Abstract

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High field superconducting magnets are a necessary component of high energy colliders, fusion devices, and other active experiments. When the wires comprising the magnet suffer structural damage, the critical current is diminished and thus the resultant magnetic field. Of the most commonly used superconductors, Nb_3Sn is characterized by the highest current density, J_σ, and a significant susceptibility to internal strain. This internal strain is greatly dependent on the existence of voids within both the copper matrix and filaments. These voids are created withing the wire during the heat treatment process. A MatLab model using randomly distributed elliptical voids will allow for a better understanding of their effects.

Describing the Effects of Voids in Nb3Sn Superconducting Wire Using Finite Element Analysis

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High field superconducting magnets are a necessary component of high energy colliders, fusion devices, and other active experiments. When the wires comprising the magnet suffer structural damage, the critical current is diminished and thus the resultant magnetic field. Of the most commonly used superconductors, Nb_3Sn is characterized by the highest current density, J_σ, and a significant susceptibility to internal strain. This internal strain is greatly dependent on the existence of voids within both the copper matrix and filaments. These voids are created withing the wire during the heat treatment process. A MatLab model using randomly distributed elliptical voids will allow for a better understanding of their effects.

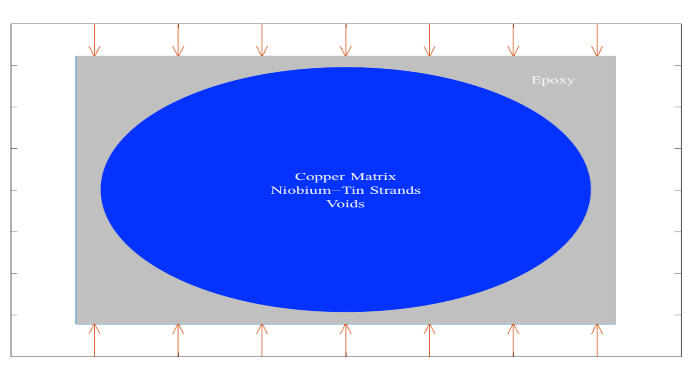


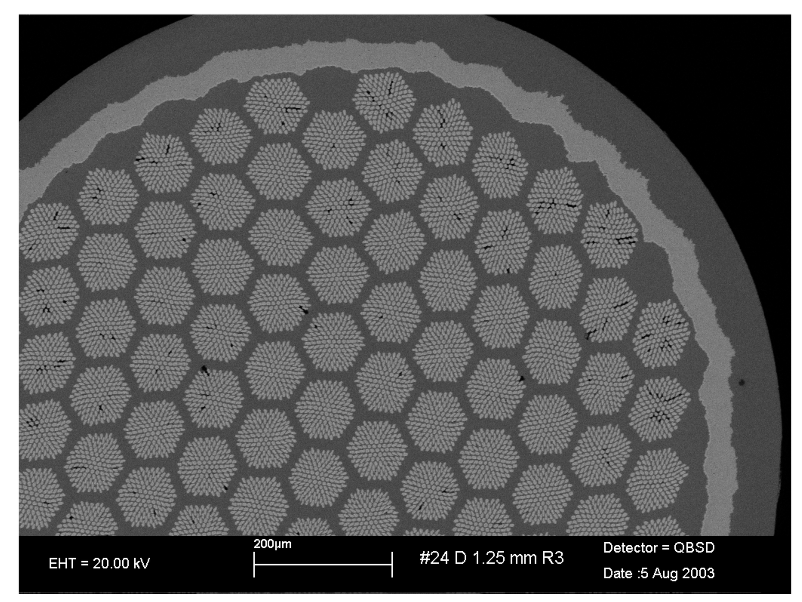
Figure 1: A diagram of the setup.

Methodology

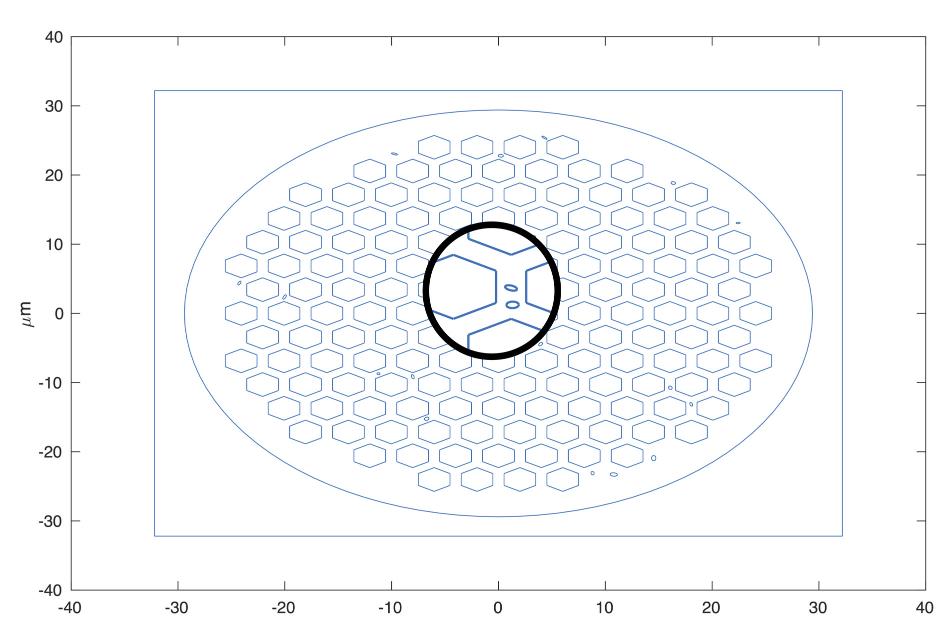
To form a complete picture of voids and their defects, we created a two dimensional model of the wire using the FEA capabilities of MatLab. Integration of the random and elliptical voids was completed using the MatLab functions *rand* and *inpolygon*. The former creates an array of random numbers between 0 and 1, while the latter returns logical values for a given set of query points and a defined set of polygons. The PDE model was then created using command line functions, rather than the GUI. A displacement of 1 μm was applied to both the top and bottom of the epoxy, while the left and right edges were kept stationary. The *pdesolve* function was then used to calculate the solution to the system of partial differential equations, u. To convert this into stresses, matrices were created defining the material properties at each position in the geometry.

Construction of the Model

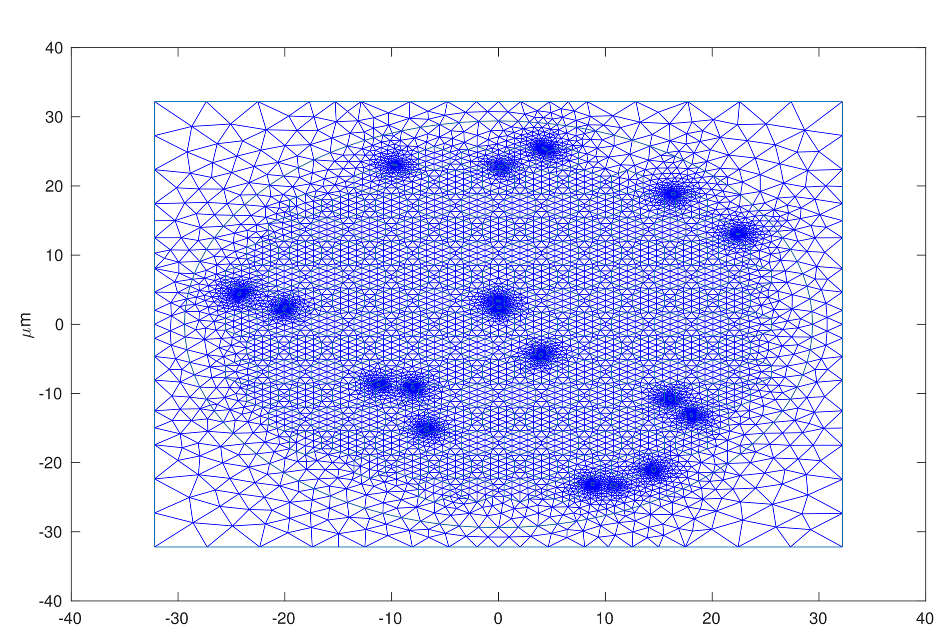
- **Cross Section**
  - The following image was the basis for the geometry of the model.



- **Geometry**
  - The geometry consists of a square, circle, and 151 hexagons. In the model the circle is represented by a hectagon.



- **Mesh**
  - The mesh image below corresponds to the specific geometry above. Rather than having the mesh contain holes where the voids exist, the voids are meshed and given material properties near 0.



Mathematical Background

In a system constrained to the xy-plane the stresses can be described by the following matrix equation involving strains, the elastic modulus (ϵ,γ) (E), and Poisson's ratio (ν).

$$\begin{bmatrix}\sigma_x\\ \sigma_y\\ \tau_{xy}\end{bmatrix} = \frac{E}{1-\nu^2}\begin{bmatrix}1 & \nu & 0\\ \nu & 1 & 0\\ 0 & 0 & \frac{1-\nu}{2}\end{bmatrix}\begin{bmatrix}\epsilon_x\\ \epsilon_y\\ \gamma_{xy}\end{bmatrix}$$

The strains are defined using the scalar displacement functions u and v.

$$\epsilon_x = \frac{\partial u}{\partial x}, \quad \epsilon_y = \frac{\partial v}{\partial y}, \quad \gamma = \frac{\partial u}{\partial y} + \frac{\partial v}{\partial x}$$

After considering the balance of forces on the planar object, the system of partial differential equations may be written as below.

$$\nabla \cdot (\mathbf{c} \otimes \nabla \mathbf{u}) = \mathbf{0}$$

The only coefficient of interest is the tensor $\mathbf{c}$ which MatLab requires to be in the form of the vector given below.

$$\bar{c} = \{2G+\mu, G, 0, G, 0, G, \mu, 0, \\ 0, \mu, G, 0, G, 0, 0, 2G+\mu\}$$

Once u and v have been determined, the principal stresses σ_1 and σ_2 may be calculated from the standard stresses. Finally the von Mises effective stress can be found using the following equation.

$$\sigma_{vm}^2 = (\sigma_1^2+\sigma_2^2)(\nu^2-\nu+1) + \sigma_1\sigma_2(2\nu^2-2\nu-1)$$

Results

The stresses calculated by this program agree with previous work[2]; stress increases locally along the major axis of the elliptical void. The following are two plots generated by MatLab illustrating the results of the our calculations on the geometry specified previously.

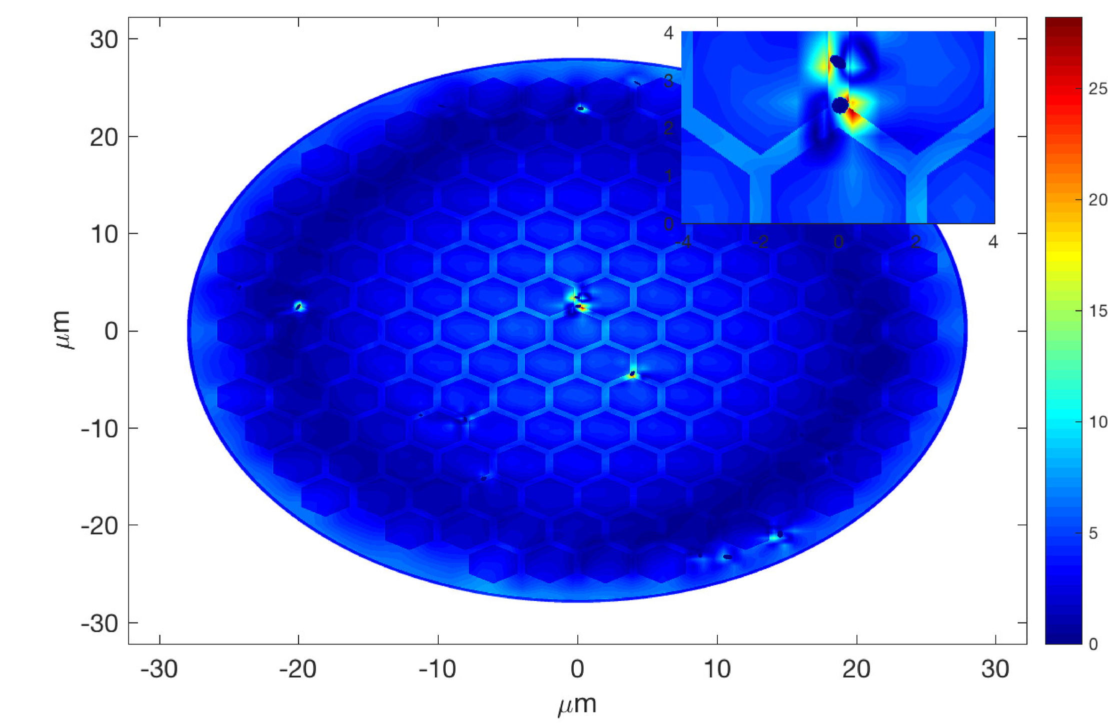


Figure 2: The von Mises stress.

Future Work

- Change the script creating the voids to allow for an exact number of voids to be created, instead of designating a maximum.
- Add a second method for creating voids that would allow experimental data as an input.
- Determine a more accurate method for stress calculations. Currently data is saved and then retrieved from a figure.
- Use this model to study correlation between data such as irreversible strain limit and the local stress concentration around the voids.

References

[1] C. Calzolaio et. al, 'Electro-mechanical properties of PIT Nb3Sn wires under transverse stress: experimental results and FEM analysis.' Supercond. Sci. Tech., Feb. 2015

[2] Y. Zhai et. al, 'Finite-Element Analysis of Transverse Compressive and Thermal Loads on Nb3Sn Wires With Voids.' IEEE Transactions on Applied Supercond, 26:4, June 2016

[3] Mathworks, (2016). Partial Differential Equations Toolbox: Users Guide (R2016a).

Acknowledgements

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