Introduction to Stellarators

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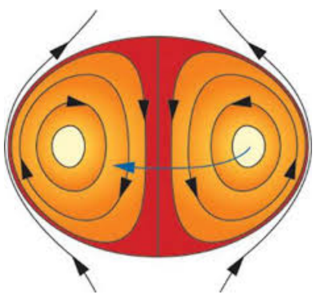
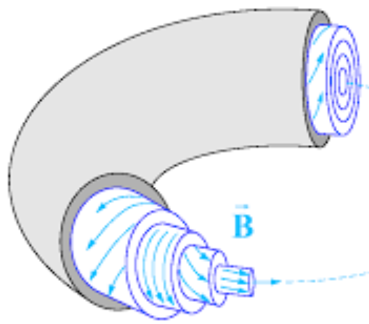
The continuum of magnetic confinement configurations

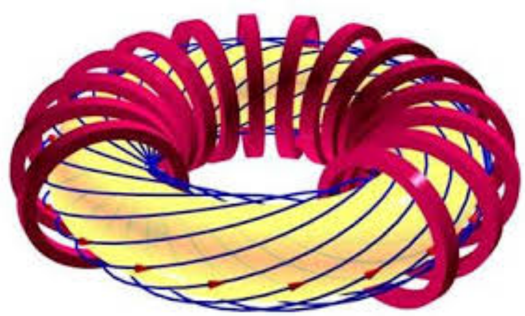
- Confining magnetic fields can be supplied/generated by internal plasma currents or external coils



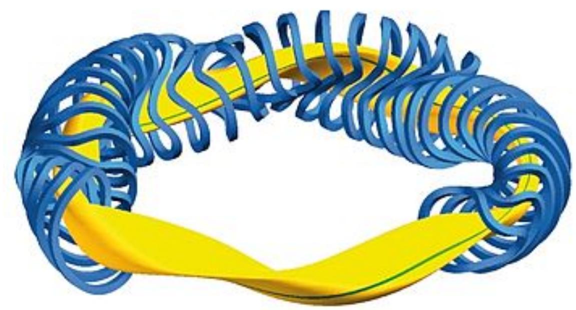
Compact tori



RFP



Tokamak



Stellarator

See M. Mauel 2015 SULI lecture for further discussion of other configurations

Stellarators were first conceived at Princeton

- Generation of rotational transform or field line twist without plasma current

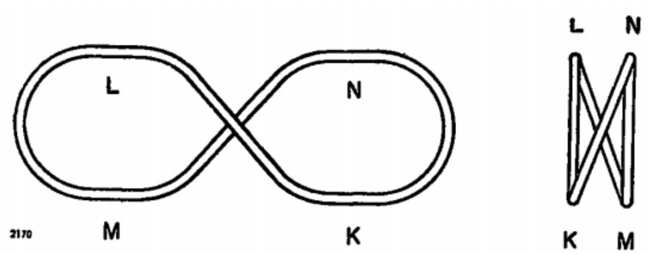


Figure 2. Top and end views of a figure-eight stellarator

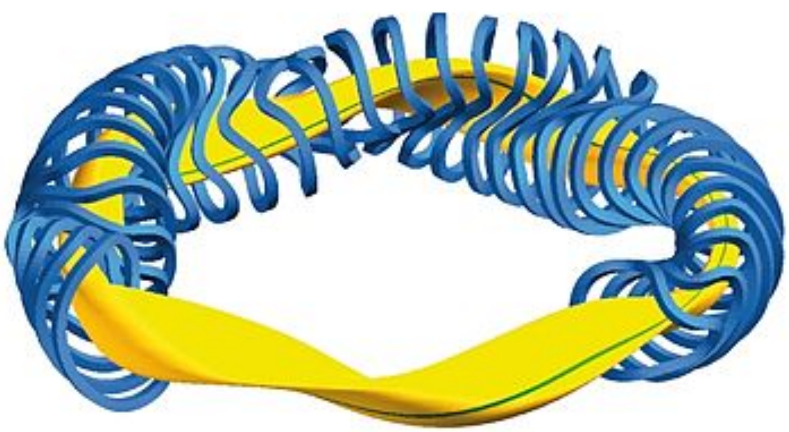
Proposed by Lyman Spitzer Jr in 1951
as part of Project Matterhorn

Called a stellarator or "star generator"

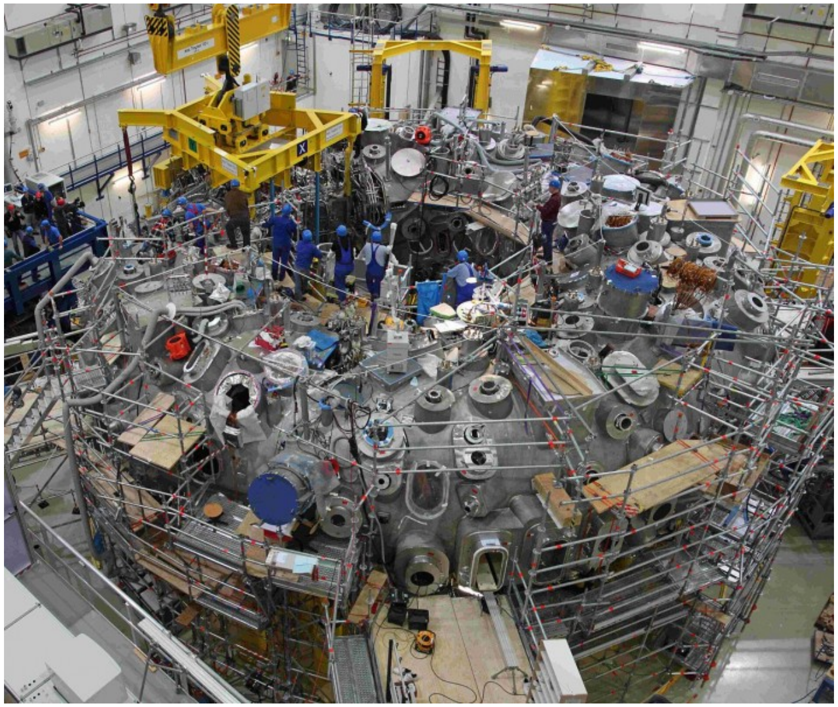
Project Matterhorn was later declassified
and renamed PPPL in 1961

It is an exciting time to be studying stellarator physics: W7-X

- First experimental test of stellarator optimization to produce tokamak like neoclassical transport



Wendelstein 7-X



Some Stellarator advantages...

- Intrinsically steady-state, without the need to drive plasma current
- Lack of plasma current removes large class of instabilities that are seen in tokamaks
- Magnetic configuration given by external coils is rigid, no disruptive loss of confinement
- Potential for greater range of designs and optimization of fusion performance

Some Stellarator disadvantages...

- Complicated coil configurations that are difficult to design, need to be precisely built, and are expensive as a result
- Achieving good particle confinement is more difficult than in tokamaks
- Divertor and heat load geometry is more complicated than in tokamaks

Outline

- Confining a plasma in a torus and the need for rotational transform
- Generating magnetic surfaces without net plasma current
- New directions in stellarator research
- Auburn University fusion program

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How do magnetic fields confine ionized matter?

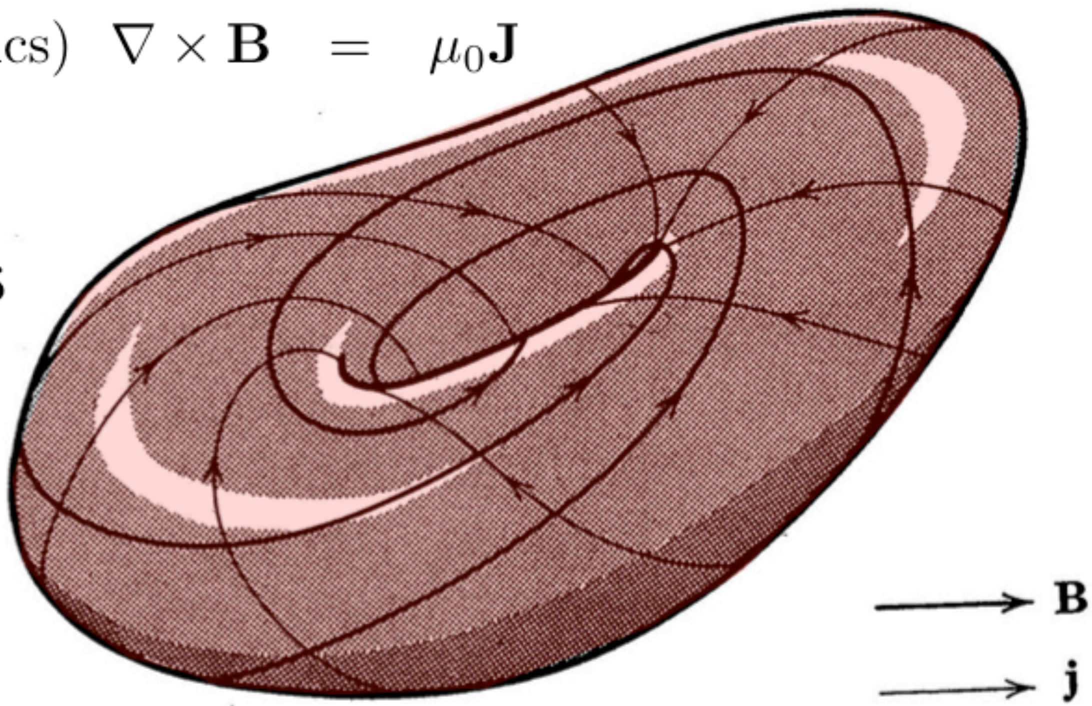
Equations of magnetic confinement…

$$\begin{aligned} \text{(No monopoles)} \quad \nabla \cdot \mathbf{B} &= 0 \\ \text{(No charge accumulation)} \quad \nabla \cdot \mathbf{J} &= 0 \\ \text{(No unbalanced forces)} \quad 0 &= -\nabla P + \mathbf{J} \times \mathbf{B} \\ \text{(Magnetostatics)} \quad \nabla \times \mathbf{B} &= \mu_0 \mathbf{J} \end{aligned}$$

Plasma Pressure (P)

Current ($\mathbf{J}$)

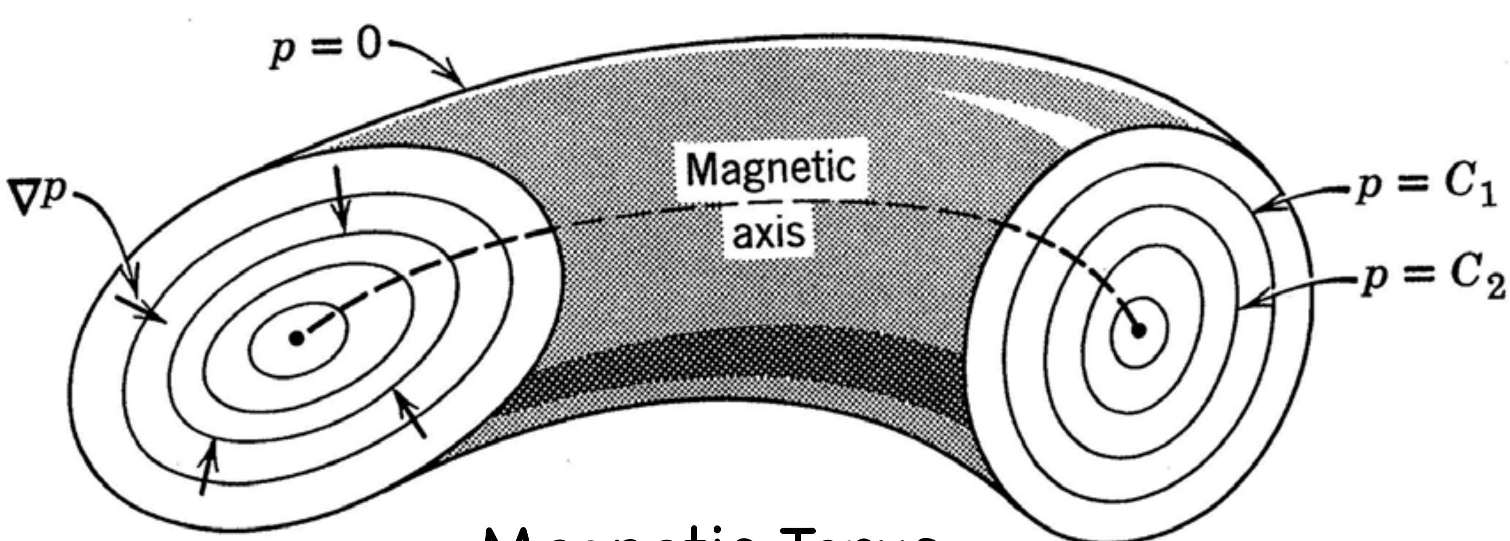
Magnetic Torus



How do magnetic fields confine ionized matter?

$$\mathbf{J} \times \mathbf{B} = \nabla P$$
$$\mathbf{B} \cdot \nabla P = 0$$
$$\mathbf{J} \cdot \nabla P = 0$$

Surfaces of constant plasma pressure form nested tori

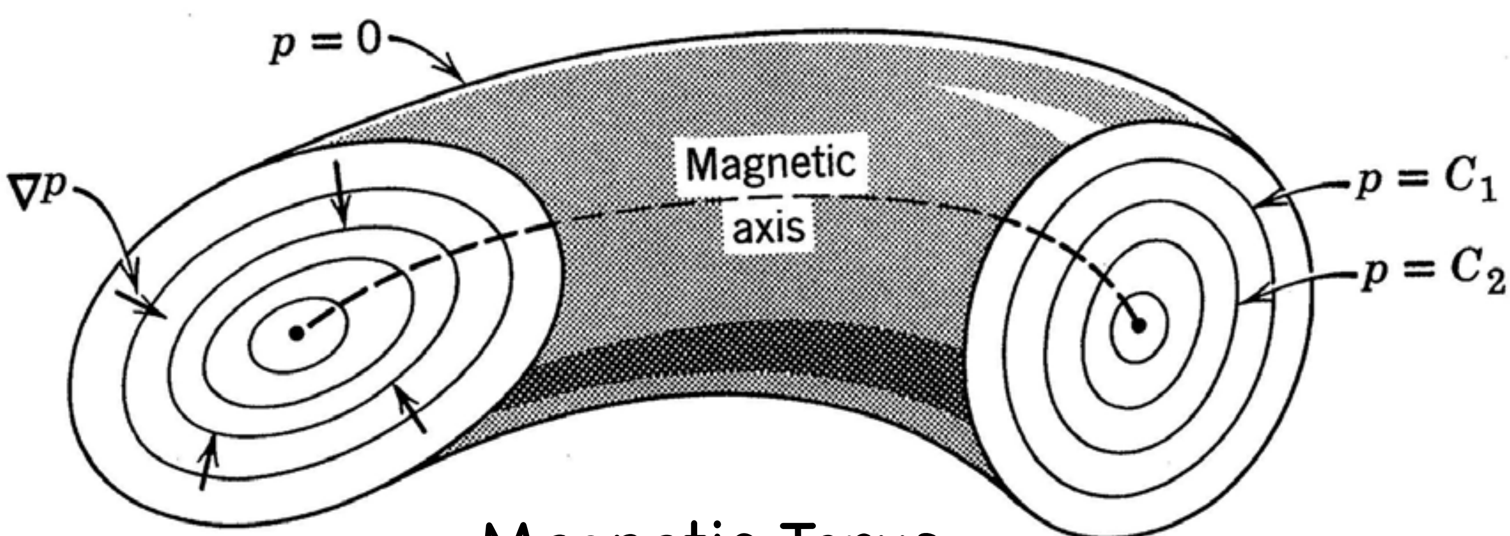


Magnetic Torus

How do magnetic fields confine ionized matter?

$$\begin{aligned} \mathbf{J} \times \mathbf{B} &= \nabla P \\ \mathbf{B} \cdot \nabla P &= 0 \\ \mathbf{J} \cdot \nabla P &= 0 \end{aligned}$$

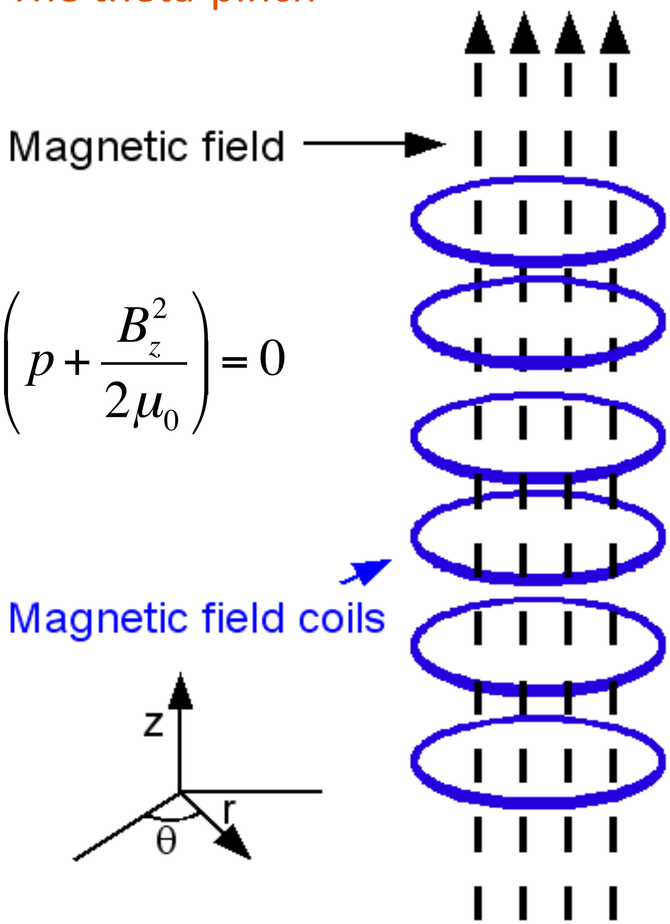
Surfaces of constant plasma pressure form nested tori



Magnetic Torus

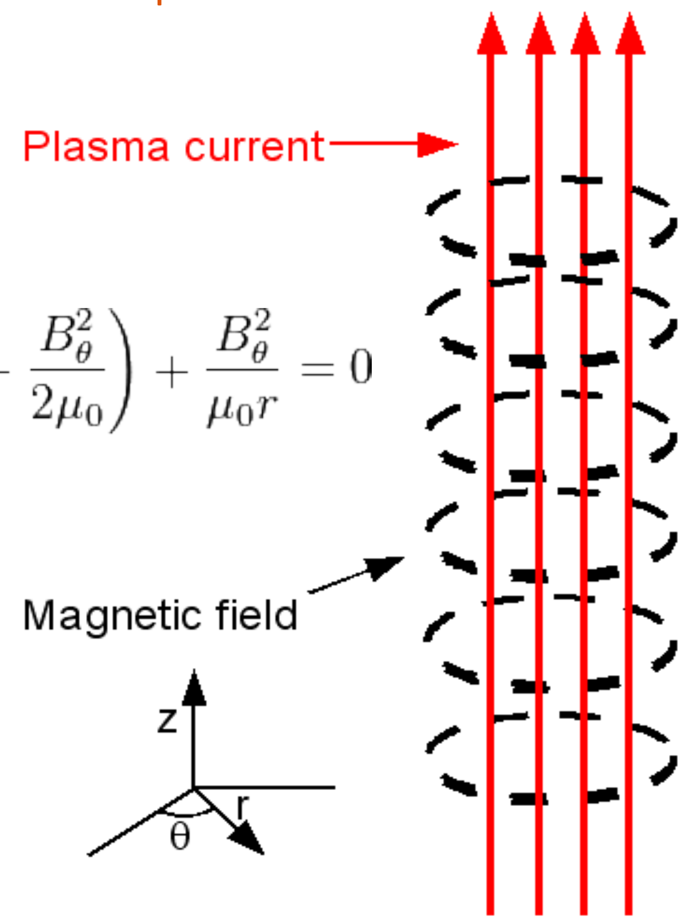
Start with simple cylindrical equilibria

The theta-pinch



$$\frac{d}{dr}\left(p + \frac{B_z^2}{2\mu_0}\right) = 0$$

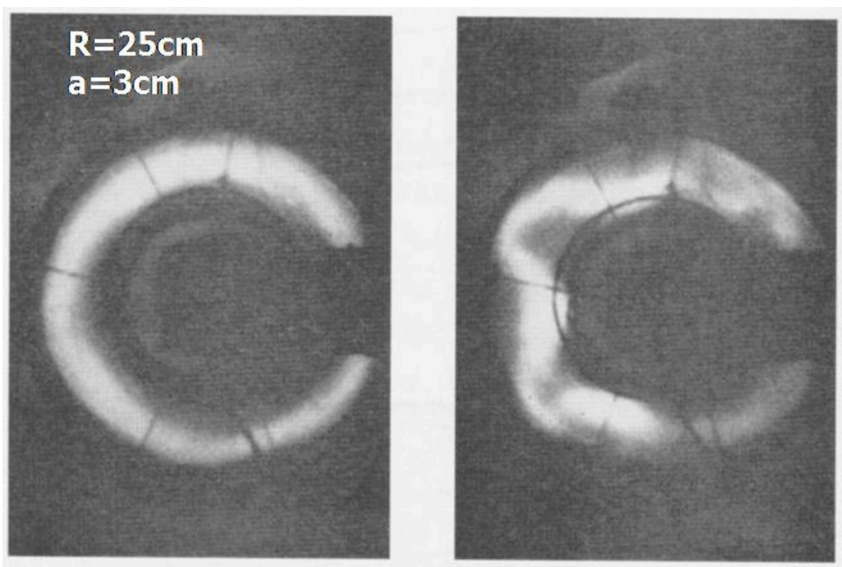
The z-pinch

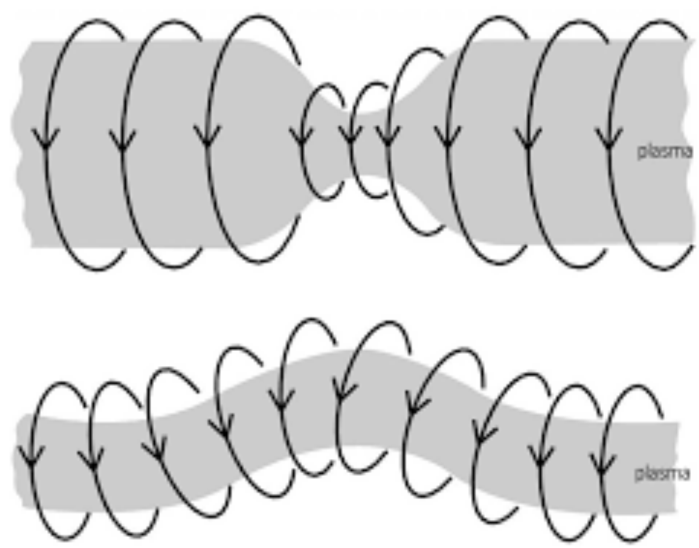


$$\frac{d}{dr}\left(p + \frac{B_\theta^2}{2\mu_0}\right) + \frac{B_\theta^2}{\mu_0 r} = 0$$

Why do we need twisting field lines for confinement?

- Bend Z pinch or Theta pinch into a torus



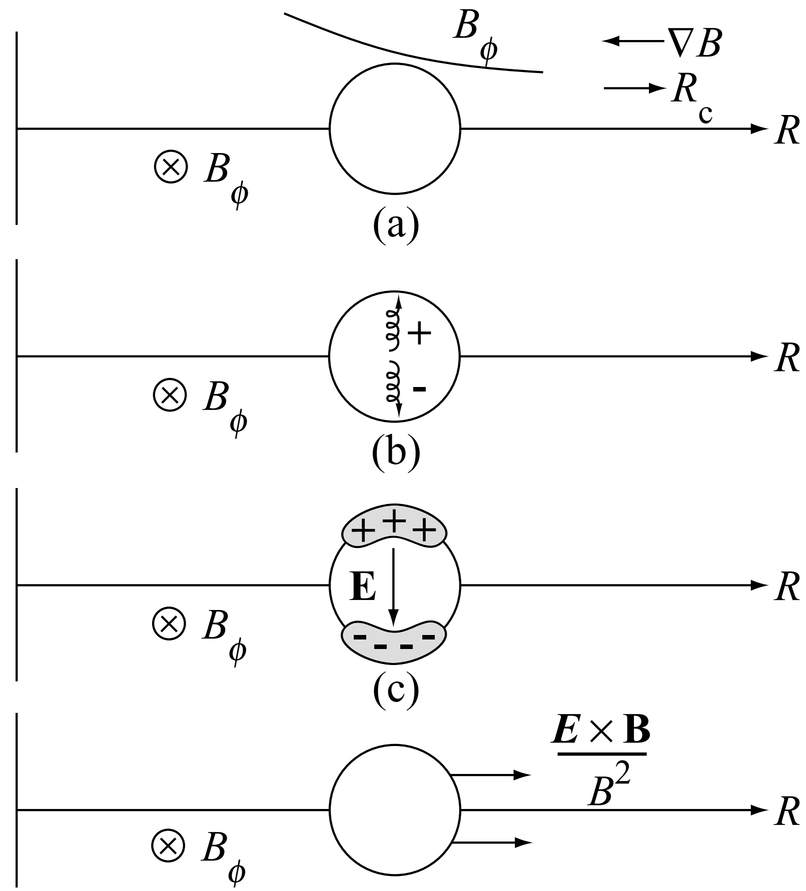


Z pinch is very MHD unstable...

Why do we need twisting field lines for confinement?

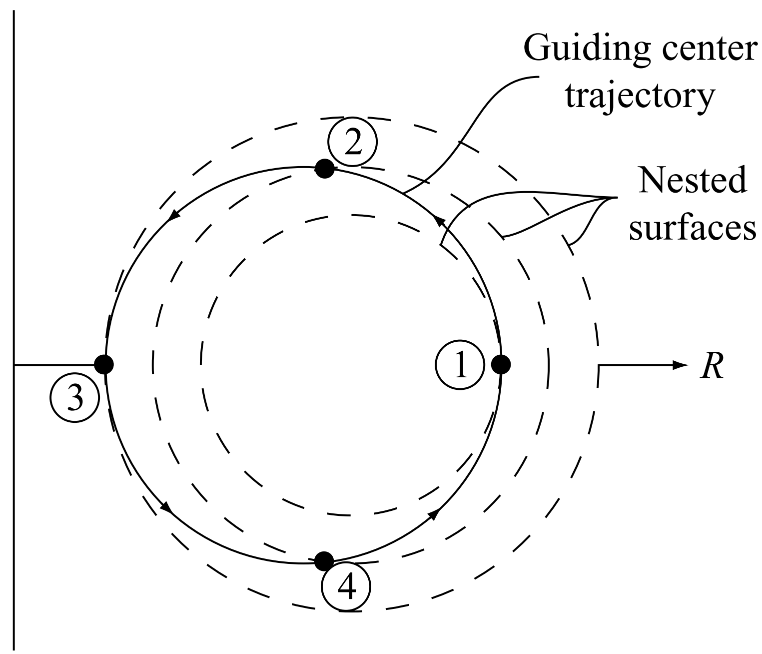
- Bend Theta pinch into a torus: guiding center picture

No equilibrium!



Why do we need twisting field lines for confinement?

- Spitzer's insight/solution



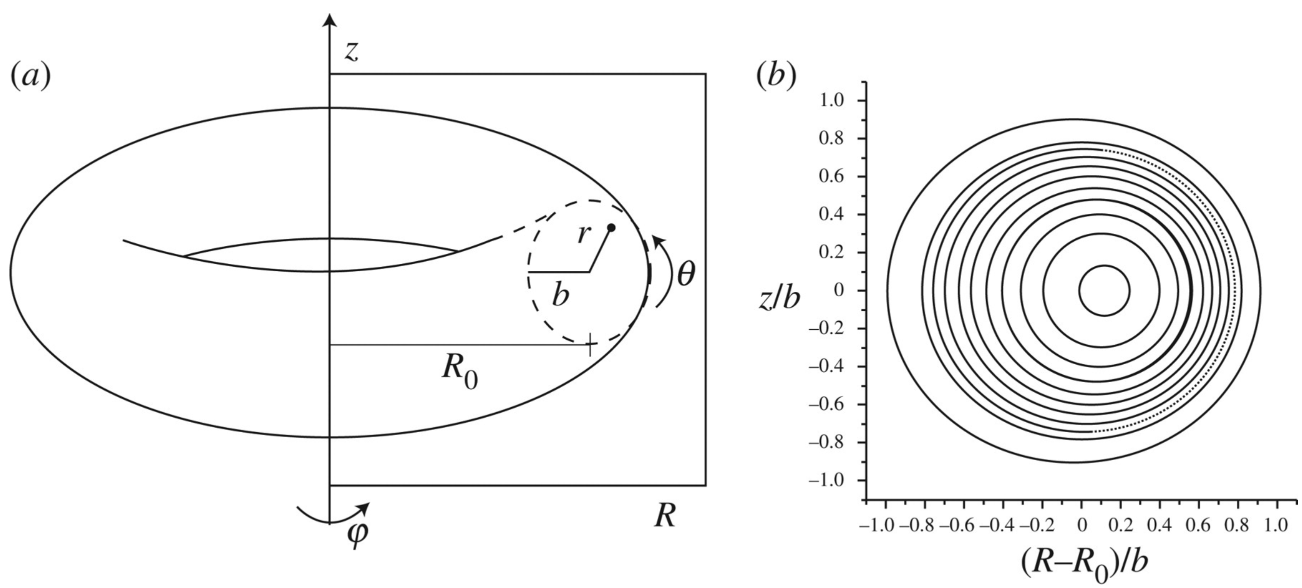
Twist causes "up" to be away from the midplane half the time and towards it the other half, thus averaging the vertical drift out

Outline

- Confining a plasma in a torus and the need for rotational transform
- Generating magnetic surfaces without net plasma current
- New directions in stellarator research
- Auburn University fusion program

Rotational transform is a measure of field line twisting

- For a screw pinch equilibrium: $\vec{B} = B_\theta(r)\hat{\theta} + B_\phi(r)\hat{\phi}$
  - Pitch: $P(r) = \dfrac{B_\theta(r)}{B_\phi(r)}$
  - Rotational transform: $\iota = \dfrac{d\theta}{d\phi} = \dfrac{R_0 B_\theta(r)}{r B_\phi(r)}$
  - Field lines and magnetic surfaces



The three ways to generate rotational transform

- Net toroidal plasma current like in the tokamak
- Torsion (non-planar) magnetic axis like the original figure eight stellarator
- Non-circular deformation of the magnetic surfaces in resonance with field line motion

Equations for field line motion

Simple example field: $B(\vec{r}) = B_0\hat{z} + \hat{z} \times \nabla f(\vec{r})$

For motion along the field direction: $\dfrac{dx}{B_x} = \dfrac{dy}{B_y} = \dfrac{dz}{B_z}$

Equations for field line motion

Simple example field: $B(\vec{r}) = B_0\hat{z} + \hat{z} \times \nabla f(\vec{r})$

For motion along the field direction: $$\frac{dx}{B_x} = \frac{dy}{B_y} = \frac{dz}{B_z}$$

Some algebra yields...

$$\frac{dx}{dz} = -\frac{1}{B_0}\frac{\partial f}{\partial y} \qquad \frac{dy}{dz} = +\frac{1}{B_0}\frac{\partial f}{\partial x}$$

Equations for field line motion

Our field line "equations of motion" for this simple model are

$$\frac{dx}{dz} = -\frac{1}{B_0}\frac{\partial f}{\partial y} \qquad \frac{dy}{dz} = +\frac{1}{B_0}\frac{\partial f}{\partial x}$$

Identifying $x \to q \quad y \to p \quad z \to t$ and setting $f = -B_0 H$

$$\Longrightarrow \qquad \dot{q} = -\frac{\partial H}{\partial p} \qquad \dot{p} = +\frac{\partial H}{\partial q}$$

Magnetic field lines are a Hamiltonian system!

Our field line "equations of motion" for this simple model are

$$\frac{dx}{dz} = -\frac{1}{B_0}\frac{\partial f}{\partial y} \qquad \frac{dy}{dz} = +\frac{1}{B_0}\frac{\partial f}{\partial x}$$

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$$\Longrightarrow \quad \boxed{\dot{q} = -\frac{\partial H}{\partial p} \qquad \dot{p} = +\frac{\partial H}{\partial q}}$$

These are Hamilton's equations of classical mechanics

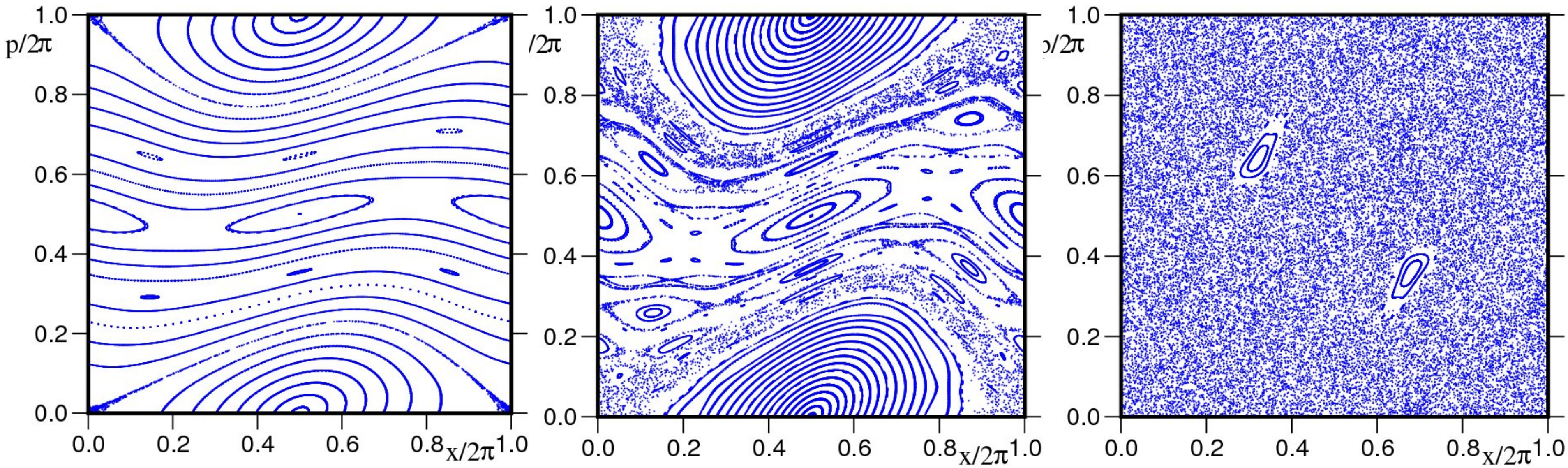
Magnetic field line structure exhibits all the complexity of Hamiltonian chaos

Magnetic surfaces are fragile unless properly made

Magnetic field line structure exhibits all the complexity of Hamiltonian chaos

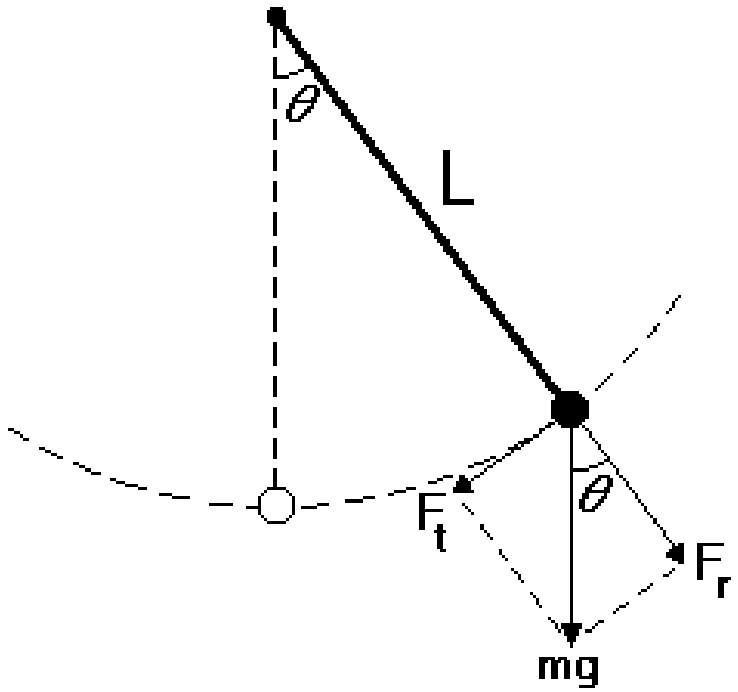
Magnetic surfaces are fragile unless properly made

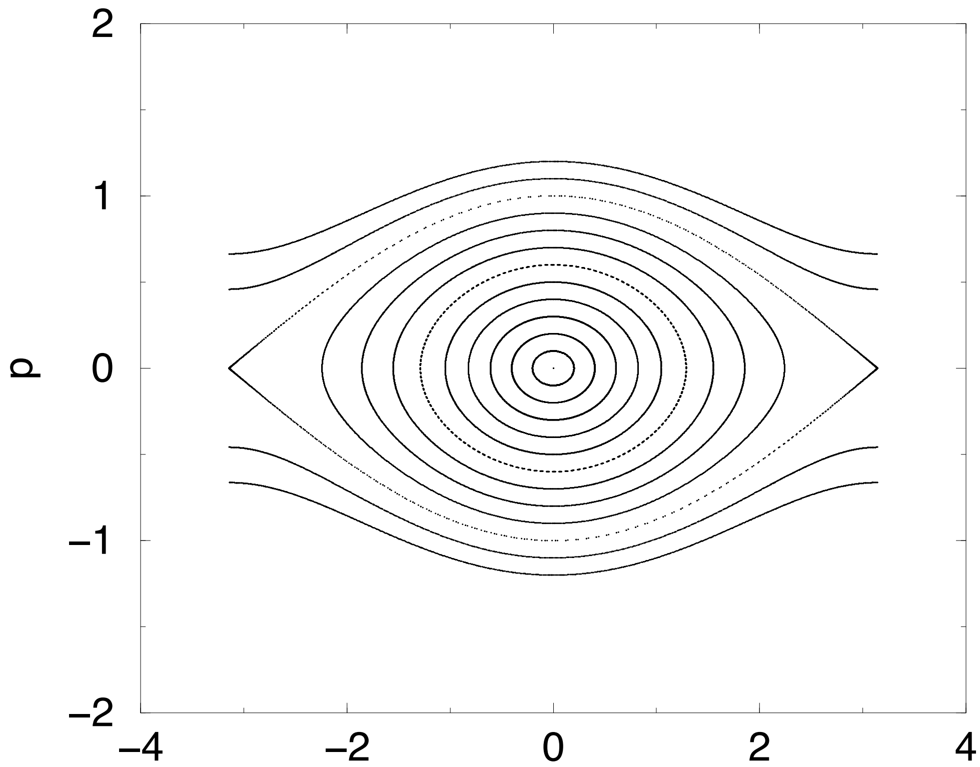
Increasing perturbation strength →



Standard map exhibits this generic behavior

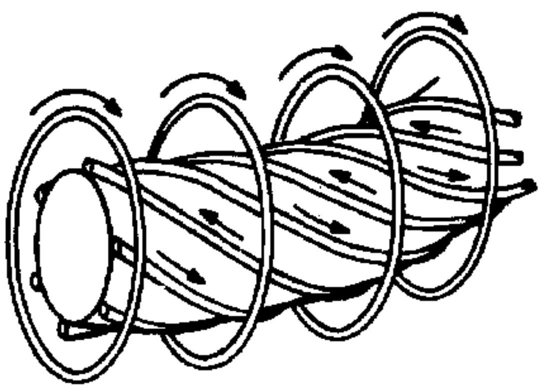
Magnetic islands and your friend the simple pendulum



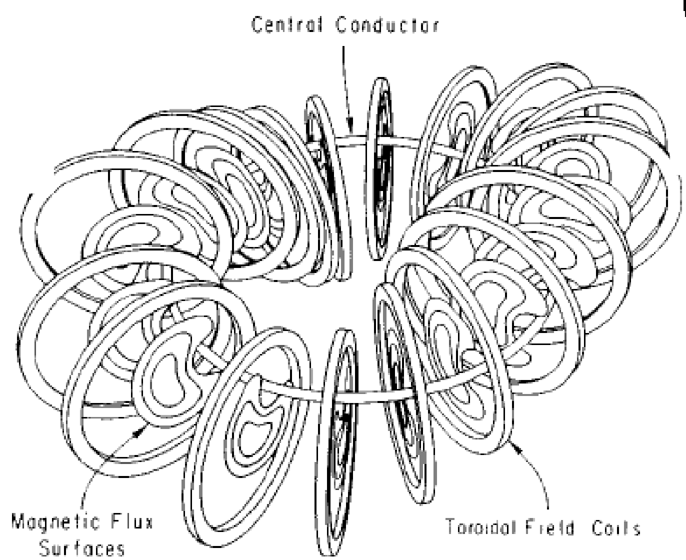


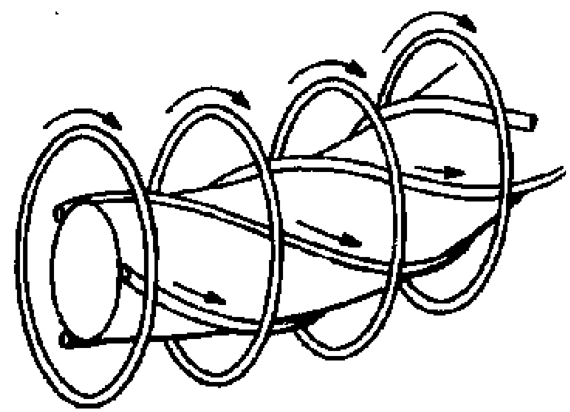
Ways to construct good magnetic surfaces without net current have been devised

- There are several "classical" stellarator device types that can do it



STELLARATOR (ℓ=3)

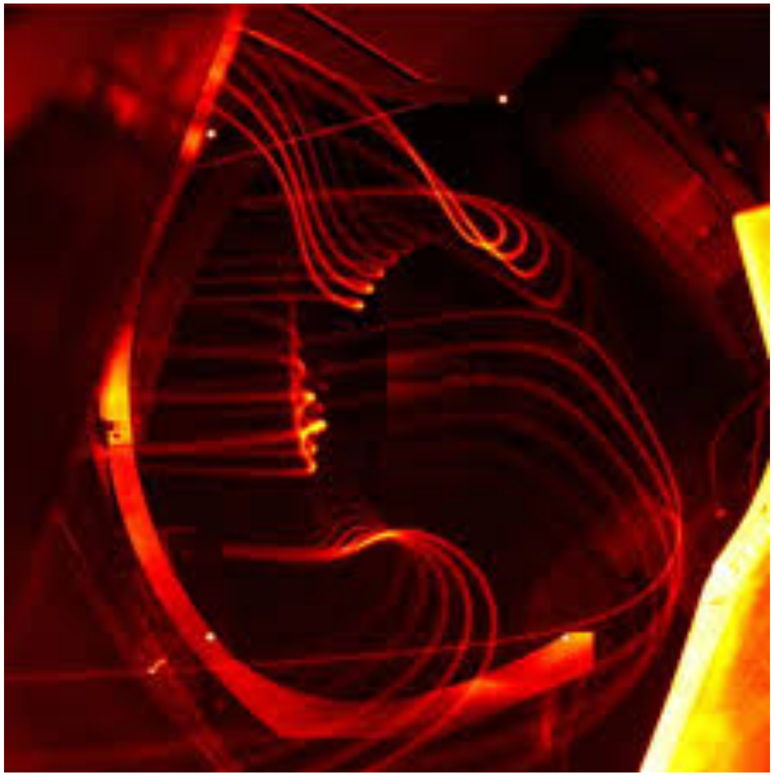




HELIOTRON (ℓ=3)

Suitably good magnetic surfaces can be constructed experimentally

- Measurement of flux surface integrity using electron beam mapping



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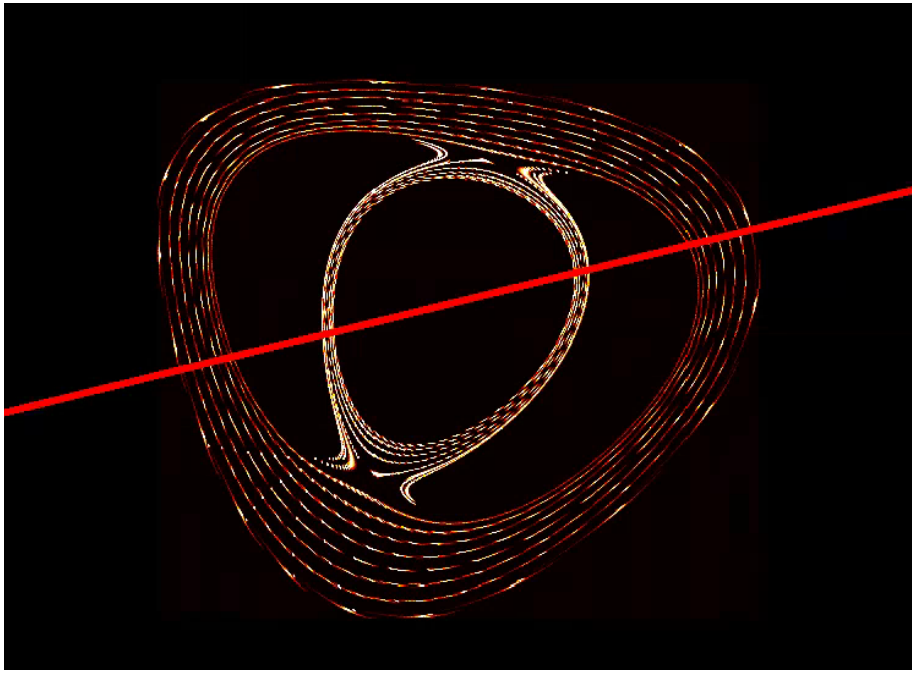
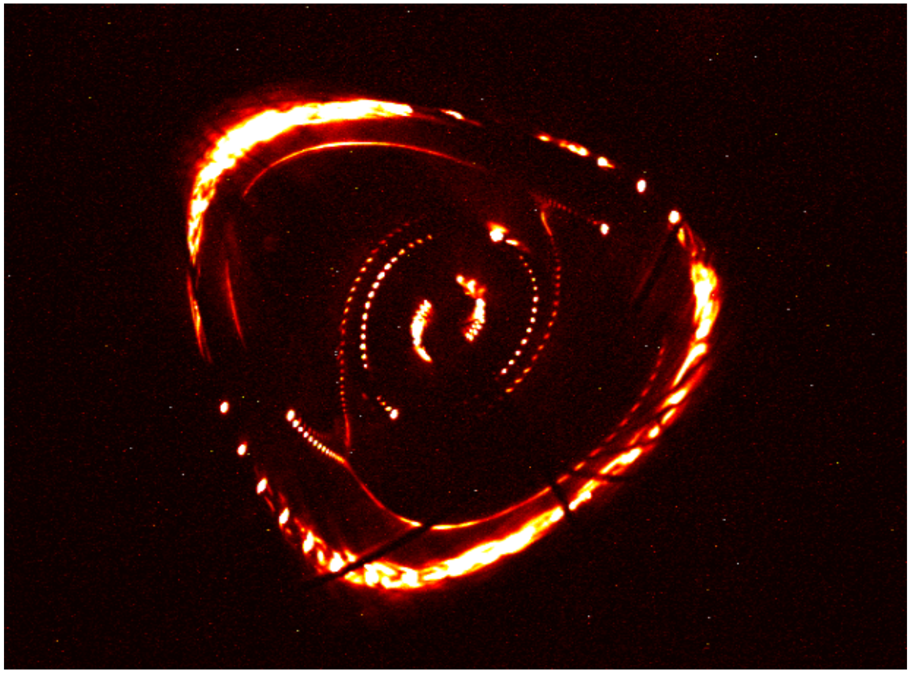


Synthetic diagnostic



Composite Image

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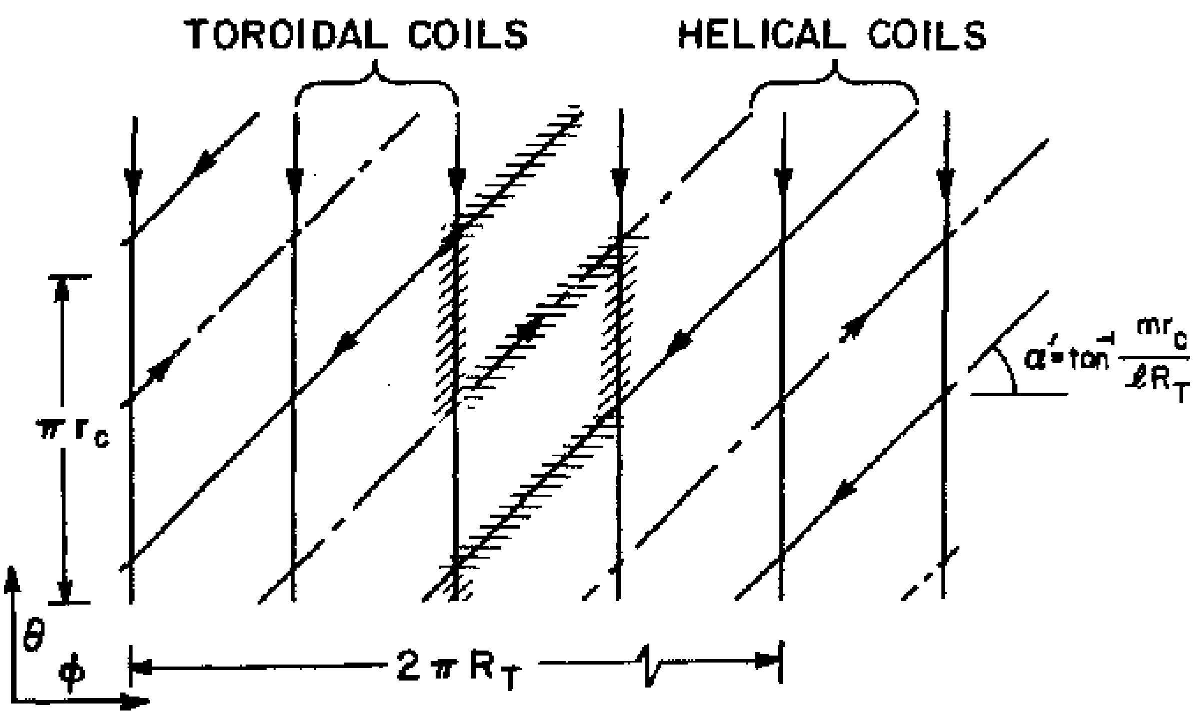
Helical coils used in classical stellarators are continously wound

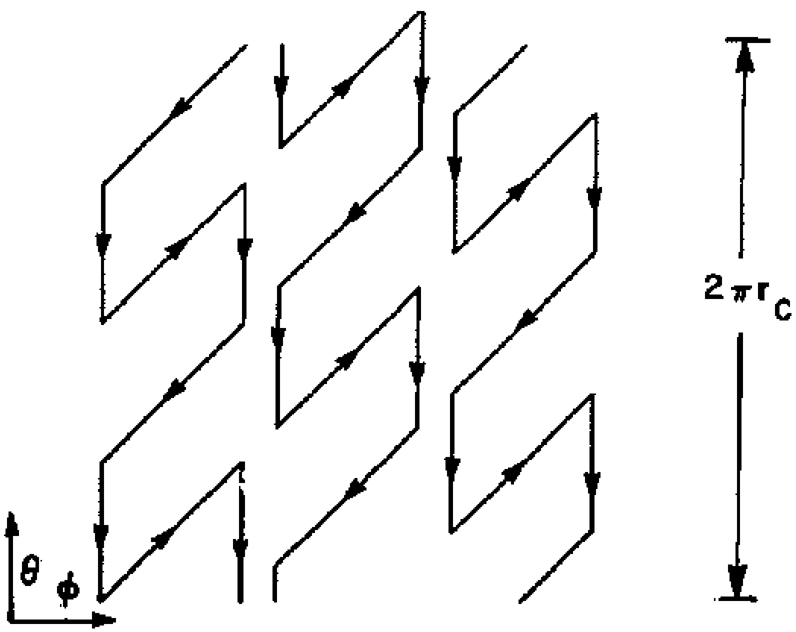
- Motivation for modular coil design



Coil currents in (θ, ϕ) space

Modular coils have advantages

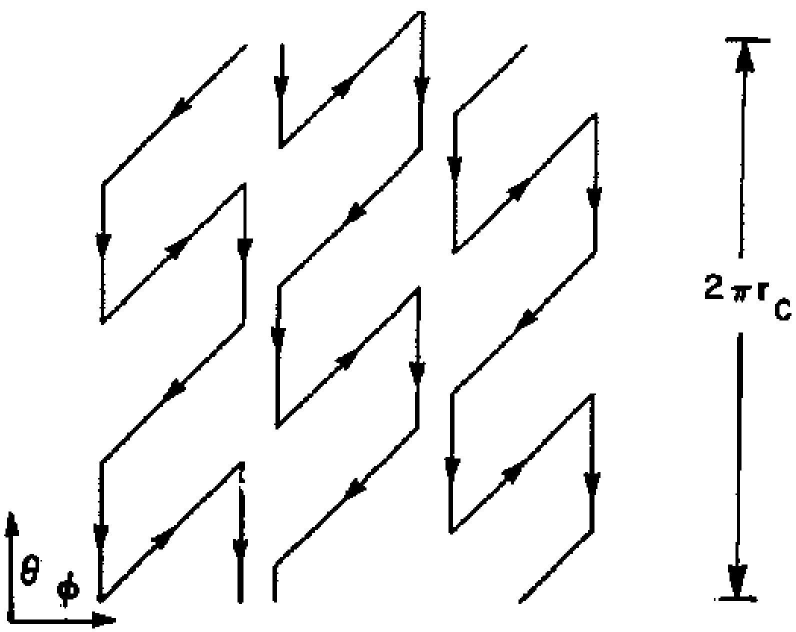
- Coils can be built independently, harder to design, but allow easier assembly/disassembly of device and access



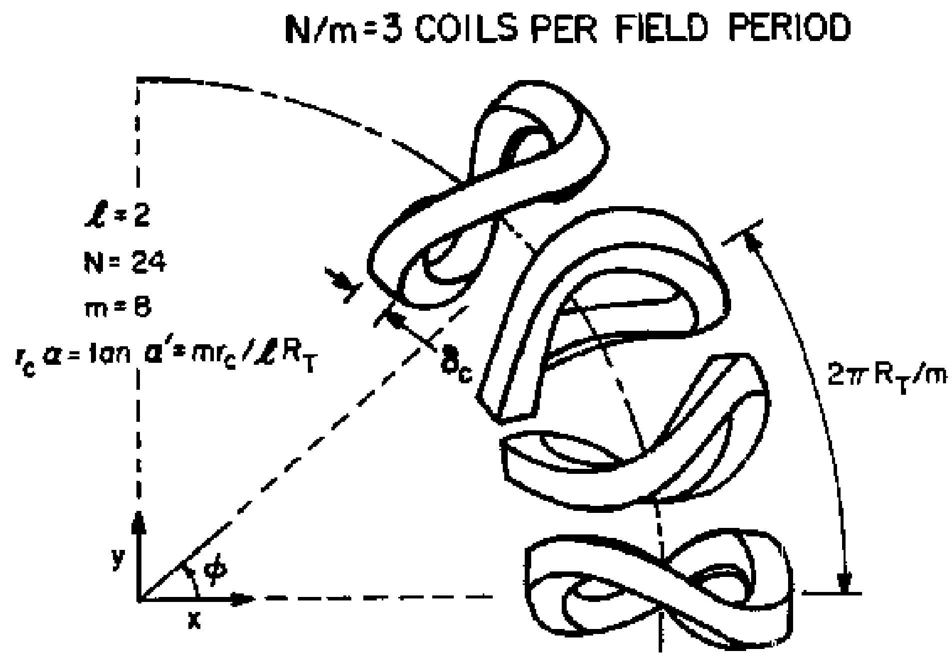
(θ, ϕ) space

Modular coils mapped back into physical space have non-trivial shape

- Coils can be built independently, harder to design, but allow easier assembly/disassembly of device and access



(θ, ϕ) space

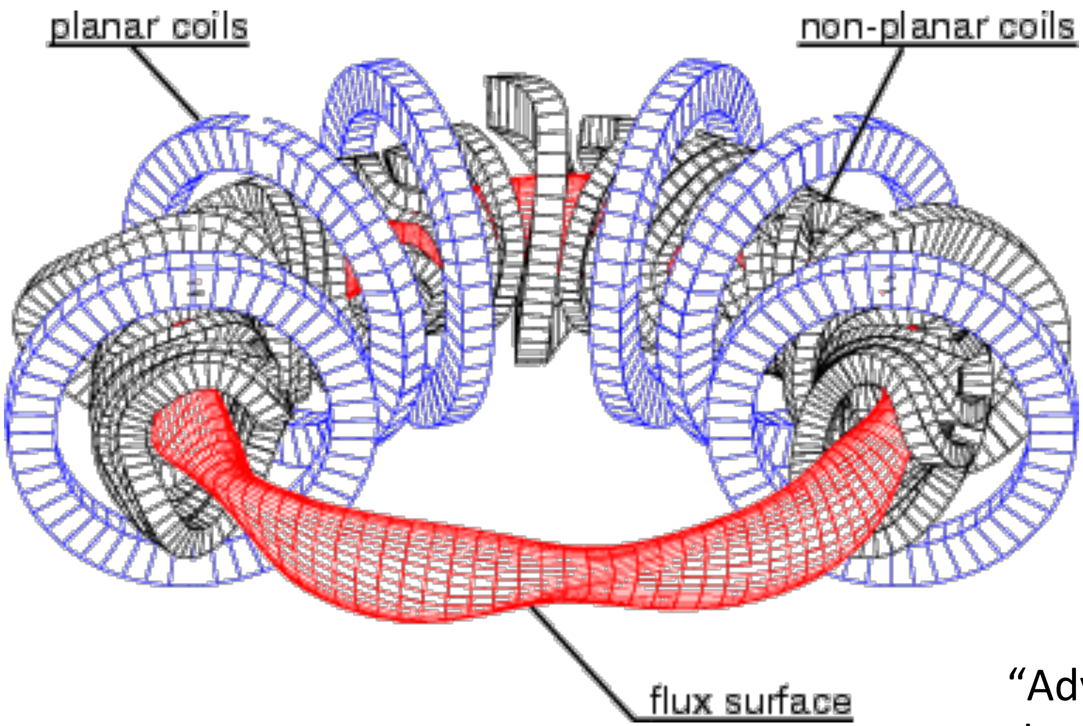


Physical space

Modular coils allow "plasma first" design

We can design the plasma equilibrium based on physics considerations (equilibrium, stability, transport) and then design a set of coils to produce the required field.

Modular coils of W7-AS



"Advanced" rather than classical stellarator

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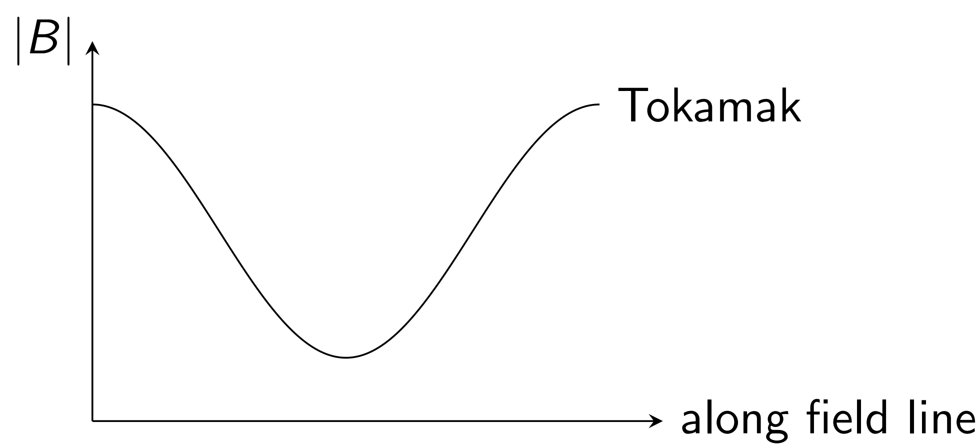
Given 3D equilibrium magnetic surfaces, how leaky are they in terms of plasma transport?

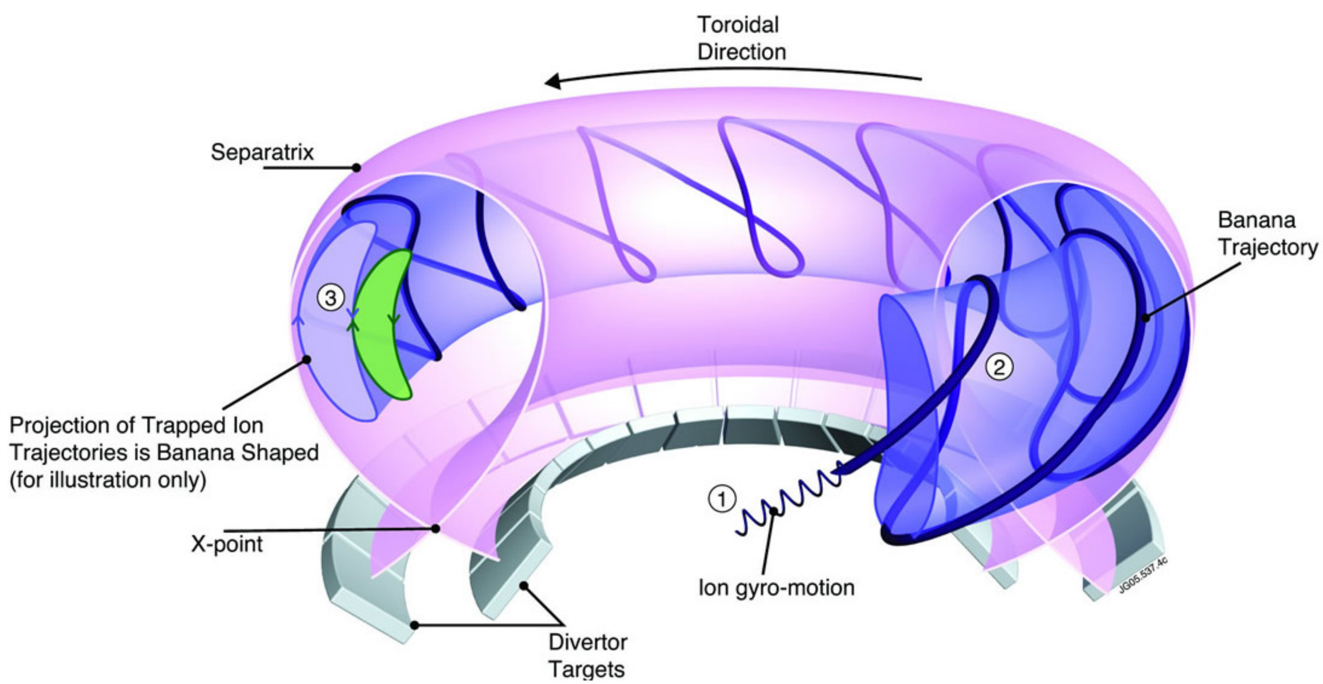
- Trajectories in axisymmetry: passing and trapped particles



- Canonical momentum conservation due to axisymmetry also bounds excursion from flux surfaces: $p_\phi = mRv_\phi + q\psi$

Particle trapping leads to so-called banana orbits in tokamaks

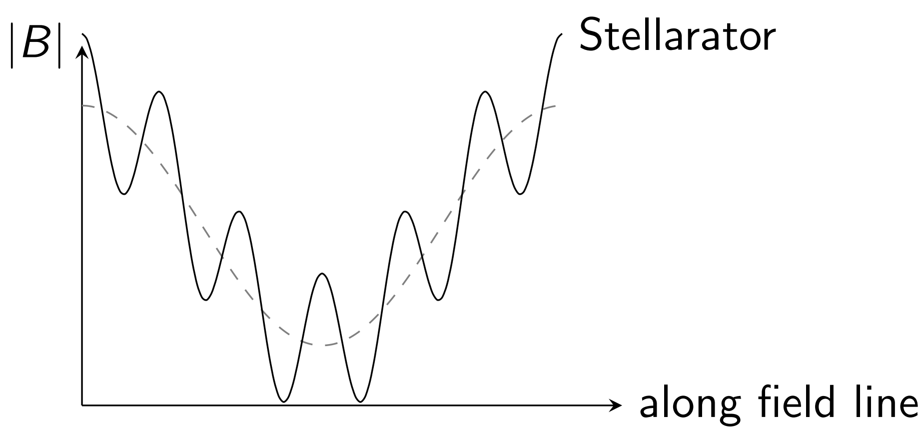
- Trapped trajectories projected to a poloidal plane are banana shaped



- Increases effective step size for collisional transport, with trapped particles dominating transport rate

Particle trajectories in a stellarator

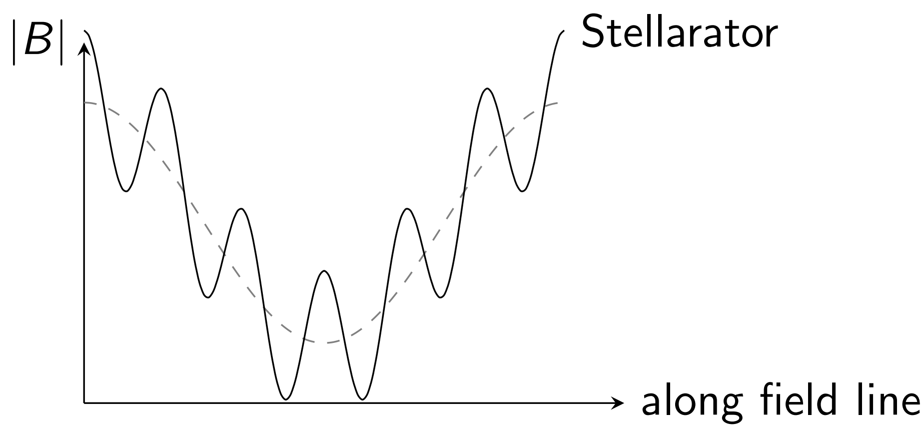
- As in a tokamak, particles can be passing or trapped due to toroidicity
- There are also particles which get trapped in local minima due to the helical periodicity



- These particles trapped in local minima are confined to regions on the upper or lower half of the flux surface

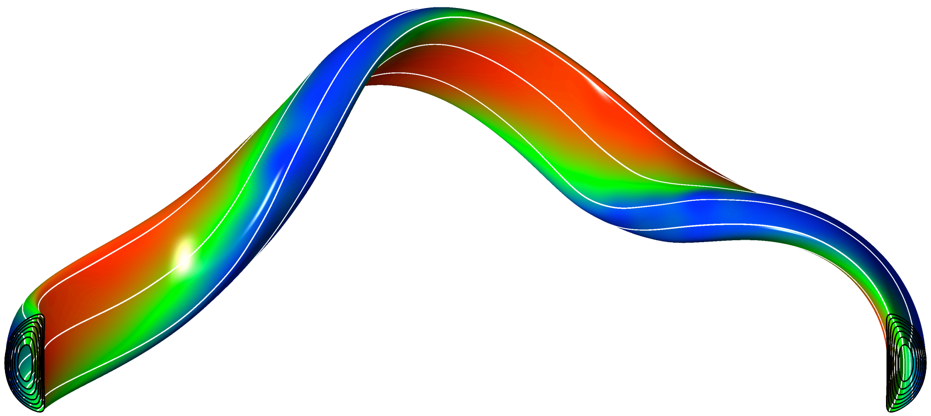
Direct orbit loss

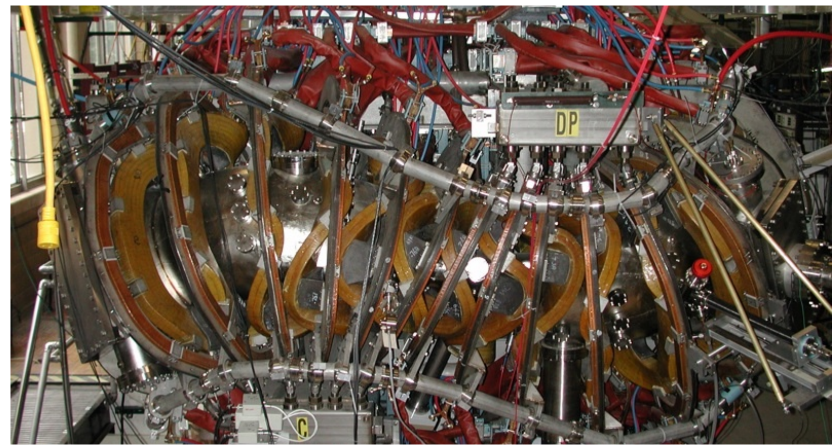
- As a result their vertical drifts don't cancel out, and they drift straight out of the machine.



- This drift is different for electrons and ions, and so leads to electric fields also
- No conserved canonical momentum without axisymmetry to help limit excursions also

Stellarator optimization: Quasi-symmetry

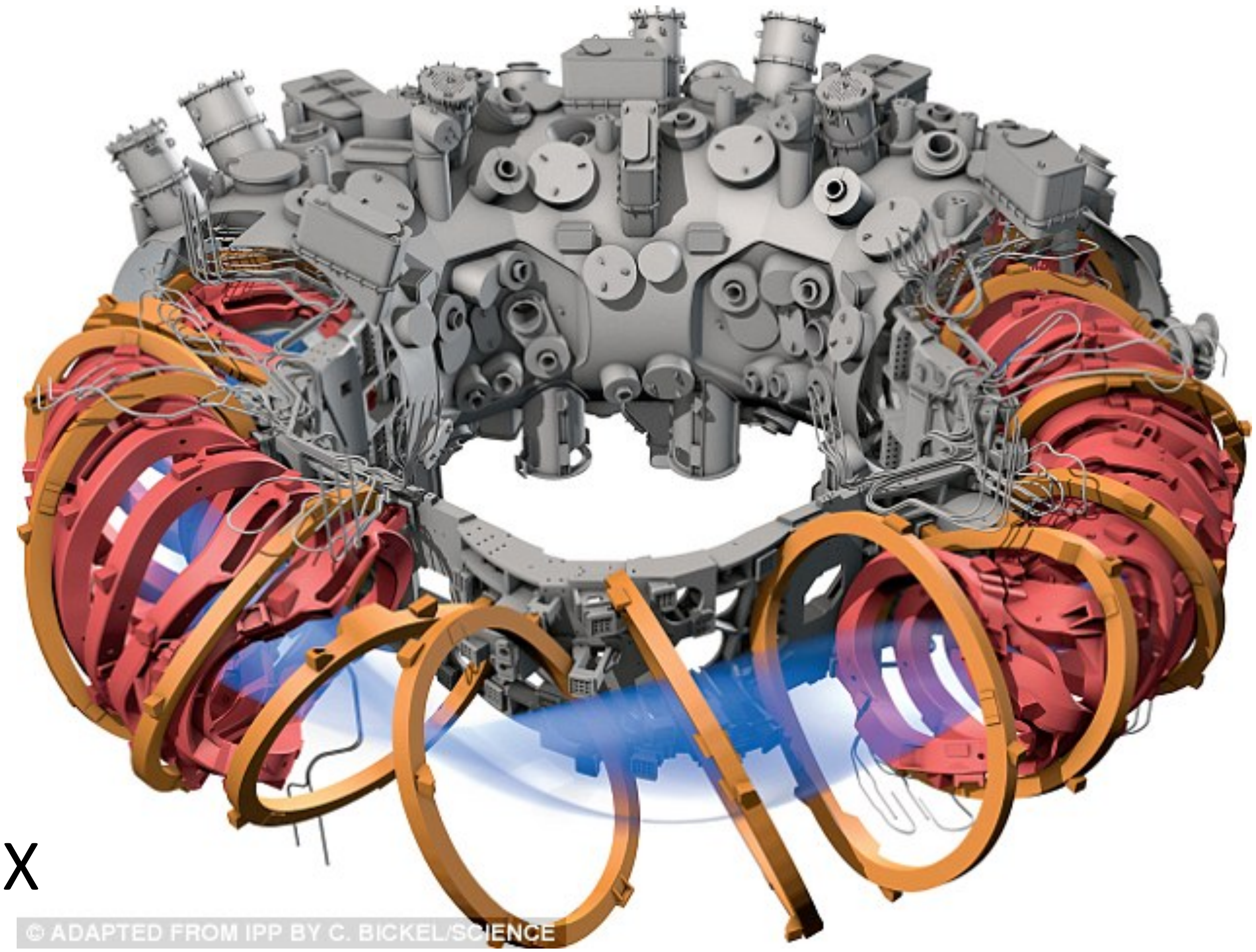




Particle drift orbits only depend on magnetic field strength, so symmetry in it gives rise to canonical momentum conservation

Helically Symmetric eXperiment (HSX) at University of Wisconsin-Madison confirmed reduction in direct loss orbits through use of quasi-symmetry

Stellarator optimization: Isodynamic



W7-X

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Disruption avoidance and mitigation essential for future current carrying tokamaks

- Context: Small amounts of 3D fields are used for a variety of purposes on present day tokamaks with $B_{3D}/B_0 \sim 10^{-3}$
- Can application higher levels of 3D magnetic shaping, $B_{3D}/B_0 \sim 0.1$, suppress tokamak instabilities and disruptions?

Work informs experimental basis for:

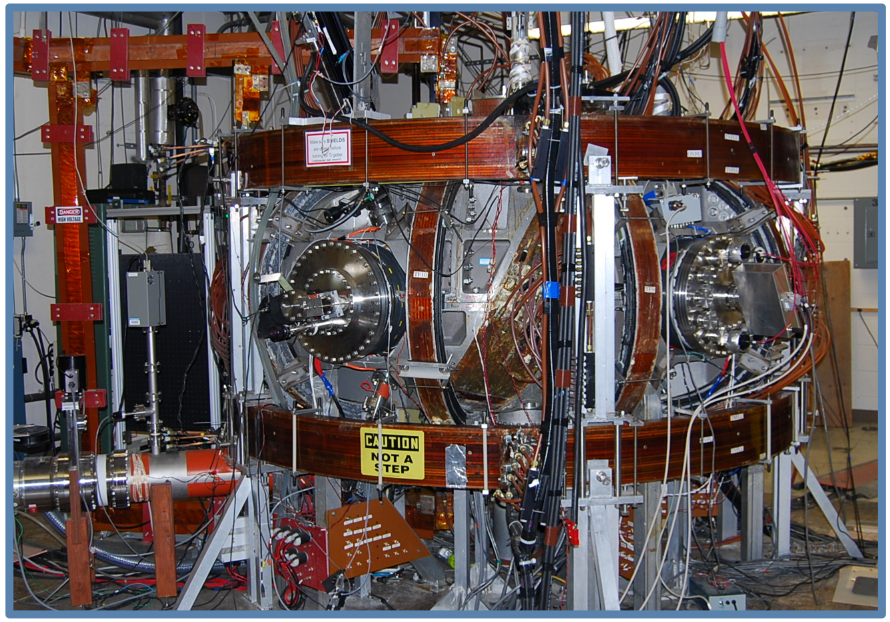
--- Stability properties of compact quasi-axisymmetric stellarators

--- Possible use of external transform on tokamak systems

--- Shed light on tokamak disruption physics and 3D MHD

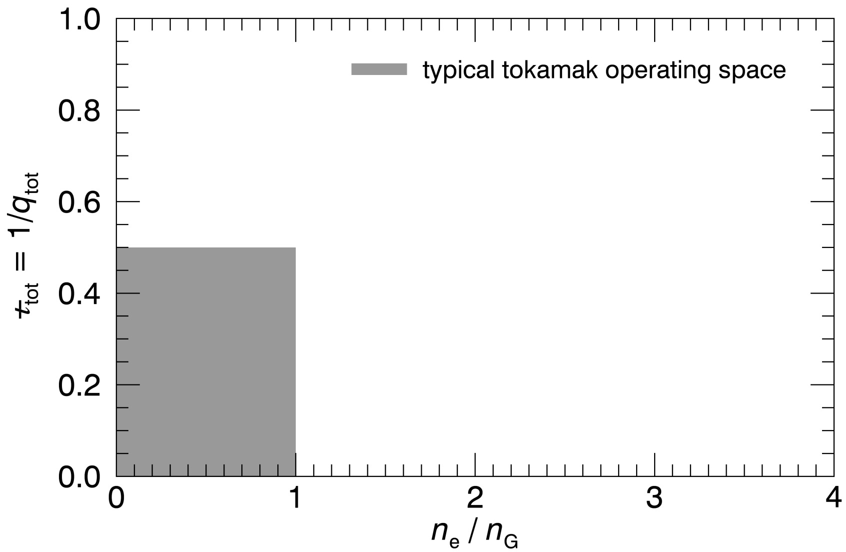
The Compact Toroidal Hybrid (CTH) was designed to address these issues

- **Hybrid**: current driven within 3D equilibrium of a stellarator plasma
- Can vary the relative amount of externally applied transform
  - I_p provides up to 95%

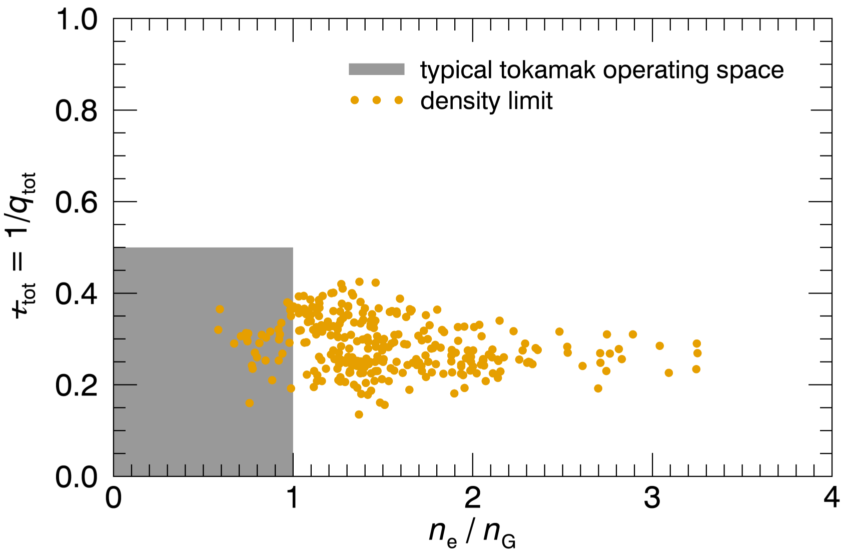


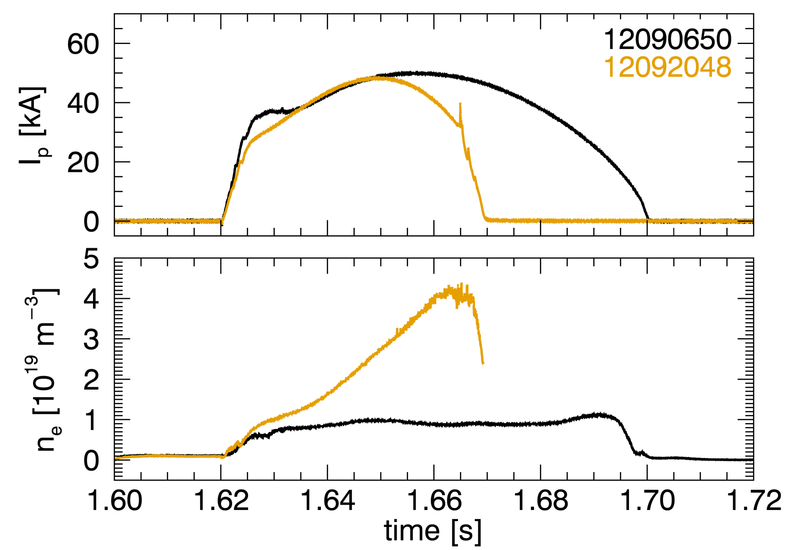
- Previous hybrids showed evidence of disruption avoidance and improved positional stability (W7-A team, Nucl. Fusion. 1980, H. Ikezi et al, Phys. Fluids. 1979)

Overview of CTH operational space and the 3 types of disruptions observed



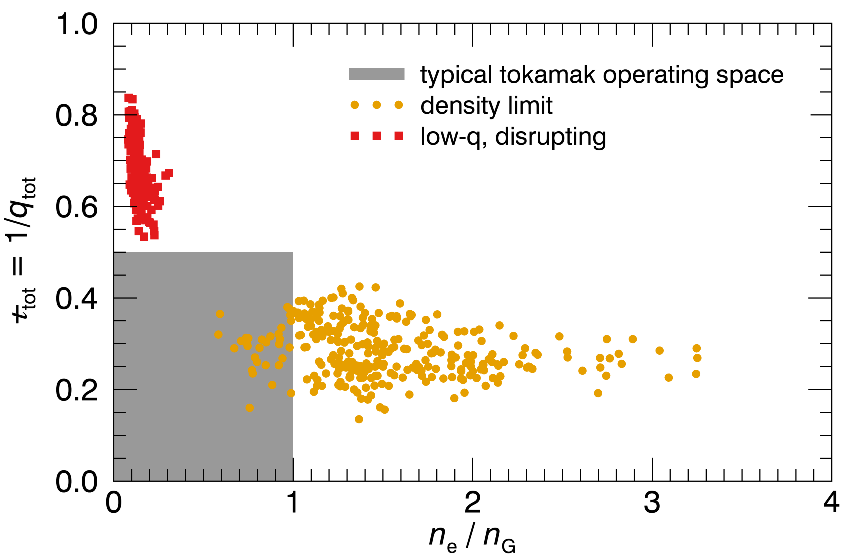
CTH can operate beyond the Greenwald density limit

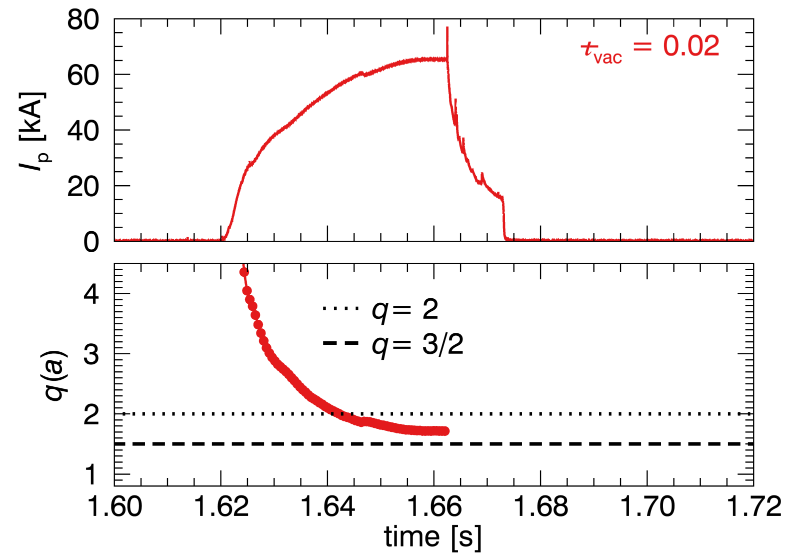




- Density-limit disruptions

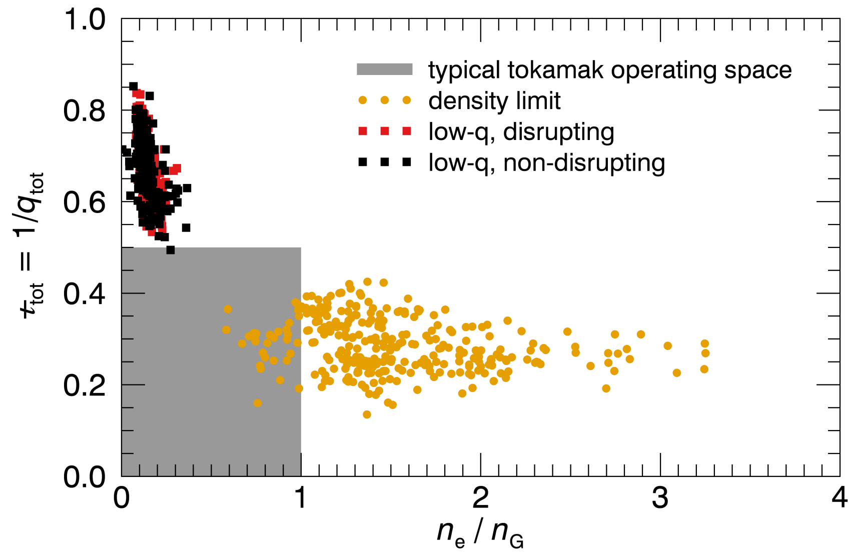
Low-q disruptions can occur when CTH operates with $q(a) < 2$

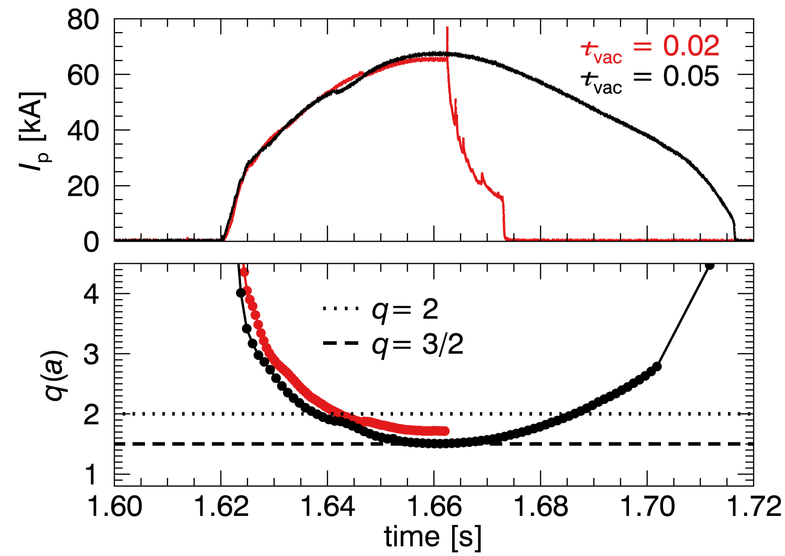




- Density-limit disruptions
- Low-q disruptions

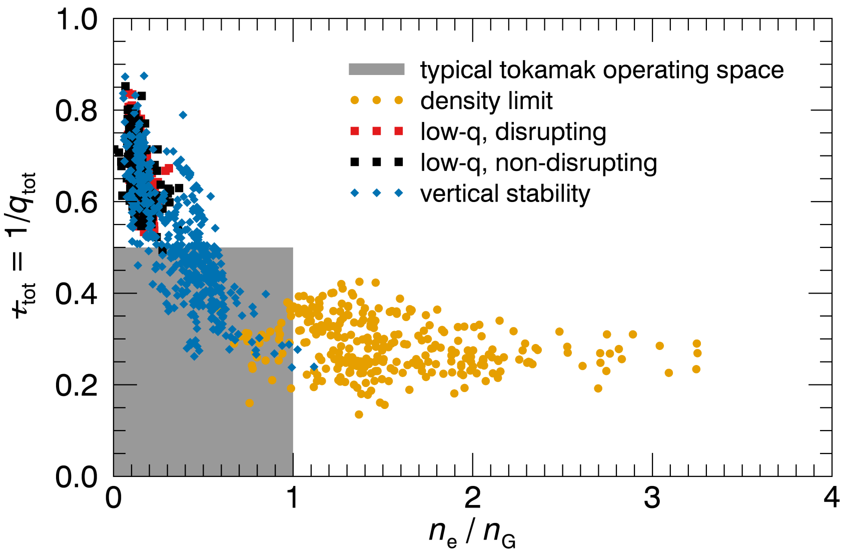
CTH can operate beyond the $q(a) = 2$ current limit, with a slight increase in ι_{vac}

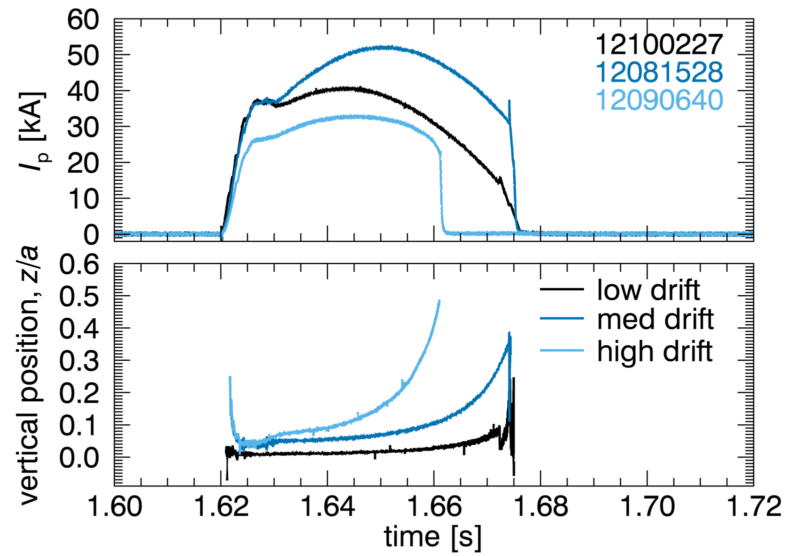




- Density-limit disruptions
- Low-q disruptions

Vertically unstable plasmas can result in a disruption if uncompensated





- Density-limit disruptions
- Low-*q* disruptions
- Vertically unstable plasmas

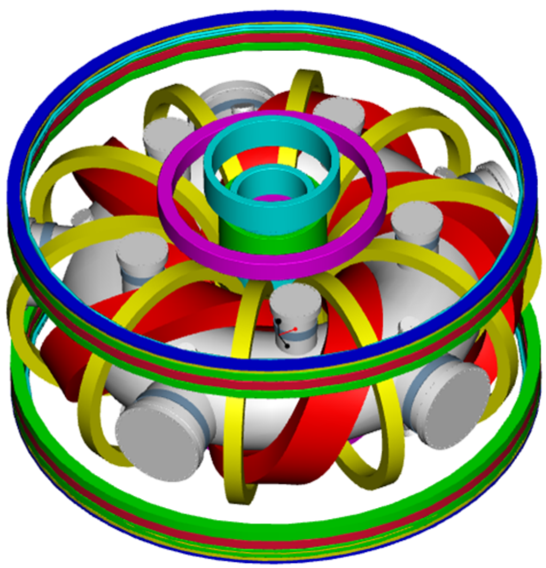
Outline

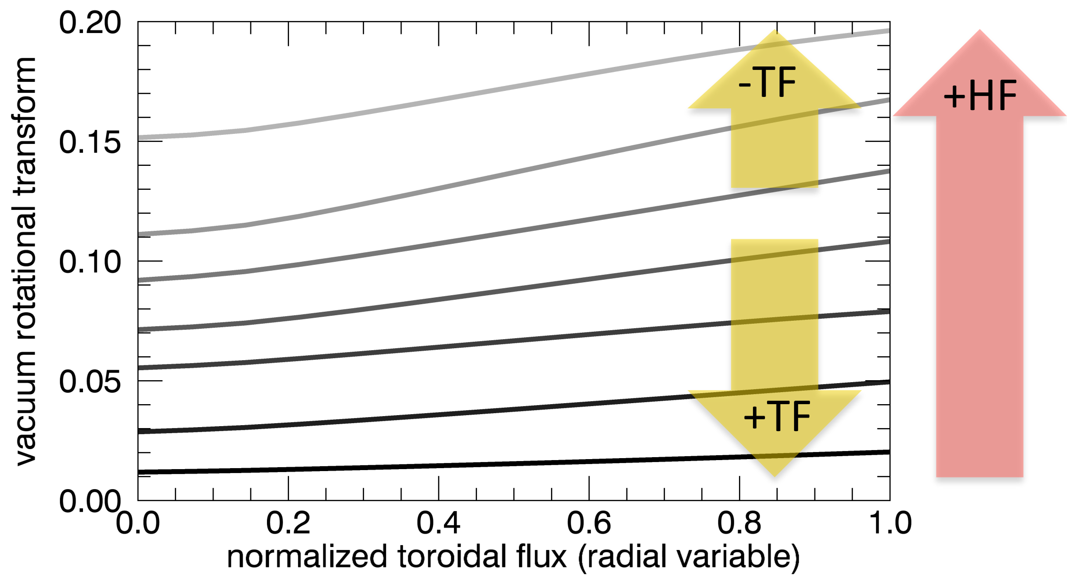
- Compact Toroidal Hybrid
- 3D equilibrium reconstruction
- Disruption avoidance:
    1. Density limit disruptions
    2. Low-*q* disruptions
    3. Vertically unstable plasmas
- Summary

CTH: Flexible magnetic configuration in low aspect ratio stellarator/tokamak hybrid

- **Helical Field coil** and **Toroidal Field coil** currents adjusted to modify vacuum rotational transform ι_{vac}

$R_0 = 0.75$ m $R/a \sim 4$ $n_e \leq 5 \times 10^{19}$ m^{-3} $T_e \leq 200$ eV $|B| \leq 0.7$ T





CTH: Flexible magnetic configuration in low aspect ratio stellarator/tokamak hybrid

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- **Shaping Vertical Field coil** varies elongation κ and shear

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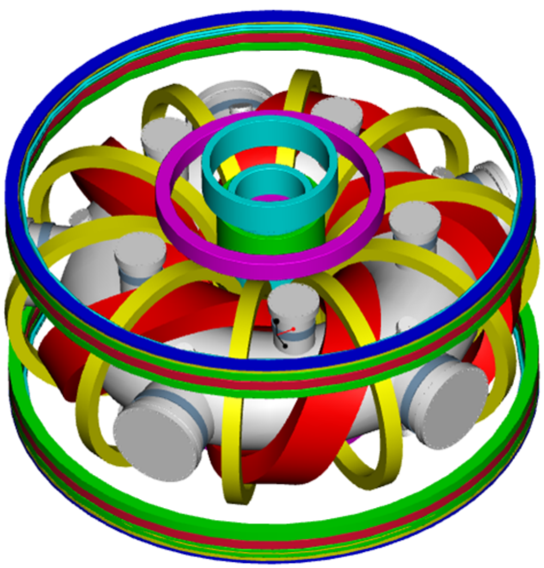

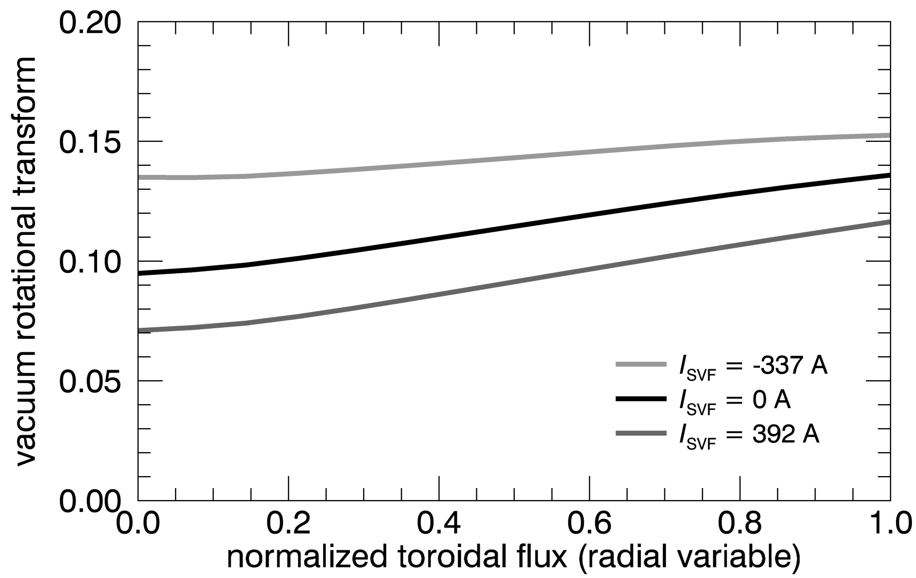

CTH: Flexible magnetic configuration in low aspect ratio stellarator/tokamak hybrid

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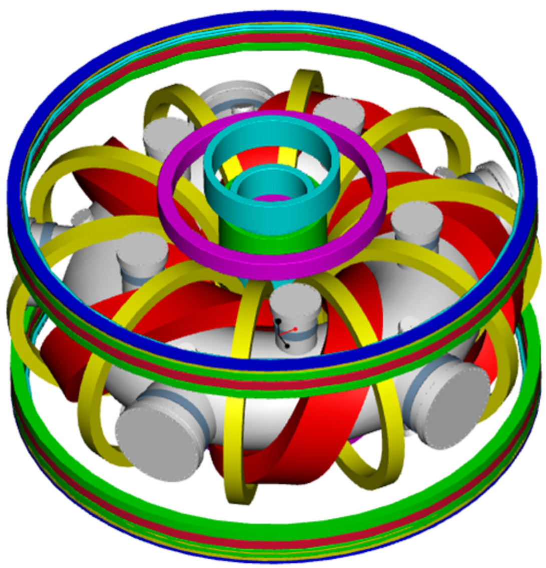
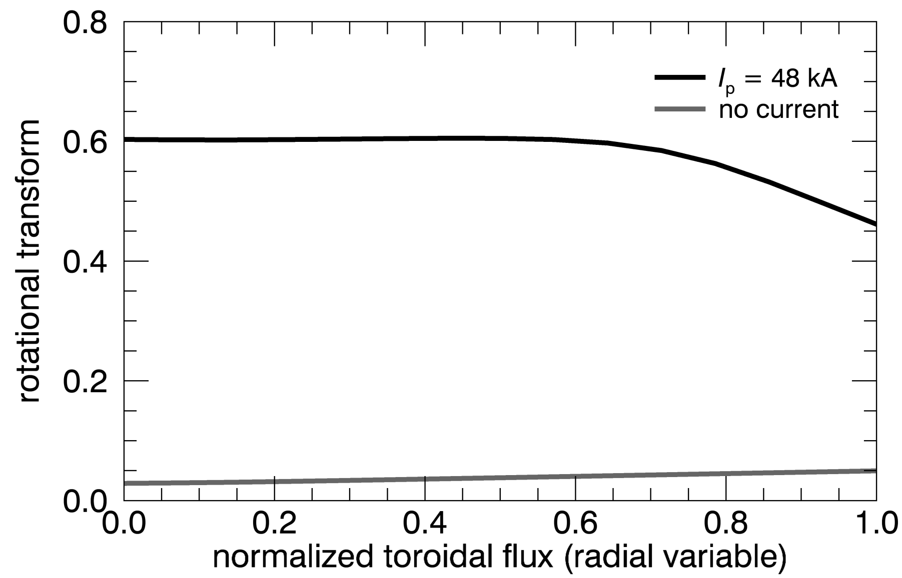
- **Central solenoid** drives $I_p \leq 80$ kA, adding to total transform

$R_0 = 0.75$ m $R/a \sim 4$ $n_e \leq 5\times10^{19}$ m^{-3} $T_e \leq 200$ eV $|B| \leq 0.7$ T

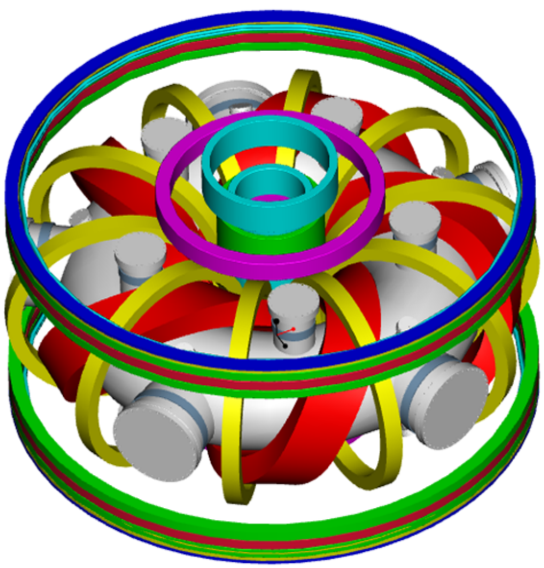


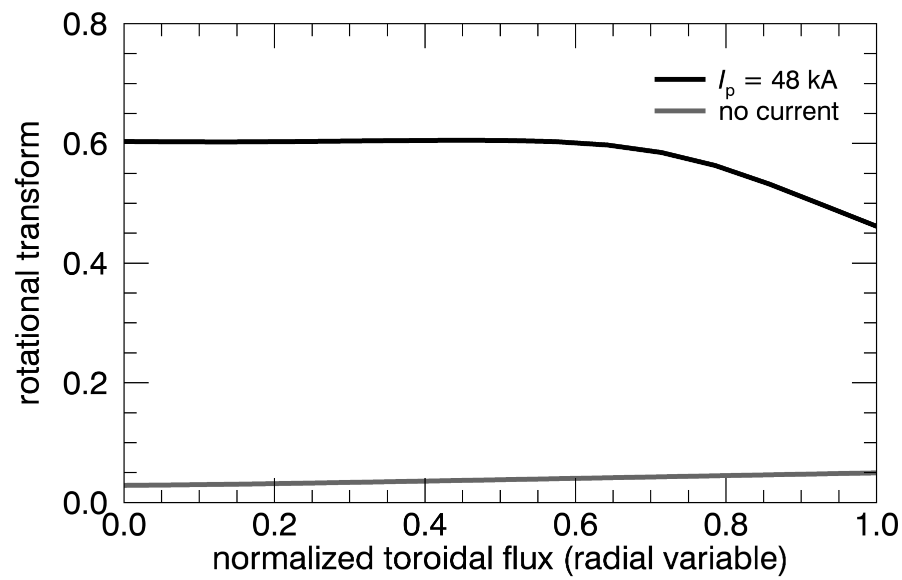


CTH: Flexible magnetic configuration in low aspect ratio stellarator/tokamak hybrid

- **Helical Field coil** and **Toroidal Field coil** currents adjusted to modify vacuum rotational transform ι_{vac}
- **Shaping Vertical Field coil** varies elongation κ and shear
- **Central solenoid** drives $I_p \leq 80$ kA, adding to total transform
- **Trim Vertical Field coil** and **Radial Field coil** control position

$R_0 = 0.75$ m $R/a \sim 4$ $n_e \leq 5\times10^{19}$ m^{-3} $T_e \leq 200$ eV $|B| \leq 0.7$ T

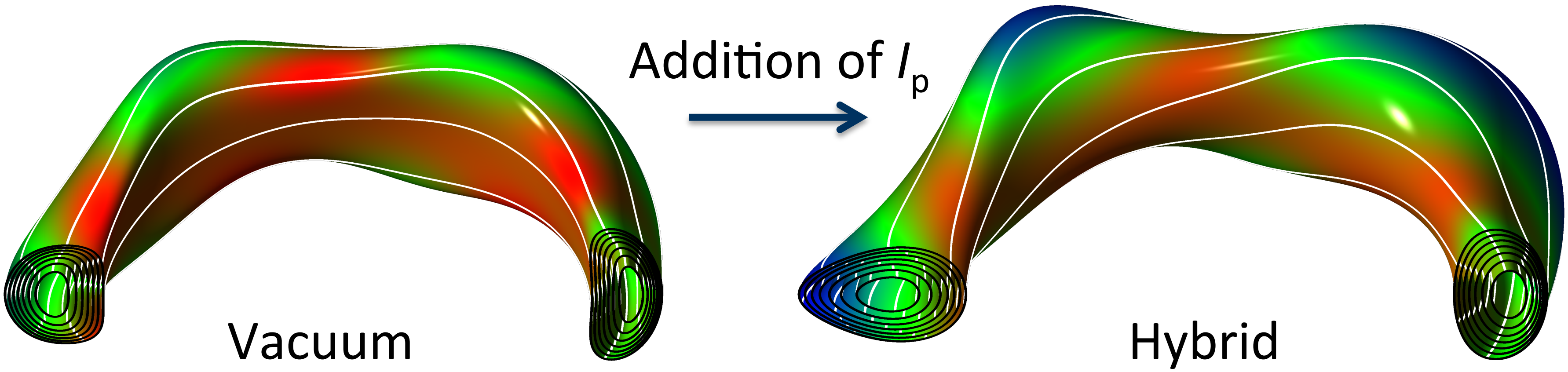




Outline

- Compact Toroidal Hybrid
- 3D equilibrium reconstruction
- Disruption avoidance:
    1. Density limit disruptions
    2. Low-*q* disruptions
    3. Vertically unstable plasmas
- Summary and conclusions

Experimental 3D equilibria reconstructed with V3FIT code (J.D. Hanson et al., Nucl. Fusion, 2009)



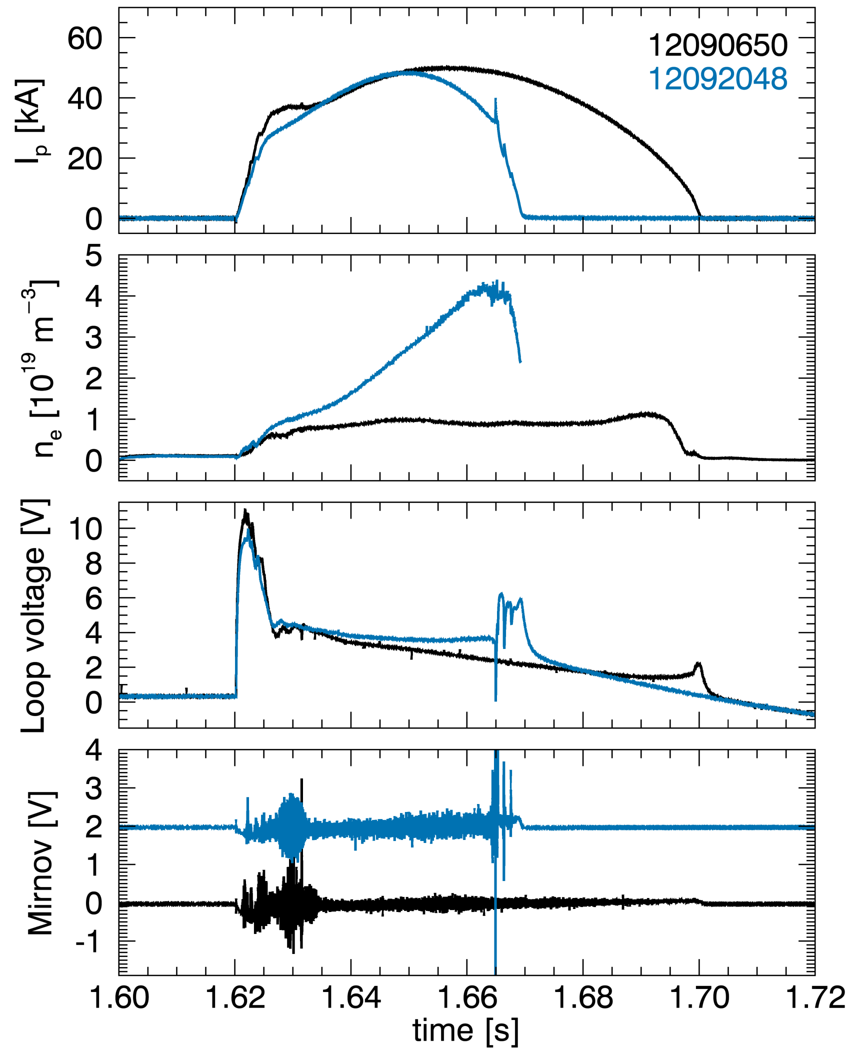
- Equilibrium strongly modified by plasma current
- Find MHD equilibrium most consistent with data
  - Over 40 external magnetic diagnostics as input
- Reconstructions using only external magnetics provide accurate information on: plasma shape, enclosed toroidal flux, rotational transform near the edge

Outline

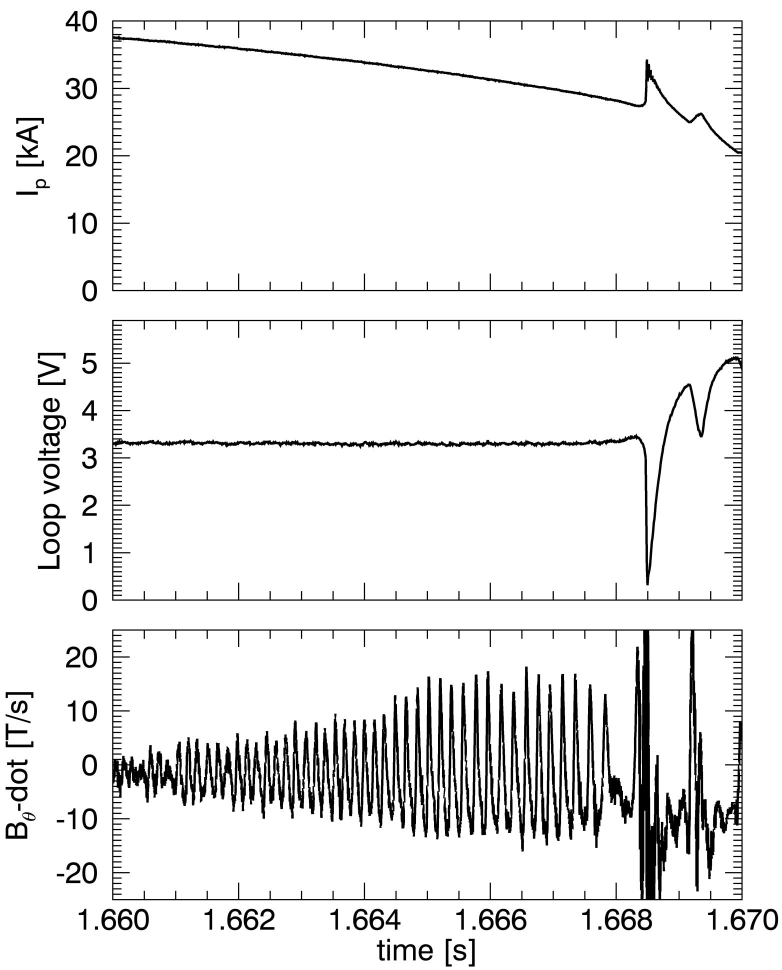
- Compact Toroidal Hybrid
- 3D equilibrium reconstruction
- Disruption avoidance and mitigation:
  1. Density limit disruptions
  2. Low $q(a)$ disruptions
  3. Vertically unstable plasmas
- Summary and conclusions

Density limit disruptions triggered by ramping density with edge fueling

- Discharges with similar low transform ι_{vac} = 0.05
- Phenomenology of hybrid discharge terminations similar to tokamak disruptions
  - Negative loop voltage spike
  - Current spike followed by rapid decay
  - Strong coherent MHD precursor

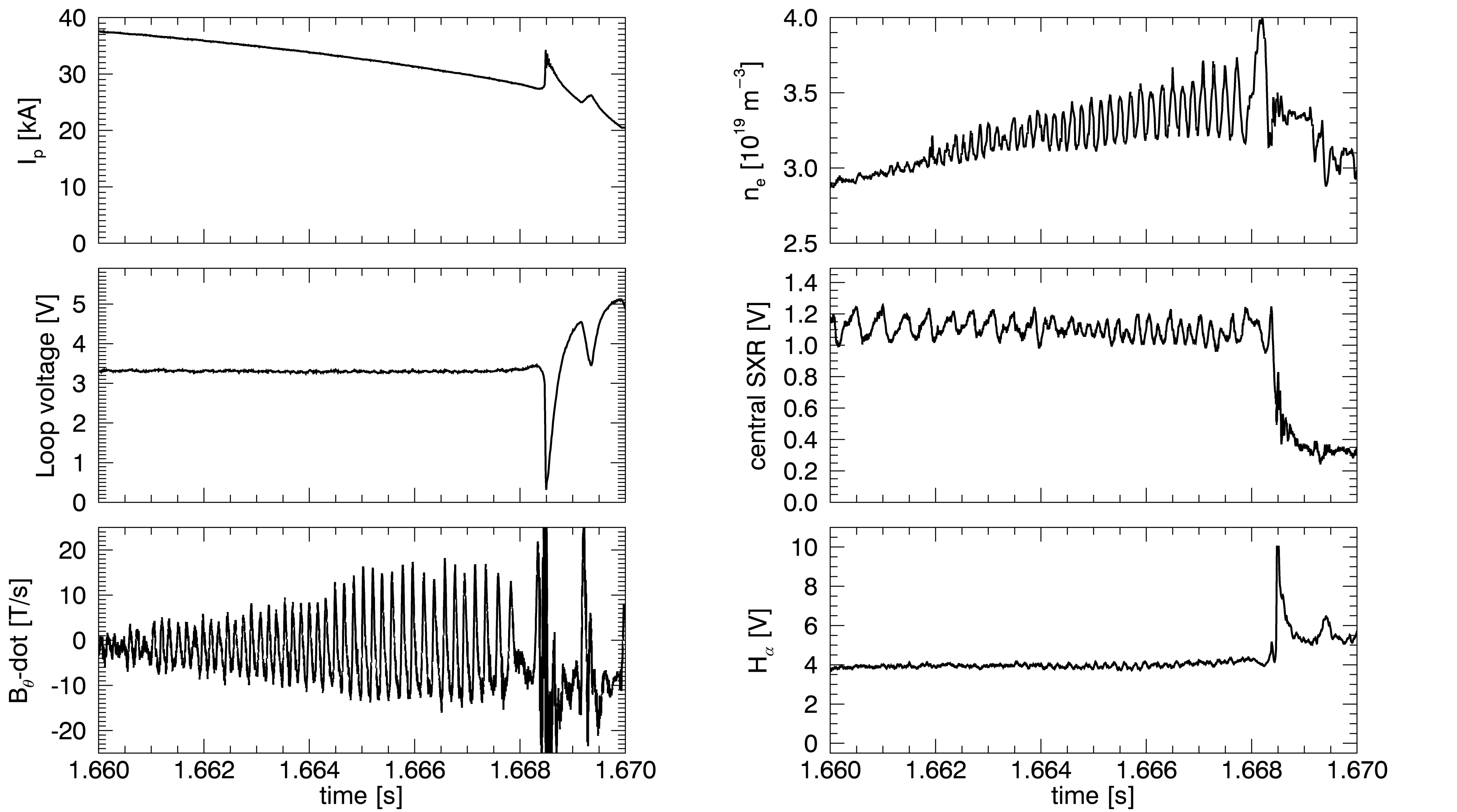


Disruption precursor fluctuations similar to those seen in tokamaks

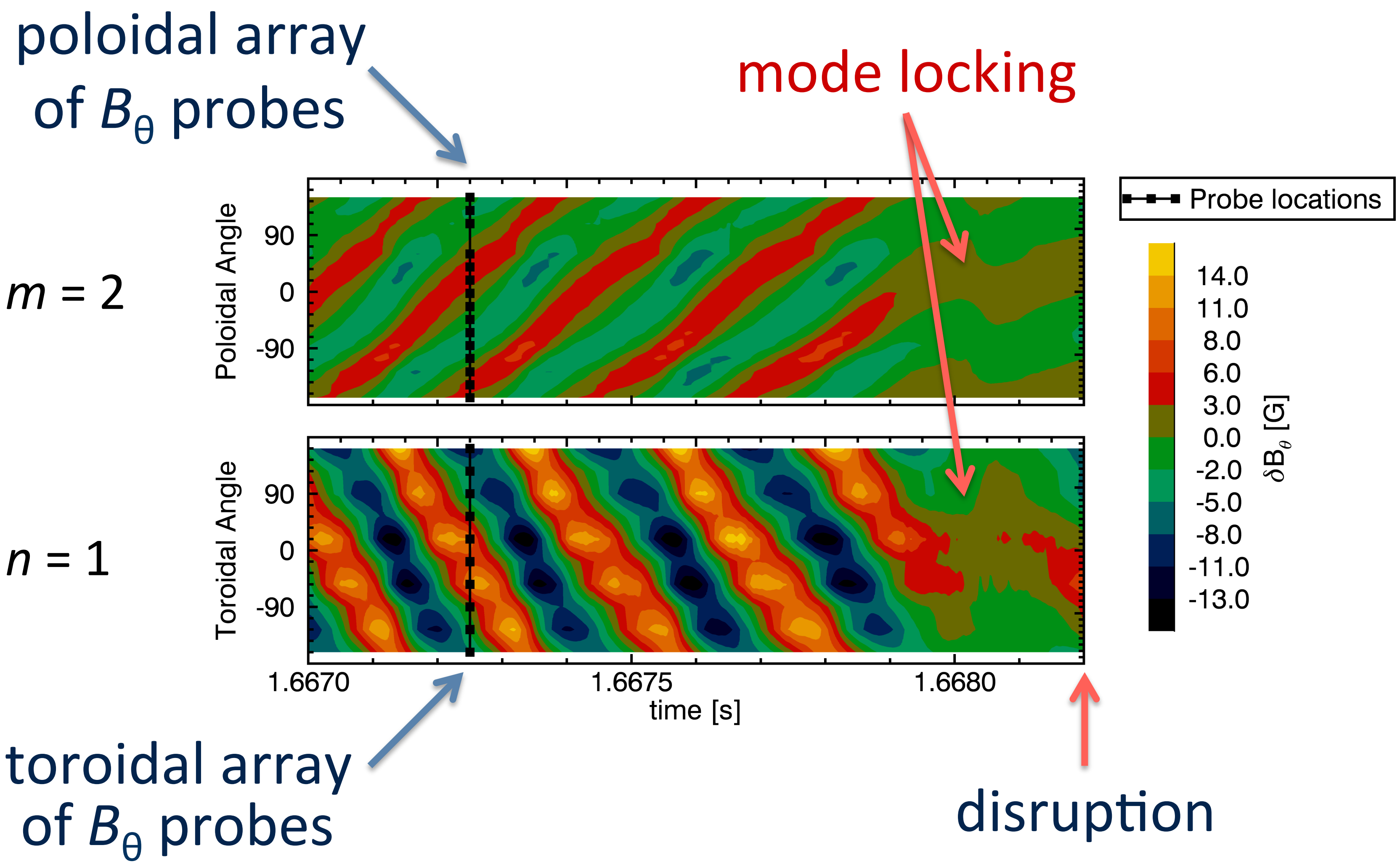


Disruption precursor fluctuations indicate internal tearing mode

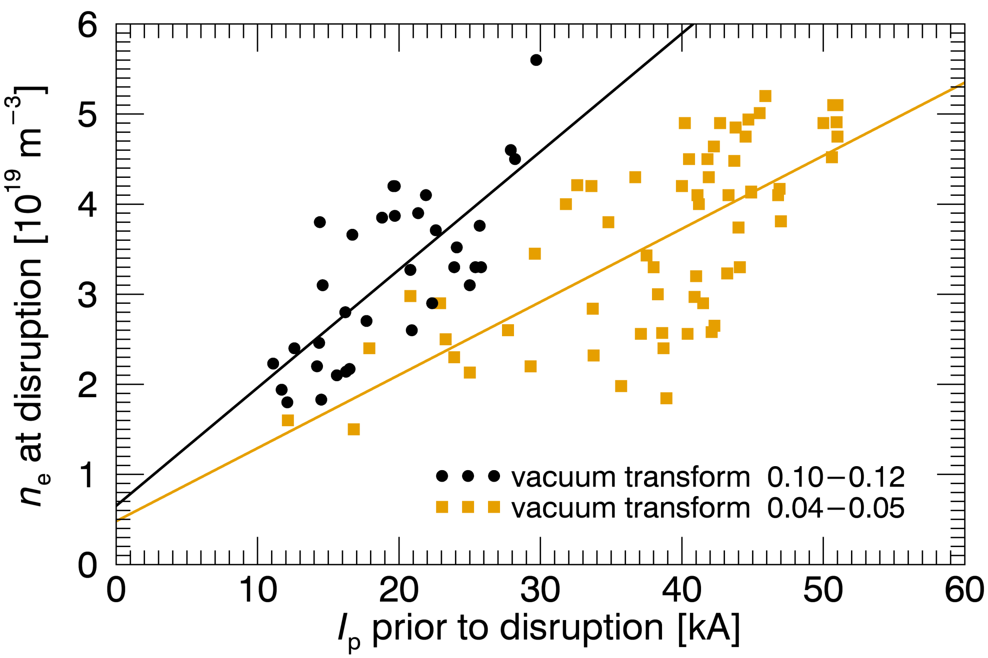
- MHD modulates density and SXR emission



Disruption preceded by rotating m/n = 2/1 tearing mode that locks



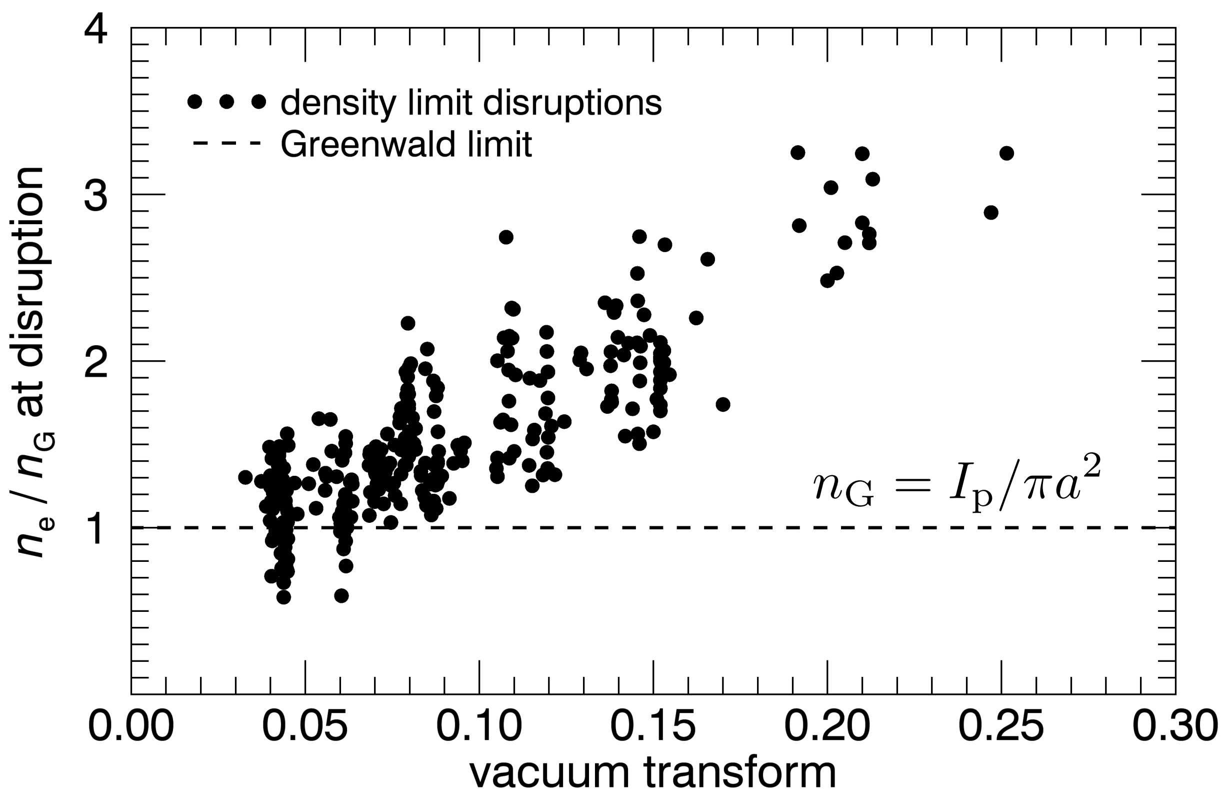
Density at disruption scales with the plasma current and vacuum transform



- Follows trend of Greenwald limiting behavior:
  - $n_{\mathrm{G}} = I_{\mathrm{p}}/\pi a^2$
- Additional dependence on applied level of vacuum transform

(M. Greenwald et al., Nucl. Fusion, 1988)

Density at disruption exceeds Greenwald limit as vacuum transform is increased



- Have not found a threshold value of vacuum transform that eliminates these disruptions

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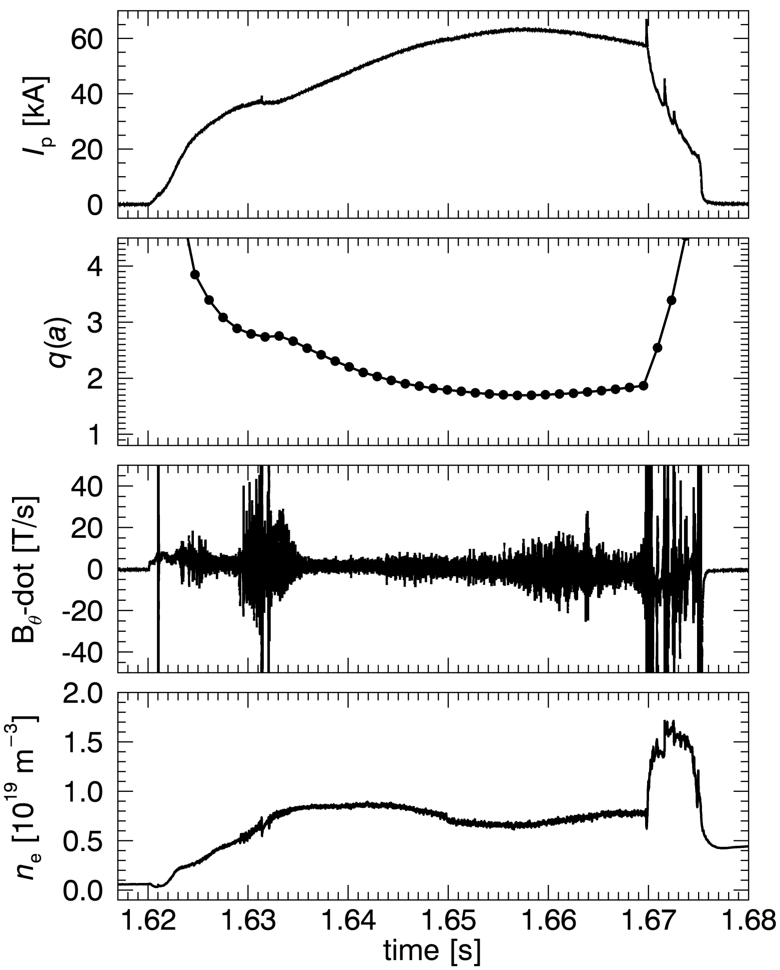
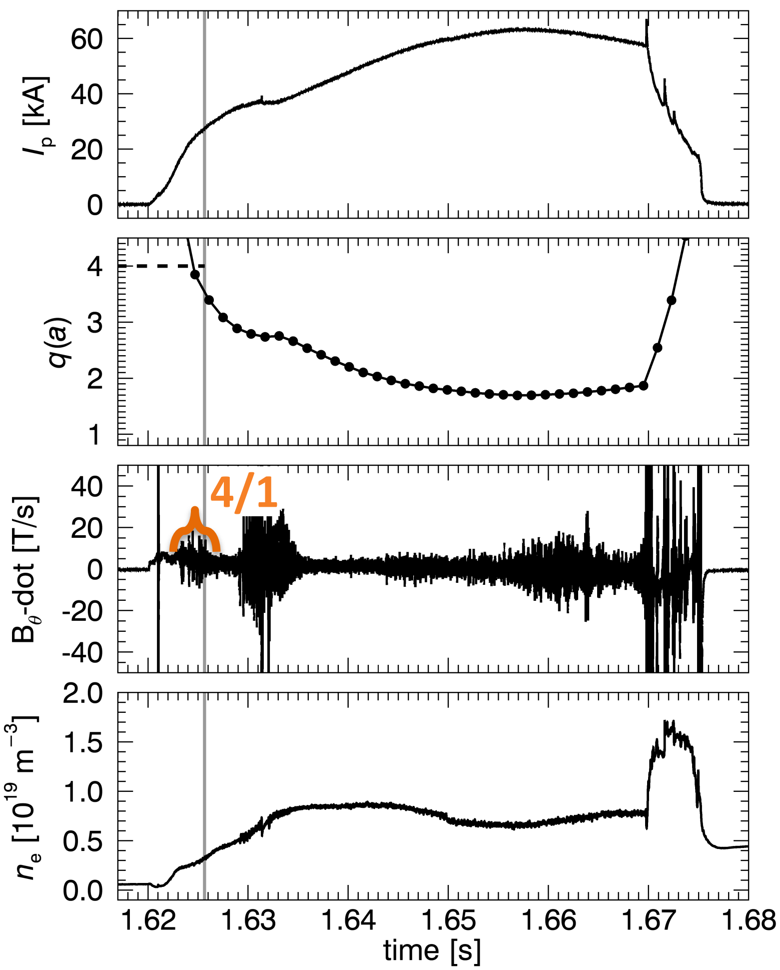
High current plasmas disrupt with $q(a)$ below 2 and vacuum transform low

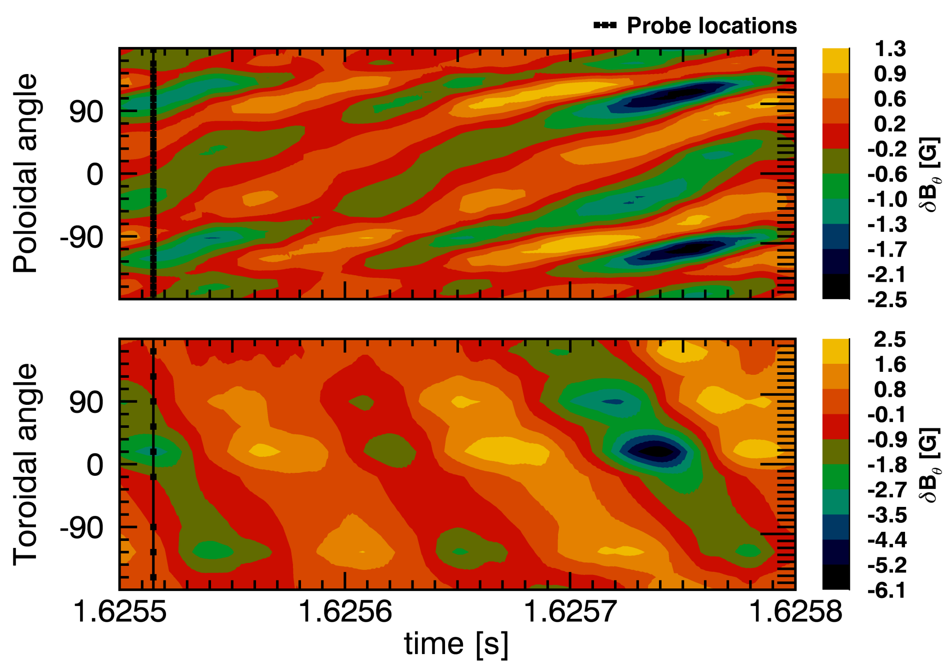


- Example with $ɫ_{vac}$ = 0.02 (q_{vac} = 50)
- Disruption does not occur on initial crossing of $q(a)$ = 2
- Bursts of magnetic fluctuations are detected throughout the discharge
- Density kept low and roughly constant

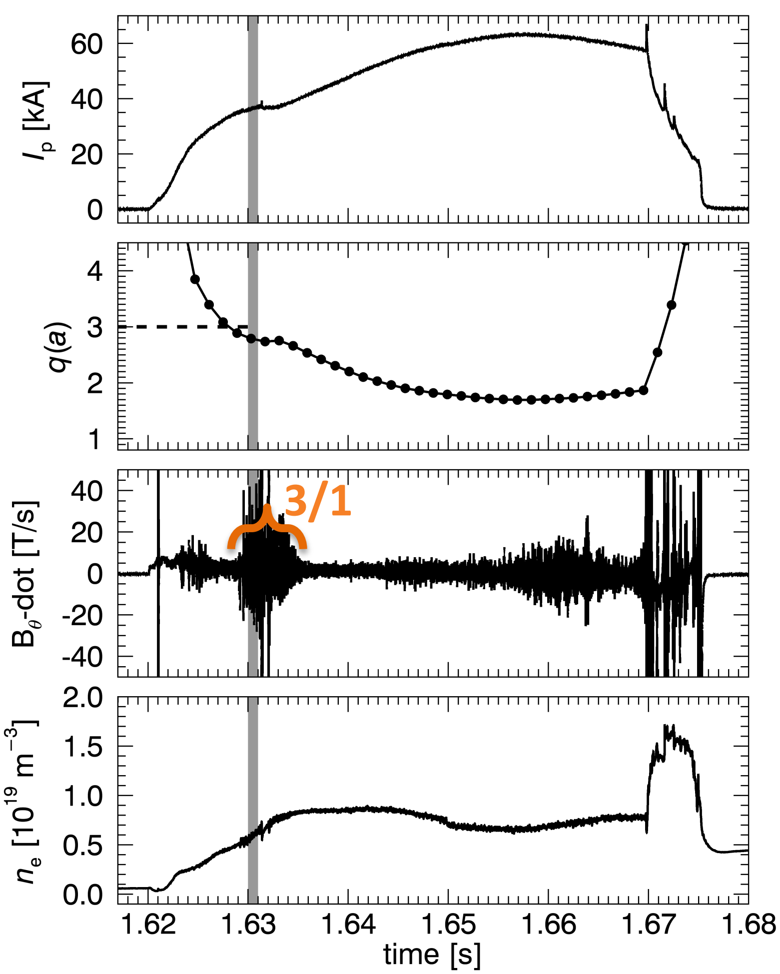
Hesitations in current rise as resonant surfaces move through the plasma edge



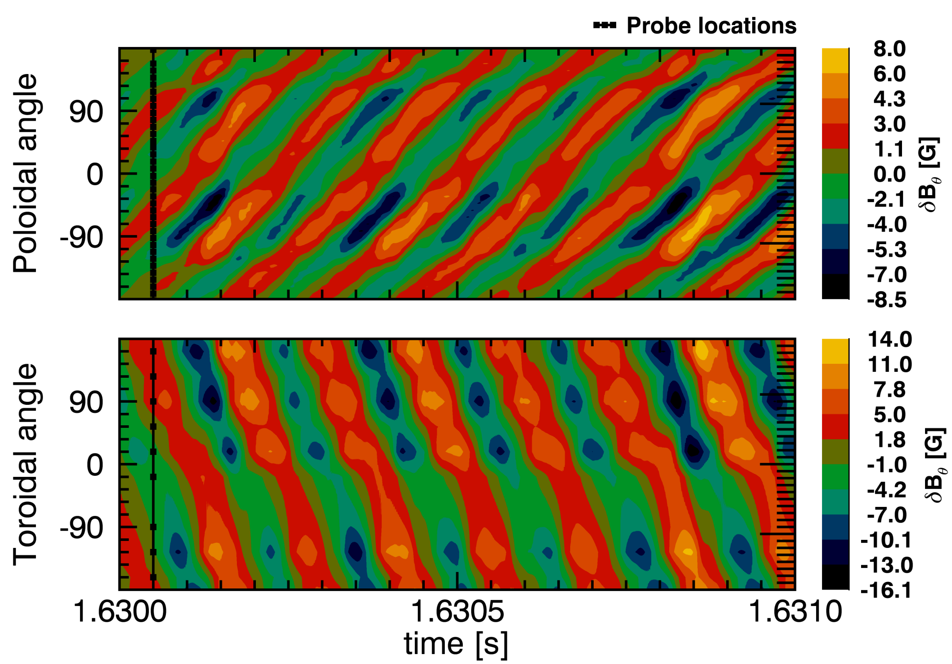
- q = 4 surface exits the plasma edge
- m = 4, n = 1 mode detected



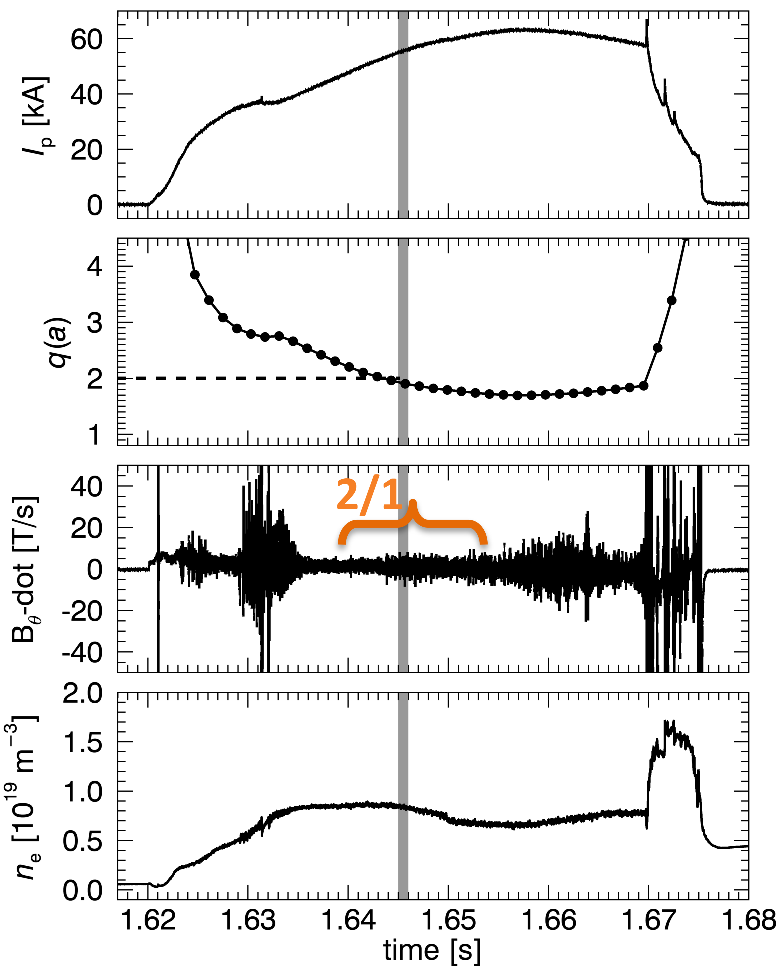
Hesitations in current rise as resonant surfaces move through the plasma edge



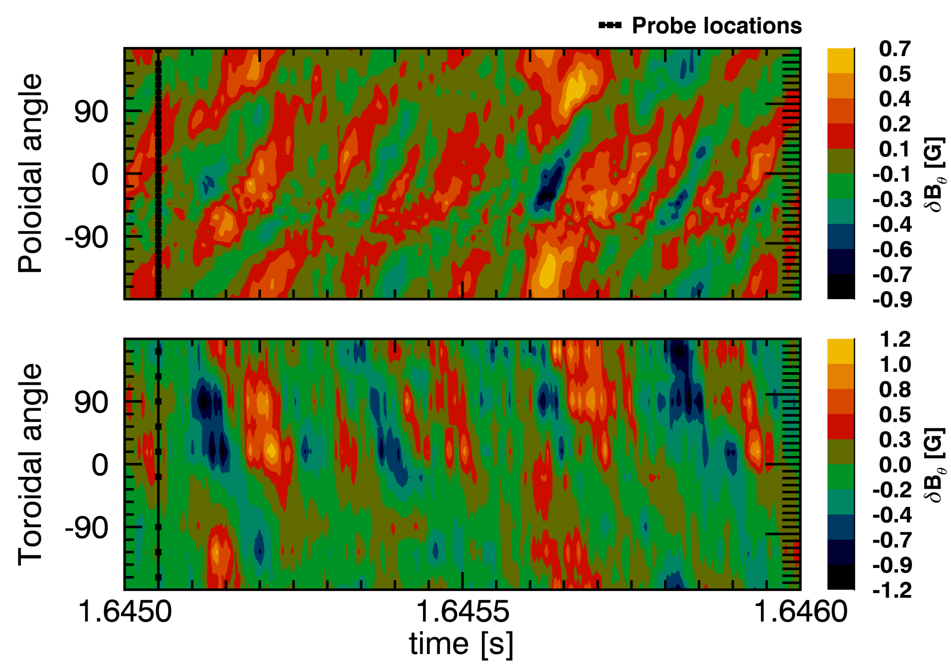
- q = 3 surface exits the plasma edge
- m = 3, n = 1 mode detected



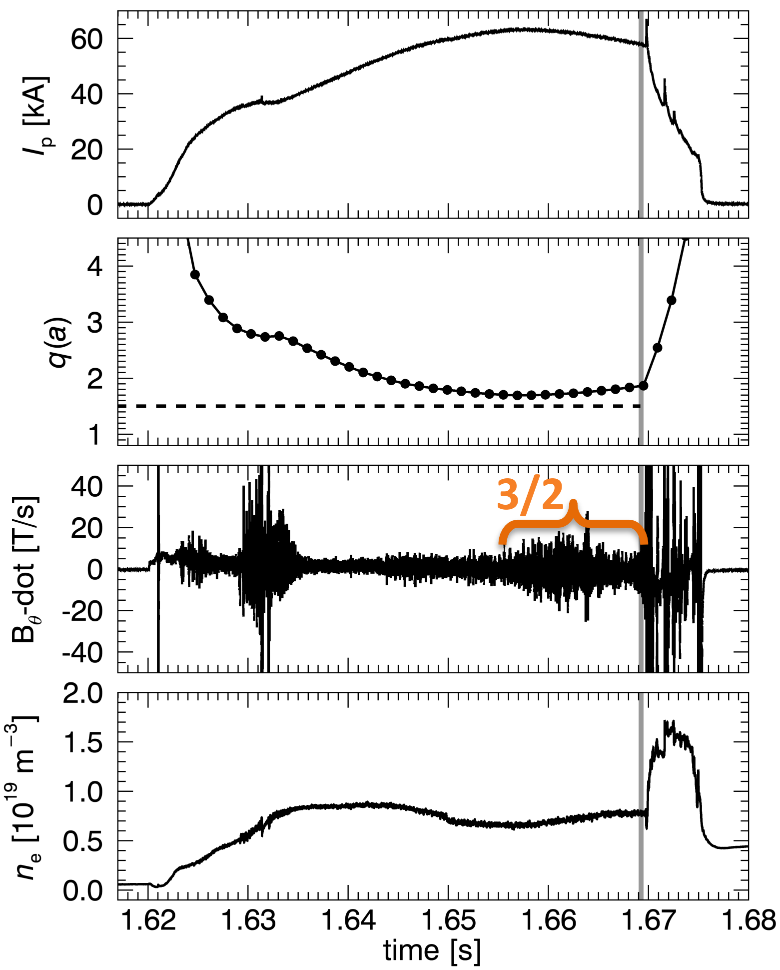
Hesitations in current rise as resonant surfaces move through the plasma edge



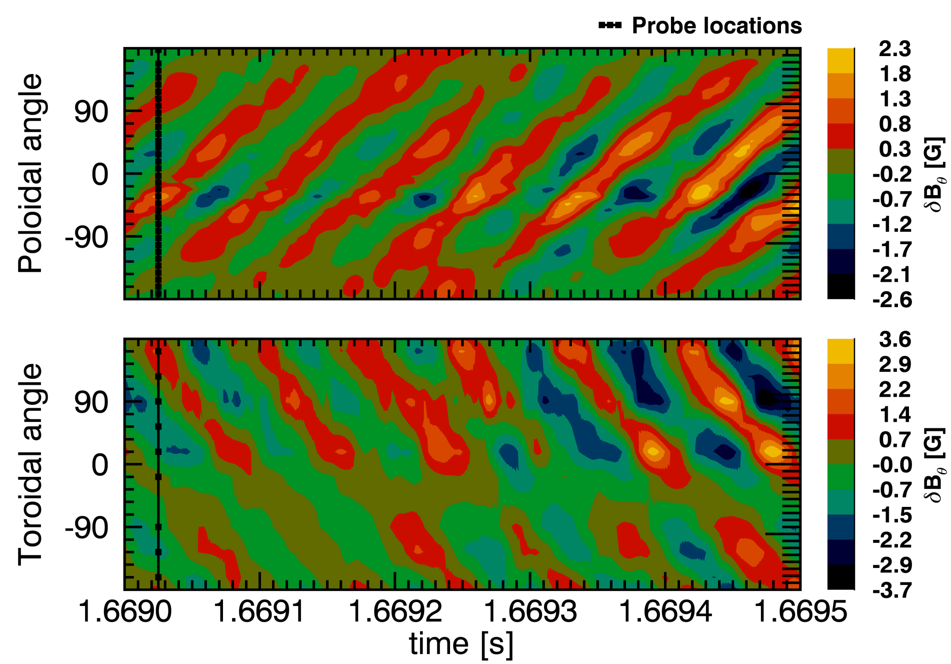
- q = 2 surface exits the plasma edge
- m = 2, n = 1 mode detected
- Remains at low amplitude



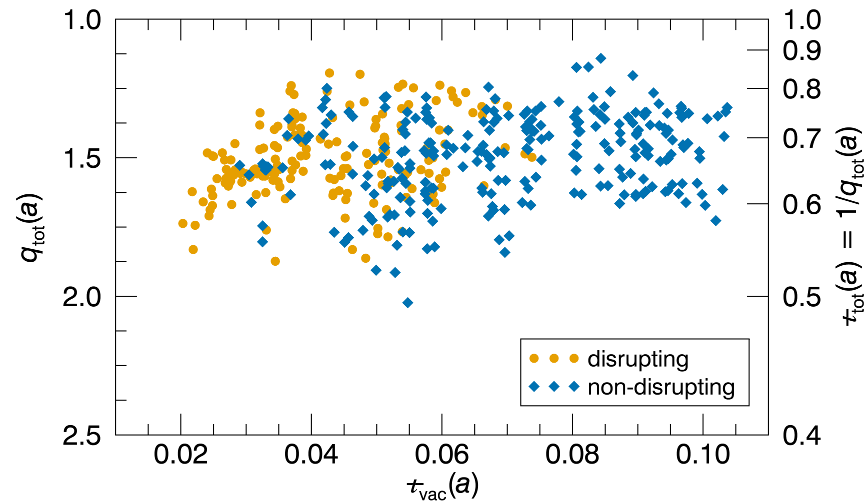
An *m* = 3, *n* = 2 mode grows to large amplitude just prior to disruption



- $q = 3/2$ surface near edge but remains inside plasma
- $m = 3$, $n = 2$ mode detected



Low-q disruptions cease to occur if vacuum transform raised above ~ 0.07



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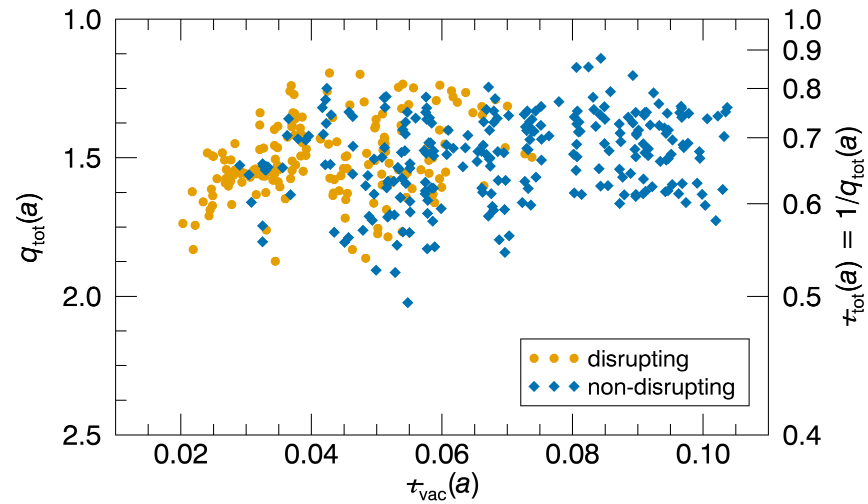

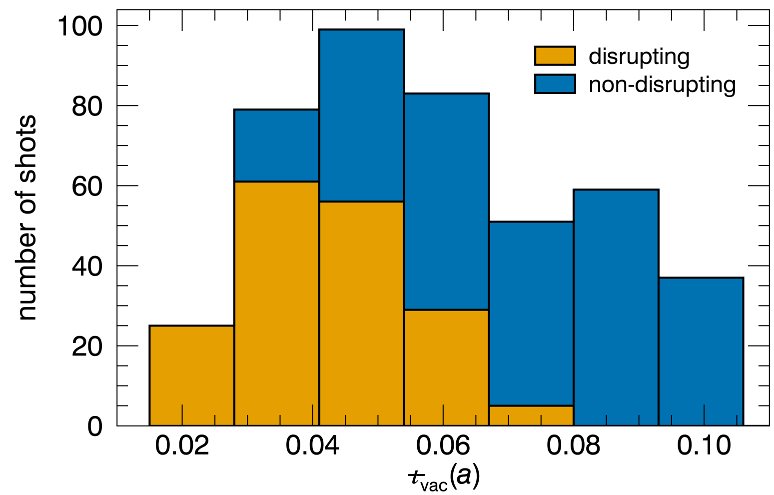

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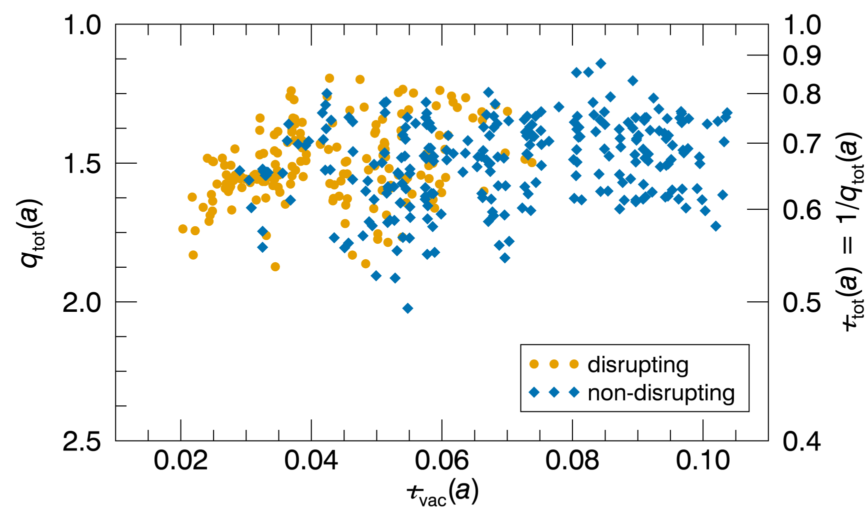

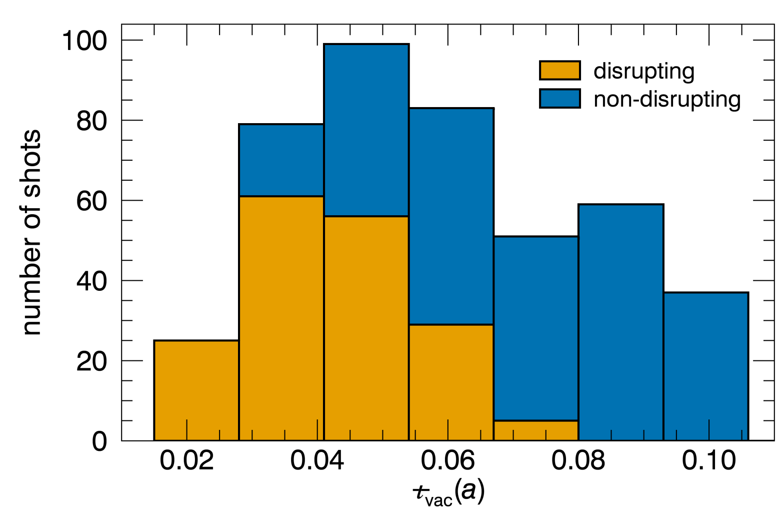

- Possible explanation: Applied ι_{vac} shifts 3/2 resonance outward to where the current profile is less steep, stabilizing the 3/2 tearing mode
  - Invoked for the stabilization of 2/1 tearing mode in W7-A (W7-A team, Nucl. Fusion. 1980)
- Lack of strong n = 1 kink mode activity seen computationally (Fu, et al., Phys. Plasmas. 2000)

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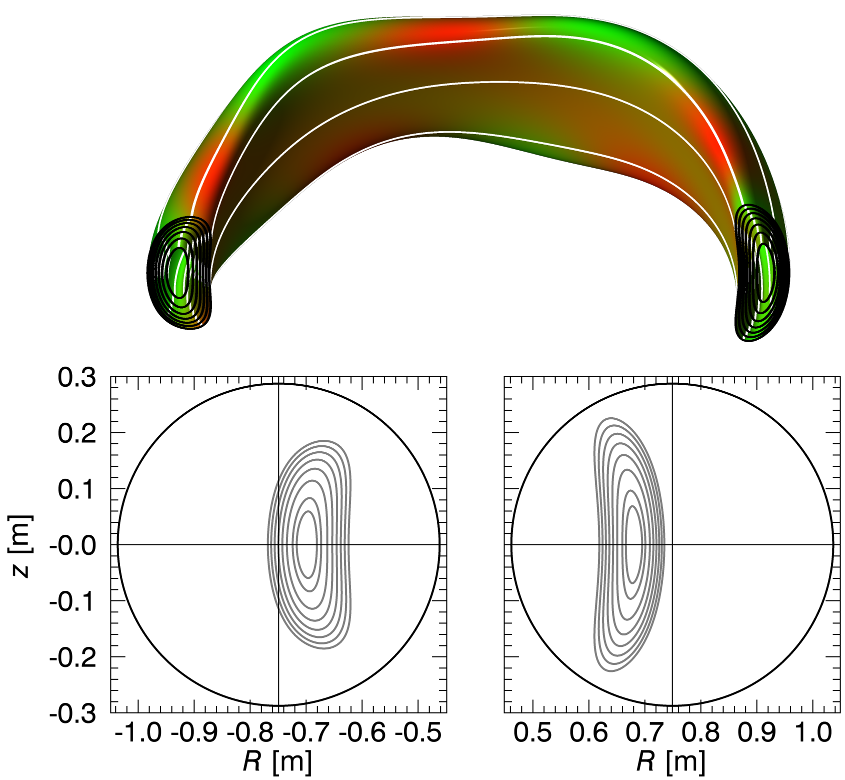
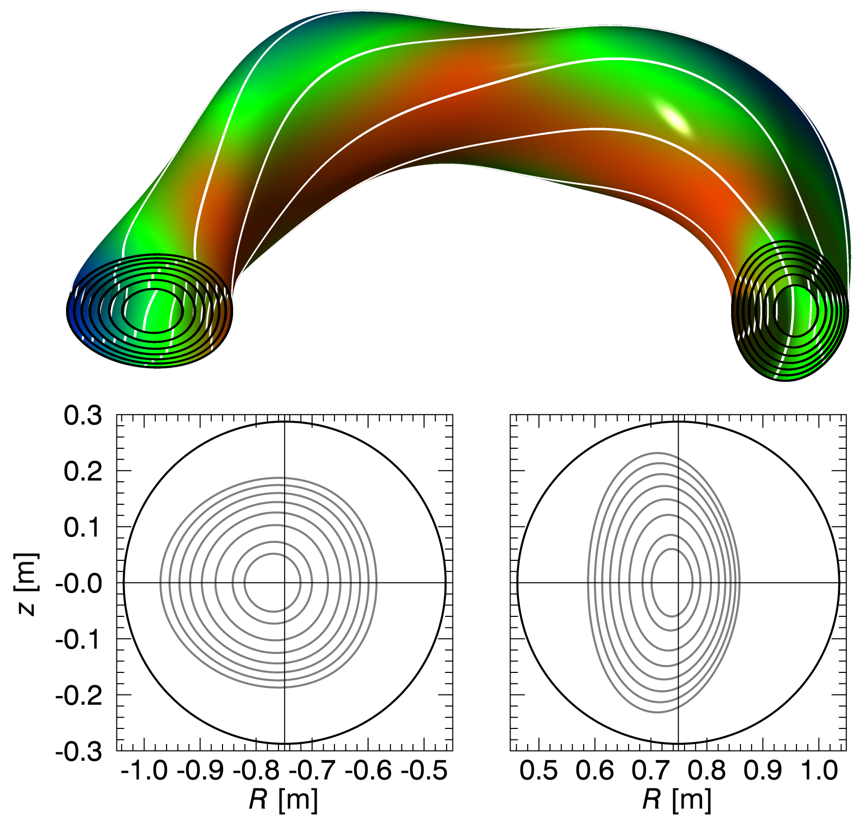
CTH discharges naturally elongated and can be susceptible to vertical instability

- ECRH plasma I_p = 0 kA
  - Mean κ = 2.77
  - Fractional transform $f = \iota_{vac}(a)/\iota_{tot}(a) = 1$

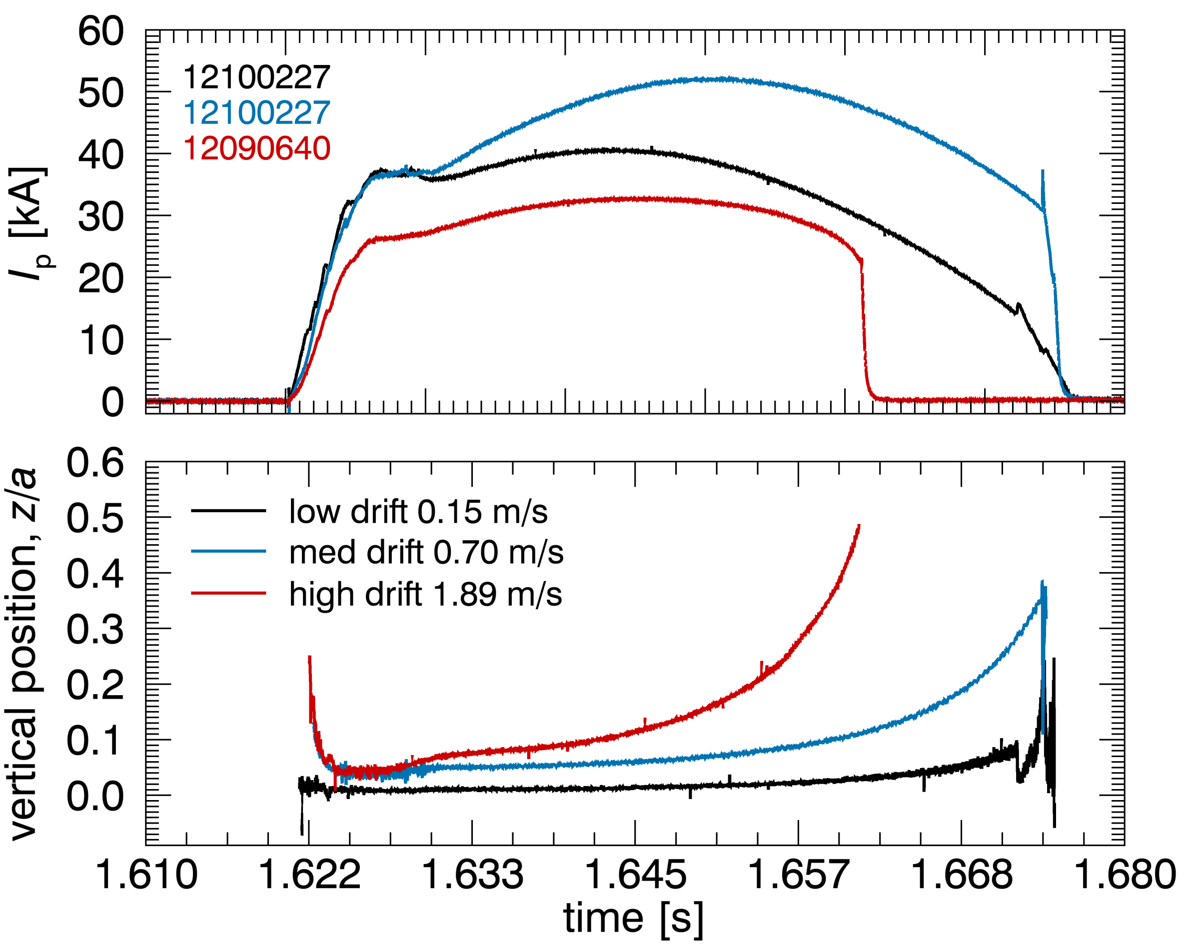


- At peak I_p = 75 kA
  - Mean κ = 1.48
  - Fractional transfrom f = 0.0634

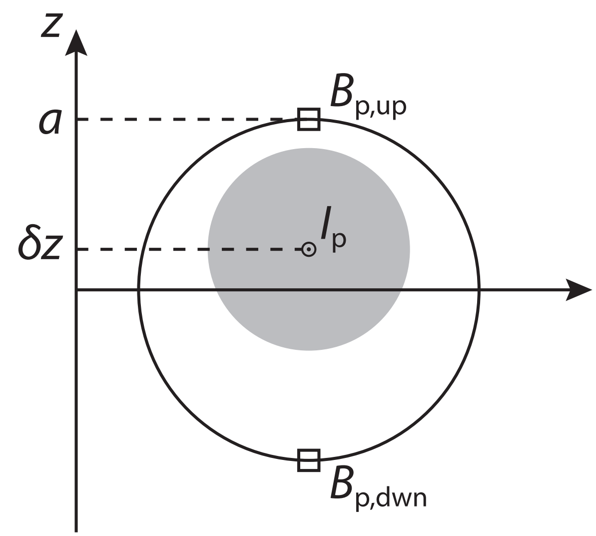


Elongated plasmas are measured to be vertically unstable

- Vertical position inferred from magnetic diagnostics

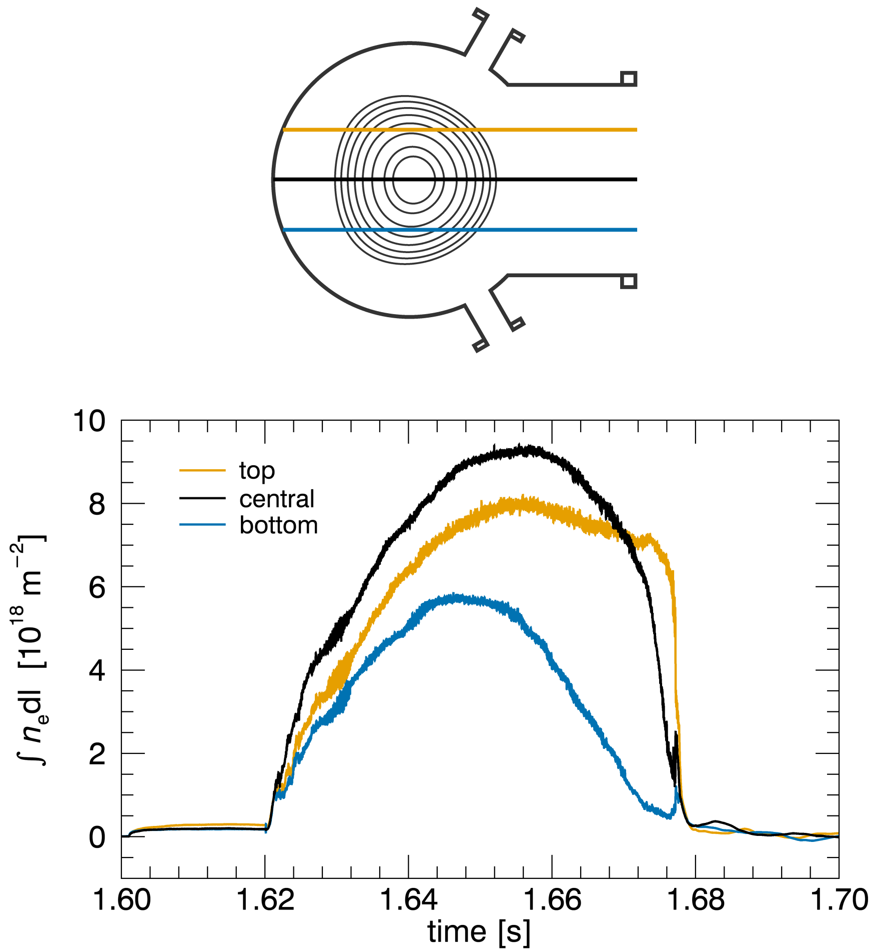


$$\frac{\delta z}{a} = \frac{B_{\mathrm{p,up}} - B_{\mathrm{p,dwn}}}{B_{\mathrm{p,up}} + B_{\mathrm{p,dwn}}}$$

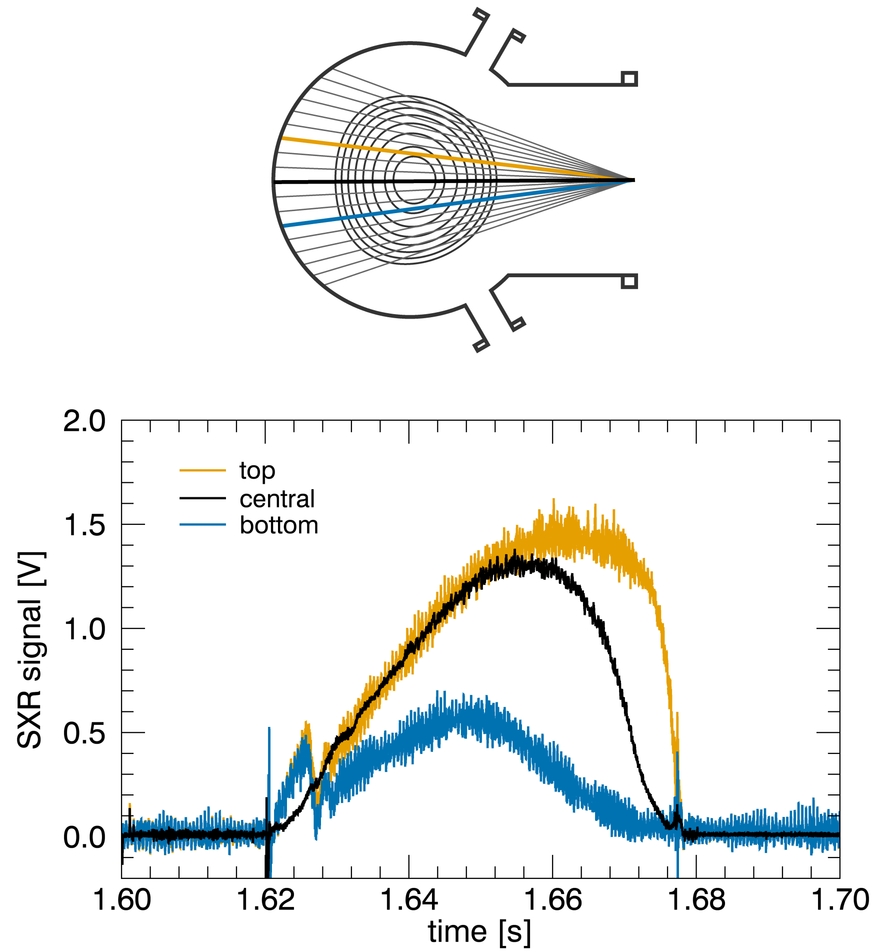


Vertical motion is also detected by interferometry and SXR cameras

1mm wave interferometer

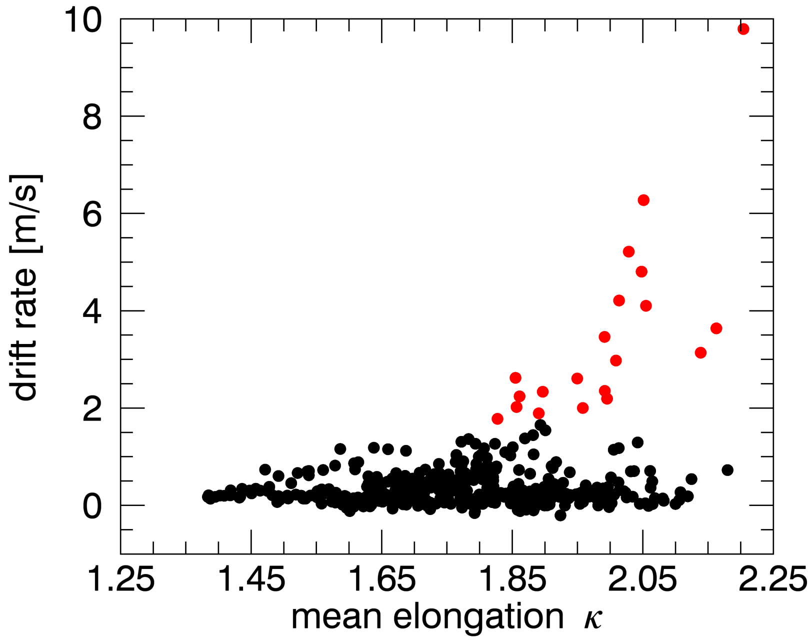


SXR pinhole camera



Discharges exhibit faster drift at high elongation and low fractional transform

- Large ensemble of discharges with varied elongation and fractional transform



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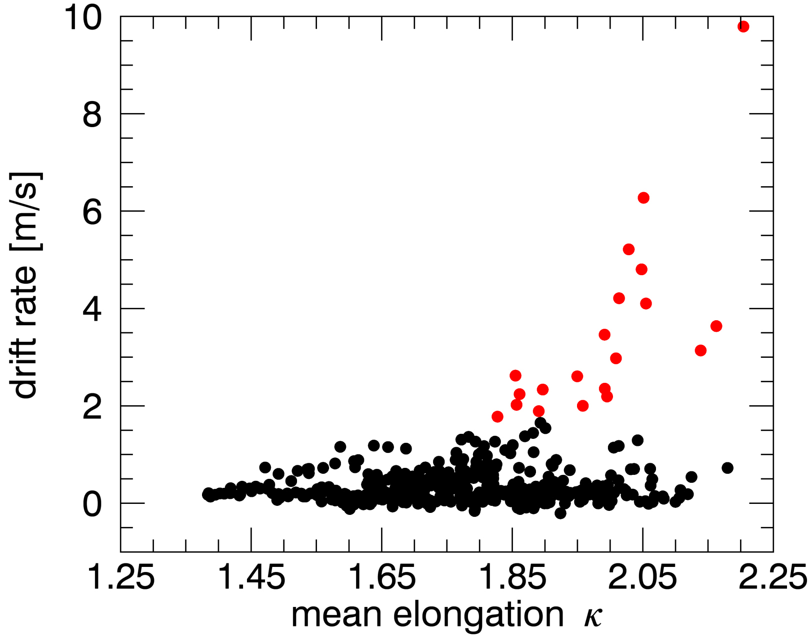
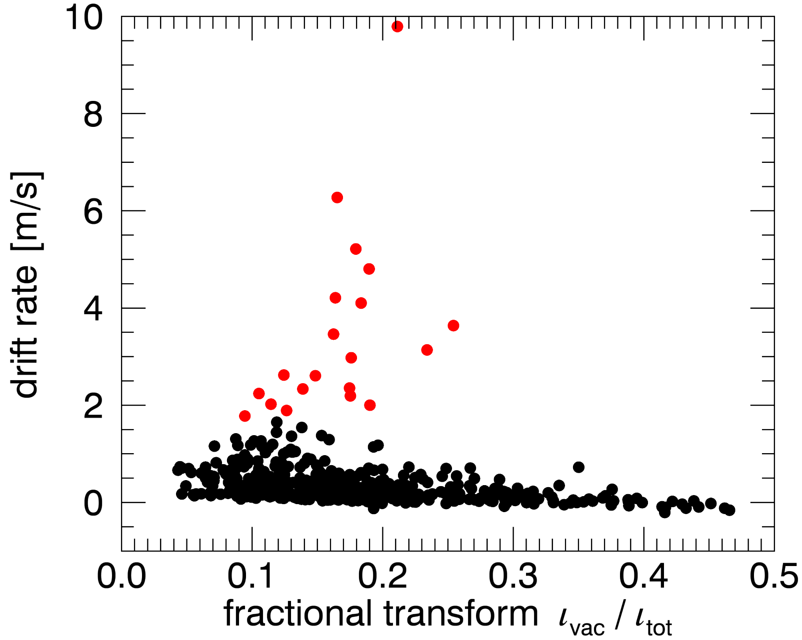
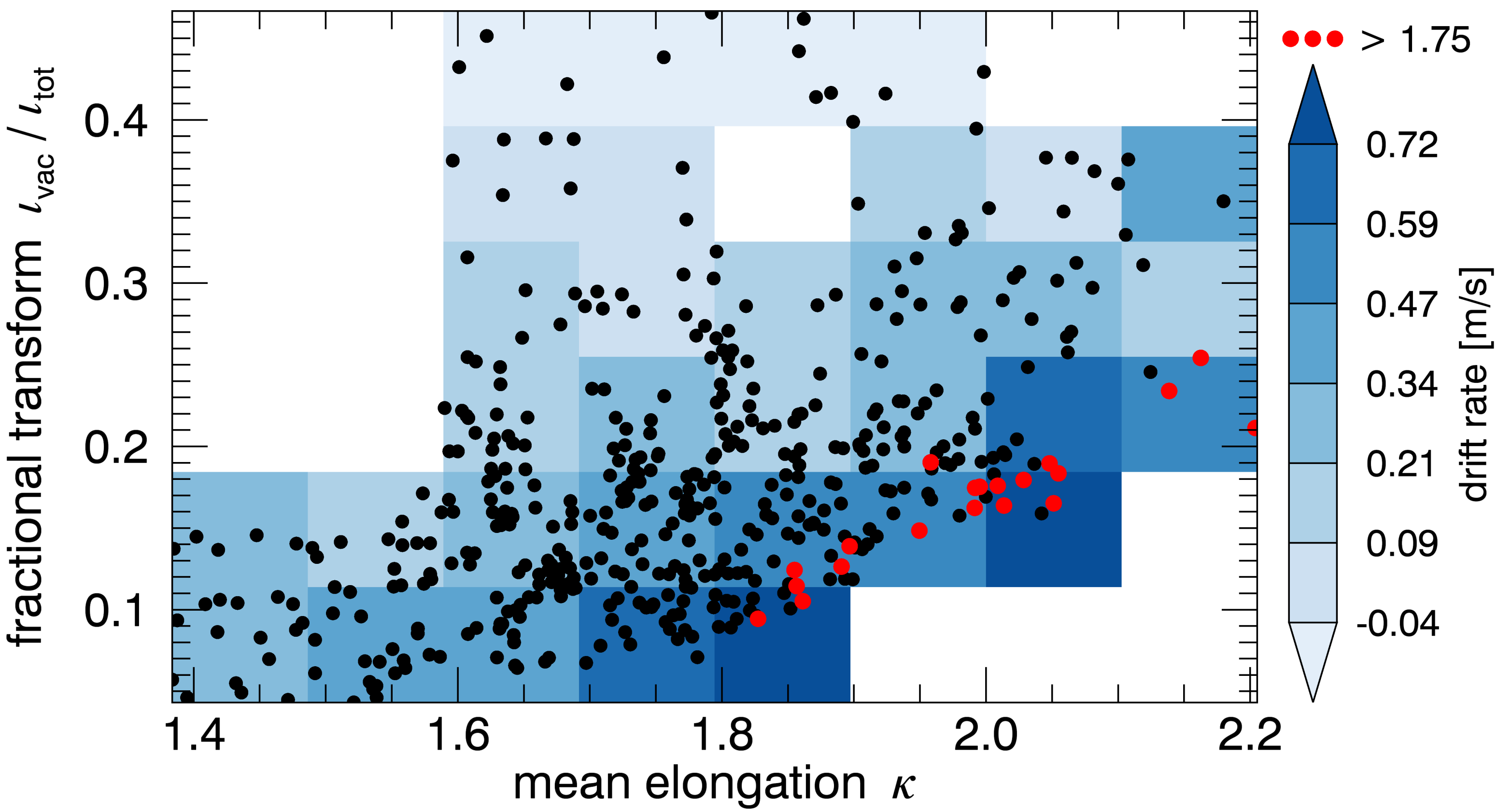




Plasmas with high elongation stabilized by addition of vacuum transform



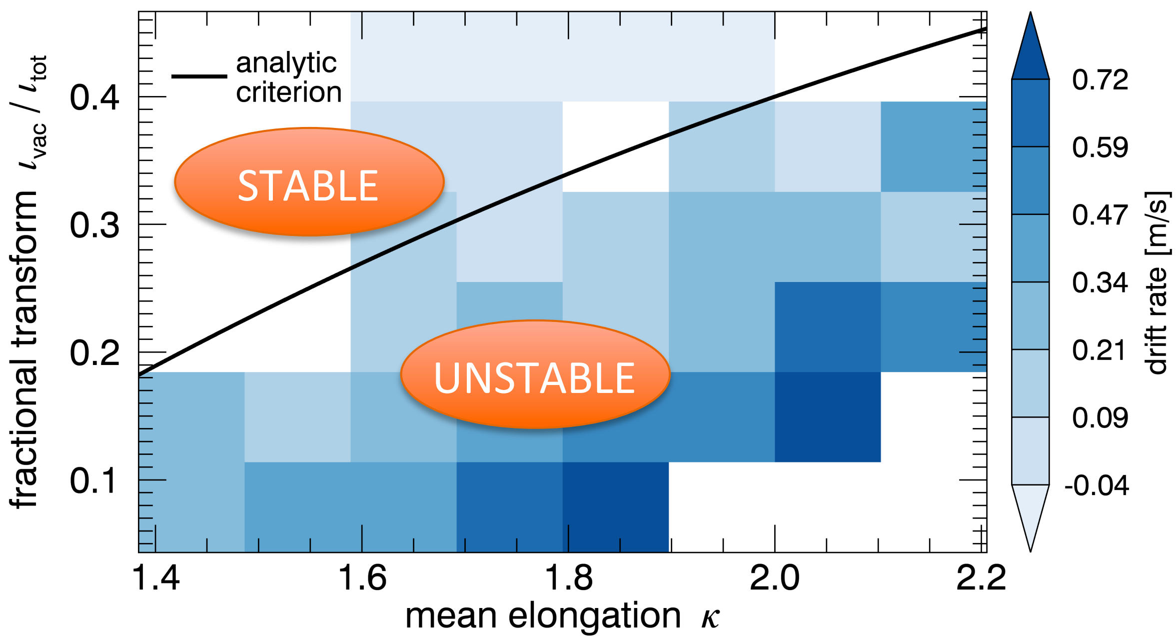
(M.C. ArchMiller, et al., Phys. Plasmas. 2014)

Qualitative agreement with analytic criterion for vertical stability

- Energy principle used to derive fraction of vacuum transform needed to stabilize vertical mode in a current-carrying stellarator (G.Y. Fu, Phys. Plasmas, 2000)

- $$f \equiv \frac{\iota_{\text{vac}}(a)}{\iota_{\text{tot}}(a)} \geq \frac{\kappa^2 - \kappa}{\kappa^2 + 1}$$

- Large aspect ratio, low-β stellarator
- Uniform profiles of current density and vacuum rotational transfrom



Summary

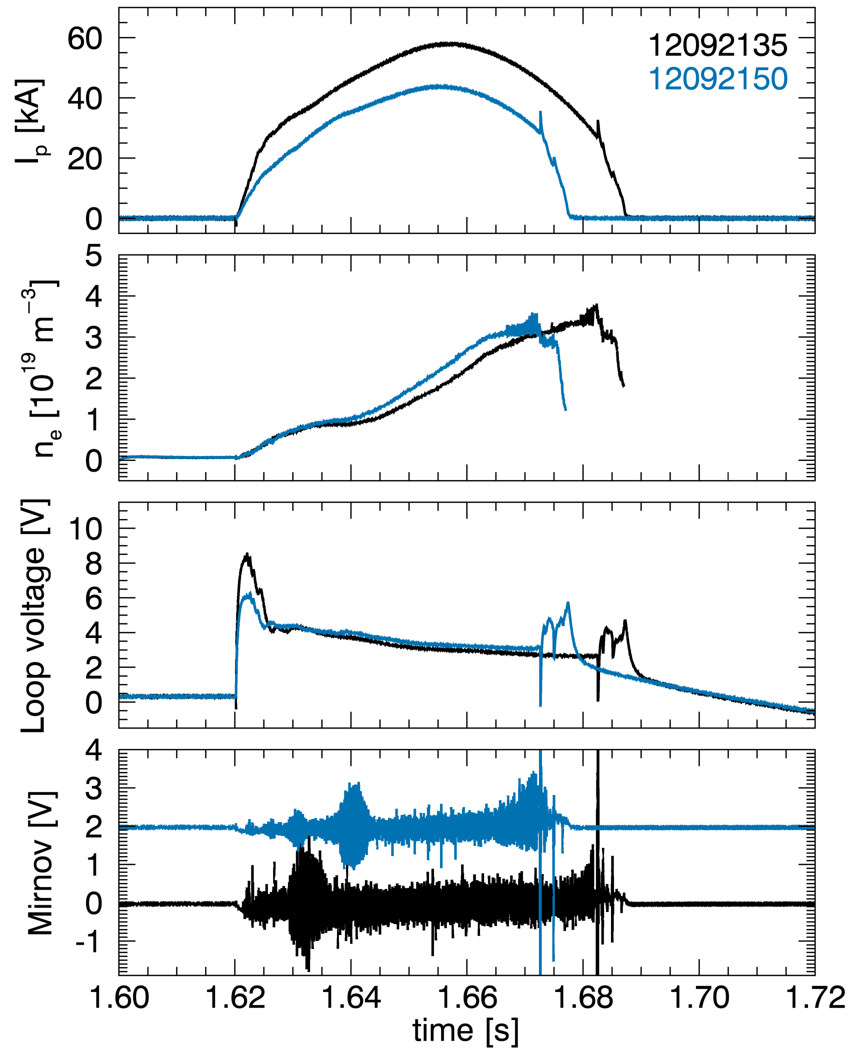
- Disruptive density limit exceeds Greenwald limit as vacuum transform is increased
  - Threshold for avoidance not observed
- Low-q disruptions cease to occur if vacuum transform raised above ~ 0.07 ($q_{vac}(a) \sim 14$)
  - $m = 2$, $n = 1$ mode not implicated in disruption
- Vertical stability of elongated plasmas improved by stellarator transform
  - Qualitative agreement with analytic theory

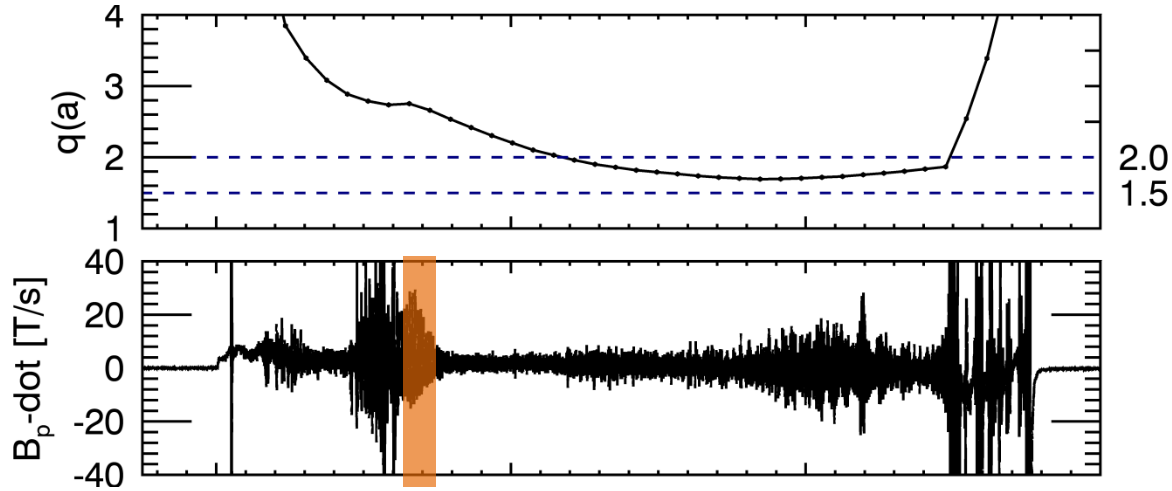
This work supported by U.S. Department of Energy grant DE-FG-02-00ER54610

Thank you

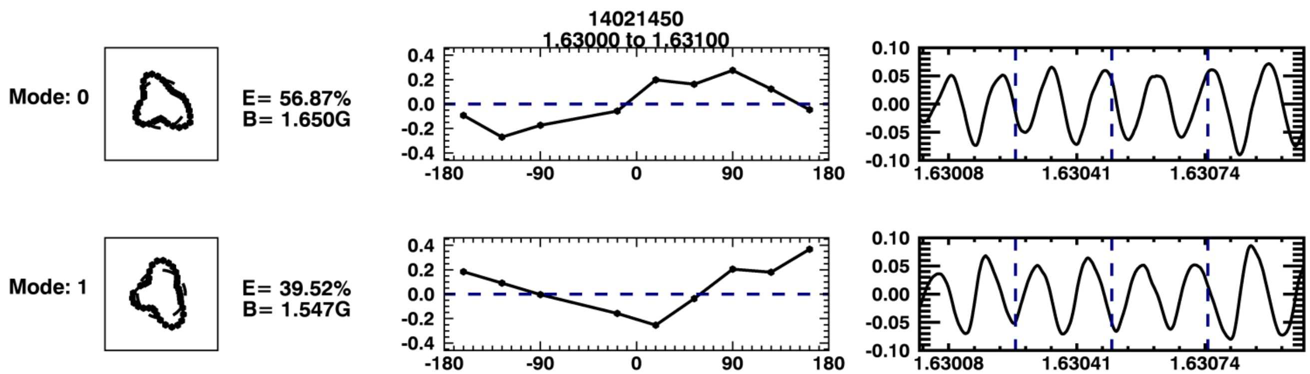
Density at disruption observed to be independent of plasma current evolution

- Discharges with similar transform $\iota_{vac} = 0.07$
- Different programmed loop voltage
- Disruption occurrence correlates with plasma current and density as in tokamaks

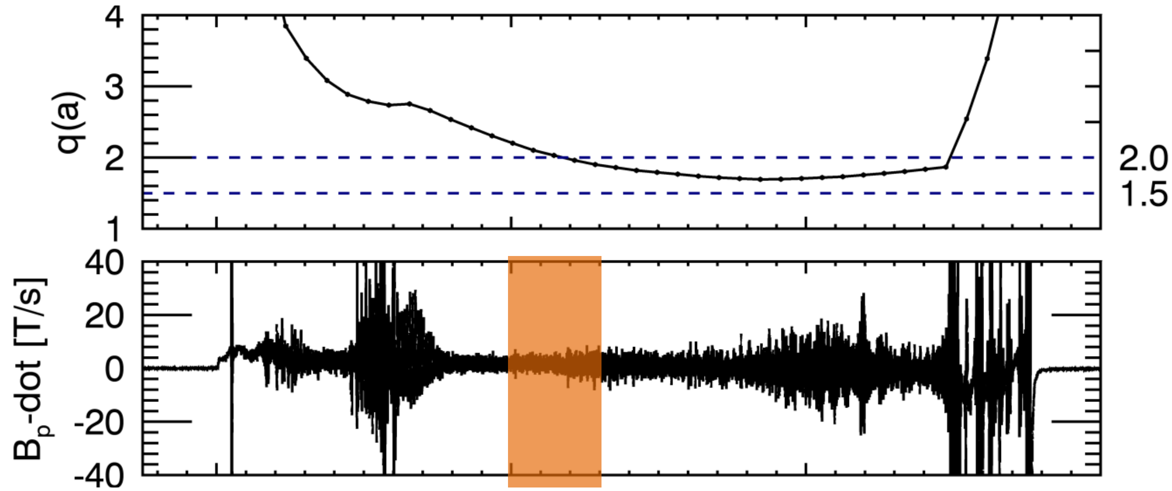


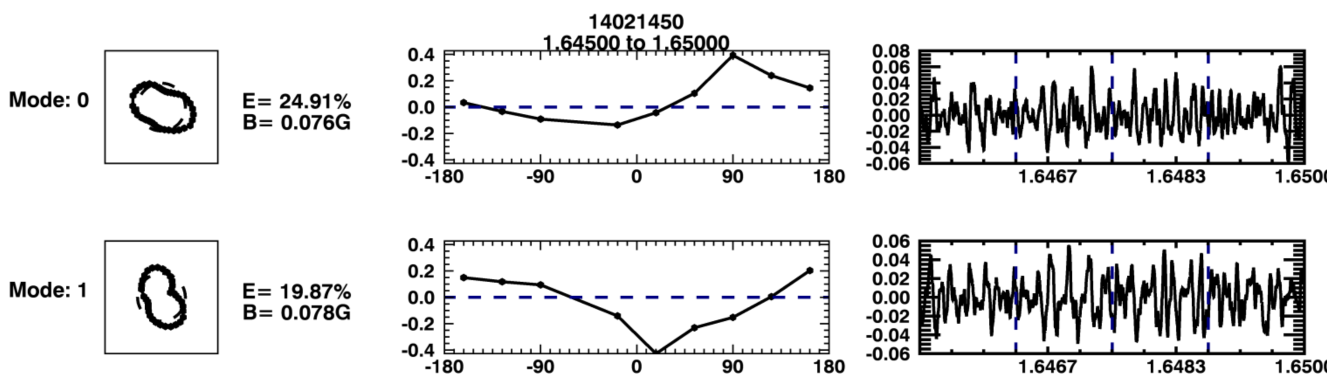


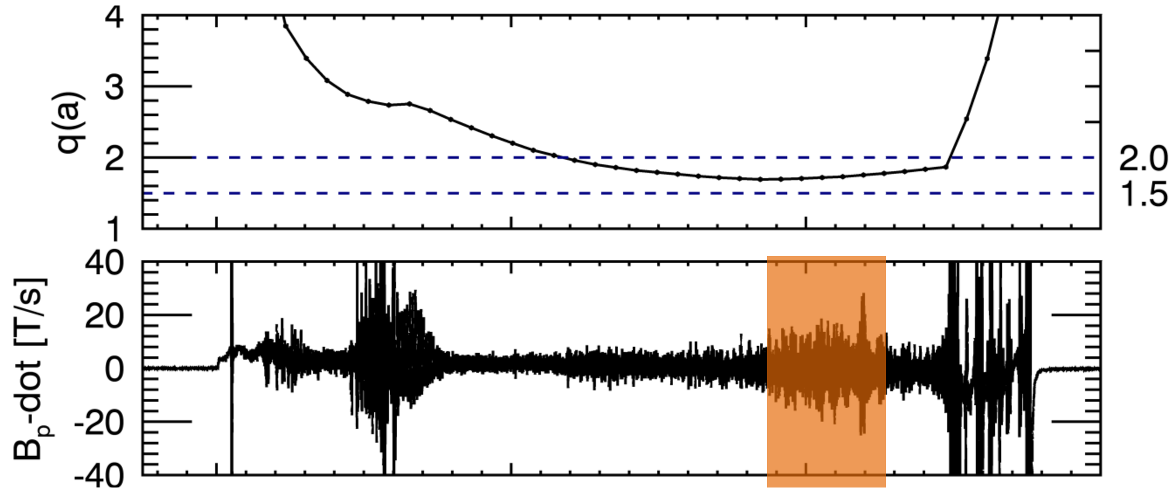
q(a)
4
3
2
1
2.0
1.5
B_p-dot [T/s]
40
20
0
-20
-40

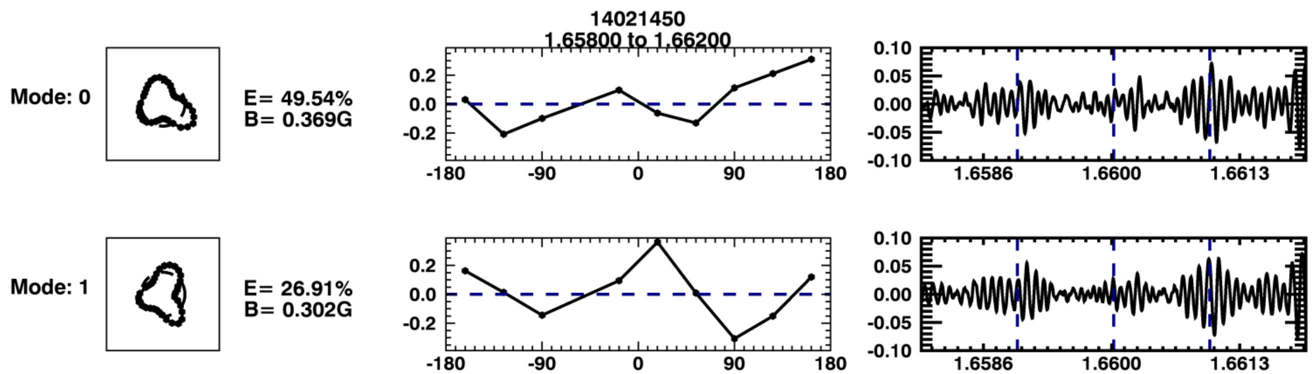


14021450
1.63000 to 1.63100
Mode: 0
E= 56.87%
B= 1.650G
Mode: 1
E= 39.52%
B= 1.547G
0.4
0.2
0.0
-0.2
-0.4
-180
-90
0
90
180
0.10
0.05
0.00
-0.05
-0.10
1.63008
1.63041
1.63074


q(a)
4
3
2
1
2.0
1.5
B_p-dot [T/s]
40
20
0
-20
-40


14021450
1.64500 to 1.65000
Mode: 0
E= 24.91%
B= 0.076G
Mode: 1
E= 19.87%
B= 0.078G
-180
-90
0
90
180
1.6467
1.6483
1.6500


q(a)
4
3
2
1
2.0
1.5
B_p-dot [T/s]
40
20
0
-20
-40


14021450
1.65800 to 1.66200
Mode: 0
E= 49.54%
B= 0.369G
Mode: 1
E= 26.91%
B= 0.302G
0.2
0.0
-0.2
-180
-90
0
90
180
0.10
0.05
0.00
-0.05
-0.10
1.6586
1.6600
1.6613

Drift orbit optimization