Computational Analysis of W7-X Field Geometry

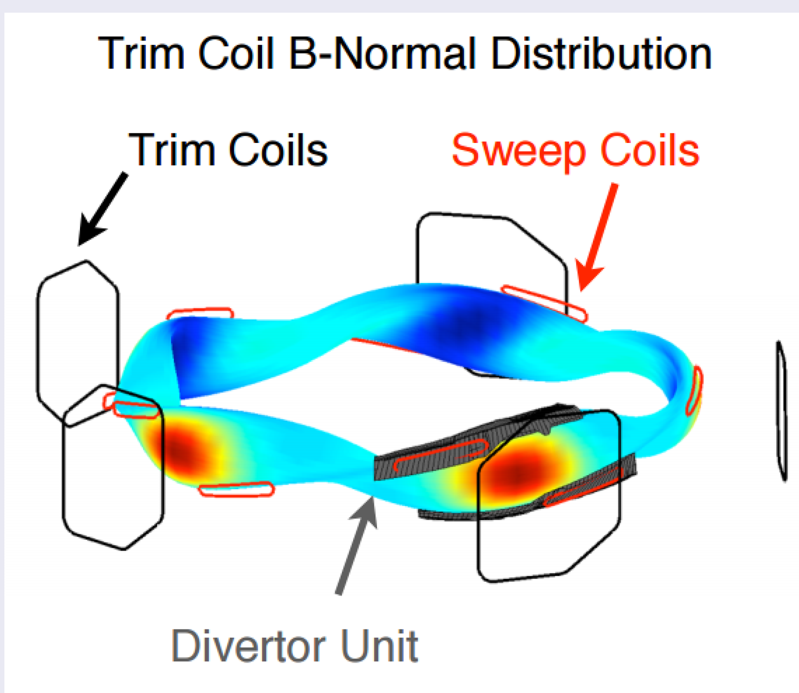
B. Y. Israeli, S. Lazerson, S. R. Hudson, D. A. Gates

PRINCETON PLASMA PHYSICS LABORATORY

COLUMBIA ENGINEERING
The Fu Foundation School of Engineering and Applied Science

Introduction

The ability of W7-X to reach high beta long-pulse operation relies on the proper operation of the island divertor. Analysis has suggested that intrinsic error fields may significantly modify the island divertor magnetic field structure in W7-X, potentially leading to the overloading of divertor plate targets.[1] Adjustment of the divertor strike points using trim coils is envisioned as a solution.



Goals

- Improve overall characterization of W7-X field geometry.
- Investigate effects of trim coils on field geometry, particularly in island divertor region.
  - Potential for perturbation of unstable manifold by induced stochastic regions.
  - Characterize adjustment of unstable manifold, which determines strike points, by varied trim coil current.

Results

- Successful implementation of rapid algorithm to determine magnetic axis and rotational transform.
  - Iterative method determines axis within predictable threshold in < 50 integration iterations.
  - Rotational transform determined on axis via simple calculation (improvement over determination from fieldline near axis).
- Trim coils shown to effectively adjust island geometry without perturbing unstable manifold.

Effect of Trim Coils on Island Divertor Geometry

Methods

- Poincaré plots were generated in order to examine variations in field geometry.
  - Fieldlines obtained via integration of magnetic field.
  - Magnetic field calculated on cylindrical grid via Biot-Savart.
- Effect of trim coils on 5/5 island chain characterized by identifying unstable manifold.
  - X-points determined via a Newton method similar to axis finder above.
  - Fieldlines traced similarly.
- Trim currents were scaled according to $\cos\Phi$, yielding a perturbation with periodicity of 1.

Results

- Several trim current magnitudes shown.
- A clear shift in the 5/5 island chain can be seen, proportional to trim current.
- The unstable manifold manifold (in red) remains intact.
- Effect of sweep coils on divertor region included for comparison.

Analysis

- Unstable manifold appears mostly unperturbed, suggesting potential negative impact of trim coils on divertor operation is minimal.
- Confirms feasibility of correcting divertor load assymetry induced by error fields using trim coils.

Location of Magnetic Axis and Calculation of ι on Axis

Methods

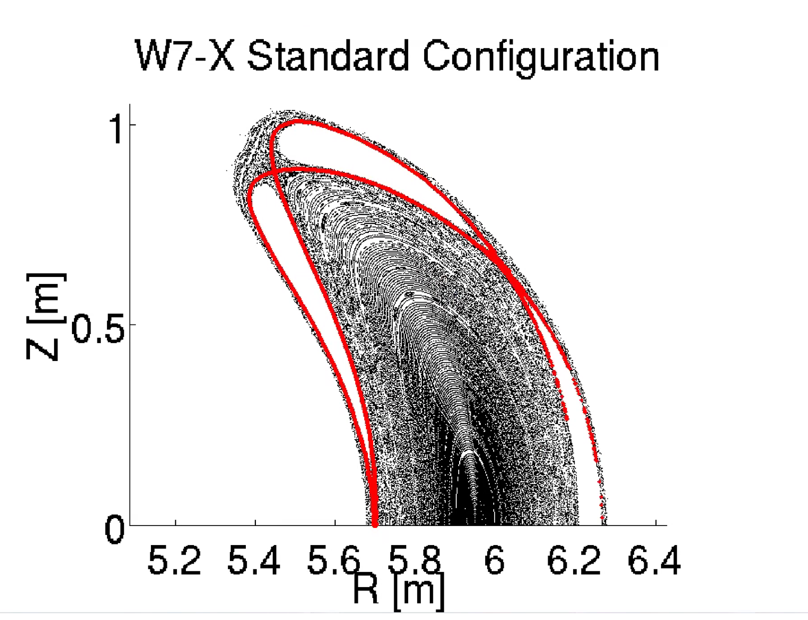
Given an initial guess for the position of the magnetic axis which is in the basin of attraction of the magnetic axis, the magnetic axis may be located and rotational transform on axis calculated. This algorithm was successfully applied to the standard configuration of W7-X.

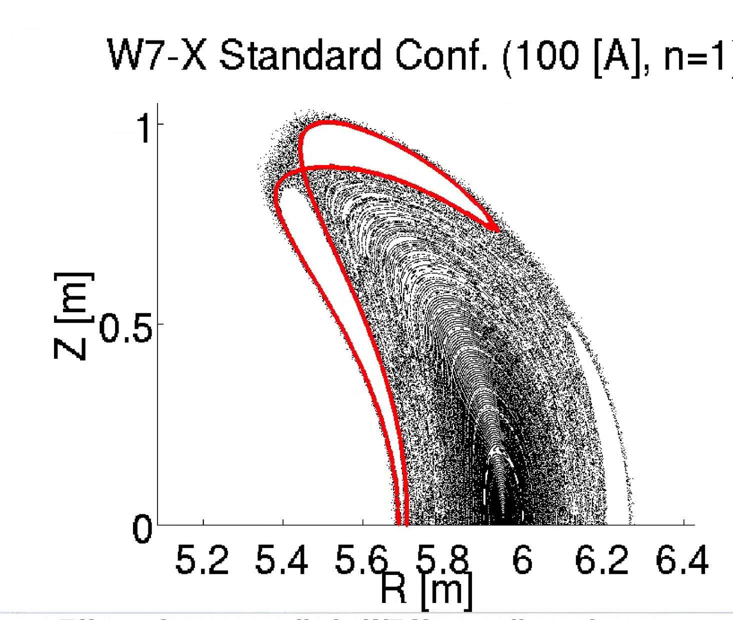
- Magnetic field line flow for one field period forms a mapping P from rz-plane at $\Phi = 0$ to rz-plane at $\Phi = 2\pi/nfp$.
- The stationary points are the periodic fieldlines.
- The magnetic axis (or potentially any periodic fieldline) may be found iteratively using a Newton method.
  $$\begin{bmatrix} r_{n+1} \\ z_{n+1} \end{bmatrix} = \nabla P^{-1} \cdot \begin{bmatrix} r_n \\ z_n \end{bmatrix}$$
  - Initial guess: (r_0, z_0)
  - Terminate when $P(r_n, z_n) - (r_n, z_n)$ is less than a defined error tolerance.
- $P(r_n, z_n)$ and the tangent map $\nabla P(r_n, z_n)$ found by integration over one field period ($\Phi \in [0, \frac{2\pi}{nfp}]$) of their derivatives with respect to Φ.
  - Noting that $\partial_\Phi r = B_r/B_\Phi$ and $\partial_\Phi z = B_z/B_\Phi$ along the fieldline:
  - $\frac{d}{d\phi}P = \begin{bmatrix} B_r/B_\Phi \\ B_z/B_\Phi \end{bmatrix}$ since P is defined as a Poincaré map of the field.
  - $\frac{d}{d\phi}\nabla P = \left(\partial_\Phi \begin{bmatrix} \partial_r r & \partial_z r \\ \partial_r z & \partial_z z \end{bmatrix}\right) \cdot \nabla P = \begin{bmatrix} \partial_r(B_r/B_\Phi) & \partial_z(B_r/B_\Phi) \\ \partial_r(B_z/B_\Phi) & \partial_z(B_z/B_\Phi) \end{bmatrix} \cdot \nabla P$
- Rotational transform on axis can be calculated from the eigenvalues of the tangent map.
  - $\iota = \tan^{-1}(\frac{b}{a})$ where $\lambda = a \pm bi$ are the eigenvalues of the tangent map.[3]
  - Easily calculated since tangent map is a 2x2 matrix.

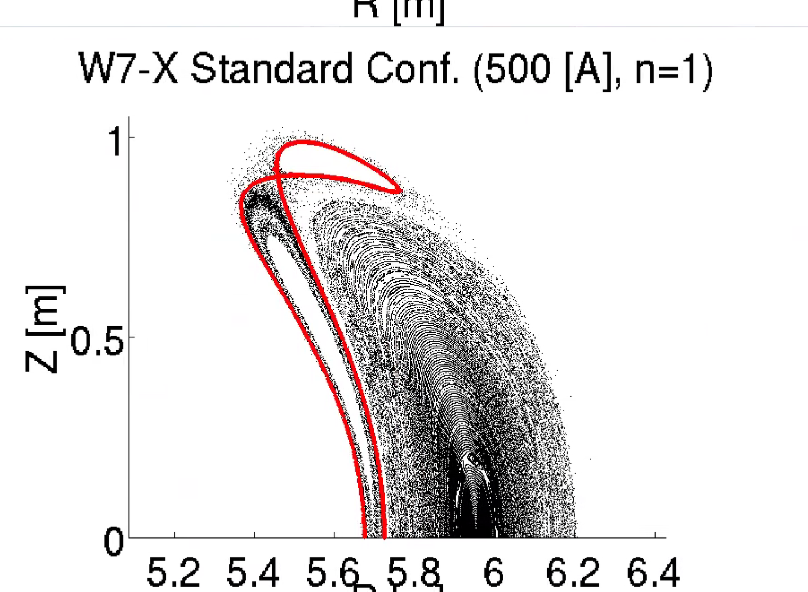
Analysis

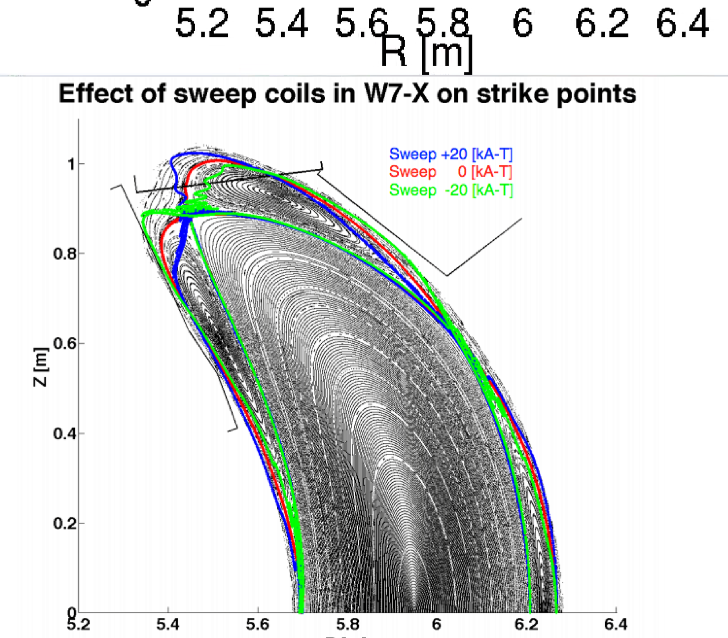
The algorithm was run until the distance between the start and end points of the fieldline in the rz-plane was less than the tolerance of the LSODE integration algorithm used. The algorithm rapidly provided an approximation of the fieldline within a clearly definable threshold, but required a significantly extended runtime beyond this threshold.

- For the primary run, the tolerance was 10^{-12}m. A run with tolerance 10^{-9}m is shown for comparison.
- For the primary run, the error distance rapidly descended monotonically below 10^{-9}m within 30 iterations, fluctuated without clear descent for the remaining iterations.
- Following the initial descent, iteration resulted in a path resembling a random walk.
- Total of 5299 iterations prior to program halt.
- In the 10^{-9}m tolerance run, the theshold for monotonic descent remained proportional to tolerance.



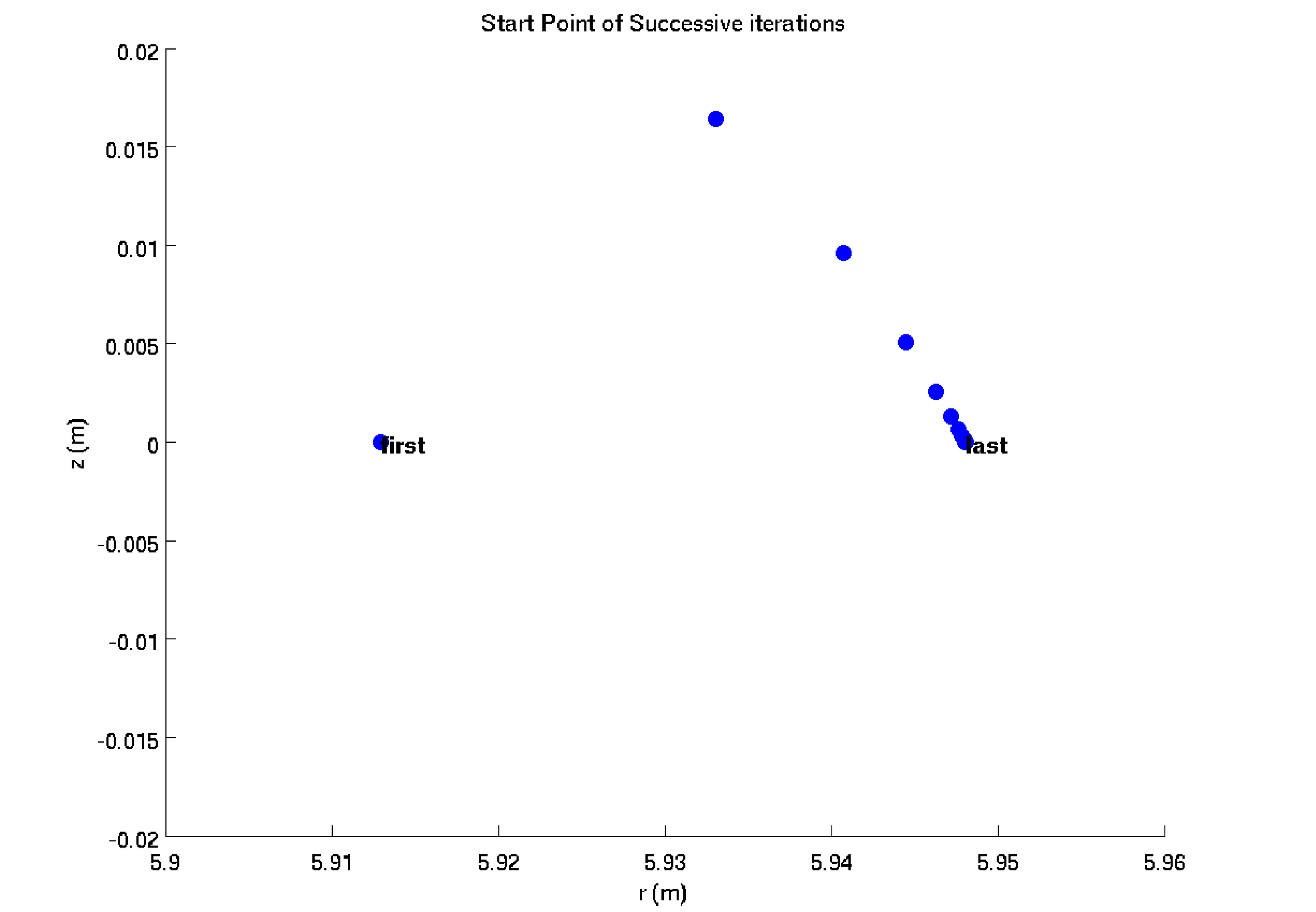


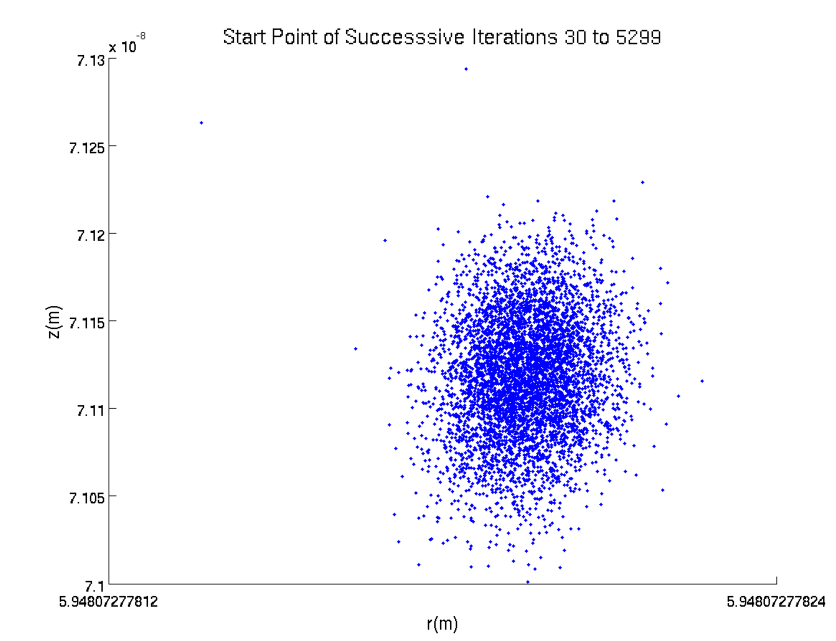


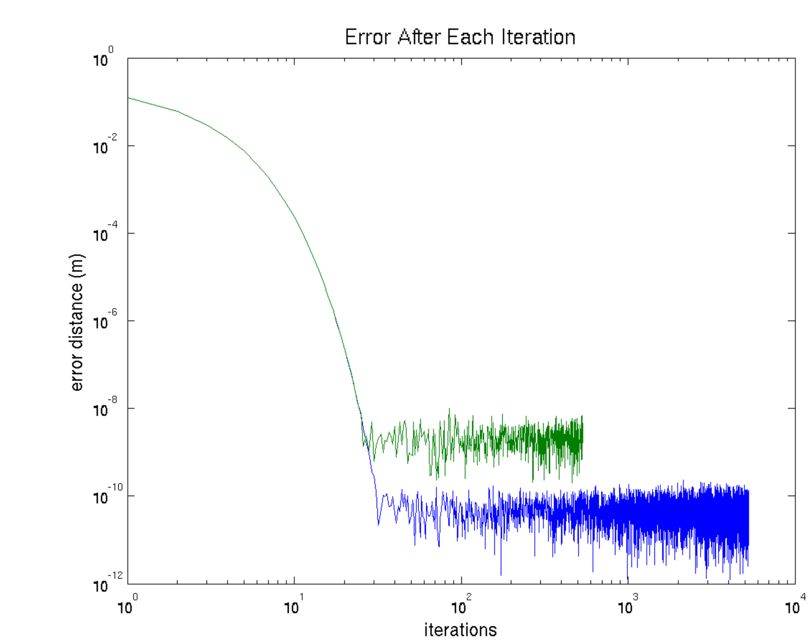


Results

- Magnetic axis: $r = 5.948073$, $z = 0.000000$ at $\Phi = 0$; $\iota = 0.857357$ on axis
- Axis finding algorithm showed good convergence up to a clearly visible theshold (approaching regime of fieldline integration tolerance), which was reached rapidly.
- Beyond this theshold, the path followed by iteration became erratic.







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Acknowledgements

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