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Introduction to Magnetic Fusion

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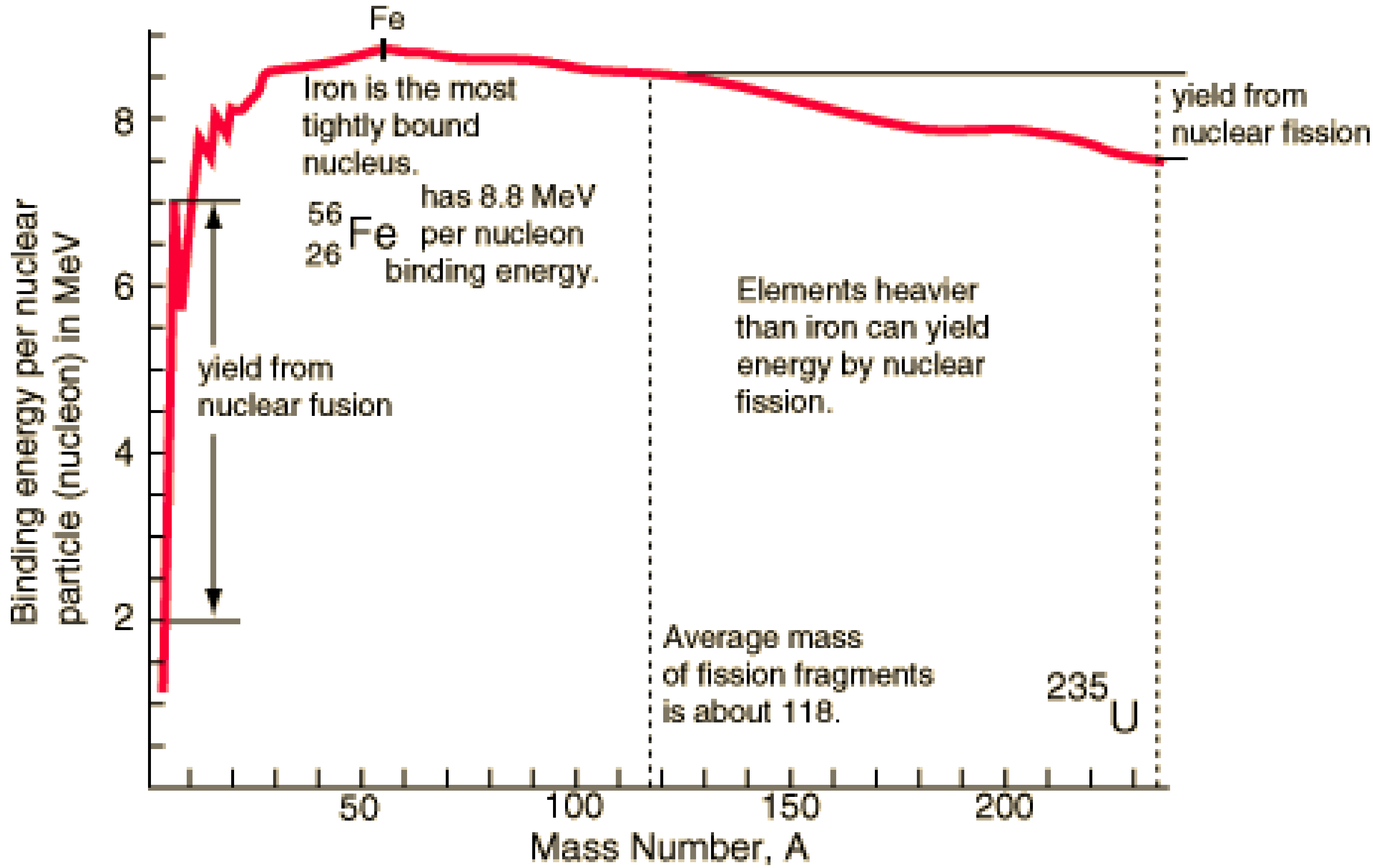
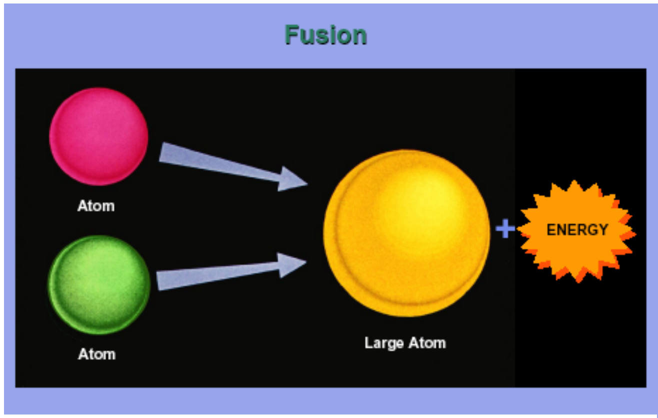
Overview: Fusion Energy

- Why?
  - To meet growing world energy demands using a safe, clean method of producing electricity.
- What?
  - Extract net energy from controlled *thermonuclear* fusion of light elements
- Who?
  - International research teams of engineers and physicists.
- Where?
  - Research is worldwide ….eventual energy source has few geographical restrictions.
- When?
  - Many feasibility issues resolved….demonstration in next decades
- How?
  - This lecture

Fusion & fission: binding energy, E = Δm c^2



Thermonuclear fusion of hydrogen powers the universe: stars



$_1H^1 + {}_1H^1 \Rightarrow {}_1H^2 + {}_1e^0$

In the first reaction, 2 protons combine to form deuterium and a positron. One of the protons is converted into a neutron and a positon

proton → neutron, positron

$_1H^2 + {}_1H^1 \Rightarrow {}_2He^3 +$ (gamma) radiation

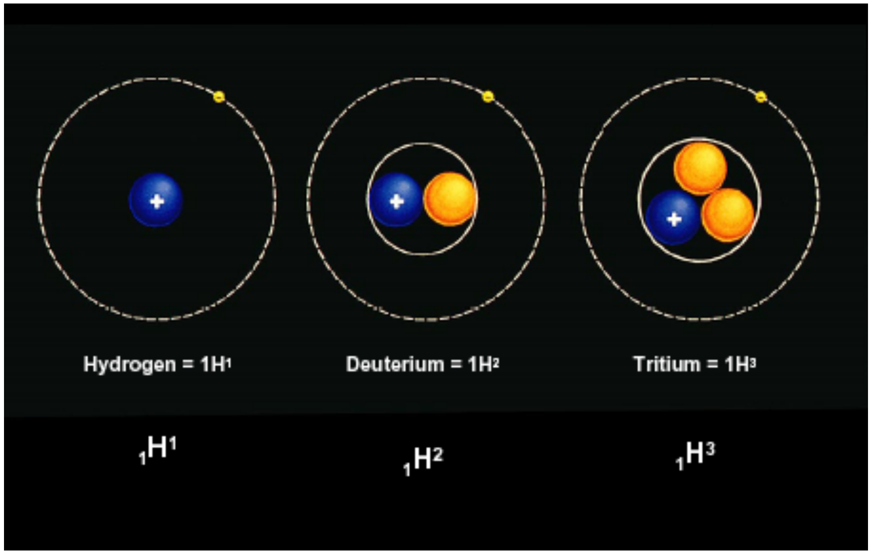
In the 2nd reaction a proton + deuterium unite to form the light isotope of helium, $_2He^3$

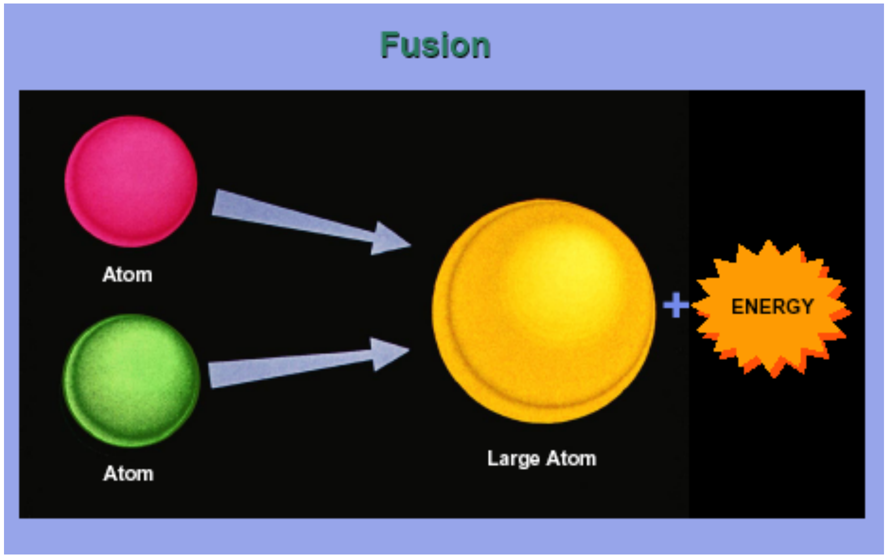
$_2He^3 + {}_2He^3 \Rightarrow {}_2He^4 + {}_1H^1 + {}_1H^1$

The first two reactions must occur twice for the 3rd reaction to occur

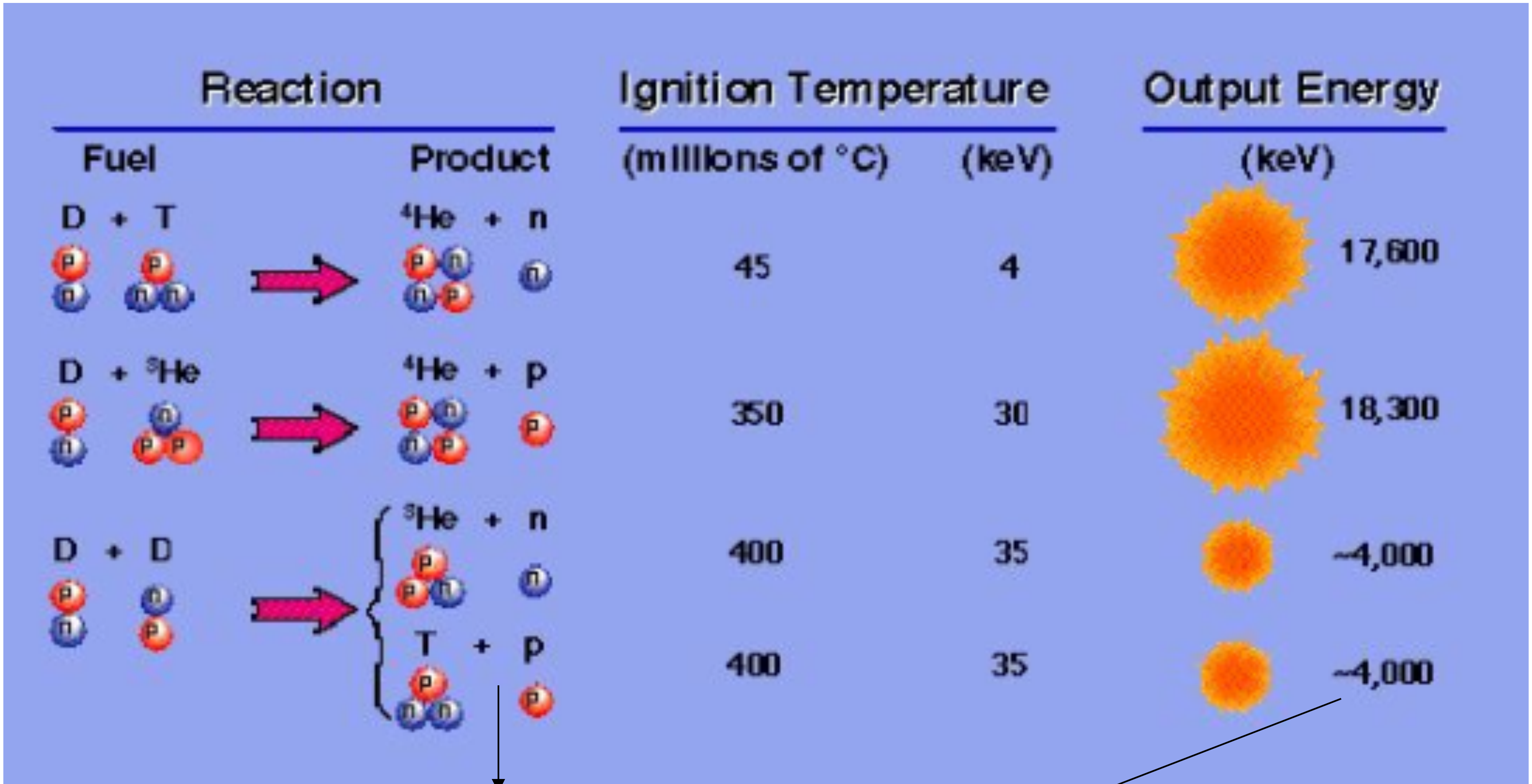
As a terrestrial energy source we are primarily interested in the exothermic fusion reactions of heavier hydrogenic isotopes and ^{3}He

- Hydrogenic species definitions:
  - Hydrogen or p (M=1), Deuterium (M=2), Tritium (M=3)





Exothermic fusion reactions of interest



Net energy gain is distributed in the Kinetic energy of the fusion products

D-T fusion facts

$$d + t \rightarrow \alpha + n \quad Q = 17.6\ MeV$$

$$^{2}_{1}H + ^{3}_{1}H \rightarrow ^{4}_{2}He + n \quad Q = 17.6\ MeV$$

Conservation of energy & linear momentum
when $E_{d,t} << Q$

$$E_\alpha \simeq \frac{m_n}{m_\alpha + m_n} Q = \frac{1}{5} Q_{DT} = 3.5\ MeV$$

$$E_n \simeq \frac{m_\alpha}{m_\alpha + m_n} Q = \frac{4}{5} Q_{DT} = 14.1\ MeV$$

A note about units

- In nuclear engineering, nuclear science and plasmas we use the unit of electron-volt "eV" for both energy and temperature
  - N.B. Energy and temperature are NOT the same thing!

- The eV, rather than Joules, is the natural energy unit when dealing with elementary particles: Energy gained by a particle with unit electron charge falling through 1 Volt of electrostatic potential energy…that's natural?

- Yes! E.g. Electrostatic particle accelerator, beam of protons fall through potential drop of 1 MV x 1 Ampere current. Beam power: V x I =1 MW..so what is the energy per proton? 10^6 eV!

$$\frac{e}{e}\bullet I\left(\frac{C}{s}\right)\bullet 10^6 V=\frac{I}{e}\left(\frac{proton}{s}\right)\bullet 10^6 e\,V=10^6\,\frac{J}{s}$$

Handy dandy unit conversions & Typical physical parameters

- 1 eV = 1.6 x 10^{-19} J (conversion between single particle & SI)
- 1 e = 1.6 x 10^{-19} C

- 1 eV ~ 11600 Kelvin ~ 10^4 K (describing temperatures)

- 10 keV ~ 10^8 K ~ 100 million K (typical fusion T)

- $v_{th} \sim 10^4 (T / M)^{1/2}$ m/s {T in eV, M in amu}
  - At typical fusion temperature thermal velocity of protons ~ 10^6 m/s

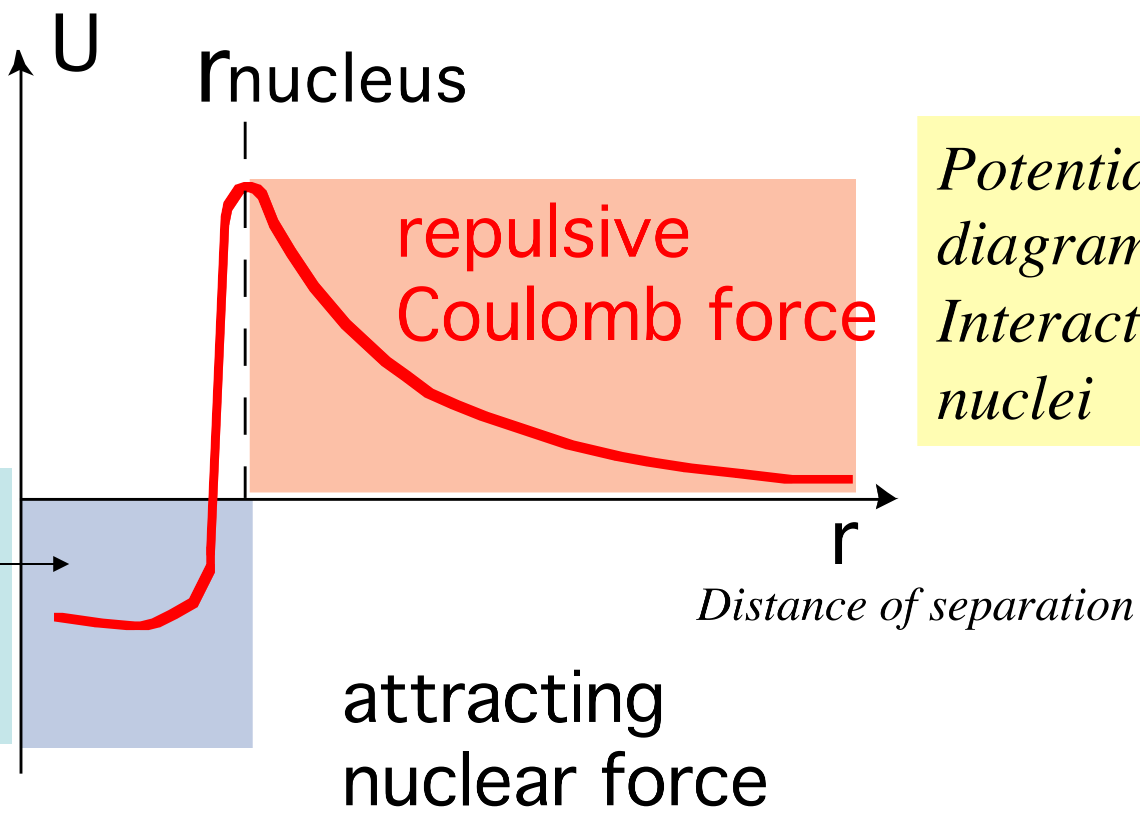
- 1 barn = 10^{-28} m^2 = 10^{-24} m^2 (nuclear cross section)

We also use eV to describe mass through energy-mass equivalence

- $E = m c^2 \rightarrow m = E / c^2$
- Electron: 0.511 MeV / c^2
- Atomic mass unit ~ proton ~ neutron: 970 MeV / c^2 ~ 10^3 MeV
- So note D-T fusion gain: ~ 20 MeV arises from relative decrease in rest mass energies of products compared to reactants
  ~ 20 MeV / (5x1000 MeV) ~ 0.4%

Fusion is a nuclear reaction…it occurs due to the strong nuclear force

- Fundamental competition: nuclear binding force vs. repulsive Coulomb force for the like + charged nuclei.



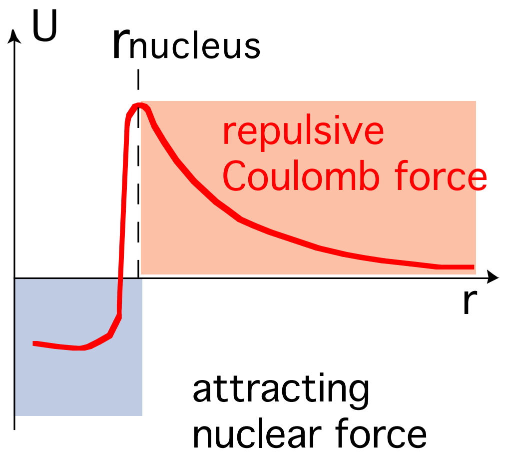
Potential diagram for Interacting nuclei

If the nuclei get Close enough Together (~ nuclei radius) then the binding nuclear force Dominates --> fusion

The Coulomb repulsion must be overcome with very high particle energies

- Repulsive force acting on two particles with same charge (ions),

$$U[J] = \frac{Z_1 Z_2 e^2}{4\pi\varepsilon_0 r}[J] \rightarrow U[eV] = \frac{Z_1 Z_2 e}{4\pi\varepsilon_0 r}$$



$$U[eV] = 1.4[eV \bullet nm]\frac{Z_1 Z_2}{r\,[nm]}$$

Z=1 → $r_{nucleus}$ ~ 4 fm = 4x10^{-6} nm → **U ~ 350,000 eV**

Compare to average atom energy at room temperature

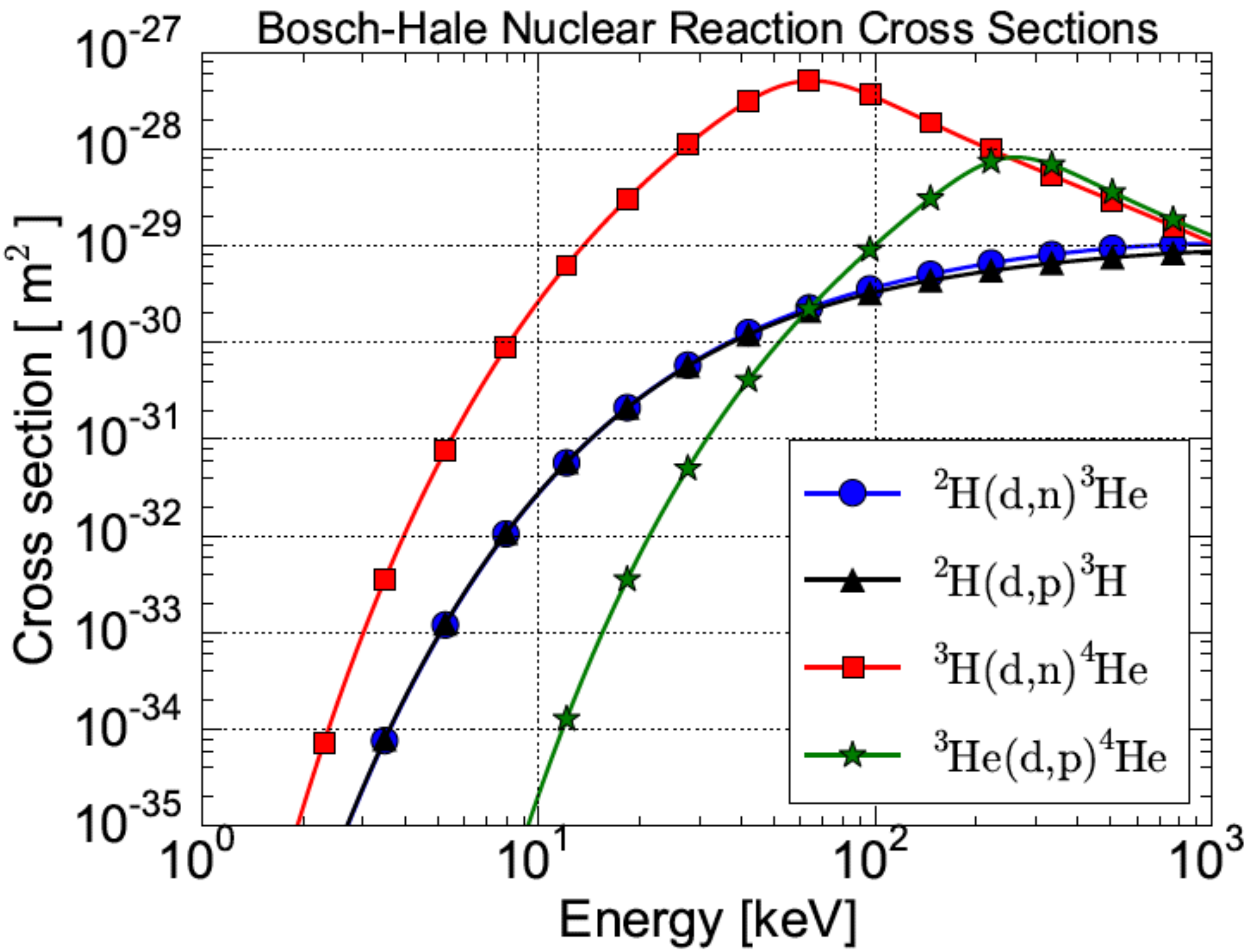
$E_{thermal}$ ~ 300 K ~ 0.03 eV !

Despite $U_{coulomb}$~350 keV, we can obtain D-T fusion at T~10 keV

- Three important effects allow this:

1. Quantum-mechanical tunneling (wave-particle nature of the interacting nuclei_ provides finite probability for energies below $U_{coulomb}$
2. Large low-energy nuclear resonance for d-t
3. Fusion achieved in thermalized population distribution with "significant" population at E > T

Fusion cross-sections: Peak of D-T reaction at 0.06 MeV → x 300 energy gain!



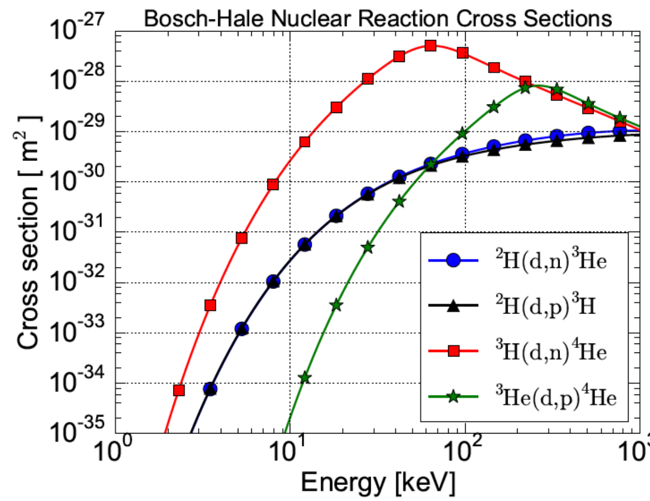
http://www.psfc.mit.edu/research/MFEFormulary/

Cross-section has units of area but is actually an expression of normalized probability of a reaction occurring per target particle

Linear reaction coefficient in homogenous medium

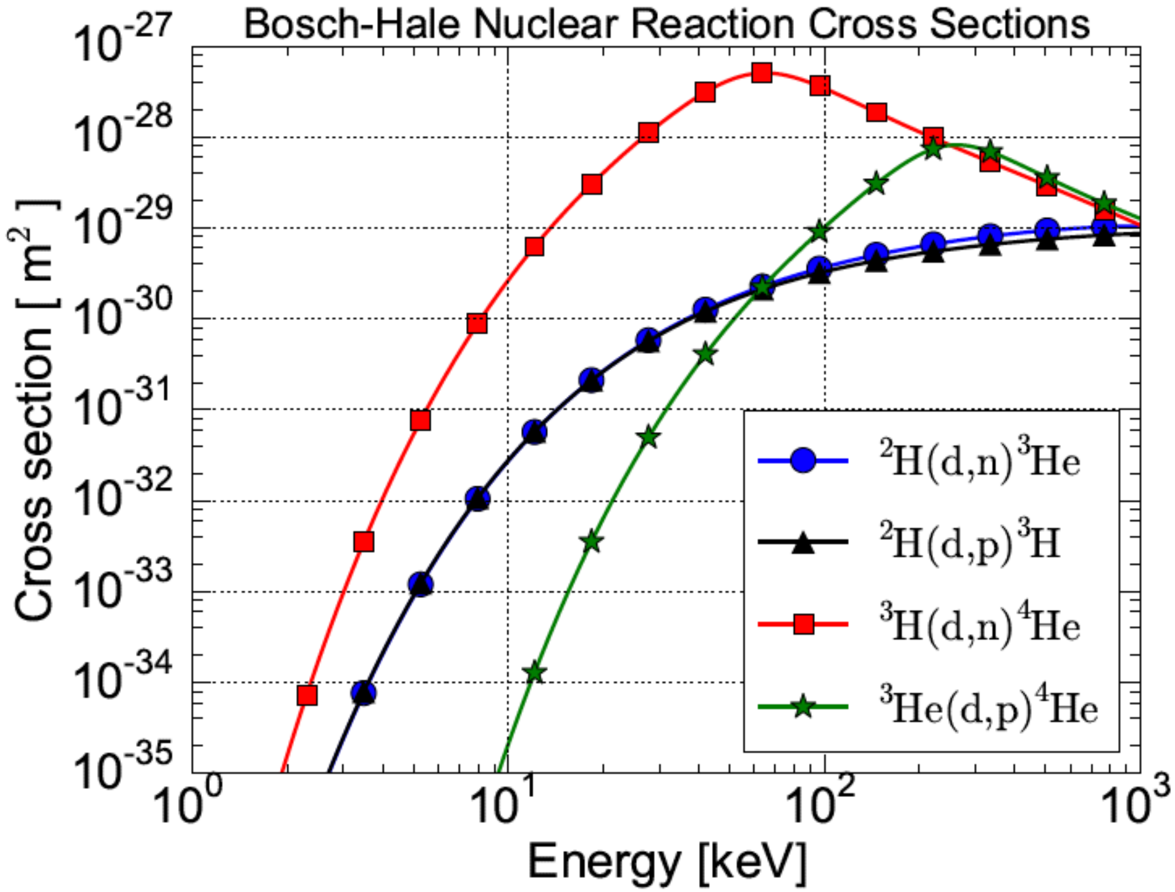
$$\mu_{d-t}[m^{-1}] \equiv \frac{dP}{dx}$$

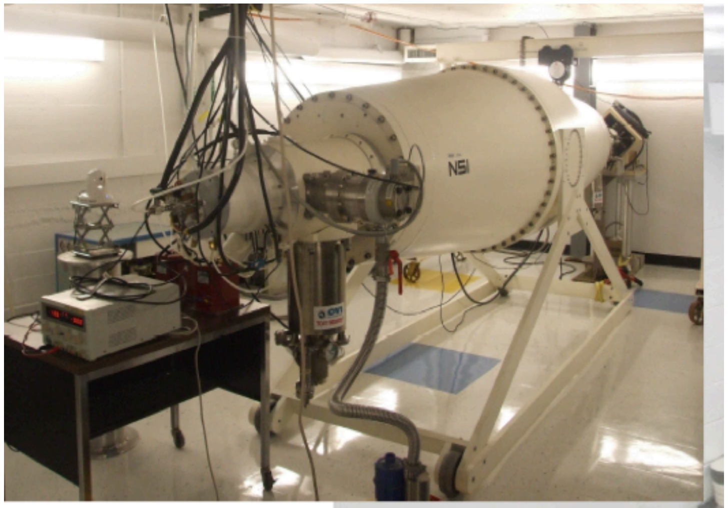
$$\sigma\left[m^2\right] \equiv \frac{\mu_{d-t}[m^{-1}]}{n_{\text{target}}[\#/m^3]}$$



For nuclear reactions, nuclei typically must approach to nuclear distances for strong-force to come into play. Thus "cross-section" of $(r_{\text{nuclesu}} \sim 10^{-14}\ \text{m})^2 \sim 10^{-28}\ \text{m}^2$ = 1 barn is quite large…

Fusion cross-sections: Peak of D-T reaction at 0.06 MeV → x 300 energy gain!



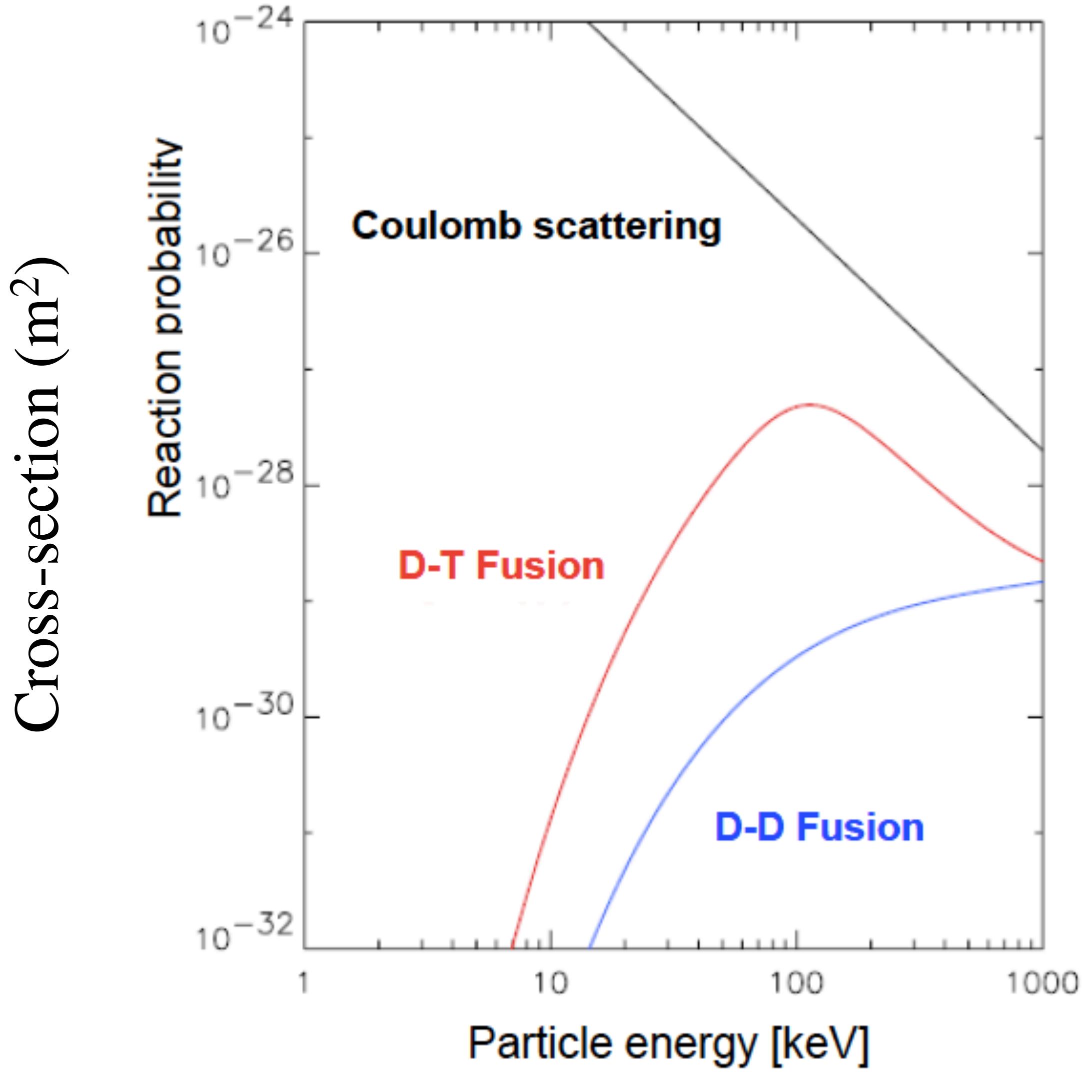


DANTE Tandem Accelerator

Beam: p, d, any negative ion

Parameters: <3 MeV; < 200 uA

Major conceptual challenge #1: Nuclei have much higher probability of undergoing non-fusion Coulomb collisions

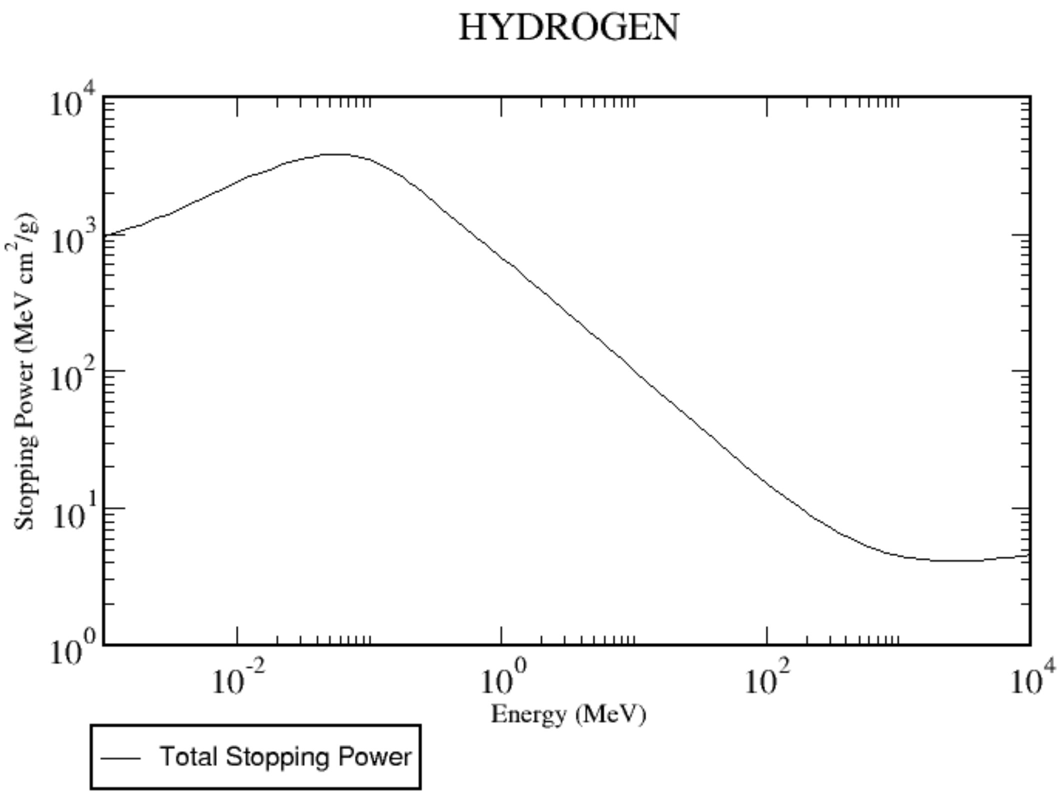


Major conceptual challenge #1: Nuclei have much higher probability of undergoing non-fusion Coulomb collisions

Coulomb collision occurs "at distance", unlike nuclear reactions

By carefully integrating over appropriate impact parameters MeV ions are dominated by energy loss to electrons in matter

This "competes" against fusion reactions because it removes energy of needed fast particles



Fusion is easy, produced all the time in accelerator labs. Fusion energy is hard because it requires thermalized population

- Let's try to make a fusion energy reactor by producing a 1 MeV beam of deuterons streaming into a tritium target

- Mass density solid hydrogen $\boldsymbol{\rho} \sim 0.1$ g/cm^3

- **From previous graph: Stopping power for the deuteron** **S ~ 10^3 MeV cm^2/g → S $\boldsymbol{\rho}$** ~ 10^2 MeV / cm

- **Distance 1 MeV deuteron travels in solid tritium before coming to rest due to slowing down by Coulomb collisions**
  $\mathbf{E_0}$ / (S $\boldsymbol{\rho}$) ~ 1 MeV / 10^2 MeV / cm ~ 10^{-2} cm (100 microns)

- Probability of fusion reaction? Average cross-section ~ 2 barn (generous!) = 2 x 10^{-24} cm^2

continued on next slide

Fusion is easy, produced all the time in accelerator labs. Fusion energy is hard because it requires thermalized population

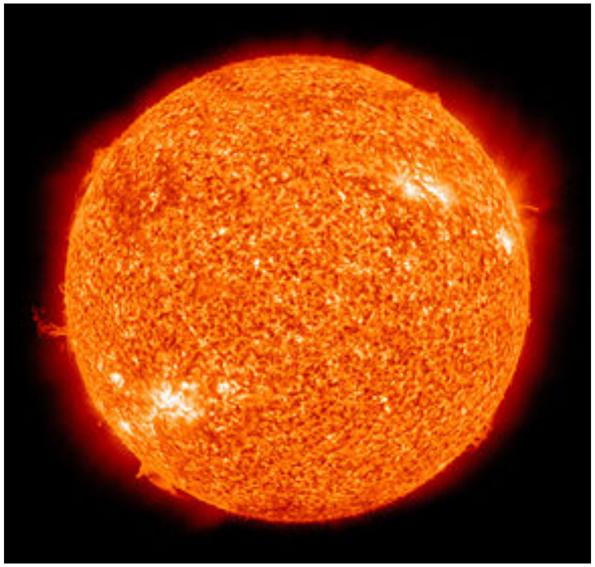
- $n \sim \rho N_A / A = 0.1 * 6e23 / 1 = 6e22\ cm^{-3}$: this is the volumetric number density of tritons in target

- Probability of fusion per unit distance in tritium target $= 2x10^{-24}\ cm^2 * 6e^{22}\ cm^{-3} = 0.1$ fusion reactions / cm^{-1}

- Approximate probability that ion fuses before full stopping: $0.1\ cm^{-1} * 10^{-2}\ cm \sim 10^{-3}$ (oh man that doesn't sound good)
  - Note this probability is independent of the target number density.

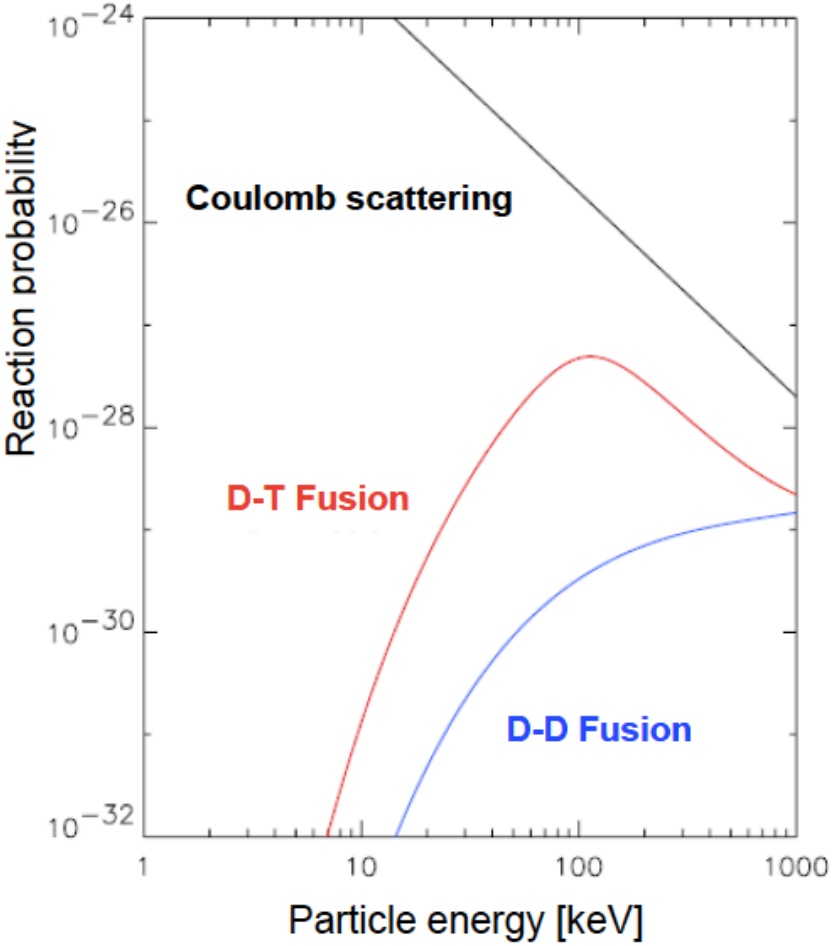
- Power balance?
  - **Power supplied by you**: 1 MeV / particle
  - **Power gained from fusion**: ~20 MeV / fusion * 10^{-3} fusion/ particle ~ 0.02 MeV/particle

Thermonuclear fusion is necessary
Coulomb collisions in thermalized population



Interior of sun ~ 15 MK
Fusion energy needs confinement
Needs Temperature

ASSURES FUSION ENERGY
MUST BE IN PLASMA STATE



Major conceptual challenge #2: There are no natural sources of tritium

- Deuterium is so abundant on earth…$f_{D/H}$ ~ 1.5×10^{-4} … as to make an essentially infinite fuel supply for fusion.
  - Think of the # water molecules in the ocean!

- But tritium does not occur naturally due to its short half-life,
  $T \rightarrow {}^3He^+ + e^-$ 1/2 life = 12.3 years.

- So Tritium must be bred. How?
  - D + n --> T + gamma (e.g. CANDU fission reactor)
  - This is the present "seed" source for fusion tritium amounting to ~20 kg

Major conceptual challenge #2: Fusion must be a breeder reactor

- For fusion we have another important set of nuclear reactions involving the neutron *product* from the D-T fusion reaction and lithium that can produce tritium.

$$^{6}\mathrm{Li} + n \rightarrow T + {}^{4}\mathrm{He} + 4.8\ \mathrm{MeV}$$

$$^{7}\mathrm{Li} + n \rightarrow T + {}^{4}\mathrm{He} + n - 2.3\ \mathrm{MeV}$$

- Lithium is highly abundant: 93% Li-7, 7% Li-6

Major conceptual challenge #2: Fusion energy system is a very simple breeder reactor

$$d + t \rightarrow \alpha + n \qquad Q = 17.6\ MeV$$

$$n + {}^{6}Li \rightarrow t + \alpha \qquad Q = 4.8\ MeV$$

$$d + {}^{6}Li \rightarrow 2\alpha \qquad Q = 22.4\ MeV$$

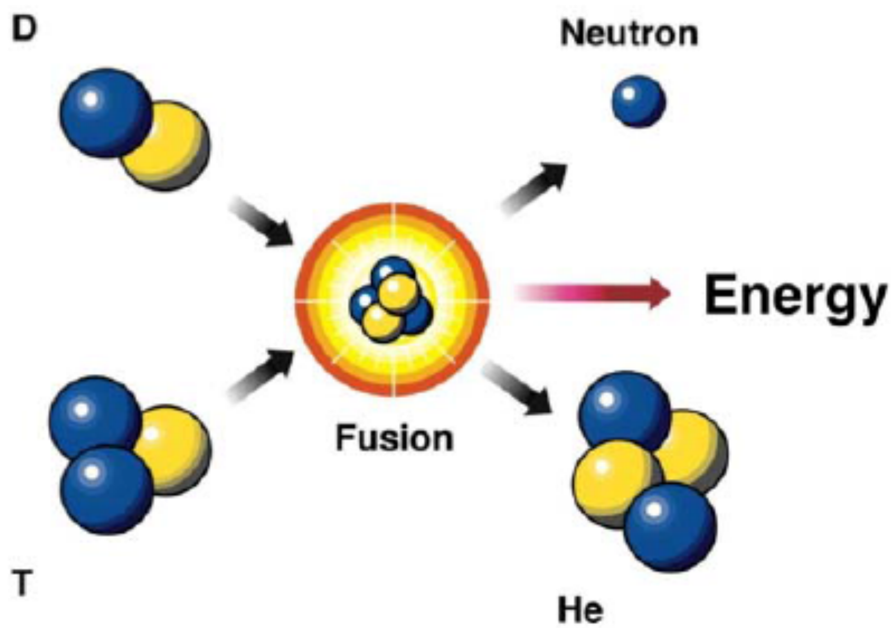
A fusion energy device heats itself and recycles neutrons internally for tritium hydrogen fuel

Energy Before (MeV)

Energy After (MeV)

~ 0.01

~0.01



14.1

3.5

Plasma physics: T=10 keV

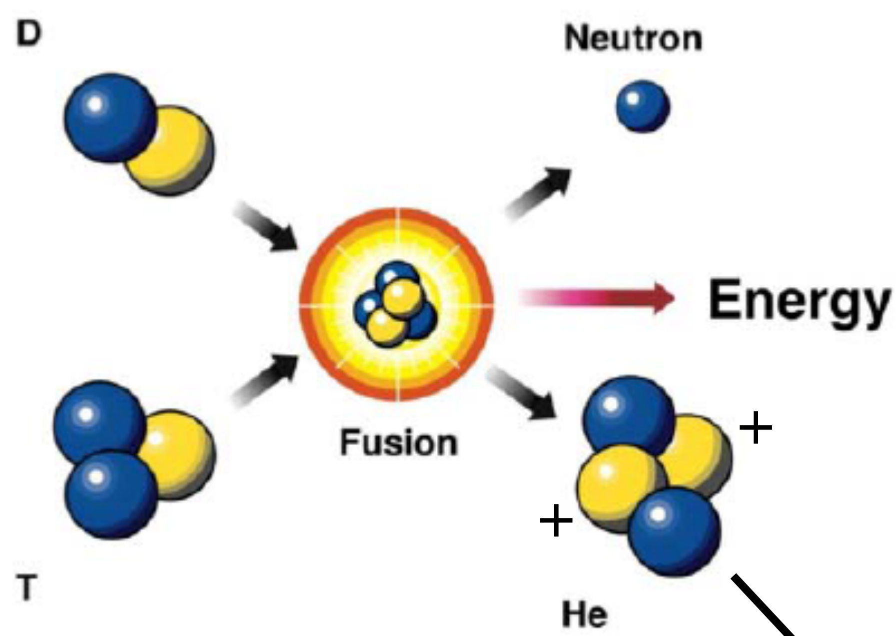
A fusion energy device heats itself and recycles neutrons internally for tritium hydrogen fuel

Energy Before

Energy After

~0.01

14.1



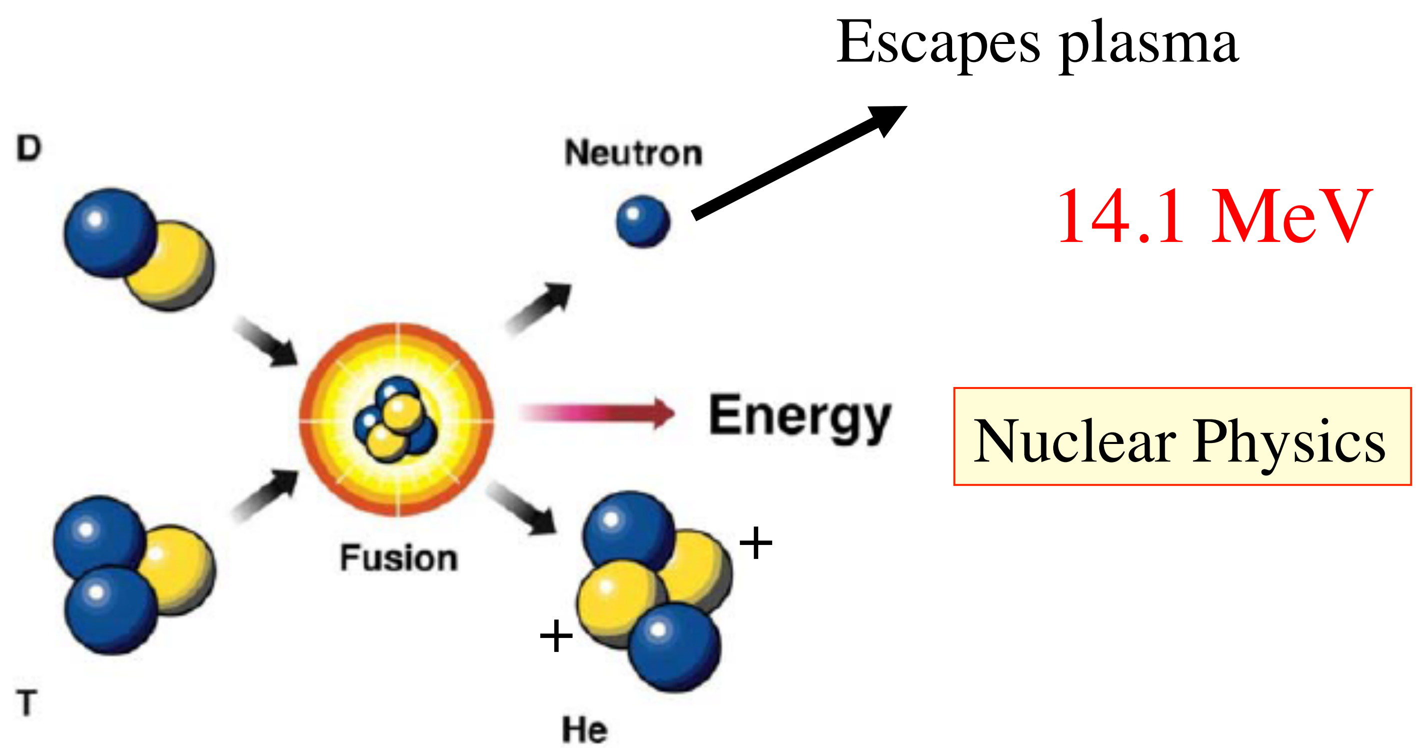
~0.01

3.5

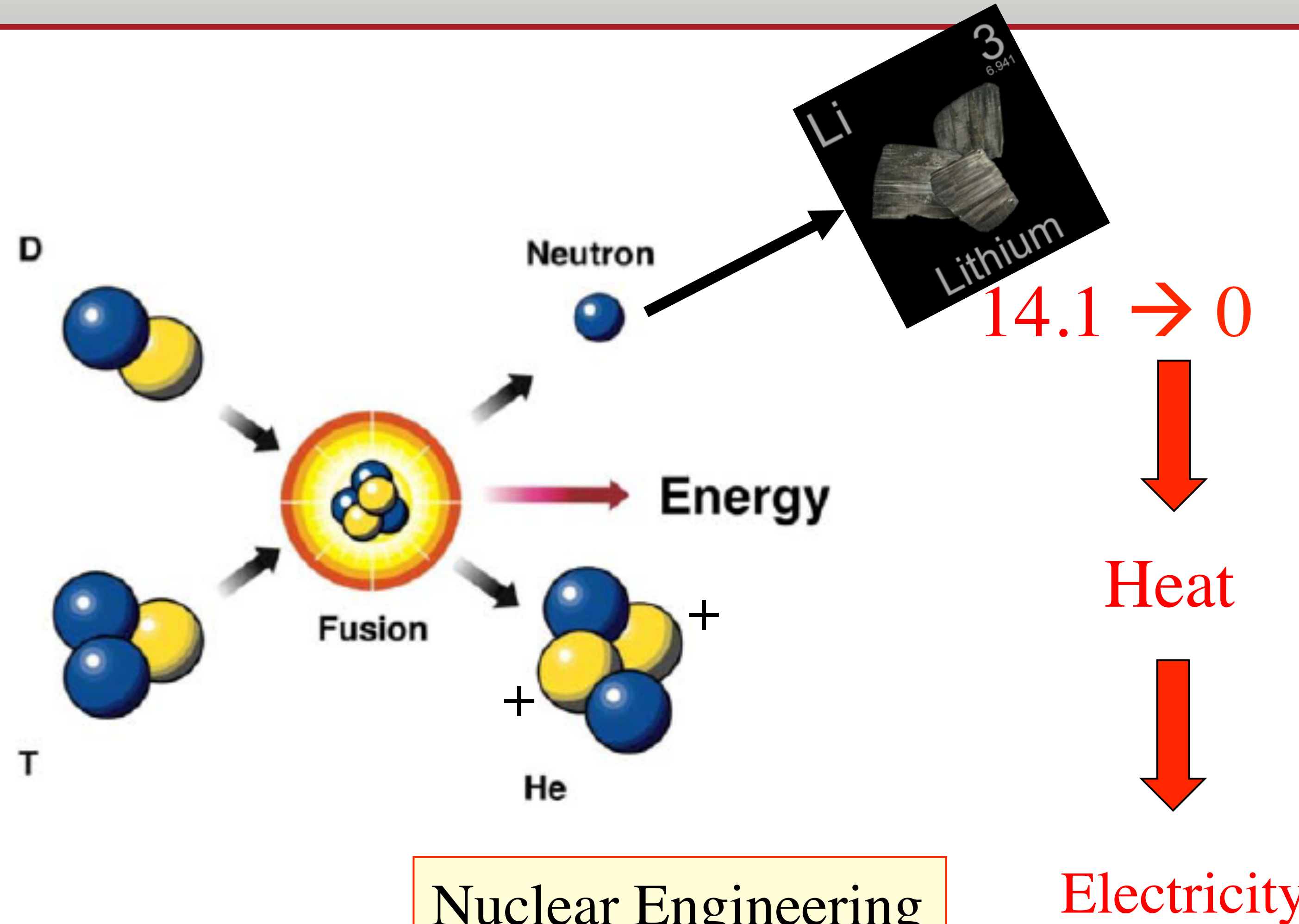
Plasma physics, $M_{Alfven} > 1$

Alphas heat plasma through scattering

A fusion energy device heats itself and recycles neutrons internally for tritium hydrogen fuel



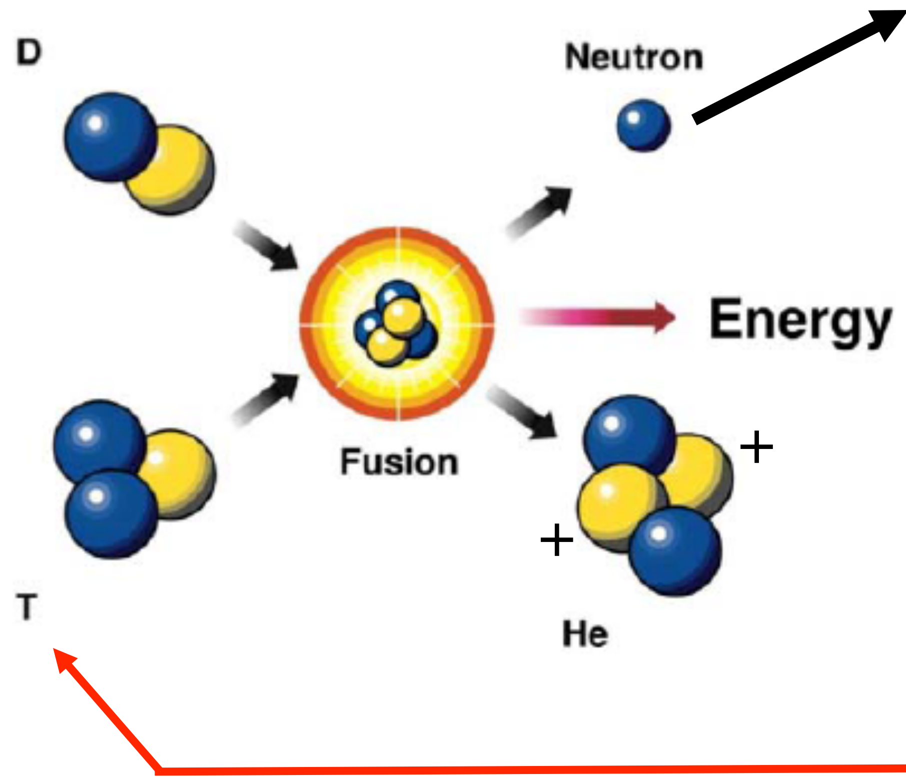
A fusion energy device heats itself and recycles neutrons internally for tritium hydrogen fuel



Nuclear Engineering

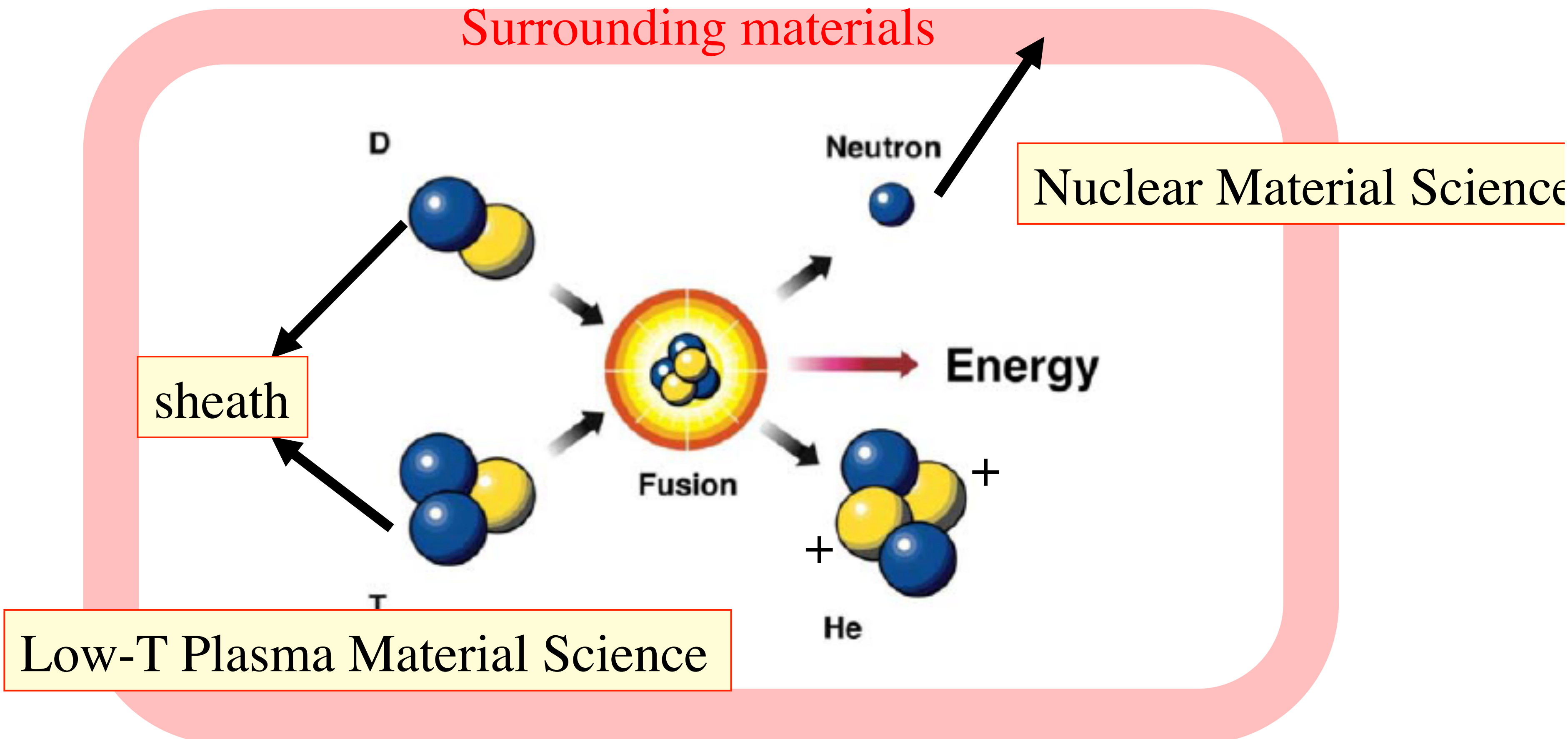
A fusion energy device heats itself and recycles neutrons internally for tritium hydrogen fuel

6-Li + n → He + **T**



Nuclear Engineering, Radiochemistry

A fusion energy device heats itself and recycles neutrons internally for tritium hydrogen fuel



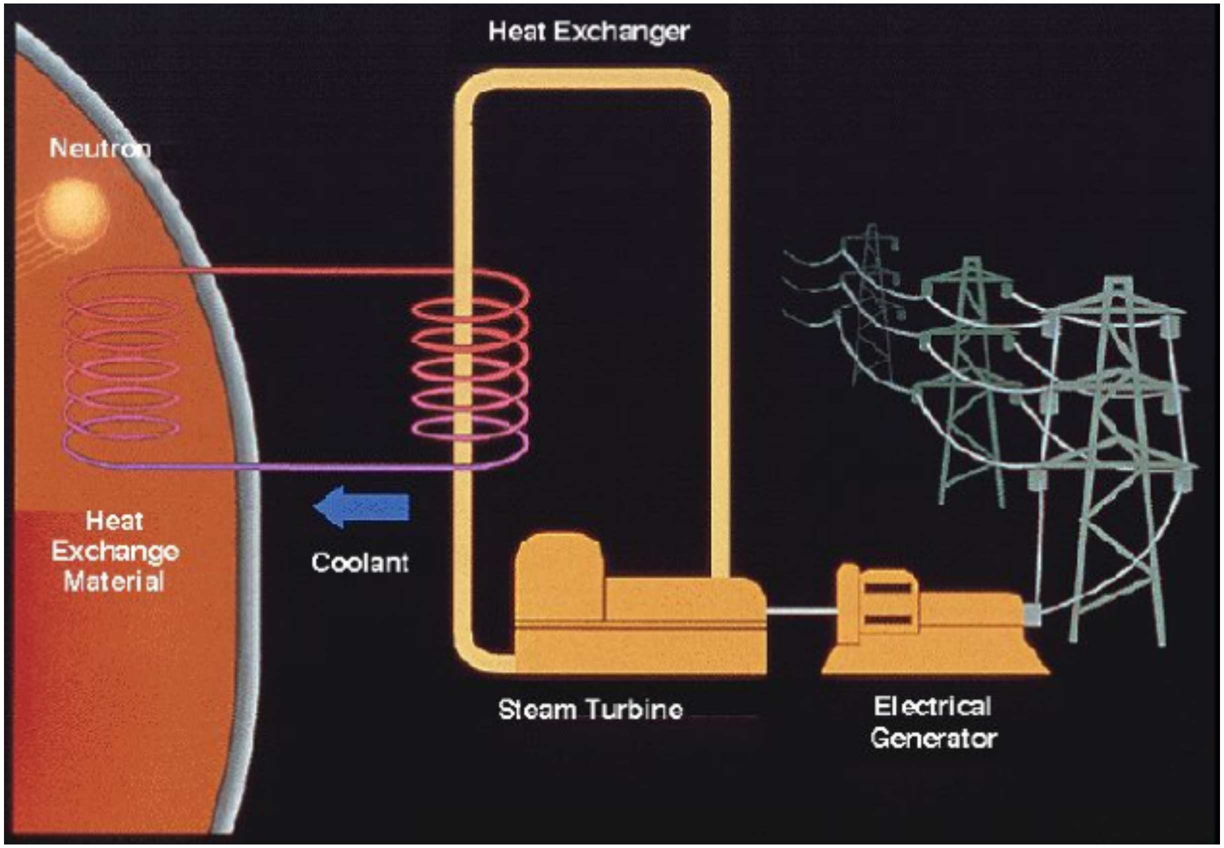
The lure of fusion

- **Nearly limitless fuel:**
  Deuterium + Lithium-6 → 2 Helium-4 + 22 MeV
- **No radioactive waste in fuel cycle**
- **Million times power density of "chemical" energy**
  - **Minimal fuel / waste stream**
  - **Minimized environmental footprint**
- **Inherently safe because it requires T > 5 keV**

The basic idea of electricity generation from fusion

- For electricity generation we absorb the power of the fusion reactants in a "blanket" and use this heating to run a turbine/ generator cycle.

- Tricky bit?
  The fusion core.
  - T > 5 MK…no direct physical container
  - Fusion core ignition has no yet been accomplished on earth in a *controlled manner*.



Fusion reaction rates in a D-T plasma are obtained by integrating through f(v)

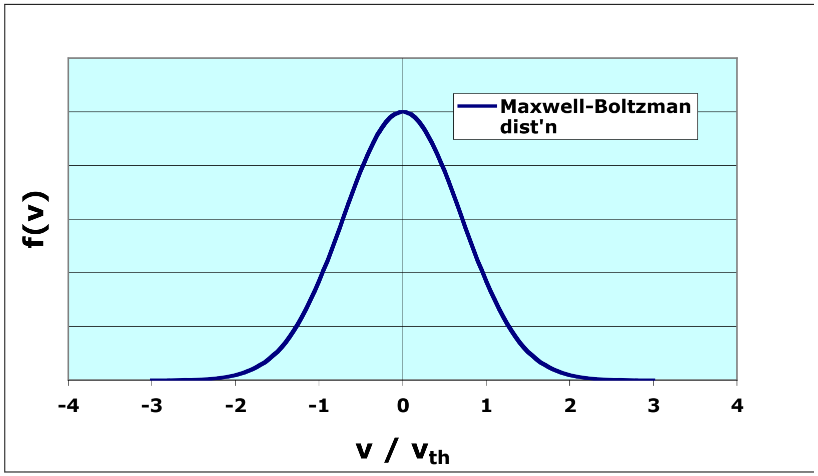
$$\sigma(v')v' f_1(\boldsymbol{v}_1) f_2(\boldsymbol{v}_2)$$

where

$$\boldsymbol{v}' = \boldsymbol{v}_1 - \boldsymbol{v}_2$$

Rate of reaction between two species with velocities v_1 and v_2
Distribution function: f

Fusion reaction rates in a D-T plasma are obtained by integrating through f(v)



If the distributions are Maxwellian,

$$f_j(v_j) = n_j \left(\frac{m_j}{2\pi T}\right)^{3/2} \exp - \frac{m_j v_j^2}{2T},$$

the total reaction rate per unit volume

$$\mathcal{R} = \iint \sigma(v')v' f_1(v_1) f_2(v_2)\, \mathrm{d}^3 v_1\, \mathrm{d}^3 v_2$$

Fusion reaction rates in a D-T plasma are obtained by integrating through f(v)

Velocity space

$$\mathcal{R} = 4\pi n_1 n_2 \left(\frac{\mu}{2\pi T}\right)^{3/2} \int \sigma(v')v'^3 \exp\left(-\frac{uv'^2}{2T}\right) \mathrm{d}v'.$$

Density product

Weighted distribution

$\varepsilon = \frac{1}{2} m_1 v'^2$ *Kinetic energy space*

$$\mathcal{R} = \left(\frac{8}{\pi}\right)^{1/2} n_1 n_2 \left(\frac{\mu}{T}\right)^{3/2} \frac{1}{m_1^2} \int \sigma(\varepsilon)\varepsilon \exp\left(-\frac{\mu\varepsilon}{m_1 T}\right) \mathrm{d}\varepsilon.$$

Fusion reaction rates in a D-T plasma

$$\mathcal{R} = \left(\frac{8}{\pi}\right)^{1/2} n_1 n_2 \left(\frac{\mu}{T}\right)^{3/2} \frac{1}{m_1^2} \int \sigma(\varepsilon)\varepsilon \exp\left(-\frac{\mu\varepsilon}{m_1 T}\right) d\varepsilon.$$

$$\downarrow$$

$$\mathcal{R} = n_{\mathrm{d}} n_{\mathrm{t}} \langle \sigma v \rangle$$

Maximum reaction rate for $n_d = n_t = 1/2\ n_{ions}$

$\langle\sigma v\rangle$ term only depends on <u>energy</u>

D-T fusion reaction rate coefficient versus plasma temperature T

$$\mathcal{R} = n_{\mathrm{d}} n_{\mathrm{t}} \langle \sigma v \rangle$$

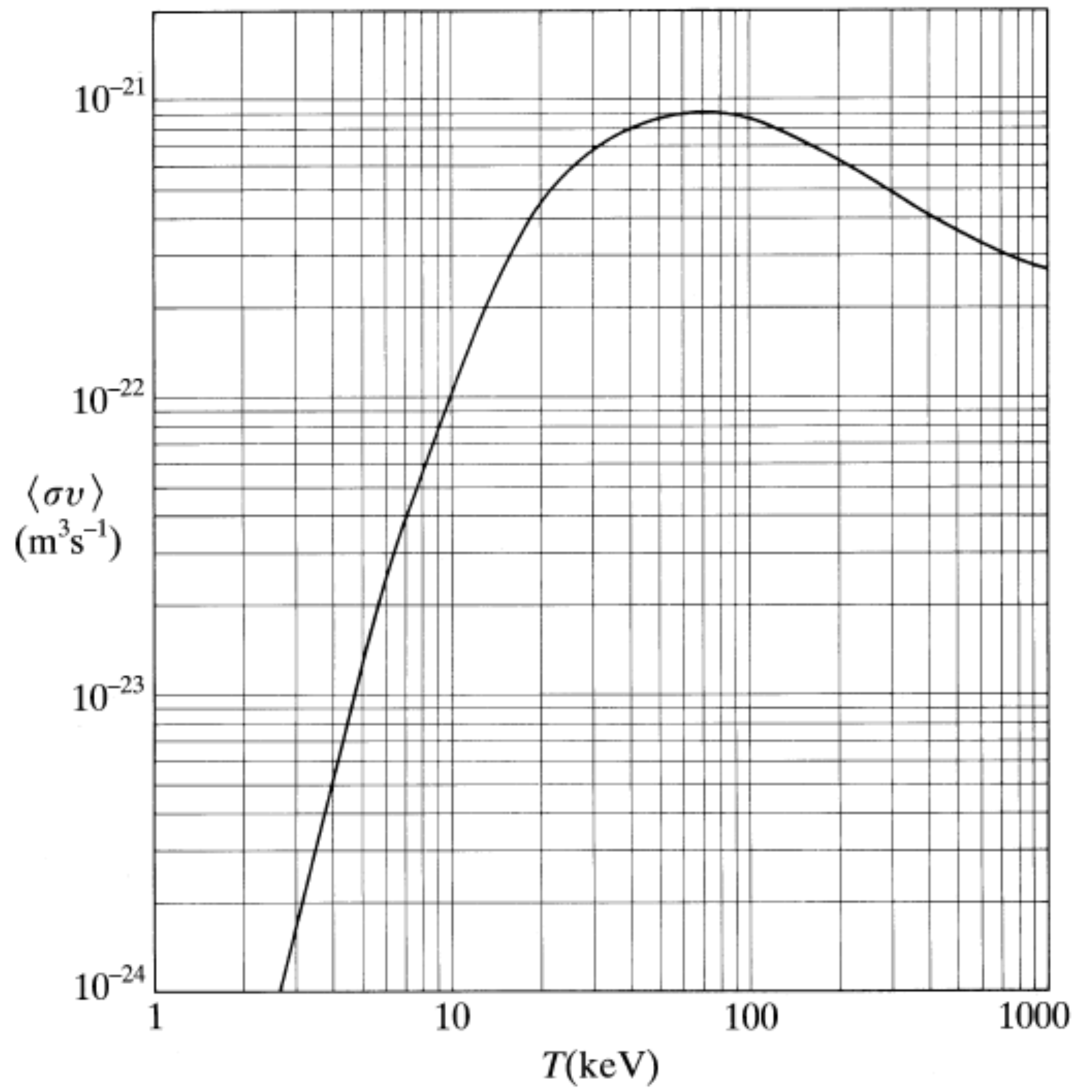
Note that for

T ~ 10 keV

$<\sigma v> \propto T^2$

So $R \propto (nT)^2$

$\mathbf{R \propto p^2}$



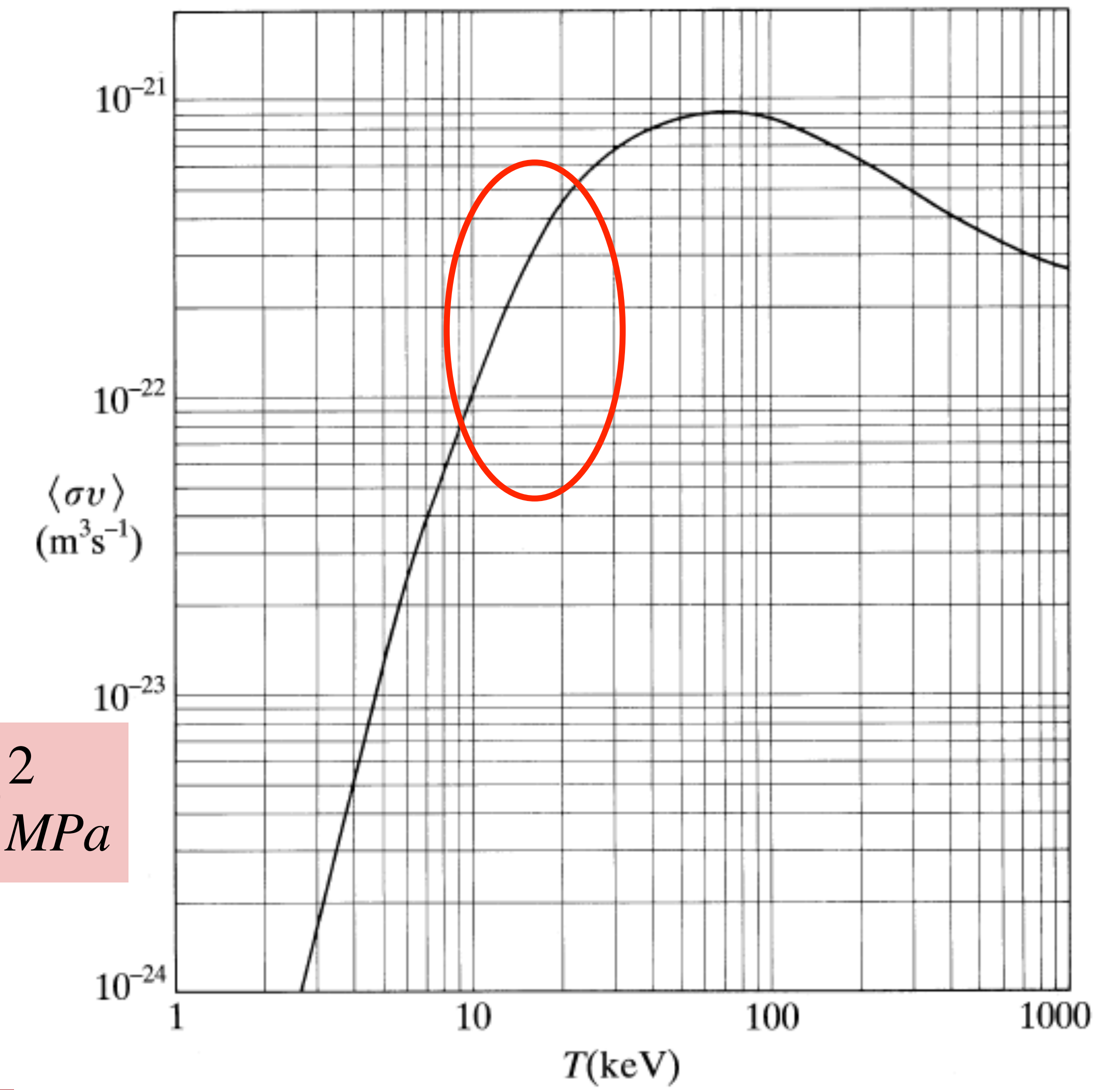
D-T fusion reaction rate coefficient versus plasma temperature T

$$\mathcal{R} = n_{\mathrm{d}} n_{\mathrm{t}} \langle \sigma v \rangle$$

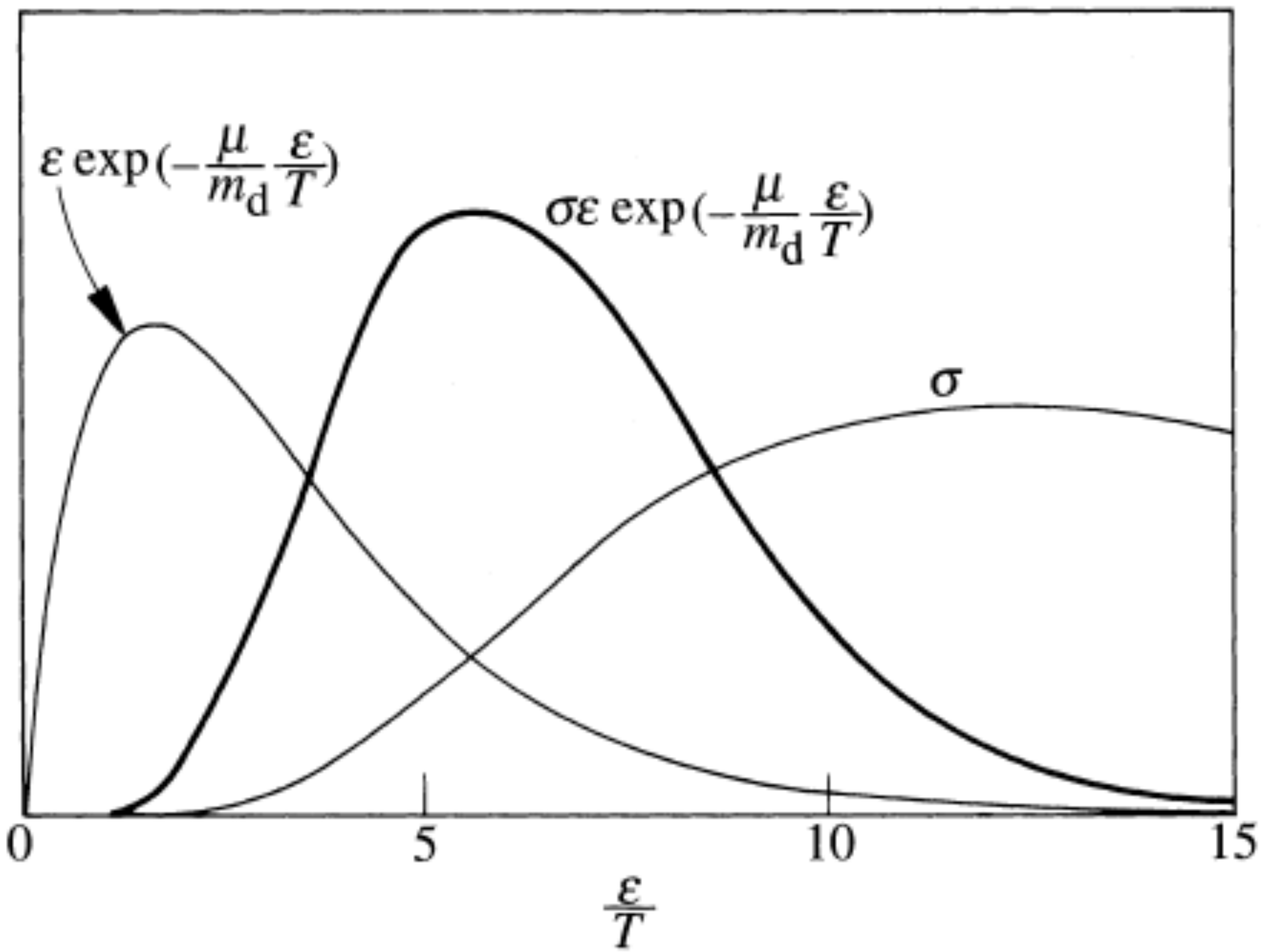
Fusion power density

$R * k_B\ Q$

$$P_{fusion}\left[MW\ m^{\overset{-3}{\cancel{-2}}}\right] \simeq 7\ p_{MPa}^{2}$$



D-T fusion is dominated by particle in energetic "tail" of distribution



Plot of weighted integrands in R equation at T=10 keV

Fusion power

$$p_{\mathrm{Tn}} = n_{\mathrm{d}} n_{\mathrm{t}} \langle \sigma v \rangle \mathcal{E},$$

Thermonuclear power per
Unit volume
$\mathcal{E}$: energy release per fusion

$$n = n_{\mathrm{d}} + n_{\mathrm{t}},$$

Total ion density

$$p_{\mathrm{Tn}} = n_{\mathrm{d}}(n - n_{\mathrm{d}}) \langle \sigma v \rangle \mathcal{E}.$$

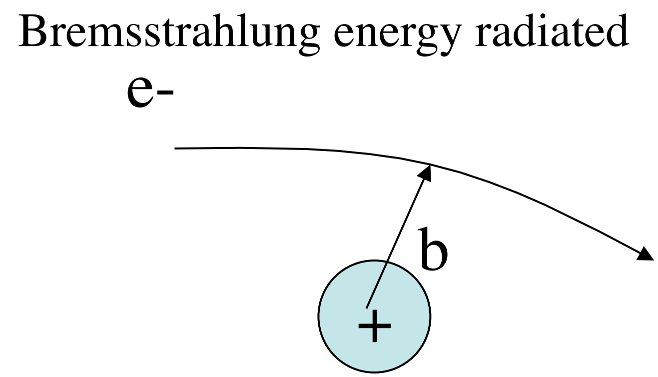
For given n, power is maximized
At $n_d = 1/2$ n

$$p_{\mathrm{Tn}} = \tfrac{1}{4} n^2 \langle \sigma v \rangle \mathcal{E}.$$

Most useful form.

For fusion plasmas bremsstrahlung radiation is an unavoidable source of power loss

Bremsstrahlung: (German for "braking radiation") is a result of acceleration of plasma electrons during collisions with ions, the acceleration causes the electron to radiate…power loss proportional to particle velocity



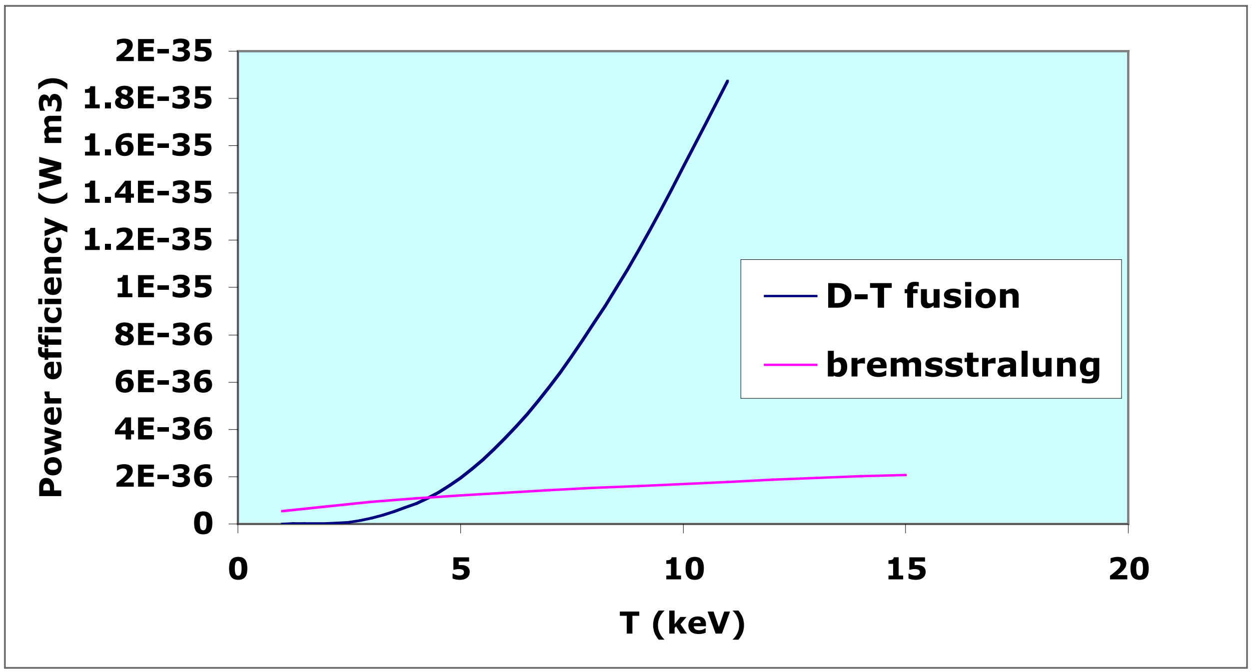
$$P_{bremsstrahlung} = 5.4x10^{-37} n^2 Z_{eff} T_e^{1/2} \quad [W / m^3]$$

Simplest power balance informs us that T > 4.5 keV is minimum temperature required for fusion energy

$$P_\alpha = P_{bremsstrahlung}$$

$$n_d n_t \langle \sigma v \rangle \left(0.2\, kQ_{d-t}\right) = 5.4x10^{-37} n^2 Z_{eff} T_e^{1/2}$$

$$\frac{n^2}{4} \langle \sigma v \rangle (T) \left(3.5 MeV\right) = 5.4x10^{-37} n^2 Z_{eff} T_e^{1/2}$$

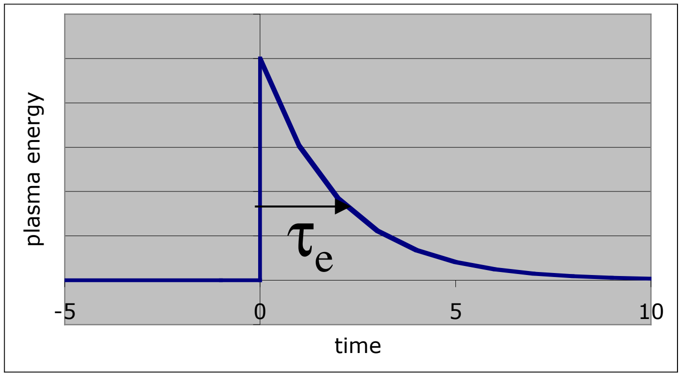


Independent
Of density

Lawson criterion: the realistic limit for reaching ignition in fusion system

Of course it is physically nonsense to take an infinitely large plasma system that undergoes no energy losses….

So let's parameterize the energy loss of our plasma by the concept of an energy confinement time, $\tau_{e,}$ which is essentially the e-folding time Of the energy containment 'efficiency" of the system



Response of system To energy input at time=0

For the moment we ignore the physical explanation for energy Confinement time (convection,conduction, etc.)

Energy loss: Continuous transport loss of energy from plasma that must be replaced

Average energy of plasma particles is 3/2 T
(1/2 T per degree of freedom)

Plasma: Equal number of ions and electrons

$n = n_i = n_e$ so energy density = 3 n T

$$W = \int 3nT \, \mathrm{d}^3x = 3\overline{nT}V,$$

Total energy in plasma of volume = V
(bar signifies volume-averaged)

Energy loss: Continuous loss of energy from plasma that must be replaced

So rate of energy loss from plasma is characterized by a confinement time which is *defined* by:

$$P_L = \frac{W}{\tau_E}.$$

In present experiments, fusion heating usually negligible, so $P_L = P_H$, ***an external heating source***

$$\tau_E = \frac{W}{P_H}.$$

Experimental definition of confinement time

Power balance in steady-state plasma (ignoring bremsstrahlung for moment)

$$P_{\mathrm{H}} + P_{\alpha} = P_{\mathrm{L}}$$

$$P_{\mathrm{H}} + \tfrac{1}{4}\overline{n^2\langle\sigma v\rangle}\mathcal{E}_{\alpha} V = \frac{3\overline{nT}}{\tau_{\mathrm{E}}} V .$$

Ignition is obtained when alpha heating is sufficient to offset losses and P_H can be removed (P_H=0)

$$P_{\mathrm{H}} = \left(\frac{3nT}{\tau_{\mathrm{E}}} - \frac{1}{4} n^2 \langle \sigma v \rangle \mathcal{E}_\alpha \right) V .$$

$$n \tau_{\mathrm{E}} > \frac{12}{\langle \sigma v \rangle} \frac{T}{\mathcal{E}_\alpha} .$$

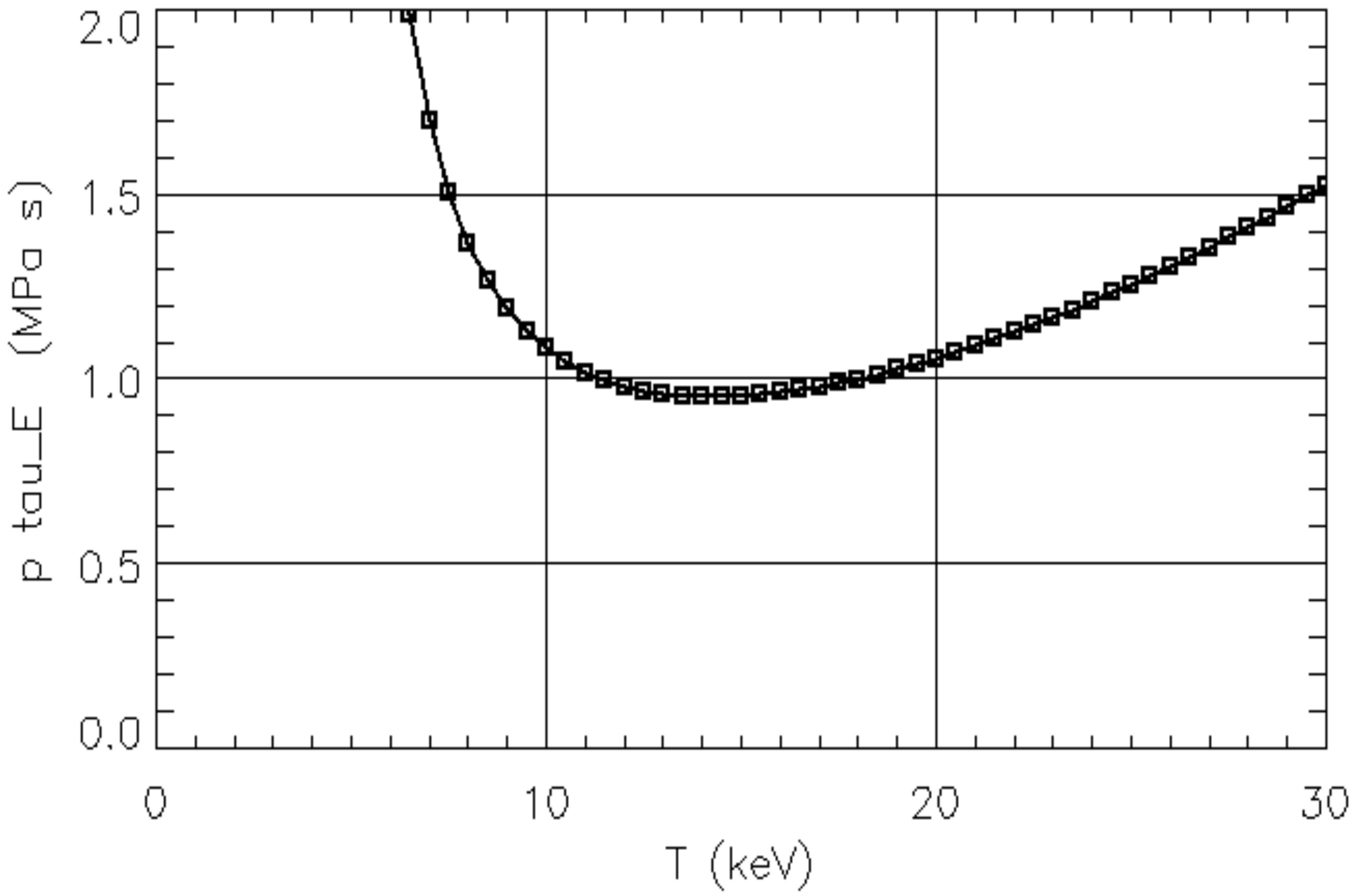
Lawson criterion

Depends only
On T

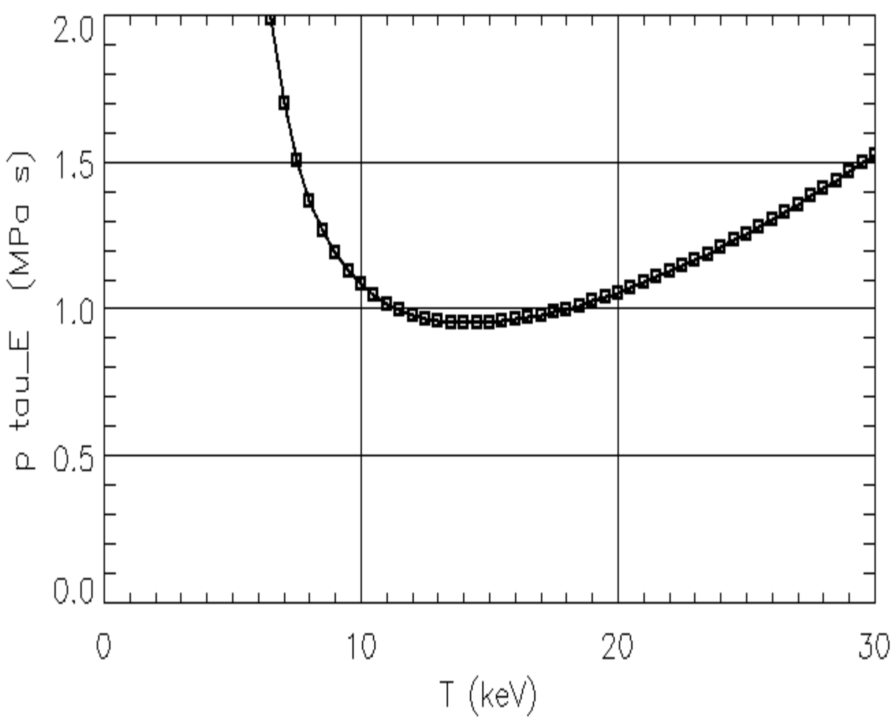
$$nT\ \tau_E \simeq \frac{T^2}{\langle \sigma v \rangle} \simeq \text{constant (T=5-25 keV)}$$

After all that the condition for fusion ignition T ~ 10-15 keV and 1 MPa-s

$$P_\alpha = P_{loss} + P_{brem}$$



Magnetic fusion reactor "design space" is highly constrained by Lawson criterion & steady-state



$$P_\alpha = P_{loss} + P_{brem}$$

Conditions?

p ~ 1 Mpa ~ 10 bar

τ_E ~ 1 second

T ~ 12 keV → n ~ $2x10^{20}$ m^{-3}

Fusion power density
5-10 MW/m^3

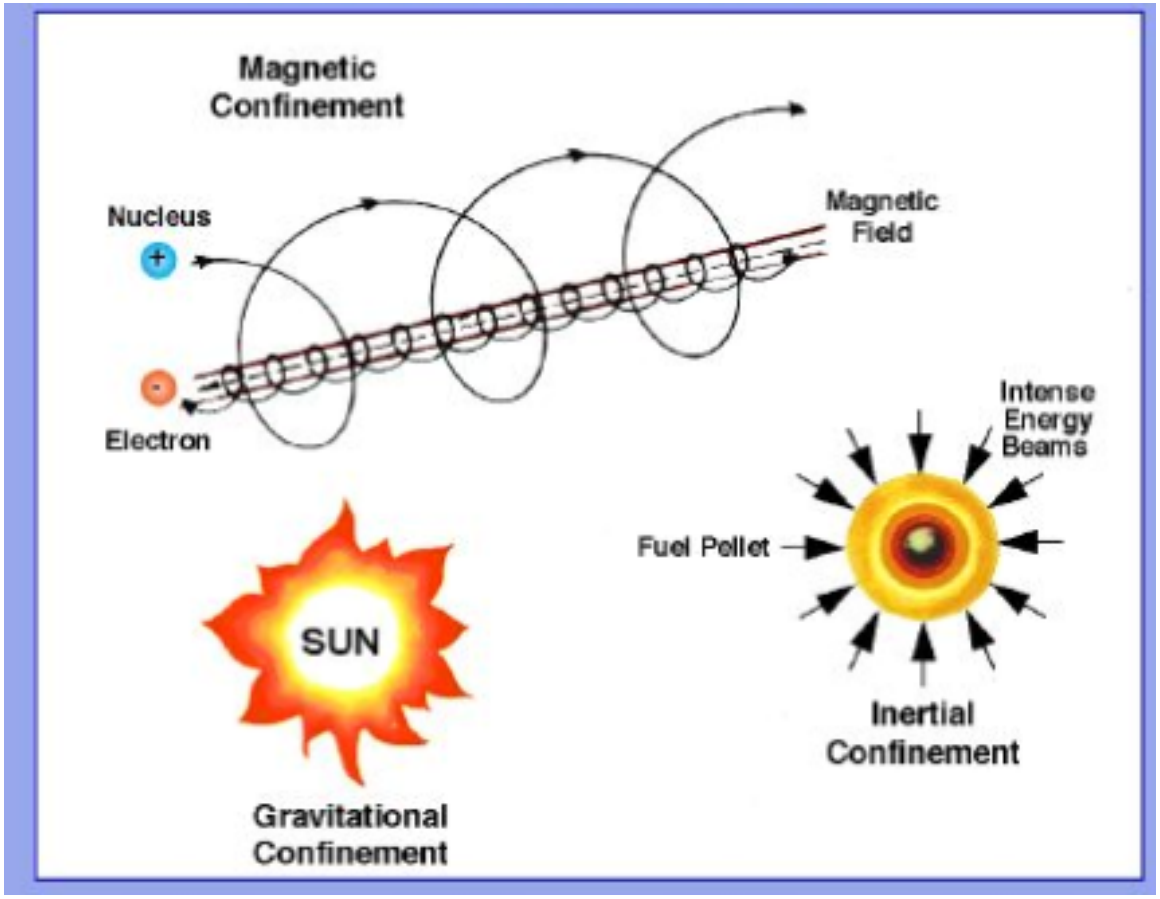
Confinement of the hot plasma is necessary to sustain the high temperatures needed for fusion

1. Gravitation field

2. Magnetic: charged particles are “trapped” by magnetic fields

$$\mathbf{F} = q\,\mathbf{v} \times \mathbf{B}$$

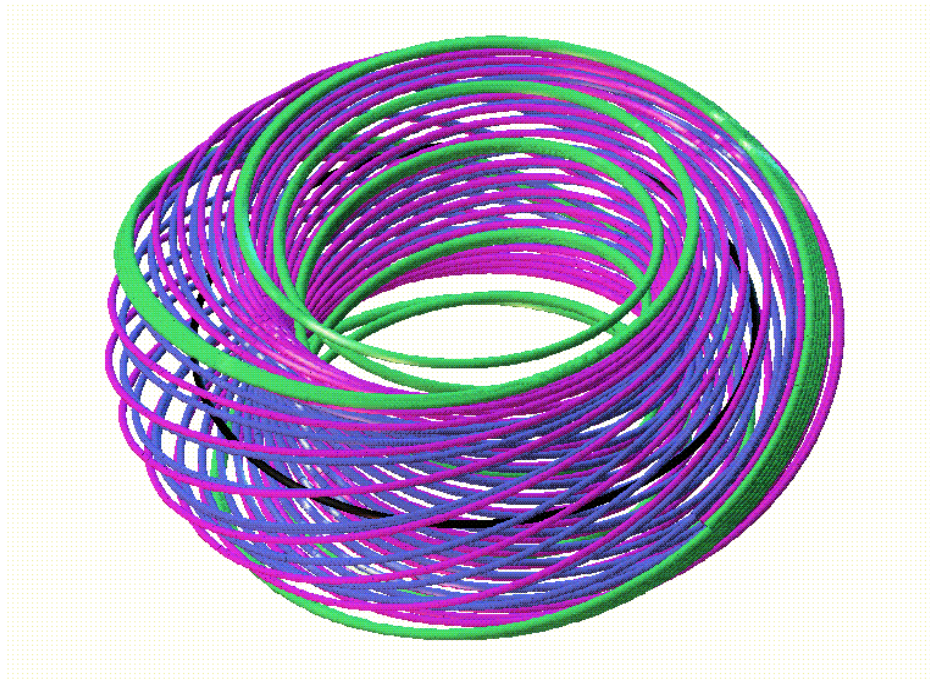
3. Inertial: the fusion occurs so quickly (ns) the fusing particles can’t get out of the way.



"Toroidal" magnetic fusion is leading concept because it removes // to B "end-losses"

The magnetic fields must be helical, i.e. they cannot be purely toroidal,

in order to short out the charged particles un-confining drifts induced by the toroidal geometry



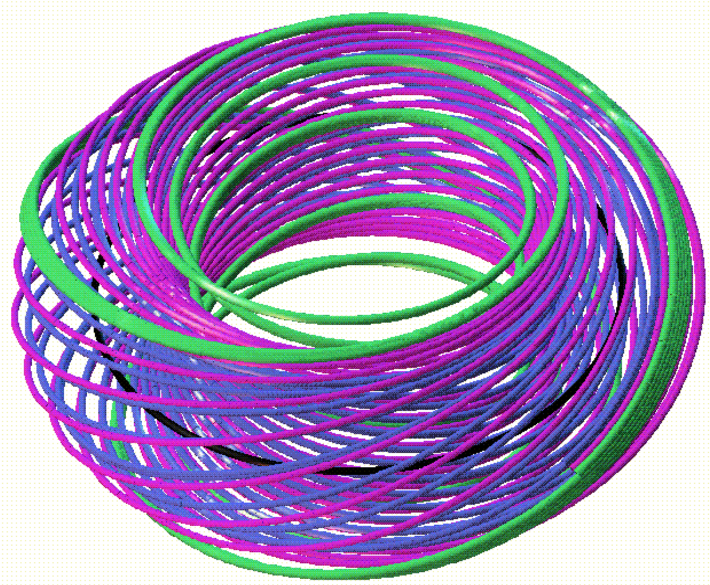
"Toroidal" magnetic fusion is leading concept because it removes // to B "end-losses"

- Magnetic field is looped continuously around in a donut or toroidal shape.
- Lorentz force produced by large magnetic fields holds energetic ions/electrons to tight orbits around field
  - For 3.5 MeV alphas, gyro-radius is 2 cm at 10 Tesla

$$\rho = v_{\perp} / \omega_c$$

$$\rho_{ion} \simeq 10^{-4} \frac{\left(T_{eV}\ \mu_{amu}\right)^{1/2}}{Z_i\ B_T}$$

Along with neutron MFP ~ 10 cm, the ions orbit size set minimum size requirements for fusion, regardless of energy confinement.



Toroidal magnetic confinement system

• Primary field in magnitude is toroidal field.

• Poloidal field (short-way around torus) provides confinement by averaging out drifts.

• Poloidal field provided by **toroidal plasma current (tokamak)** or by **external coil windings** (stellarator).

Wesson, *Tokamaks*

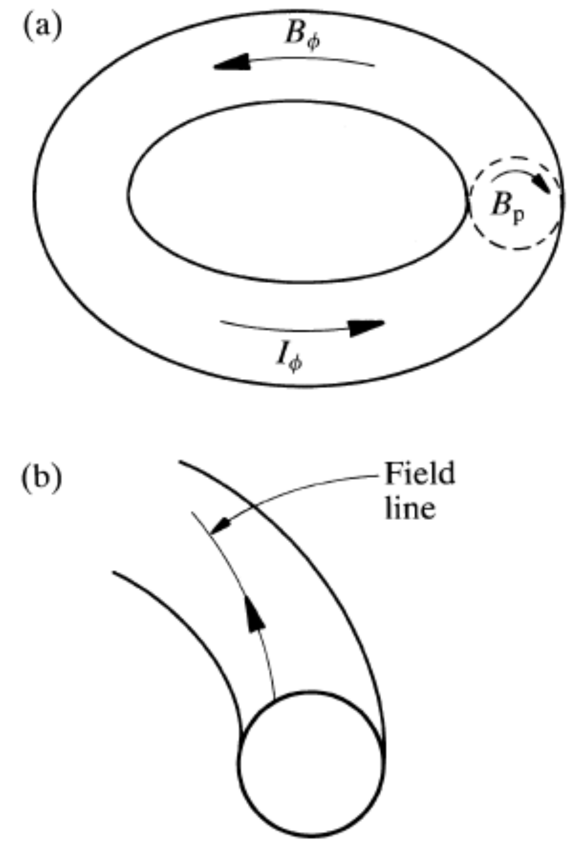
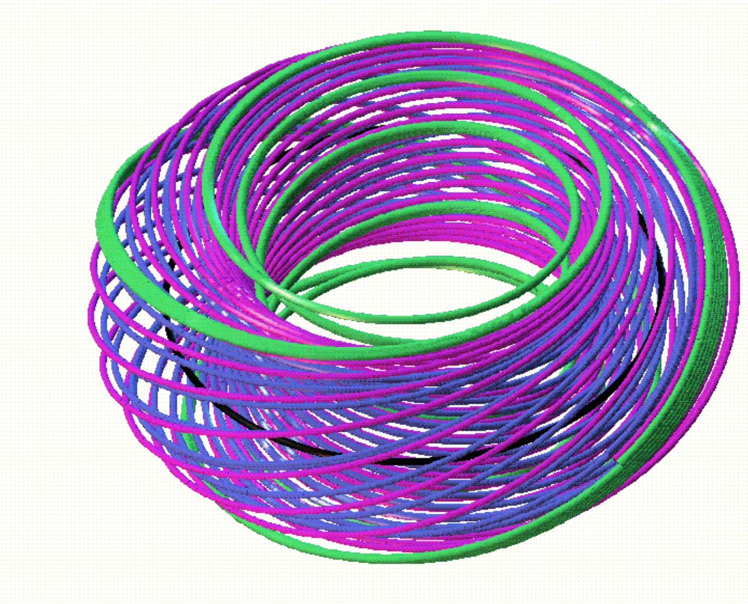
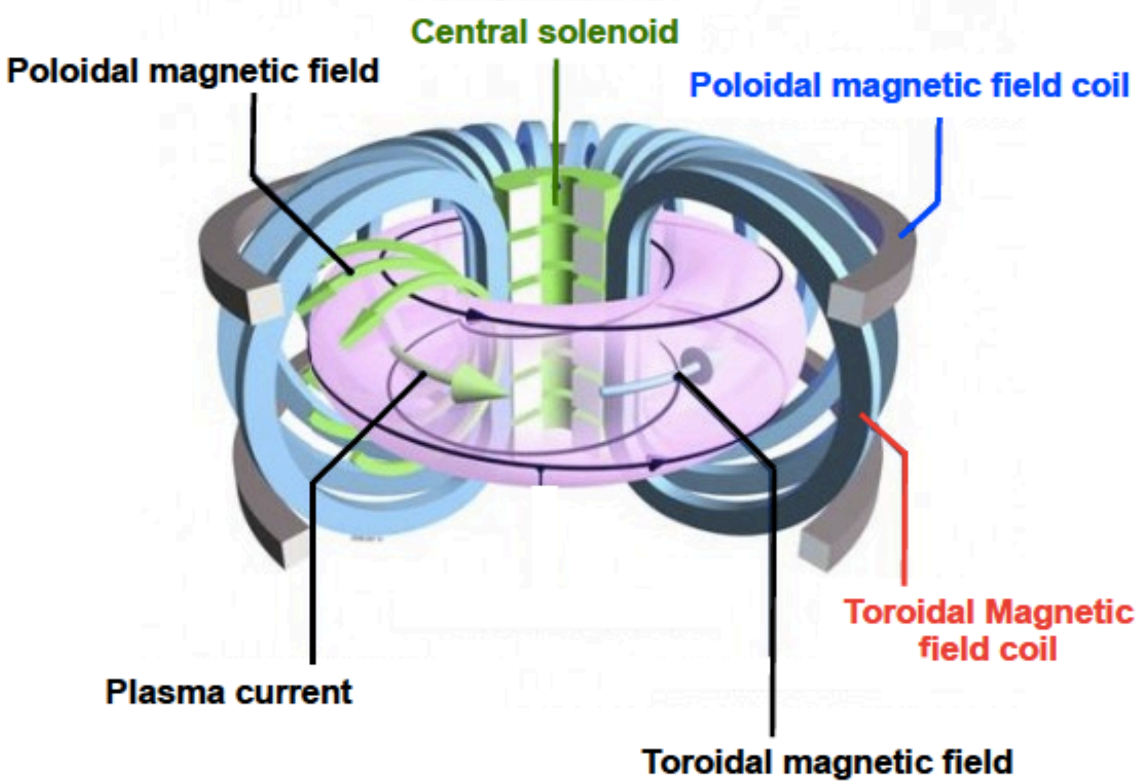


Fig. 1.6.1 (a) Toroidal magnetic field B_ϕ, and poloidal magnetic field B_p due to toroidal current I_ϕ. (b) Combination of B_ϕ and B_p causes field lines to twist around plasma.

The tokamak magnetic confinement configuration has historically provided the best fusion performance in gain & power density





External electromagnetic coils provide magnetic field

- Due to toroidal arrangement of coils the B decreases linearly with distance from toroidal axis.

- Typical fields considered for fusion energy B ~ 5-10 T Why?
  - Cooling limits (copper)
  - Field limits (superconductors)

$$\frac{P_{fusion}}{V} \sim p_{th}^2 \sim \beta^2 B^4$$

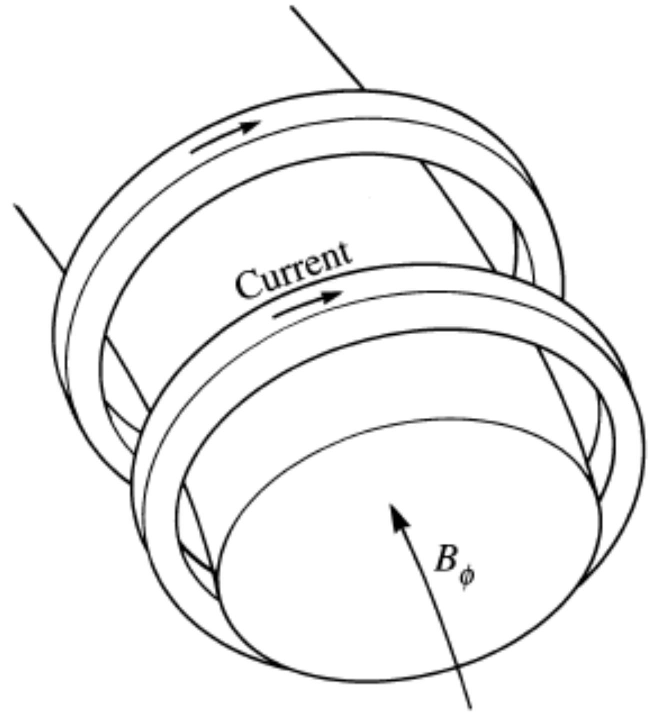
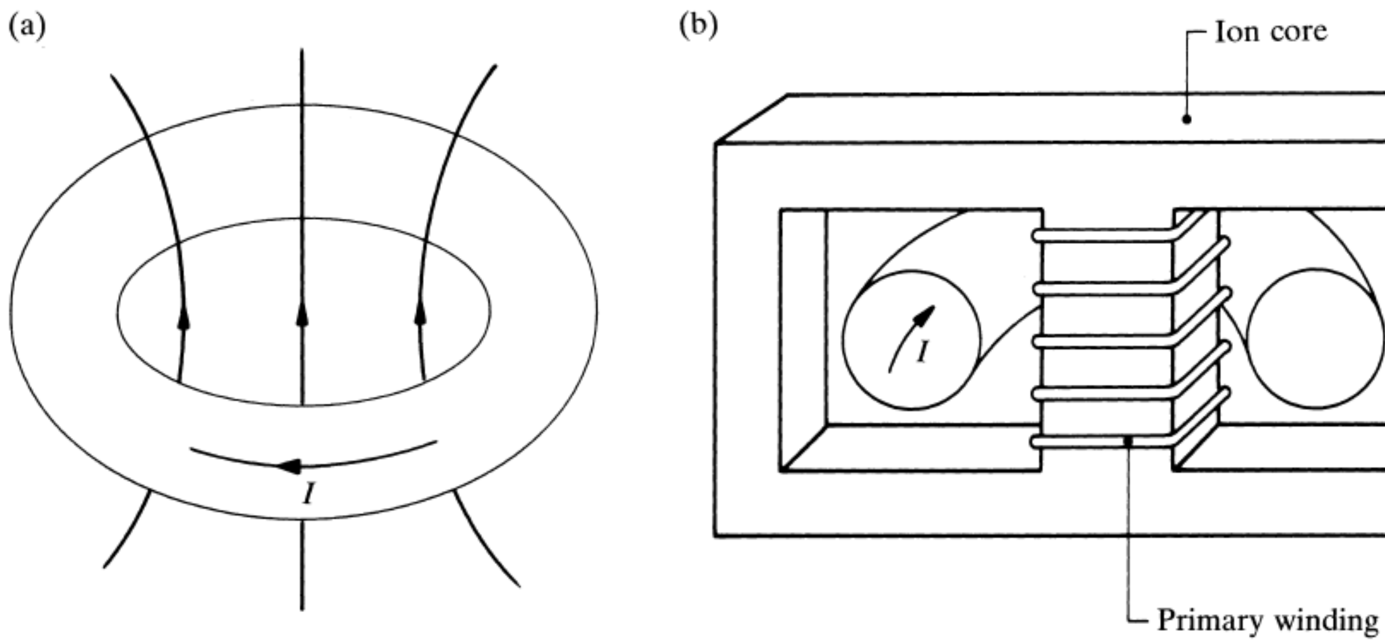


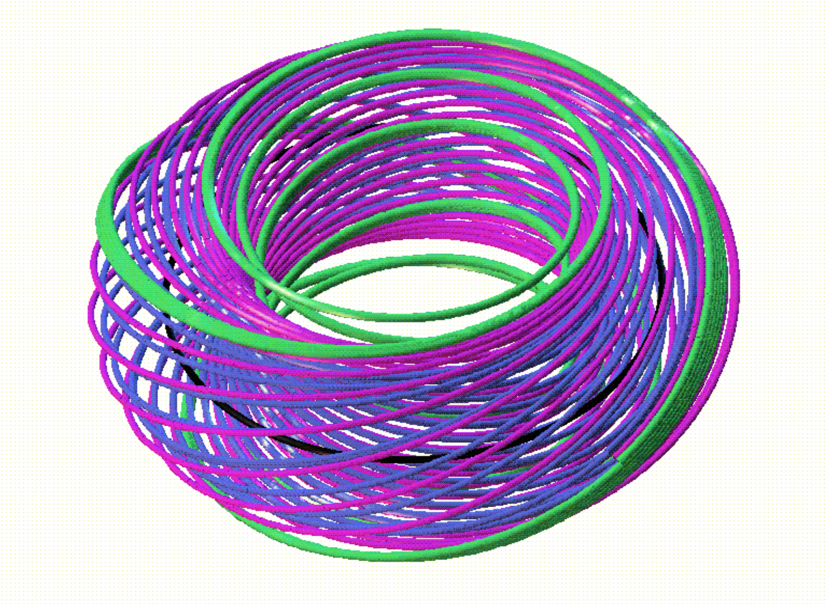
Fig. 1.6.2 The toroidal magnetic field is produced by current in external coils.

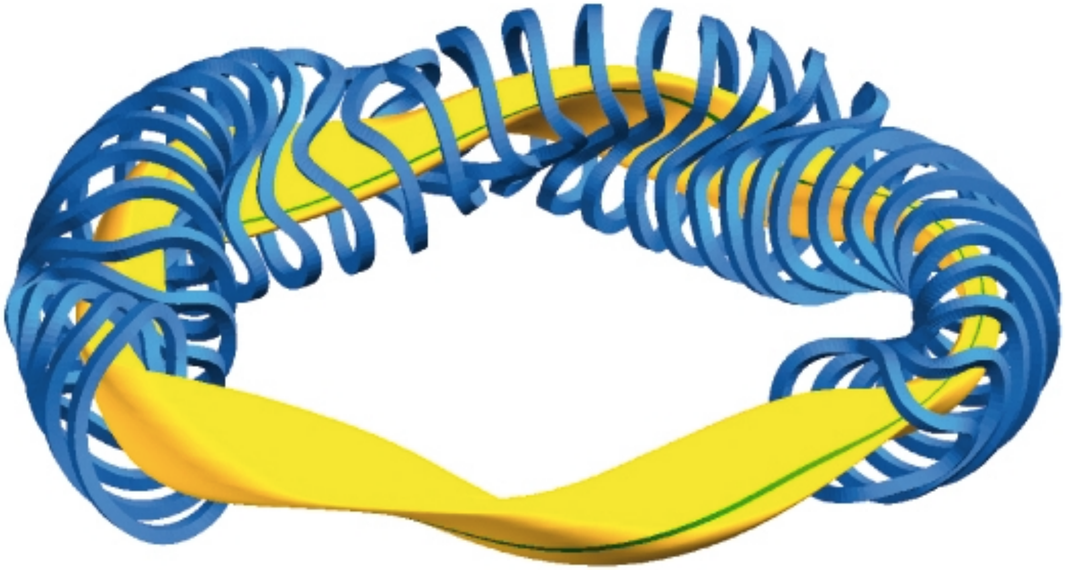
In a standard tokamak, toroidal current is driven inductively



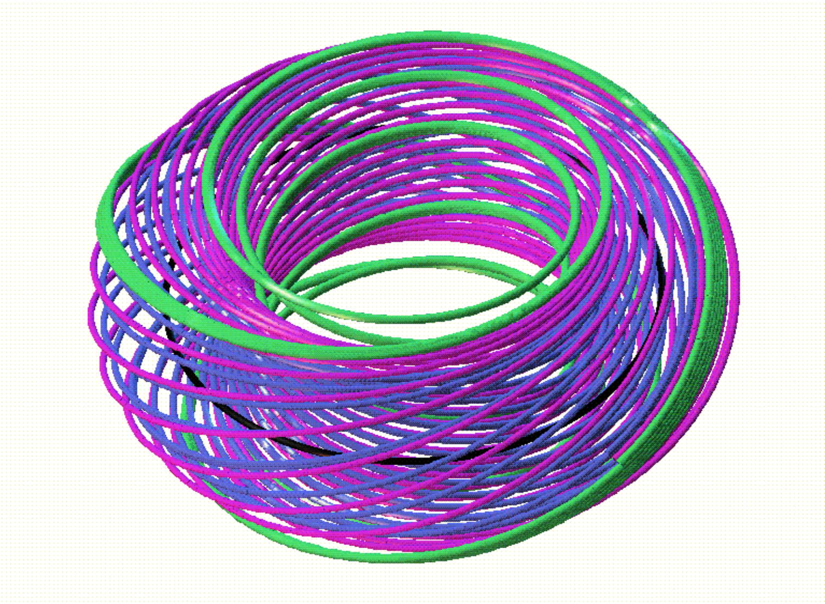
- Changing flux through torus center induces toroidal electric field that drives > MA current in the conducting plasma.
- Plasma is heated "ohmically" : $P_{ohmic} = R\,I^2$
- **Indcutive current is inherently AC or pulsed.**

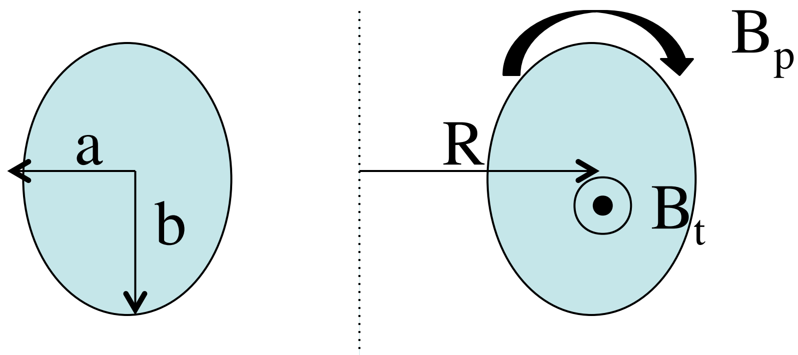
The stellarator configuration uses external coils to produce the helical field





Geometry and magnetic field strength from external coils drive design





Basic design choices

R: major radius

a: minor radius

R/a: aspect ratio

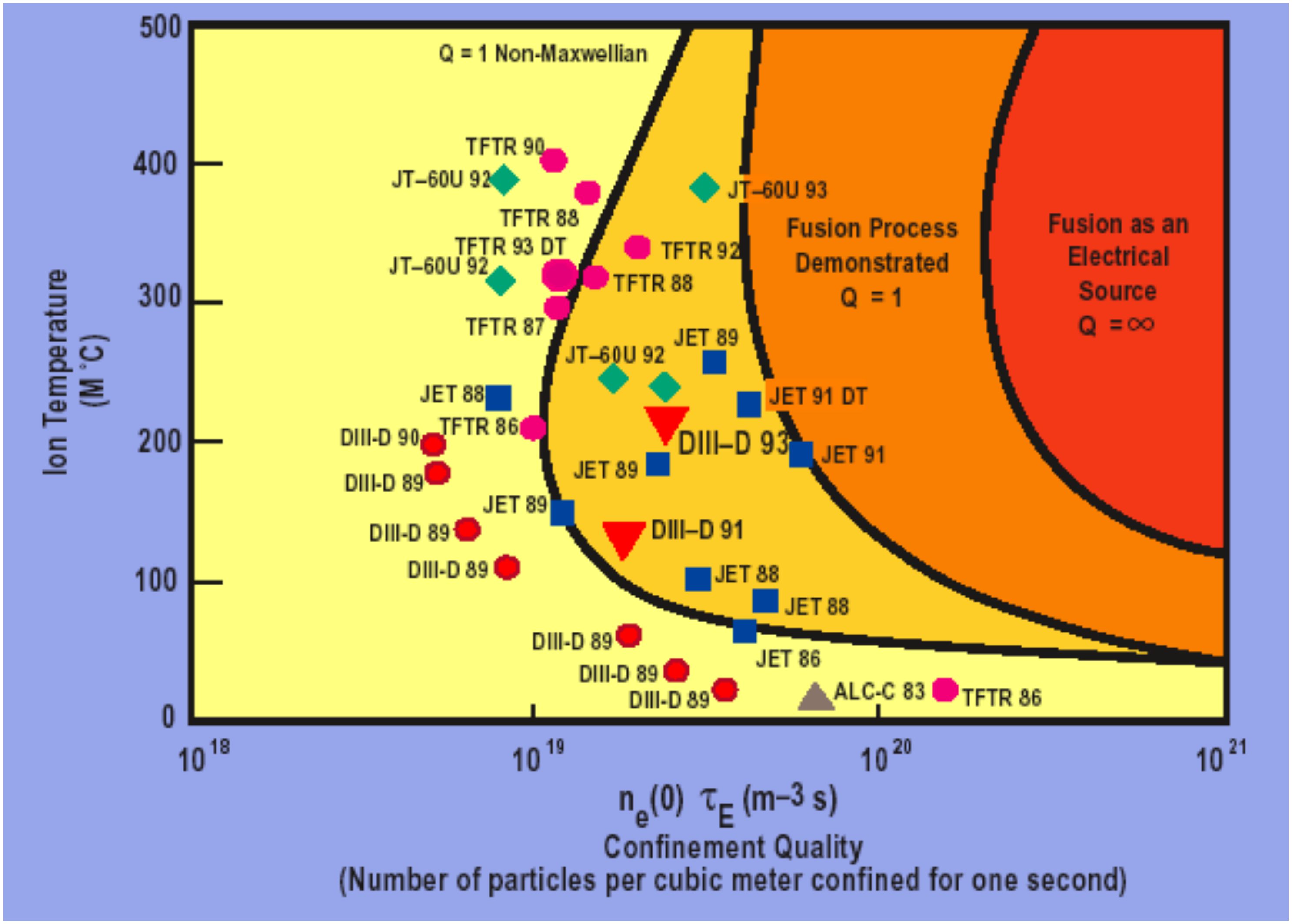
b/a: elongation, κ

B_t: toroidal magnetic field (T)

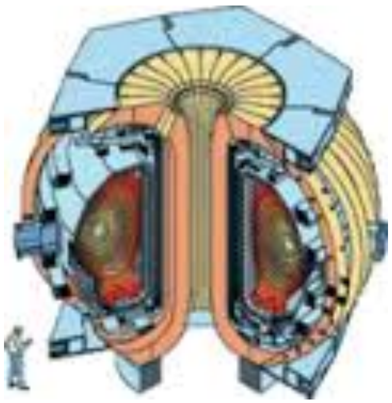
B_p: poloidal field

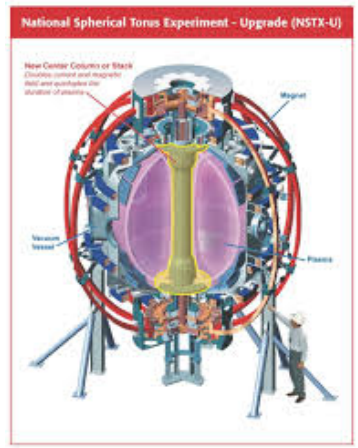
$(B_t/B_p)(a/R)$: magnetic winding

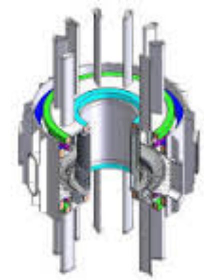
Worldwide magnetic fusion research has made great progress with experiments approaching ignition conditions



US has variety of configurations to study magnetic confinement

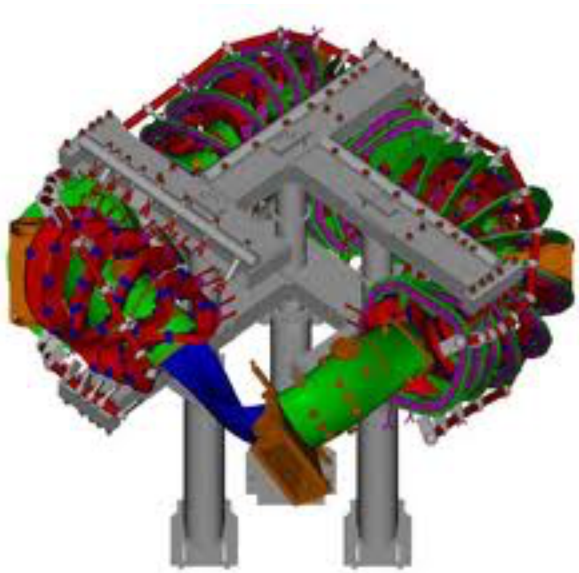


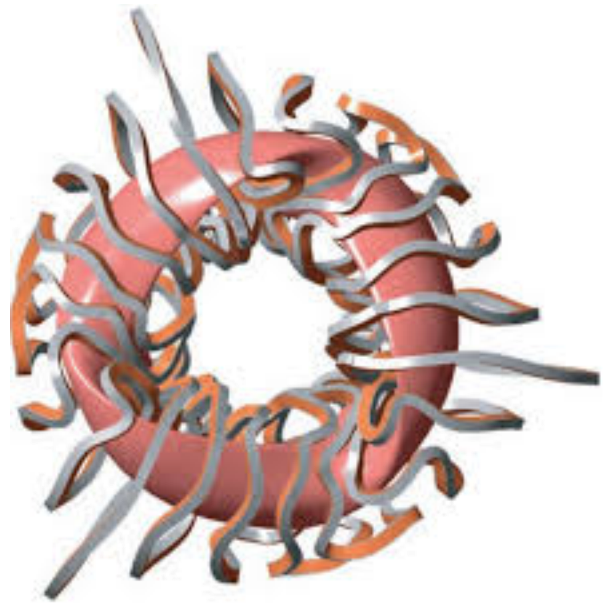




DIII-D	NSTX-U	Alcator C-Mod
B ~ 2 T	B ~ 1 T	B ~ 5-8 T
R = 1.6 m	R = 1.6 m	R = 0.7 m
R/a ~ 2.8	R/a ~ 1.8	R/a ~ 3

US has variety of configurations to study magnetic confinement

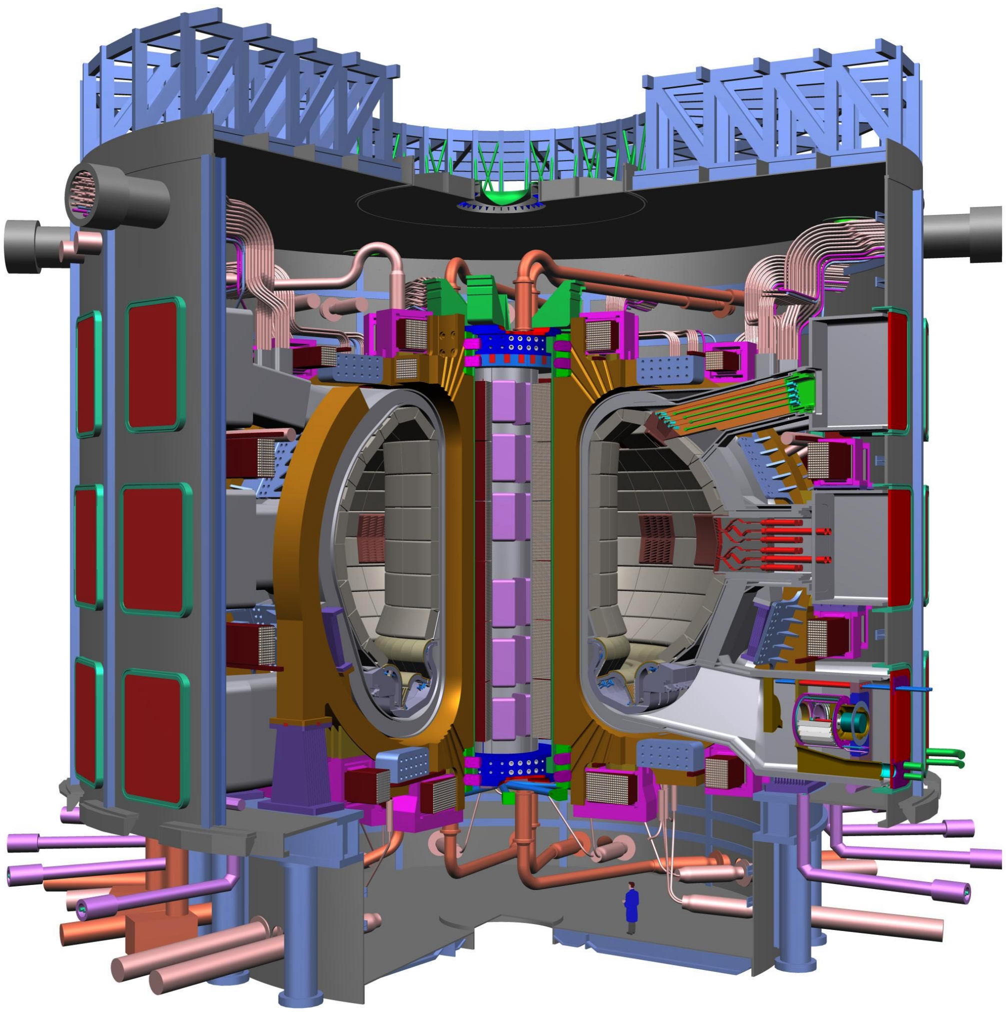




HSX
B ~ 1.2 T
R = 1.2 m
R/a ~ 8

NCSX
B ~ 2 T
R = 1.4 m
R/a ~ 4.5

ITER is being built as the first magnetic fusion experiment to obtain fusion burn and approach ignition where the plasma is self-sustained



$Q = 10$

$P_{alpha} = 2\ P_{external}$

$R = 6.2$ m

Volume ~ 800 m^3

$B = 5.3$ Tesla

Superconducting

Tokamak

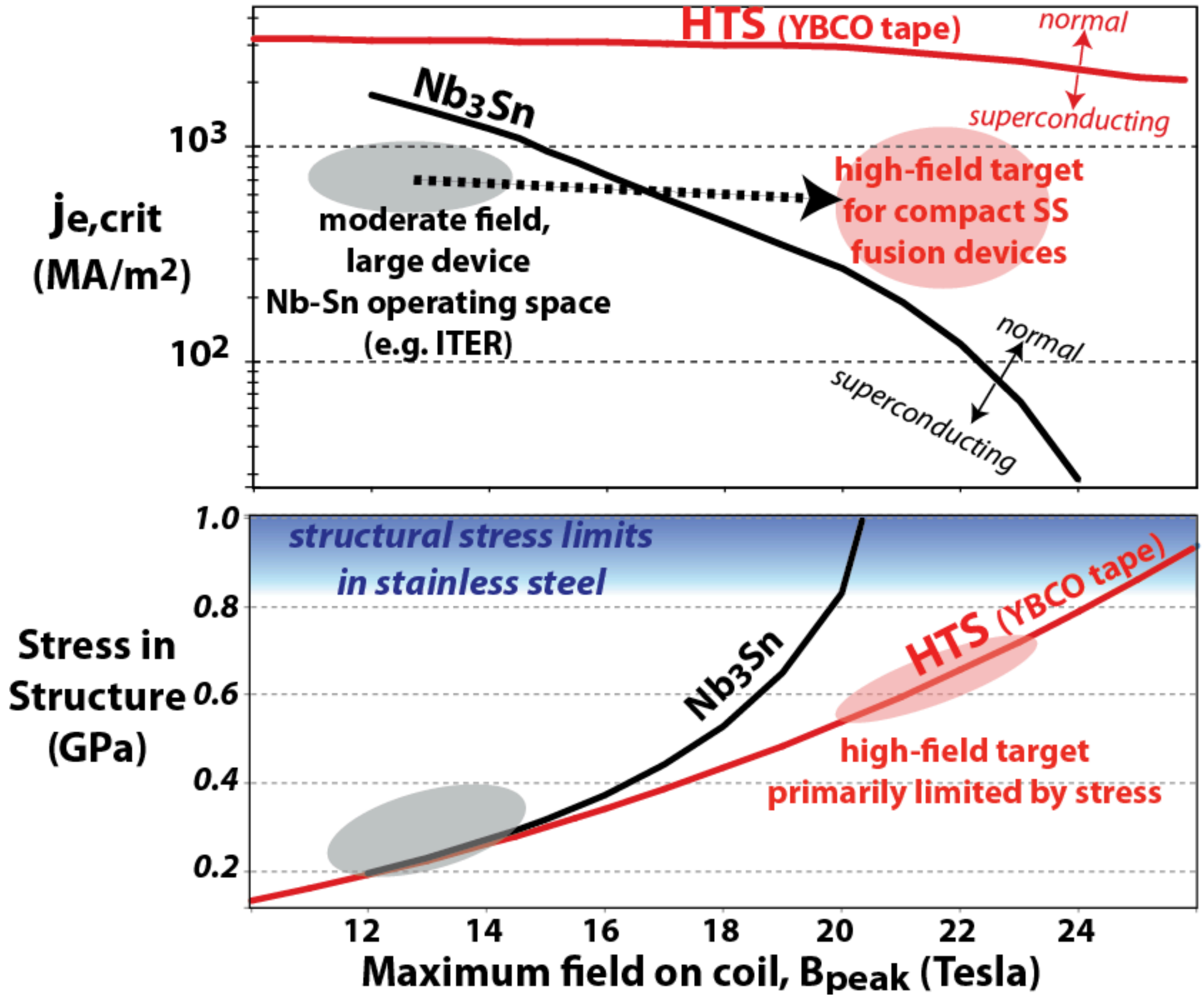
Superconductor coils produce the primary B field without loss → Net energy from fusion

- **Zero resistance at temperatures < 70 K.**

$$\nabla x B = \mu_o J$$

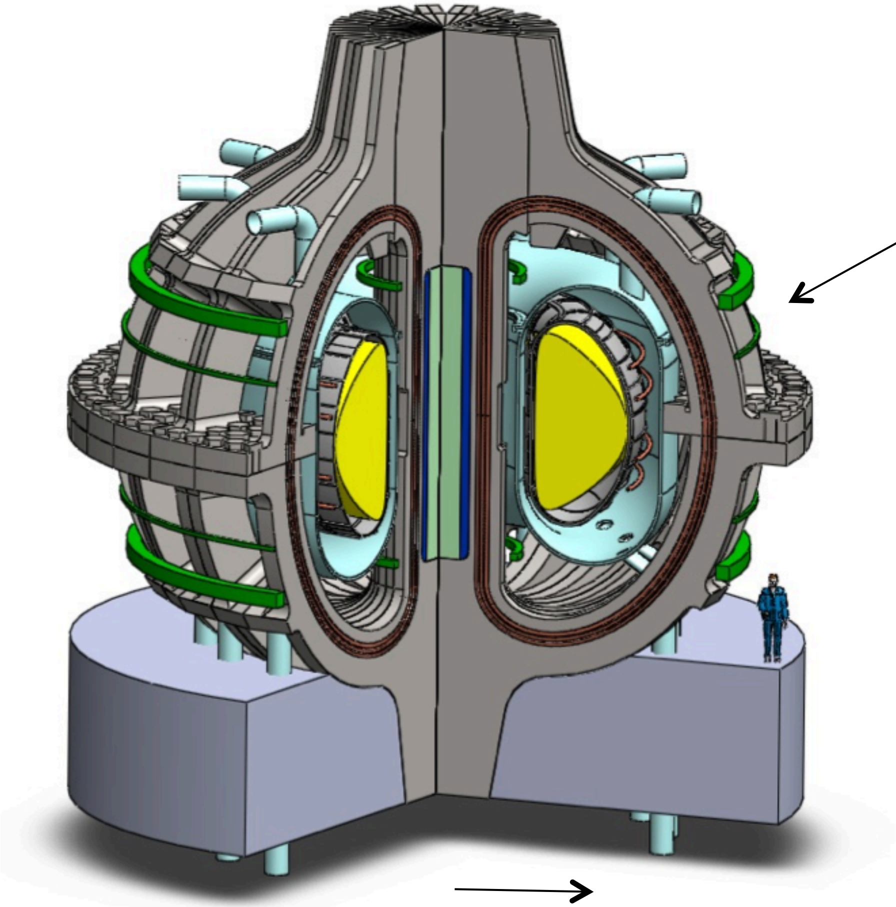
$$p_{magnetic} = \frac{B^2}{2\mu_0}$$

$$p_{mag}[MPa] \simeq 0.4\ B^2$$



The ARC tokamak conceptual design: Example of student-driven innovation!

B. Sorbom et al Fusion Eng Design



Integrated new HT superconductor tape + coil structure

R=3.2 meters

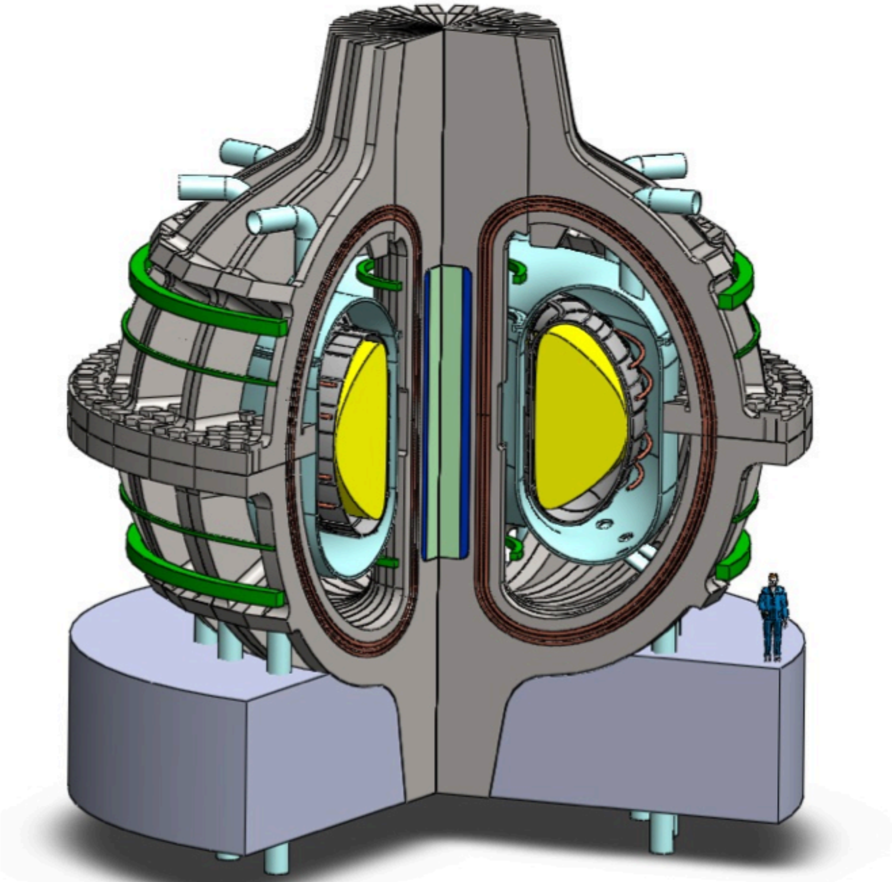
500 million watts fusion power

~200 million watts electricity

~ 4 Million watts per square meter

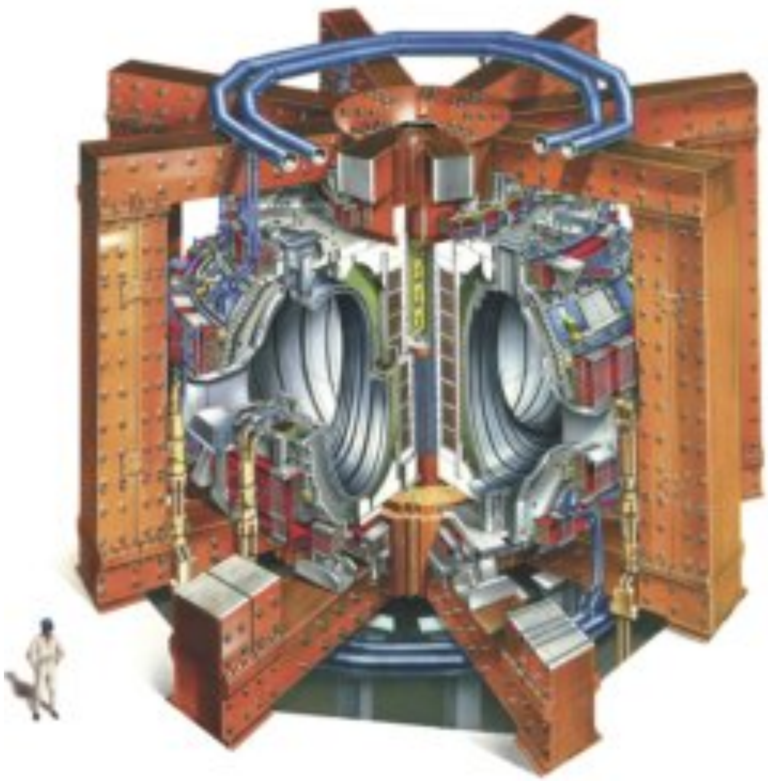
The ARC conceptual design: Fusion power at device scale already developed

B = 9.2 T



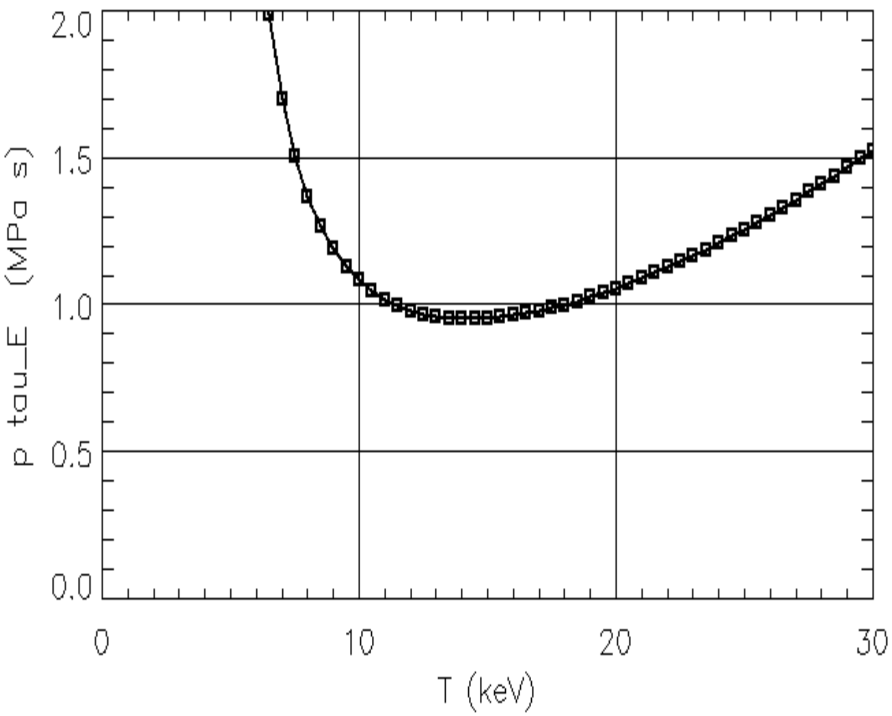
ARC: R ~ 3.2 m

B = 3.5 T



JET (UK): R ~ 3 m

How do we get to these different shapes, sizes, etc.? There are multiple ways to achieve requirements



$$P_\alpha = P_{loss} + P_{brem}$$

Conditions?

p ~ 1 Mpa ~ 10 bar

τ_E ~ 1 second

T ~ 12 keV → n ~ 2x10^{20} m^{-3}

Fusion power density
5-10 MW/m^3

How do we get to these different shapes, sizes, etc.? There are multiple ways to achieve requirements within limits established by MFE research

Fusion power density, pressure

$$P_{fusion}\left[MW/m^3\right] \simeq 7\ p_{MPa}^2$$

$$\beta \equiv \frac{p_{plasma}}{p_{magnetic}} = \frac{p_{MPa}}{0.4\ B^2}$$

$$\beta_{\max} \simeq \beta_N \left(\frac{I_p}{aB_t}\right) \propto \frac{\beta_N \epsilon}{q_*}$$

$$P_{fusion} \sim \frac{\beta_N{}^2 \epsilon^2}{q_*^2} B^4$$

Magnetic winding safety factor

$$q_* \propto \frac{B_T}{B_{pol}} \frac{P_{poloidal}}{P_{toroidal}} \propto \frac{B_T}{B_{pol}} \frac{2\pi a\sqrt{\kappa}}{2\pi R}$$

$$\propto \frac{B_T}{I_p} \frac{a^2\sqrt{\kappa}}{R}$$

Inverse aspect ratio

$$\epsilon \equiv \frac{a}{R}$$

How do we get to these different shapes, sizes, etc.? There are multiple ways to achieve requirements within limits established by MFE research

Energy confinement time from empirical fits

$$\boxed{\tau_E = 0.08\; H\; n_{20}^{0.1}\;\; I_{p,MA}^{0.85}\;\; R^{1.5}\epsilon^{0.3}\;\; \kappa^{0.5}\;\; B^{0.2}\;\; P^{-0.5}}$$

Energy confinement time in terms of global parameters

$$\tau_E = \frac{0.072\; H\; n_{20}^{0.1}\; B^{1.05} R^{1.35}\; \epsilon^{1.5} \left\{\kappa^{0.5}(1+\kappa^2)^{0.6}\right\}}{q^{0.85}\left(\dfrac{P_f}{S}\right)^{1/2}\left(1+5/Q_p\right)^{0.5}}$$

$$\tau_E \propto \frac{H\; B\; R^2 f\left(\epsilon,\kappa\right)}{q\left(P_f/S\right)^{1/2}}$$

Optimization of aspect ratio for a tokamak (and stellarator) is complicated and is a significant topic of present research

Fusion power density

$$\frac{P_f}{S}(\epsilon) \propto [\beta_N(\epsilon)]^2 \epsilon^3 \kappa (1+\kappa^2)^{3/2} \left(1-\epsilon-\frac{\Delta_b}{R}\right)^4 \left(\ln\left(\left(1+\epsilon+\frac{\Delta_b}{R}\right)\left(1-\epsilon-\frac{\Delta_b}{R}\right)^{-1}\right)\right)^{-2}$$

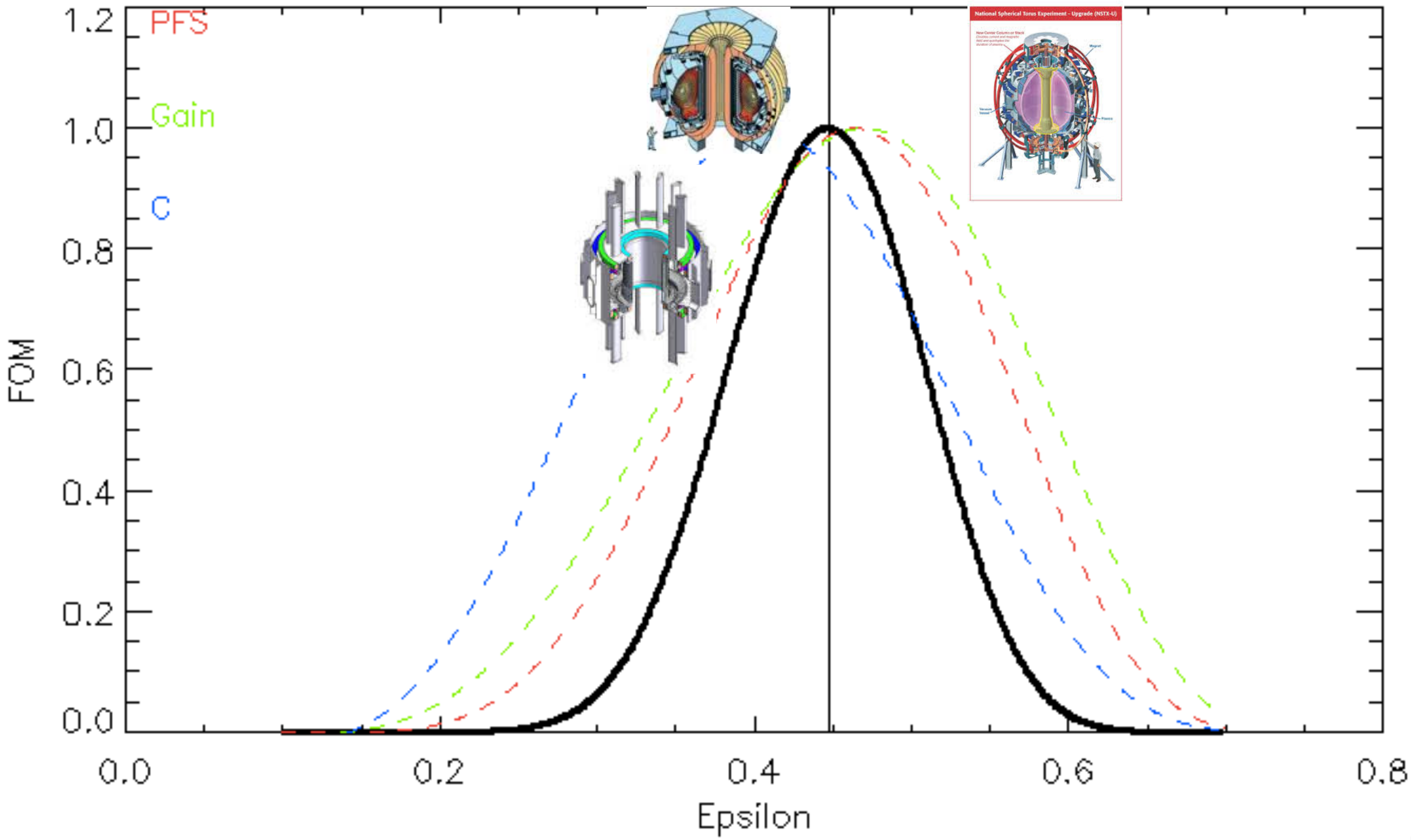
Gain / Lawson

$$p_{th}\tau_E(\epsilon) \propto \beta_N(\epsilon)\epsilon^{2.5}\kappa^{0.5}(1+\kappa^2)^{1.6}\left(1-\epsilon-\frac{\Delta_b}{R}\right)^{3.05}\left(\ln\left(\left(1+\epsilon+\frac{\Delta_b}{R}\right)\left(1-\epsilon-\frac{\Delta_b}{R}\right)^{-1}\right)\right)^{-1.525}$$

Robust SS

$$C \equiv Q\frac{I_{CD}}{I} \propto \beta_N(\epsilon)\epsilon^{5/2}\kappa(1+\kappa^2)(1+\epsilon/2)\left(1-\epsilon-\frac{\Delta_b}{R}\right)^4\left(\ln\left(\left(1+\epsilon+\frac{\Delta_b}{R}\right)\left(1-\epsilon-\frac{\Delta_b}{R}\right)^{-1}\right)\right)^{-2}$$

Example of optimization of aspect ratio: Blanket width / R ~ 0.5 m / 2 m ~ 0.25



Lawson criterion, nuclear fusion rate coefficients and plasma operational limits constrain the requirements for producing an MFE reactor

Lawson Criterion

$$p_{MPa}\tau_E \propto \frac{\beta_N H}{q_*^2} R^{1.3} B^3$$

β = *normalized plasma pressure*
H = *norm. confinement time*
q = *safety factor to instability*

POWER DENSITY

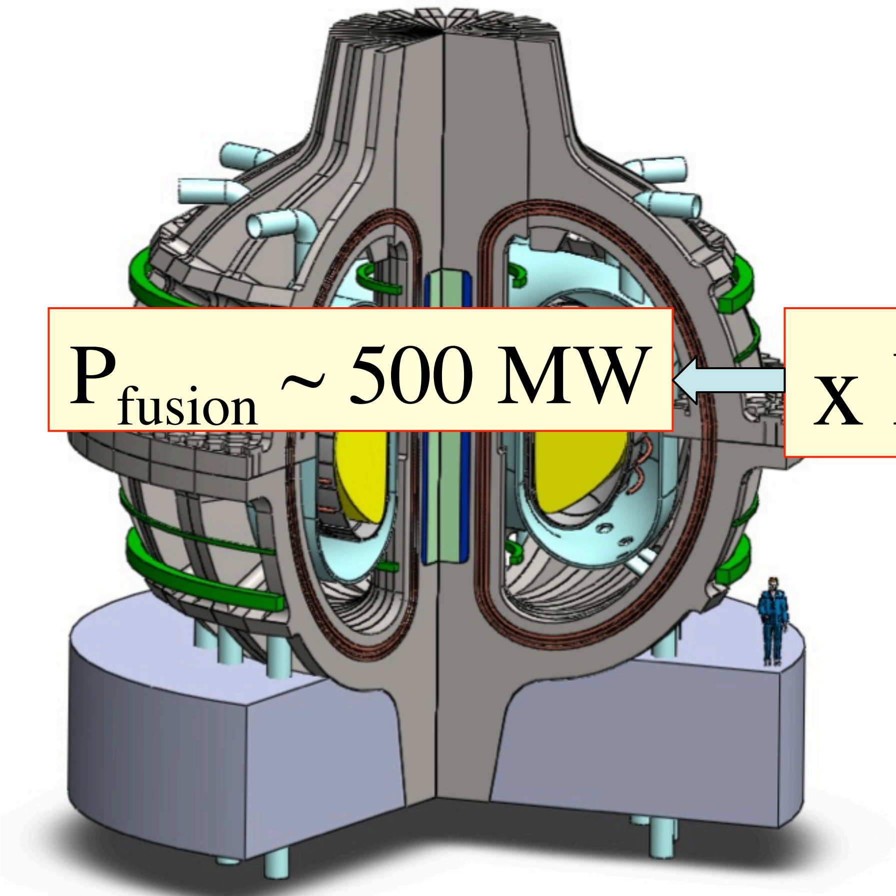
$$\frac{P_{fusion}}{S_{blanket}} \propto \frac{\beta_N^{\ 2}}{q_*^2} R B^4$$

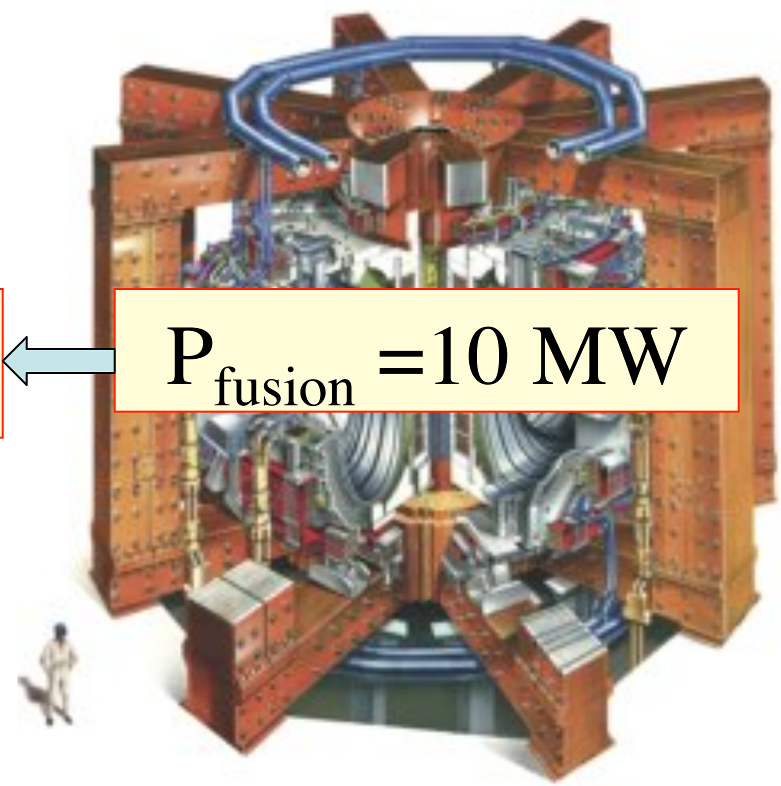
R = *linear size*
B = *magnetic field strength*

The ARC conceptual design: Exploiting improved superconductors

B = 9.2 T

B = 3.5 T





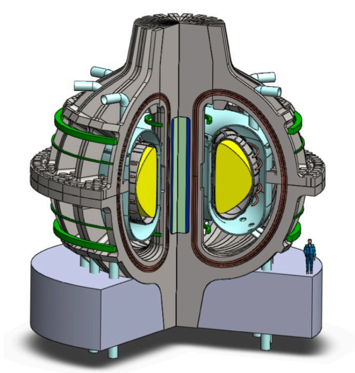
$P_{fusion} \sim 500$ MW ← x B^4 ← $P_{fusion} = 10$ MW

ARC: R ~ 3.2 m

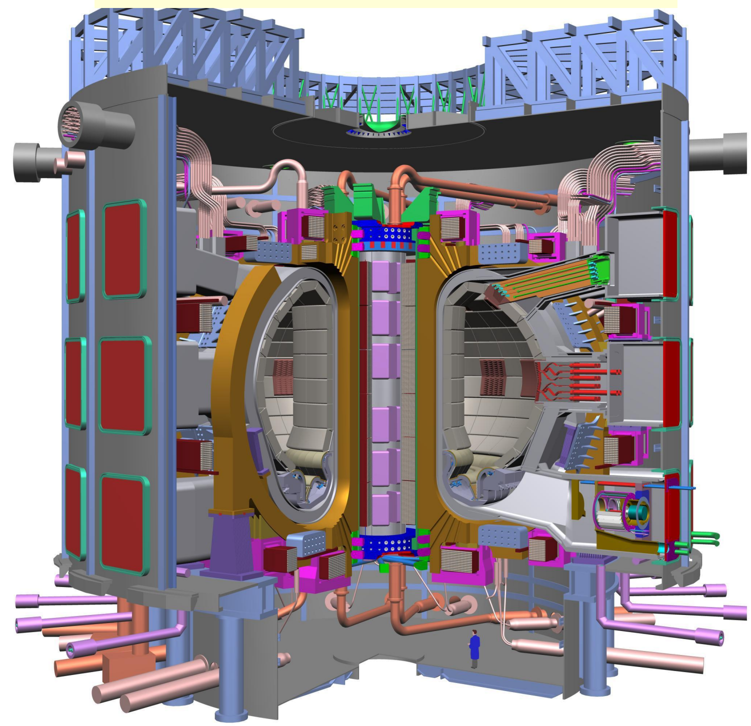
JET (UK): R ~ 3 m

The allowed value of B strongly impacts the size of MFE devices

Fusion power: 500 MW
Q > 10
B = 9.2 T



Fusion power: 500 MW
Q = 10
B= 5.3 T



Summary

- Magnetic fusion uses the Lorentz force to confine a thermonuclear plasma
- D-T fusion cycle is by far the easiest…only requires
  - 1 MPa – s product of plasma pressure and energy confinement time
  - ~1 MPa pressure produces ~ 10 MW/m^3
- Plasma physics is a necessary field of study for fusion but does not provide fusion energy by itself
  - Nuclear engineering, superconductor coils, materials
- <u>Significant</u> optimization of MFE still to be done in terms of geometry, B technology and nuclear engineering.