Heat Diffusion and Pfirsch-Schluter Currents Near the Divertor Separatrix

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Introduction

In Reiman, Radhakrishnan [1], and previously in Reiman [2], an analytical calculation of the Pfirsch-Schluter current in and near a small magnetic island showed a logarithmic singularity at the X-line for non-symmetric geometries, which arises due to non-constant pressure near the separatrix. This project extends this work to a tokamak divertor. In large-scale fusion experiments, the ratio $\chi_{\|}/\chi_{\perp}$ can become very large, which leads to numerical errors corrupting perpendicular transport [3]. Here, we

- Obtain a numerical solution of heat diffusion near the divertor separatrix.
- Calculate the corresponding pressure-driven current.

Numerical Method

We implement the symmetric, finite difference scheme in Gunter, et al [3], to solve the steady state, heat diffusion equation $\nabla \cdot \mathbf{q} = Q$ for power sources Q, and where q is given by

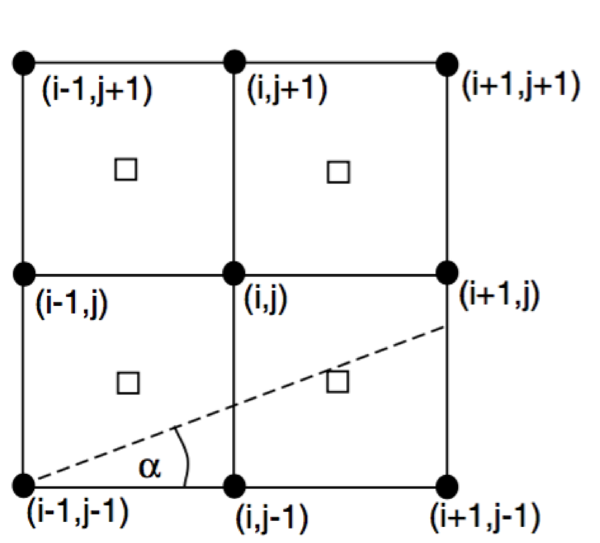
$$\mathbf{q} = [\chi_{\|}\hat{\mathbf{b}}\hat{\mathbf{b}} + \chi_{\perp}(\mathbf{I} - \hat{\mathbf{b}}\hat{\mathbf{b}})] \cdot \nabla T \tag{1}$$

- **Magnetic Field** We run a current through two adjacent wires and sum the fields, which go like $1/r_1 + 1/r_2$.
- **Diffusivities** $\chi_{\text{pol}}/\chi_{\perp} \propto \mathbf{B}^2_{\text{pol}}$, where $\mathbf{B}^2_{\text{pol}} = B_x^2 + B_y^2$; we solve for temperature. Then we iteratively solve a nonlinear equation, taking $\chi_{\text{pol}}/\chi_{\perp} \propto \mathbf{B}^2_{\text{pol}} T^{5/2}$. We increase the coefficient to 10^6.
- **Symmetric Scheme** This discretized formulation, shown below, maintains the self-adjointness of the operator $\nabla \cdot \nabla_{\|}$. The temperature gradient is then

$$\begin{cases} \frac{\partial}{\partial x} T\big|_{i+\frac{1}{2},j+\frac{1}{2}} = \frac{1}{2(\Delta x)}((T_{i+1,j+1} + T_{i+1,j}) - (T_{i,j+1} + T_{i,j})) \\ \frac{\partial}{\partial y} T\big|_{i+\frac{1}{2},j+\frac{1}{2}} = \frac{1}{2(\Delta x)}((T_{i+1,j+1} + T_{i,j+1}) - (T_{i+1,j} + T_{i,j})) \end{cases} \tag{2}$$

With the field and diffusivities defined on the half-grid (refer to figure 1a), we use the expression for $\nabla \cdot \mathbf{q}_{\|}\big|_{i,j}$ presented in [3] and get a 9-diagonal matrix of coefficients for $\mathbf{T}_{\text{num}} = (T_1, \ldots, T_N)$, where $N = N_x N_y$. We utilize symmetry about $x = 0$, setting $T = 0$ at the boundary on all other sides of the grid.

1a)



1b)

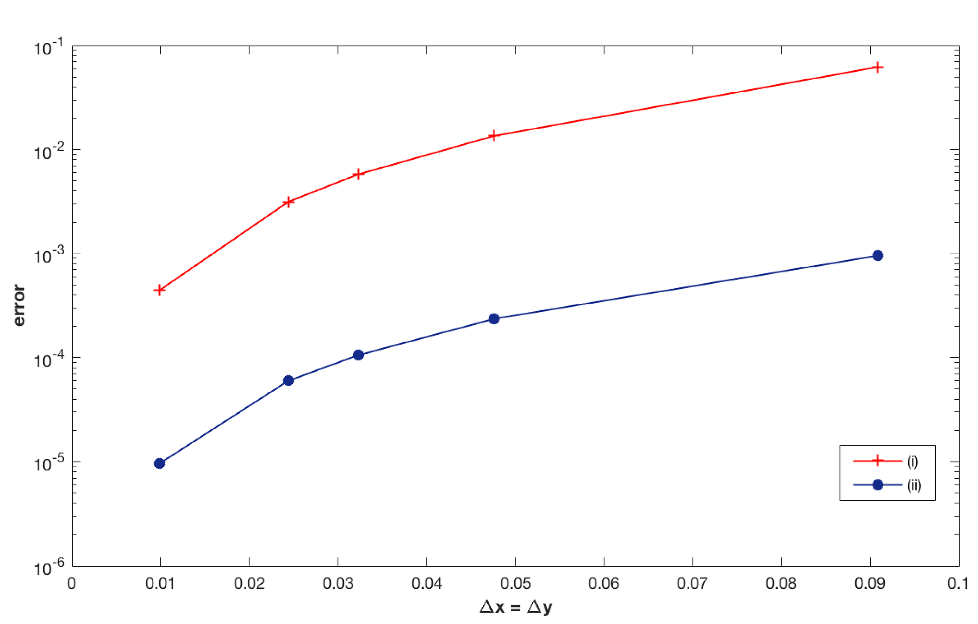


Figure 1 - Diagram of Symmetric Scheme

a) Diagram of grid structure [3]. T is given on the grid points, while the magnetic field and diffusivities are specified on the half-grid. α is an Auerbach-Boozer coordinate [4].
b) Scaling of error, given by err $= |T_{\text{num}}(0,0) - T_{\text{anal}}(0,0)|/|T_{\text{anal}}(0,0) - T_{\text{anal}}(0.5,0)|$ [3], for manufactured solution of $T(r) = 1 - r^3$, $\chi = 10^9$, and circular magnetic field centered at (i) $x_0 = y_0 = 0$ and (ii) $x_0 = y_0 = -0.5$.

Auerbach-Boozer Coordinates

In order to calculate the current in the plasma, we interpolate our solution for pressure onto an Auerbach-Boozer grid [4]. The magnetic surfaces $(\rho/a)^2 = \cos(2\alpha) \pm \sqrt{k^2 - \sin(2\alpha)}$ solve the equation

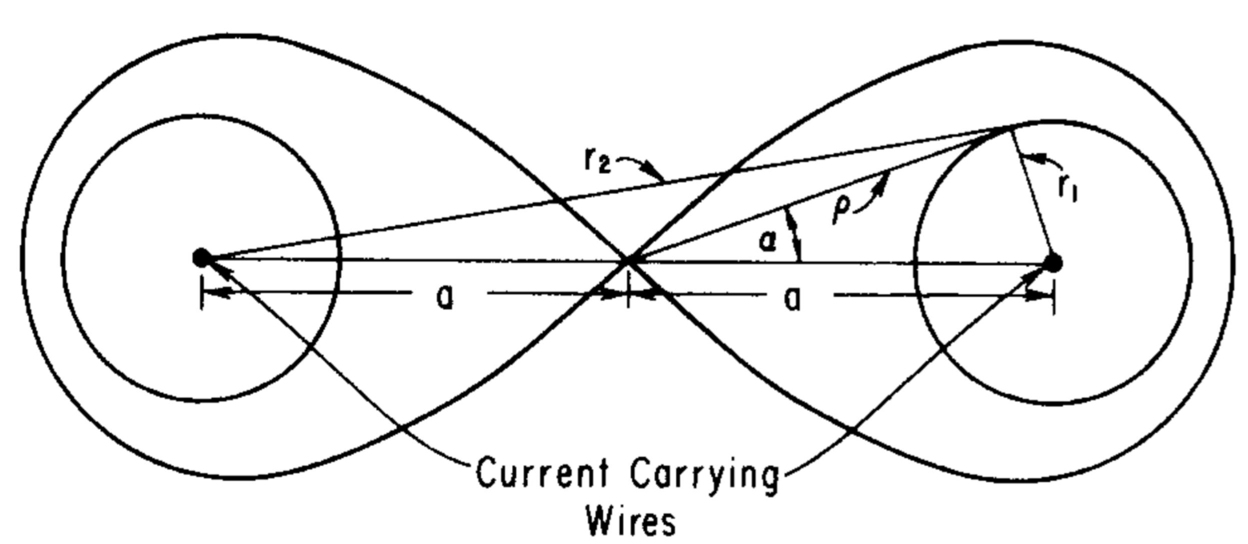
$$(\rho/a)^2 - 2(\rho/a)^2 \cos(2\alpha) + 1 = k^2 \tag{3}$$

- The separatrix surface corresponds to magnetic flux surface label $k = 1$, and (3) becomes $(\rho/a)^2 = 2\cos(2\alpha)$.
- Inside the plasma boundary, when $k < 1$, α is constrained by $\sin^2(2\alpha) \le k^2$.
- In a similar treatment to that of the current calculation inside a small magnetic island [1], [2], we introduce a new angle η mapped from α by the relation $\sin(\eta) = \sin(2\alpha)/k$.

Finally, we relate back to Cartesian coordinates with

$$x(k,\eta) = -\rho(k,\eta)\sin\alpha(k,\eta), \;\; y(k,\eta) = \rho(k,\eta)\cos\alpha(k,\eta) \tag{4}$$

2a)



2b)

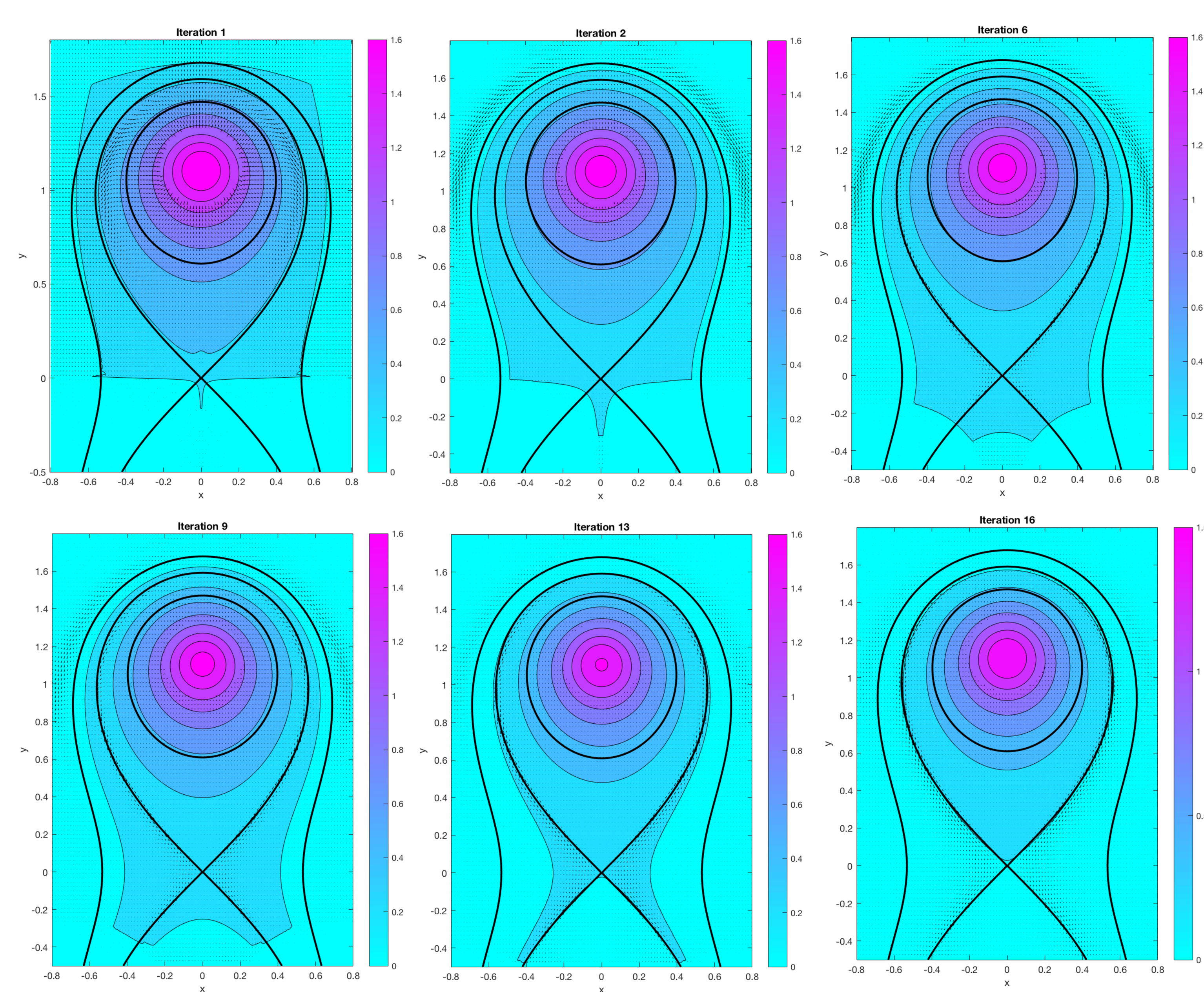


Figure 2 - Solution of Nonlinear Diffusion Equation

a) Diagram of Auerbach-Boozer coordinate system (k, α) [4]. At the divertor separatrix, $-\pi/2 \le \alpha \le \pi/2$, and α changes rapidly as it approaches the X-point.
b) Convergence of heat diffusion with diffusivity $\chi_{\text{pol}}/\chi_{\perp} = 10^6 \mathbf{B}^2_{pol} T^{5/2}/\max(\mathbf{B}^2_{pol} T^{5/2})$. As $\chi_{\text{pol}}/\chi_{\perp}$ increases, the converged solution better aligns with the magnetic surfaces.

Equilibrium Current Calculation

The variation of pressure on flux surfaces in the region near the divertor separatrix leads to a nonzero $\mathbf{B} \cdot \nabla p$ at the X-line, and a large Pfirsch-Schluter current near the X-line. [1], [2]

- Using the field configuration $\mathbf{B} = B_0 R_0 \nabla\phi + B_p \nabla\hat{\theta}$, we have

$$\nabla \cdot \mathbf{j}_{\perp} = -(\mathbf{B} \times \nabla p) \cdot \nabla\left(\frac{1}{B^2}\right) \tag{5}$$

where $\nabla(1/B^2) = 2\hat{x}/(R_0 B_0^2)$.

- Consider magnetic differential equation $\mathbf{B} \cdot \nabla\lambda = f$, where

$$f(k,\eta) := \nabla \cdot \mathbf{j}_{\perp} \tag{6}$$

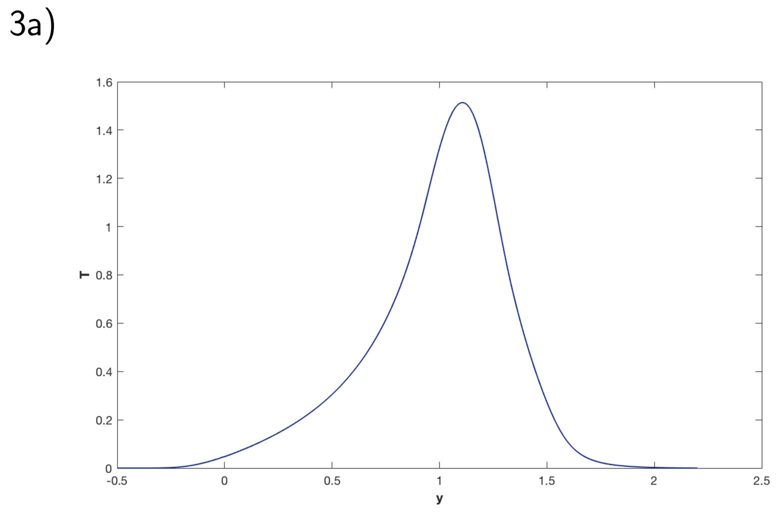
- Using the chain rule,

$$\mathbf{B} \cdot \nabla\eta \frac{\partial\lambda}{\partial\eta} + \mathbf{B} \cdot \nabla k \frac{\partial\lambda}{\partial k} = f(k,\eta) \tag{7}$$

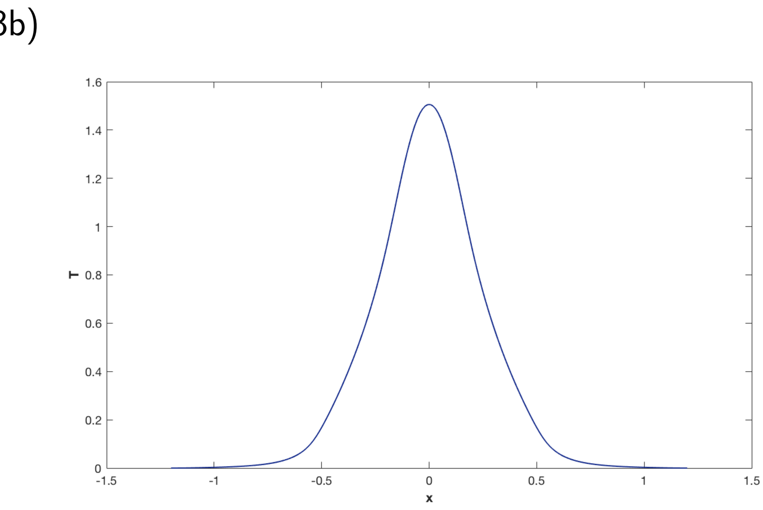
- Since $\frac{\partial\lambda}{\partial k} = 0$, we use $\mathbf{B} \cdot \nabla\eta = \left(\mathbf{B} \cdot \nabla x \frac{\partial x}{\partial\alpha} + \mathbf{B} \cdot \nabla y \frac{\partial y}{\partial\alpha}\right)\left(\frac{2\cos(2\alpha(k,\eta))}{k\cos\eta}\right)$ to obtain the solution

$$\lambda = \int_0^{\eta} \frac{f(k,\eta')}{\mathbf{B} \cdot \nabla\eta'} d\eta' \tag{8}$$

3a)



3b)



3c)

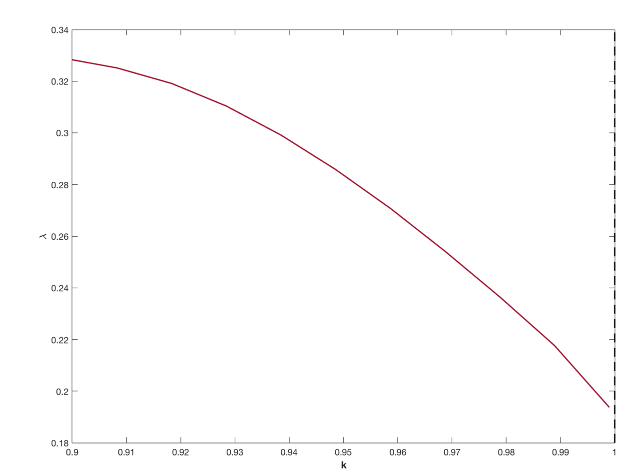


Figure 3 - Temperature and Pressure-Driven Current Plots

a) $T(y)$ for fixed $x = 0$
b) $T(x)$ for fixed $y = 1.125$ (upper wire where power is deposited)
c) $\lambda(k)$ for fixed $\eta = \pi/2$

References

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4. Auerbach, S. P., Boozer, A. H. (1980). Classical diffusion in the presence of an X point. The Physics of Fluids, 23(12), 2396-2412.

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