Computational Analysis of W7-X Field Geometry
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Introduction
The ability of W7-X to reach high beta long-pulse operation relies on the proper operation of the island divertor. Analysis has suggested that intrinsic error fields may significantly modify the island divertor magnetic field structure in W7-X, potentially leading to the overloading of divertor plate targets. Adjustment of the leading to the overloading of divertor field structure in W7-X, potentially modify the island divertor magnetic intrinsic error fields may significantly

Results

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Methods

Given an initial guess for the position of the magnetic axis which is in the basin of attraction of the magnetic axis, the magnetic axis may be located and rotational transform on axis calculated. This algorithm was successfully applied to the standard configuration of W7-X.

Magnetic field line flow for one field period forms a mapping $P$ from rz-plane at $\Phi = 0$ to rz-plane at $\Phi = 2\pi$/nfp.

The stationary points are the periodic fieldlines.

The magnetic axis (or potentially any periodic fieldline) may be found iteratively using a Newton method:

$$\begin{bmatrix} r_n+1 \\ z_n+1 \end{bmatrix} = \mathbf{P}^{-1} \begin{bmatrix} r_n \\ z_n \end{bmatrix}$$

Initial guess: $(r_0, z_0)$

Terminate when $\mathbf{P}(r_n, z_n) - (r_n, z_n)$ is less than a defined error tolerance.

$\mathbf{P}(r_n, z_n)$ and the tangent map $\mathbf{P}'(r_n, z_n)$ found by integration over one field period ($\Phi \in [0, \frac{2\pi}{nfp}]$) of their derivatives with respect to $\Phi$.

Noting that $\partial r/\partial \Phi = B_r/B_z$ and $\partial z/\partial \Phi = B_z/B_r$ along the fieldline:

$$d = \frac{d}{d\Phi} \begin{bmatrix} B_r \\ B_z \end{bmatrix}$$

since $\mathbf{P}$ is defined as a Poincaré map of the field.

$$d = \frac{d}{d\Phi} \mathbf{P} = \begin{bmatrix} \partial r/\partial \Phi \\ \partial z/\partial \Phi \end{bmatrix} \mathbf{P}' = \begin{bmatrix} \partial (B_r/B_z) \\ \partial (B_z/B_r) \end{bmatrix} \mathbf{P}$$

Rotational transform on axis can be calculated from the eigenvalues of the tangent map.

$$\lambda = \tan^{-1}(\ell)$$

Easily calculated since tangent map is a 2x2 matrix.

Analysis

The algorithm was run until the distance between the start and end points of the fieldline in the rz-plane was less than the tolerance of the LSODE integration algorithm used. The algorithm rapidly provided an approximation of the fieldline within a clearly definable threshold, but required a significantly extended runtime beyond this threshold.

For the primary run, the tolerance was $10^{-12}$m. A run with tolerance $10^{-16}$m is shown for comparison.

For the primary run, the error distance rapidly descended monotonically below $10^{-2}$m within 30 iterations, fluctuated without clear descent for the remaining iterations.

Following the initial descent, iteration resulted in a path resembling a random walk.

Total of 5299 iterations prior to program halt.

In the $10^{-16}$m tolerance run, the threshold for monotonic descent remained proportional to tolerance.

References


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