Simulation of Collisions in Slab Geometry Approximation of Wave Driven Rotating Torus (WDRT)

PRINCETON PLASMA PHYSICS LABORATORY

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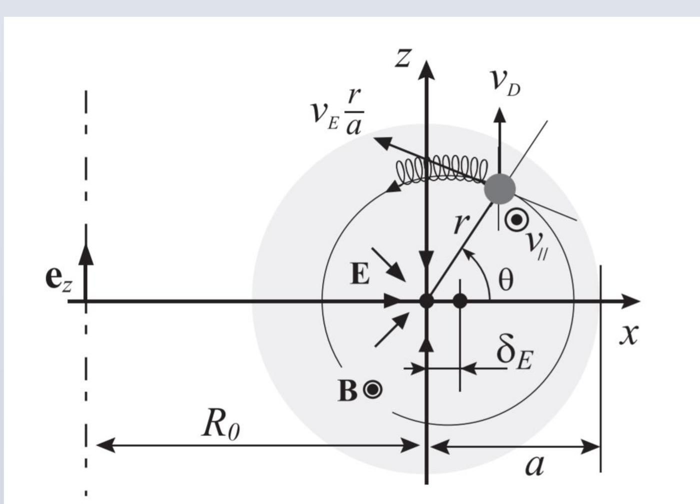
Goals

- Develop computational simulation algorithm to simulate single particle motion in given electric and magnetic fields
- Use this simulation to provide model where collisions time is low compared to system traversal time
  - Contrast to Fluid models

Background: WDRT Magnetic Confinement



- There are a number of papers detailing mathematical analysis of the WDRT[1]
- I wish to examine a simplified "slab" version of the WDRT geometry where
  - Toroidal angle (φ) replaced by z
  - Radial (r) direction replace by y
  - Poloidal angle (θ) replaced by x
- Adopt the following model:

$$\boldsymbol{B} = B_0(0, 0, 1 + \varepsilon \sin kx)$$

$$\boldsymbol{E} = E_0(0,1,0)$$

- Want to investigate poloidal variation of magnetic field
- Constant E field approximation

Parameter Regime

- Thermal Energy larger than $\boldsymbol{E}\times\boldsymbol{B}$ drift energy
- Background particles traveling with $\boldsymbol{E}\times\boldsymbol{B}$ drift
- Gyro-period much shorter than traversal time

Algorithm Development

Boris Algorithm

- Conserves volume in phase space[2]
- Designed for charged particles
- Offset computation of velocity and position

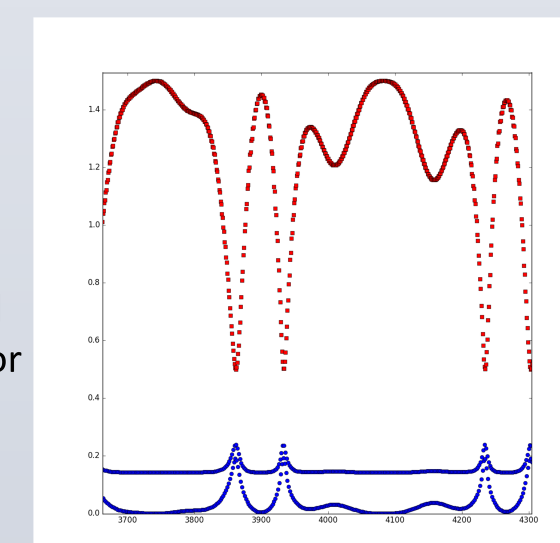
$$\boldsymbol{v}^- = \boldsymbol{v}_k + \frac{q}{m}\boldsymbol{E}_k\frac{\Delta t}{2}$$

$$\frac{\boldsymbol{v}^+ - \boldsymbol{v}^-}{\Delta t} = \frac{q}{2m}(\boldsymbol{v}^+ + \boldsymbol{v}^-)\times\boldsymbol{B}_k$$

$$\boldsymbol{v}_{k+1} = \boldsymbol{v}^+ + \frac{q}{m}\boldsymbol{E}_k\frac{\Delta t}{2}$$

Dynamic Time Step

- Want time step to be small compared to gyro-period, therefore scale negatively with magnetic field strength
- More difficult to implement in Boris Algorithm[3]
- $dt = dt_0({}^{B_0}/_{B})$



Magnetic field strength (red) and time step (blue) for each step

Langevin Model for Collisions

- Standard approximation for collisions impact on single particle trajectories[4]

$$m\frac{d\boldsymbol{v}}{dt} = q(\boldsymbol{E} + \boldsymbol{v}\times\boldsymbol{B}) - \gamma(\boldsymbol{v} - \boldsymbol{v}_{\boldsymbol{E}\times\boldsymbol{B}}) + \boldsymbol{L}(t)$$

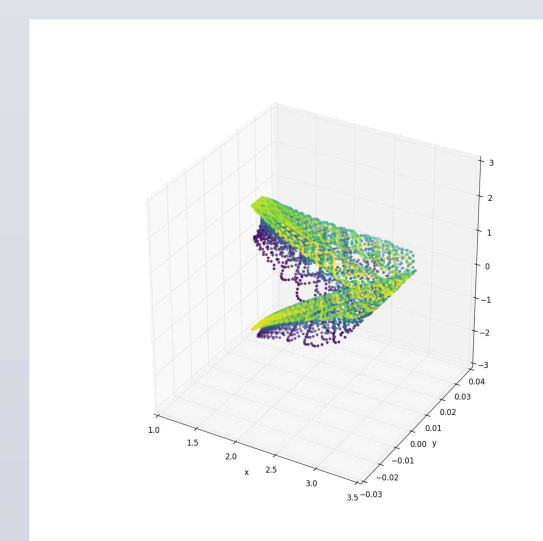
$$\langle L_i(t)L_j(t')\rangle = 2\gamma k_B T\delta_{ij}\delta(t - t')$$

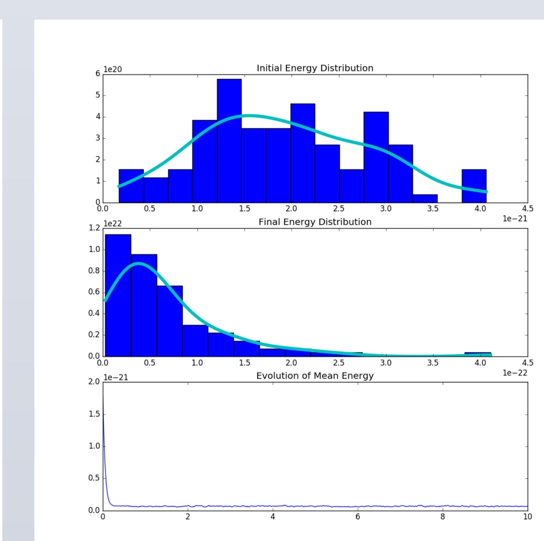
- Where $\boldsymbol{L}(t)$ is a multivariate Gaussian
- We want collision time greater than traversal time

$$\gamma = 0.1 \times v_{\boldsymbol{E}\times\boldsymbol{B}}/L$$

Application of Algorithm

- Predicts the trajectory of particles
- Can track the evolution of a random distribution of initial conditions



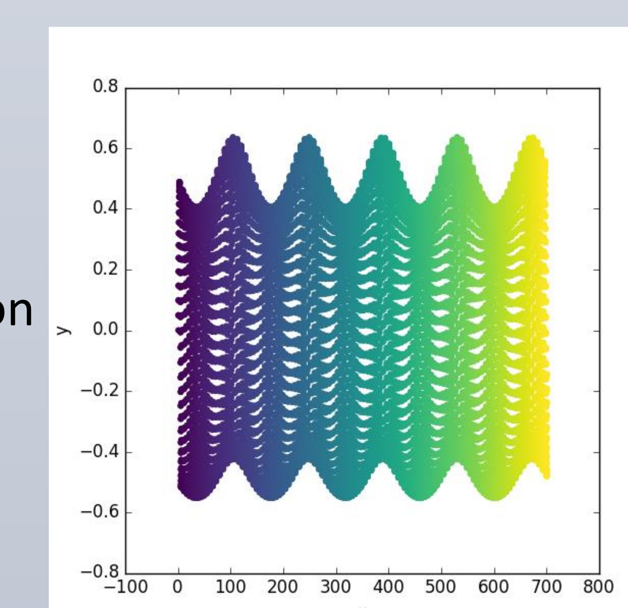


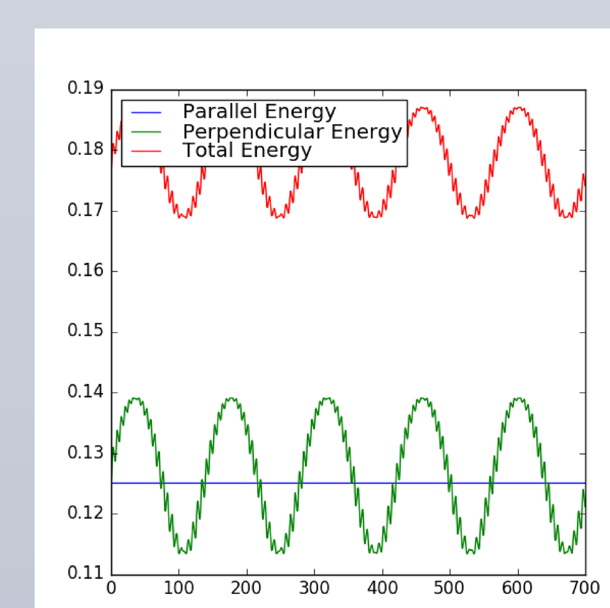
Simulations of WDRT

$E_0 = B_0 = 1$
$m = q = 1$
$\varepsilon = 0.1$
$\mathbf{r}_0 = (1,\ 0,\ 0)$
$\mathbf{v}_0 = (1,\ 0.5,\ 0.5)$
$k_B T = 25 m v_{E\times B}^2 = 25$
$L = \frac{2\pi}{k} = 20 r_L = 141.42$
$\gamma = (0.1)\frac{v_{E\times B}}{L} = 7.07\times 10^{-4}$
$dt_0 = 0.01$

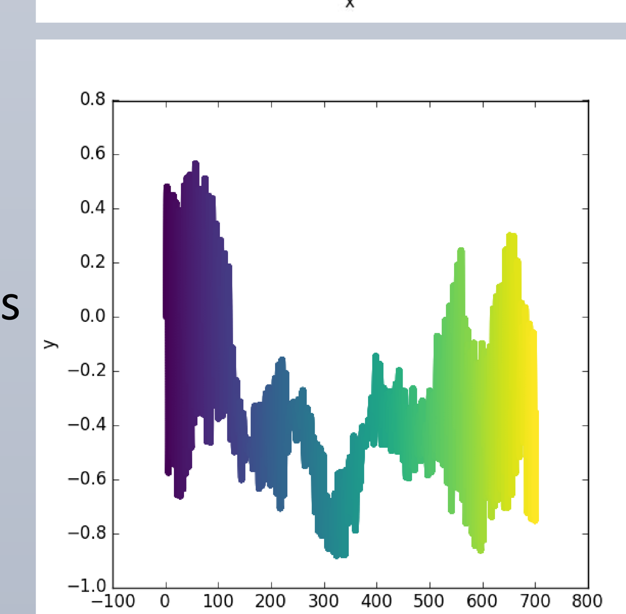
Trajectory of Single Particle | Energy vs Time, Single Particle

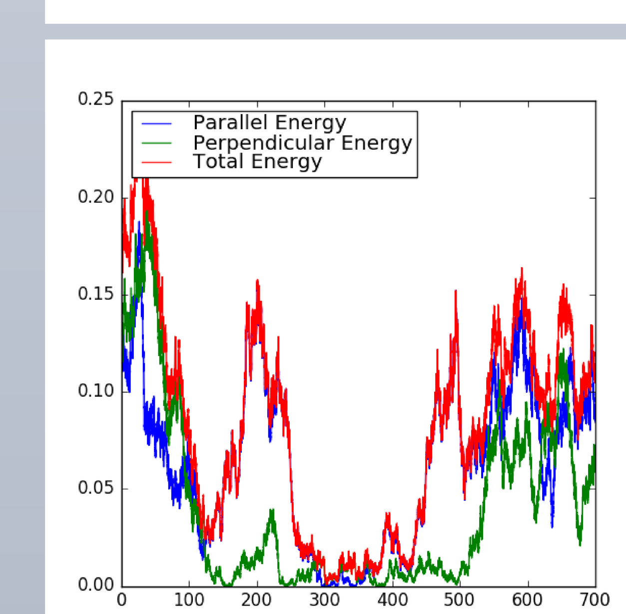
Collision Free





Collisions

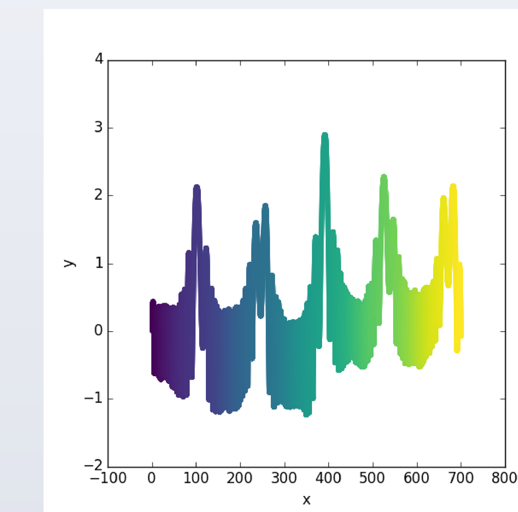




Conclusions/Further Research

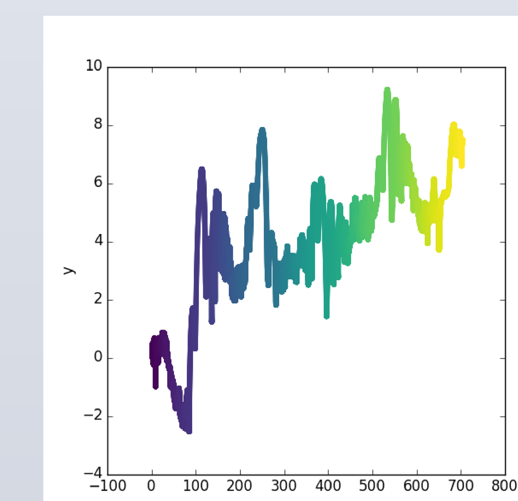
- Epsilon dependence

$\varepsilon = 0.5$



- Comparison to fluid-like model

$$\gamma = 10 \times v_{\boldsymbol{E}\times\boldsymbol{B}}/L$$



- Temperature Anisotropy
  - Parallel and Perpendicular velocities evolve differently

References

[1] J.M. Rax, R. Gueroult, N.J Fisch. Efficiency of wave-driven rigid body rotation toroidal confinement. Physics of Plasmas 24(3), 032504 (2017)
[2] Hong Qin et. al. Why is Boris Algorithm So Good? Physics of Plasmas 20(8), 084503 (2013)
[3] Chandrasekhar, S. Reviews of Modern Physics 15, 1 (1943)
[4] Toggweiler, Matthias. An adaptive time integration method for more efficient simulation of particle accelerators. ETH Zurich (2011)

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