SULI Introductory Course in Plasma Physics

Plasma Astrophysics

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The Plasma Universe

- In most of the universe, plasma is the "first' state of matter, and by a very large margin. Out there, fully intact atoms are much rarer. Practically all the visible contents of the cosmos – not just stars, but even regions of interstellar dust containing barely a million particles per cubic meter – are in a plasma state.
- *Tools of discovery*

  __ Light (photons) from space- and ground-based instruments

  __ *In situ* satellites

  __ Theoretical models and computer simulations

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Equations of MHD

Newton

$$\frac{m\mathbf{a}}{vol} = \frac{\mathbf{F}}{vol} \longrightarrow \rho \frac{d\mathbf{v}}{dt} = -\nabla P + \mathbf{J} \times \mathbf{B}$$

$$\rho\left(\frac{\partial \mathbf{v}}{\partial t} + \mathbf{v} \cdot \nabla \mathbf{v}\right) = -\nabla P + \mathbf{J} \times \mathbf{B}$$

Ohm's Law:

$$V = IR \longrightarrow \mathbf{E}' = \mathbf{E} + \mathbf{v} \times \mathbf{B} = \eta \mathbf{J}$$

Maxwell's equations:

$$\nabla \times \mathbf{B} = \mu_0 \mathbf{J} \quad \text{and} \quad \nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t}$$

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What is Magnetic Reconnection?

If a plasma is perfectly conducting, that is, it obeys the ideal Ohm's law,

$$\mathbf{E} + \mathbf{v} \times \mathbf{B}/c = 0$$

B-lines are frozen in the plasma, and no reconnection occurs.

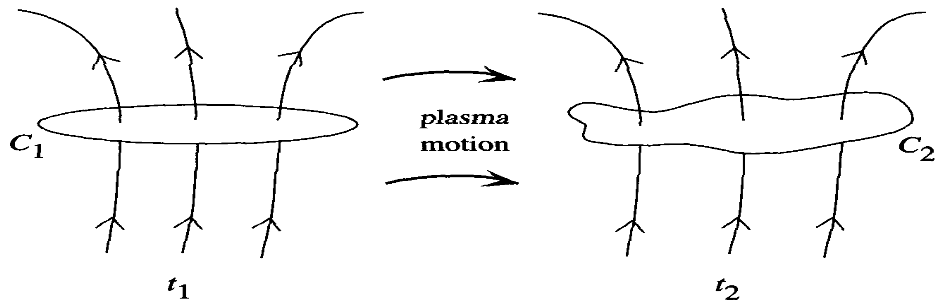

Fig. 1.6. Magnetic flux conservation: if a curve C_1 is distorted into C_2 by plasma motion, the flux through C_1 at t_1 equals the flux through C_2 at t_2.

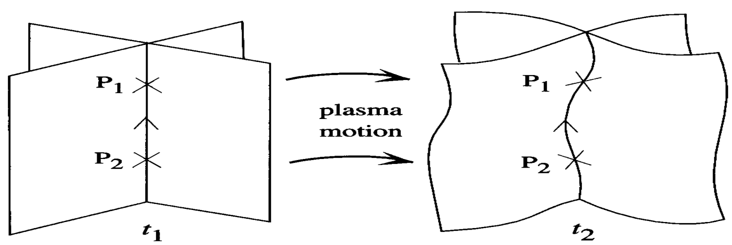

Fig. 1.7. Magnetic field-line conservation: if plasma elements P_1 and P_2 lie on a field line at time t_1, then they will lie on the same line at a later time t_2.

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Magnetic Reconnection: Working Definition

Departures from ideal behavior, represented by

$$\mathbf{E} + \mathbf{v} \times \mathbf{B}/c = \mathbf{R}, \quad \nabla \times \mathbf{R} \neq \mathbf{0}$$

allow field lines to break and reconnect.

In the generalized Ohm's law for weakly collisional or collisionless plasmas, **R** contains resistivity, Hall current, electron inertia and pressure.

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Magnetic Island Formation Due To Reconnection

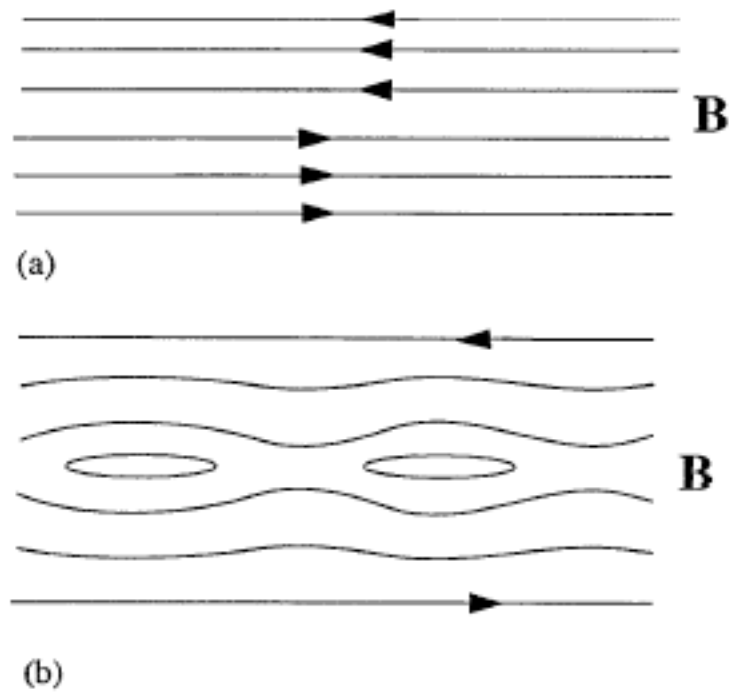

FIG. 1. (a) The topology of field lines in the Harris equilibrium $\mathbf{B}=B_0 \tanh(z/a)\hat{\mathbf{x}}$. (b) The topology of field lines when the perturbation $\mathbf{b}=b \sin(kx)\hat{\mathbf{z}}$ is imposed on the Harris equilibrium.

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Why is magnetic reconnection important?

- Magnetic reconnection enables a system to access states of lower energy by topological relaxation of the magnetic field. The energy thus liberated can be converted to the kinetic energy of particles and heat. Since the Universe is permeated by magnetic fields (in Nature and the laboratory), magnetic reconnection is a ubiquitous mechanism wherever such phenomena occur, including eruptive stellar/solar flares, magnetospheric storms, and disruptions in fusion plasmas.
- By allowing small-scale, tangled fields to reconnect and forming larger-scale fields, reconnection plays a critical role in the "dynamo effect"----the mechanism most widely invoked on how large-scale magnetic fields in the Universe are spontaneously generated from various types of plasma turbulence. Understanding of fast reconnection is central to the question: "Why is the Universe magnetized?"

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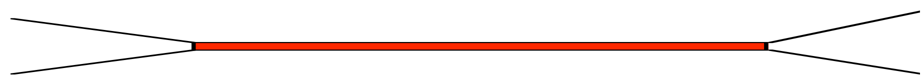
Classical (2D) Steady-State Models of Reconnection

Sweet-Parker [Sweet 1958, Parker 1957]



Geometry of reconnection layer : Y-points

Length of the reconnection layer is of the order of the system size >> width Δ

Reconnection time scale

$$\tau_{SP} = \left(\tau_A \tau_R\right)^{1/2} = S^{1/2}\tau_A$$

Solar flares: $S \sim 10^{12}, \quad \tau_A \sim 1s$

$$\Rightarrow \tau_{SP} \sim 10^{6}\, s$$

Too long to account for solar flares!

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Impulsive Reconnection: The Onset/Trigger Problem

Dynamics exhibits an impulsiveness, that is, a sudden change in the time-derivative of the reconnection rate.

The magnetic configuration evolves slowly for a long period of time, only to undergo a sudden dynamical change over a much shorter period of time.

Dynamics is characterized by the formation of near-singular current sheets which need to be resolved in computer simulations: a classic multi-scale problem coupling large scales to small.

Examples

Magnetospheric substorms

Impulsive solar/stellar flares

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Substorms in the Earth's Magnetosphere

Photograph of the aurora by Dirk Obudzinski, near the Yukon River in Circle, Alaska (August 29, 2000).

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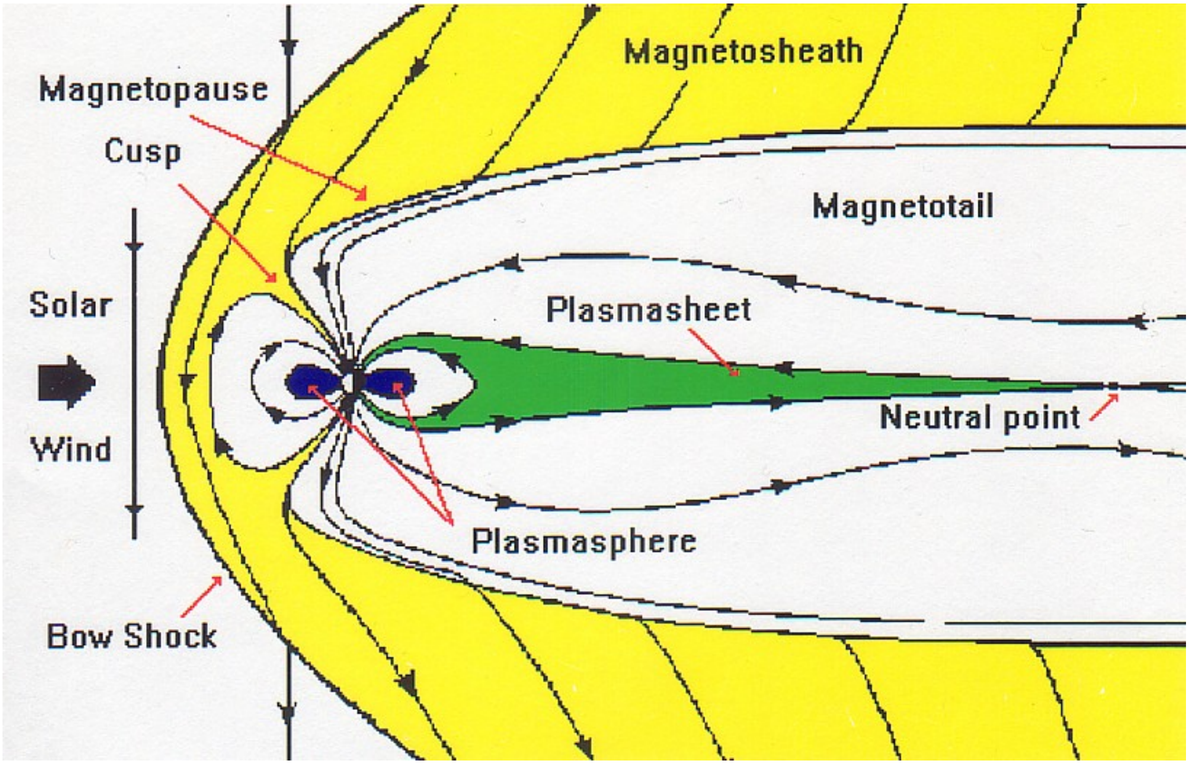
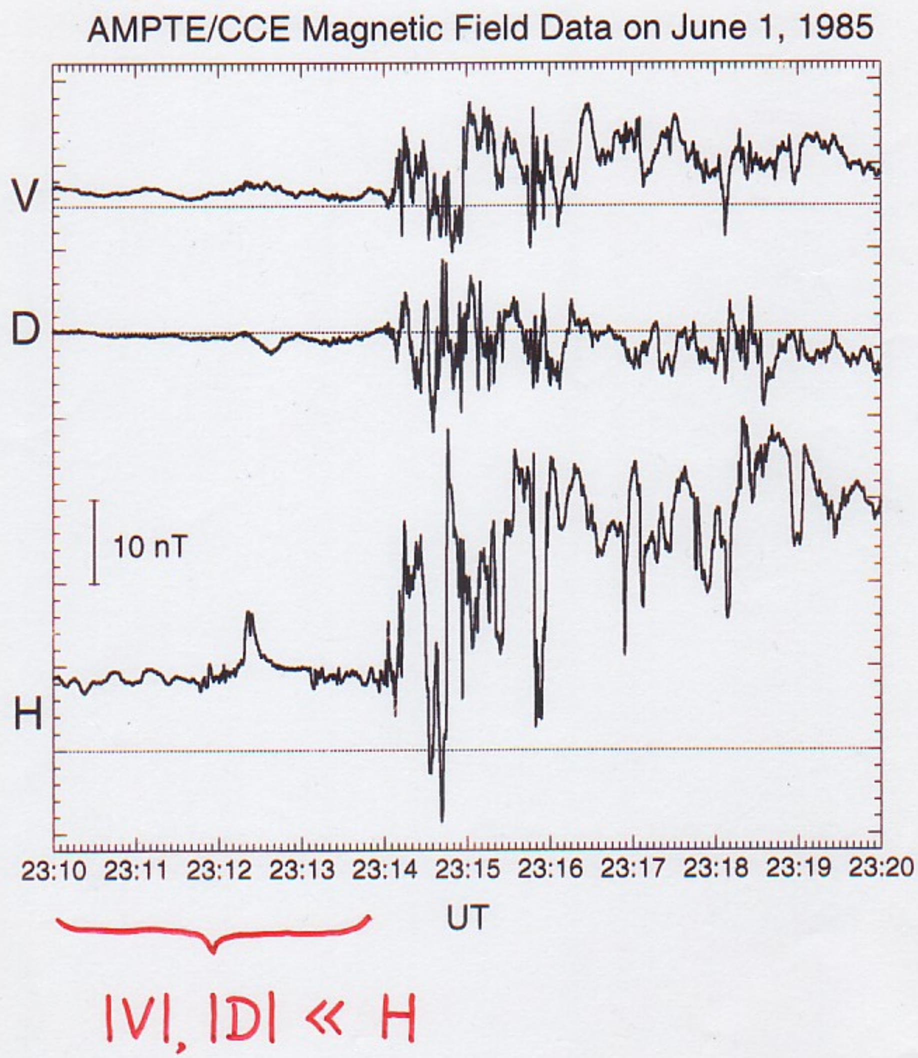
Substorms : a cartoon



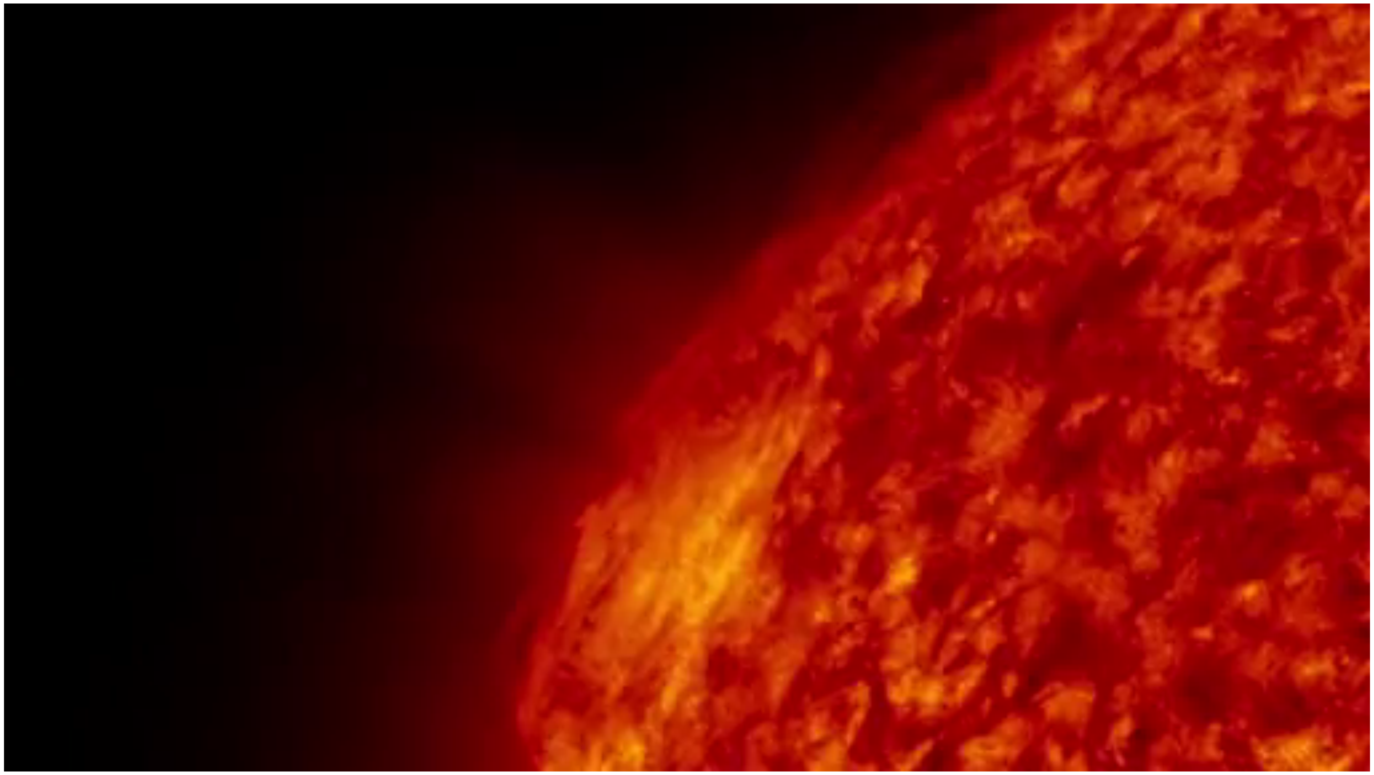
R = 8.79 R_E

MLT = 0.27



See Lui et al. [1988, 1992]

The Flaring Sun



Courtesy: The Solar Dynamics Observatory

Generalized Ohm's law

What's really important?

$$\mathbf{E} = \underbrace{-\mathbf{v}_i \times \mathbf{B}}_{\text{(i)}} \underbrace{- \frac{m_e}{e}\frac{d\mathbf{v}_e}{dt}}_{\text{(ii)}} \underbrace{- \frac{1}{en_e}\nabla\cdot \overleftrightarrow{P}_e}_{\text{(iii)}} + \underbrace{\frac{1}{en_e}\mathbf{J}\times\mathbf{B}}_{\text{(iv)}} + \underbrace{\eta_e \mathbf{J}}_{\text{(v)}}$$

$$\frac{\text{(i)}}{\text{(v)}} = \frac{vB}{\eta_e B/\mu_0 \ell} = \frac{\ell v \mu_0}{\eta_e} \equiv R_m$$

$$\frac{\text{(i)}}{\text{(ii)}} = \frac{vB}{m_e v B/\mu_0 e^2 n_e \ell^2} = \frac{\ell^2}{m_e/\mu_0 e^2 n_e} = \left(\frac{\ell}{c/\omega_{pe}}\right)^2$$

$$\frac{\text{(i)}}{\text{(iv)}} = \frac{vB}{B^2/\mu_0 e n_e \ell} = \frac{v\ell}{B/\mu_0 e n_e} = \left(\frac{v}{v_A}\right)\left(\frac{\ell}{c/\omega_{pi}}\right)$$

$$\frac{\text{(i)}}{\text{(iii)}} = \frac{vB}{p_e/e n_e \ell} = \frac{v\ell}{k_B T/eB} = \left(\frac{v}{c_s}\right)\left(\frac{\ell}{\rho_i}\right)$$

Importance of term depends on length scale of solution

On large scales plasma is ideal conductor

Hall MHD Model and the Generalized Ohm's Law

In high-S plasmas, when the width Δ_η of the thin current sheet satisfies

$$\Delta_\eta < c/\omega_{pi}$$

"collisionless" terms in the generalized Ohm's law cannot be ignored.

Generalized Ohm's law (dimensionless form)

$$\mathbf{E} + \mathbf{v} \times \mathbf{B} = \frac{1}{S}\mathbf{J} + d_e{}^2 \frac{d\mathbf{J}}{dt} + \frac{d_i}{n}\left(\mathbf{J} \times \mathbf{B} - \nabla \bullet \overleftrightarrow{p}_e\right)$$

Electron skin depth	$d_e \equiv L^{-1}\left(c/\omega_{pe}\right)$
Ion skin depth	$d_i \equiv L^{-1}\left(c/\omega_{pi}\right)$
Electron beta	β_e

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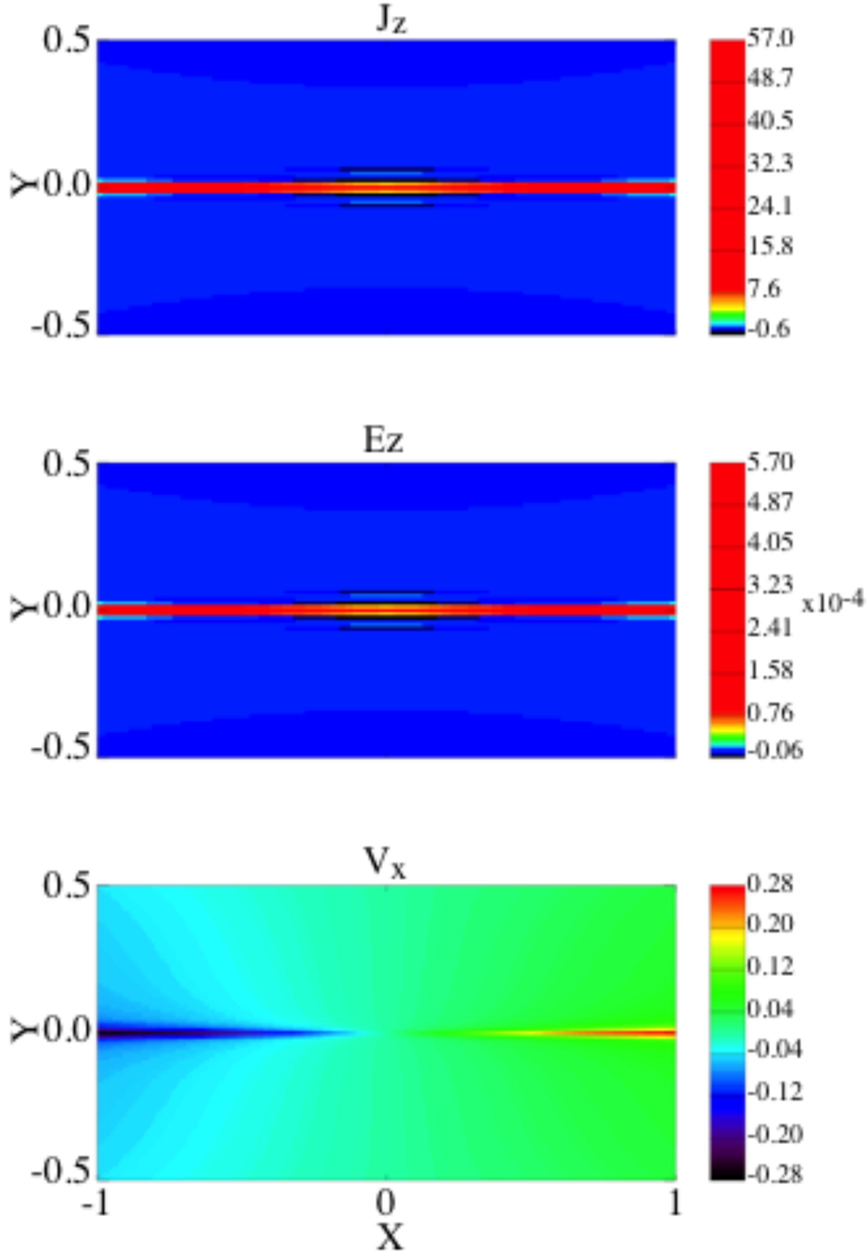
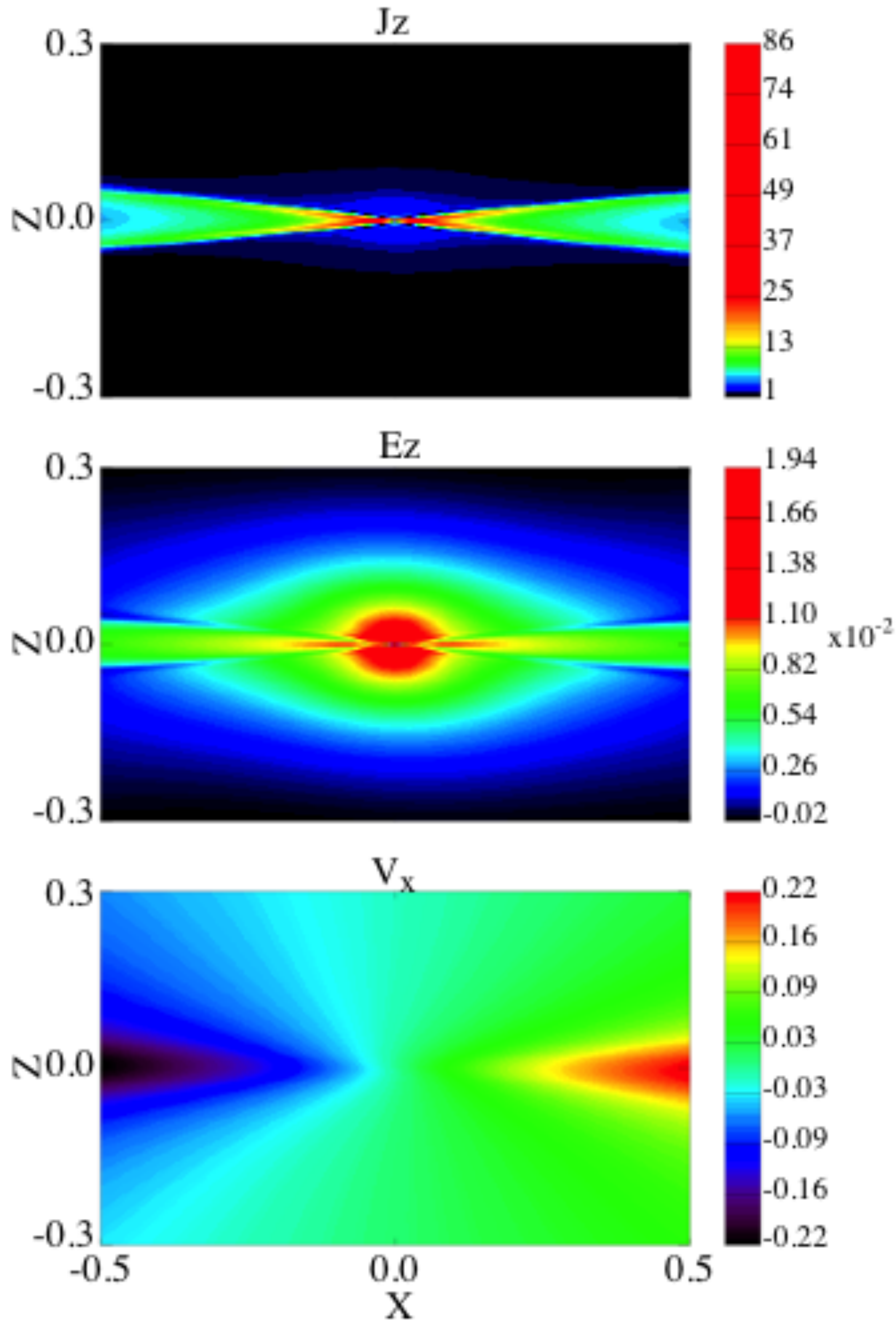


Jz
0.5
Y 0.0
-0.5
57.0
48.7
40.5
32.3
24.1
15.8
7.6
-0.6
Ez
0.5
Y 0.0
-0.5
5.70
4.87
4.05
3.23
2.41
1.58
0.76
-0.06
x10-4
Vx
0.5
Y 0.0
-0.5
0.28
0.20
0.12
0.04
-0.04
-0.12
-0.20
-0.28
-1
0
1
X



Jz
0.3
Z0.0
-0.3
86
74
61
49
37
25
13
1
Ez
0.3
Z0.0
-0.3
1.94
1.66
1.38
1.10
0.82
0.54
0.26
-0.02
x10^-2
Vx
0.3
Z0.0
-0.3
0.22
0.16
0.09
0.03
-0.03
-0.09
-0.16
-0.22
-0.5
0.0
0.5
X

Transition from Collisional to Collisionless Regimes in MRX

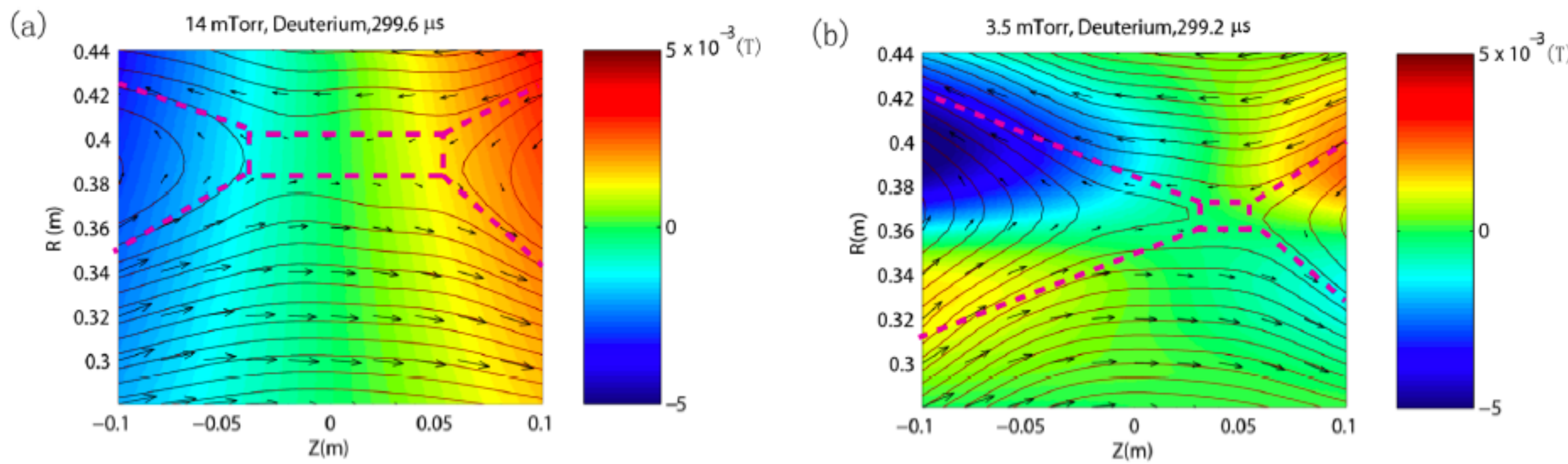


***Fig.2**; Comparison of neutral sheet configuration described by measured magnetic field vectors and flux counters for high (collisional) and low density cases; (a) Collisional regime (λ_{mfp} ~1mm << δ)); (b) Nearly collisionless regime (λ_{mfp} ~1cm ~ δ). Out-of plane fields are depicted by the color codes ranged -50 G <Bt <50 G.*

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Linkage between space and laboratory plasmas

System	L (cm)	B (G)	$d_i = c/\omega_{pi}$(cm)	δ_{sp} (cm)	d_i/δ_{sp}
MRX	10	100-500	1-5	0.1-5	.2-100
Tokamak	100	10^4	10	0.1	100
Magnetosphere	10^9	10^{-3}	10^7	10^4	1000
Solar flare	10^9	100	10^4	10^2	100
ISM	10^{18}	10^{-6}	10^7	10^{10}	0.001
Proto-star	$d_i/\delta_s >> 1$				

$$d_i/\delta_{sp} \sim 5(\lambda_{mfp}/L)^{1/2}$$

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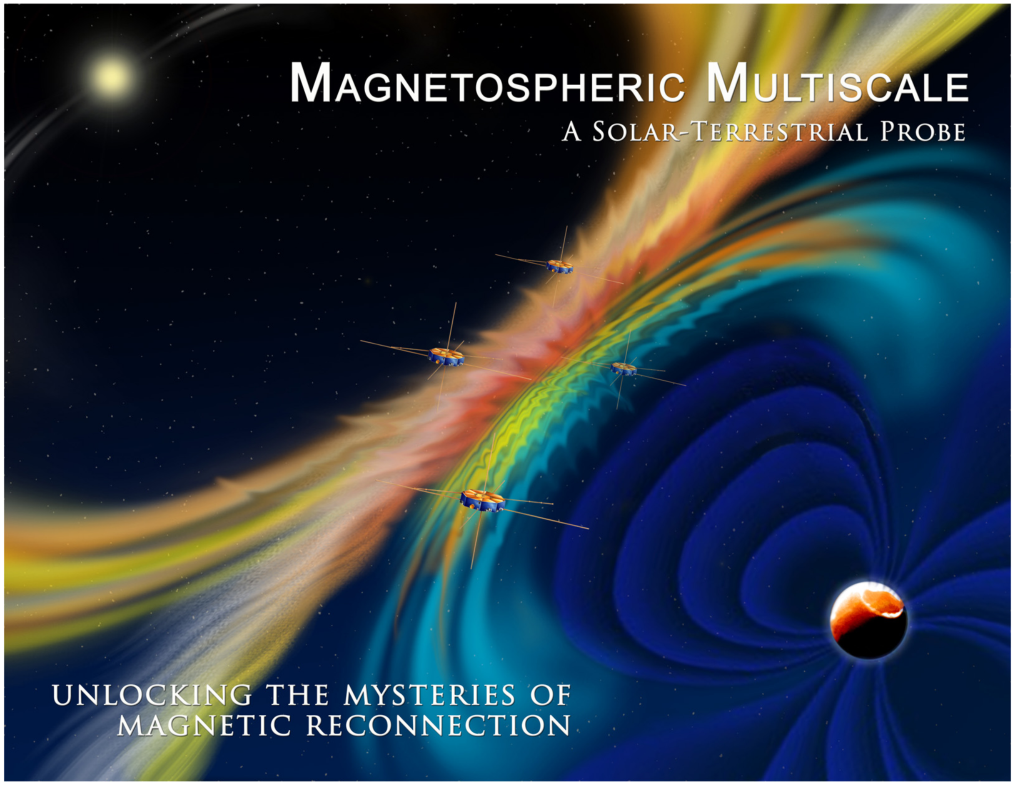
Magnetospheric Multiscale Mission

- The MMS Mission science will be conducted by the SMART (Solving Magnetospheric Acceleration, Reconnection and Turbulence) Instrument Suite Science Team and a group of three Interdiscliplinary Science (IDS) teams.
- Launched in March 2015.

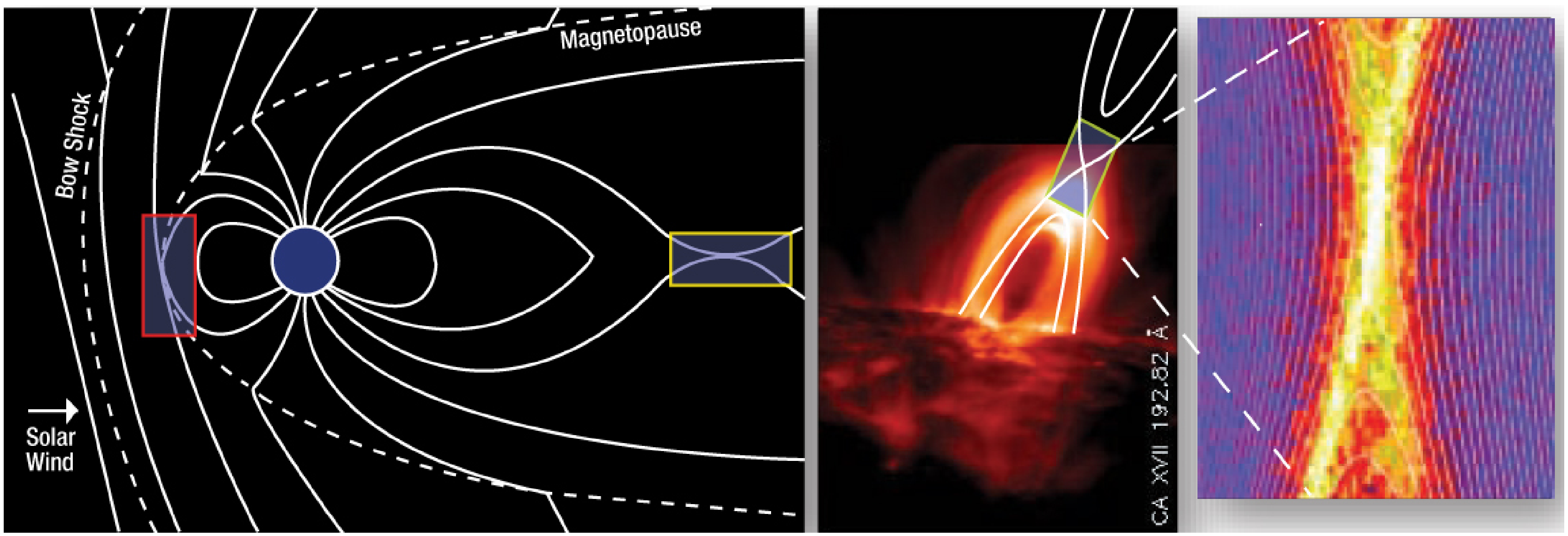
http://mms.space.swri.edu



(Courtesy: J. Burch, SWRI)

A Fundamental Universal Process

(a) (b) (c)



Magnetic reconnection is important in the (a) Earth's magnetosphere, (b) in the solar corona (solar flares and CMEs) and throughout the universe (high energy particle acceleration). Simulations (c) guide the MMS measurement strategy.

(Courtesy: J. Burch, SWRI)

Plasmoid Instability of *Large-Scale* Current Sheets

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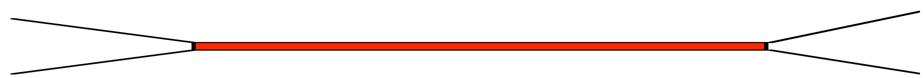

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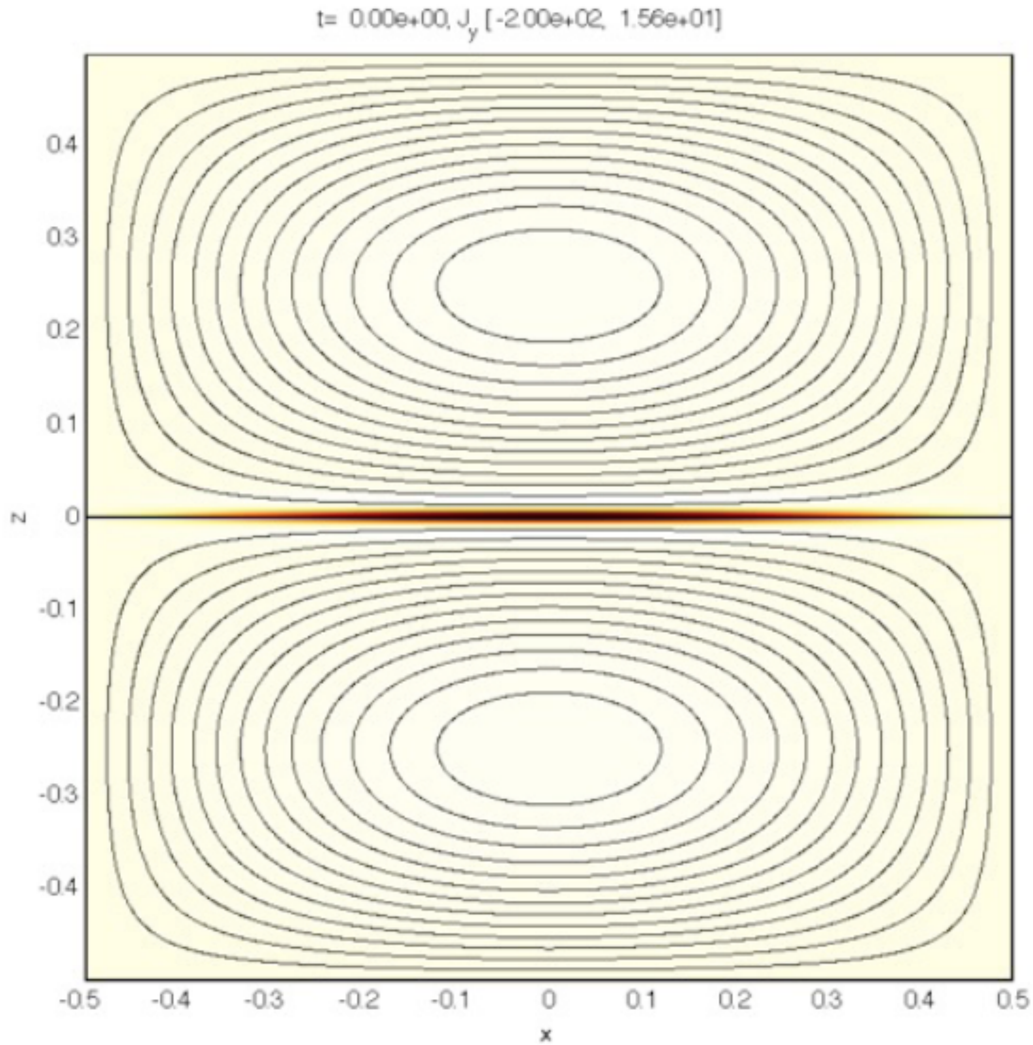
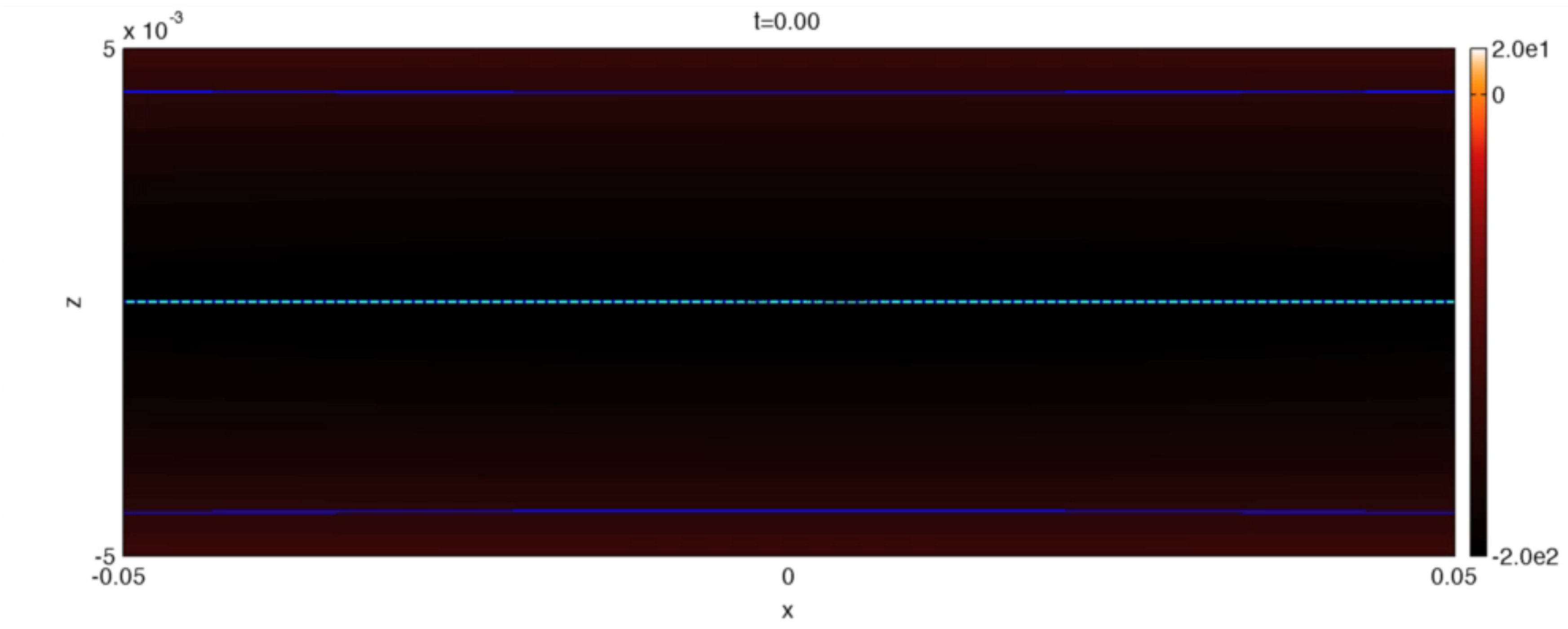


A Low Amplitude Random Forcing is Added

$$\partial_t(\rho\mathbf{v}) + \nabla\cdot(\rho\mathbf{v}\mathbf{v}) = -\nabla p - \nabla\psi\nabla^2\psi + \epsilon\mathbf{f}(\mathbf{x},t)$$

$$\langle f_i(\mathbf{x},t)f_j(\mathbf{x}',t')\rangle \sim \delta_{ij}\delta(\mathbf{x}-\mathbf{x}')\delta(t-t')$$

The thin current sheet is explosively stable over a critical Lundquist number, forming, ejecting, and coalescing a hierarchy of plasmoids.



B., Huang, Rogers and Yang 2009, Huang and B. 2010

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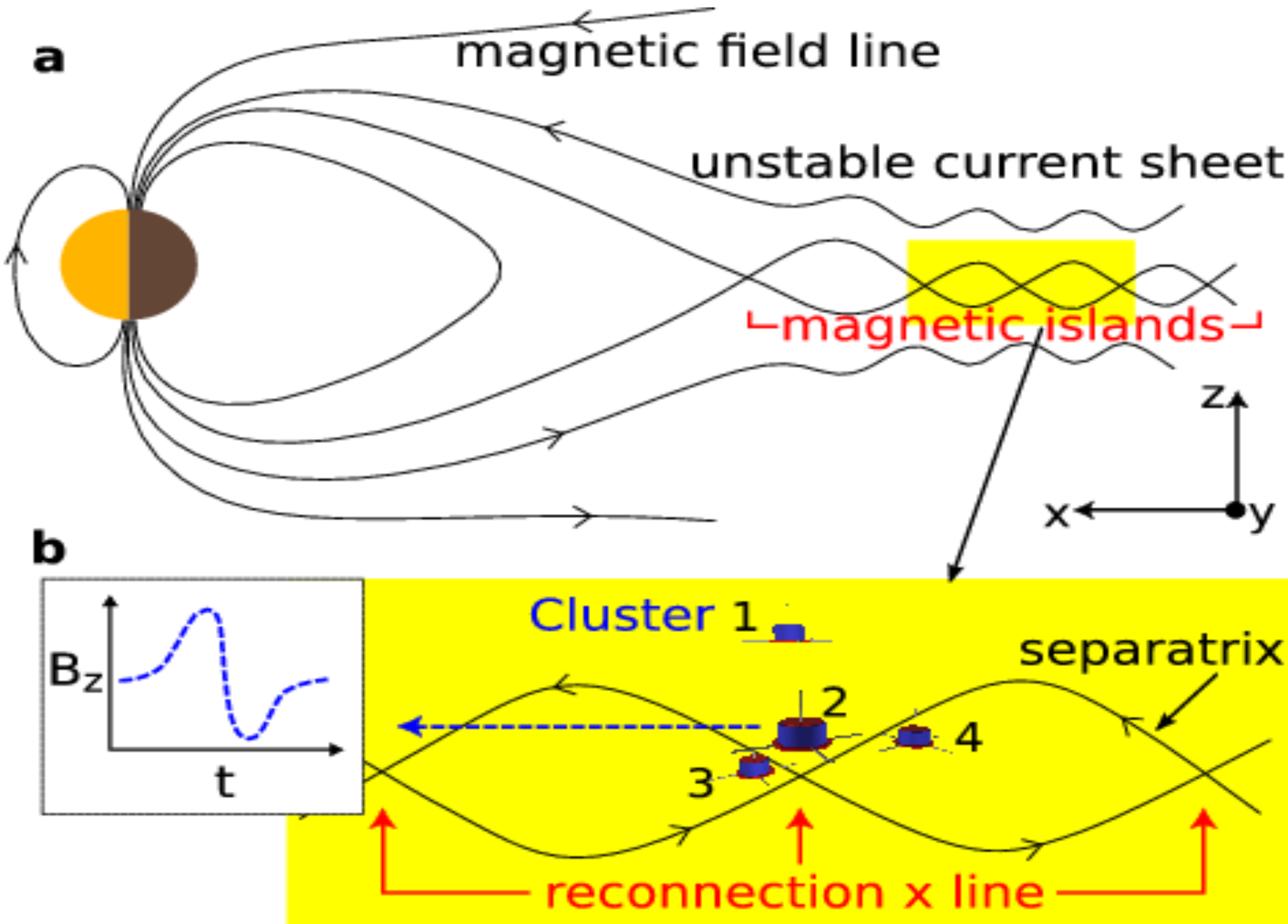
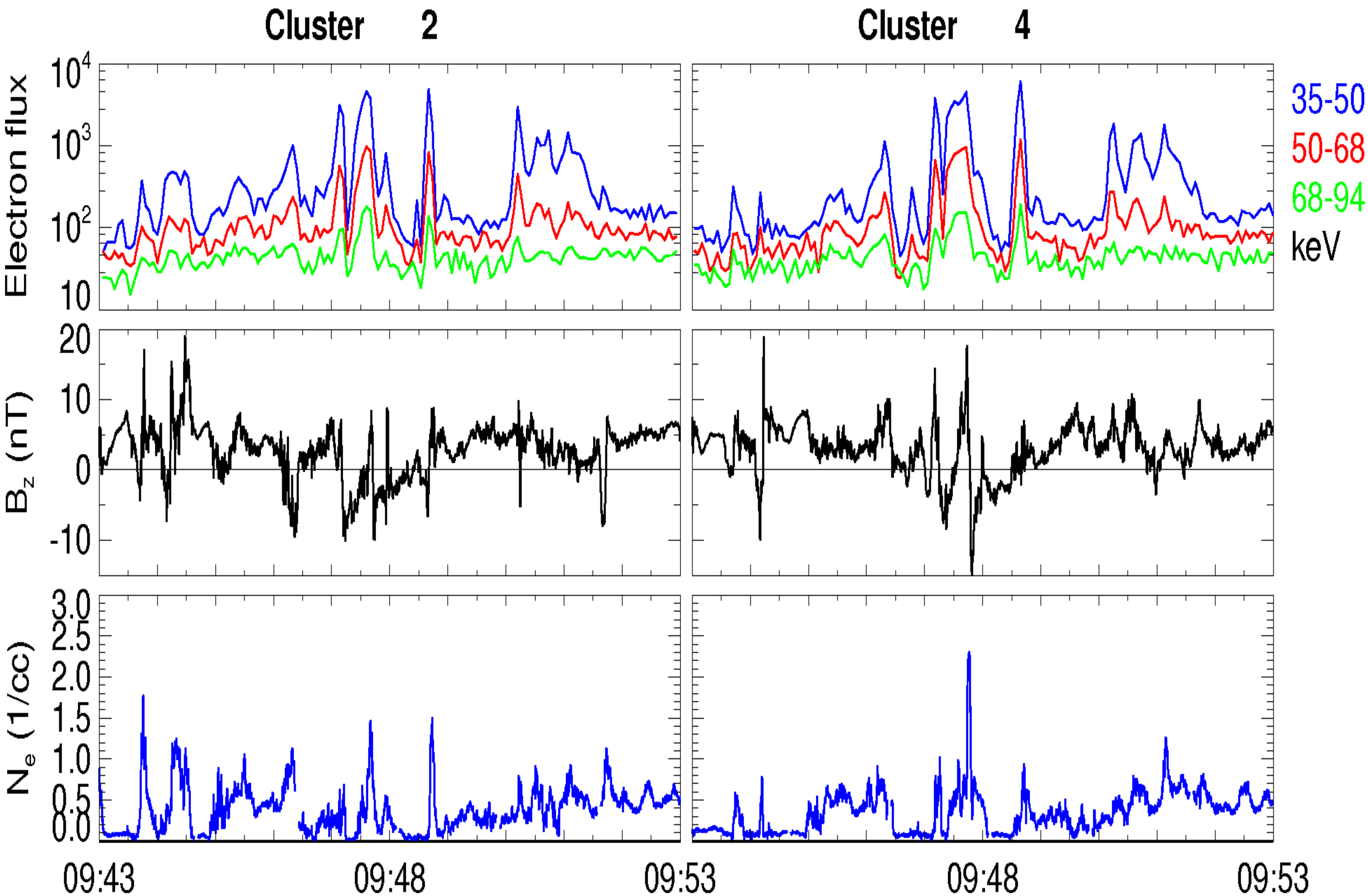
Fluxes of energetic electrons peak within magnetic islands

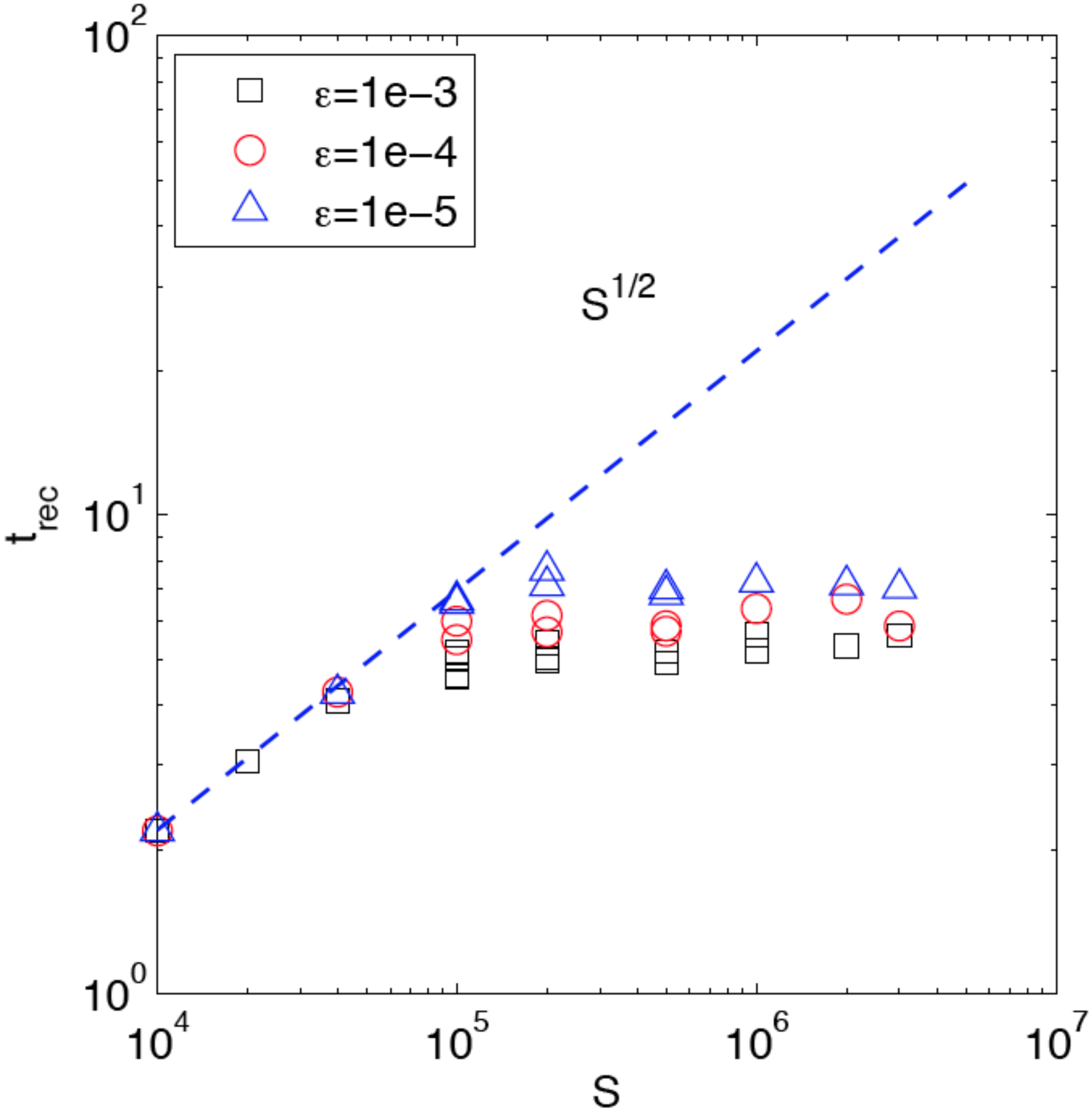
[Chen et al., Nature Physics, 2008]



e bursts & bipolar Bz & Ne peaks
~10 islands within 10 minutes



Reconnection Time of 25% of Initial Flux



$$< \frac{1}{V_A B}\frac{d\psi}{dt} > \sim 0.01$$

$$< u_i > \sim 0.01 V_A$$

What is Turbulence?

- Webster's 1913 Dictionary: "*The quality or state of being turbulent; a disturbed state; tumult; disorder, agitation.*"
- Alexandre J. Chorin, Lectures on Turbulence Theory (Publish or Perish, Inc., Boston 1975): "*The distinguishing feature of turbulent flow is that its velocity field appears to be random and varies unpredictably. The flow does, however, satisfy a set of……equations, which are not random. This contrast is the source of much of what is interesting in turbulence theory.*"

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Leonardo da Vinci (1500)

Navier-Stokes Equation: Fundamental Equation for Fluid Turbulence

$$\frac{\partial v}{\partial t} + v \cdot \nabla v = -\nabla p + R^{-1}\nabla^2 v + F$$

$$\nabla \cdot v = 0$$

R (= LV/viscosity) is called the Reynolds number.

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Turbulence: Spatial Characteristics

- Turbulence couples large scales and small scales.
- The process of development of turbulence often starts out as large-scale motion by the excitation of waves of long wavelength that quickly produces waves of small wavelength by a domino effect.
- Wavelength and wavenumber

$$\lambda, \; k = 2\pi / \lambda$$

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Millennium Prize Problems

P versus NP

The Hodge Conjecture

The Poincaré Conjecture

The Riemann Hypothesis

Yang–Mills Existence and Mass Gap

Navier–Stokes Existence and Smoothness

The Birch and Swinnerton–Dyer Conjecture

Announced 16:00, on Wednesday, May 24, 2000
Collège de France

Statement from the Directors and Scientific Advisory Board

Rules for the CMI Millennium Prize Problems

Historical Context

Press Statement

Press Reaction

Remarks

Information US: +1 617 868-8277 (Clay Mathematics Institute)

Hydrodynamic turbulence

Kolmogorov (1941): can get energy spectrum by dimensional analysis

Assumptions:
- isotropy
- local interaction in k-space
  (energy moves from one k-shell to the next)

Energy cascade rate: $\varepsilon(k) \propto k^{\alpha} \underbrace{E(k)}^{\beta}$ = constant

Energy spectrum: $\int E(k)dk =$ total energy

$$\Rightarrow \alpha = 5/2, \quad \beta = 3/2$$

Kolmogorov spectrum: $E(k) \sim C_{\mathrm{K}} \varepsilon^{2/3} k^{-5/3}$

Kolmogorov constant: $C_{\mathrm{K}} \sim 1.4 - 2$

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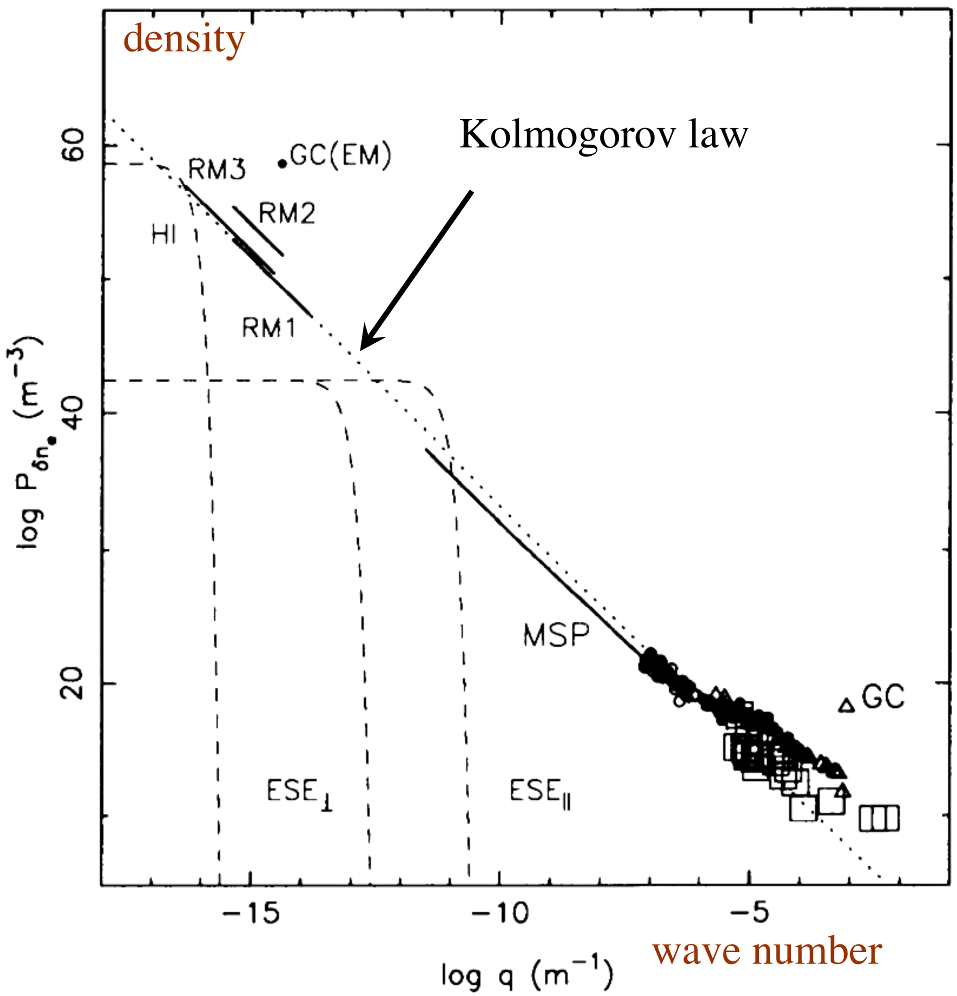
Interstellar turbulence

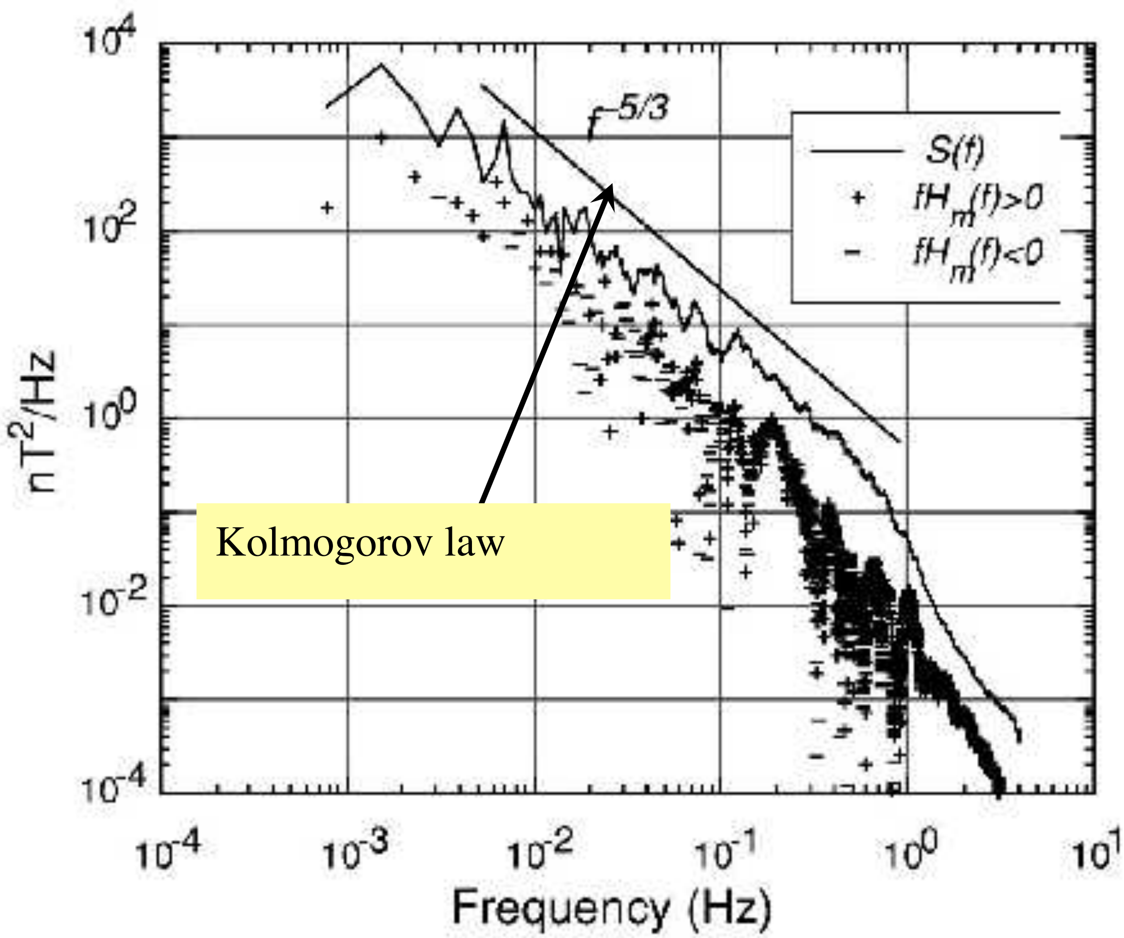
Observation:
power law relation
between electron
density spectrum
and spatial scales



From Cordes (1999)

Solar wind turbulence

Observation: power law in magnetic energy spectrum



From Goldstein & Roberts (1999)

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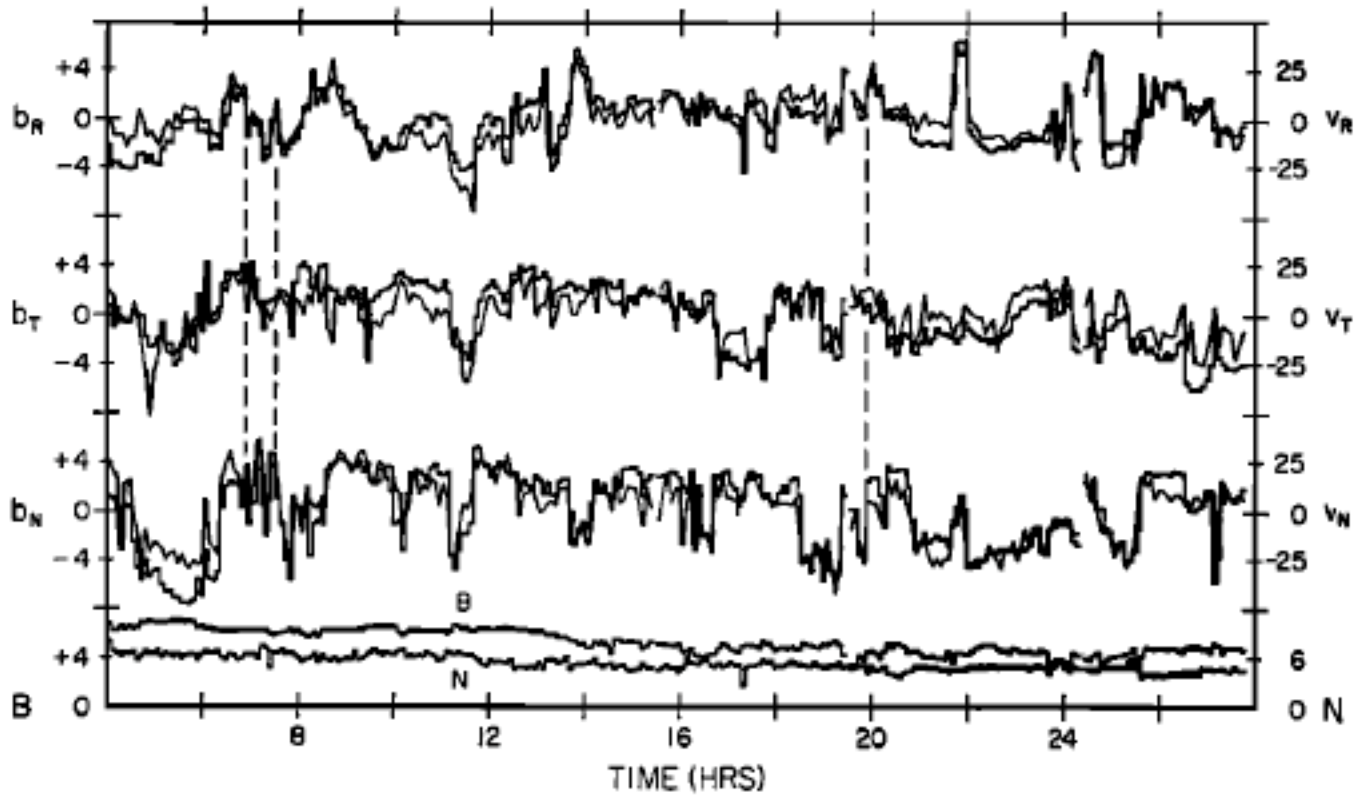
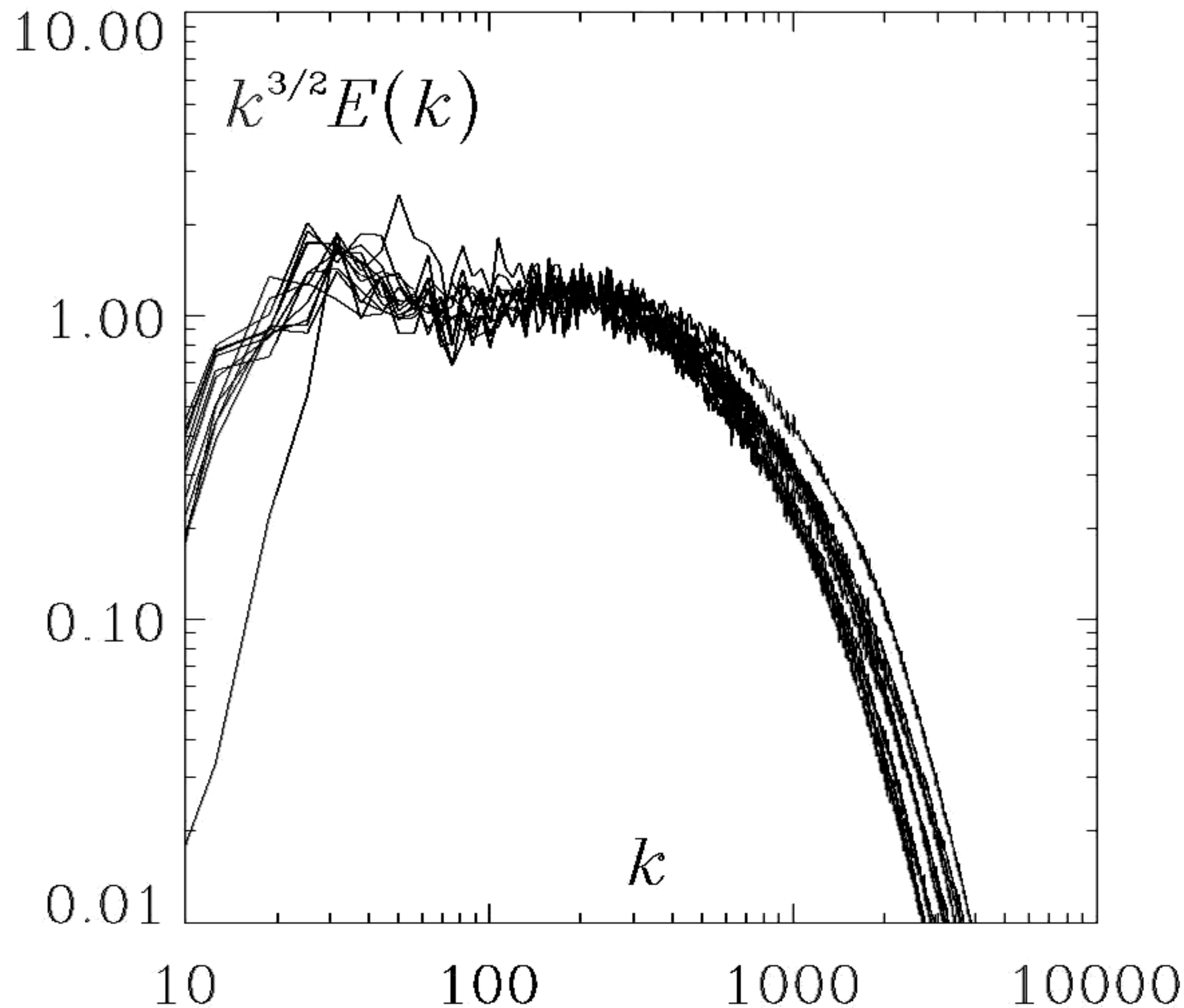
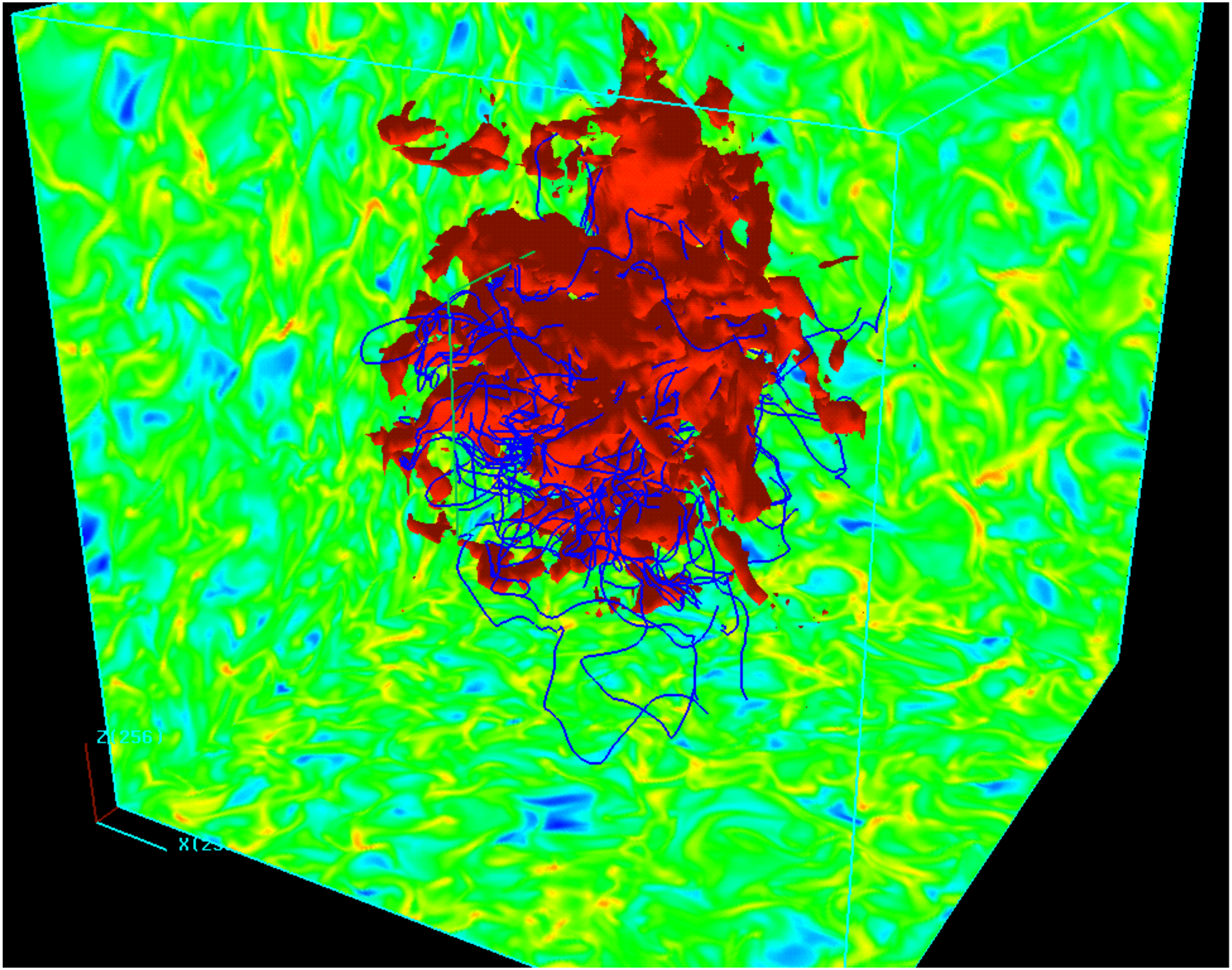


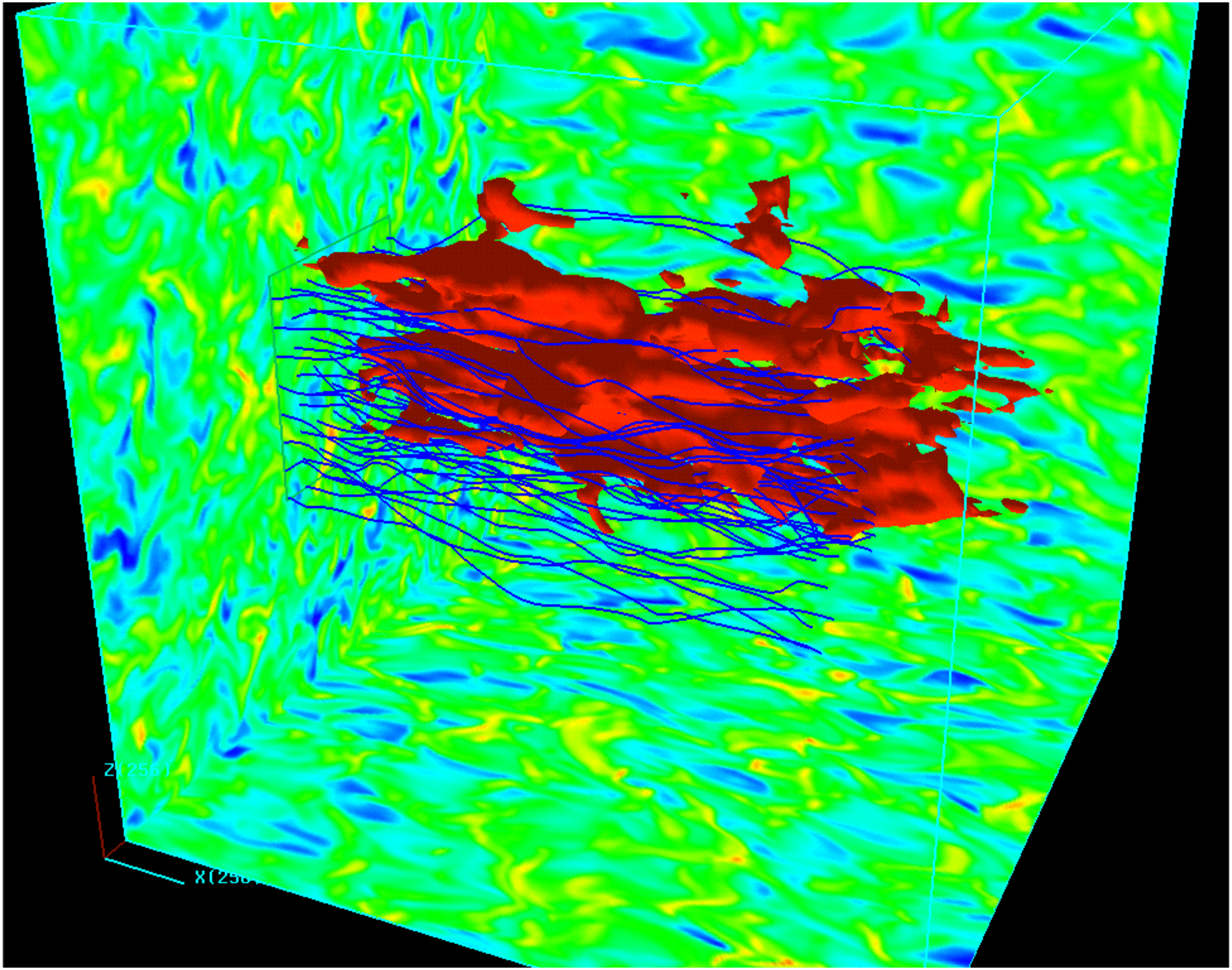
Fig. 1. Twenty-four hours of magnetic field and plasma data demonstrating the presence of nearly pure Alfvén waves. The upper six curves are 5.04-min bulk velocity components in km/sec (diagonal lines) and magnetic field components averaged over the plasma probe sampling period, in gammas (horizontal and vertical lines). The lower two curves are magnetic field strength and proton number density.

$k^{-3/2}$ energy spectra

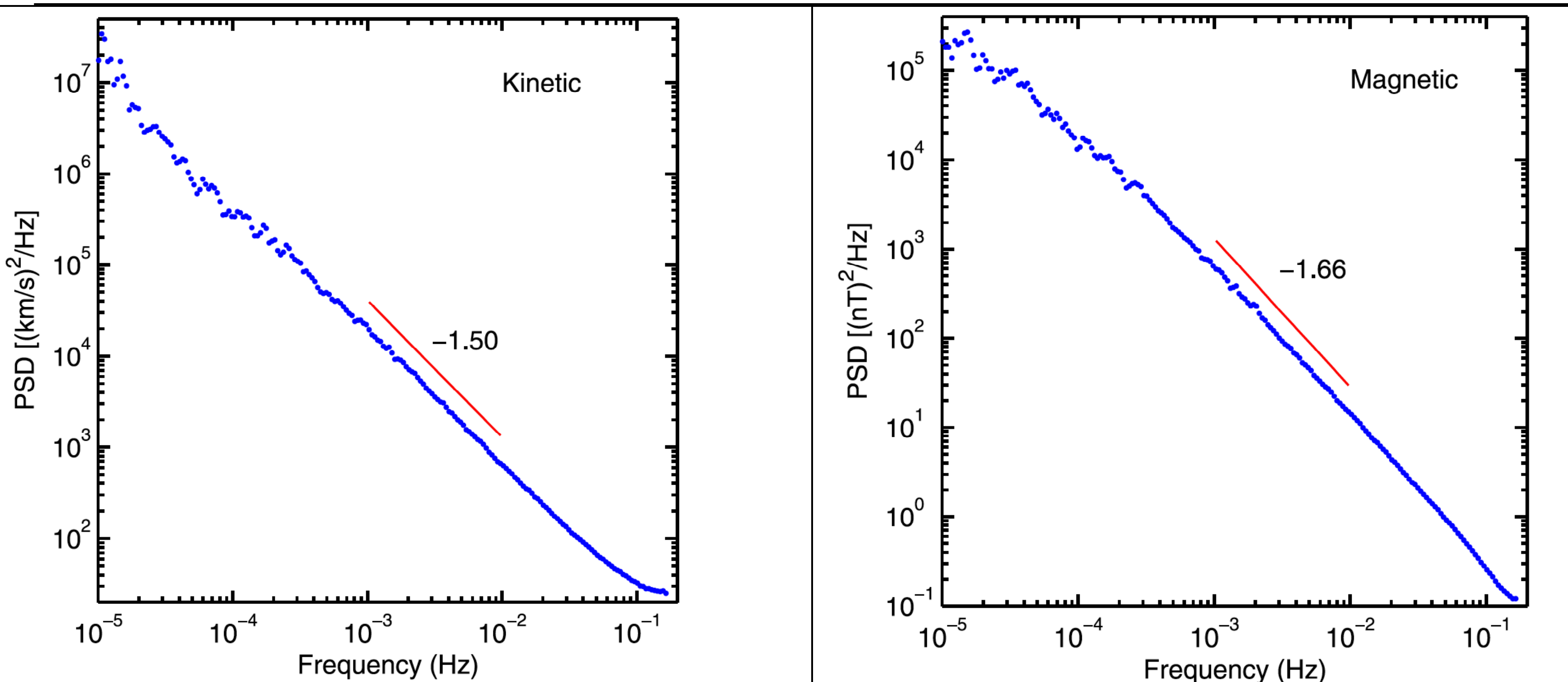
- overlapping spectra over a period of large scale Alfvén time




Z(256)


Z(256)
X(25

Kinetic vs magnetic spectrum -- observations



(a) (b)

From [*Podesta, Roberts, & Goldstein* 2006], power spectra for the total kinetic energy (a) and for the total magnetic energy (b) The best fit straight line over the interval is indicated by the short red line segment.

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