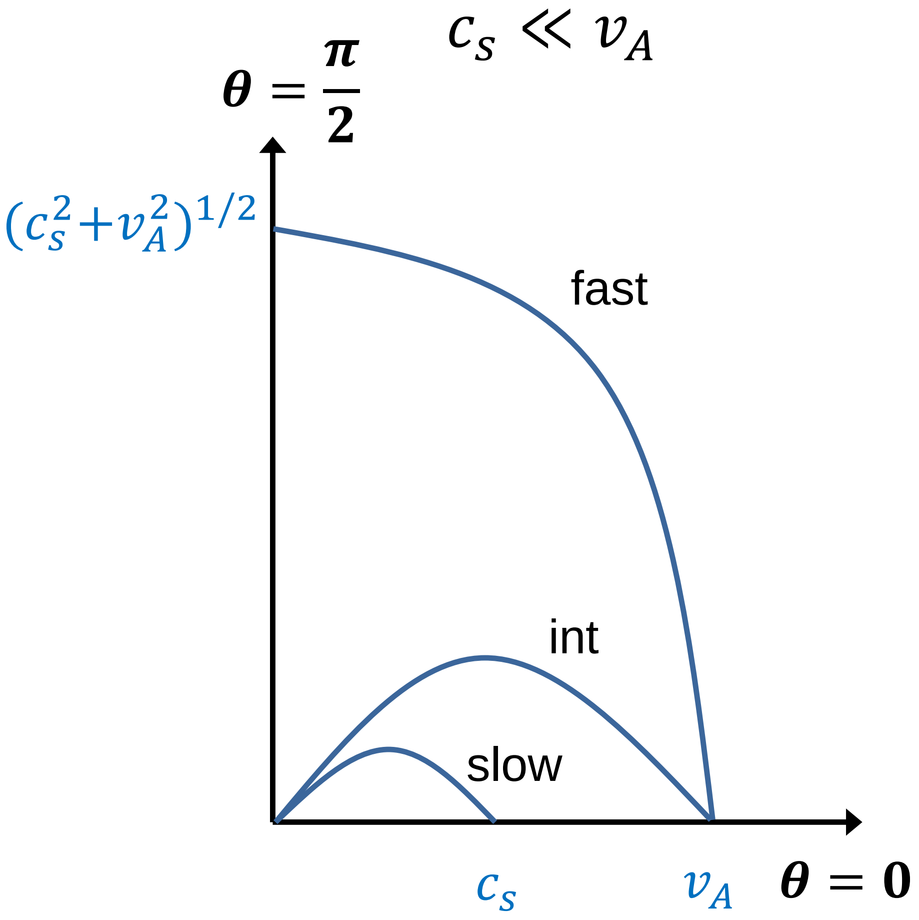
	$\nabla \cdot v = 0$	$\nabla \cdot v \neq 0$	
$\boldsymbol{k} = k\hat{\boldsymbol{x}}$	Shear Alfven	Acoustic	Parallel propagation
$\boldsymbol{k} = k\hat{\boldsymbol{z}}$		Magnetosonic	Perpendicular propagation

MHD waves a summary

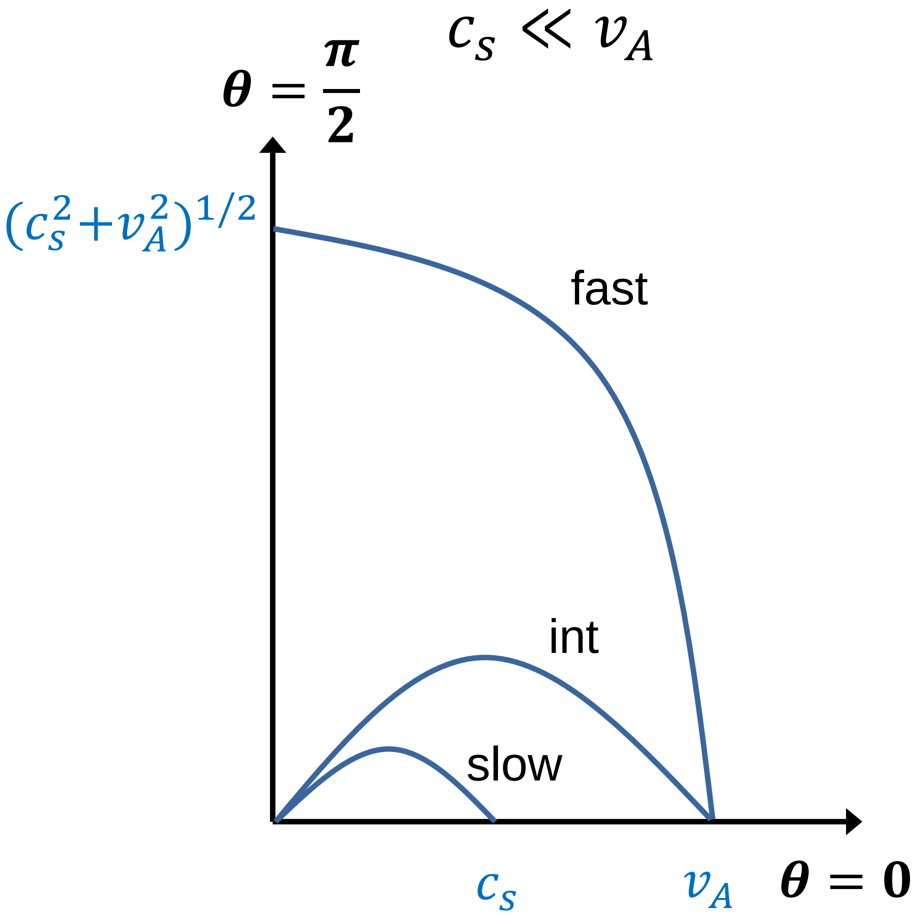
$c_s \ll v_A$

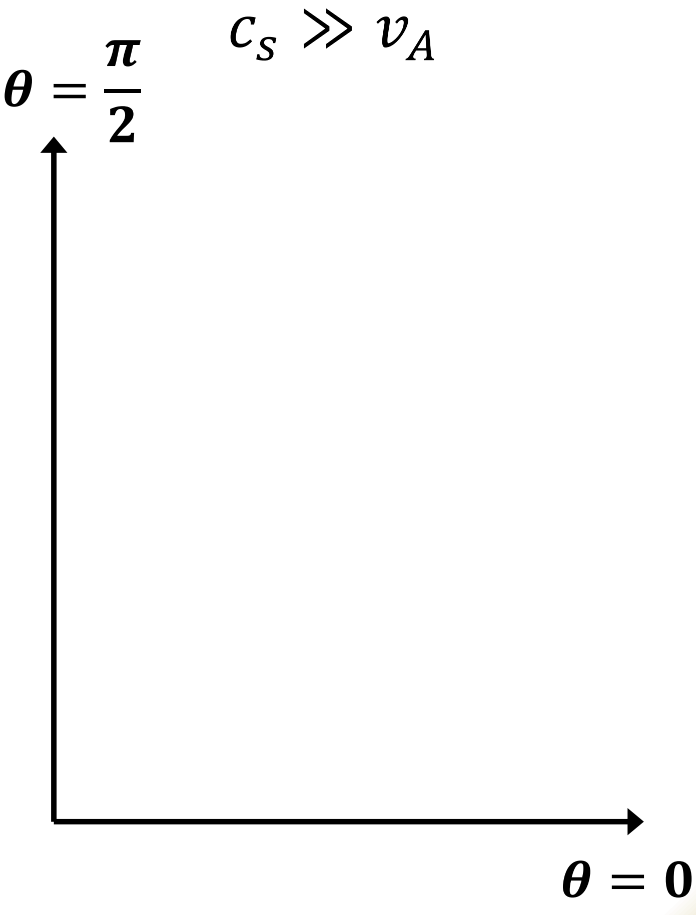


MHD waves a summary

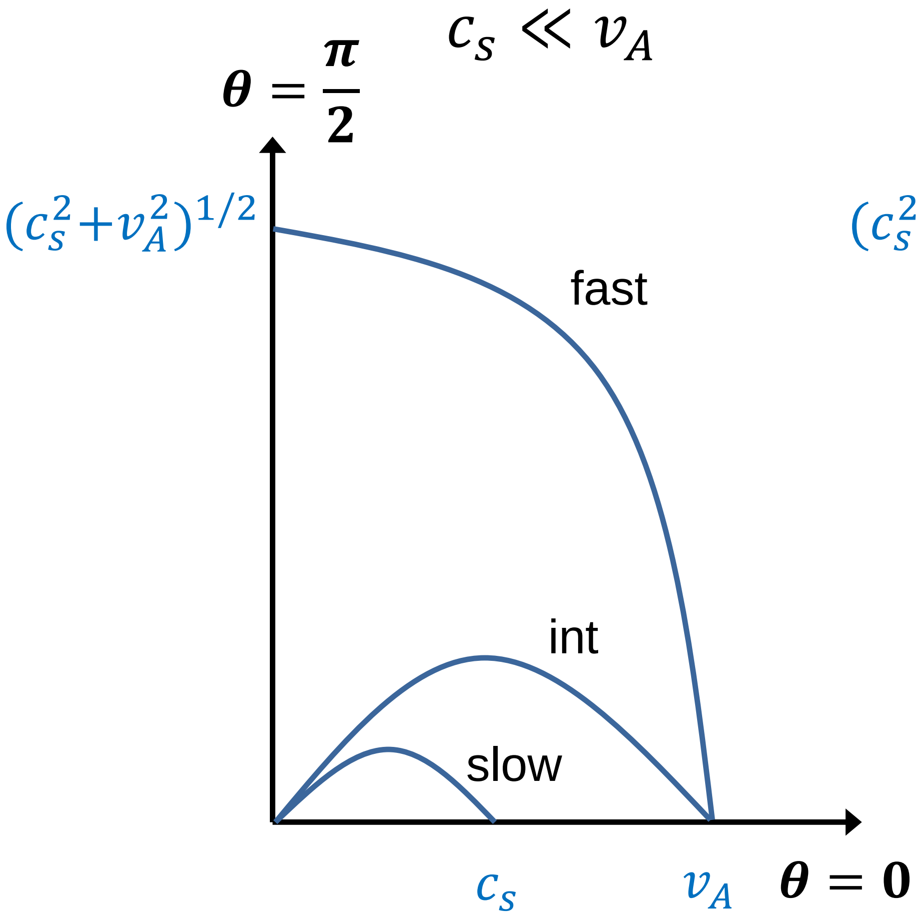


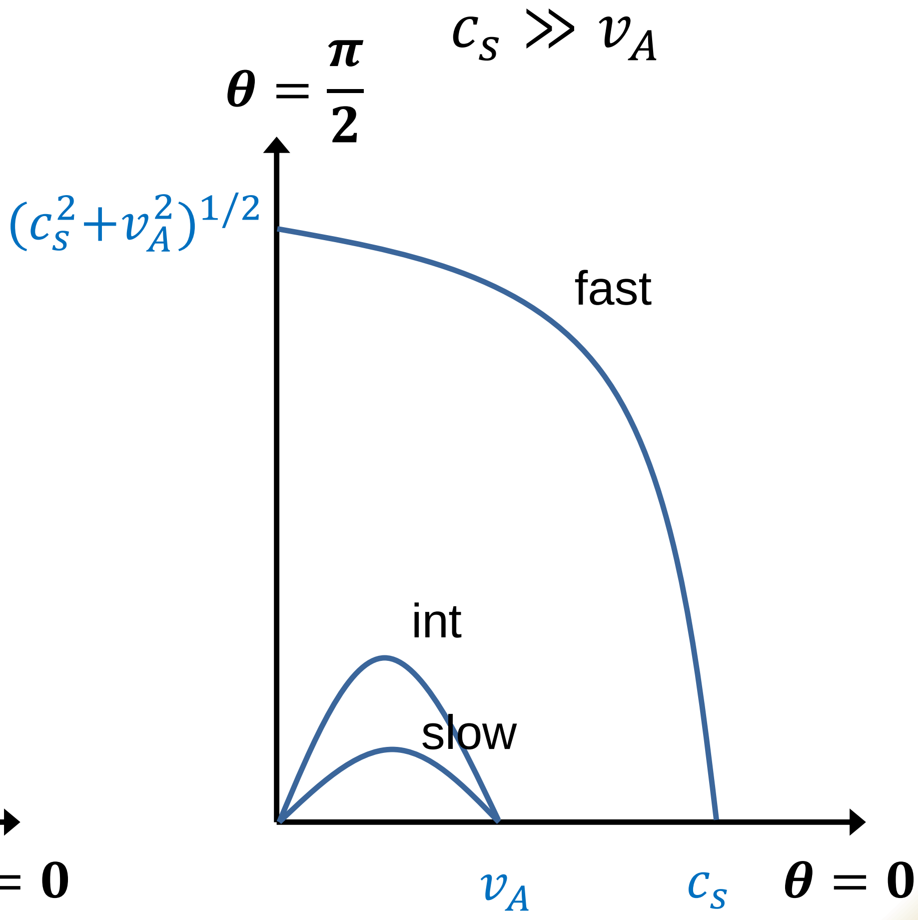
MHD waves a summary





MHD waves a summary





Summary of waves in plasmas

- Langmuir wave
  - Electron and Ions are cold
  - Electrons are not cold
  - Use as a diagnostic
- Waves can exchanges energy with particles and vice-versa
- MHD waves
  - Shear Alfven wave
  - Acoustic Wave
  - Magnetosonic Wave