Single Particle Motion

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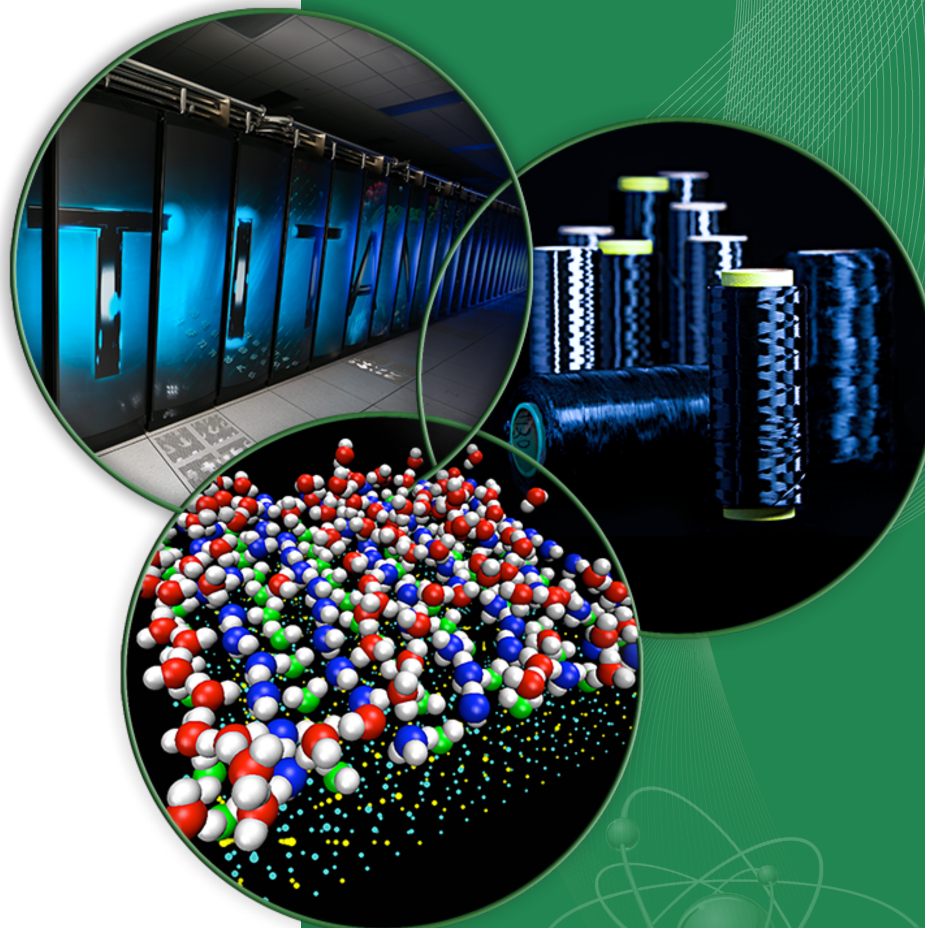


Presented:

June 8th 2015

SULI Introductory Course in Plasma Physics

Motivation: Single particle motion and collisions limit plasma confinement

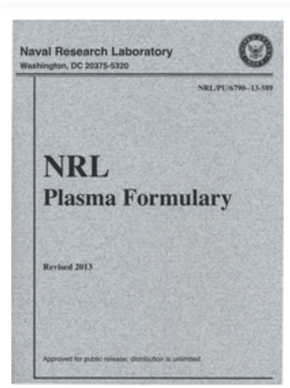
- Plasma transport can be described as a “random walk”
  - single particle motion gives the step size
  - collision frequency tells you how often steps are taken

- If care isn’t taken to optimize your magnetic field geometry particles can have trajectories which cause them to leave the system even without collisions
  - High energy particles are the most susceptible and the most damaging

The 3 things you should walk away from this talk with:

1. Charged particles move freely along a constant magnetic field, but any velocity perpendicular to the field causes them to orbit around the field lines
2. When the magnetic field strength isn't constant it will change parallel velocity of the charged particle and cause it to drift off the field line
3. A force perpendicular to your field lines will cause the particle to move perpendicular to both the force and the magnetic field lines

Resources Available Online



- NRL Plasma Formulary
  - www.nrl.navy.mil/ppd/content/nrl-plasma-formulary
- Magnetic Fusion Energy Formulary
  - www.psfc.mit.edu/research/MFEFormulary/
- Fusionwiki
  - fusionwiki.ciemat.es/
- Introduction to Plasma Physics and Controlled Fusion by F. Chen

Section 1: Neutral particles

Section 1:
Neutral particles

Ballistic.

Section 1: Neutral particles travel ballistically

- Not confined by magnetic fields
  - Neutrons generated by the D+T fusion process travel directly to the wall
- Neutral particles are very relevant near material interfaces
  - But their single particle motion is not very interesting

Section 2:(AKA The rest of this talk) CHARGED particles

- Electrons
- Ions
- Very highly charged, extremely small dust particles

Lorentz force equation forms the basis for single particle motion

- Lorentz force describes the forces on a charged particle moving in a the presence of an electric field and magnetic field

$$\vec{F} = q\left(\vec{E} + \vec{v} \times \vec{B}\right)$$

$\vec{F}$ =force on the particle

q =charge on the particle

$\vec{E}$ = electric field

$\vec{v}$ =velocity of the particle

$\vec{B}$ =magnetic field

Newton’s law second law of motion also important

Newton’s second law of motion

$$\vec{F} = m\vec{a}$$

$\vec{F}$=force on the particle

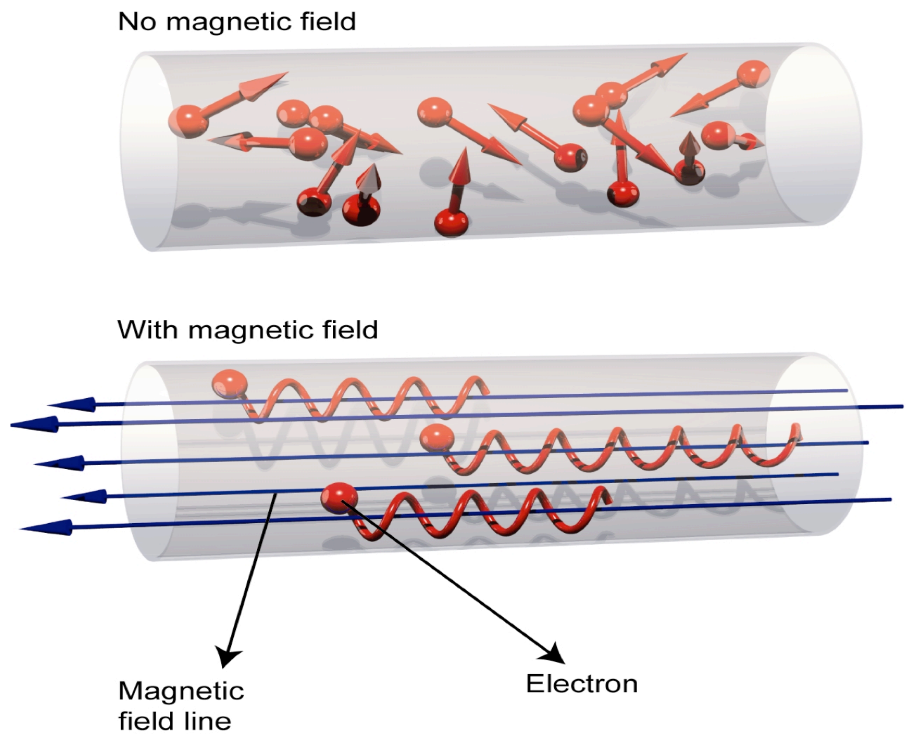
m=mass of that particle

$\vec{a}$ = acceleration of that particle

Charged particles move freely ALONG magnetic fields

$$\vec{F} = q\left(\cancelto{0}{\vec{E}} + \vec{v} \times \vec{B}\right)$$

$$\vec{F} = q\vec{v} \times \vec{B}$$



- Particles are free to stream along magnetic field lines
  - …when the magnetic field is constant

Figure from: http://iter.rma.ac.be/en/img/MagneticConfinement.jpg

Cyclotron-frequency and Larmor radius

Cyclotron-frequency (aka gyro-frequency) the frequency at which a particle moves around the magnetic field:

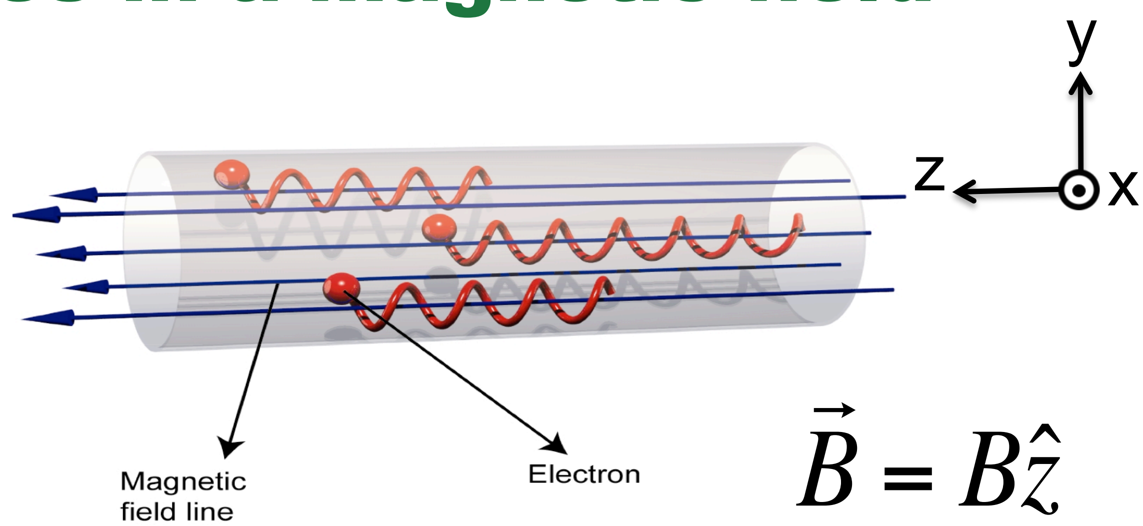
$$\omega_c \equiv \frac{|q|B}{m}$$

Larmor radius the radius of the orbit the particle:

$$r_L \equiv \frac{v_\perp}{\omega_c}$$

Charged particles in a magnetic field

$$\vec{F} = q\vec{v} \times \vec{B} = m\vec{a}$$



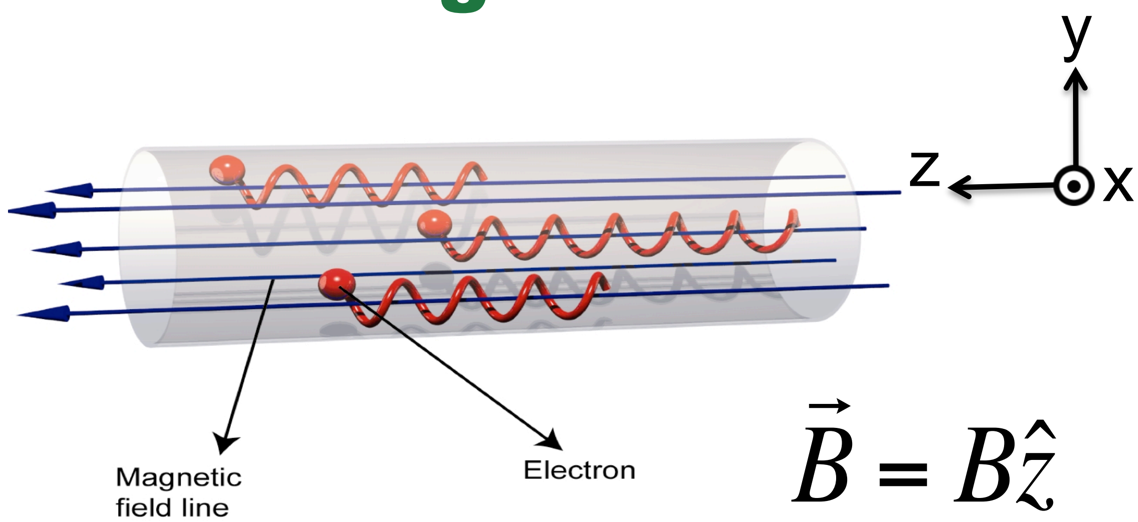
$$\vec{B} = B\hat{z}$$

Figure from: http://iter.rma.ac.be/en/img/MagneticConfinement.jpg

Charged particles in a magnetic field

$$\vec{F} = q\vec{v} \times \vec{B} = m\vec{a}$$

$$\vec{v} \equiv v_x\hat{x} + v_y\hat{y} + v_z\hat{z}$$



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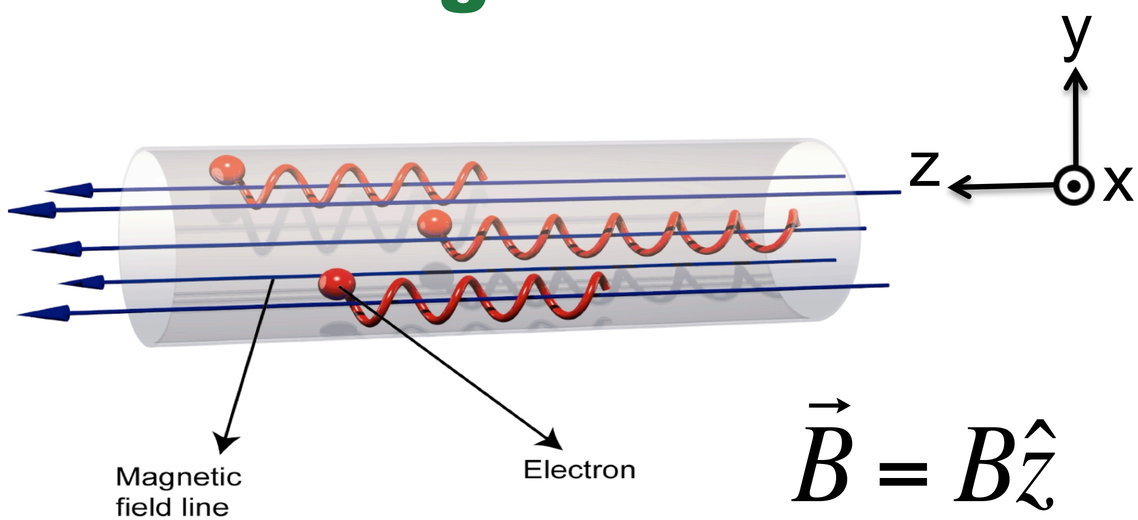

$$\vec{v} \equiv v_x\hat{x} + v_y\hat{y} + v_z\hat{z}$$

$$\vec{B} = B\hat{z}$$

$$\vec{v}\,' \equiv \vec{a} = v'_x\,\hat{x} + v'_y\,\hat{y} + v'_z\,\hat{z}$$

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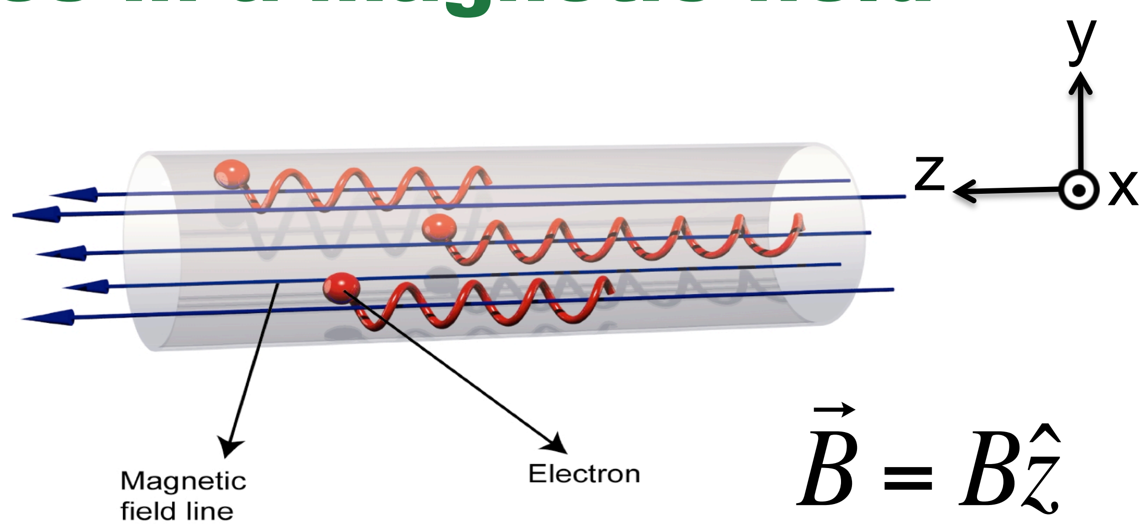
Charged particles in a magnetic field

$$\vec{F} = q\vec{v} \times \vec{B} = m\vec{a}$$



$$\vec{B} = B\hat{z}$$

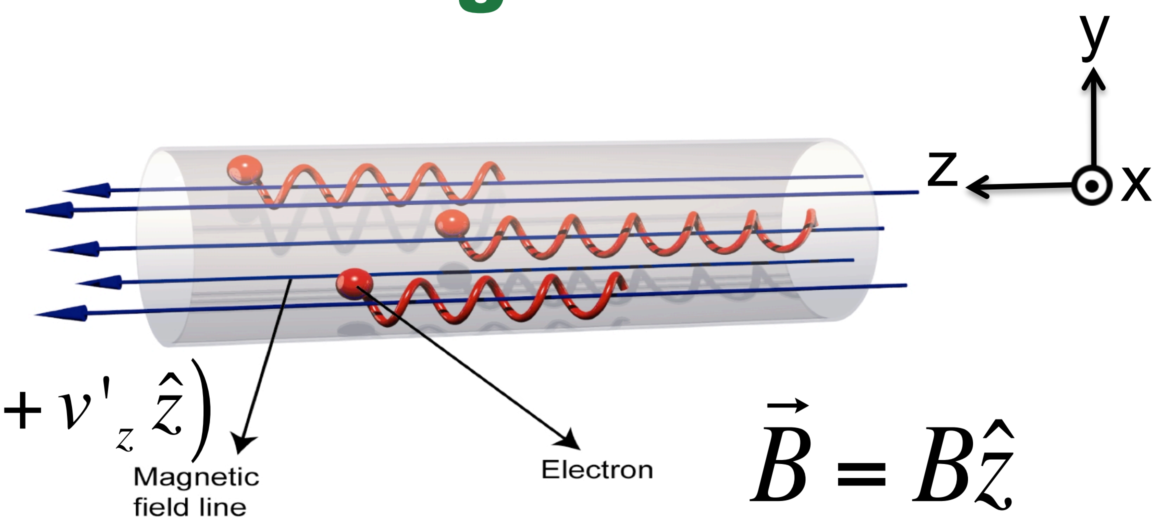
$$\vec{v} \equiv v_x\hat{x} + v_y\hat{y} + v_z\hat{z}$$

$$\vec{v}\,' \equiv \vec{a} = v'_x\,\hat{x} + v'_y\,\hat{y} + v'_z\,\hat{z}$$

$$q\vec{v} \times \vec{B} = m\left(v'_x\,\hat{x} + v'_y\,\hat{y} + v'_z\,\hat{z}\right)$$

Figure from: http://iter.rma.ac.be/en/img/MagneticConfinement.jpg

Charged particles in a magnetic field



$$q\vec{v}\times\vec{B} = m\left(v'_x\,\hat{x} + v'_y\,\hat{y} + v'_z\,\hat{z}\right)$$

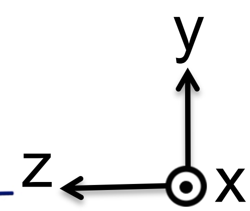
$$\vec{B} = B\hat{z}$$

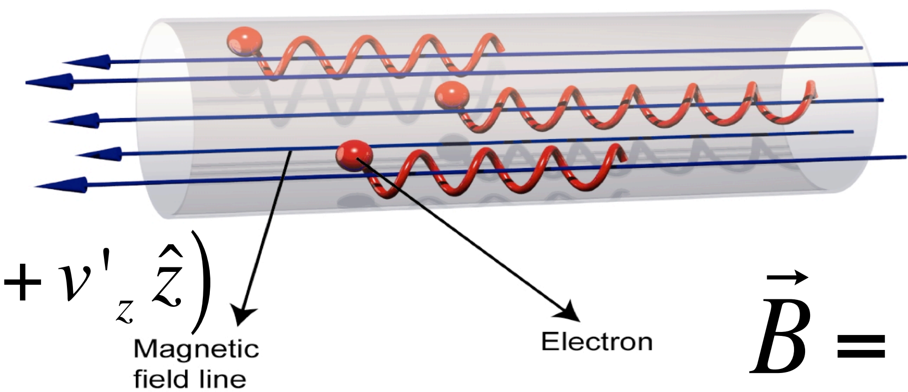
$$v'_x\,\hat{x} + v'_y\,\hat{y} + v'_z\,\hat{z} = \frac{q\vec{v}\times\vec{B}}{m} = \frac{-qv_xB\hat{y} + qv_yB\hat{x}}{m}$$

$$v'_x\,\hat{x} = \frac{qv_yB\hat{x}}{m} \qquad v'_y\,\hat{y} = \frac{-qv_xB\hat{y}}{m} \qquad v'_z\,\hat{z} = 0$$

Figure from: http://iter.rma.ac.be/en/img/MagneticConfinement.jpg

Charged particles in a magnetic field





$$q\vec{v}\times\vec{B} = m\left(v'_x\,\hat{x} + v'_y\,\hat{y} + v'_z\,\hat{z}\right)$$

$$\vec{B} = B\hat{z}$$

$$v'_x\,\hat{x} + v'_y\,\hat{y} + v'_z\,\hat{z} = \frac{q\vec{v}\times\vec{B}}{m} = \frac{-qv_xB\hat{y} + qv_yB\hat{x}}{m}$$

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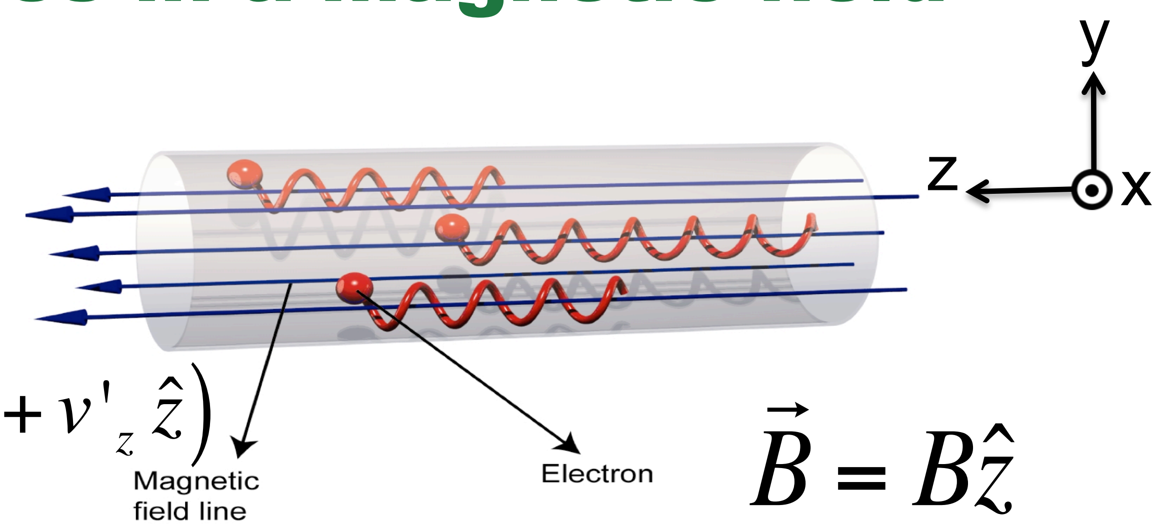
$$v'_z\,\hat{z} = 0$$

Charged particles move freely along the magnetic field!

Figure from: http://iter.rma.ac.be/en/img/MagneticConfinement.jpg

Charged particles in a magnetic field



$$q\vec{v}\times\vec{B} = m\left(v'_x\,\hat{x} + v'_y\,\hat{y} + v'_z\,\hat{z}\right)$$

$$\vec{B} = B\hat{z}$$

$$v'_x\,\hat{x} + v'_y\,\hat{y} + v'_z\,\hat{z} = \frac{q\vec{v}\times\vec{B}}{m} = \frac{-qv_xB\hat{y} + qv_yB\hat{x}}{m}$$

$$v'_x\,\hat{x} = \frac{qv_yB\hat{x}}{m} \qquad v'_y\,\hat{y} = \frac{-qv_xB\hat{y}}{m}$$

$$\cancel{v'_z\,\hat{z} = 0}$$

Figure from: http://iter.rma.ac.be/en/img/MagneticConfinement.jpg

Motion perpendicular to the magnetic field

$$v'_x \hat{x} = \frac{q v_y B \hat{x}}{m}$$

$$v'_y \hat{y} = \frac{-q v_x B \hat{y}}{m}$$

Drop the unit vectors

$$v'_x \hat{x} = \frac{q v_y B \hat{x}}{m} \rightarrow v'_x = \frac{q v_y B}{m}$$

$$v'_y \hat{y} = \frac{-q v_x B \hat{y}}{m} \rightarrow v'_y = \frac{-q v_x B}{m}$$

Take another derivative with respect to time

$$v'_x \hat{x} = \frac{qv_y B\hat{x}}{m} \qquad v'_x = \frac{qv_y B}{m} \longrightarrow v''_x = \frac{qv'_y B}{m}$$

$$v'_y \hat{y} = \frac{-qv_x B\hat{y}}{m} \qquad v'_y = \frac{-qv_x B}{m} \longrightarrow v''_y = \frac{-qv'_x B}{m}$$

The goal: differential equations which involve a single spatial coordinate

$$v'_x \hat{x} = \frac{qv_y B\hat{x}}{m}$$

$$v'_x = \frac{qv_y B}{m}$$

$$v''_x = \frac{qv'_y B}{m}$$

$$v'_y \hat{y} = \frac{-qv_x B\hat{y}}{m}$$

$$v'_y = \frac{-qv_x B}{m}$$

$$v''_y = \frac{-qv'_x B}{m}$$

$$v''_y = \frac{-q^2 B^2 v_y}{m^2}$$

Repeat for the terms involving x

$$v'_x \hat{x} = \frac{qv_y B\hat{x}}{m}$$

$$v'_x = \frac{qv_y B}{m}$$

$$v''_x = \frac{qv'_y B}{m}$$

$$v'_y \hat{y} = \frac{-qv_x B\hat{y}}{m}$$

$$v'_y = \frac{-qv_x B}{m}$$

$$v''_y = \frac{-qv'_x B}{m}$$

$$v''_x = \frac{-q^2 B^2 v_x}{m^2}$$

$$v''_y = \frac{-q^2 B^2 v_y}{m^2}$$

2 equations each with involving a single spatial coordinate

$$v'_x \hat{x} = \frac{qv_y B\hat{x}}{m} \qquad v'_x = \frac{qv_y B}{m} \qquad v''_x = \frac{qv'_y B}{m}$$

$$v'_y \hat{y} = \frac{-qv_x B\hat{y}}{m} \qquad v'_y = \frac{-qv_x B}{m} \qquad v''_y = \frac{-qv'_x B}{m}$$

$$v''_x = \frac{-q^2 B^2 v_x}{m^2}$$

$$v''_y = \frac{-q^2 B^2 v_y}{m^2}$$

2 differential equations can be solved using sines and cosines

$$v''_x = \frac{-q^2 B^2 v_x}{m^2} \qquad v''_y = \frac{-q^2 B^2 v_y}{m^2}$$

$$v_x = -v_\perp \cos\left(\frac{qB}{m}t + \phi_0\right) \qquad v_y = v_\perp \sin\left(\frac{qB}{m}t + \phi_0\right)$$

- $v_\perp$ could be an arbitrary coefficient, but is constrained by the initial velocity perpendicular to $\vec{B}$
- ϕ_0 An arbitrary phase is used to match the initial velocity

Integrate to find the position

$$v''_x = \frac{-q^2 B^2 v_x}{m^2} \qquad v''_y = \frac{-q^2 B^2 v_y}{m^2}$$

$$v_x = v_\perp \cos\left(\frac{qB}{m}t + \phi_0\right) \qquad v_y = v_\perp \sin\left(\frac{qB}{m}t + \phi_0\right)$$

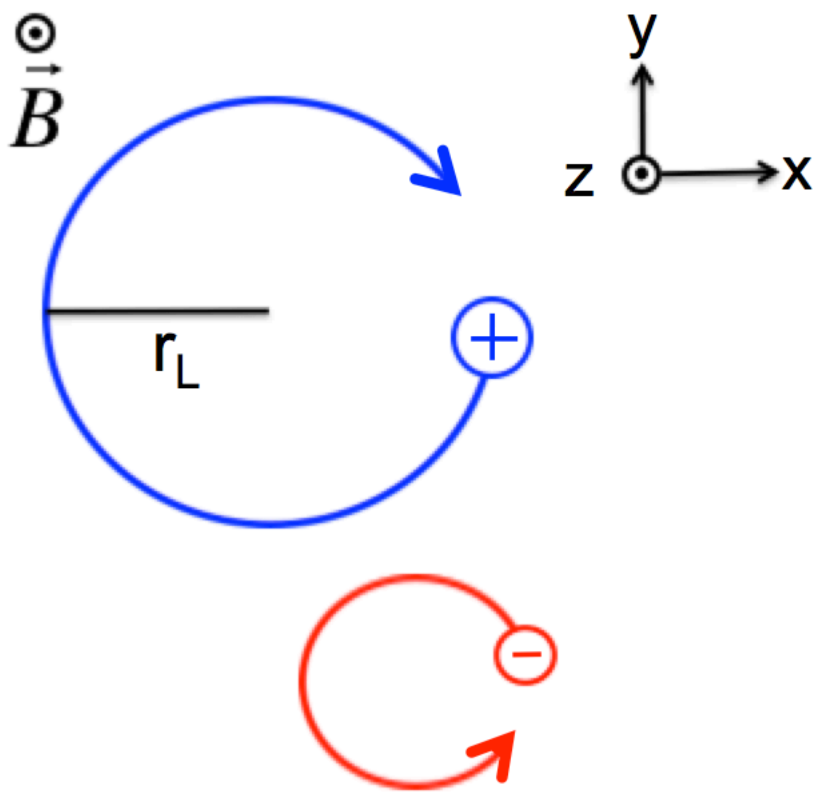
$$\downarrow \qquad \downarrow$$

$$x = -\frac{v_\perp m}{qB}\sin\left(\frac{qB}{m}t + \phi_0\right) + x_0 \qquad y = -\frac{v_\perp m}{qB}\cos\left(\frac{qB}{m}t + \phi_0\right) + y_0$$

- 2 equations to describe the particle's position in the plane perpendicular to the magnetic field

Particles make circular orbits, with a handedness that depends on charge

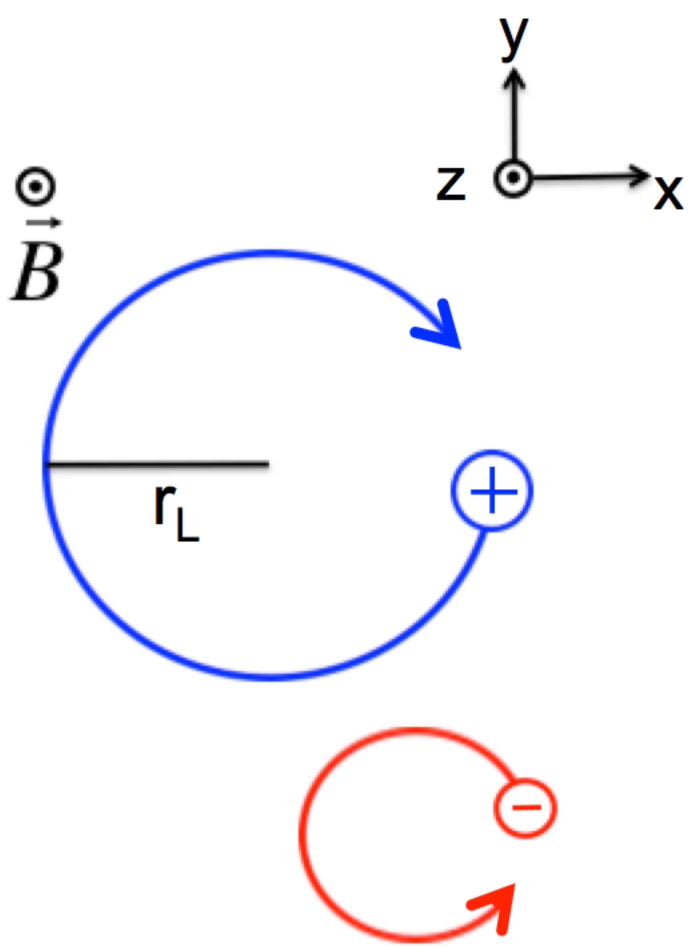
$$x = -\frac{v_\perp m}{qB}\sin\left(\frac{qB}{m}t + \phi_0\right) + x_0 \qquad y = -\frac{v_\perp m}{qB}\cos\left(\frac{qB}{m}t + \phi_0\right) + y_0$$

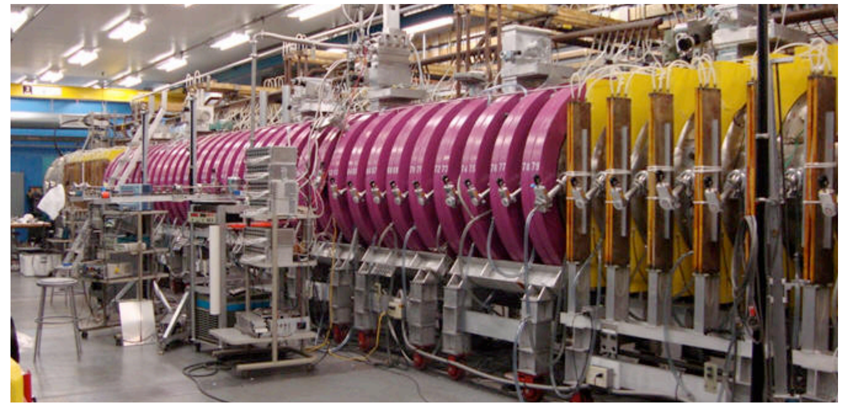


$$\omega_c \equiv \frac{|q|B}{m}$$

$$r_L \equiv \frac{v_\perp}{\omega_c}$$

Larmor radius set the MINIMUM size for a confinement device





- Devices must be much larger than the Larmor radii of the particles they are confining

Picture: LAPD linear plasma device

Ions are a lot heavier than electrons

$$\frac{m_{proton}}{m_{electron}} \approx \frac{m_{bowlingball}}{m_{pingpongball}}$$

- protons are about 1800 times heavier than electrons

Ions generally have much larger Larmor radii than electrons

$$E_{electron} \equiv \frac{1}{2} m_{electron} v_{electron}^2 = \frac{1}{2} m_{proton} v_{proton}^2 = E_{proton}$$

Energy tends to equilibrate in the system

$$r_L = \frac{v_\perp}{\omega_c} = \frac{\sqrt{2E}\sqrt{m}}{|q|B}$$

$$\frac{r_{L\_\mathrm{Proton}}}{r_{L\_Electron}} = \frac{\sqrt{m_{\mathrm{Proton}}}}{\sqrt{m_{Electron}}} \approx 43$$

Cyclotron frequency

$$\omega_c \equiv \frac{|q|B}{m}$$

- Important for particle heating

$$f_c = \frac{\omega_c}{2\pi} = \frac{|q|B}{m\pi}$$

- Electron cyclotron frequency in a 1 Tesla magnetic fields is 28.0 GHz
- Ion cyclotron frequency in a 1 Tesla magnetic fields is 14.2 MHz
- Your microwave oven operates at 2.45 GHz
- The FM radio band is from 88 to 108 MHz

Electric field parallel to magnetic field

$$\vec{F} = q\left(\vec{E} + \vec{v} \times \vec{B}\right) = m\left(v'_x \hat{x} + v'_y \hat{y} + v'_z \hat{z}\right)$$

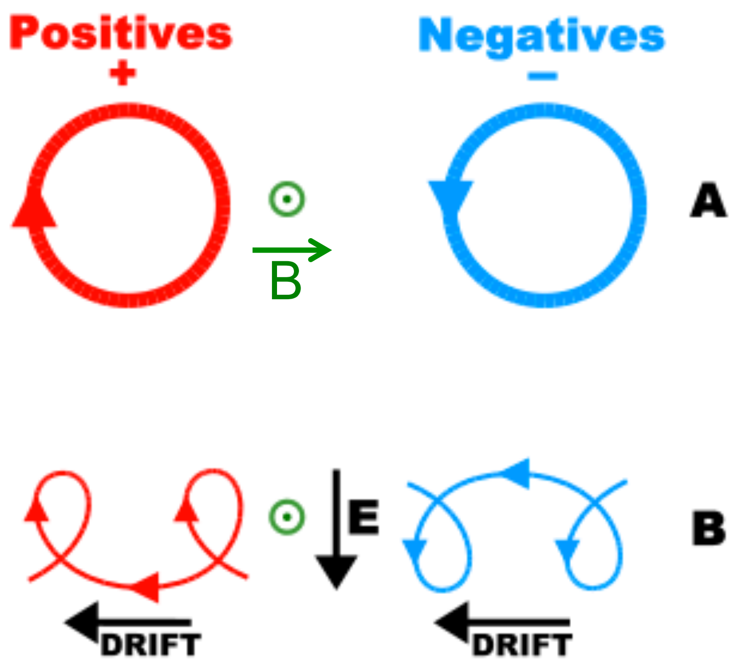
$$q\left(E\hat{z} + \vec{v} \times B\hat{z}\right) = m\left(v'_x \hat{x} + v'_y \hat{y} + v'_z \hat{z}\right)$$

$$qE\hat{z} = mv'_z \hat{z}$$

$$\frac{q}{m}E = v'_z$$

- If the electric field is parallel to the magnetic field the charge particles are accelerated just like they would be if there was no magnetic field

Electric field PERPENDICULAR to the magnetic field



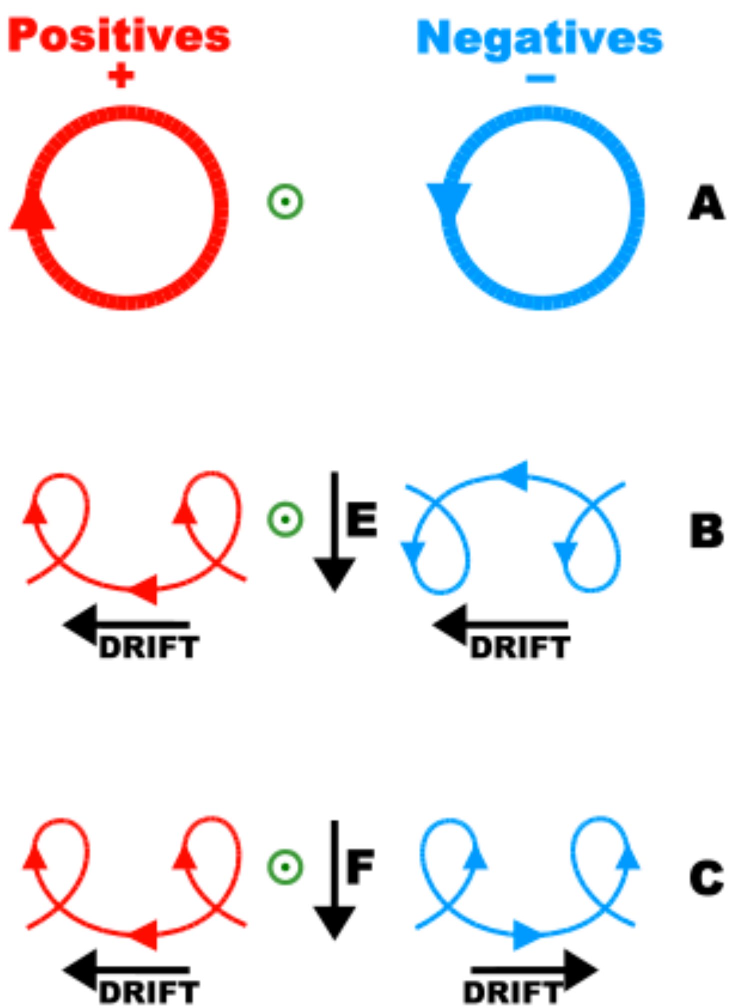
- Particles are accelerated when traveling along the electric field direction and decelerated when traveling against it

- This alters the gyromotion causing the particles to drift

$$v_E = \frac{\left(\vec{E} \times \vec{B}\right)}{B^2}$$

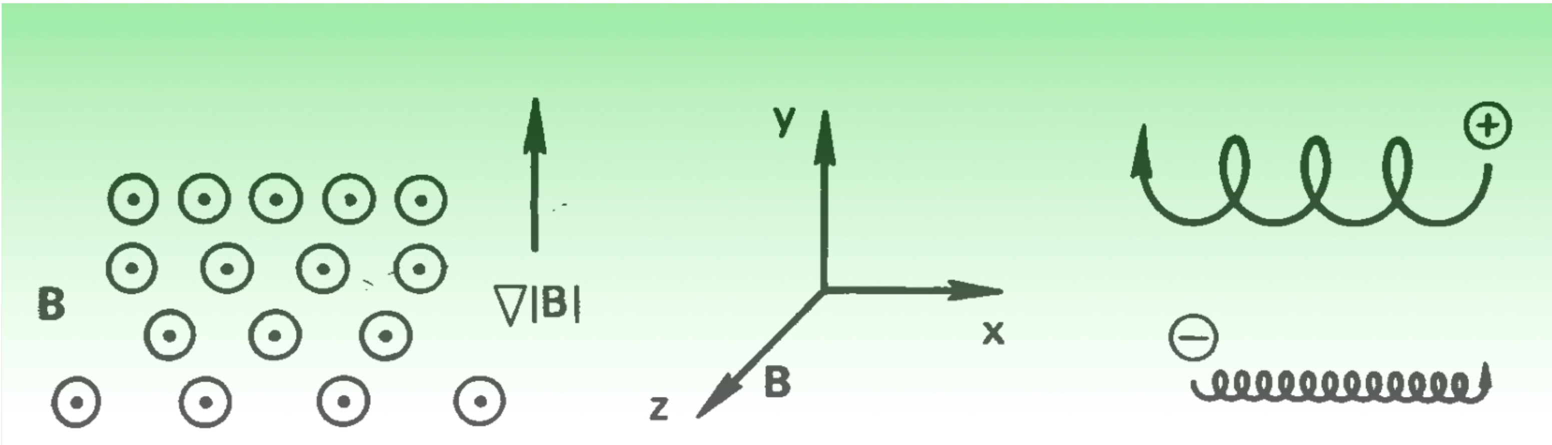
Diagram from: http://www.newworldencyclopedia.org/entry/File:Charged-particle-drifts.svg#file

Other forces cause drifts which push positive and negative particles in opposite directions



- One example of this is drift caused by the gravitational force: $$F_g = m\vec{g}$$
- Is given by:
$$v_g = \frac{m}{q}\frac{\vec{g}\times\vec{B}}{B^2}$$
- …but will generally be much weaker than the other forces in your system

Particle motion in a non-uniform magnetic field



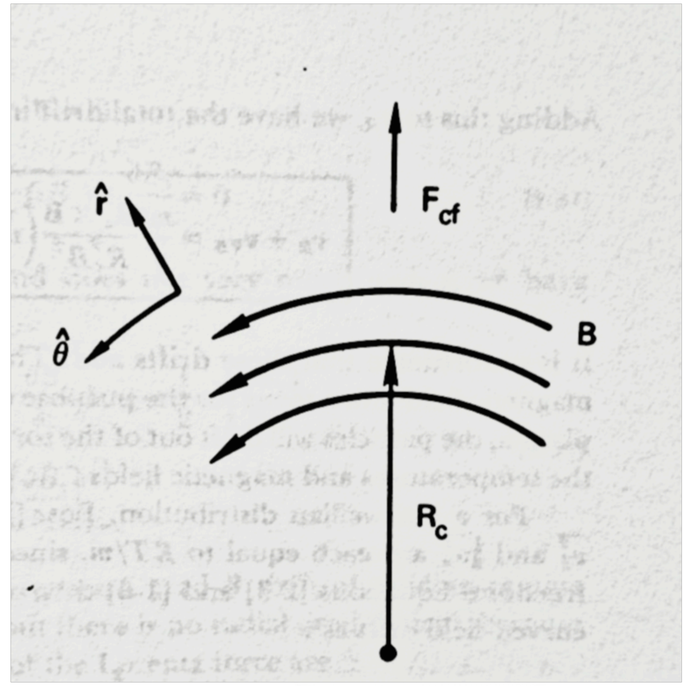
- In gyro-radius will be larger where the field is weaker and smaller where the field is stronger
- The resulting velocity is described by:

$$\vec{v}_{\nabla B} = \pm \frac{1}{2} v_{\perp} r_L \frac{\vec{B} \times \nabla B}{B^2}$$

Total particle motion in a curved magnetic field

- The resulting velocity is described by:

$$\vec{v}_R = \frac{1}{q}\frac{\vec{F}_{cf} \times \vec{B}}{B^2} = \frac{mv_{\parallel}^2}{qB^2}\frac{\vec{R}_c \times \vec{B}}{R_c^2}$$



3 invariant quantities

- These quantities don't change unless something does work on the particle
- Magnetic fields do not do work on the particles

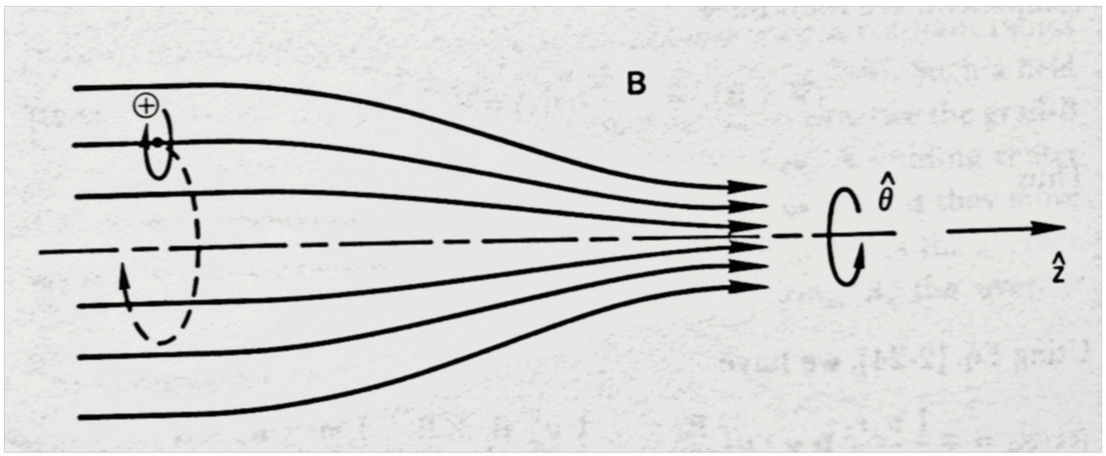
Energy: $$\frac{1}{2}mv^2 = \frac{1}{2}m\left(v_\perp^2 + v_\parallel^2\right)$$

Magnetic moment: $$\mu \equiv \frac{1}{2}\frac{mv_\perp^2}{B}$$

Adiabatic invariant: $$J \equiv \int_a^b v_\parallel \, ds$$

The magnetic moment of a gyrating particle is a conserved quantity

Magnetic moment $$\mu \equiv \frac{1}{2}\frac{mv_{\perp}^{2}}{B}$$



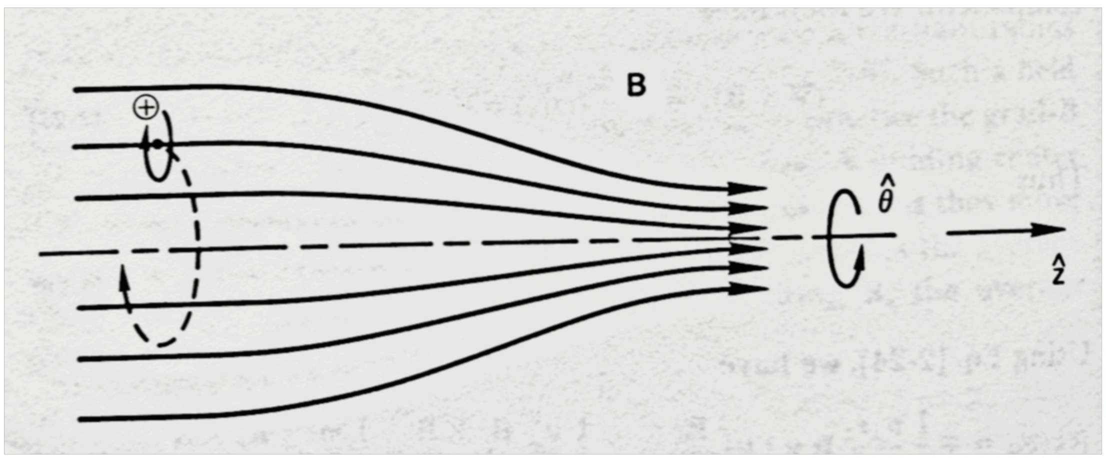
- As a particle moves to a region of larger magnetic field, the particles velocity perpendicular to the field must also increase

The conservation of the magnetic moment and conservation of energy creates the mirror effect

The particle's initial energy

$$\frac{1}{2}mv_0^2 = \frac{1}{2}m\left(v_\perp^2 + v_\parallel^2\right) = \frac{1}{2}mv^2$$

The particle's energy at any given time



- As the particle's $v_\perp$ increase to conserve μ the particle's $v_\parallel$ must decrease to conserve energy

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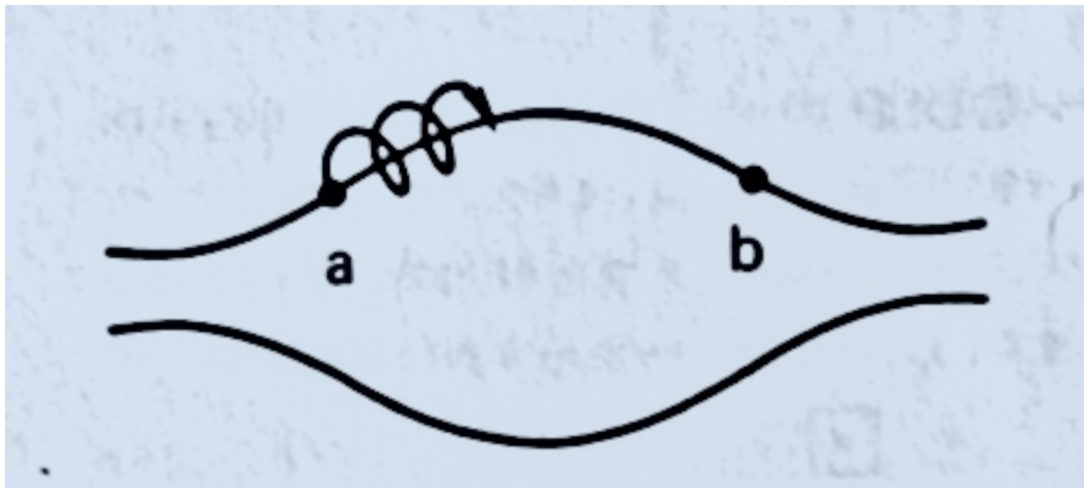
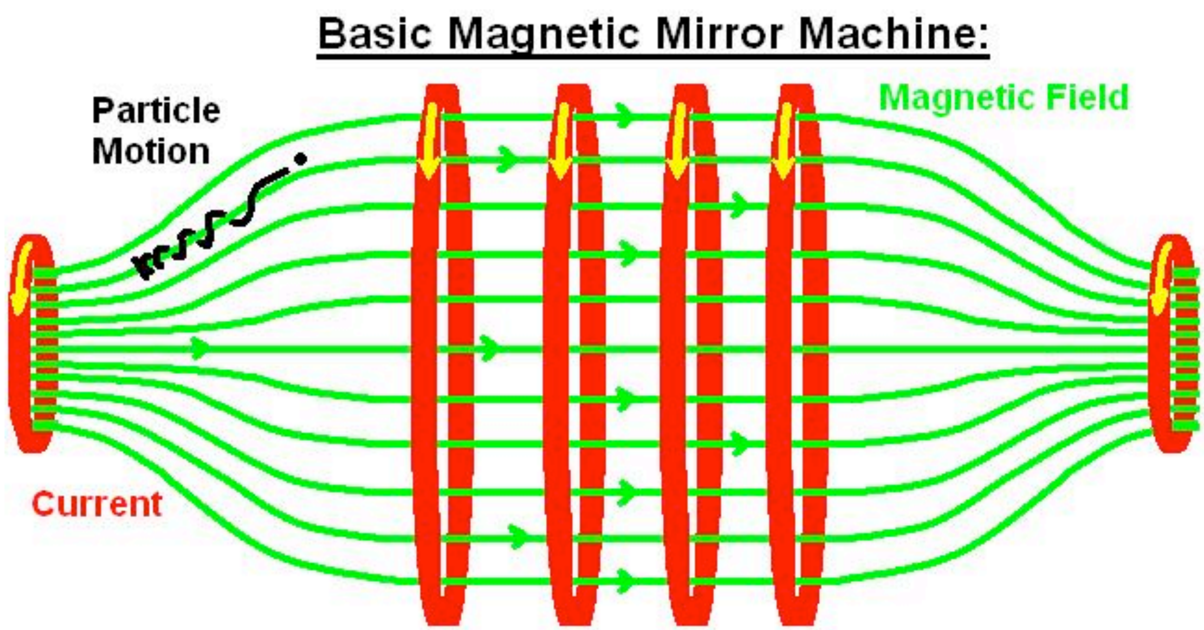
Adiabatic invariant

$$J \equiv \int_a^b v_{\parallel} ds$$



- The integral of the parallel velocity as a particle bounces between two points stays constant

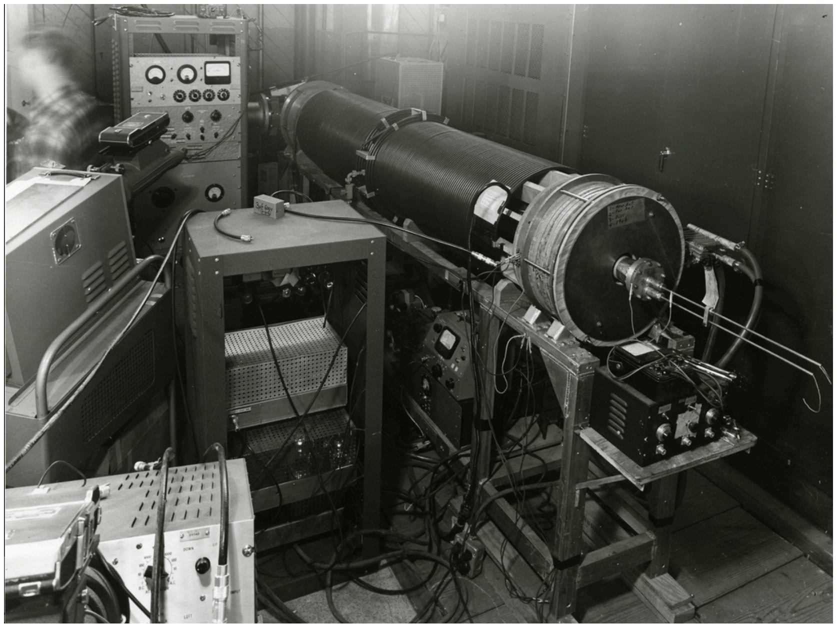
Magnetic mirrors were one of the first plasma confinement devices



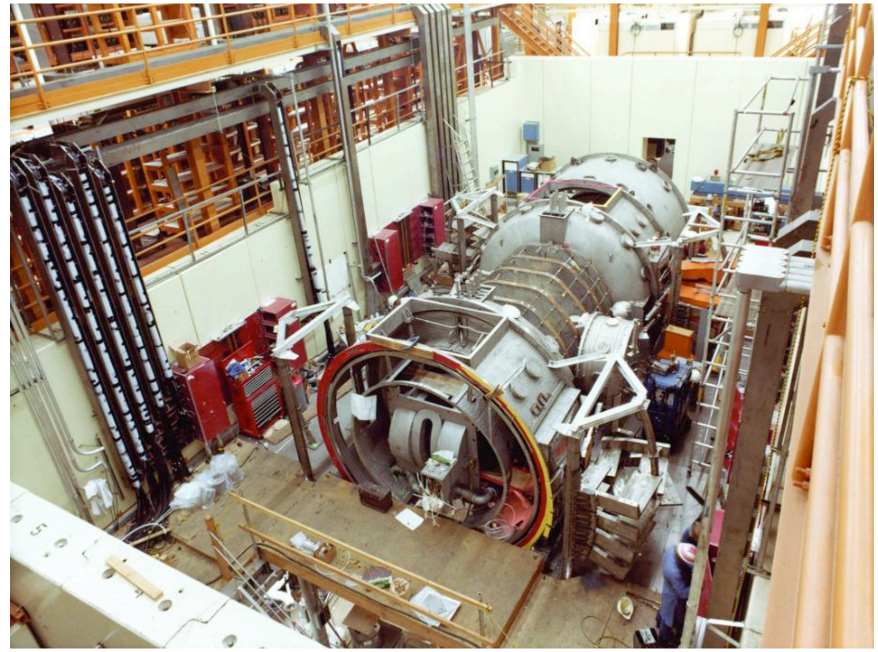
- The magnetic field prevents the particles from traveling radially towards the wall
- Particles are reflected by in the regions of higher magnetic field

Picture from http://en.wikipedia.org/wiki/Magnetic_mirror#/media/File:Basic_Magnetic_Mirror.jpg

Non-uniform magnetic fields in mirror machines can be used to confine particles



Q-cumber 1955



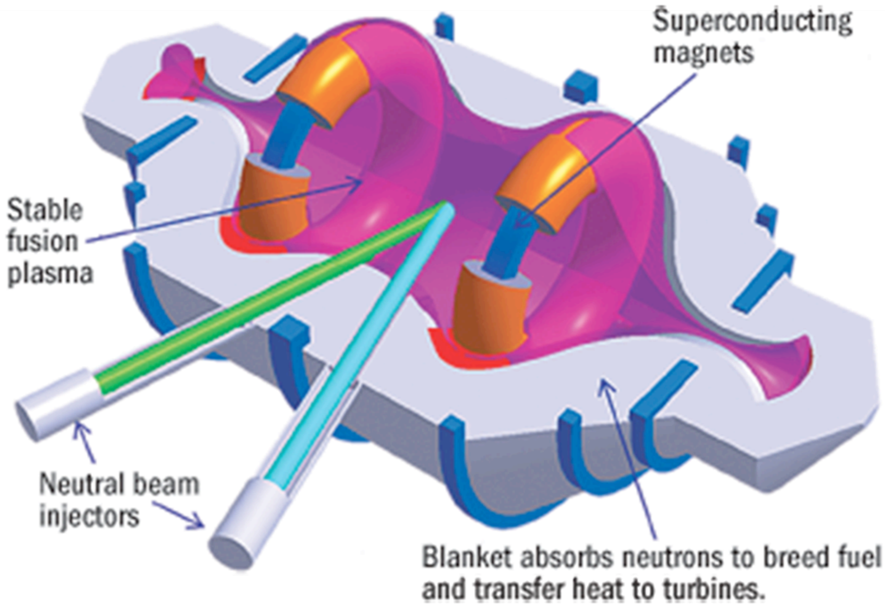
Tandem mirror 1979

- Both machines were at Lawrence Livermore National Lab

http://en.wikipedia.org/wiki/Tandem_Mirror_Experiment

https://www.flickr.com/photos/llnl/page4

Lockhead Martin's "compact fusion reactor"



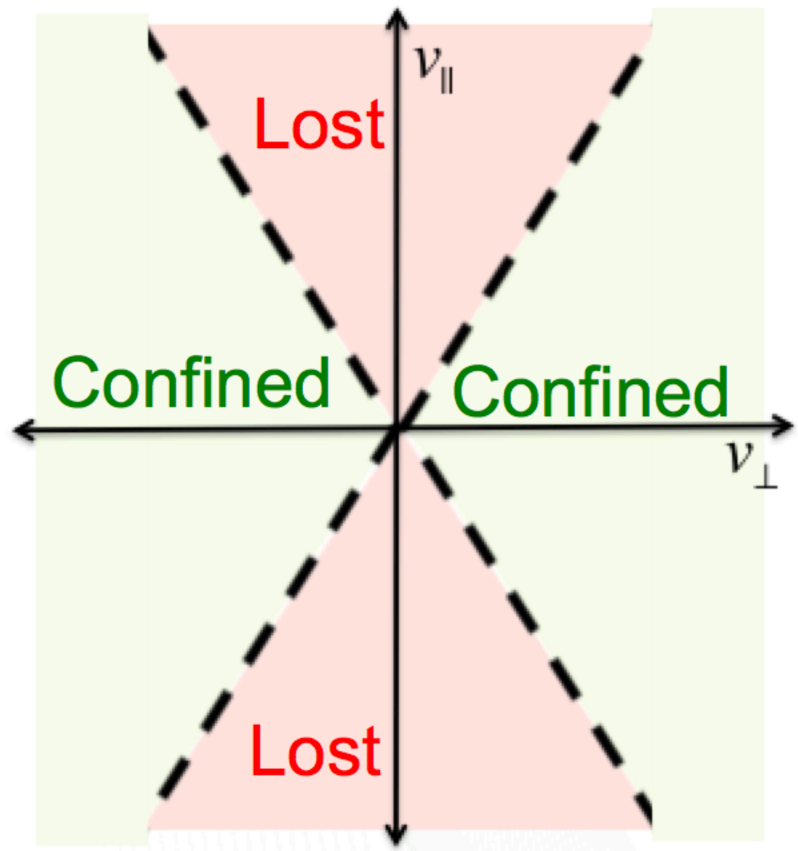
- Figures released show a confinement scheme similar to previous magnetic mirror confinement devices

Pictures from:

http://aviationweek.com/technology/skunk-works-reveals-compact-fusion-reactor-details

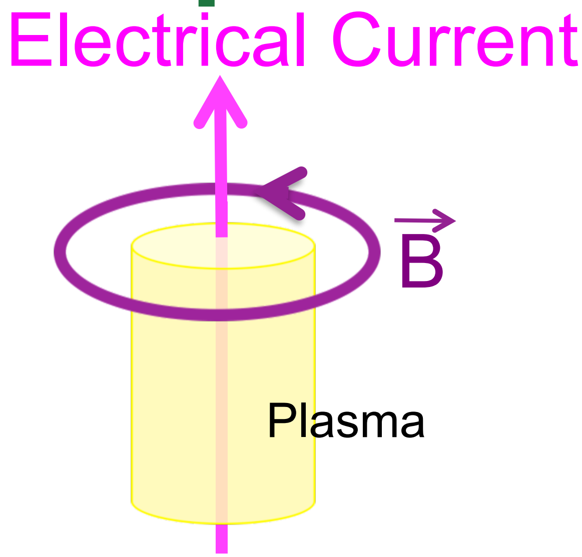
Some particles will always escape

- Any particles with sufficient parallel velocity will escape
- Loss cone can be narrowed by increasing the magnetic field at the throat of the device
- Collisions constantly replenish the loss cone

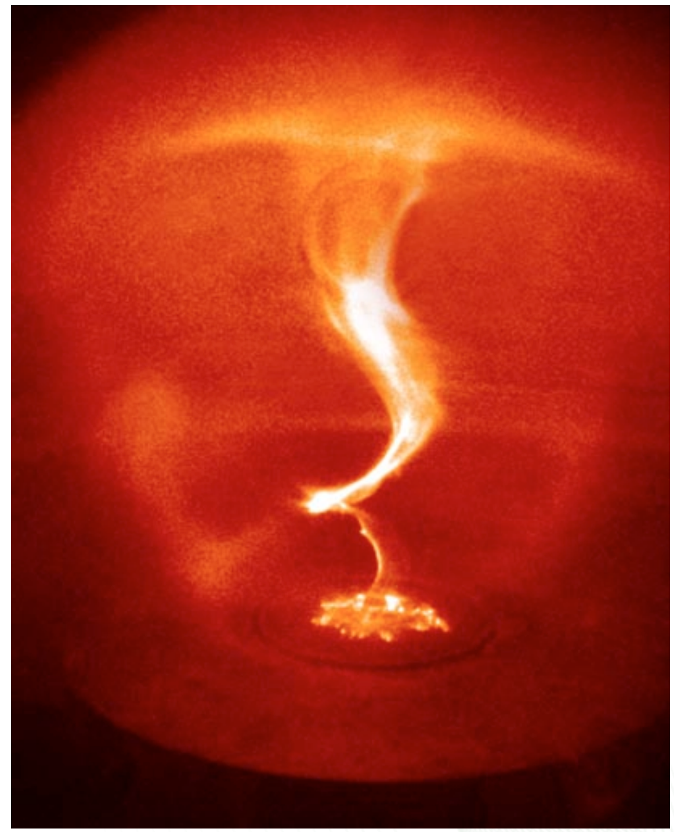


Picture from: http://en.wikipedia.org/wiki/Magnetic_mirror#/media/File:Basic_Magnetic_Mirror.jpg

A circular magnetic field doesn't have end loss problems…but it does have other problems



A kink instability



- Z-pinch: a current is driven in the plasma to create a the confining magnetic field
- These configurations are unstable to kink modes

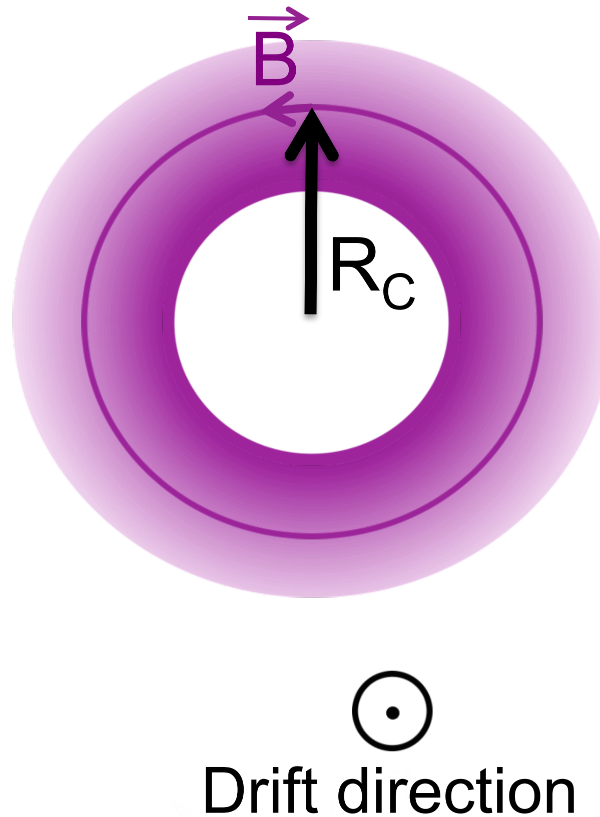
Picture from: https://www.thunderbolts.info

Curved fields cause particles to drift off the field lines

$$\vec{v}_{\nabla B} = \pm \frac{1}{2} v_{\perp} r_L \frac{\vec{B} \times \nabla B}{B^2}$$

$$\vec{v}_R = \frac{1}{q} \frac{\vec{F}_C \times \vec{B}}{B^2} = \frac{m v_{\parallel}^2}{q B^2} \frac{\vec{R}_c \times \vec{B}}{R_c^2}$$

$$\vec{v}_R + \vec{v}_{\nabla B} = \frac{m}{q} \frac{\vec{R}_c \times \vec{B}}{R_c B^2} \left(v_{\parallel}^2 + \frac{1}{2} v_{\perp}^2 \right)$$



- Particles will drift in a direction normal to both the magnetic field and the radius of curvature
- These drifts always add

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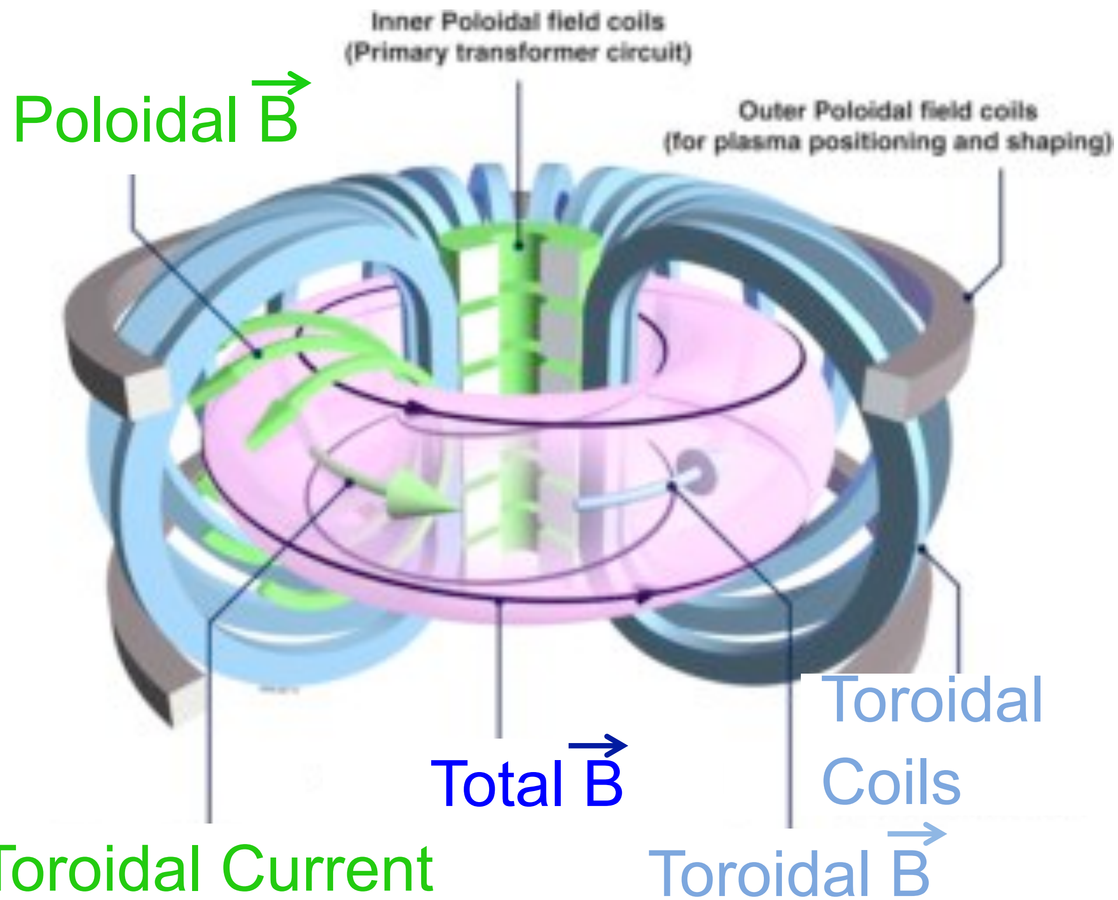
Magnetic fields that wrap both the short and long way around a toroidal device compensate for drifts off a surface

- Tokamaks have both and externally generated toroidal field
- And a poloidal field generated by driving a current through the plasma



Picture from: http://www.alternative-energy-action-now.com/tokamak-fusion-reactor.html

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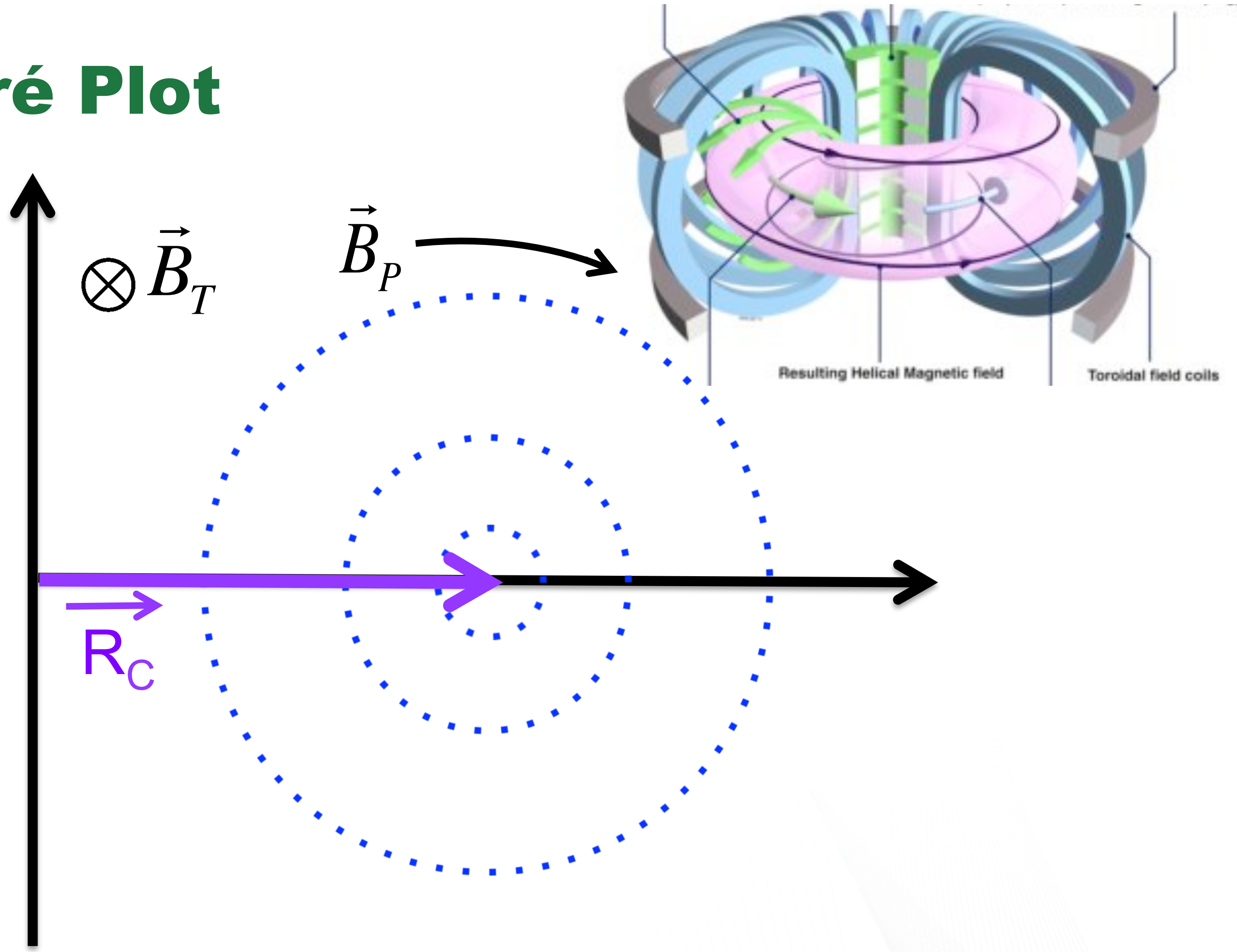
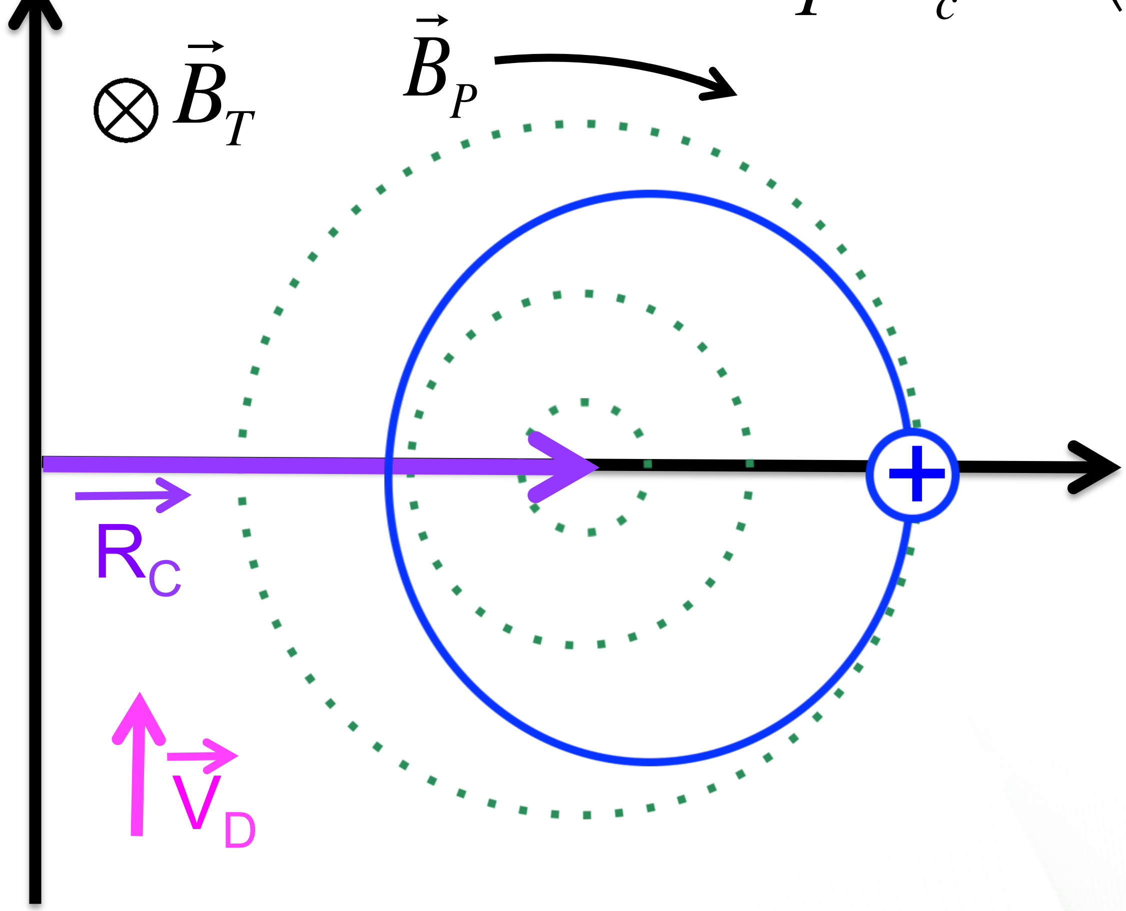
Poinecaré Plot



- A slice of the tokamak at a given toroidal angle

Passing particle

$$\vec{v}_D = \vec{v}_R + \vec{v}_{\nabla B} = \frac{m}{q}\frac{\vec{R}_c \times \vec{B}}{R_c B^2}\left(v_\parallel^2 + \frac{1}{2}v_\perp^2\right)$$

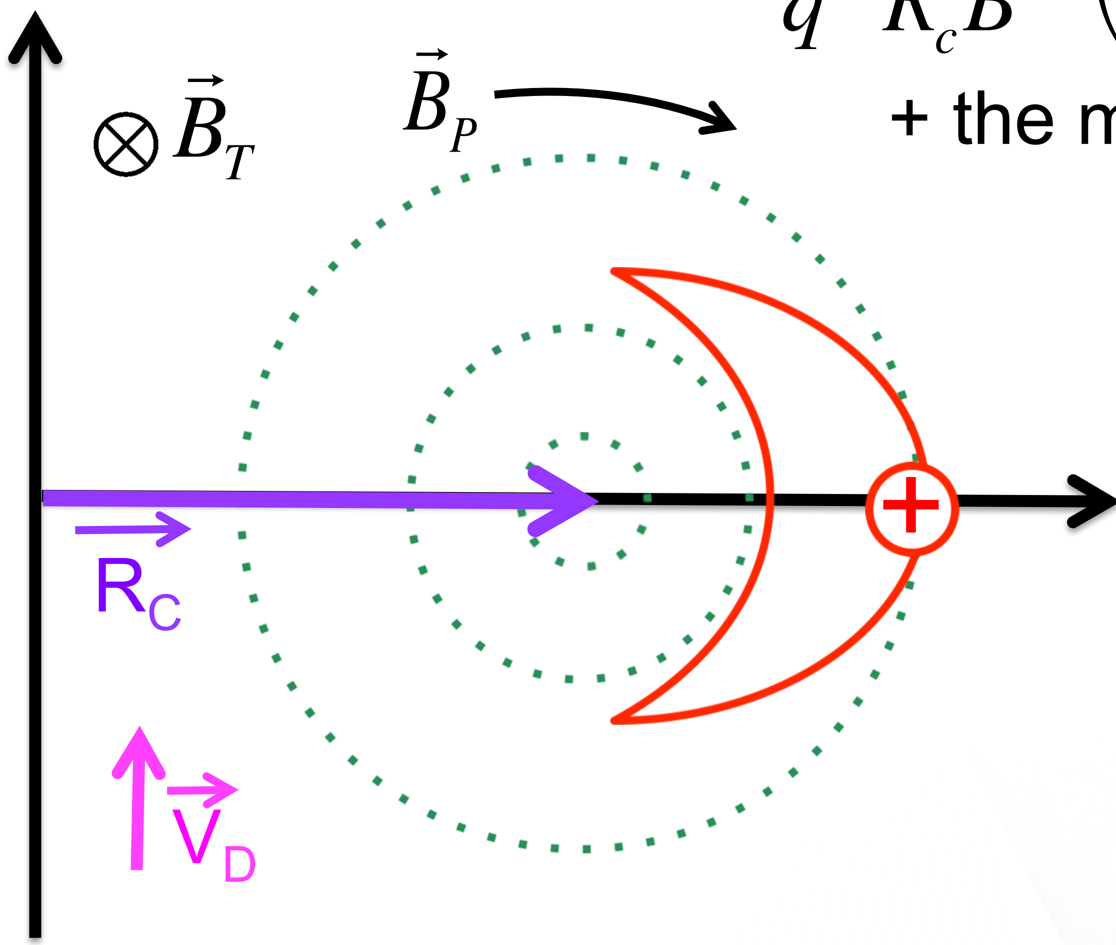


- Particles with sufficient velocity will leave their original flux surface but will make a full poloidal transit

Trapped particles

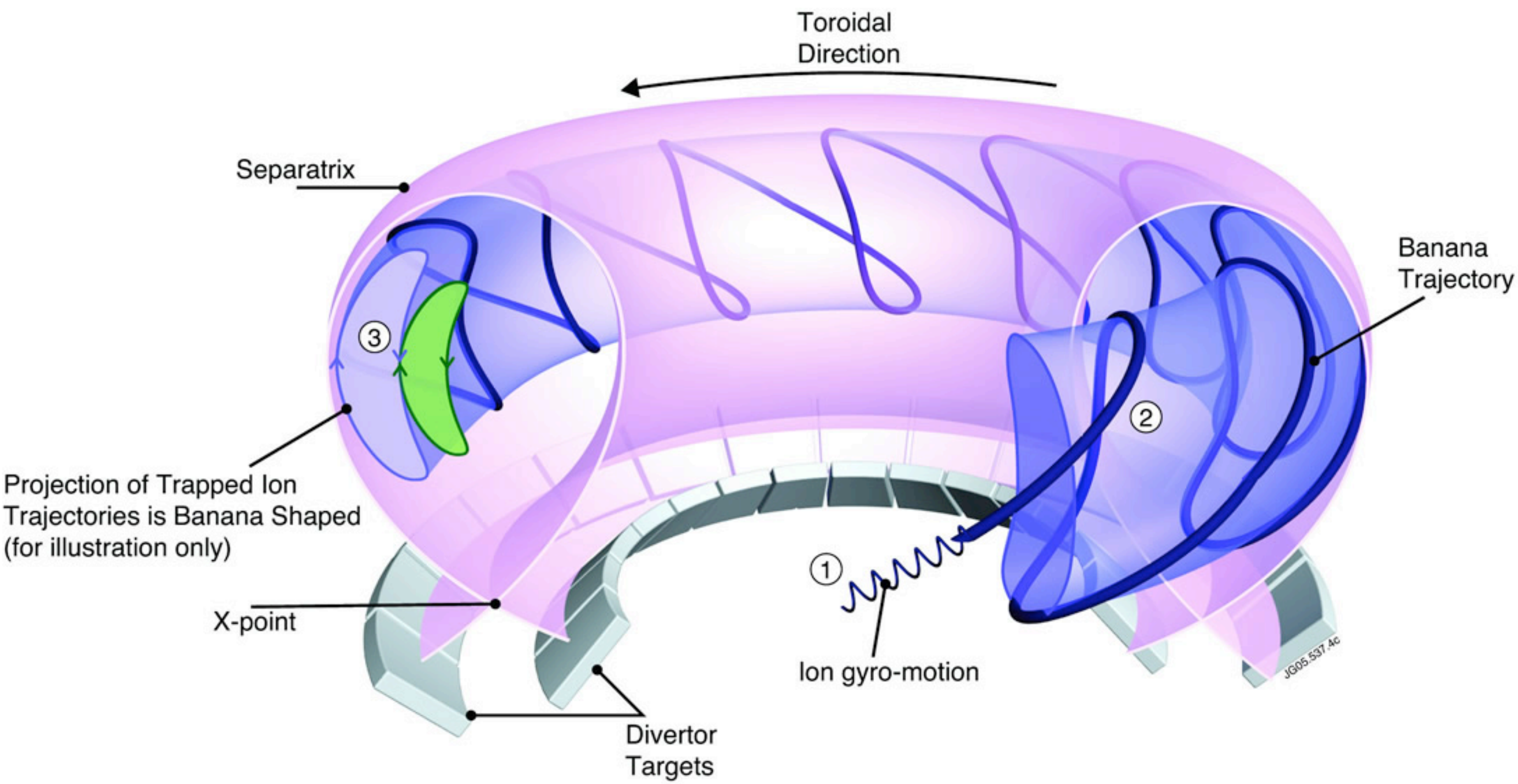
$$\vec{v}_D = \vec{v}_R + \vec{v}_{\nabla B} = \frac{m}{q}\frac{\vec{R}_c \times \vec{B}}{R_c B^2}\left(v_\parallel^2 + \frac{1}{2}v_\perp^2\right)$$

+ the mirror force



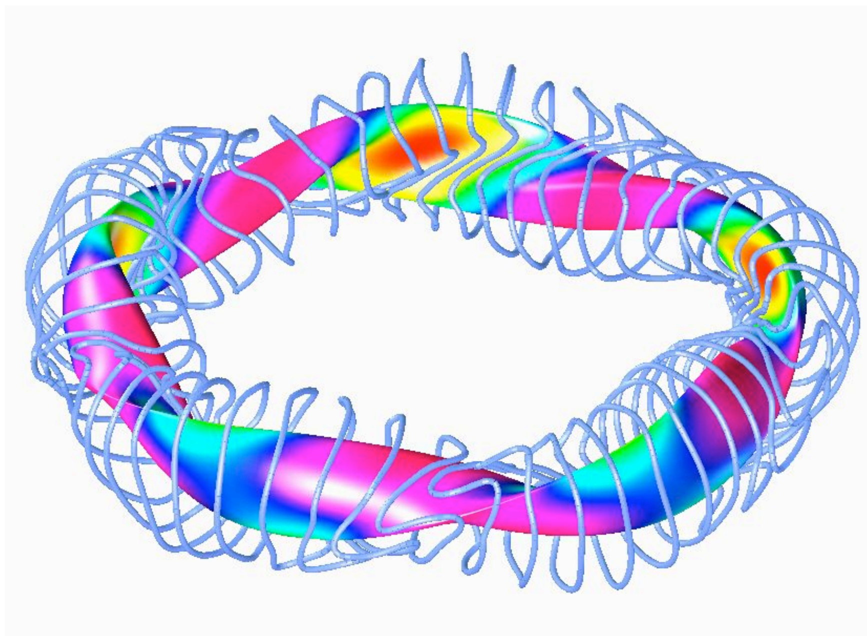
- Particles with lower parallel velocity will become trapped in "banana" orbits
- This increases plasma diffusion

Banana orbits move around the torus



Picture from: https://www.euro-fusion.org/wpcms/wp-content/uploads/2011/09/jg05-537-4c.jpg

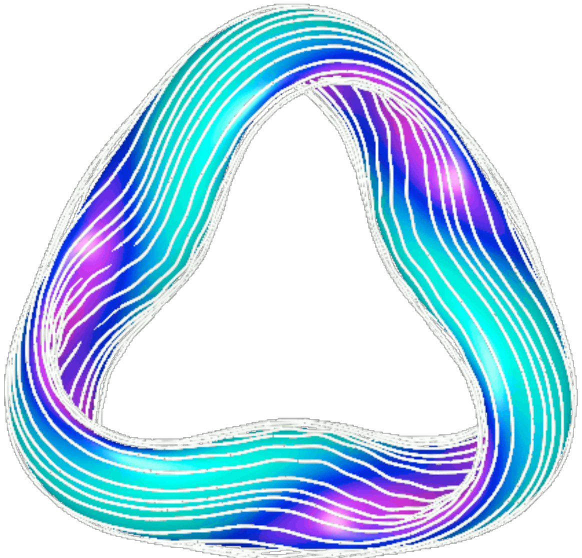
Stellarator: both poloidal and toroidal field are generated using external coils



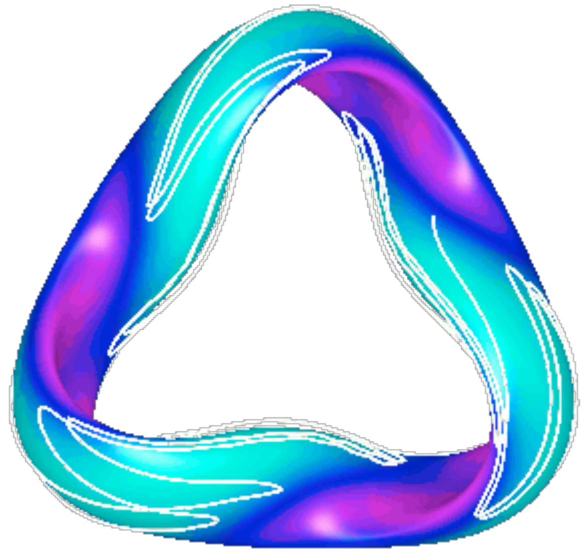
- Complicated coil structure leads to more complicated field structure and more complicated particle orbits

Picture W7-X: physics.ucla.edu/icnsp/Html/spong/spong.htm

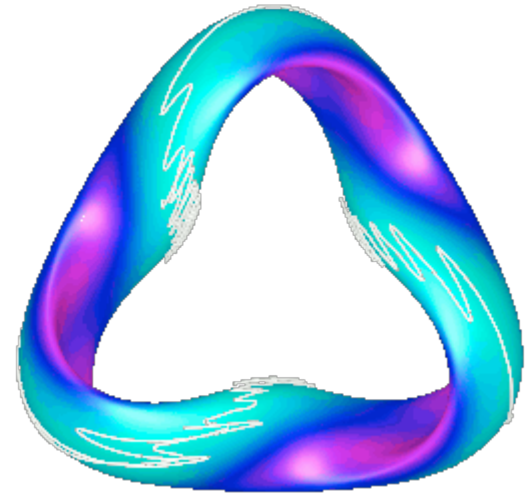
Stellarator have super bananas



Passing



Trapped



Deeply Trapped

- Trapped particles can precess poloidally around stellarators
- If stellarator fields are not optimized these particles will end up trajectories which intersect the wall

Picture from: http://web.ornl.gov/sci/fed/mhd/QOS_Orbits.html

Conclusion

- Although particles are free to move along magnetic field lines, drifts push particles way from their original field lines

- Toroidal confinement devices have magnetic fields in both the poloidal and toroidal direction to compensate for these drifts

- Regions of strong magnetic fields can reflect particles with with insufficient velocity to overcome the mirror effect