Simulation of Atmospheric Plasma Arc Characteristics

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Magnetohydrodynamic (MHD) Power

- MHD generators rely on the Lorentz force: $\vec{F} = q\left(\vec{E} + \vec{v} \times \vec{B}\right)$
- Plasma forced through magnetic field, where Lorentz force causes current across electrodes (perpendicular to velocity)
- More efficient than turbine generators, but high temperature arcs from plasma can damage electrodes quickly
- New (liquid) electrode design in progress, requiring a detailed analysis of plasma characteristics

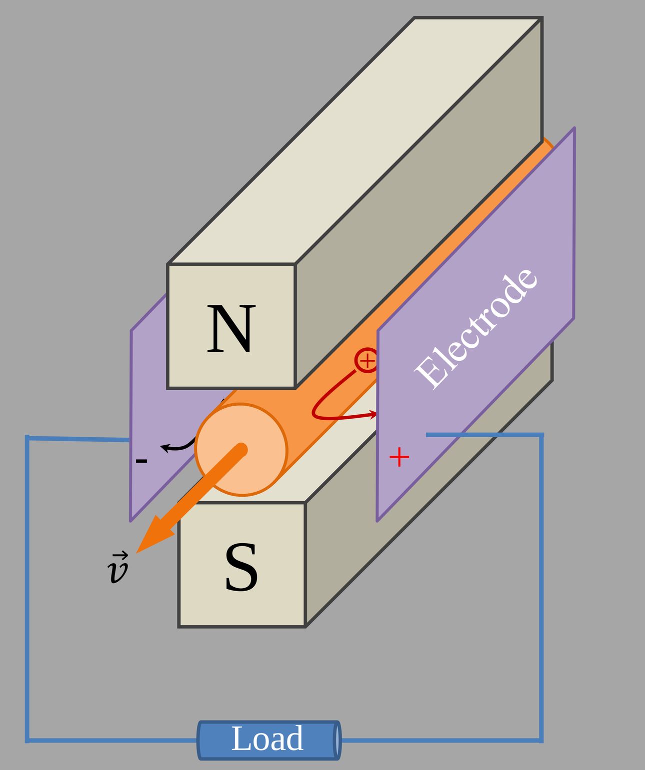


Figure 1: MHD generator schematic

Motivations

- Arc generator technology allows for the analysis of plasma properties without the need for involved fuel handling technologies or combustor development
- Simulation of arc characteristics necessary to successfully model arc-electrode interactions
- Composition data obtained from simulation can be used to as a predictive tool for other diagnostic systems

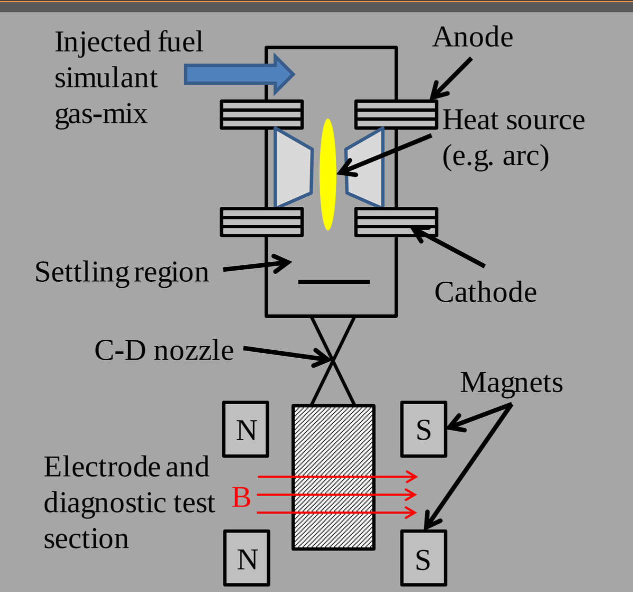


Figure 2: Arc generator schematic

Chemical Equilibrium with Applications (CEA)

NASA software calculates the equilibrium properties of complex gas mixtures at specified temperatures and pressures:

- Mole fractions, including charged species and electrons
- Thermodynamic properties
- Thermal transport properties (including thermal conductivity)

Gibbs energy minimization method of calculating chemical equilibria is rapid, allowing for the timely simulation of both simple and complex fuel mixtures

- Pure hydrogen arcs
- Argon/CO_2 arcs
- Simulated coal burning compositions

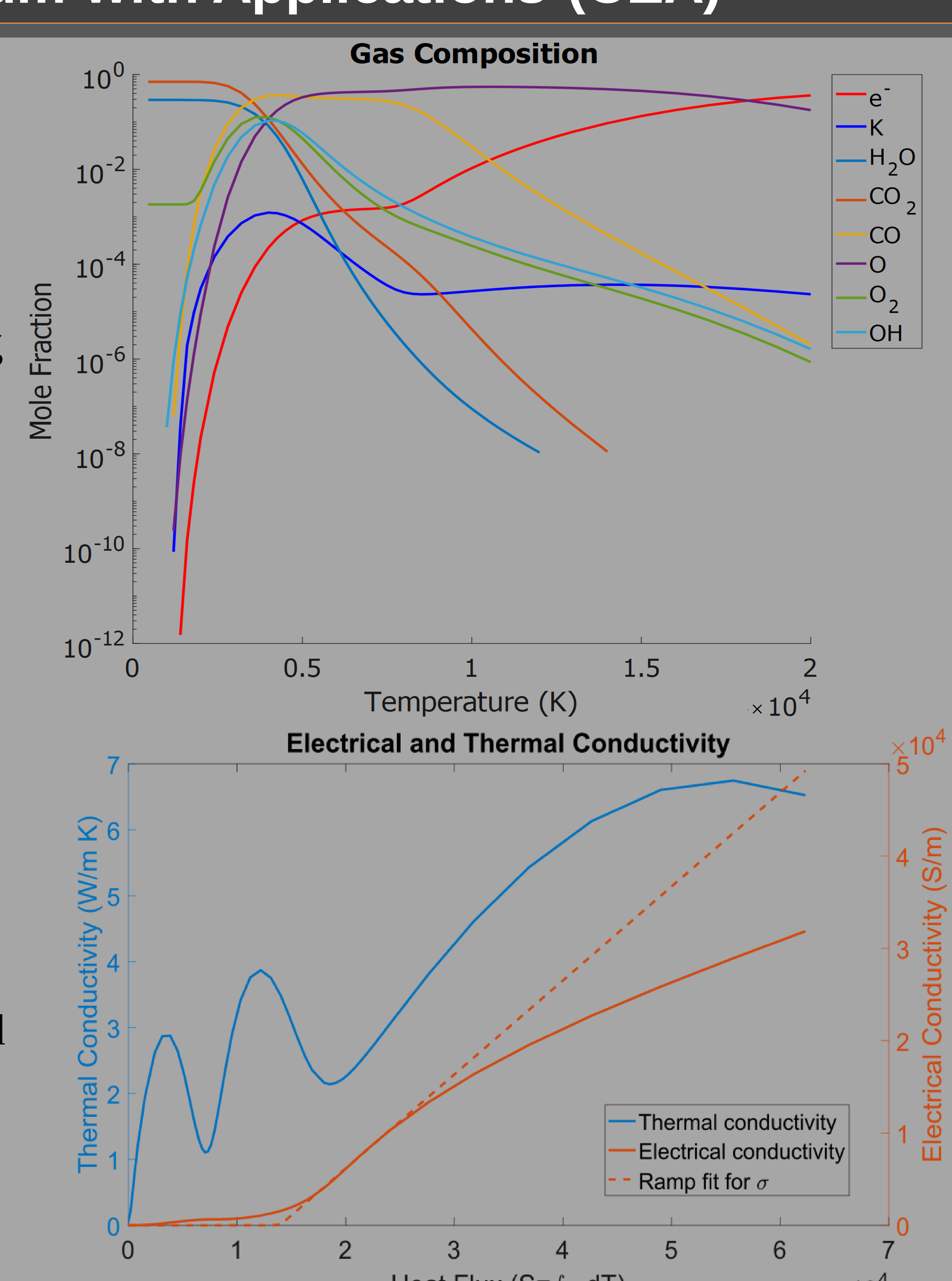


Figure 3: CEA data for coal burn

Analytic Calculations

We seek solutions to the Elenbaas-Heller equation in the form of radial temperature profiles $T(r)$ for a given arc current:

$$\underbrace{\sigma}_{\text{Electrical conductivity}} \underbrace{E^2}_{\text{Electric field}} + \frac{1}{r}\frac{d}{dr}\left(r \underbrace{\kappa}_{\text{Thermal conductivity}} \frac{dT}{dr}\right) - \underbrace{4\pi\epsilon_n}_{\text{Radiative loss term (neglected)}} = 0$$

CEA data include thermal conductivity and allow for calculation of electrical conductivity over a wide range of temperatures.

Electrical conductivity is parameterized in terms of heat flux ($S = \int \kappa \, dT$) and fitted to a function of the form $\sigma(S) = \mathcal{R}(B(S - S_1))$ where $\mathcal{R}(x) = \max(x, 0)$ is the ramp function and B, S_1 are constants. Solving for $S(r)$ with appropriate boundary conditions yields solutions of the form

$$S_{\text{in}}(r) = S_1 - \frac{S_1 J_0\left(\frac{\beta}{\rho R} r\right)}{\beta J_1(\beta) \log(\rho)} \qquad (0 \le r < \rho R)$$

$$S_{\text{out}}(r) = S_1 + \frac{S_1}{\log(\rho)} \log\left(\frac{1}{\rho R} r\right) \qquad (\rho R \le r \le R)$$

where: $J_n(x)$ is the n-th order Bessel function of the first kind; β is the first zero of $J_0(x)$; S_1 and B are defined by $\sigma(S)$ as above; R is the radius of the arc; and ρ is a dimensionless, current-dependent quantity given by

$$\rho(I) = \frac{\beta}{2\pi S_1 \sqrt{B} R} I \, W\left(\frac{2\pi S_1 \sqrt{B} R}{I\beta}\right)$$

where $W(x)$ is product logarithm function. Current-voltage relations also follow from the solutions, yielding the electric field $E(I)$.

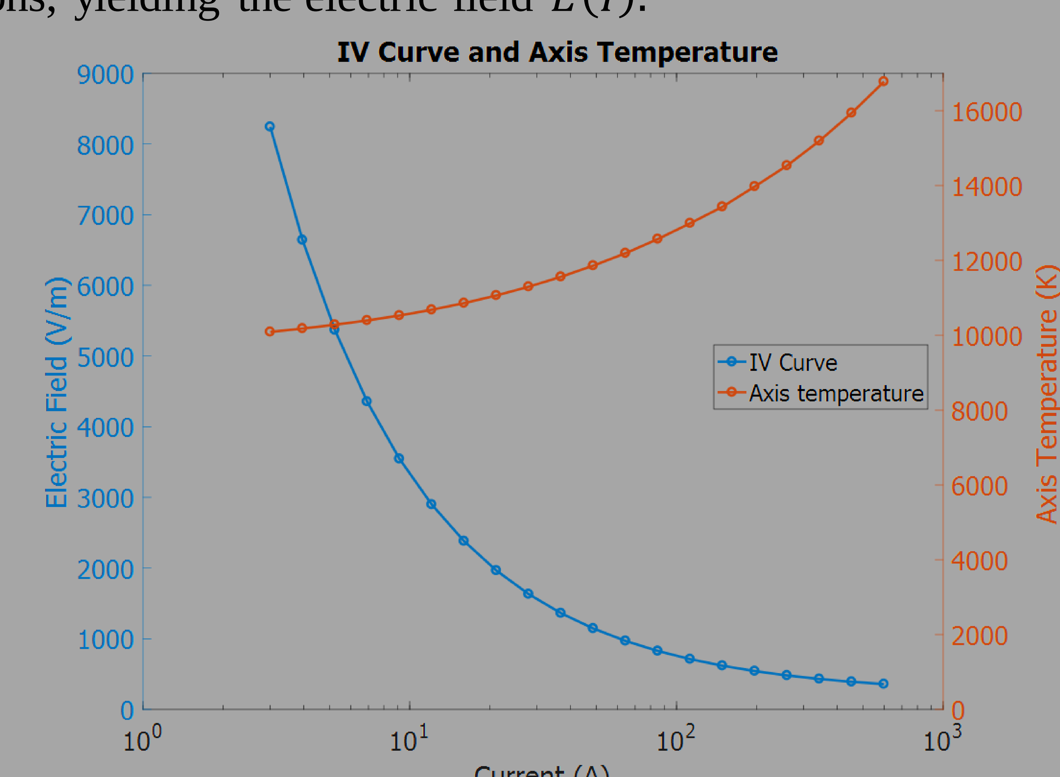
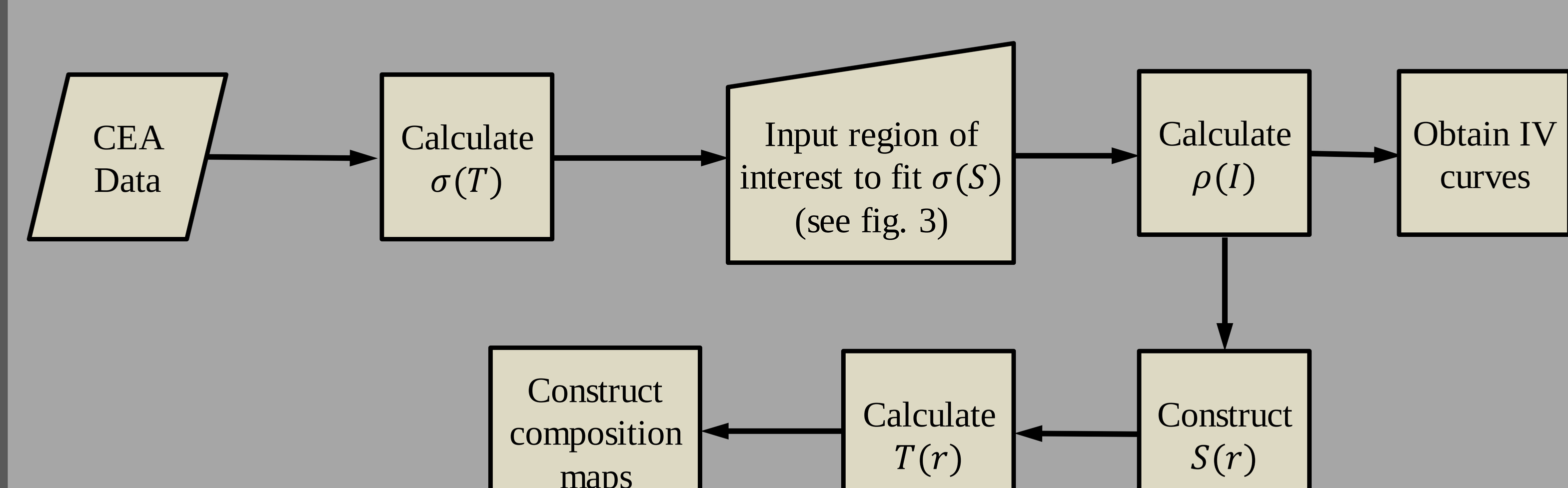


Figure 5: IV curve and axis temperature for simulated coal burn

Simulation Procedure

Program written in Matlab calculates plasma arc characteristics from composition data and momentum transfer cross sections:



Results

From $S(r)$, we can also calculate $T(r)$ and composition maps for the arc:

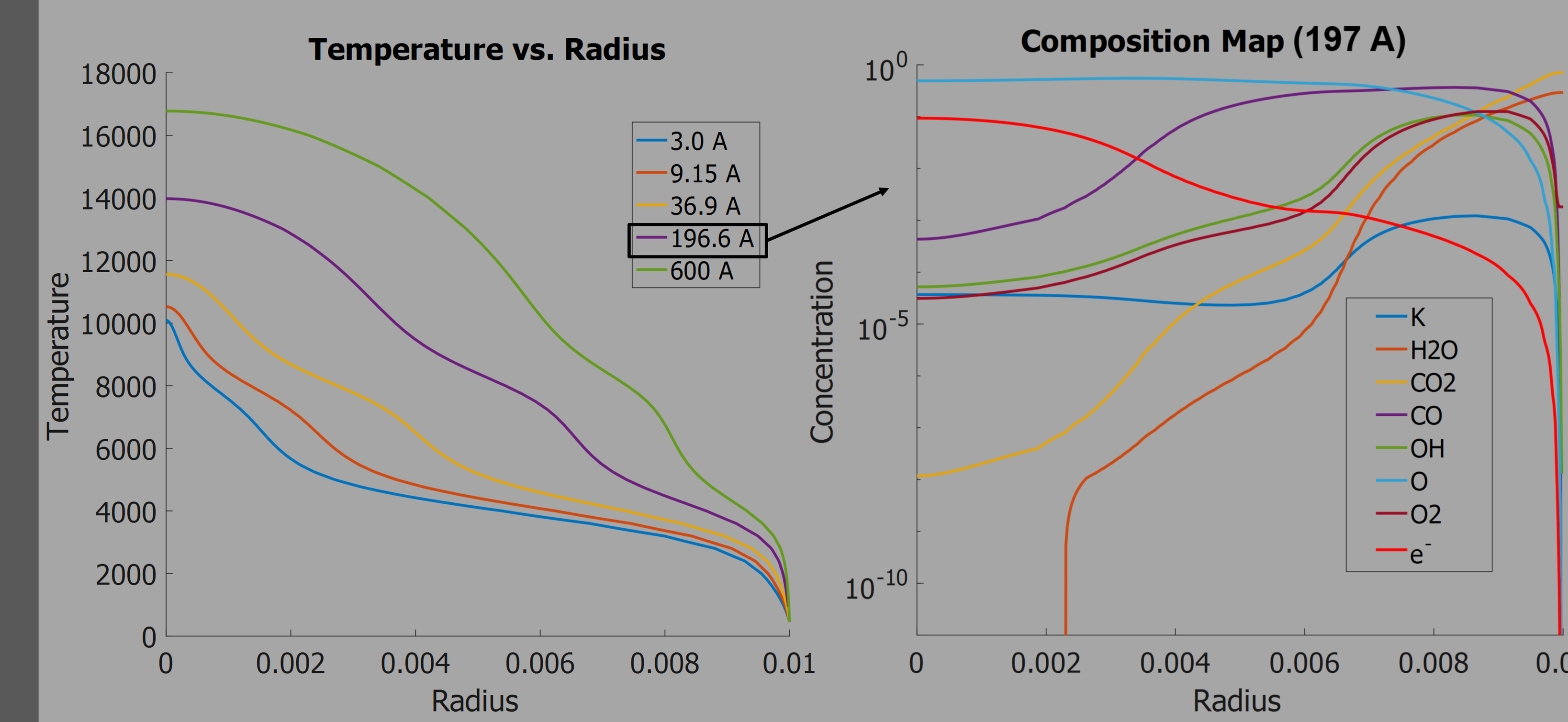


Figure 6: Temperature profile and composition map of simulated coal burn

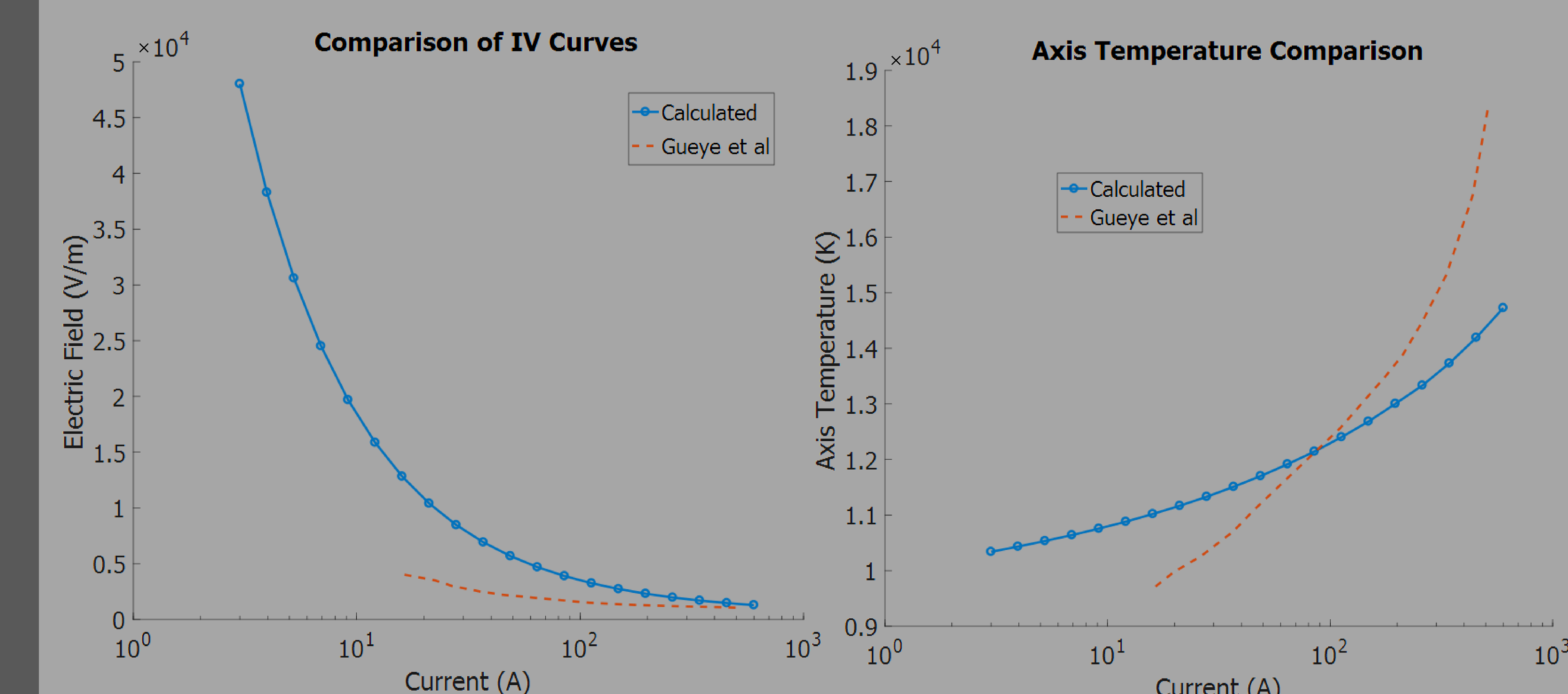


Figure 7: Comparison of results for a hydrogen arc with literature. Gueye et. al. use different methods for calculating thermal transport properties, leading to discrepancy.

Future Directions

A variety of avenues for future study in this area of research are open:

- Additional summer work involved construction of a schlieren optical diagnostic system; index of refraction simulations will dovetail with experimental measurements
- More detailed modeling of electrical conductivity $\sigma(S)$ is desirable, as more complex gas mixtures have more complex behavior
- Detailed molecular spectroscopy diagnostics (e.g. FTIR) could be used to test the results of simulations
- Modeling of the radiative loss term is desirable for arcs at high temperatures where a large proportion of power is lost to radiation

References

1. Gueye, P., Cressault, Y., Rohani, V. & Fulcheri, L. A simplified model for the determination of current-voltage characteristics of a high pressure hydrogen plasma arc. *Journal of Applied Physics* **121,** 73302 (2017).

2. Rosa, R. J. *Magnetohydrodynamic energy conversion*. (Hemisphere Pub. Corp, 1987).

3. Gordon, S. and McBride, B. J., "Computer Program for Calculation of Complex Chemical Equilibrium Compositions and Applications," NASA Reference Publication 1311 (1996).

4. Jaworski, M. et al, "Collaborative Possibilities for PPPL and NETL," October 2016

Acknowledgements

This work was made possible by funding from the Department of Energy for the Summer Undergraduate Laboratory Internship (SULI) program. This work is supported by the US DOE Contract No. DE-AC02-09CH11466