Extreme Light-Matter Interactions Laboratory

Department of Mechanical and Aerospace Engineering, Princeton University, NJ, USA

Relativistic Laser Plasmas

Julia Mikhailova

Mechanical and Aerospace Engineering

Princeton University

j.mikhailova@princeton.edu

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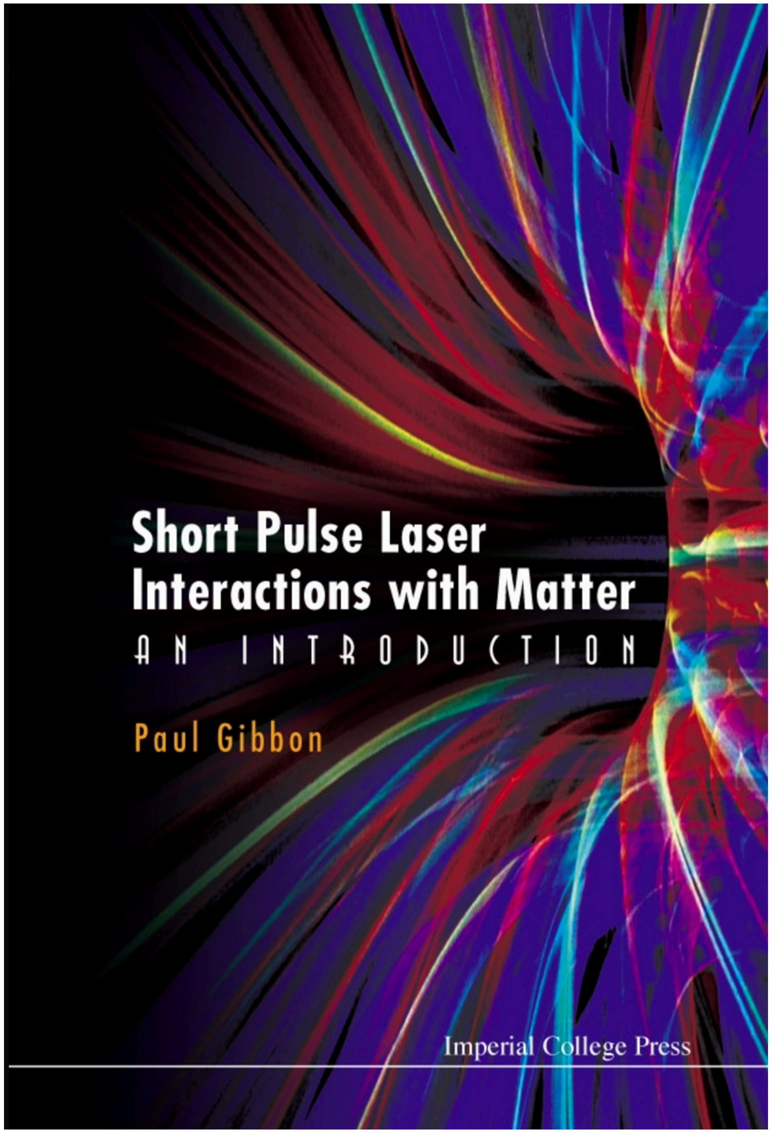
Textbook

Paul Gibbon,
"Short Pulse Laser Interactions with Matter", Imperial College Press, London U.K. (2005)



http://www.fz-juelich.de/ias/jsc/EN/AboutUs/Organisation/ComputationalScience/Simlabs/slpp/Teaching/SPLIM/_node.html

High-field science on a tabletop

Fundamental questions of high-field science and astrophysics

- How does matter behave in extreme fields?
- What is the physics of stellar atmospheres, relativistic astrophysical jets, supernovae, etc?
- Can we observe the effects of radiation reaction and nonlinear QED in plasma dynamics?
- Can we generate electron-positron pairs from the Dirac Sea in vacuum by intense photon interactions?

Applications

- Can we shrink kilometer-scale accelerator/synchrotron facilities to fit on a laboratory tabletop?
- Ultrafast sources of intense light and high-energy particles
  - Ultra-intense laser light (Raman/Brillouin amplification in plasmas)
  - Attosecond (10^{-18} s) x-ray pulses for time-resolved studies
  - High-energy (GeV) electron beams (Laser Wakefield Acceleration)
  - High-energy ion/proton beams (Solid foils)
- Laser Fusion

Laser fields

Lasers are the most intense sources of electromagnetic radiation available in laboratory conditions for high-field science experiments.

Power

$$P = \frac{energy}{time}$$

Irradiance or intensity

$$I = \frac{energy}{time \times area}$$

$$I_L = \frac{1}{2}\varepsilon_0 c E_0^2$$

- *SI* units of *I* are W/m^2, but W/cm^2 is used in laser science.

- Intensity defines the fields and e/m force.

$I = 10^{20}$ W/cm^2 ➔ $E = 3\times10^{13}$ V/m, $B = 10^5$ T

E-field is 10^5 times higher than in conventional RF accelerators

Sunlight near Earth surface: $I \approx 0.1$ W/cm^2

- This laser light converts cold target matter (gas, liquid, solid) almost instantaneously into plasma and drives huge currents.

- Lasers of similar powers may aim at very different experiments:

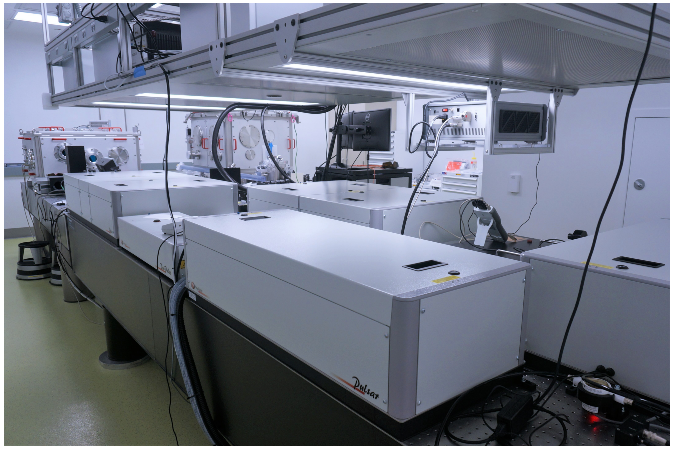
Image:LLNL

NIF: 1.8 MJ (192 beams at 351nm), ns, ~mm

$P \sim 500\text{TW} \Rightarrow I < 10^{16}\ \text{W/cm}^2$

Rep.rate – one shot per day

Laser diode chip ≈ less than 1 mm



Princeton: 500 mJ (800nm), 25 fs, few μm spot size

$P \sim 20\ \text{TW} \Rightarrow I > 10^{19}\ \text{W/cm}^2$

Rep.rate – 10 Hz

Ultrafast light sources

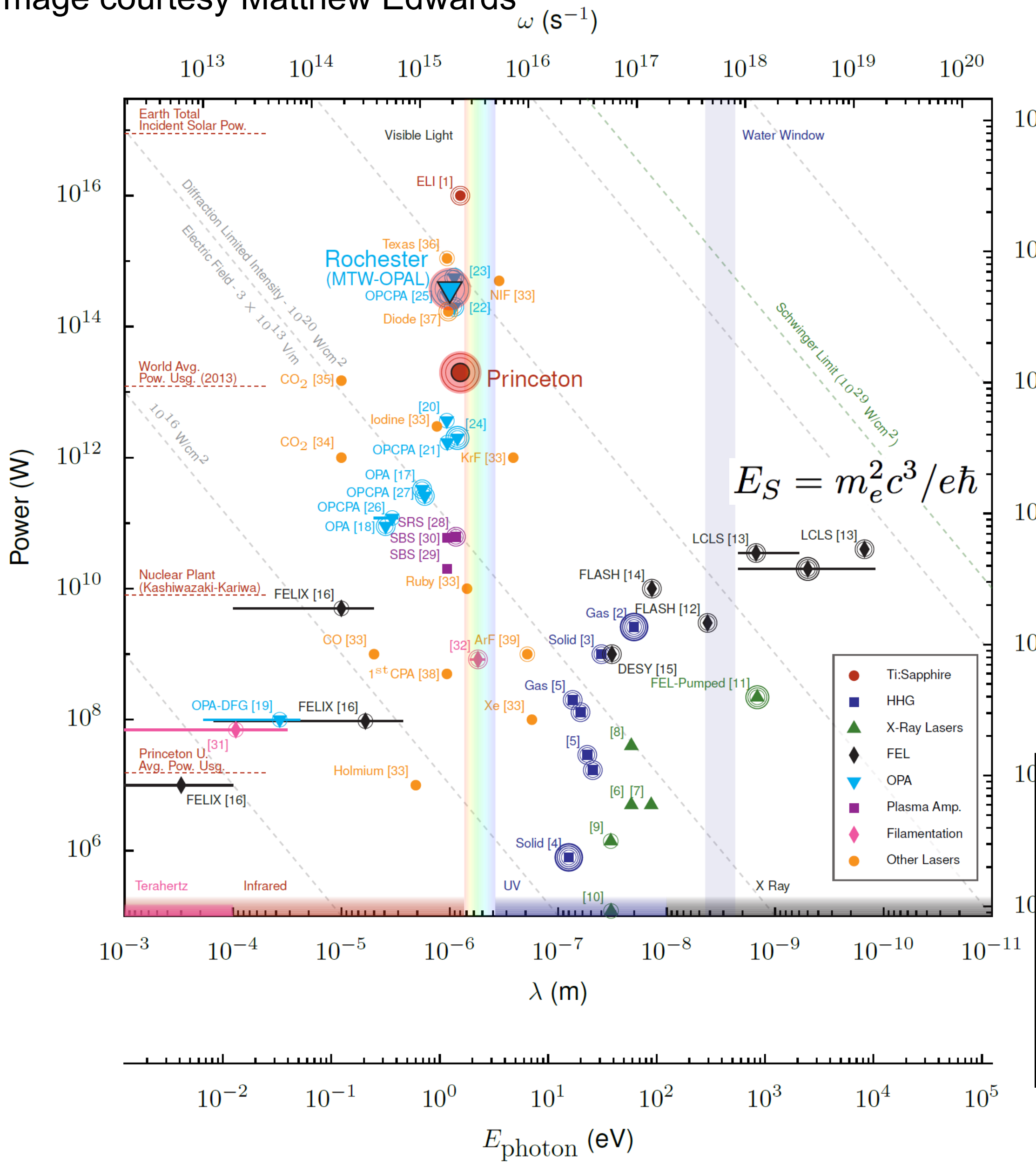
○ < 1 ps
◎ < 100 fs
◎ < 10 fs
◎ < 1 fs

ps $=10^{-12}$ s

fs $=10^{-15}$ s

as $=10^{-18}$ s

Image courtesy Matthew Edwards



The shortest optical pulse generated is single-cycle.
(tiny energy,
yet to be demonstrated on a terawatt scale)

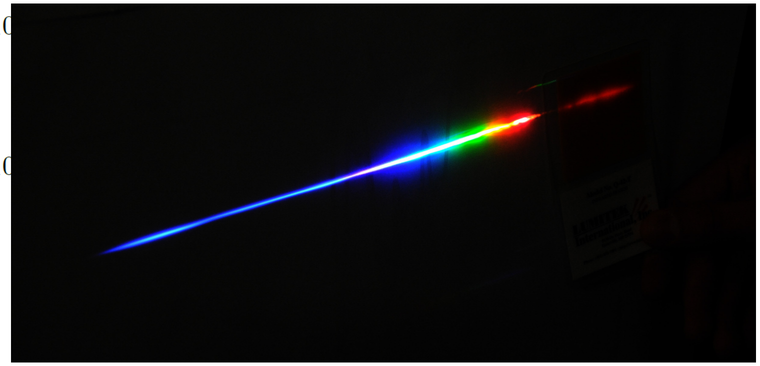
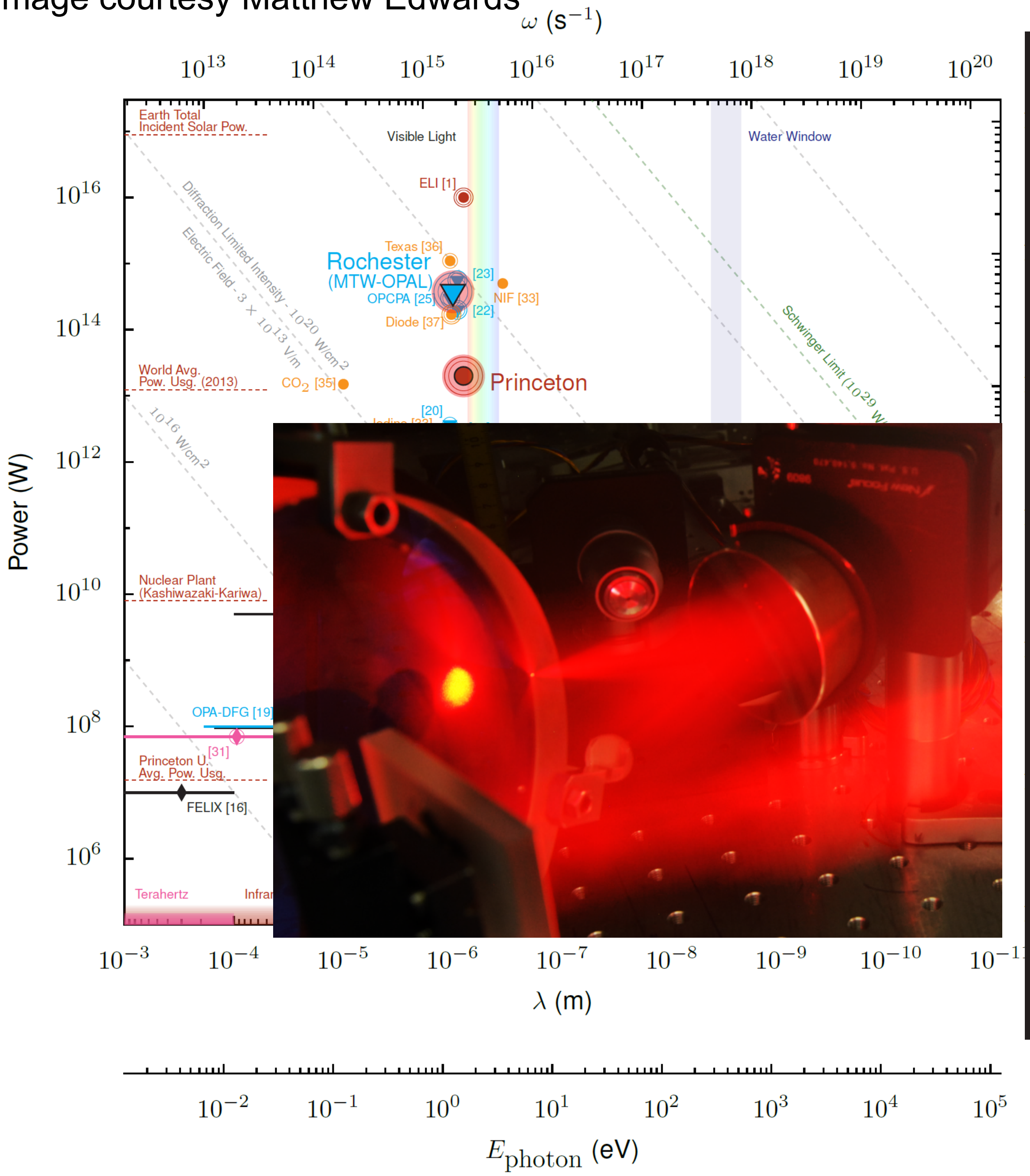


Image: MPQ

Ultrafast light sources

Image courtesy Matthew Edwards



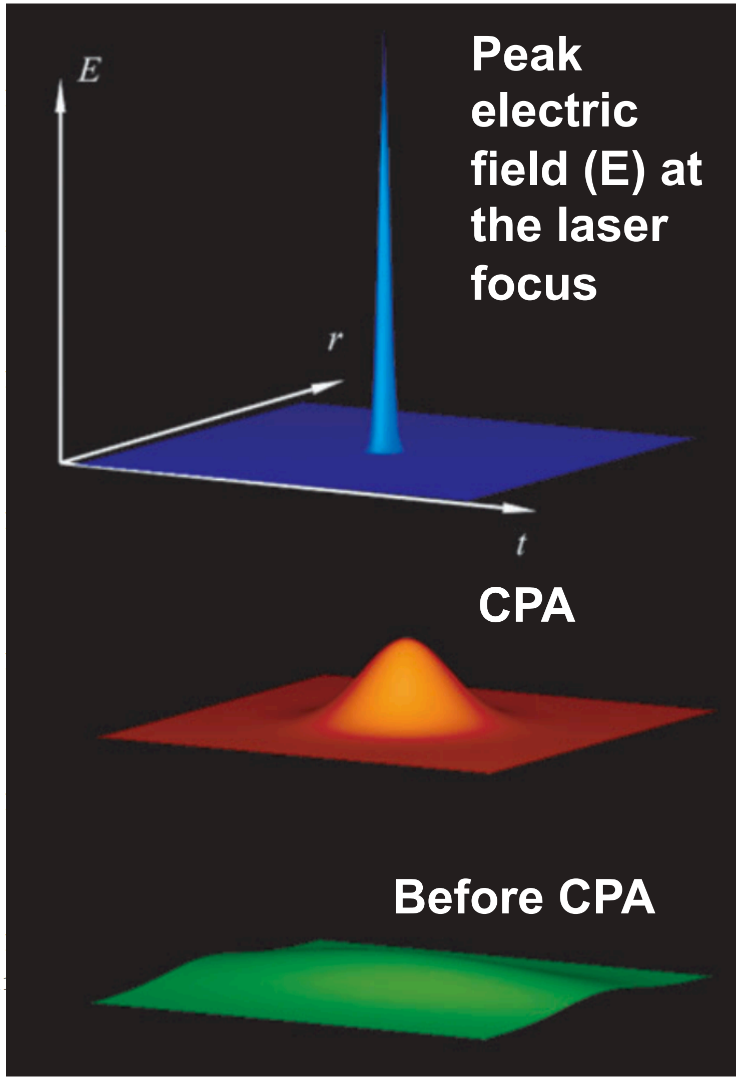


Image: D. Umstadter, J. Phys. D: Appl. Phys. 36, R151, 2003

High-Power Lasers

High power from high energy

– Flash-lamp-pumped glass-type, wavelength around 1μm (or 2w, 3w)

– Energies 10 - 1000 Joules

– Durations > 500 fs

– Titan (LLNL), Trident (LANL), MTW (LLE) and others

– Repetition rate < few shots per day

High power from short pulse durations

– Laser-pumped Ti:Sapphire, wavelength = 800 nm

- OPCPA, broadband 650-1100nm

– Energies 0.1 – 100 Joules

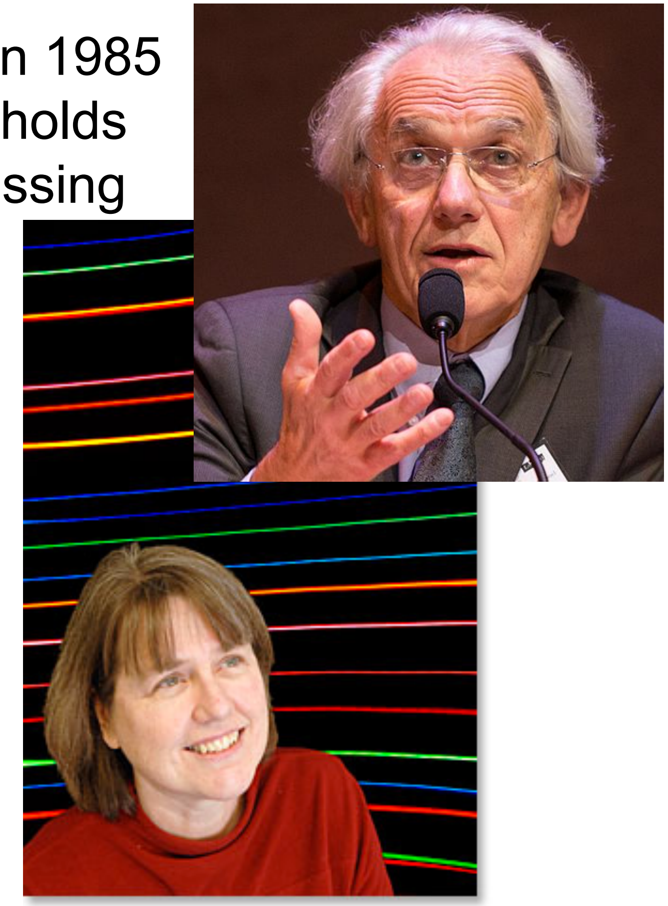
– Durations ~25 – 200 fs

– Hercules (UMichigan), Scarlet (OSU), Callisto (LLNL), MTW-OPAL (LLE), Princeton, etc

– Repetition rate - up to 10 Hz

Chirped pulse amplification

- Invented by Gerard Mourou and Donna Strickland in 1985
- Way of increasing intensities beyond damage thresholds amplifying longer pulses (100 ps – 1 ns) and compressing them to <1ps after amplification.



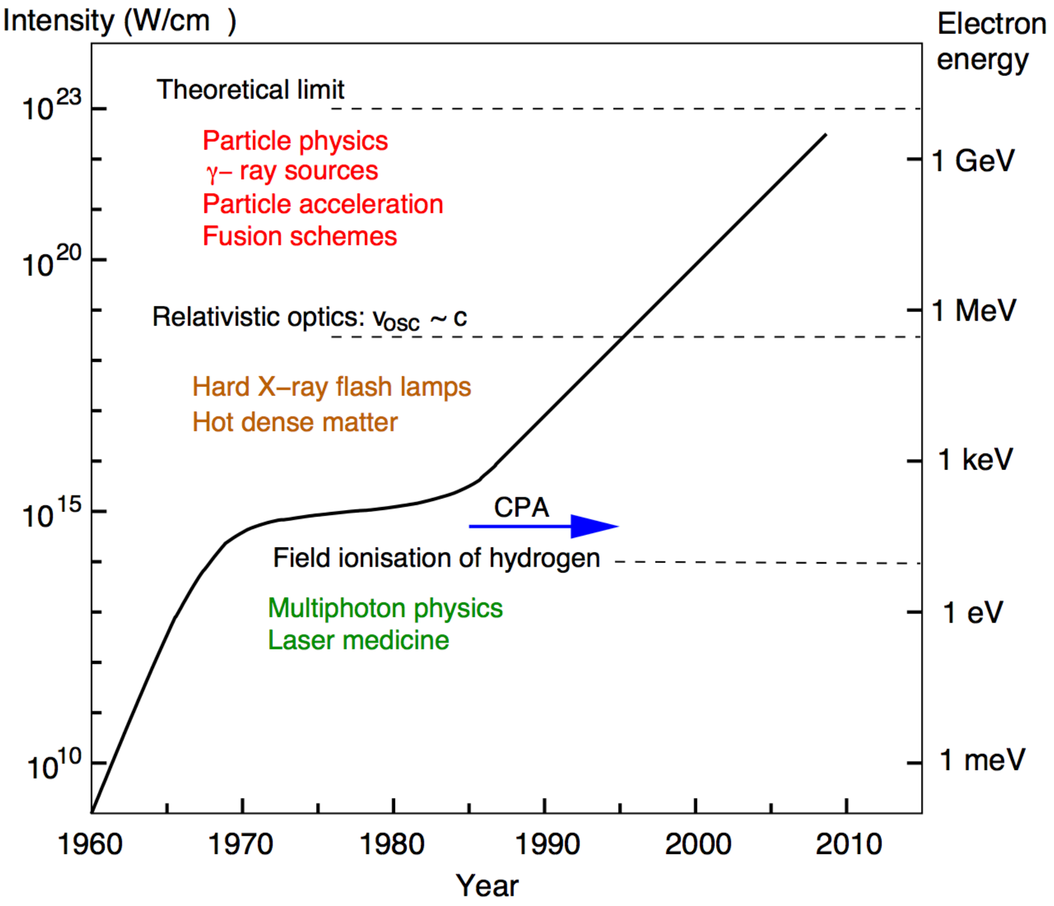
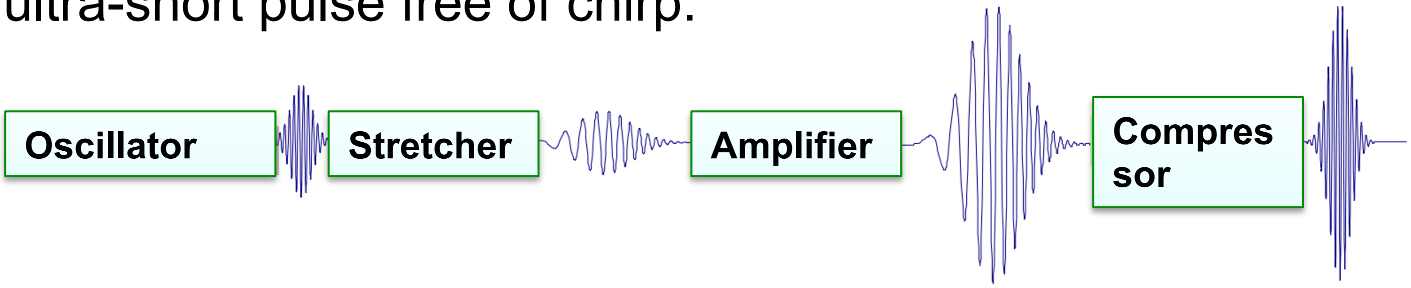


Figure: Paul Gibbon "Short Pulse Laser Interactions with Matter", Lectures

Chirped pulse amplification

- Invented by Gerard Mourou and Donna Strickland in 1985
- Way of increasing intensities beyond damage thresholds amplifying longer pulses (100 ps – 1 ns) and compressing them to <1ps after amplification.

1. **Oscillator:** generates fs, broadband, low-energy pulse
2. **Stretcher:** converts fs pulse to >100 ps. Creates different optical paths for each wavelength of the spectrum. Produces a “chirped” pulse.
3. **Amplifier:** increase the pulse energy by a factor of 10^{7}–10^{10}
4. **Compressor:** performs optical inverse of the stretcher to deliver an amplified fs pulse. The result is a high-intensity ultra-short pulse free of chirp.



Oscillator **Stretcher** **Amplifier** **Compressor**

Chirped pulse amplification

Typical femtosecond TW laser system

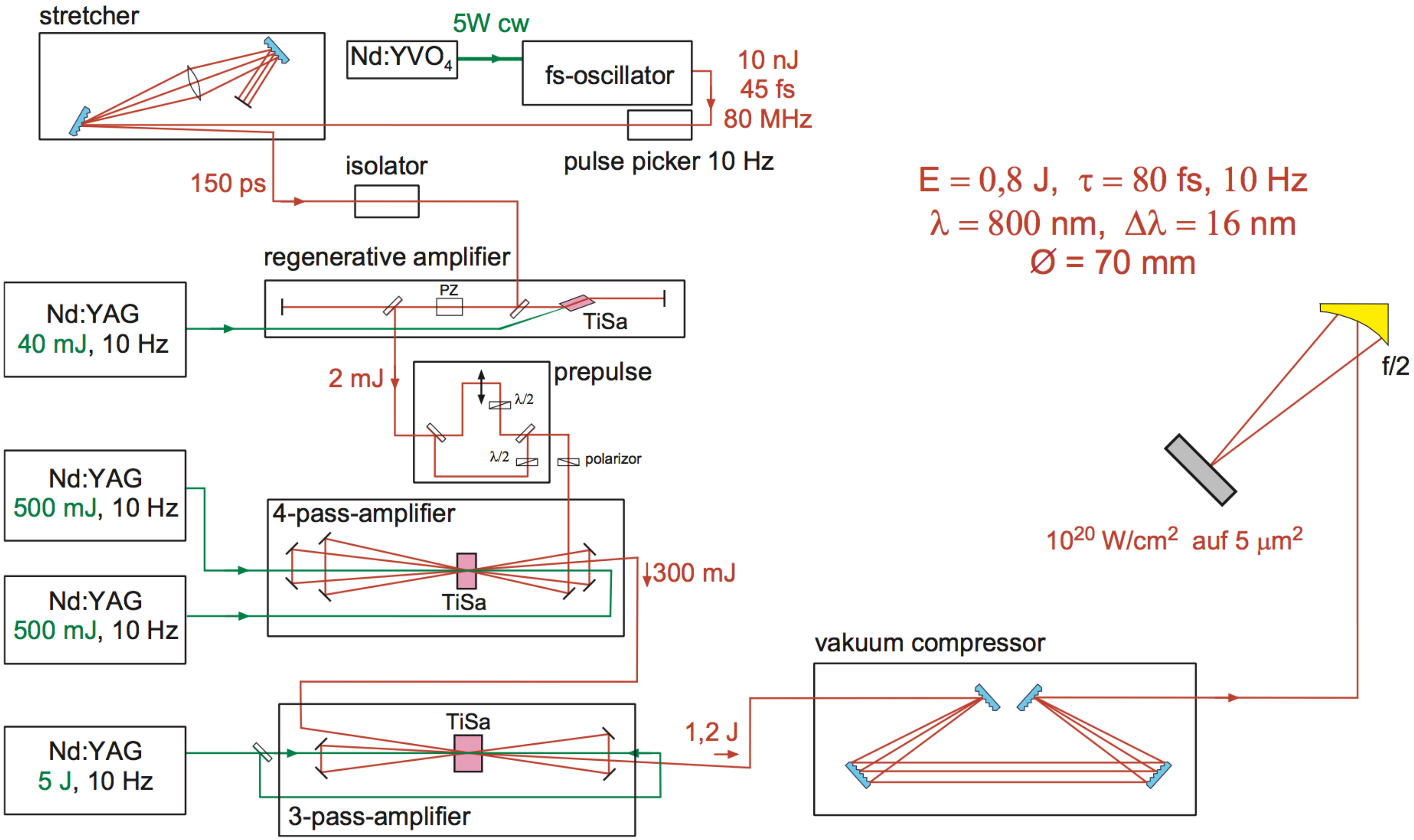


Image from Paul Gibbon "Short Pulse Laser Interactions with Matter", Lectures

Single electron in a plane wave

Assumptions:

- Electron is classical (no spin)
- Neglect back reaction force of electron’s radiation
- Electron and wave are in vacuum, there are no other charges, potential electric field = 0

Charge

Electric field

Electron velocity

Magnetic field

Relativistic momentum

$$\frac{dp}{dt} = -e\mathrm{E} - \frac{e}{c}[v \times \mathrm{H}]$$

Speed of light

$$\frac{dr}{dt} = v = \frac{p}{\gamma m}$$

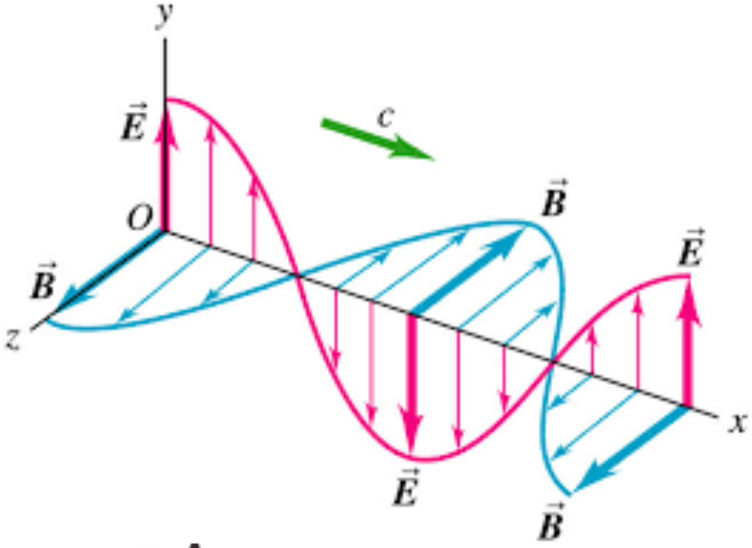
Lorentz factor

$$\gamma = 1/\sqrt{1-(v/c)^2} = \sqrt{1+(p/mc)^2}$$

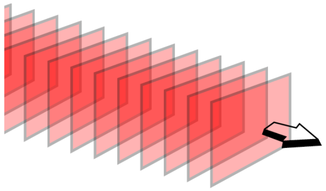
Rest mass

Relativistic mass - the mass of an electron in motion

Plane electromagnetic wave



$$\frac{\partial^2 E}{\partial x^2}-\frac{1}{c^2}\frac{\partial^2 E}{\partial t^2}=0$$

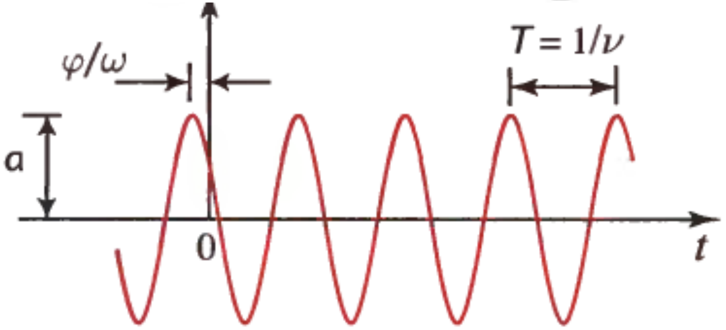


Sinusoidal plane wave is propagating along x-axis, its wavefronts are parallel planes.

$$E(x,t)=E(t-x/c)+E(t+x/c)$$

$$\longrightarrow \qquad \longleftarrow$$

$$x$$



It is convenient to use the field vector potential:

$$A(t-x/c)$$

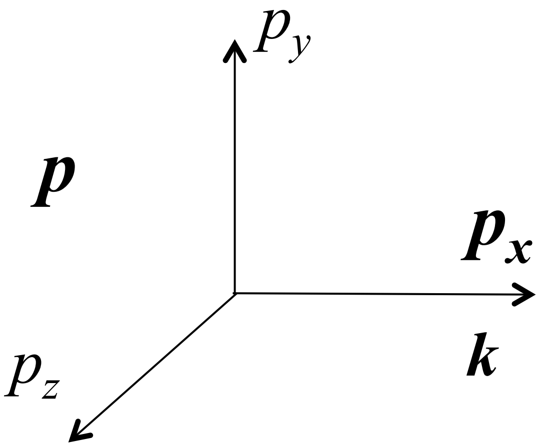
$$\mathrm{E}=-\partial A/c\partial t \qquad B=\nabla\times A$$

A plane wave is useful, because:

- Any function, which is smooth and rapidly decreasing in infinity, can be represented as a superposition of plane waves - Fourier transform in time and space.

- If a wavefront curvature radius of an arbitrary wave >> than linear dimensions of a given volume, then inside this volume the wave is almost plane. E.g. atomic radius is about 0.1 nm << wavelength of visible light (400–700 nm).

Single electron in a plane wave.
Low field - Non-relativistic motion



$$\vec{E}(\vec{r},t) = \vec{E}_o(\vec{r})\, cos\omega t$$

$$\frac{dp}{dt} = -e\mathrm{E} - \frac{e}{c}\xcancel{[v\times\mathrm{H}]}cos\omega t$$

Transverse motion

$$m\,\frac{d}{dt}\vec{v}^{(1)} = -e\vec{E}_o cos\omega t$$

$$\vec{v}^{(1)} = \frac{e\vec{E}_o sin\omega t}{m\omega}$$

$$\vec{r}^{(1)} = \frac{e\vec{E}_o cos\omega t}{m\omega^2}$$

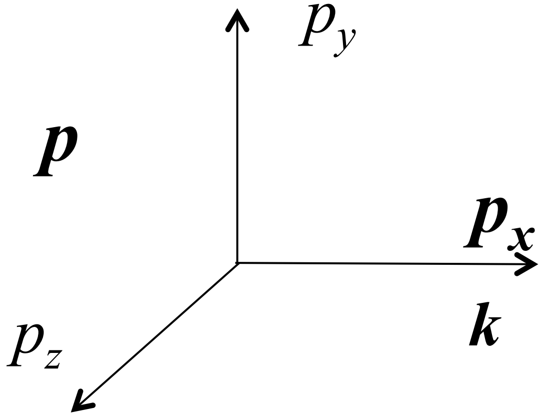
Longitudinal drift motion

There is no drift in the direction of laser propagation

Electron oscillates at the laser frequency in the direction parallel to the E field vector.

The amplitude of electron oscillation is less than the light wavelength.

Single electron in a plane wave. High field - Relativistic motion



$$\frac{dp}{dt} = -e\mathrm{E} - \frac{e}{c}[v \times \mathrm{H}]$$

$$A(t - x/c)$$

$$\mathrm{E} = -\partial A / c\partial t \qquad B = \nabla \times A$$

Transverse motion

$$p = f + eA/c$$

Longitudinal drift motion

$$p_x = p_{x0} + \frac{feA}{E_0 - cp_{x0}} + \frac{e^2A^2}{2c(E_0 - cp_{x0})}$$

***f* = *const*, *p*$_{x0}$ =const, *E*$_0$=const**
are the initial electron momenta and
energy before the interaction with a wave.

This drift motion originates from

$$v \times B \propto E^2\hat{k}$$

Single electron in a plane wave. High field - Relativistic motion

Note:

Transverse motion

$$p \sim A$$

Longitudinal drift motion

$$p_x \sim A^2$$

For small A, $p >> p_x$. For large A, $p_x >> p$.

$$E = E_0 + c\frac{efA}{E_0 - cp_{x0}} + \frac{e^2A^2}{2(E_0 - cp_{x0})}$$

When the field is vanishing (A=0), the electron momenta and energy return to their initial values (f, p_{x0}, E_0).

Electron's coordinates can be found in the parametric form:

$$r(\xi) = \frac{c^2}{E_0 - cp_{x0}}\int(f + \frac{e}{c}A)d\xi$$

$$x(\xi) = \frac{c^2}{E_0 - cp_{x0}}\int p_x(\xi)d\xi$$

$$t(\xi) = \xi + x(\xi)/c$$

Special case

Electron at rest before the wave arrival: f=0, p_{x0}=0

Transverse motion

$$p = \frac{e}{c}A$$

$$v = \frac{1}{\gamma m}\frac{e}{c}A$$

Longitudinal drift motion

$$p_x = \frac{e^2A^2}{2mc^3}$$

$$E = mc^2 + \frac{e^2A^2}{2mc^2}$$

$$v_x = c\frac{\gamma - 1}{\gamma}$$

$$\gamma = 1 + \frac{e^2A^2}{2m^2c^4}$$

$$A_{rel} = mc^2 / e \qquad A = aA_{rel}$$

$$\gamma = 1 + a^2/2$$

$\boldsymbol{a}$ is a dimensionless figure of merit of the laser vector potential

At $\boldsymbol{a} = \mathbf{1}$, electron kinetic energy $= \dfrac{mc^2}{2}$

Relativistic intensity of light

$$a_0 = A_0/A_{rel} = E_0/E_{rel} = \sqrt{I/I_{rel}}$$

$$A_{rel} = mc^2 / e \qquad E_{rel} = mc\omega/e$$

$$I_L = \frac{1}{2}\varepsilon_0 c E_0^2; \quad \lambda_L = \frac{2\pi c}{\omega}$$

$$a_0 \simeq 0.85(I_{18}\lambda_\mu^2)^{1/2}, \quad I_{18} = \frac{I_L}{10^{18}\ \mathrm{Wcm}^{-2}}; \quad \lambda_\mu = \frac{\lambda_L}{\mu m}$$

$$I_{\mathrm{W/cm^2}} = 1.37 \times 10^{18} \left(\frac{a_0}{\lambda_{\mu\mathrm{m}}}\right)^2$$

$$800\ \mathrm{nm:}\ \ I_{\mathrm{rel}} = 2.14 \times 10^{18}\ \mathrm{W/cm^2}$$

$$10\ \mu\mathrm{m:}\ \ I_{\mathrm{rel}} = 1.37 \times 10^{16}\ \mathrm{W/cm^2}$$

Special case

Electron at rest before the wave arrival: f=0, $p_{x0}=0$

$$A_{rel}=mc^2/e \qquad A=aA_{rel}$$

$$\gamma=1+a^2/2$$

Transverse motion

$$v=c\frac{a}{1+a^2/2}$$

$$v\le c/\sqrt{2}$$

$$v=c/\sqrt{2} \quad a=\sqrt{2}$$

Longitudinal drift motion

$$v_x=c\frac{a^2/2}{1+a^2/2}$$

v_x becomes close to ***c*** with increasing ***a***

Special case

Electron at rest before the wave arrival: f=0, p_{x0}=0

Transverse motion

$$r = \frac{e}{mc}\int_{-\infty}^{\xi} A\,d\xi' + r_0$$

If $A = A_{amp}\sin\xi$, then

$$r \approx r_0 - r_{amp}\cos\xi$$

$$r_{amp} = \frac{\lambda}{2\pi}a_{amp}$$

Longitudinal drift motion

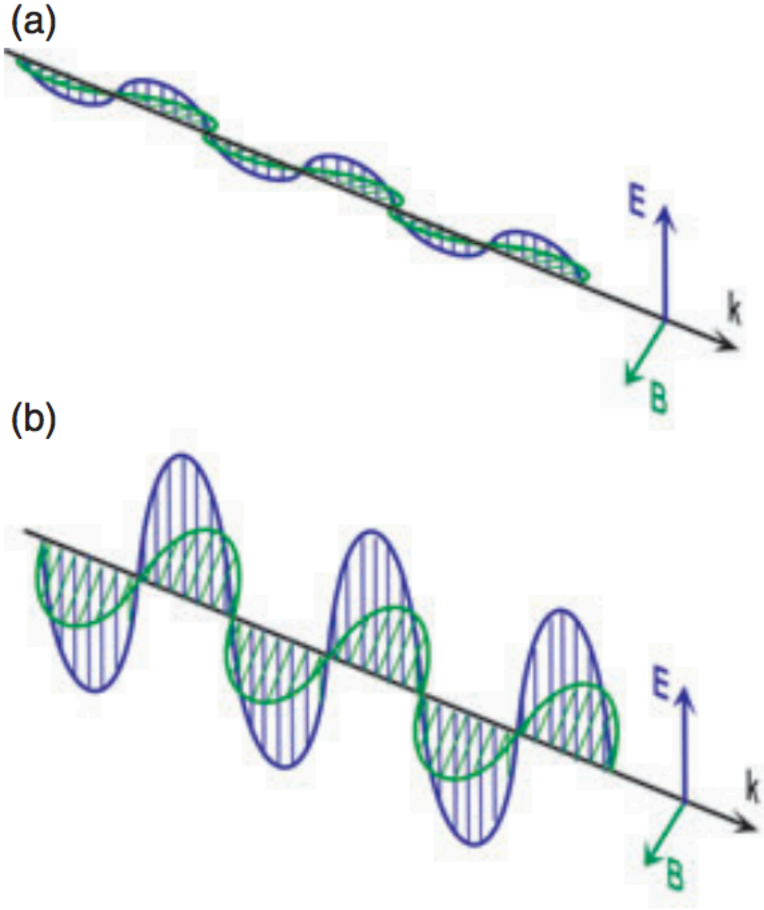
$$x = \frac{e^2}{2m^2c^3}\int_{-\infty}^{\xi} A^2\,d\xi' + x_0$$

$$t = \xi + \frac{e^2}{2m^2c^4}\int_{-\infty}^{\xi} A^2\,d\xi' + x_0/c$$

$$a_{amp} = \frac{A_{amp}}{A_{rel}} = \frac{E_{amp}}{E_{rel}} = \sqrt{I/I_{rel}}$$

The amplitude of electron oscillation can be **much higher than the laser wavelength**.

Single electron in a plane wave



(a) Non-relativistic case: the wave amplitude is small, only the **E-field** acts on the electron, electron oscillates in the direction of the E field at the light's frequency, **velocity << c, displacement amplitude < λ**.
There is no displacement along the wave propagation direction.

(b) Relativistic case: the wave amplitude is very large, **B-field** becomes important, the combined action of the E and B-fields pushes the electron forward. Transverse **velocity is limited by *c*/√2, longitudinal velocity is close to *c***.

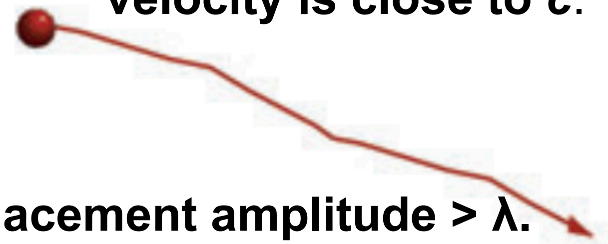
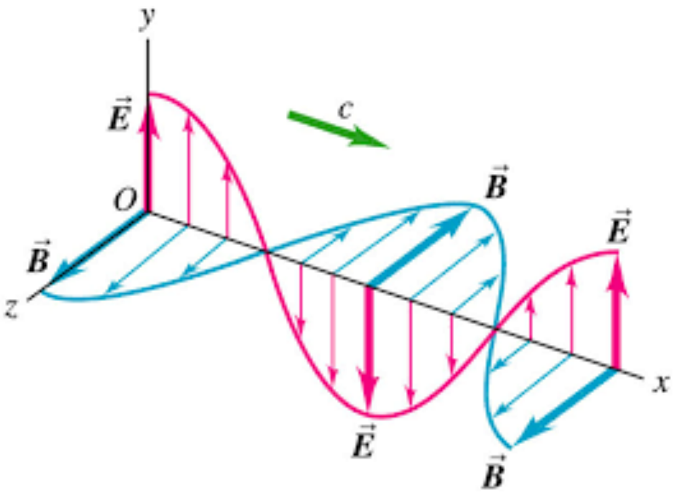
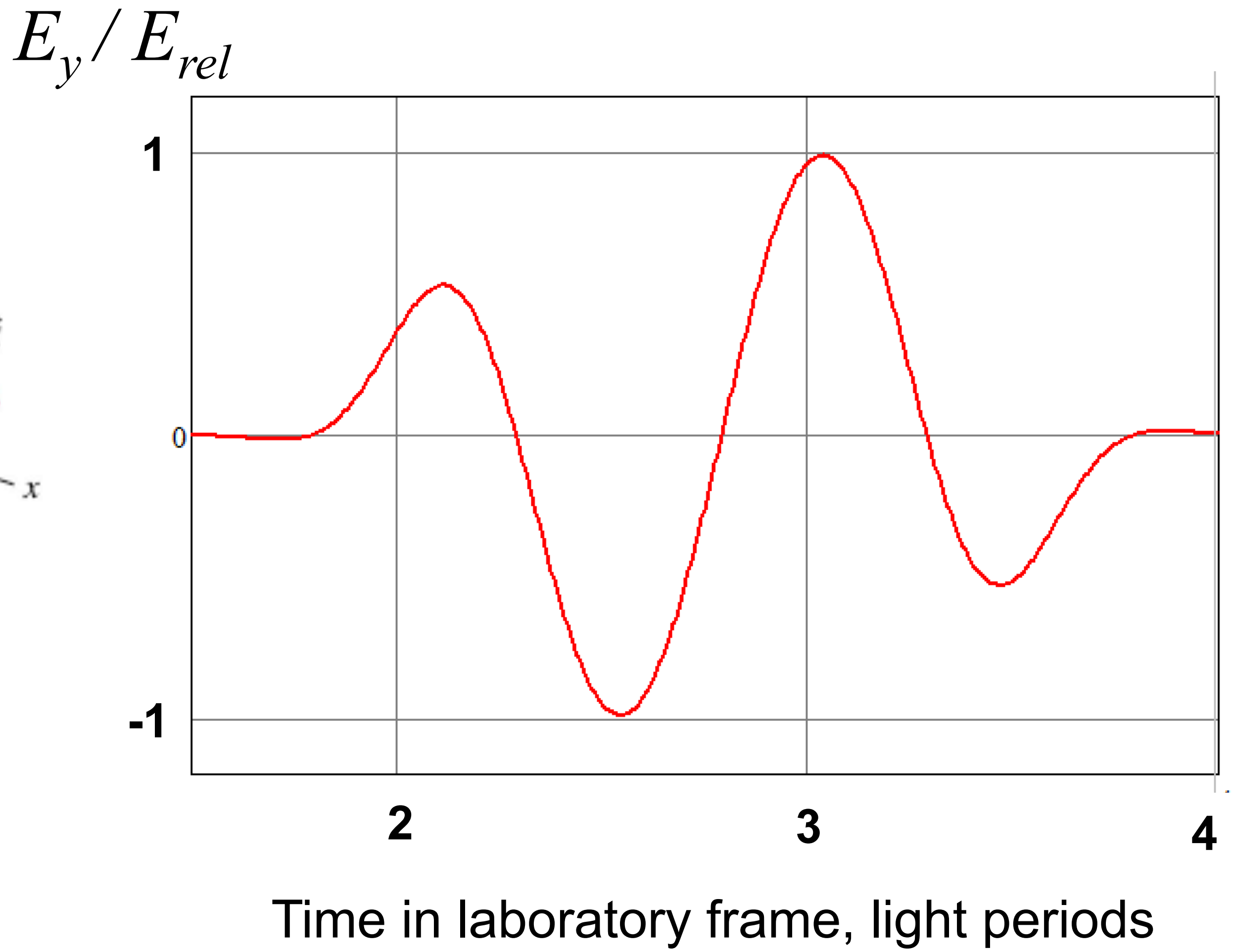


Image: D. Umstadter, J. Phys. D: Appl. Phys. 36, R151, 2003

Displacement amplitude > λ.
Thus, the plane wave approximation may fail for finite-size beams.

Single electron in a laser pulse





Plot the solution of the equation of motion for an electron, which was at rest before the laser pulse arrival.

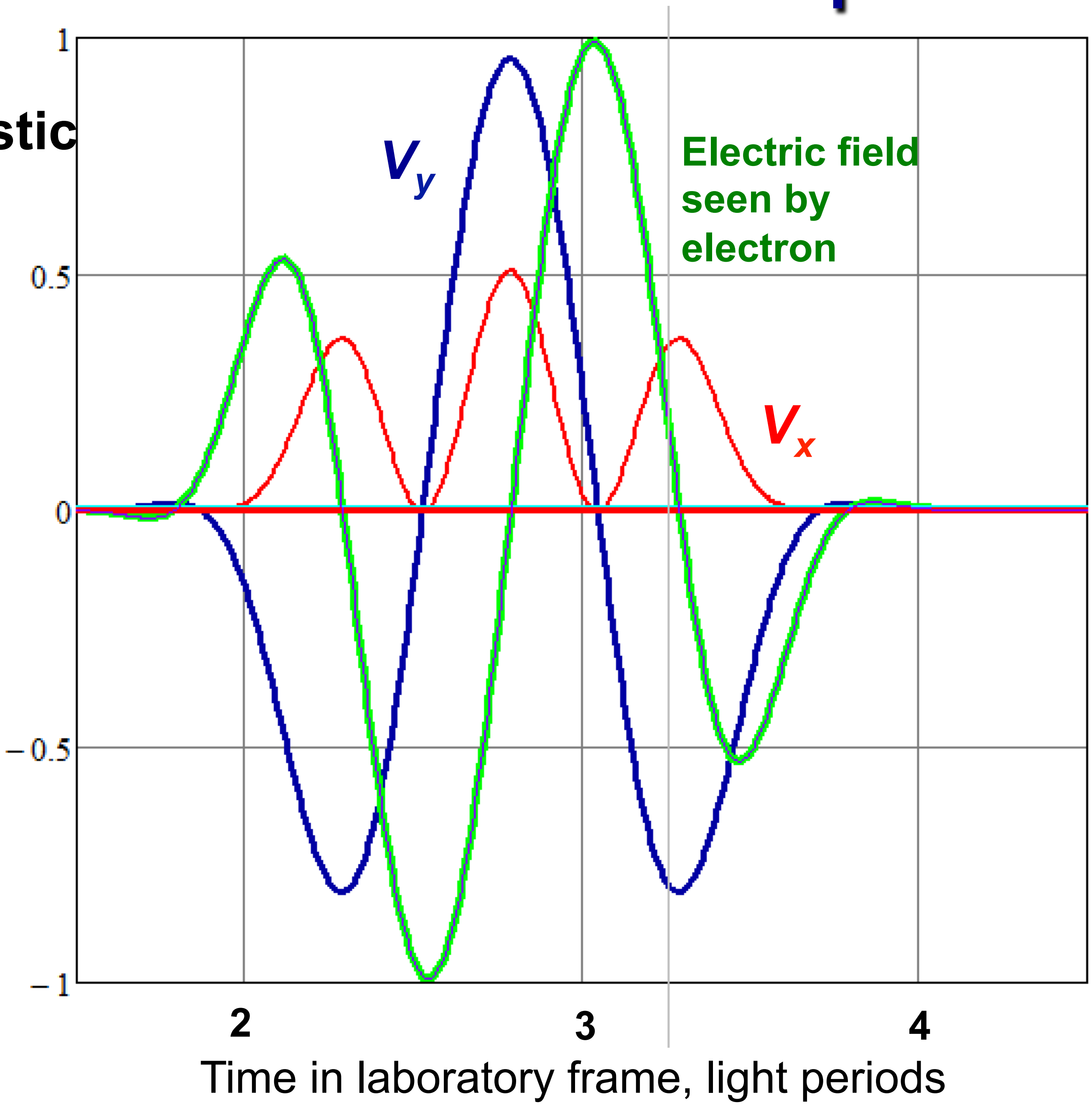
Single electron in a laser pulse

Non-Relativistic case

$a_0=0.0001$

$(V_y/c)\times 10^4$

$(V_x/c)\times 10^8$



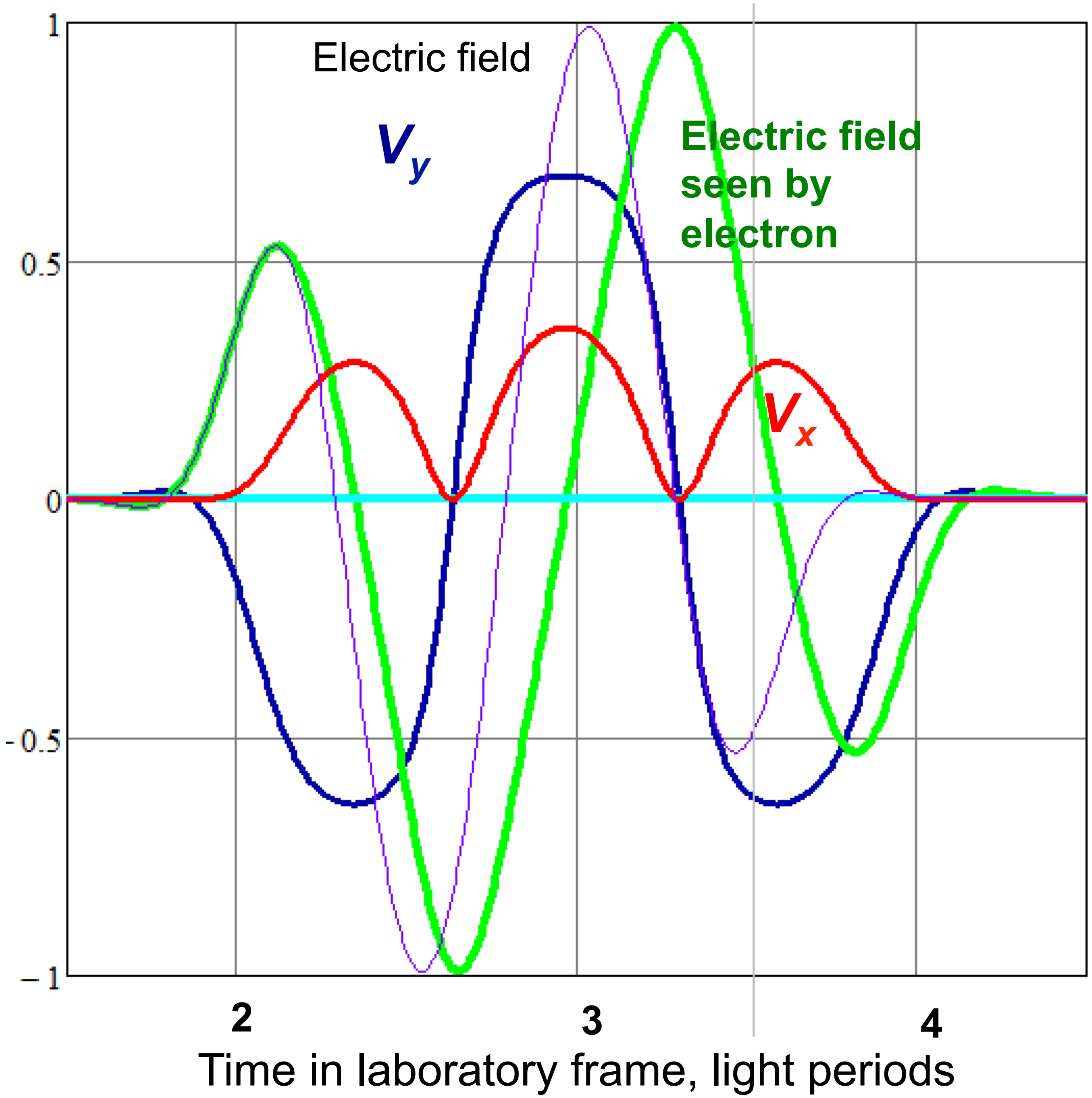
Single electron in a laser pulse

Onset of Relativistic effects

a_0=1.0

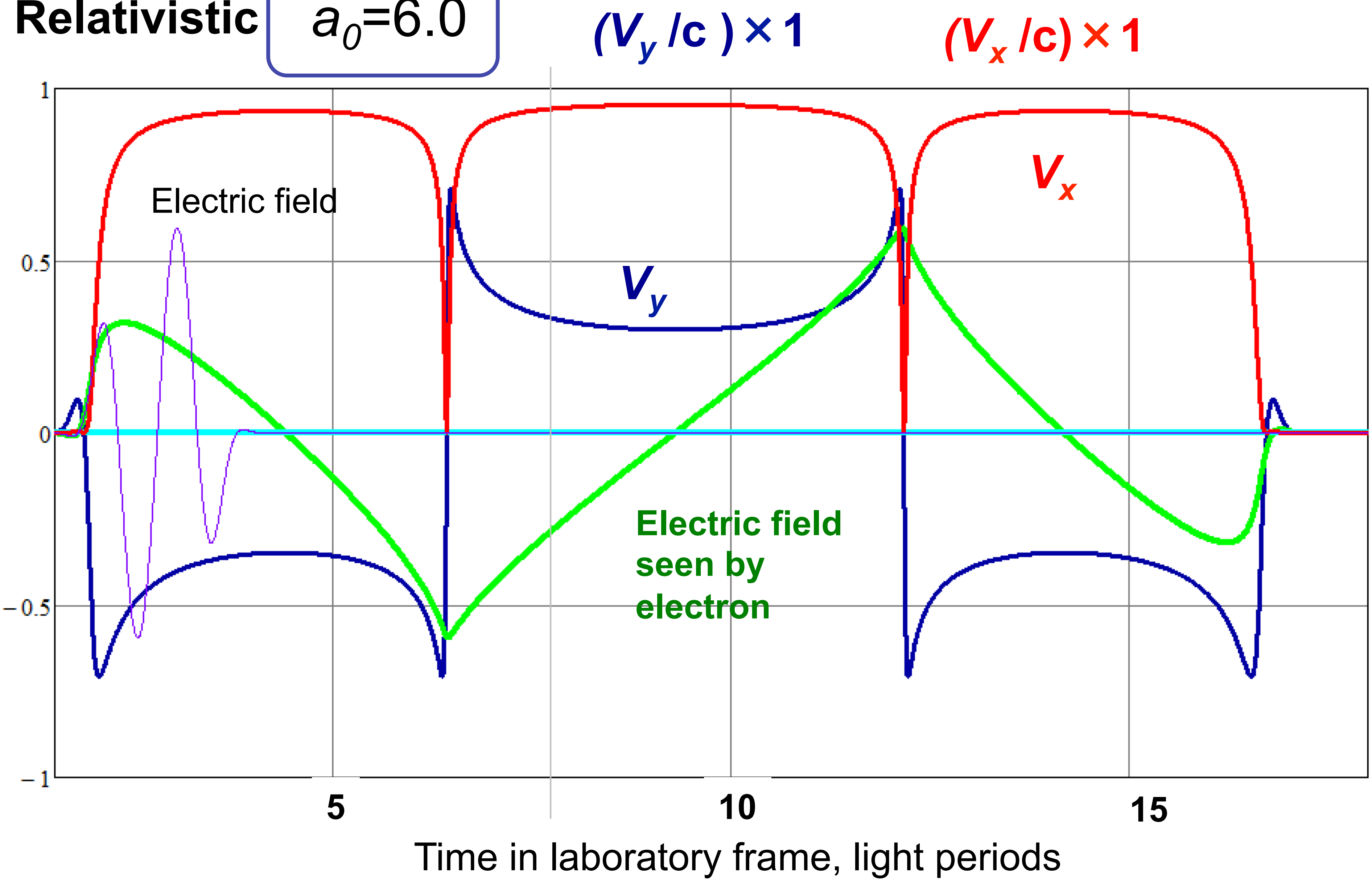
$(V_y/c)\times 1$

$(V_x/c)\times 1$



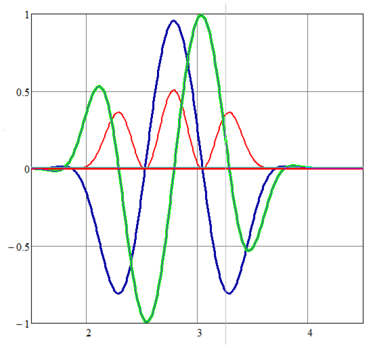
Single electron in a laser pulse

Relativistic $a_0=6.0$

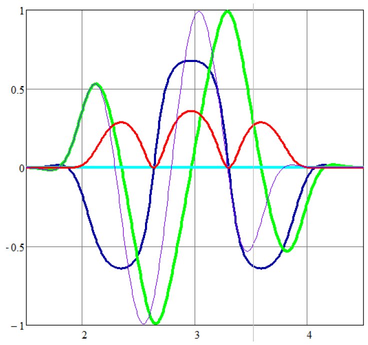


Single electron in a laser pulse

Non-Relativistic

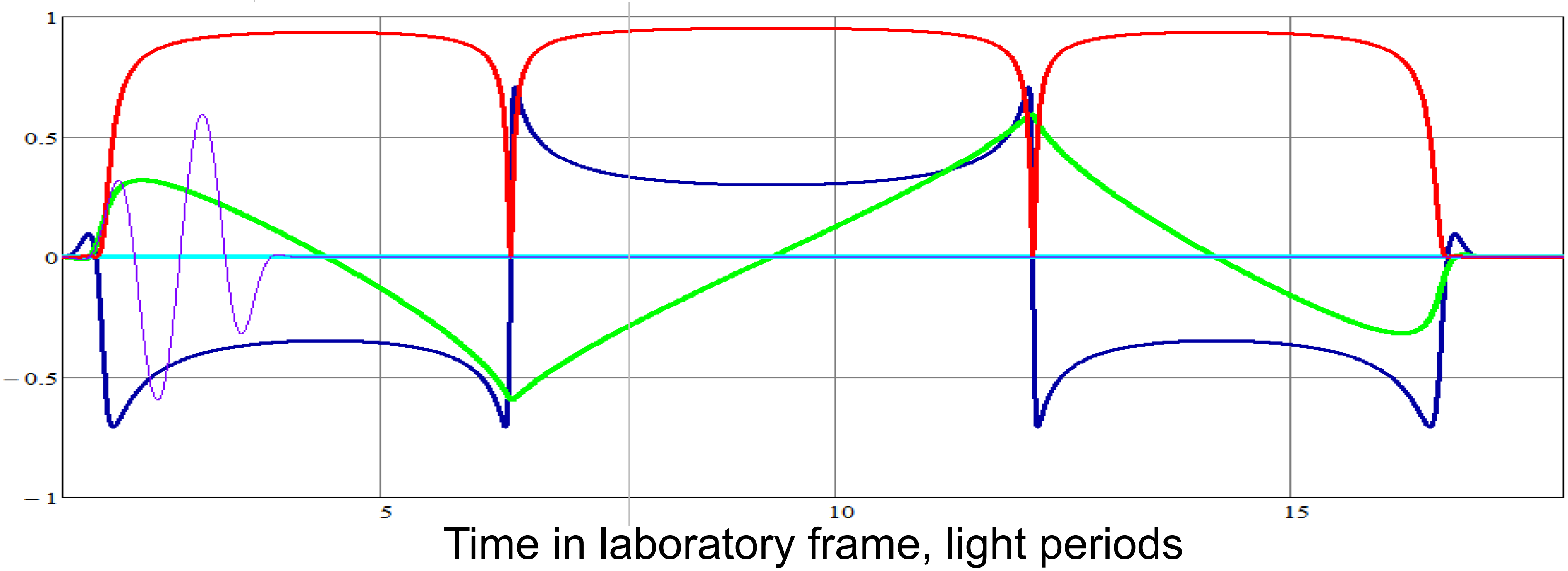


a_0=0.0001

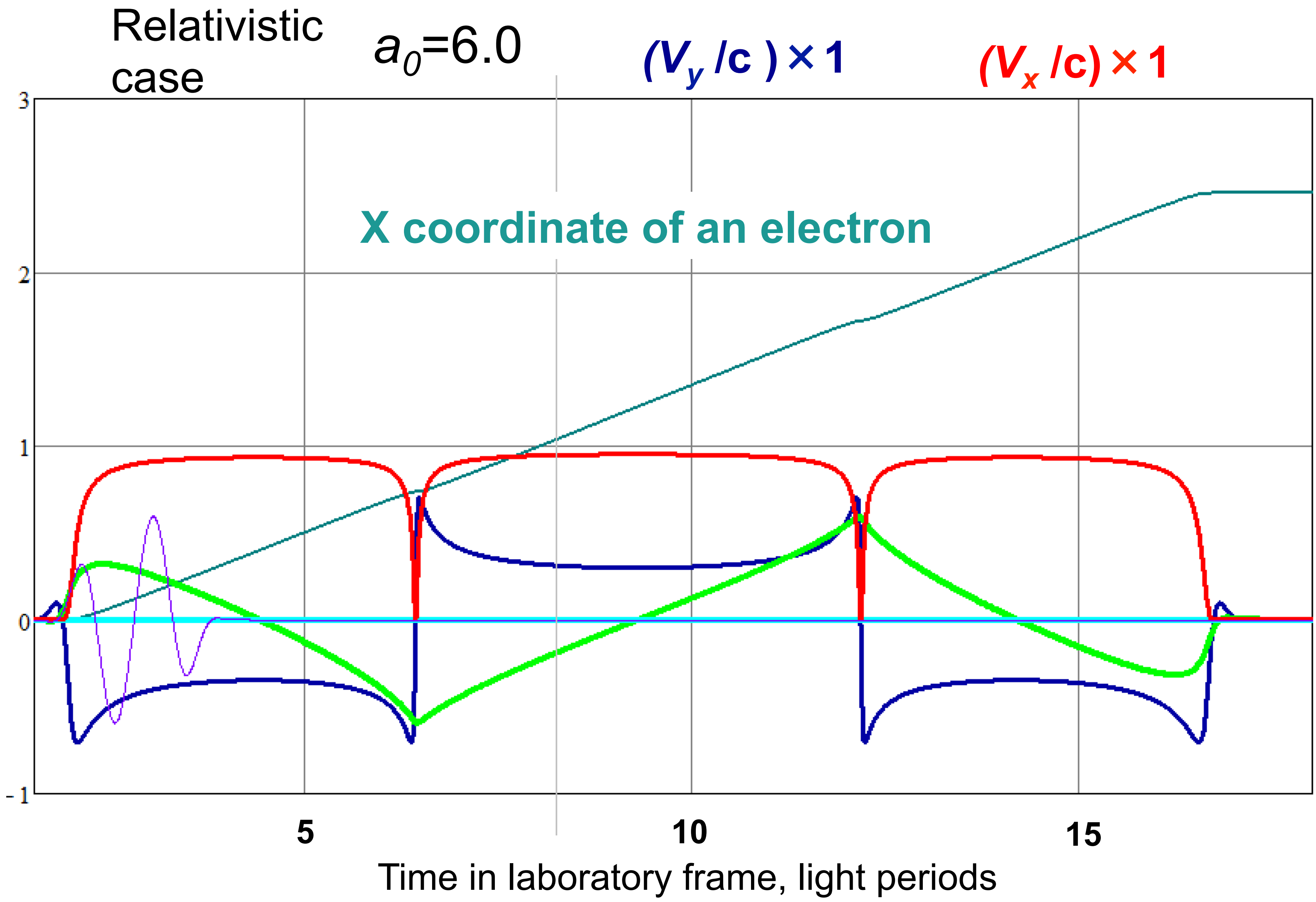


a_0=1.0

a_0=6.0 Relativistic



Single electron in a laser pulse



What if an electron sits in dense plasmas?

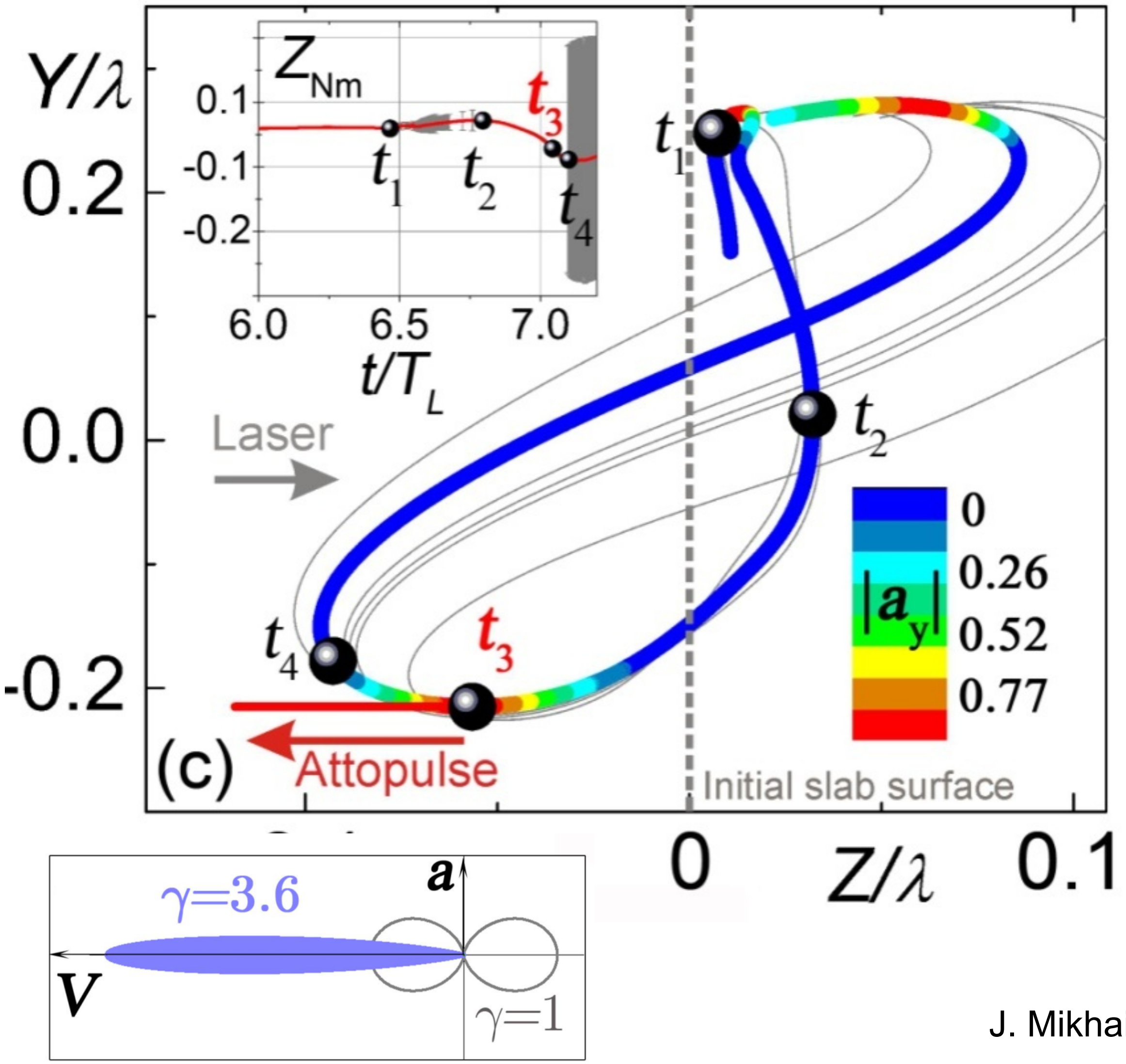
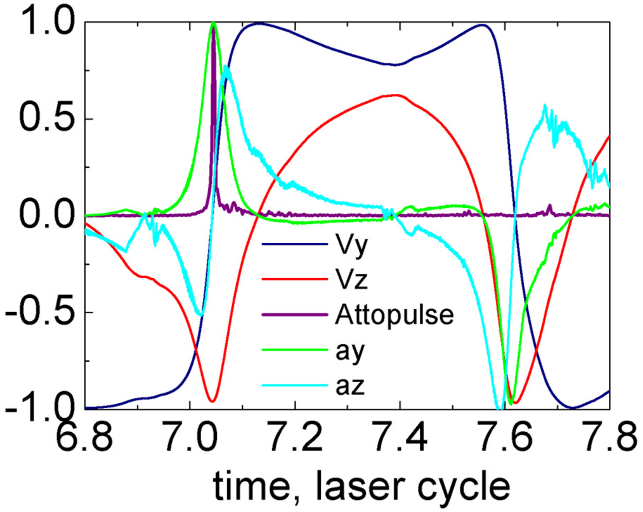


Figure-8
trajectory in
laboratory frame



J. Mikhailova *et al, PRL, 109, 245005 (2012)*

Relativistic Laser - Plasma Interactions

- Cold matter — solid, liquid or gas — is rapidly ionized when subjected to strong laser fields, far exceeding binding fields of atoms.

$$I_a = \frac{\varepsilon_0 c E_a^2}{2} \simeq 3.51 \times 10^{16} \ \mathrm{Wcm}^{-2}$$

laser intensity of $I_L > I_a$ will *guarantee ionization* for any target material, though in fact this can occur well below this threshold value via other ionization mechanisms

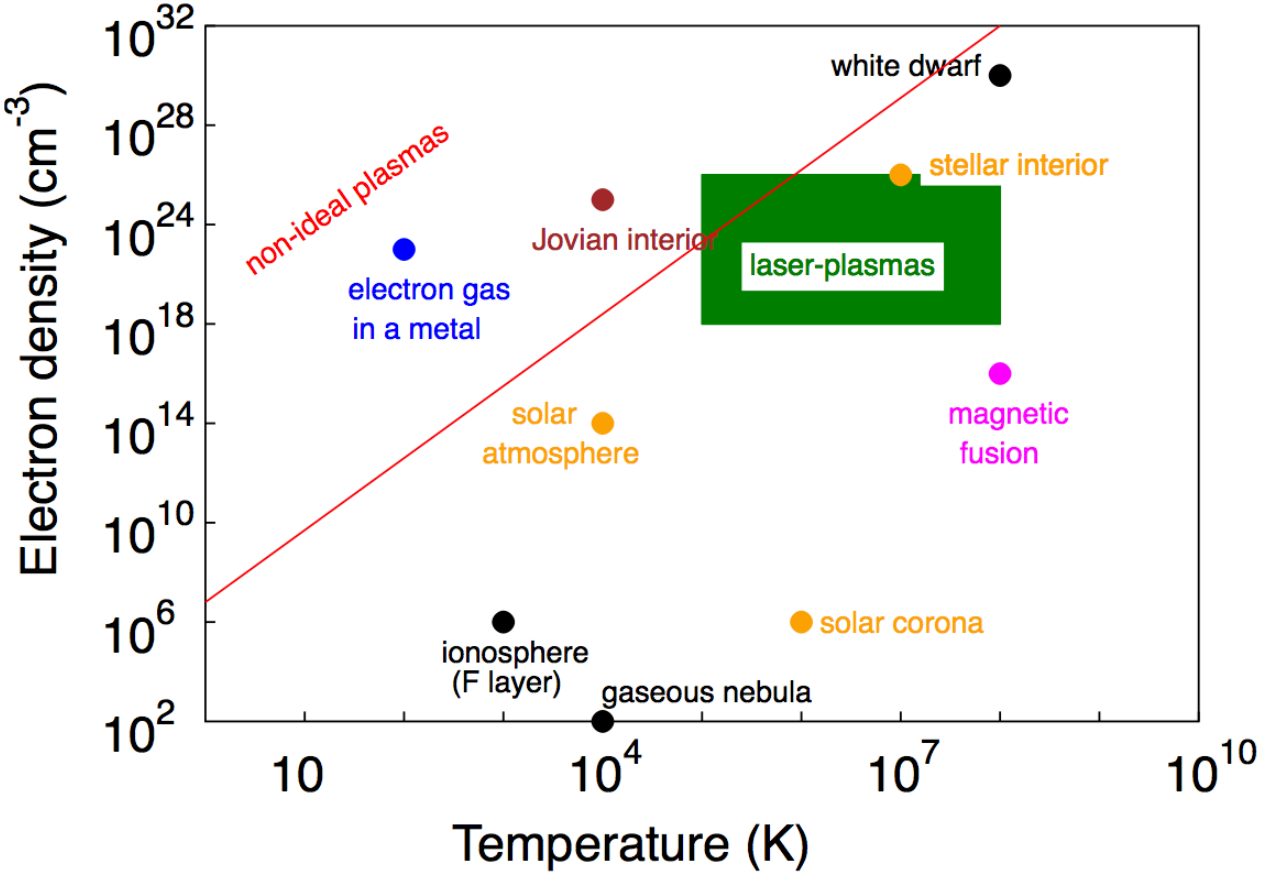
- Electrons released are immediately caught in the laser field, oscillate and drift - field dominated physics.

- Capability for laser-based particle acceleration (GeV), and short-wavelength (x-ray) radiation sources.

Plasma classification: Ideal and Non-ideal

An *ideal* plasma has many particles per Debye sphere:

$$N_D \equiv n_e \frac{4\pi}{3} \lambda_D^3 \gg 1.$$

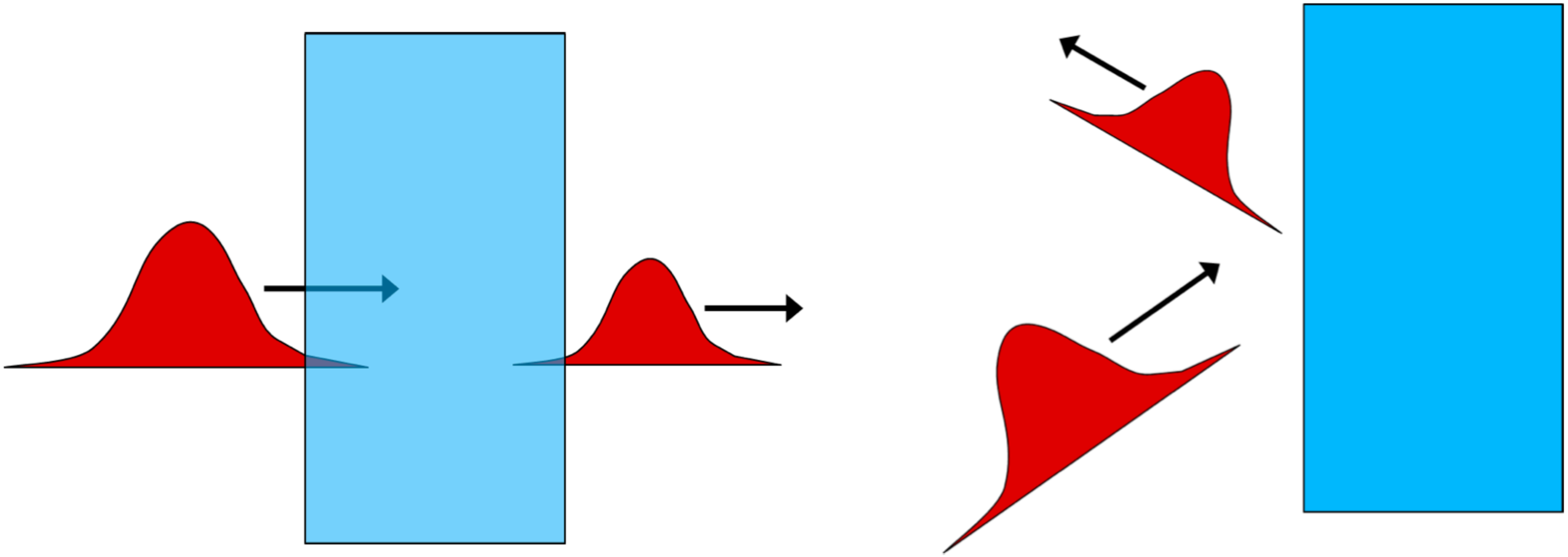


Paul Gibbon "Short Pulse Laser Interactions with Matter", Lectures

Underdense and Overdense

If the plasma response time is shorter than the period of a external electromagnetic field (such as a laser), then this radiation will be *shielded out*.

$$\omega_p \equiv \left(\frac{e^2 n_e}{\varepsilon_0 m_e}\right)^{1/2} \simeq 5.6 \times 10^4 \left(\frac{n_e}{\mathrm{cm}^{-3}}\right)^{1/2} \mathrm{s}^{-1}$$



Underdense, $\omega > \omega_p$:
plasma acts as nonlinear refractive medium

Overdense, $\omega < \omega_p$:
plasma acts like mirror

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Critical plasma density

Normalized target density

$$N = \frac{n_e}{n_c}$$

$$\frac{\omega_p^2}{\omega^2} = \frac{e^2 n_e}{\varepsilon_0 m_e} \cdot \frac{\lambda^2}{4\pi^2 c^2} \quad =1$$

$$\boxed{n_c \simeq 10^{21} \lambda_\mu^{-2} \text{ cm}^{-3}}$$

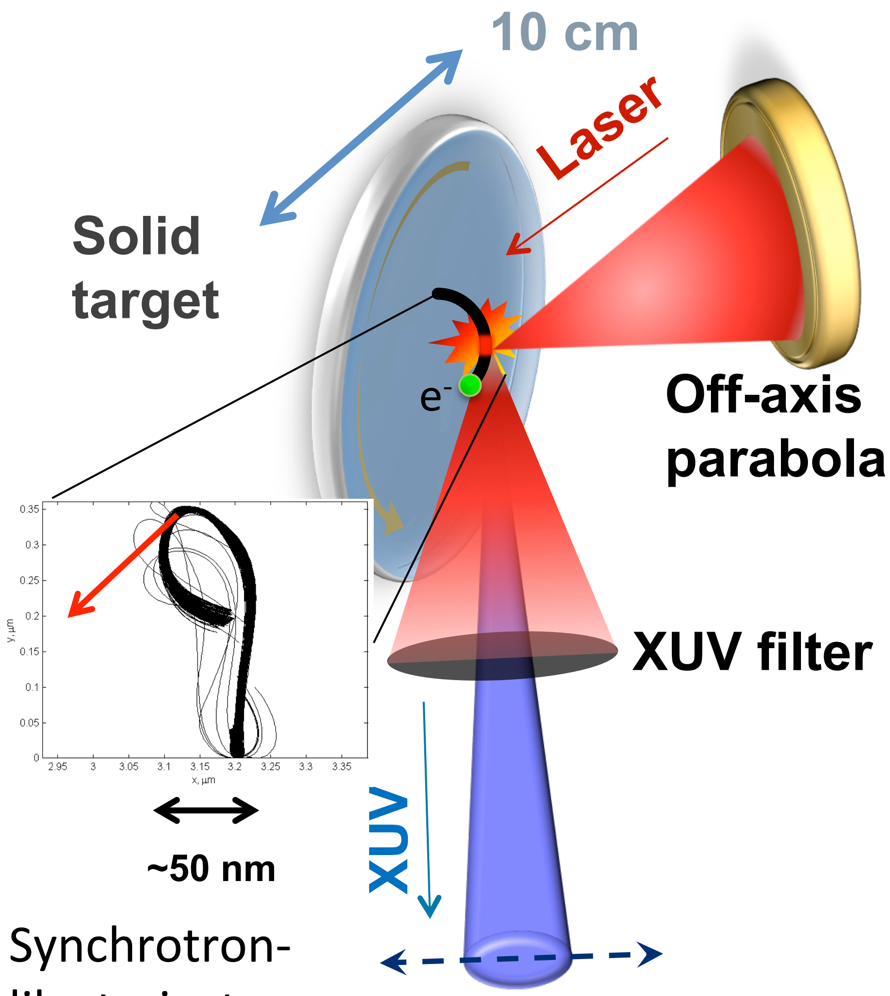
$$800 \text{ nm: } n_c = 1.75 \times 10^{21} \text{ cm}^{-3}$$

$$10\ \mu\text{m: } n_c = 1.12 \times 10^{19} \text{ cm}^{-3}$$

Target material	Electron density n_e (cm^{-3})	n_e/n_c (800nm)
Capilliary discharge	10^{16}	10^{-5}
Gas jet	10^{18}	10^{-3}
Foam/aerogel	10^{21}	$0.1 - 5$
Frozen H	10^{22}	36
CH foil	5×10^{23}	600

Paul Gibbon "Short Pulse Laser Interactions with Matter", Lectures

High-Intensity Laser-Solid Interaction



Synchrotron-like trajectory of electrons

- High intensity laser pulse hitting solid target creates a dense plasma and accelerates electrons at the surface to relativistic speeds.
- Relativistic electron motion is highly-nonlinear, allowing the generation of high harmonics of the incident laser.
- This emission can, according to theory, be phase-locked to produce attosecond x-ray pulses.

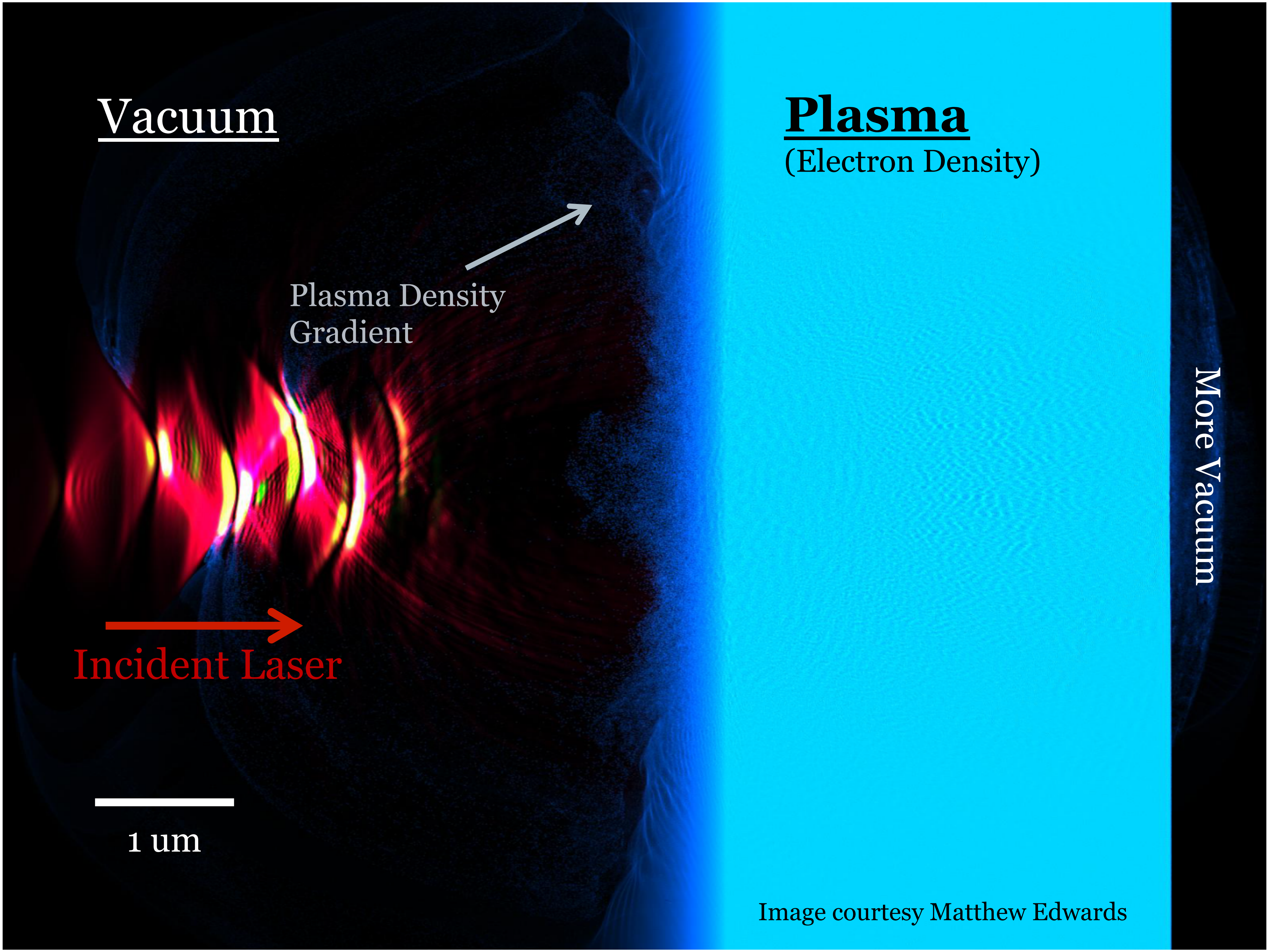


Image courtesy Matthew Edwards

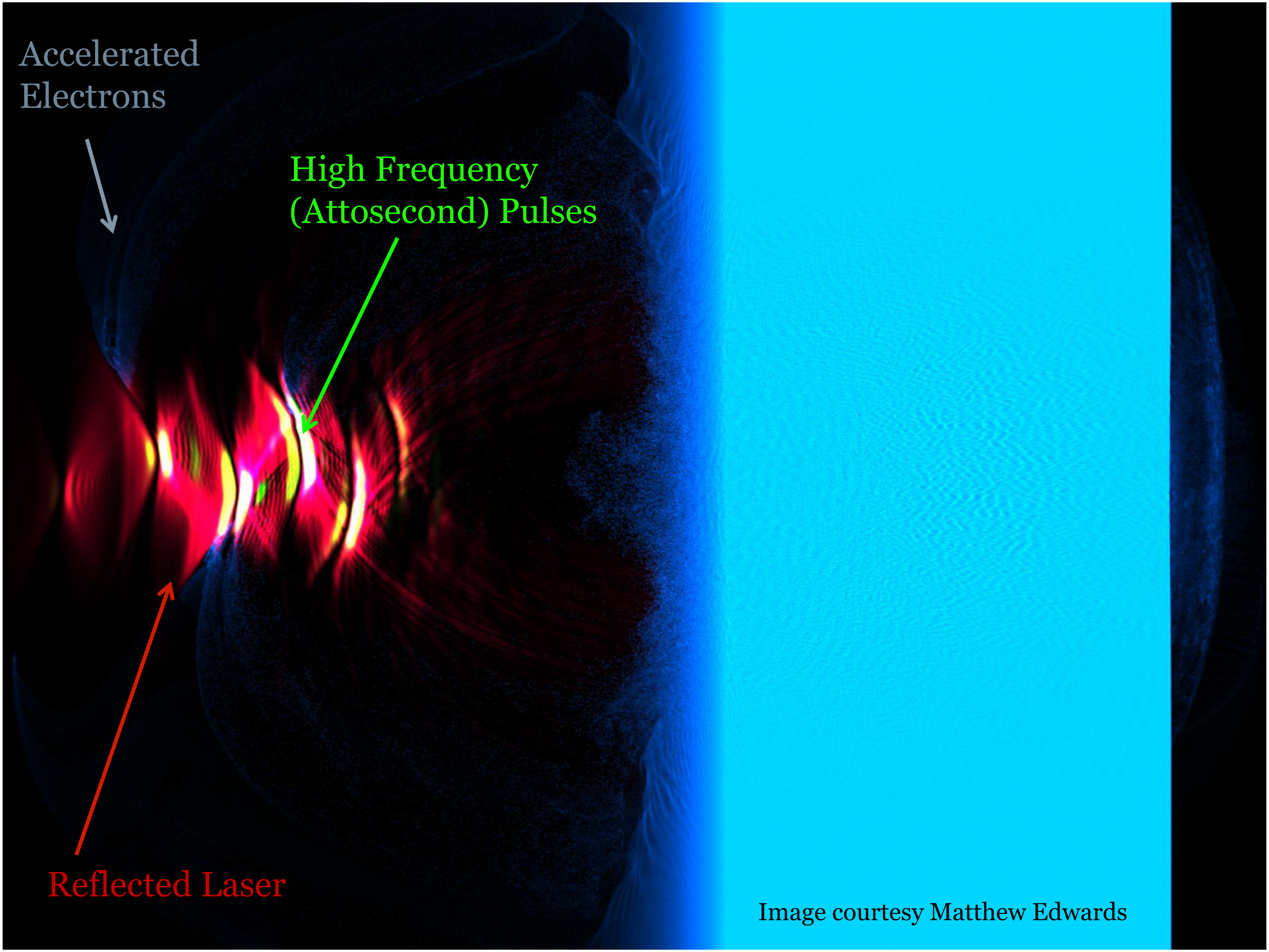
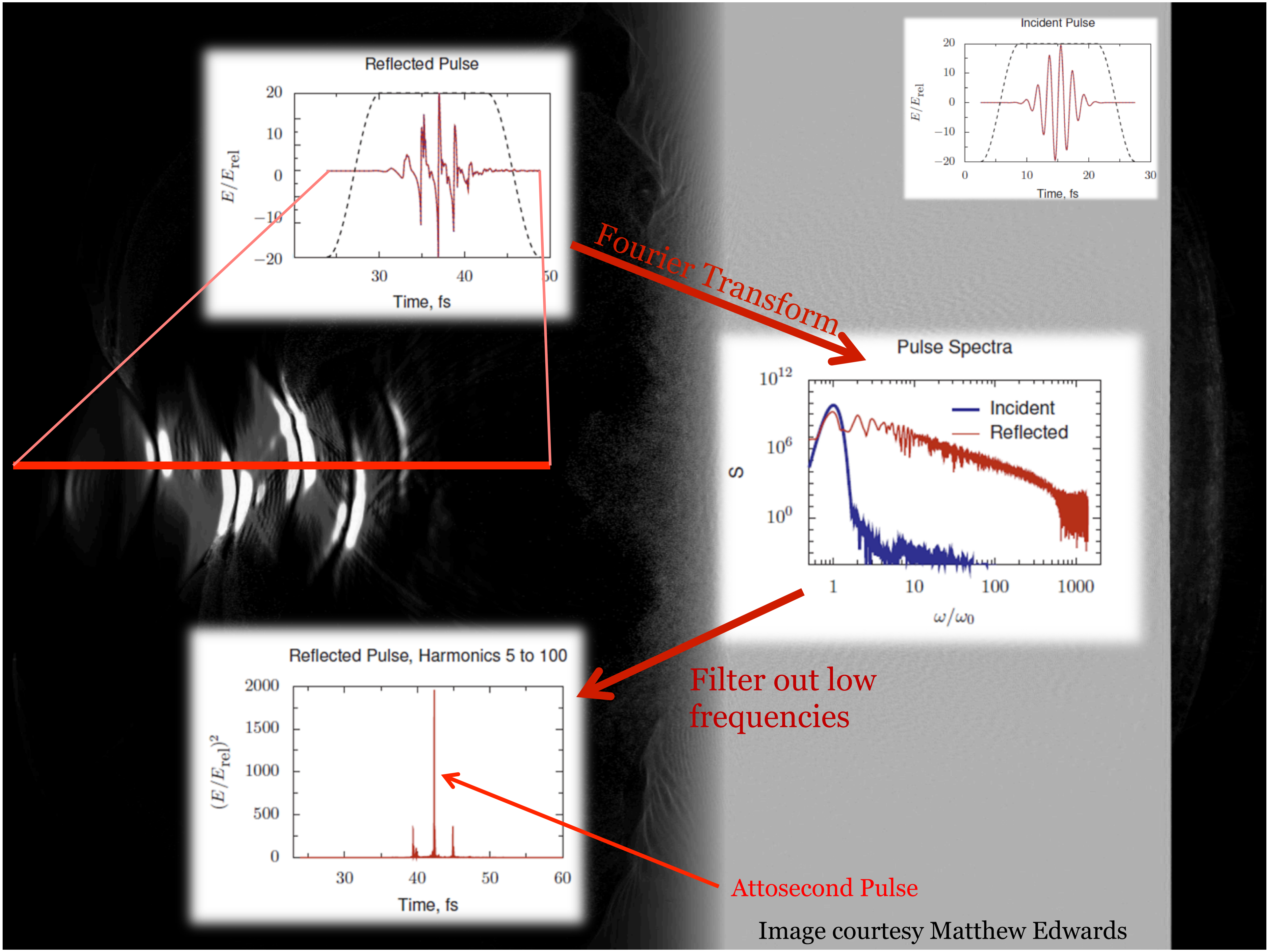


Image courtesy Matthew Edwards



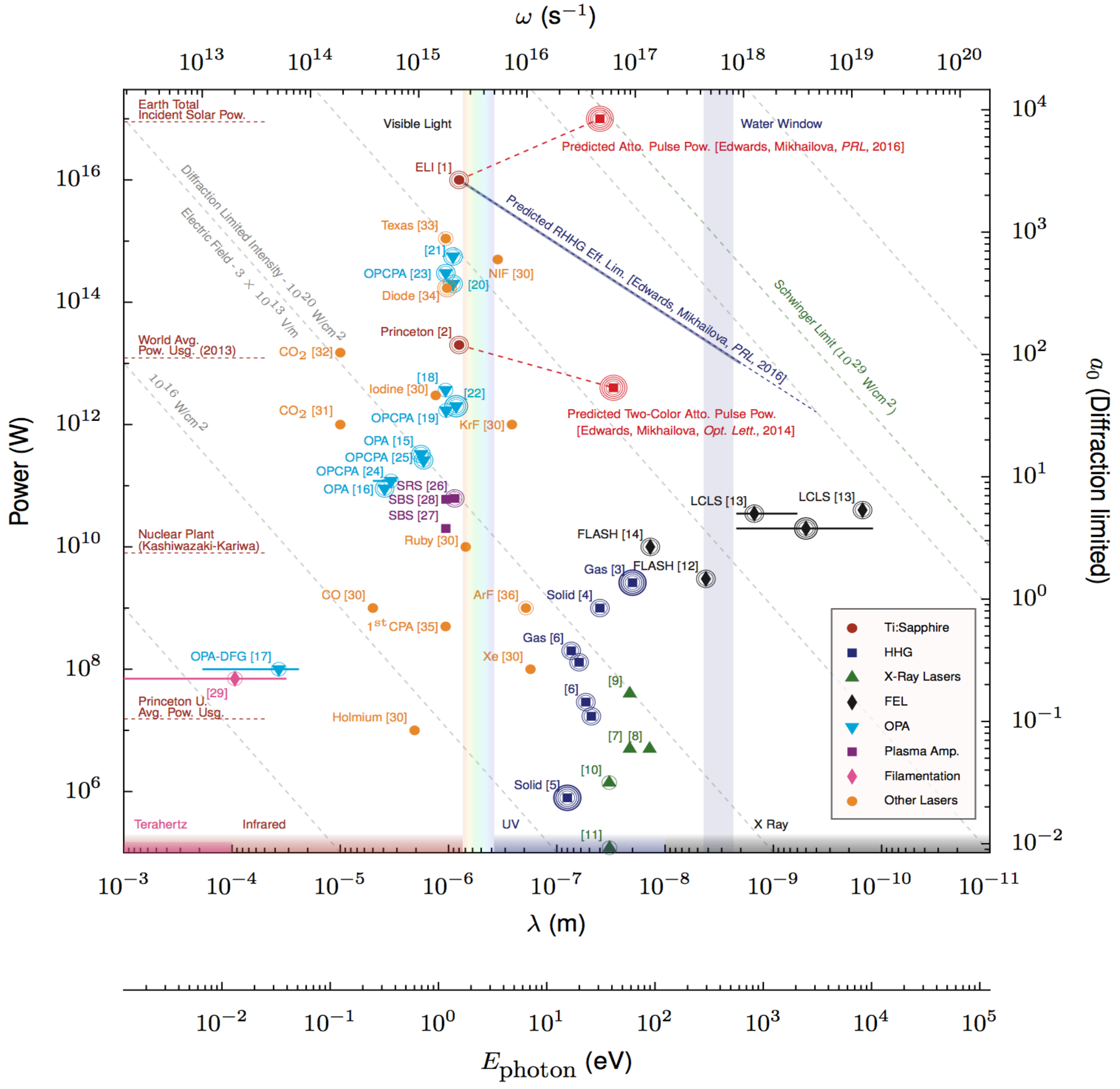
Fourier Transform

Filter out low frequencies

Attosecond Pulse

Image courtesy Matthew Edwards

Ultrafast light sources



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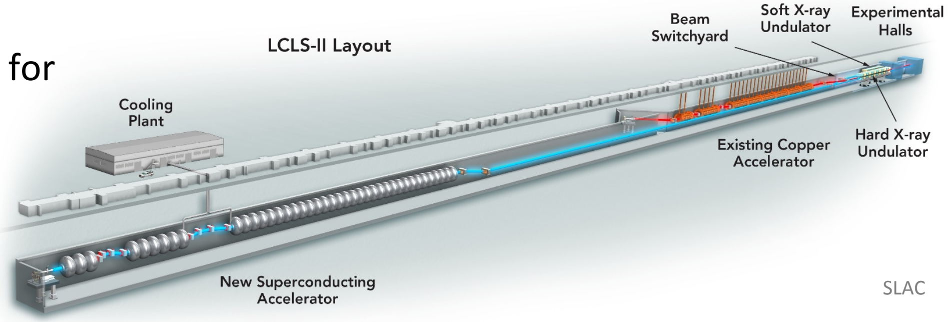
X-ray Free Electron Lasers

- Capabilities (LCLS)
  - ~50 fs (pulse duration)
  - 40 GW (peak power)
  - 0.12 – 22 nm (wavelength)
    - Note that in principle FELs can be used for infrared through x-ray wavelengths.
  - 10^{18} W/cm^2 at 1 nm wavelength

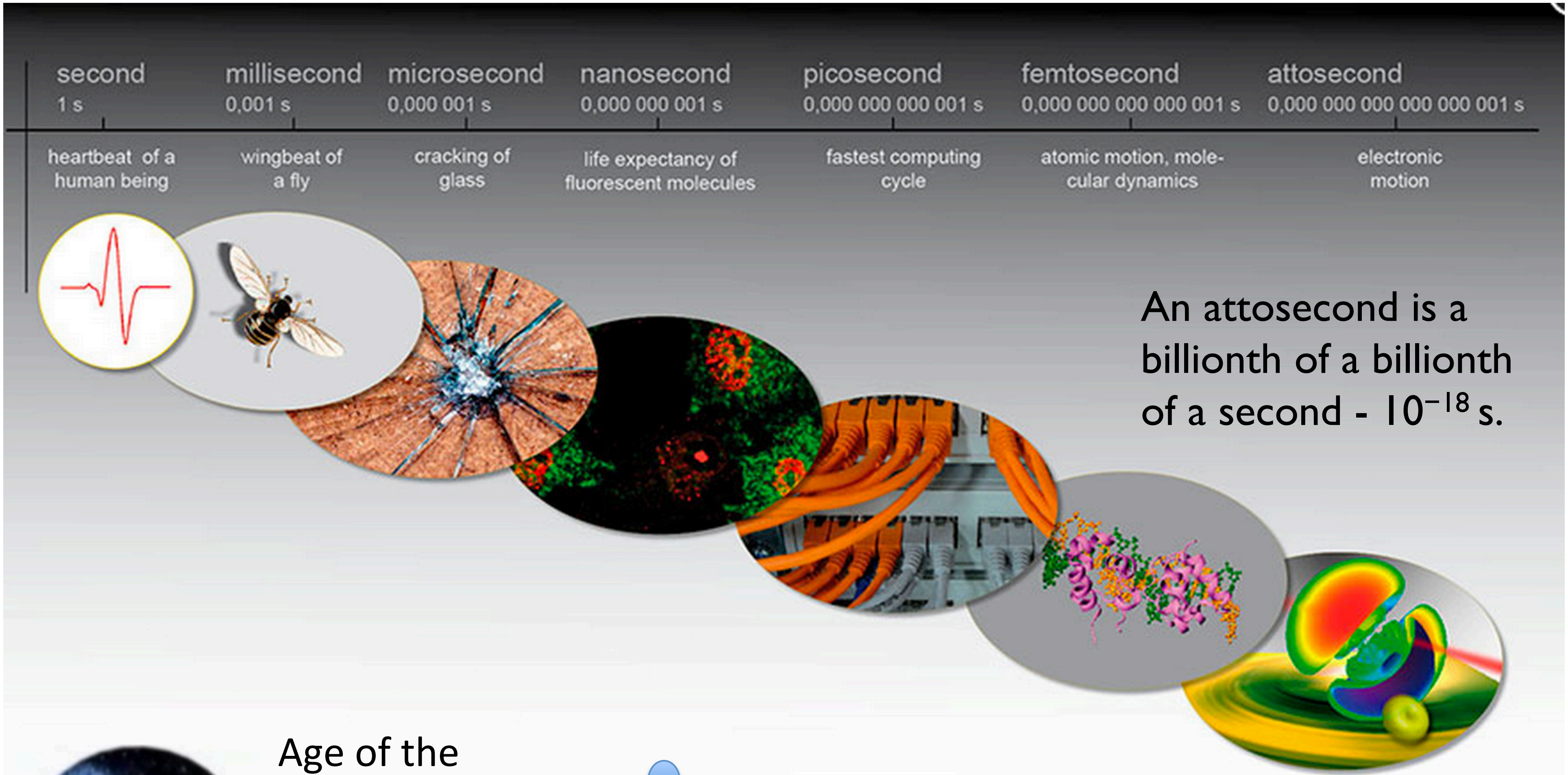


SLAC

- Major Facilities
  - LCLS – SLAC National Accelerator Laboratory
  - FLASH – DESY (Hamburg)
  - European XFEL – DESY (Hamburg)
  - SACLA – RIKEN (Japan)

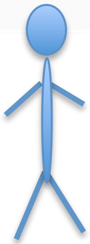
European XFEL

The time scale



An attosecond is a billionth of a billionth of a second - 10^{-18} s.

Age of the Universe is of the order of 10^{17} s



Human life is of the order of 10^{9} s

Image: Ferenc Krausz, MPQ

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Prof. Platonenko Victor Trifonovich

1938 – 2017

Moscow State University, Russia

Thank you for your attention.