Development of a Novel Method for Determination of Momentum Transport Parameters

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Abstract

The toroidal momentum pinch velocity V_ϕ and diffusivity χ_ϕ in NSTX were previously determined from the transient response of the toroidal rotation W following applied n=3 magnetic perturbations that brake the plasma [1,2]. Assuming P=nmR^2($-\chi_\phi \nabla\Omega + V_\phi \Omega$), where the momentum flux Π is determined using TRANSP, these local analyses used fits to Ω and $\nabla\Omega$ to obtain χ_ϕ and V_ϕ one flux surface at a time. This work attempts to improve the accuracy of the inferred χ_ϕ(r) and V_ϕ(r) profiles by utilizing many flux surfaces simultaneously. We employ nonlinear least-squares minimization that compares the entire perturbed rotation profile evolution Ω(r,t) against the profile evolution generated by solving the momentum transport equation. We compare the local and integrated approaches and discuss their limitations.

Importance of Momentum Transport Analysis

- Rotation profile and shear important for stability
- Rotation profile influenced by momentum transport effects, namely momentum diffusion and convective pinch
- A convective momentum pinch has been found to be important in many tokamaks
- The following work attempts to improve understanding and expand applicability of perturbative analysis methods (following work of Solomon, PRL 2008; Kaye, NF 2009)

Steady State Measurements Insufficient

- Momentum transport equation:

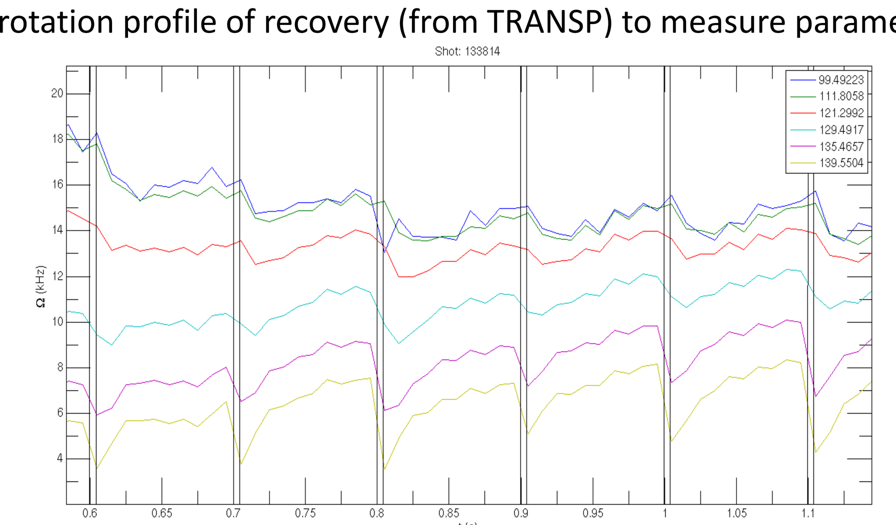
$$nmR^2 \frac{\partial\Omega(r,t)}{\partial t} = -\frac{1}{r}\frac{\partial}{\partial r}(r\Pi) + T_{inj} - T_{loss}$$

$$\Pi = nmR^2\left(-\chi_\phi(r)\frac{\partial\Omega(r,t)}{\partial r} + V_\phi(r)\Omega(r,t)\right)$$

- Actually used flux-surface-averaged version from Goldston (Varenna, 1985)
- Π = Momentum flux (kg/s^2)
- χ_ϕ = Momentum diffusivity (m^2/s)
- V_ϕ = Toroidal pinch velocity (m/s)
- Problem: In steady state χ_ϕ and V_ϕ are correlated, making them impossible to simultaneously measure

Analysis Made Possible Through Perturbations

- Solution: Use perturbed plasma state
- Apply n = 3 RMPs to brake plasma
- Use rotation profile of recovery (from TRANSP) to measure parameters

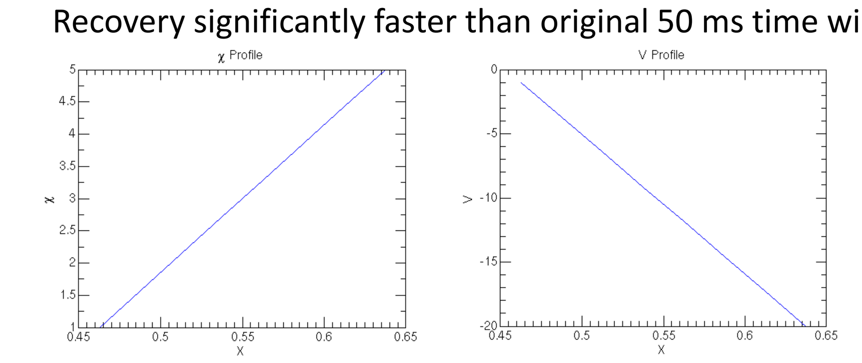


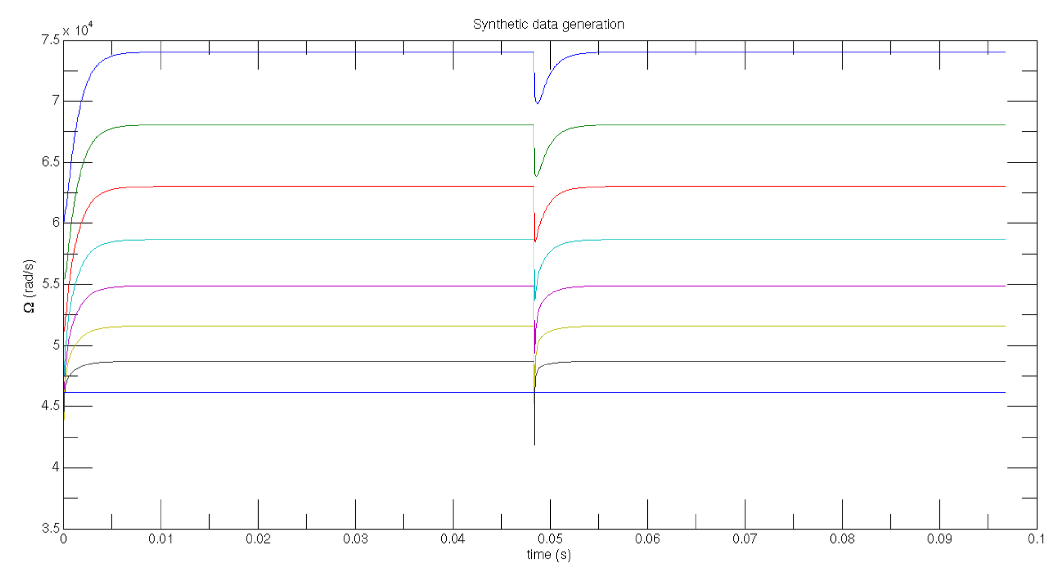
Applying PDE Fit

- Nonlinear least-squares fit comparing rotation profile to profile generated by solving momentum transport equation
- Initial guess: χ and V profiles from local analysis
- Boundary conditions:
  - Dirichlet at outer edge (Ω at outer edge = Ω from TRANSP)
  - Flux matching at inner edge (flux at inner edge = flux from TRANSP)
- PDE fit requires making use of all flux surfaces at once

Using Synthetic Data Allows for Meaningful Tests of Method

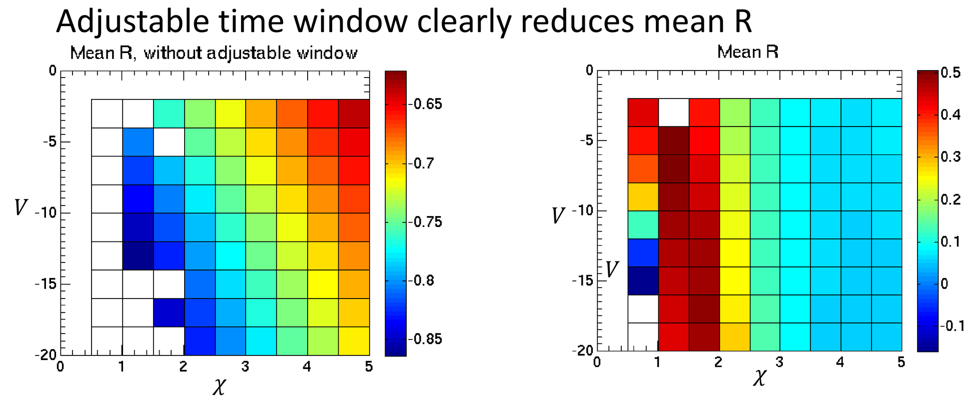
- Example: using linearly increasing profiles
- Recovery significantly faster than original 50 ms time window





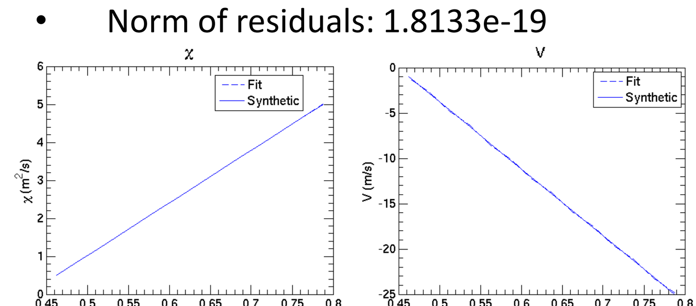
Adjustable Time Window Increases Accuracy

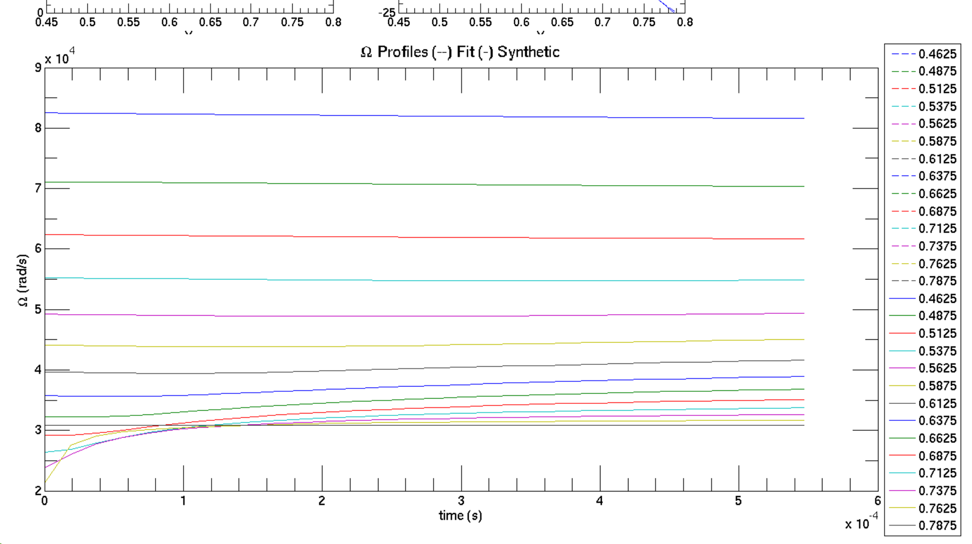
- Obtaining Correlation Fit Metric:
  - Calculate R for each radius
  - Average over all radii (Mean R)
- Using linear profiles:
  - χ from 0.5 to row value, V from -2 to column value
- Adjustable time window clearly reduces mean R



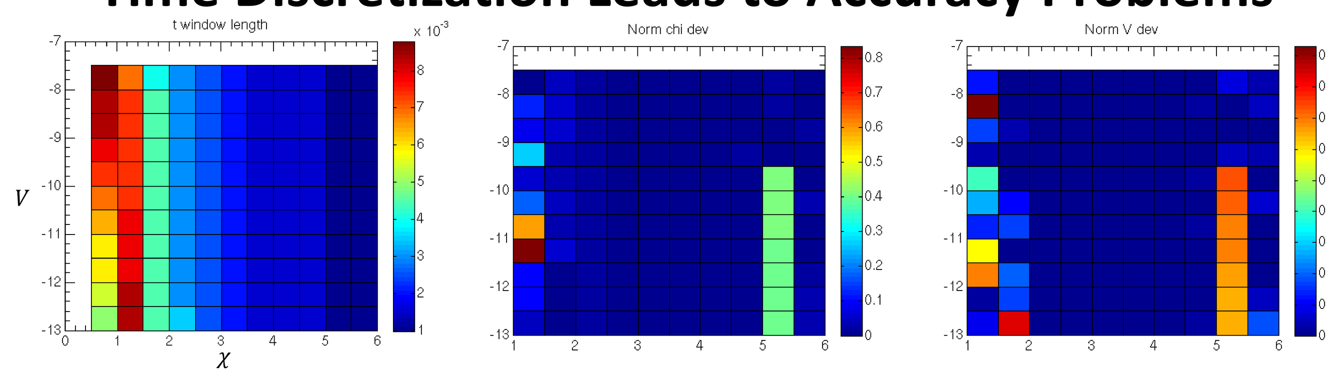
Results: Example of Fit

- Normalized χ deviation: .003
- Normalized V deviation: .0515
- Norm of residuals: 1.8133e-19

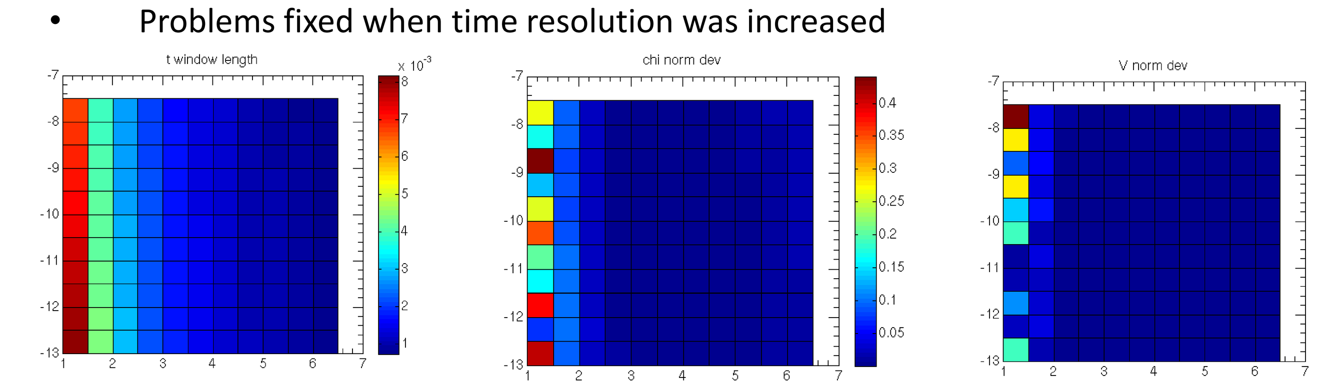




Time Discretization Leads to Accuracy Problems

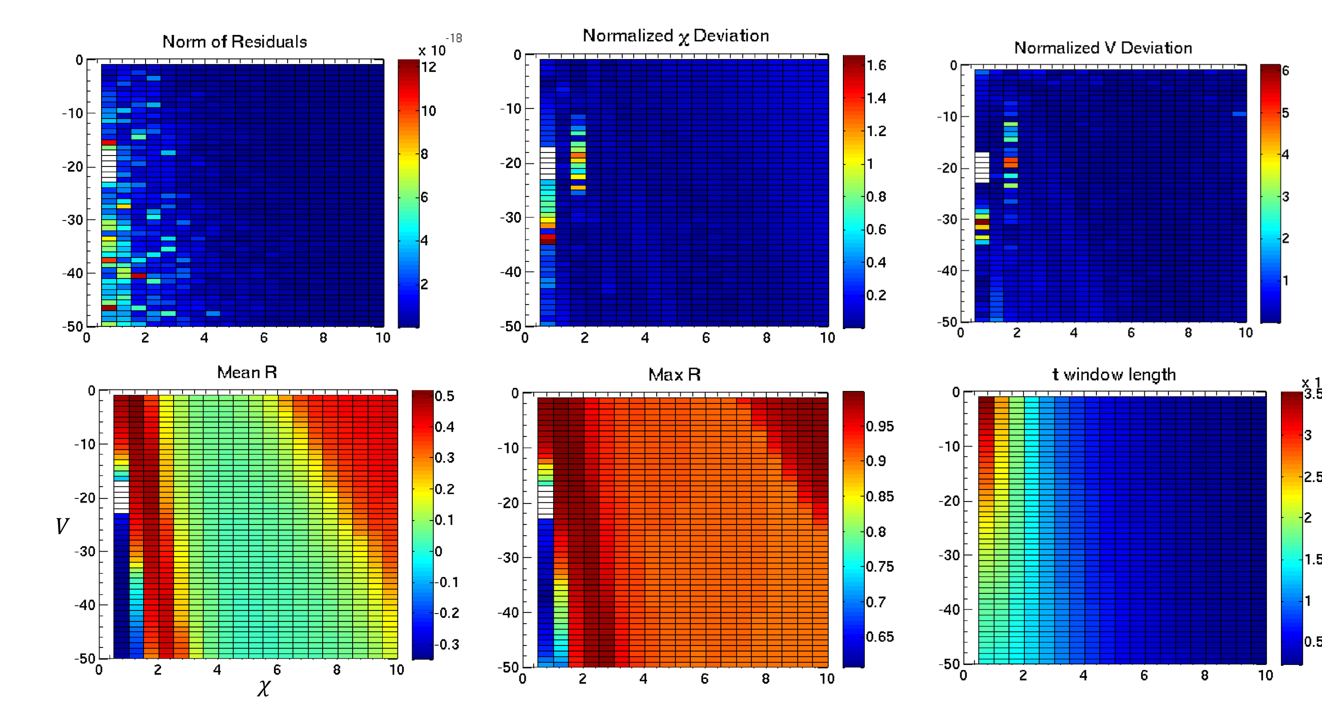


- In above, time resolution is that of TRANSP data (~1.1 ms between time points)
- Region of bad behavior at a discrete jump in time window length (caused by a very small number of time points)
- Problems fixed when time resolution was increased



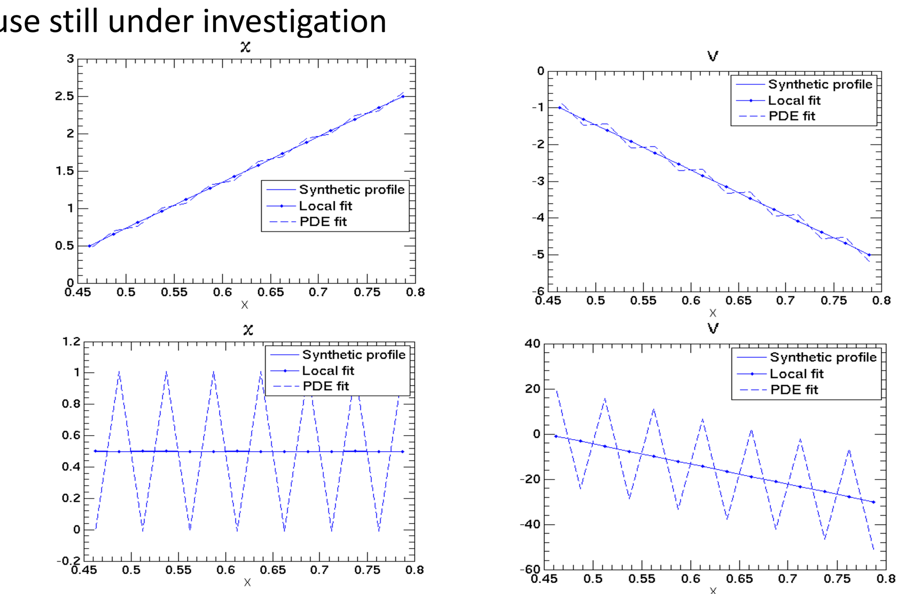
- In above, time resolution increased to 1000 time points per fit window, giving ~.05 ms between time points
- Conclusion: Higher time resolution necessary for complete confidence in method
  - CHERS time resolution is ~10 ms, not sufficient to eliminate odd behavior

Results: Synthetic Linear Profiles



Limitation of PDE Fit: Spurious Oscillations

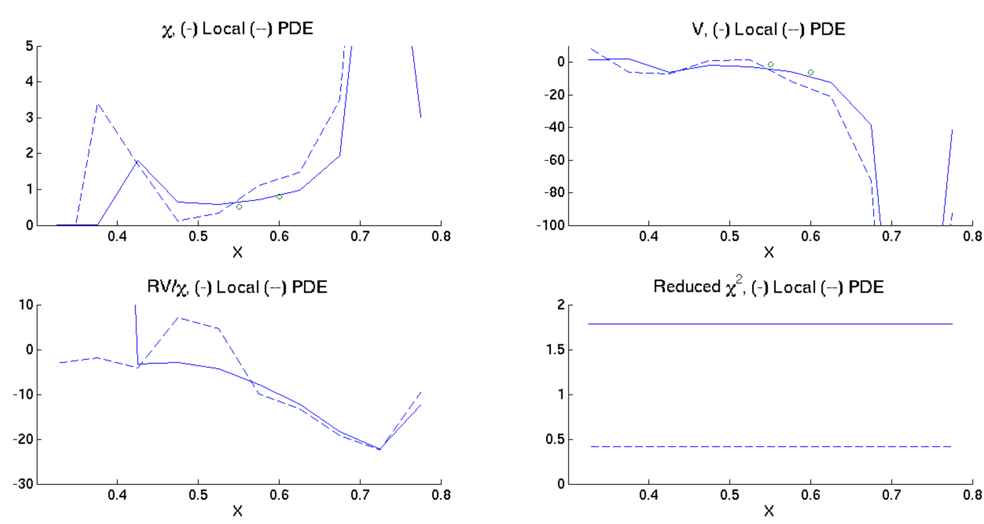
- PDE fit oscillates about synthetic profile
- Oscillation much worse when in "failure" region (i.e. low χ)
- Average still correct
- Cause still under investigation



Fit Metrics

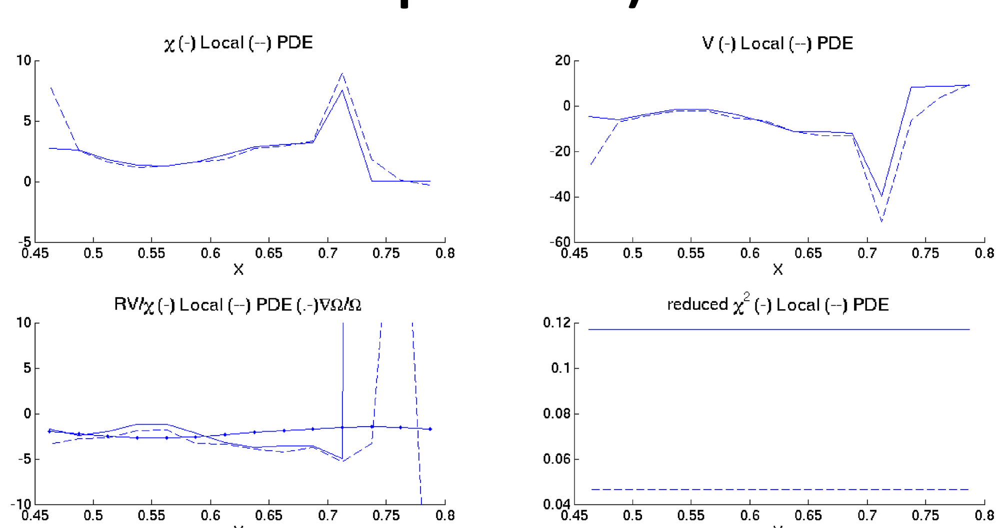
- Norm of residuals $= \sum F^2$, where F = objective function
- $\chi_{dev} = \sum\left[\left(\frac{\chi_{synth}-\chi_{fit}}{\chi_{synth}}\right)^2\right]$; $V_{dev} = \sum\left[\left(\frac{V_{synth}-V_{fit}}{\chi_{synth}}\right)^2\right]$
- Correlation coefficient between Ω and $\nabla\Omega$

Results from Experiment: 134783 A01



- Reproduces Solomon results relatively well, and adds to fit range
- Differences: No smoothing (vs. 20-ms smoothing) and 20-ms window (vs. 50-ms window)
- PDE fit reduces statistical χ^2

Results: 133814 A15 (ELM Pacing Experiment)



- ELM pacing -> more RMP current, shorter time interval
  - Results in clearer perturbations
- Reasonable values in at least X=0.5 to 0.7

Conclusion

- Implemented PDE Fit, while adding:
  - Synthetic data generation and scanning
  - Auto-adjusting time window (for synthetic data)
  - Speed-related improvements
- Time window, time resolution are important
- In principle, PDE Fit should generate accurate parameter profiles
  - Limitations: Spurious oscillations, time discretization
- When applied to experimental data, PDE Fit increases the reliable range over which a profile can be inferred
  - Still issues where χ approaches zero and V becomes positive
- Further Applications:
  - Add statistical noise
  - Implement radius-dependent time window length
  - Time-evolution of state variables
  - Test for feasibility of measurement of residual stress (Π_{RS})
  - Test Dependence of Accuracy on Perturbation Size/Shape